Macroeconomics: an Introduction

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The Scope of Macroeconomics

- Microeconomics: Object of interest is a single (or small number of) household or firm.

- Macroeconomics: Object of interest is the entire economy. We care mostly about:

  1. Growth.

  2. Fluctuations.

Relation between Macro and Micro

- Micro and Macro are consistent applications of standard neoclassical theory.

- Unifying theme, EQUILIBRIUM APPROACH:

  1. Agents optimize given preferences and technology.

  2. Agents' actions are compatible with each other.

- This requires:

  1. Explicit about assumptions.

  2. Models as abstractions.

What are the Requirements of Theory?

- Well articulated models with sharp predictions.
- Good theory cannot be vague: predictions must be falsifiable.
- Internal Consistency.
- Models as measurement tools.

All this is Scientific Discipline.

Why should we care about Macroeconomics?

- Self Interest: macroeconomic aggregates affect our daily life.

- Cultural Literacy: understanding our world.

- Common Welfare: Essential for policymakers to do good policy.

- Civic Responsibility: Essential for us to understand our politicians.

A Brief Overview of the History of Macroeconomics I

- Classics (Smith, Ricardo, Marx) did not have a sharp distinction between micro and macro.

- Beginning of the XX century: Wicksell, Pigou.

- J.M. Keynes, The General Theory of Employment, Interest, and Money (1936).

- 1945-1970, heyday of *Neoclassical Synthesis*: Samuelson, Solow, Klein.

- Monetary versus Fiscal Policy: Friedman, Tobin.

A Brief Overview of the History of Macroeconomics II

- 1972, Rational Expectations Revolution: Lucas, Prescott, Sargent.

- 1982, Real Business Cycles: Kydland and Prescott.

- 1990's, Rich dynamic equilibrium models.

- Future?

Why do Macroeconomist Disagree?

- Most research macroeconomist agree on a wide set of issues.
- There is wide agreement on growth theory.
- There is less agreement on business cycle theory.
- Normative issues.
- Are economist ideologically biased? Caplan (2002).

National Income and Product Accounts

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A Guide to NIPA's

- What is the goal?

- When did it begin? Role of Simon Kuznets:

  1. Nobel Prize in Economics 1971.

  2. Prof. at Penn during the key years of NIPA creation.

- Gigantic intellectual achievement.

- Elaborated by Bureau of Economic Analysis and published in the *Survey of Current Business*. `http://www.bea.gov/`

Question: How are macroeconomic aggregates measured?

Gross Domestic Product (GDP)

Can be measured in three different, but equivalent ways:

1. Production Approach.
2. Expenditure Approach.
3. Income Approach.

Nominal GDP

- For 2003, nominal GDP was:

$$\$11,004,000,000,000$$

- Population, July 2003 was:

$$290,788,976$$

- Nominal GDP per capita is roughly:

$$\$37,842$$

Computing GDP through Production

- Calculate nominal GDP by adding value of production of all industries: production surveys.

- Problem of double-counting: i.e. USX and GM.

- Value Added=Revenue−Intermediate Goods.

- Nominal GDP=Sum of Value Added of all Industries.

Computing GDP through Expenditure

$$
\begin{aligned}
C &= \text{Consumption} \\
I &= \text{(Gross Private) Investment} \\
G &= \text{Government Purchases} \\
X &= \text{Exports} \\
M &= \text{Imports} \\
Y &= \text{Nominal GDP}
\end{aligned}
$$

$$Y \equiv C + I + G + (X - M)$$

Consumption (C)

- Durable Goods: 3 years rule.

- Nondurable Goods.

- Services.

Gross Private Investment (I)

- Nonresidential Fixed Investment.
- Residential Fixed Investment.
- Inventory Investment.
- Stocks vs. Flows.

Investment and the Capital Stock

- Capital Stock: total amount of physical capital in the economy

- Depreciation: the part of the capital stock that wears out during the period

- Capital Stock at end of this period=Capital Stock at end of last period+Gross Investment in this period−Depreciation in this period

- Net Investment=Gross Investment−Depreciation=Capital Stock, end this per.−Capital Stock, end of last per.

Inventory Investment

- Why included in GDP?
- Inventory Investment=Stock of Inventories at end of this year−Stock of Inventories at the end of last year
- Final Sales=Nominal GDP−Inventory Investment

Government Purchases (G)

- Sum of federal, state, and local purchases of goods and services.
- Certain government outlays do not belong to government spending: transfers (SS and Interest Payments).
- Government Investment.

Exports (E) and Imports (M)

- Exports: deliveries of US goods and services to other countries.
- Imports: deliveries of goods and services from other countries to the US.
- Trade Balance=Exports−Imports
- Trade Deficit: if trade balance negative.
- Trade Surplus: if trade balance positive

Composition of GDP - Spending	in billion $	in % of GDP
Total Nom. GDP	11,004.0	100.0%
Consumption	7,760.0	70.5%
Durable Goods	950.7	8.6%
Nondurable Goods	2,200.1	20.0%
Services	4,610.1	41.9%
Gross Private Investment	1,667.0	15.1%
Nonresidential	1,094.7	9.9%
Residential	572.3	5.2%
Changes in Inventory	−1.2	−0.0%
Government Purchases	2,075.5	18.9%
Federal Gov.	752,2	6.8%
State & Local Gov.	1,323.3	12.2%
Net Exports	-498.1	-4.5%
Exports	1,046.2	9.5%
Imports	1,544.3	14.0%
Gross National Product	11,059.2	100.5%

Computing GDP through Income

National Income: broadest measure of the total incomes of all Americans

Gross Domestic Product (11,004.0)

+Factor Inc. from abroad (329.0) − Factor Inc. to abroad (273.9)

= Gross National Product (11,059.2)

−Depreciation (1,359.9)

= Net National Product (9,705.2)

−Statistical Discrepancy (25.6)

= National Income (9,679.6)

Distribution of National Income

1. Employees' Compensation: wages, salaries, and fringe benefits.

2. Proprietors' Income: income of noncorporate business.

3. Rental Income: income that landlords receive from renting, including "imputed" rent less expenses on the house, such as depreciation.

4. Corporate Profits: income of corporations after payments to their workers and creditors.

5. Net interest: interests paid by domestic businesses plus interest earned from foreigners.

Labor and Capital Share

- Labor share: the fraction of national income that goes to labor income

- Capital share: the fraction of national income that goes to capital income.

- Labor Share$= \frac{\text{Labor Income}}{\text{National Income}}$

- Capital Share$= \frac{\text{Capital Income}}{\text{National Income}}$

- Proprietor's Income?

Distribution of National Income

	Billion $US	% of Nat. Inc.
National Income less Prod. Tax.	8,841.0	100.0%
Compensation of Employees	6,289.0	71.1%
Proprietors' Income	834.1	9.4%
Rental Income	153.8	1.7%
Corporate Profits	1021.1	11.6%
Net Interest	543.0	6.1%

Composition of National Income

Industries	Val. Add.	in %
National Income'	9,396.6	100.0%
Agr., Forestry, Fish.	75.8	0.8%
Mining	94.9	1.0%
Construction	476.5	5.1%
Manufacturing	1,113.1	11.8%
Public Utilities	156.0	1.7%
Transportation	259.9	2.8%
Wholesale Trade	569,6	6.1%
Retail Trade	752.8	7.7%
Fin., Insur., Real Est.	1,740.8	18.5%
Services	1,893.6	20.1%
Government	1,182.8	12.6%
Rest of the World	55.1	0.6%

Other Income Concepts: Personal Income

- Income that households and noncorporate businesses receive

$$\text{National Income } (9{,}679.6) - \text{Corporate Profits } (1021.1)$$
$$-\text{Net Taxes on Production and Imports } (751.3)$$
$$-\text{Net Interest } (543.0) - \text{Contributions for Social Insurance } (773.1)$$
$$+\text{Personal Interest Income } (1{,}322.7)$$
$$+\text{Personal Current Transfer Receipts } (1{,}335.4)$$
$$= \text{Personal Income } (9{,}161.8)$$

Other Income Concepts: Disposable Personal Income

- Income that households and noncorporate businesses can spend, after having satisfied their tax obligations

$$\begin{gathered}\text{Personal Income (9,161.8)}\\ -\text{Personal Tax and Nontax Payments (1,001.9)}\\ =\text{Disposable Personal Income (8,159.9)}\end{gathered}$$

Investment and Saving

- Private Saving (S): gross income minus consumption and taxes plus transfers

- From income side $Y = C + S + T - TR + NFP$

- From expenditure side $Y = C + I + G + X - M$

$$\underbrace{I}_{\text{Private Inv.}} = \underbrace{S}_{\text{Private Saving}} + \underbrace{T - TR - G}_{\text{Public Saving}} + \underbrace{M - X + NFP}_{\text{Foreign Saving}}$$

Some Nontrivial Issues

- Releases of Information and revisions.
- Methodological Changes.
- Technological Innovation.
- Underground Economy.
- Non-market activities.
- Welfare.

Price Indices

Question: How to compute the price level?

Idea: Measure price of a particular basket of goods today versus price of same basket in some *base* period

Example: Economy with 2 goods, hamburgers and coke

$$
\begin{aligned}
h_t &= \text{\# of hamburgers produced, period } t \\
p_{ht} &= \text{price of hamburgers in period } t \\
c_t &= \text{\# of coke produced, period } t \\
p_{ct} &= \text{price of coke in period } t \\
(h_0, p_{h0}, c_0, p_{c0}) &= \text{same variables in period 0}
\end{aligned}
$$

Laspeyres price index

$$L_t = \frac{p_{ht}h_0 + p_{ct}c_0}{p_{h0}h_0 + p_{c0}c_0}$$

Paasche price index

$$Pa_t = \frac{p_{ht}h_t + p_{ct}c_t}{p_{h0}h_t + p_{c0}c_t}$$

Problems with Price Indices

- Laspeyres index tends to overstate inflation.
- Paasche index tends to understate inflation.
- Fisher Ideal Index: geometric mean: $(L_t \times Pa_t)^{0.5}$.
- Chain Index.

From Nominal to Real GDP

- Nominal GDP: total value of goods and services produced.

- Real GDP: total production of goods and services in physical units.

- How is real GDP computed in practice, say in 2004?

1. Pick a base period, say 1996

2. Measure dollar amount spent on hamburgers.

3. Divide by price of hamburgers in 2004 and multiply by price in 1996. (this equals the number of hamburgers sold in 2004, multiplied by the price of hamburgers in 1996 -the base period).

4. Sum over all goods and services to get real GDP.

For our example ...

$$\begin{aligned}\text{Nominal GDP in 2004} &= h_{2004}p_{h2004} + c_{2004}p_{c2004}\\ \text{Real GDP in 1996} &= h_{2000}p_{h1996} + c_{2004}p_{c1996}\end{aligned}$$

Note that

$$\begin{aligned}\text{GDP deflator} &= \frac{\text{Nominal GDP}}{\text{Real GDP}}\\ &= \frac{h_{2004}p_{h2004} + c_{2004}p_{c2004}}{h_{2004}p_{h1996} + c_{2004}p_{c1996}}\end{aligned}$$

Measuring Inflation I

- $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ where P_t is the "Price Level".

- GDP deflator: basket corresponds to current composition of GDP.

- Consumer Price Index (CPI): basket corresponds to what a typical household bought during a typical month in the base year

$$\text{CPI} = \frac{h_{1996}p_{h2004} + c_{1996}p_{c2004}}{h_{1996}p_{h1996} + c_{1996}p_{c1996}}$$

- CPI important because of COLA's.

Measuring Inflation II

- CPI may overstate inflation: Boskin Commission, New Goods.

- How to measure new technologies? David Cutler's example:

1. Average heart attack in mid-1980's costs about $\$_{1999}$12,000 to treat.

2. Average heart attack in late 1990's costs about $\$_{1999}$22,000 to treat.

3. Average life expectancy in late 1990's is one year higher than in mid-1980's.

4. Is health care more expensive now?

Inflation over History I

- How much is worth \$1 from 1789 in 2003?

    1. \$20.76 using the Consumer Price Index

    2. \$21.21 using the GDP deflator.

- How much is worth \$1 from 1861 in 2003?

    1. \$20.76 using the Consumer Price Index

    2. \$17.61 using the GDP deflator.

Inflation over History II

- How much is worth $1 from 1929 in 2003?

  1. $10.73 using the Consumer Price Index
  2. $8.83 using the GDP deflator.

- How much is worth $1 from 1985 in 2003?

  1. $1.71 using the Consumer Price Index
  2. $1.52 using the GDP deflator.

More on Growth Rates

- Growth rate of a variable Y (say nominal GDP) from period $t-1$ to t is given by

$$g_Y(t-1,t) = \frac{Y_t - Y_{t-1}}{Y_{t-1}}$$

- Growth rate between period $t-5$ and period t is given by

$$g_Y(t-5,t) = \frac{Y_t - Y_{t-5}}{Y_{t-5}}$$

- Suppose that GDP equals Y_{t-1} in period $t-1$ and it grows at rate $g_Y(t-1,t)$. How big is GDP in period t?

$$\begin{aligned} g_Y(t-1,t) &= \frac{Y_t - Y_{t-1}}{Y_{t-1}} \\ g_Y(t-1,t) * Y_{t-1} &= Y_t - Y_{t-1} \\ g_Y(t-1,t) * Y_{t-1} + Y_{t-1} &= Y_t \\ (1 + g_Y(t-1,t))Y_{t-1} &= Y_t \end{aligned}$$

Hence GDP in period t equals GDP in period $t-1$, multiplied by 1 plus the growth rate.

- Example: If GDP is \$1000 in 2004 and grows at 3.5%, then GDP in 2005 is

$$Y_{2005} = (1 + 0.035) * \$1000 = \$1035$$

- Suppose GDP grows at a constant rate g over time. Suppose at period 0 GDP equals some number Y_0 and GDP grows at a constant rate of g% a year. Then in period t GDP equals

$$Y_t = (1+g)^t Y_0$$

- Example: If Octavio Augustus would have put 1 dollar in the bank at year 0AD and the bank would have paid a constant real interest rate of 1.5%, then in 2000 he would have:

$$Y_{2000} = (1.015)^{2000} * \$1 = \$8,552,330,953,000$$

which is almost the US GDP for last year.

- Reverse question: Suppose we know GDP at 0 and at t. Want to know at what constant rate GDP must have grown to reach Y_t, starting from Y_0 in t years.

$$
\begin{aligned}
Y_t &= (1+g)^t Y_0 \\
(1+g)^t &= \frac{Y_t}{Y_0} \\
(1+g) &= \left(\frac{Y_t}{Y_0}\right)^{\frac{1}{t}} \\
g &= \left(\frac{Y_t}{Y_0}\right)^{\frac{1}{t}} - 1
\end{aligned}
$$

- Example: In 1900 a country had GDP of \$1,000 and in 2000 it has GDP of \$15,000. Suppose that GDP has grown at constant rate g. How big must this growth rate be? Take 1900 as period 0, 2000 as period 100, then

$$\begin{aligned} &= \left(\frac{\$15,000}{\$1,000}\right)^{\frac{1}{100}} - 1 \\ &= 0.027 = 2.7\% \end{aligned}$$

- Question: We know GDP of a country in period 0 and its growth rate g. How many time periods it takes for GDP in this country to double (to triple and so forth).

$$\begin{aligned} Y_t &= (1+g)^t Y_0 \\ (1+g)^t &= \frac{Y_t}{Y_0} \end{aligned}$$

Since $\log\left(a^b\right) = b * \log(a)$

$$\begin{aligned} \log\left((1+g)^t\right) &= \log\left(\frac{Y_t}{Y_0}\right) \\ t * \log(1+g) &= \log\left(\frac{Y_t}{Y_0}\right) \\ t &= \frac{\log\left(\frac{Y_t}{Y_0}\right)}{\log(1+g)} \end{aligned}$$

- Suppose we want to find the number of years it takes for GDP to double, i.e. the t such $\frac{Y_t}{Y_0} = 2$. We get

$$t = \frac{\log(2)}{\log(1+g)}$$

- Example: with $g = 1\%$ it takes 70 years, with $g = 2\%$ it takes 35 years, with $g = 5\%$ it takes 14 years.

Transactions with the Rest of the World

$$\text{Trade Balance} = \text{Exports} - \text{Imports}$$

$$\text{Current Account Balance} = \text{Trade Balance} + \text{Net Unilateral Transfers}$$

- Unilateral transfers: include aid to poor countries, interest payments to foreigners for US government debt, and grants to foreign researchers or institutions.

- Net wealth position of the US: difference between what the US is owed and what it owes to foreign countries.

- Capital account balance: equals to the change of the net wealth position of the US

$$
\begin{aligned}
&\text{Capital Account Balance this year} \\
&= \text{Net wealth position at end of this year} \\
&- \text{Net wealth position at end of last year}
\end{aligned}
$$

Unemployment Rate

- Labor force: number of people, 16 or older, that are either employed or unemployed but actively looking for a job.

- Current Population Survey.

- Unemployment Rate$= \frac{\text{number of unemployed people}}{\text{labor force}}$

- What is the current unemployment rate now?

Interest Rates

- Important as they determine how costly it is to borrow money

- Suppose in period $t-1$ you borrow the amount $\$B_{t-1}$. The loan specifies that in period t you have to repay $\$B_t$. Nominal interest rate on the loan from period $t-1$ to period t, i_t, is

$$i_t = \frac{B_t - B_{t-1}}{B_{t-1}}$$

- Real interest rate r_t

$$r_t = i_t - \pi_t$$

- Example: In 2003 you borrow $\$15,000$ and the bank asks you to repay $\$16,500$ exactly one year later. The yearly nominal interest rate from 2003 to 2004 is

$$i_{2004} = \frac{\$16,500 - \$15,000}{\$15,000} = 0.1 = 10\%$$

Now suppose the inflation rate is 3% in 2004. Then the real interest rate equals $10\% - 3\% = 7\%$.

Introduction to Growth Theory

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Growth Theory

I do not see how one can look at figures like these without seeing them as representing *possibilities*. Is there some action a government could take that would lead the Indian economy to grow like Indonesia's or Egypt's? If so, *what* exactly? If not, what is it about the "nature of India" that makes it so? The consequences for human welfare involved in questions like these are simply staggering: Once one starts to think about them, it is hard to think about anything else (Lucas 1988, p. 5).

Some Motivation

- Differences across countries:

  1. Out of 6.4 billion people, 0.8 do not have access to enough food, 1 to safe drinking water, and 2.4 to sanitation.

  2. Life expectancy in rich countries is 77 years, 67 years in middle income countries, and 53 million in poor countries.

- Differences across time:

  1. Japanese boy born in 1880 had a life expectancy of 35 years, today 81 years.

  2. An American worked 61 hours per week in 1870, today 34.

History of Economic Growth Theory: a Roadmap

1. Smith, Ricardo, Malthus and Mill had little hope for sustained growth.

2. Forgotten for a long while. Ill attempted in UK (Harrod and Domar).

3. Robert Solow (MIT, Nobel 1987): two main papers: 1956 and 1957.

4. Completed by David Cass (Penn) and Tjalling Koopmans (Nobel 1971).

5. 80's and 90's: Paul Romer (Stanford, Nobel 20??) and Robert Lucas (Chicago, Nobel 1995).

Growth Facts (Nicholas Kaldor)

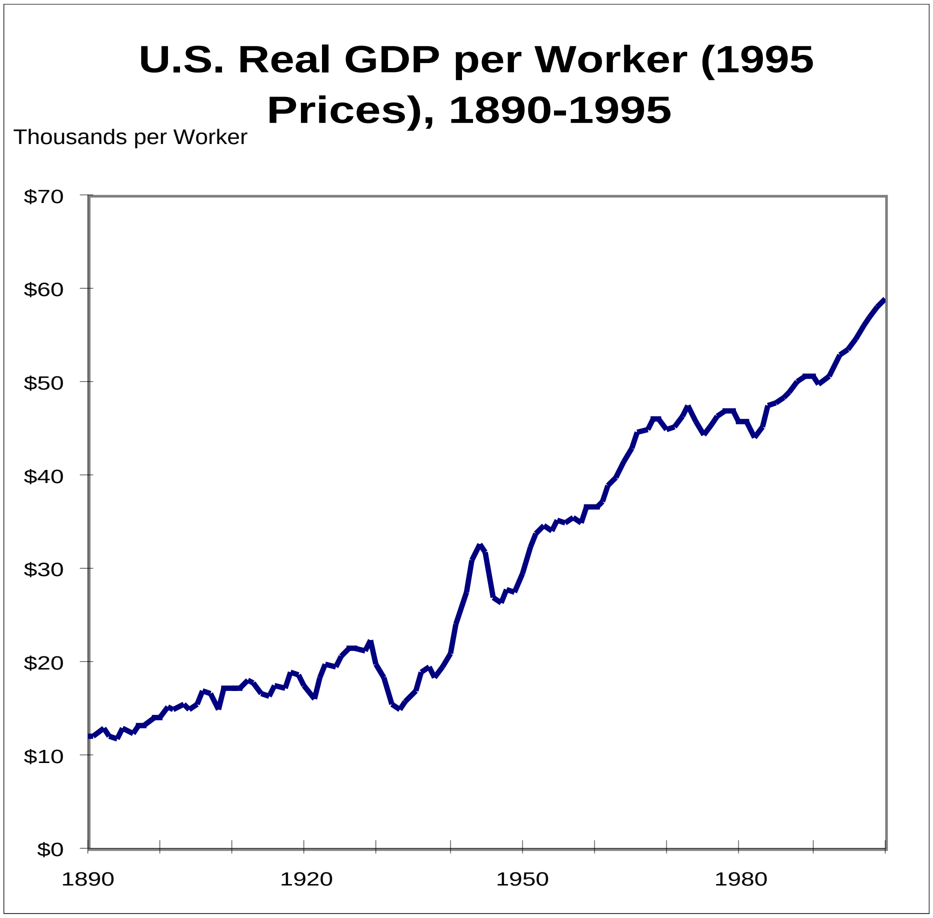
Stylized growth facts (empirical regularities of the growth process) for the US and for most other industrialized countries

1. Output (real GDP) per worker $y = \frac{Y}{L}$ and capital per worker $k = \frac{K}{L}$ grow over time at relatively constant and positive rate.

2. They grow at similar rates, so that the ratio between capital and output, $\frac{K}{Y}$ is relatively constant over time

3. The real return to capital r (and the real interest rate $r - \delta$) is relatively constant over time.

4. The capital and labor shares are roughly constant over time.


U.S. Real GDP per Worker (1995 Prices), 1890-1995
Thousands per Worker
$70
$60
$50
$40
$30
$20
$10
$0
1890
1920
1950
1980

Data

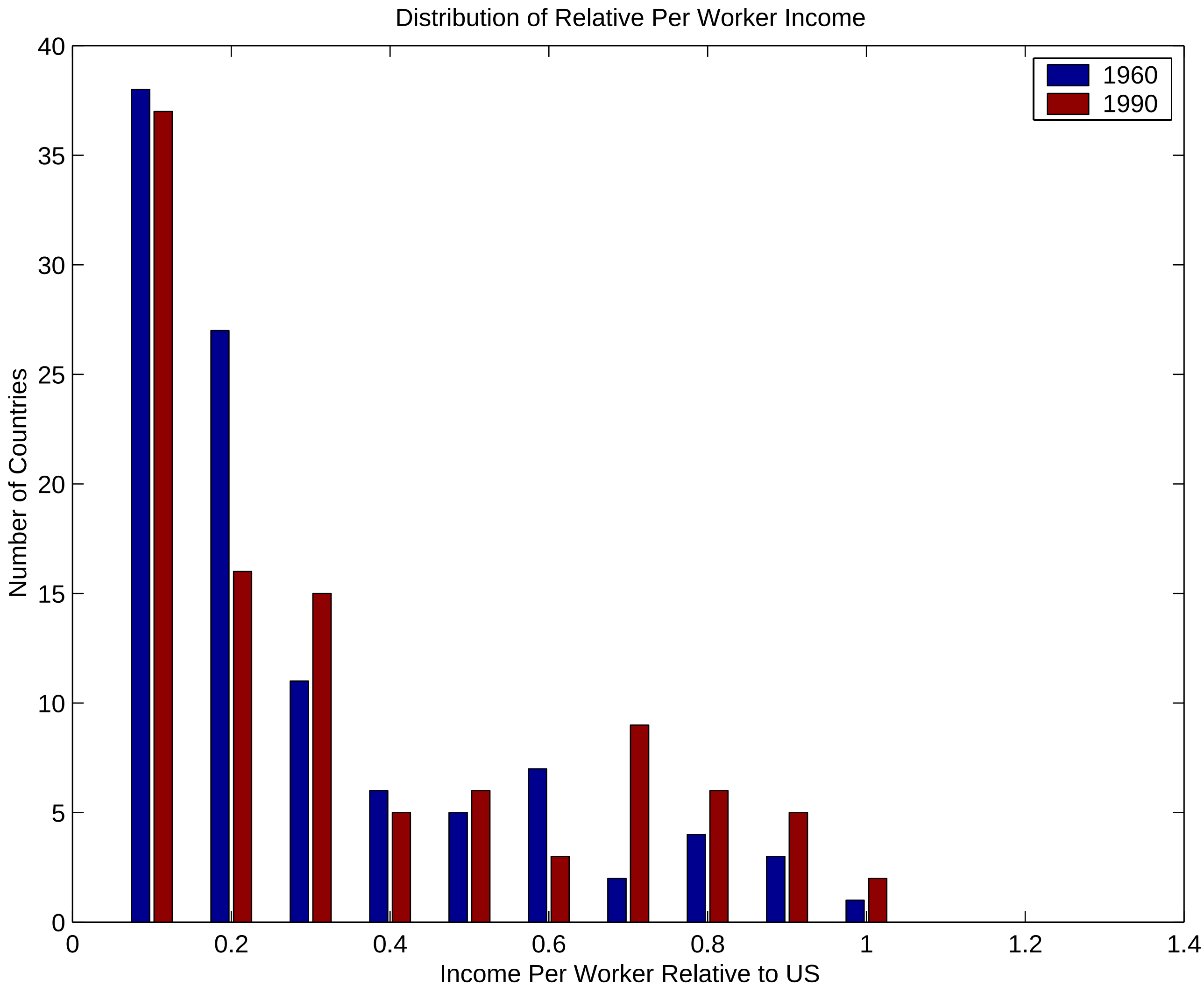
- How do incomes and growth rates vary across countries.
- Summers-Heston data set at Penn: follow 104 countries over 30 years.
- Focus on income (GDP) per worker.
- Measure income (GDP) using PPP-based exchange rates.

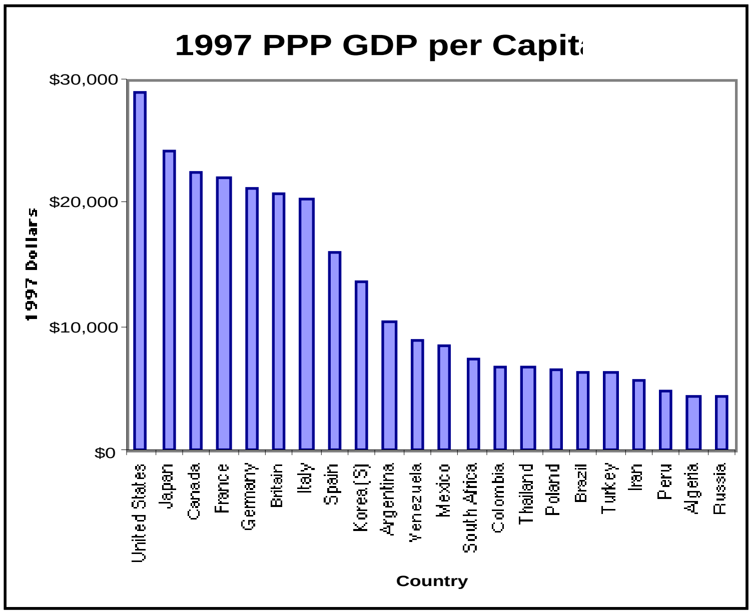
Development Facts I

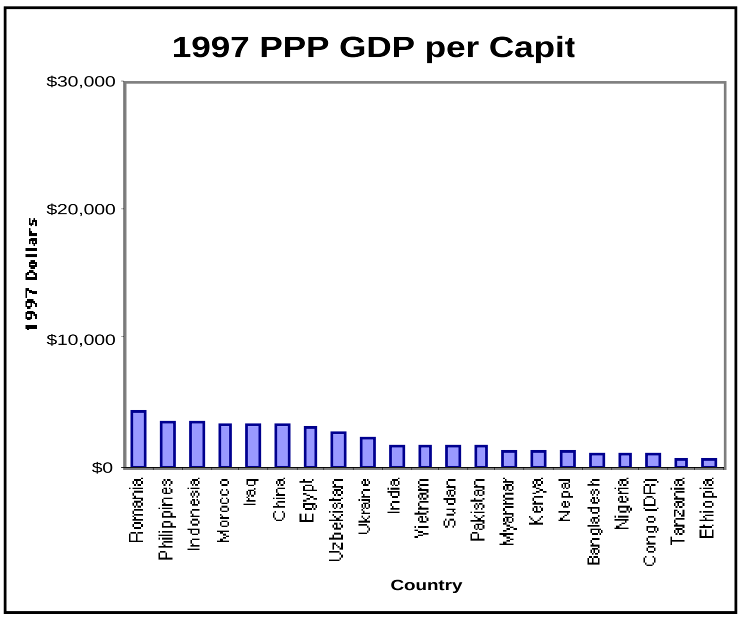
1. Enormous variation of per worker income across countries.

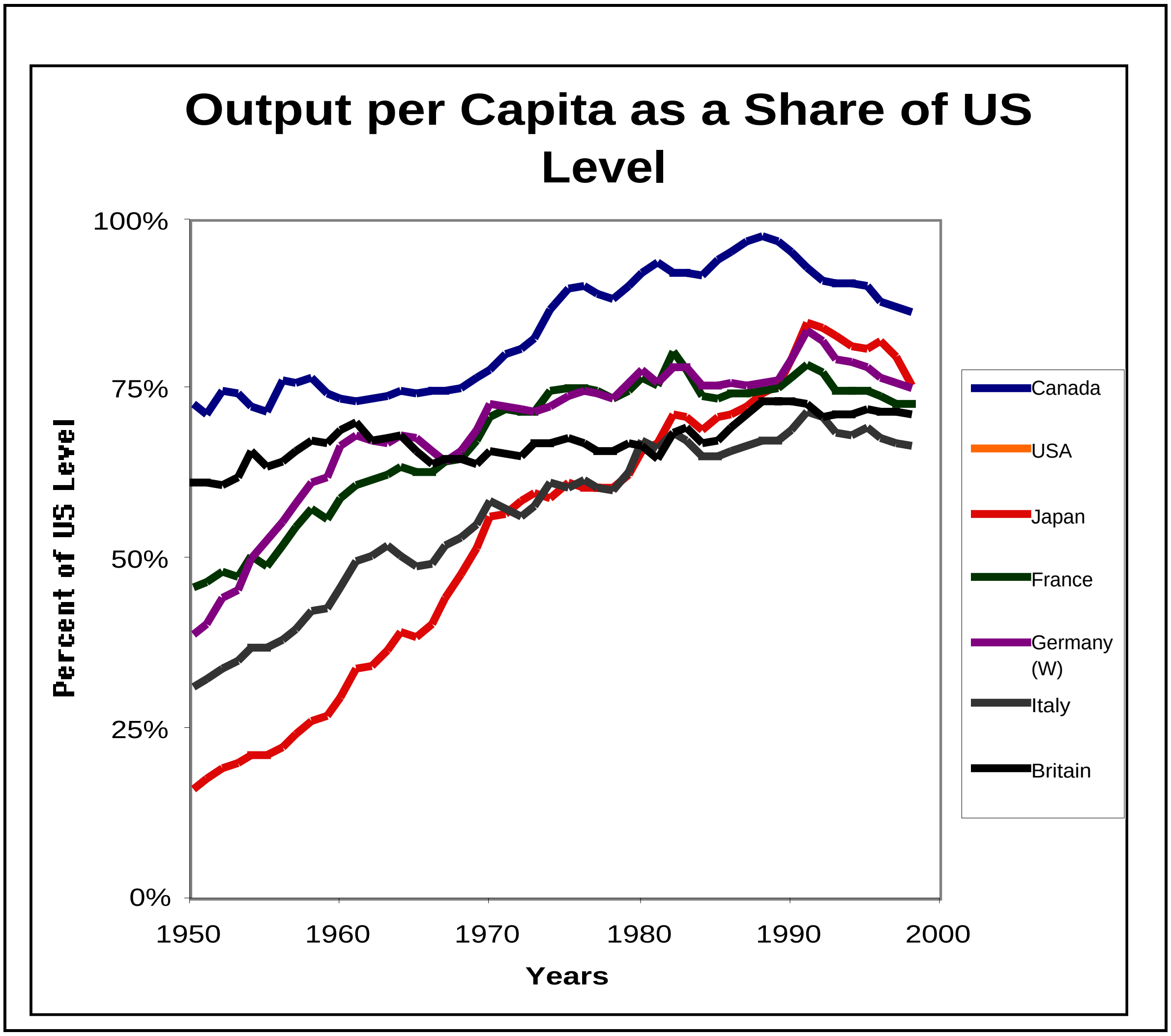
2. Enormous variation in growth rates of per worker income across countries.

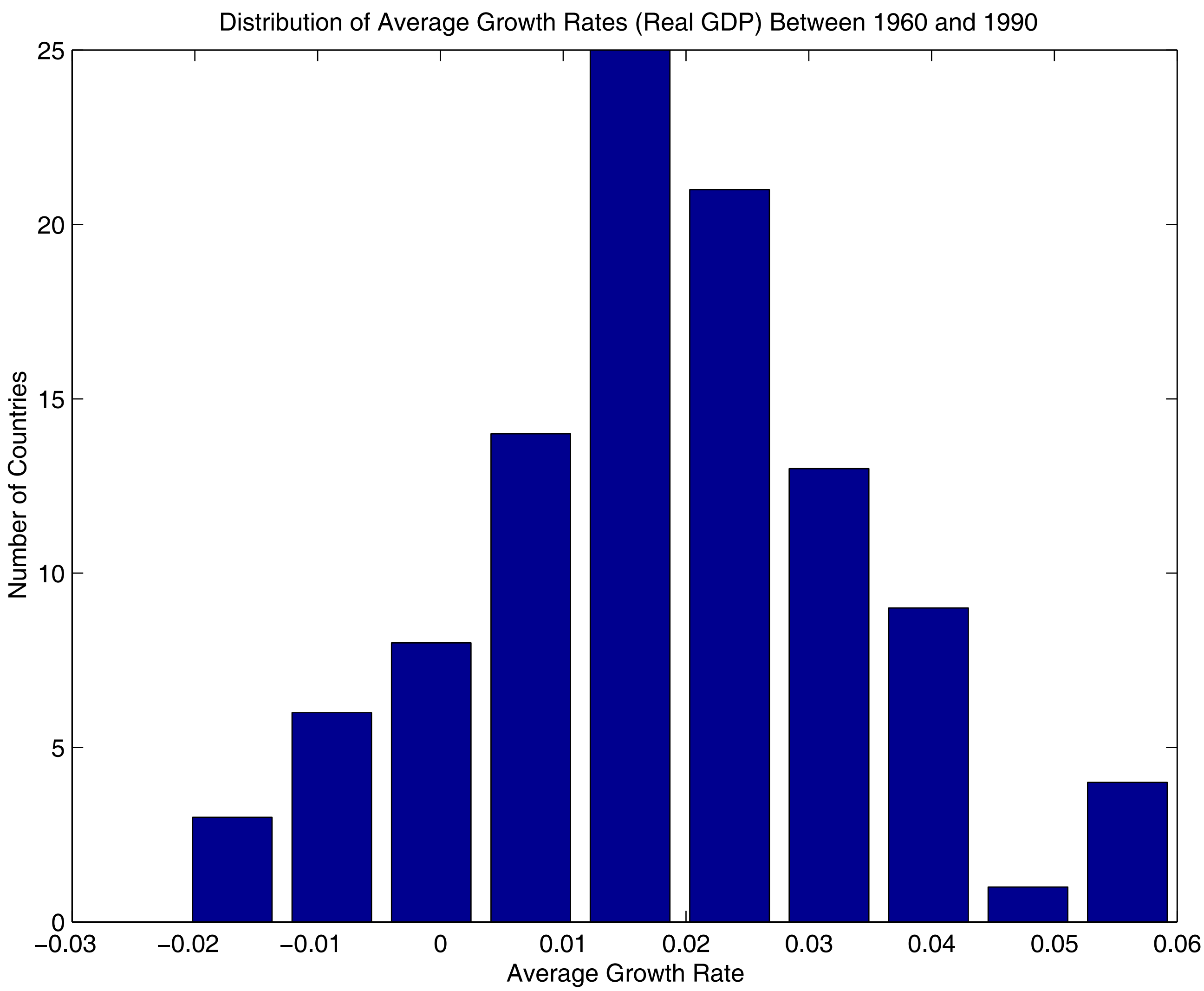
Growth "Miracles"	g_{60-97}
South Korea	5.9%
Taiwan	5.2%
Growth "Disasters"	
Venezuela	-0.1%
Madagascar	-1.4%


Distribution of Relative Per Worker Income
Number of Countries
Income Per Worker Relative to US
1960
1990
0
5
10
15
20
25
30
35
40
0
0.2
0.4
0.6
0.8
1
1.2
1.4


1997 PPP GDP per Capita
$30,000
$20,000
$10,000
$0
1997 Dollars
United States
Japan
Canada
France
Germany
Britain
Italy
Spain
Korea (S)
Argentina
Venezuela
Mexico
South Africa
Colombia
Thailand
Poland
Brazil
Turkey
Iran
Peru
Algeria
Russia
Country


1997 PPP GDP per Capit
$30,000
$20,000
$10,000
$0
1997 Dollars
Romania
Philippines
Indonesia
Morocco
Iraq
China
Egypt
Uzbekistan
Ukraine
India
Vietnam
Sudan
Pakistan
Myanmar
Kenya
Nepal
Bangladesh
Nigeria
Congo (DR)
Tanzania
Ethiopia
Country


Output per Capita as a Share of US Level
100%
75%
50%
25%
0%
Percent of US Level
1950
1960
1970
1980
1990
2000
Years
Canada
USA
Japan
France
Germany (W)
Italy
Britain



Distribution of Average Growth Rates (Real GDP) Between 1960 and 1990
Number of Countries
Average Growth Rate
0
5
10
15
20
25
−0.03
−0.02
−0.01
0
0.01
0.02
0.03
0.04
0.05
0.06

Development Facts II

3. Growth rates are not constant over time for a given country.

4. Countries change their relative position in the international income distribution.

Development Facts III

5. Growth in output and growth in international trade are closely related.

6. Demograhic transition.

7. International migration.

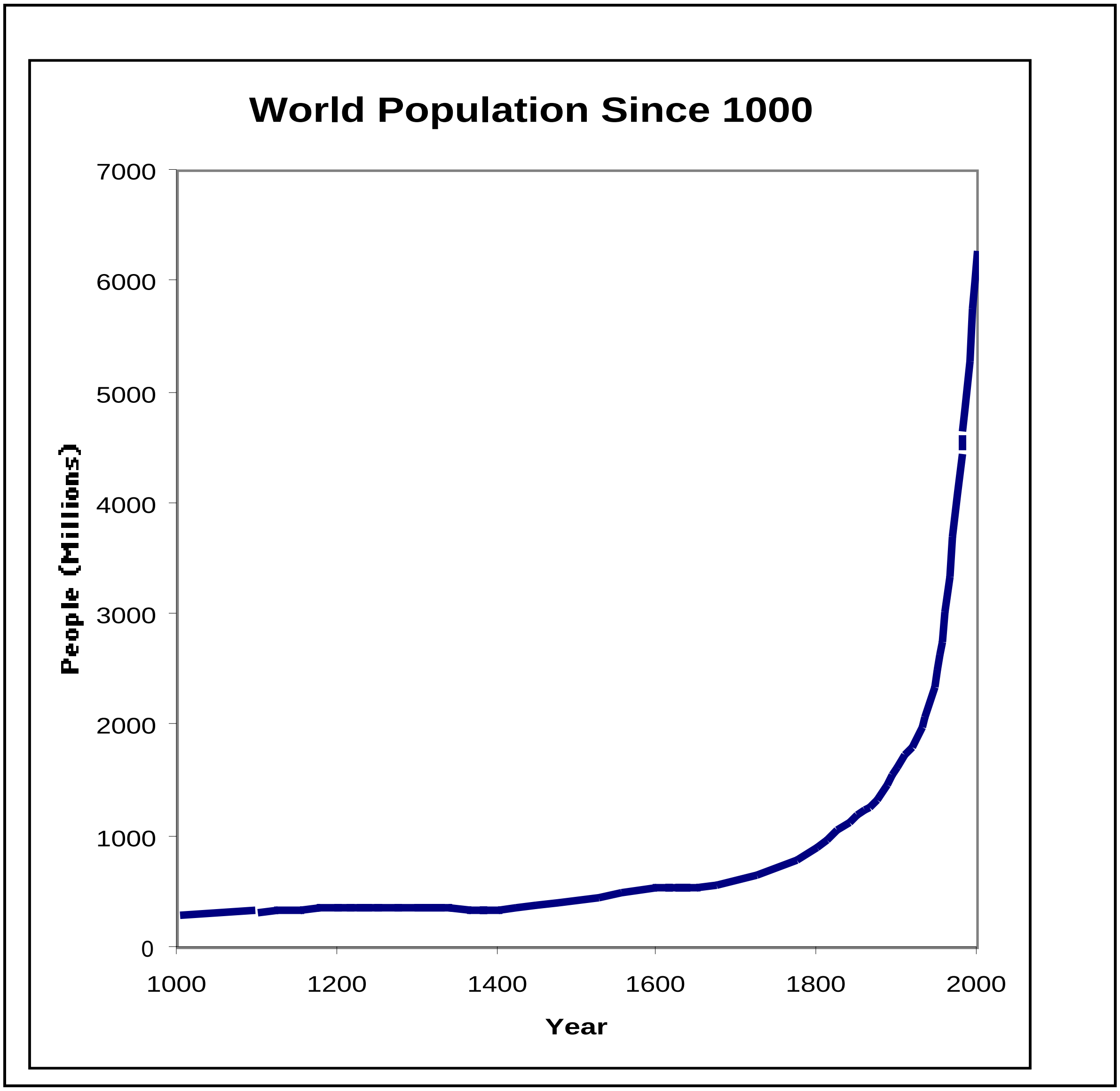
8. “La longue durée”.

Year	Population*	GDP per Capita**
-5000	5	$130
-1000	50	$160
1	170	$135
1000	265	$165
1500	425	$175
1800	900	$250
1900	1625	$850
1950	2515	$2030
1975	4080	$4640
2000	6120	$8175

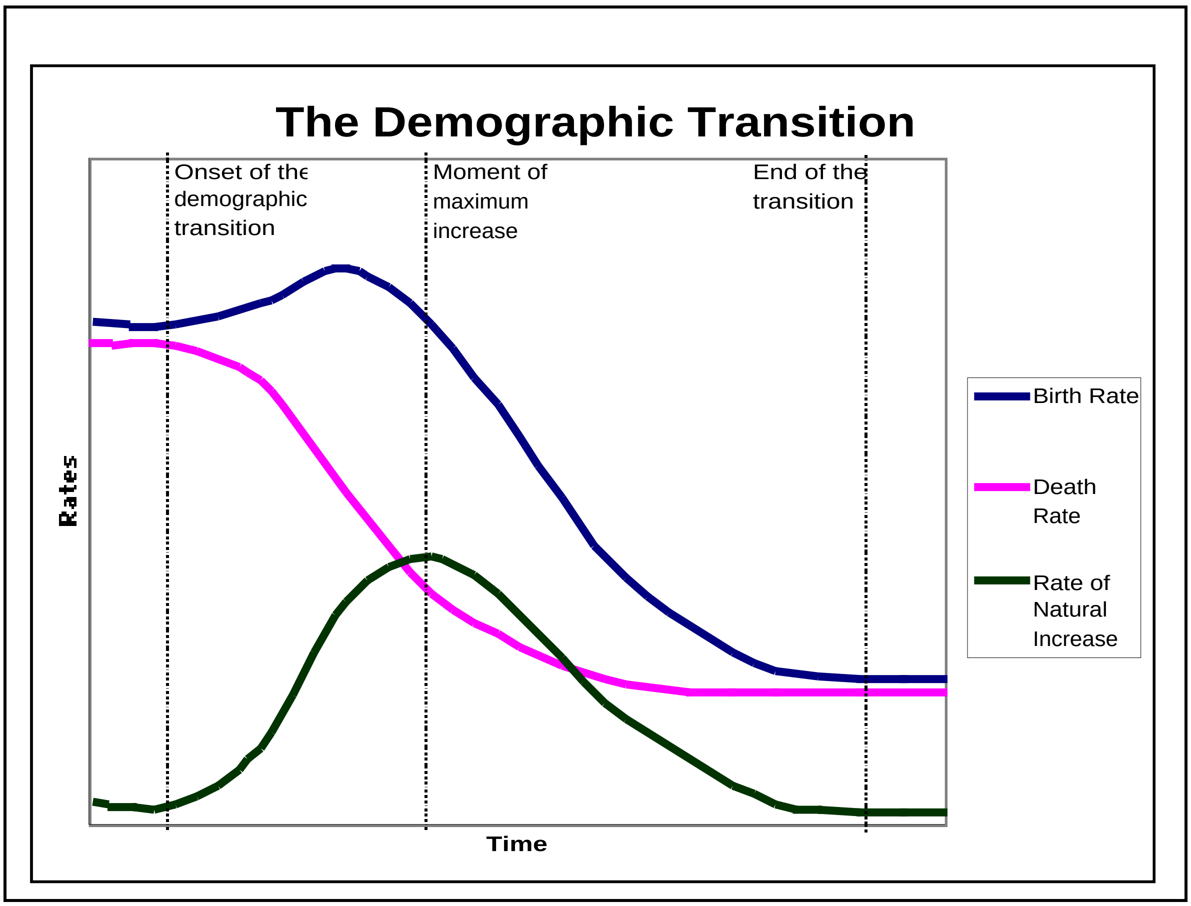
*Millions

**In year-2000 international dollars.

Population Growth Since 1000



Stylized Picture of the Demographic Transition



Growth Accounting

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Growth Accounting

- Output is produced by inputs capital K and labor L, in combination with the available technology A

- Want: decompose the growth rate of output into the growth rate of capital input, the growth rate of labor input and technological progress. This exercise is called growth accounting.

- Why?

Aggregate production function

- Maps inputs into output:

$$Y = F(A, K, L)$$

  A is called total factor productivity (TFP).

- Cobb-Douglas example:

$$Y = AK^{\alpha}L^{1-\alpha}$$

- Interpretation.

Discrete vs. Continuous Time

- In discrete time a variable is indexed by time: x_t.

- In continuous time a variable is a function of time: $x(t)$.

- We observe the world only in discrete time...

- but it is often much easier to work with continuous time!

Growth Rates and Logarithms I

- Remember:

$$g_x(t-1,t) = \frac{x_t - x_{t-1}}{x_{t-1}}$$
$$1 + g_x(t-1,t) = \frac{x_t}{x_{t-1}}$$

- Take logs on both sides:

$$\log\left(1 + g_x(t-1,t)\right) = \log\left(\frac{x_t}{x_{t-1}}\right)$$

Growth Rates and Logarithms II

- Taylor series expansion of $\log(1+y)$ around $y=0$:

$$\log(1+y)|_{y=0} = \ln 1 + \frac{1}{1!}y + \text{higher order terms} \simeq y$$

- Then:

$$\begin{aligned} \log(1+g_x(t-1,t)) &\simeq g_x(t-1,t) \simeq \log\left(\frac{x_t}{x_{t-1}}\right) \\ g_x(t-1,t) &\simeq \log x_t - \log x_{t-1} = \Delta \log x_t \end{aligned}$$

- Remember from calculus that validity of Taylor series expansion is local: g small!

Moving between Continuous and Discrete Time I

- Let $x(t)$ be a variable that depends of t.

- Notation:

$$\dot{x}(t) \equiv \frac{dx(t)}{dt}$$

- Take $\log x(t)$. Then:

$$\frac{d \log((x(t))}{dt} = \frac{\dot{x}(t)}{x(t)} = g_x(t)$$

- Why is this useful?

Moving between Continuous and Discrete Time II

- The definition of time derivative is:

$$\dot{x}(t) = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

- Then:

$$\frac{\dot{x}(t)}{x(t)} = \frac{\lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}}{x(t)}$$

- When Δt is small (let's say a year):

$$g_x(t) = \frac{\dot{x}(t)}{x(t)} \simeq \frac{x(t+1) - x(t)}{x(t)} = g_x(t-1, t) \simeq \Delta \log x_t$$

Growth Rates of Ratios I

- Suppose $k(t) = \frac{K(t)}{L(t)}$. What is $g_k(t)$?

- Step 1: take logs

$$\log(k(t)) = \log(K(t)) - \log(L(t))$$

- Step 2: differentiate with respect to time

$$\begin{aligned}
\frac{d\log((k(t))}{dt} &= \frac{d\log(K(t))}{dt} - \frac{d\log(L(t))}{dt} \\
\frac{\dot{k}(t)}{k(t)} &= \frac{\dot{K}(t)}{K(t)} - \frac{\dot{L}(t)}{L(t)} \\
g_k(t) &= g_K(t) - g_L(t)
\end{aligned}$$

Growth Rates of Ratios II

- Growth rate of a ratio equals the difference of the growth rates:

$$g_k(t) = g_K(t) - g_L(t)$$

- Ratio constant over time requires that both variables grow at same rate:

$$g_k(t) = 0 \Rightarrow g_K(t) = g_L(t)$$

Growth Rates of Weighted Products I

- Suppose

$$Y(t) = K(t)^{\alpha} L(t)^{1-\alpha}$$

What is $g_Y(t)$?

- Step 1: take logs

$$\log(Y(t)) = \alpha \log(K(t)) + (1 - \alpha) \log(L(t))$$

Growth Rates of Weighted Products II

- Step 2: differentiate

$$
\begin{aligned}
\frac{d\,\mathsf{log}(Y(t))}{dt} &= \alpha\frac{d\,\mathsf{log}(K(t))}{dt} + (1-\alpha)\frac{d\,\mathsf{log}(L(t))}{dt} \\
\frac{\dot{Y}(t)}{Y(t)} &= \alpha\frac{\dot{K}(t)}{K(t)} + (1-\alpha)\frac{\dot{L}(t)}{L(t)} \\
g_Y(t) &= \alpha g_K(t) + (1-\alpha)g_L(t)
\end{aligned}
$$

- Growth rate equals weighted sum, with weights equal to the share parameters

Growth Accounting I

- Observations in discrete time.

- Production Function: $Y(t) = F\left(A(t), K(t), L(t)\right)$

- Differentiating with respect to time and dividing by $Y(t)$

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{F_A A(t)}{Y(t)}\frac{\dot{A}(t)}{A(t)} + \frac{F_k K(t)}{K(t)}\frac{\dot{K}(t)}{K(t)} + \frac{F_L L(t)}{Y(t)}\frac{\dot{L}(t)}{L(t)}$$

Growth Accounting II

- Useful benchmark: Cobb-Douglas $Y(t) = A(t)K(t)^{\alpha}L(t)^{1-\alpha}$.

- Why?

- Taking logs and differentiating with respect to time gives

$$g_Y(t) = g_A(t) + \alpha g_K(t) + (1-\alpha)g_L(t)$$

- g_A is called TFP growth or multifactor productivity growth.

Doing the Accounting

- Pick an α (we will learn that α turns out to be the capital share).

- Measure g_Y, g_K and g_L from the data.

- Compute g_A as the residual:

$$g_A(t) = g_Y(t) - \alpha g_K(t) - (1 - \alpha) g_L(t)$$

- Therefore g_A is also called the Solow residual.

- Severe problems if mismeasurement (g_K is hard to measure).

Data for the US

- We pick $\alpha = \frac{1}{3}$

Per.	g_Y	αg_K	$(1-\alpha) g_L$	TFP (g_A)
$48 - 98$	2.5	0.8 (32%)	0.2 (8%)	1.4 (56%)
$48 - 73$	3.3	1.0 (30%)	0.2 (6%)	2.1 (64%)
$73 - 95$	1.5	0.7 (47%)	0.3 (20%)	0.6 (33%)
$95 - 98$	2.5	0.8 (32%)	0.3 (12%)	1.4 (56%)

- Key observation: Productivity Slowdown in the 70's

- Note: the late 90's look much better

Reasons for the Productivity Slowdown

1. Sharp increases in the price of oil in 70's

2. Structural changes: more services and less and less manufacturing goods produced

3. Slowdown in resources spent on R&D in the late 60's.

4. TFP was abnormally high in the 50's and 60's

5. Information technology (IT) revolution in the 70's

Growth Accounting for Other Countries

- One key question: was fast growth in East Asian growth miracles mostly due to technological progress or mostly due to capital accumulation?

- Why is this an important question?

Country	Per.	g_Y	α	αg_K	$(1-\alpha)g_L$	g_A
Germany	60-90	3.2	0.4	59%	−8%	49%
Italy	60-90	4.1	0.38	49%	3%	48%
UK	60-90	2.5	0.39	52%	−4%	52%
Argentina	40-80	3.6	0.54	43%	26%	31%
Brazil	40-80	6.4	0.45	51%	20%	29%
Chile	40-80	3.8	0.52	34%	26%	40%
Mexico	40-80	6.3	0.63	41%	23%	36%
Japan	60-90	6.8	0.42	57%	14%	29%
Hong Kong	66-90	7.3	0.37	42%	28%	30%
Singapore	66-90	8.5	0.53	73%	31%	−4%
South Korea	66-90	10.3	0.32	46%	42%	12%
Taiwan	66-90	9.1	0.29	40%	40%	20%

Neoclassical Growth Model

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Models and Assumptions I

- What is a model? A mathematical description of the economy.

- Why do we need a model? The world is too complex to describe it in every detail. A model abstracts from details to understand clearly the main forces driving the economy.

- What makes a model successful? When it is simple but effective in describing and predicting how the world works.

Models and Assumptions II

- A model relies on simplifying assumptions. These assumptions drive the conclusions of the model. When analyzing a model it is crucial to spell out the assumptions underlying the model.

- Realism may not a the property of a good assumption.

- An assumption is good when it helps us to build a model that accounts for the observations and predicts well.

Basic Assumptions of the Neoclassical Growth Model

1. Continuous time.

2. Single good in the economy produced with a constant technology.

3. No government or international trade.

4. All factors of production are fully employed.

5. Labor force grows at constant rate $n = \frac{\dot{L}}{L}$.

6. Initial values for capital, K_0 and labor, L_0 given.

Production Function I

- Neoclassical (Cobb-Douglas) aggregate production function:

$$Y(t) = F(K(t), L(t)) = K(t)^{\alpha} L(t)^{1-\alpha}$$

- To save on notation write:

$$Y = K^{\alpha} L^{1-\alpha}$$

where the dependency on t is understood implicitly.

Properties of the Technology I

- Constant returns to scale:

$$\lambda Y = (\lambda K)^{\alpha} (\lambda L)^{1-\alpha} = \lambda K^{\alpha} L^{1-\alpha}$$

- Inputs are essential:

$$F(0,0) = F(K,0) = F(0,L) = 0$$

- Marginal productivities are positive:

$$\begin{aligned} \frac{\partial F}{\partial K} &= \alpha A K^{\alpha-1} L^{1-\alpha} > 0 \\ \frac{\partial F}{\partial L} &= (1-\alpha) A K^{\alpha} L^{-\alpha} > 0 \end{aligned}$$

Properties of the Technology II

- Marginal productivities are decreasing,

$$\begin{aligned}\frac{\partial^2 F}{\partial K^2} &= (\alpha - 1)\,\alpha K^{\alpha-2} L^{1-\alpha} < 0 \\ \frac{\partial^2 F}{\partial L^2} &= (\alpha - 1)\,\alpha K^{\alpha} L^{-\alpha-1} < 0\end{aligned}$$

- Inada Conditions,

$$\begin{aligned}\lim_{K\to 0} \alpha K^{\alpha-1} L^{1-\alpha} &= \infty, \quad \lim_{K\to\infty} \alpha K^{\alpha-1} L^{1-\alpha} = 0 \\ \lim_{L\to 0} (1-\alpha)\,K^{\alpha} L^{-\alpha} &= \infty, \quad \lim_{L\to\infty} (1-\alpha)\,K^{\alpha} L^{-\alpha} = 0\end{aligned}$$

Per Worker Terms

- Define $x = \frac{X}{L}$ as a per worker variable.

- Then

$$y = \frac{Y}{L} = \frac{K^{\alpha} L^{1-\alpha}}{L} = \left(\frac{K}{L}\right)^{a} \left(\frac{L}{L}\right)^{1-\alpha} = k^{\alpha}$$

- Per worker production function has decreasing returns to scale.

Capital Accumulation I

- Capital accumulation equation:

$$\dot{K} = sY - \delta K$$

- Important additional assumptions:

  1. Constant saving rate

  2. Constant depreciation rate

Capital Accumulation II

- Dividing by K in the capital accumulation equation:

$$\frac{\dot{K}}{K} = s\frac{Y}{K} - \delta$$

- Some Algebra:

$$\frac{\dot{K}}{K} = s\frac{Y}{K} - \delta = s\frac{\frac{Y}{L}}{\frac{K}{L}} - \delta = s\frac{y}{k} - \delta$$

Capital Accumulation III

- Now remember that:

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} = \frac{\dot{K}}{K} - n \Rightarrow \frac{\dot{K}}{K} = \frac{\dot{k}}{k} + n$$

- We get

$$\frac{\dot{k}}{k} + n = s\frac{y}{k} - \delta \Rightarrow \dot{k} = sy - (\delta + n)\,k$$

- Fundamental Differential Equation of Neoclassical Growth Model:

$$\boxed{\dot{k} = sk^{\alpha} - (\delta + n)\,k}$$

Graphical Analysis

- Change in k, $\dot{k}$ is given by difference of sk^{α} and $(\delta + n)k$
- If $sk^{\alpha} > (\delta + n)k$, then k increases.
- If $sk^{\alpha} < (\delta + n)k$, then k decreases.
- Steady state: a capital stock k^{*} where, when reached, $\dot{k} = 0$
- Unique positive steady state in Neoclassical Growth model.
- Positive steady state (locally) stable.

Close-Form Solution I

- $\dot{k} = sk^{\alpha} - (\delta + n)\, k$ is a Bernoulli Equation.

- Change of variable:

$$z = k^{1-\alpha}$$

- Then:

$$\dot{z} = (1-\alpha)\, k^{-\alpha}\dot{k} \Rightarrow \dot{k} = \dot{z}\,(1-\alpha)^{-1}\, k^{\alpha}$$

Close-Form Solution II

- Some algebra

$$\dot{z}\,(1-\alpha)^{-1}\,k^{\alpha} = sk^{\alpha} - (\delta+n)\,k$$

$$\dot{z}\,(1-\alpha)^{-1} = s - (\delta+n)\,k^{1-\alpha} = s - (\delta+n)\,z$$

$$\dot{z} = (1-\alpha)\,s - (1-\alpha)\,(\delta+n)\,k^{1-\alpha}$$

$$\boxed{\dot{z} + \lambda z = (1-\alpha)\,s}$$

where $\lambda = (1-\alpha)\,(\delta+n)$.

Close-Form Solution III

- We have a linear, first order differential equation with constant coefficients.

- Integrating with respect to $e^{\lambda t}dt$:

$$\int (\dot{z} + \lambda z)\, e^{\lambda t} dt = \int (1-\alpha)\, s e^{\lambda t} dt$$

we get

$$z e^{\lambda t} = \frac{(1-\alpha)\, s}{\lambda} e^{\lambda t} + b$$

where b is an integrating constant.

Close-Form Solution IV

- Then:

$$z(t) = \frac{(1-\alpha)\,s}{\lambda} + be^{-\lambda t}$$

- Substituting back: $z = k^{1-\alpha}$ we get the general solution:

$$k(t) = \left(\frac{s}{\delta + n} + be^{-\lambda t}\right)^{\frac{1}{1-\alpha}}$$

- To find the particular solution note that

$$z(0) = \frac{(1-\alpha)\,s}{\lambda} + be^{-\lambda 0} = \frac{s}{\delta + n} + b = z_0 \Rightarrow b = z_0 - \frac{s}{\delta + n}$$

Close-Form Solution V

- Then:

$$z(t) = \frac{s}{\delta + n} + \left(z_0 - \frac{s}{\delta + n}\right) e^{-\lambda t}$$

- Interpretation of λ.

- Substituting back $z = k^{1-\alpha}$ we get:

$$\boxed{k(t) = \left(\frac{s}{\delta+n} + \left(k_0^{1-\alpha} - \frac{s}{\delta+n}\right) e^{-\lambda t}\right)^{\frac{1}{1-\alpha}}}$$

$$\boxed{y(t) = \left(\frac{s}{\delta+n} + \left(k_0^{1-\alpha} - \frac{s}{\delta+n}\right) e^{-\lambda t}\right)^{\frac{\alpha}{1-\alpha}}}$$

Steady State Analysis

- Steady State: $\dot{k} = 0$

- Solve for steady state

$$0 = s\,(k^*)^{\alpha} - (n+\delta)k^* \Rightarrow k^* = \left(\frac{s}{n+\delta}\right)^{\frac{1}{1-\alpha}}$$

- Steady state output per worker $y^* = \left(\frac{s}{n+\delta}\right)^{\frac{\alpha}{1-\alpha}}$

- Steady state output per worker depends positively on the saving (investment) rate and negatively on the population growth rate and depreciation rate.

Comparative Statics

- Suppose that of all a sudden saving rate s increases to $s' > s$. Suppose that at period 0 the economy was at its old steady state with saving rate s.

- $(n+\delta)k$ curve does not change.

- $sk^{\alpha} = sy$ shifts up to $s'y$.

- New steady state has higher capital per worker and output per worker.

- Monotonic transition path from old to new steady state.

Evaluating the Basic Neoclassical Growth Model: the Good

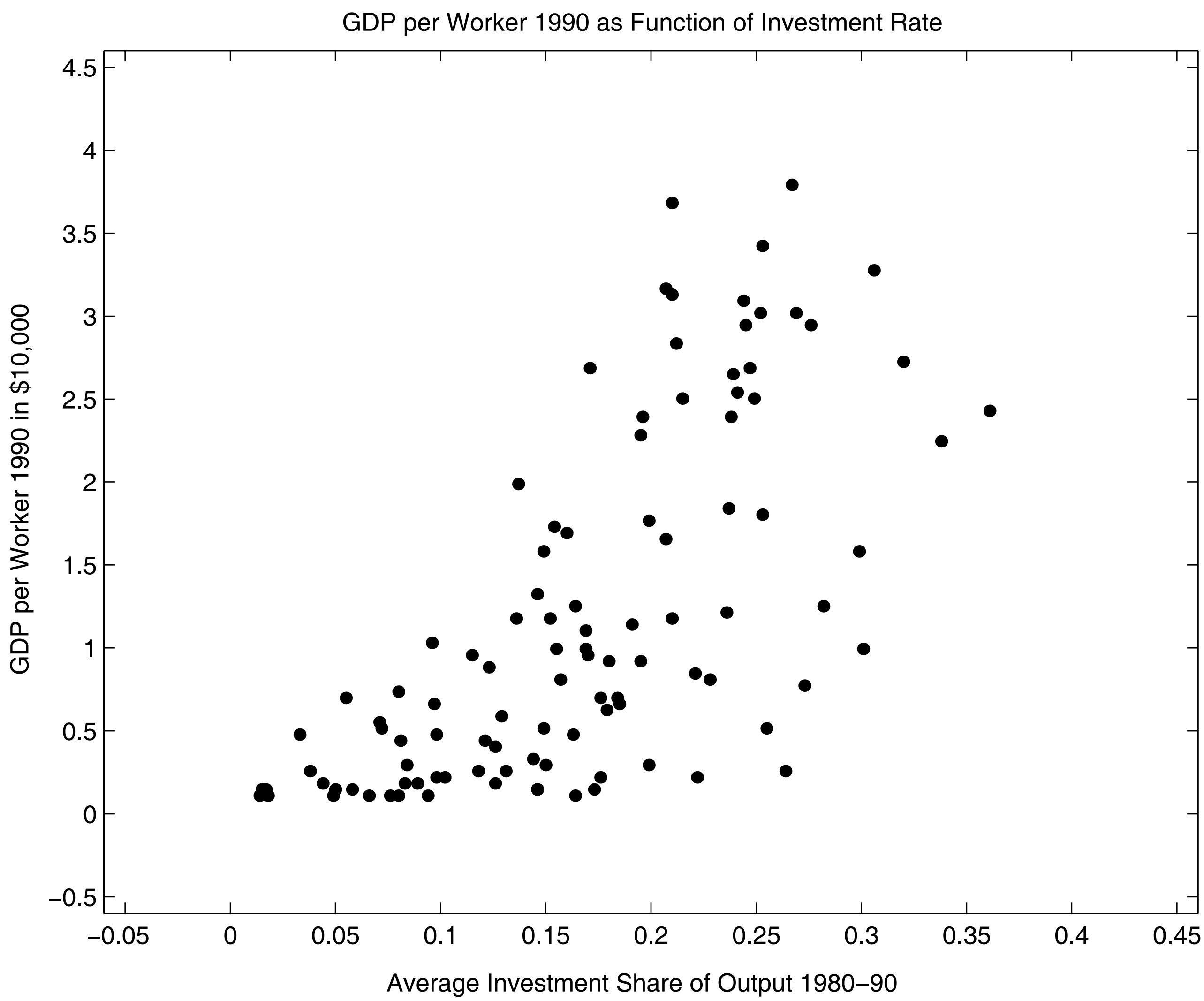
- Why are some countries rich (have high per worker GDP) and others are poor (have low per worker GDP)?

- Neoclassical Growth model: if all countries are in their steady states, then:

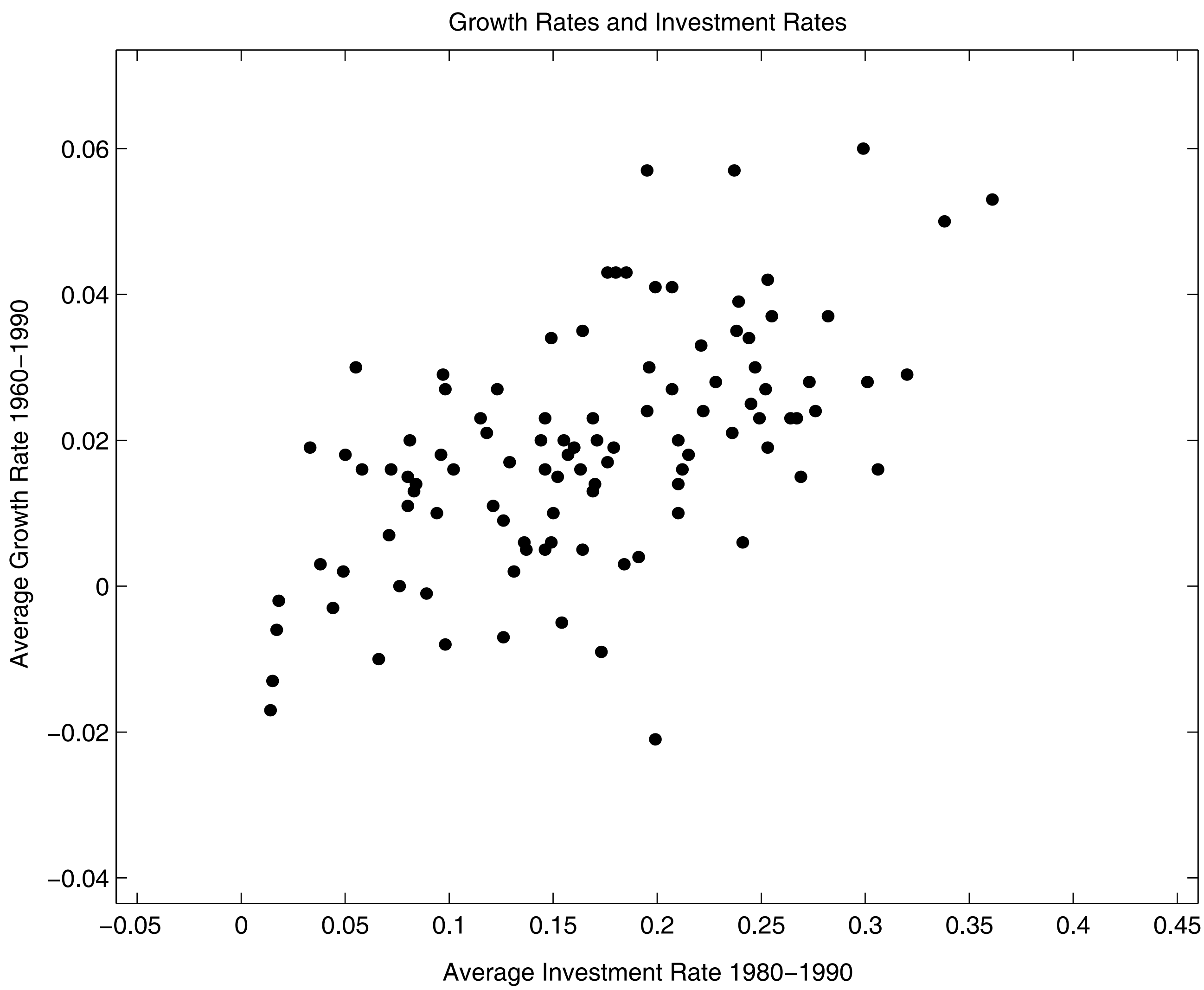
  1. Rich countries have higher saving (investment) rates than poor countries.

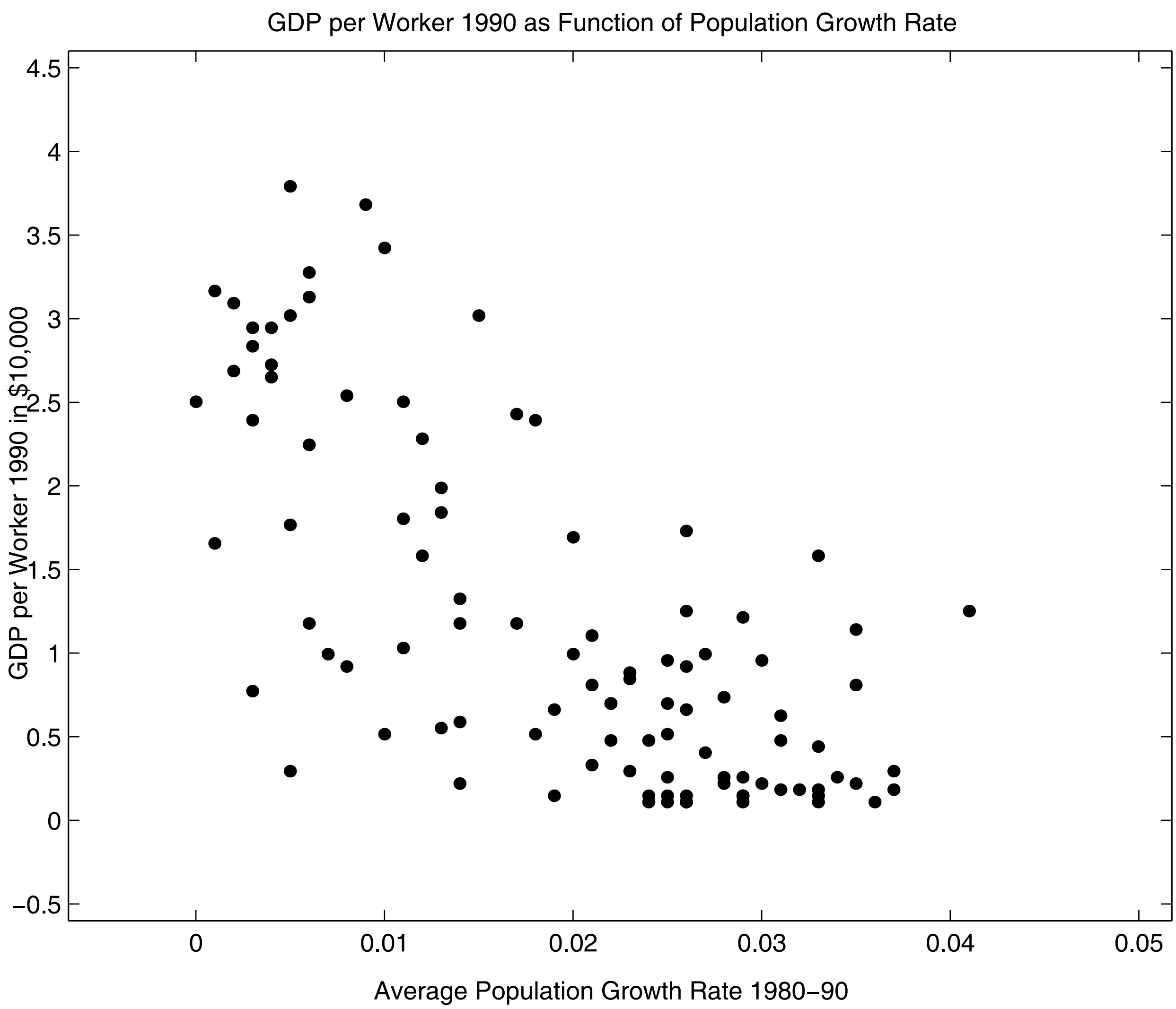
  2. Rich countries have lower population growth rates than poor countries.

- Data seem to support this prediction of the Neoclassical Growth model.



GDP per Worker 1990 as Function of Investment Rate
GDP per Worker 1990 in $10,000
Average Investment Share of Output 1980−90
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
−0.5
−0.05
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45


Growth Rates and Investment Rates
Average Growth Rate 1960–1990
0.06
0.04
0.02
0
−0.02
−0.04
−0.05
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
Average Investment Rate 1980–1990


GDP per Worker 1990 as Function of Population Growth Rate
GDP per Worker 1990 in $10,000
Average Population Growth Rate 1980−90
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
−0.5
0
0.01
0.02
0.03
0.04
0.05

Evaluating the Basic Neoclassical Growth Model: the Bad

- Are saving and population growth rates exogenous?

- Are the magnitude of differences created by the model right?

$$
\begin{aligned}
y_{ss}^{us} &= \left(\frac{0.2}{0.01+0.06}\right)^{\frac{1}{2}} = 1.69 \\
y_{ss}^{chad} &= \left(\frac{0.05}{0.02+0.06}\right)^{\frac{1}{2}} = 0.79
\end{aligned}
$$

- No growth in the steady state: only transitional dynamics.

The Neoclassical Growth Model and Growth

- We can take the absence of growth as a positive lesson.

- Illuminates why capital accumulation has an inherit limitation as a source of economic growth:

  1. Soviet Union.

  2. Development theory of the 50's and 60's.

  3. East Asian countries today?

- Tells us we need to look some place else: technology.

Introducing Technological Progress

- Aggregate production function becomes

$$Y = K^{\alpha} (AL)^{1-\alpha}$$

- A : Level of technology in period t.

- Key assumption: constant positive rate of technological progress:

$$\frac{\dot{A}}{A} = g > 0$$

- Growth is exogenous.

Balanced Growth Path

- Situation in which output per worker, capital per worker, and consumption per worker grow at constant (but potentially different) rates

- Steady state is just a balanced growth path with zero growth rate

- For Neoclassical Growth model, in BGP: $g_y = g_k = g_c$

Proof

- Capital Accumulation Equation $\dot{K} = sY - \delta K$

- Dividing both sides by K yields $g_K \equiv \frac{\dot{K}}{K} = s\frac{Y}{K} - \delta$

- Remember that $g_k \equiv \frac{\dot{k}}{k} = \frac{\dot{K}}{K} - n$

- Hence

$$g_k \equiv \frac{\dot{k}}{k} = s\frac{Y}{K} - (n + \delta)$$

- In BGP g_k constant. Hence $\frac{Y}{K}$ constant. It follows that $g_Y = g_K$. Therefore $g_y = g_k$

What is the Growth Rate?

- Output per worker

$$y = \frac{Y}{L} = \frac{K^{\alpha}(AL)^{1-\alpha}}{L} = \frac{K^{\alpha}}{L^{\alpha}}\frac{(AL)^{1-\alpha}}{L^{1-\alpha}} = k^{\alpha}A^{1-\alpha}$$

- Take logs and differentiate $g_y = \alpha g_k + (1-\alpha)g_A$

- We proved $g_k = g_y$ and we use $g_A = g$ to get

$$g_k = \alpha g_k + (1-\alpha)g = g = g_y$$

- BGP growth rate equals rate of technological progress. No TP, no growth in the economy.

Analysis of Extended Model

- In BGP variables grow at rate g. Want to work with variables that are constant in long run. Define:

$$\begin{aligned}\tilde{y} &= \frac{y}{A} = \frac{Y}{AL} \\ \tilde{k} &= \frac{k}{A} = \frac{K}{AL}\end{aligned}$$

- Repeat the analysis with new variables:

$$\begin{aligned}\tilde{y} &= \tilde{k}^{\alpha} \\ \dot{\tilde{k}} &= s\tilde{y} - (n+g+\delta)\tilde{k} \\ \dot{\tilde{k}} &= s\tilde{k}^{\alpha} - (n+g+\delta)\tilde{k}\end{aligned}$$

Close-Form Solution

- Repeating all the steps than in the basic model we get:

$$\boxed{\tilde{k}(t) = \left(\frac{s}{\delta+n+g} + \left(\tilde{k}_0^{1-\alpha} - \frac{s}{\delta+n+g}\right) e^{-\lambda t}\right)^{\frac{1}{1-\alpha}}}$$

$$\boxed{\tilde{y}(t) = \left(\frac{s}{\delta+n+g} + \left(\tilde{k}_0^{1-\alpha} - \frac{s}{\delta+n+g}\right) e^{-\lambda t}\right)^{\frac{\alpha}{1-\alpha}}}$$

- Interpretation.

Balanced Growth Path Analysis I

- Solve for $\tilde{k}^*$ analytically

$$
\begin{aligned}
0 &= s{\tilde{k}^*}^{\alpha} - (n+g+\delta)\tilde{k}^* \\
\tilde{k}^* &= \left(\frac{s}{n+g+\delta}\right)^{\frac{1}{1-\alpha}}
\end{aligned}
$$

- Therefore

$$
\tilde{y}^* = \left(\frac{s}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}}
$$

Balanced Growth Path Analysis II

$$
\begin{aligned}
k(t) &= A(t)\left(\frac{s}{n+g+\delta}\right)^{\frac{1}{1-\alpha}} \\
y(t) &= A(t)\left(\frac{s}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}} \\
K(t) &= L(t)A(t)\left(\frac{s}{n+g+\delta}\right)^{\frac{1}{1-\alpha}} \\
Y(t) &= L(t)A(t)\left(\frac{s}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}}
\end{aligned}
$$

Evaluation of the Model: Growth Facts

1. Output and capital per worker grow at the same constant, positive rate in BGP of model. In long run model reaches BGP.

2. Capital-output ratio $\frac{K}{Y}$ constant along BGP

3. Interest rate constant in balanced growth path

4. Capital share equals α, labor share equals $1 - \alpha$ in the model (always, not only along BGP)

5. Success of the model along these dimensions, but source of growth, technological progress, is left unexplained.

Evaluation of the Model: Development Facts

1. Differences in income levels across countries explained in the model by differences in s, n and δ.

2. Variation in growth rates: in the model *permanent* differences can only be due to differences in rate of technological progress g. *Temporary* differences can be explained by transition dynamics.

3. That growth rates are not constant over time for a given country can be explained by transition dynamics and/or shocks to n, s and δ.

4. Changes in relative position: in the model countries whose s moves up, relative to other countries, move up in income distribution. Reverse with n.

Interest Rates and the Capital Share

- Output produced by price-taking firms
- Hire workers L for wage w and rent capital K from households for r
- Normalization of price of output to 1.
- Real interest rate equals $r - \delta$

Profit Maximization of Firms

$$\max_{K,L} K^{\alpha} (AL)^{1-\alpha} - wL - rK$$

- First order condition with respect to capital K

$$\begin{aligned} \alpha K^{\alpha-1} (AL)^{1-\alpha} - r &= 0 \\ \alpha \left(\frac{K}{AL}\right)^{\alpha-1} &= r \\ \alpha \tilde{k}^{\alpha-1} &= r \end{aligned}$$

- In balanced growth path $\tilde{k} = \tilde{k}^*$, constant over time. Hence in BGP $r$constant over time, hence $r - \delta$ (real interest rate) constant over time.

Capital Share

- Total income $= Y$, total capital income $= rK$

- Capital share

$$\begin{aligned} \text{capital share} &= \frac{rK}{Y} \\ &= \frac{\alpha K^{\alpha-1}(AL)^{1-\alpha} K}{K^{\alpha}(AL)^{1-\alpha}} \\ &= \alpha \end{aligned}$$

- Labor share $= 1 - \alpha$.

Wages

- First order condition with respect to labor L

$$
\begin{aligned}
(1-\alpha)K^{\alpha}(LA)^{-\alpha}A &= w \\
(1-\alpha)\tilde{k}^{\alpha}A &= w
\end{aligned}
$$

- Along BGP $\tilde{k} = \tilde{k}^{*}$, constant over time. Since $A$is growing at rate g, the wage is growing at rate g along a BGP.

Human Capital and Growth

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Introduction to Human Capital

- Education levels are very different across countries.

- Rich countries tend to have higher educational levels than poor countries.

- We have the intuition that education (learning skills) is an important factor in economic growth.

Production Function

- Cobb-Douglas aggregate production function:

$$Y = K^{\alpha} H^{\beta} (AL)^{1-\alpha-\beta}$$

- Again we have constant returns to scale.

- Human capital and labor enter with a different coefficient.

Inputs Accumulation

- Society accumulates human capital according to:

$$\dot{H} = s_h Y - \delta H$$

- Capital accumulation equation:

$$\dot{K} = s_k Y - \delta K$$

- Technology progress: $\frac{\dot{A}}{A} = g > 0$.

- Labor force grows at constant rate: $\frac{\dot{L}}{L} = n > 0$.

Rewriting the Model in Efficiency Units

- Redefine the variables in efficiency units:

$$\tilde{x} \equiv \frac{X}{AL}$$

- Then, diving the production function by AL:

$$\tilde{y} = \tilde{k}^{\alpha} \tilde{h}^{\beta}$$

- Decreasing returns to scale in per efficiency units.

Human Capital Accumulation

- The evolution of inputs is determined by:

$$\begin{aligned}\dot{\tilde{k}} &= s_k \tilde{k}^{\alpha} \tilde{h}^{\beta} - (n+g+\delta)\tilde{k} \\ \dot{\tilde{h}} &= s_h \tilde{k}^{\alpha} \tilde{h}^{\beta} - (n+g+\delta)\tilde{h}\end{aligned}$$

- System of two differential equations.

- Solving it analytically it is bit tricky so we will only look at the BGP.

Phase Diagram

- Solving the system analytically it is bit tricky.

- Alternatives:

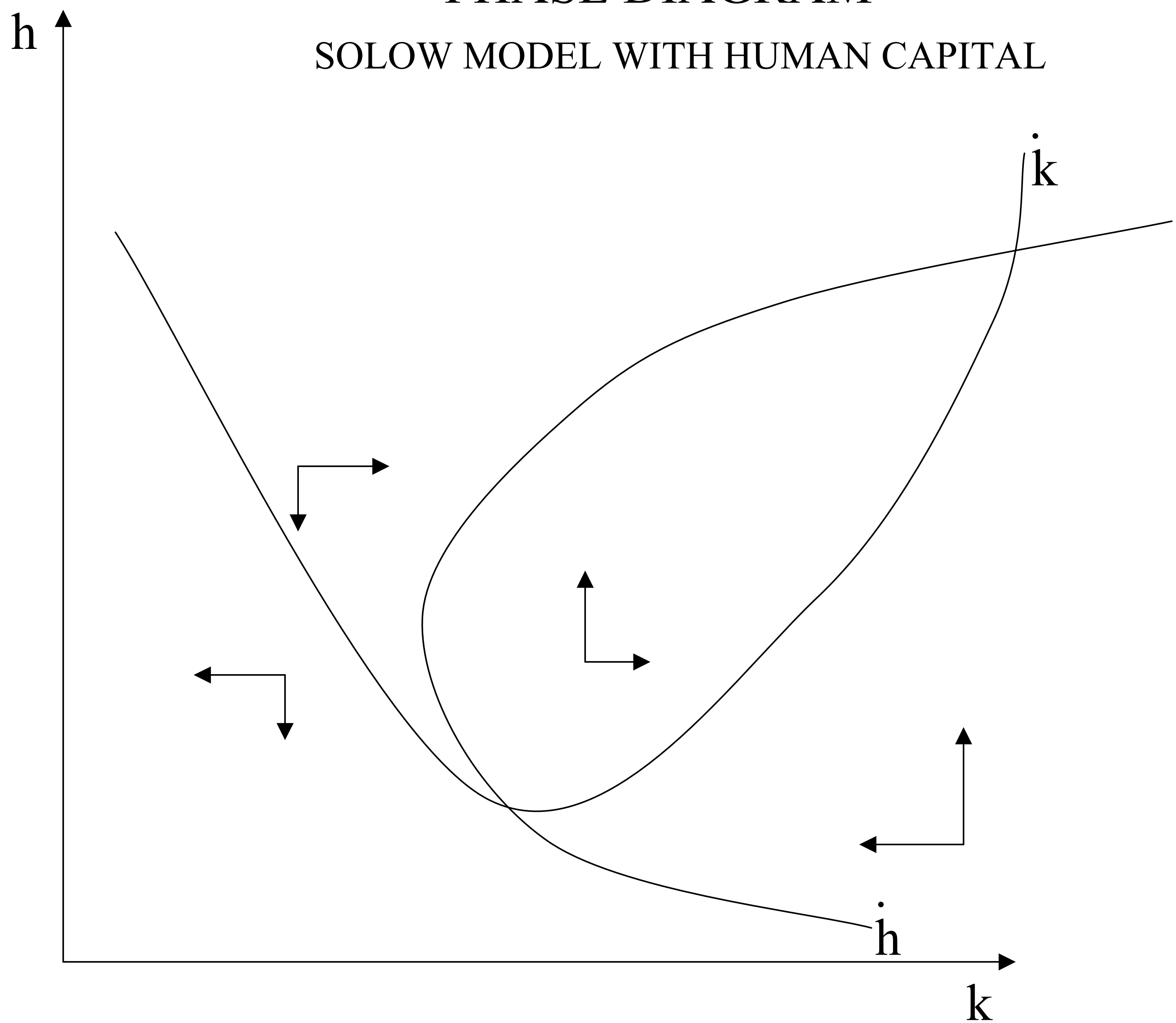
  1. Use numerical methods.

  2. Linearize the system.

  3. Phase diagram.

PHASE DIAGRAM

SOLOW MODEL WITH HUMAN CAPITAL



Balanced Growth Path Analysis I

- To find the BGP equate both equations to zero:

$$\begin{aligned} s_k \tilde{k}^{*\alpha} \tilde{h}^{*\beta} - (n+g+\delta)\tilde{k}^* &= 0 \\ s_h \tilde{k}^{*\alpha} \tilde{h}^{*\beta} - (n+g+\delta)\tilde{h}^* &= 0 \end{aligned}$$

- From first equation:

$$\tilde{h}^* = \left(\frac{(n+g+\delta)}{s_k} \tilde{k}^{*1-\alpha} \right)^{\frac{1}{\beta}}$$

Balanced Growth Path Analysis II

- Plugging it in the second equation

$$s_h \tilde{k}^{*\alpha} \frac{(n+g+\delta)}{s_k} \tilde{k}^{*1-\alpha} - (n+g+\delta)\left(\frac{n+g+\delta}{s_k}\tilde{k}^{*1-\alpha}\right)^{\frac{1}{\beta}} = 0 \Rightarrow$$

$$\frac{s_h}{s_k}\tilde{k}^{*} = \left(\frac{n+g+\delta}{s_k}\tilde{k}^{*1-\alpha}\right)^{\frac{1}{\beta}}$$

- Work with the expression.

Some Algebra

$$\frac{s_h}{s_k}\tilde{k}^* = \left(\frac{n+g+\delta}{s_k}\tilde{k}^{*1-\alpha}\right)^{\frac{1}{\beta}} \Rightarrow$$

$$\tilde{k}^{*1-\frac{1-\alpha}{\beta}} = \tilde{k}^{*-\frac{1-\alpha-\beta}{\beta}} = \frac{s_k}{s_h}\left(\frac{(n+g+\delta}{s_k}\right)^{\frac{1}{\beta}} \Rightarrow$$

$$\boxed{\tilde{k}^* = \left(\frac{s_k^{1-\beta}s_h^{\beta}}{n+g+\delta}\right)^{\frac{1}{1-\alpha-\beta}}}$$

$$\boxed{\tilde{h}^* = \left(\frac{s_k^{\alpha}s_h^{1-\alpha}}{n+g+\delta}\right)^{\frac{1}{1-\alpha-\beta}}}$$

Evaluating the Model I

- Using the production function:

$$\tilde{y} = \frac{Y}{AL} = \tilde{k}^{\alpha}\tilde{h}^{\beta} = \left(\frac{s_k^{1-\beta} s_h^{\beta}}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{s_k^{\alpha} s_h^{1-\alpha}}{n+g+\delta}\right)^{\frac{\beta}{1-\alpha-\beta}} \Rightarrow$$

$$y = \frac{Y}{L} = \left(\frac{s_k^{1-\beta} s_h^{\beta}}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{s_k^{\alpha} s_h^{1-\alpha}}{n+g+\delta}\right)^{\frac{\beta}{1-\alpha-\beta}} A$$

- Given some initial value of technology A_0 we have:

$$y = \left(\frac{s_k^{1-\beta} s_h^{\beta}}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{s_k^{\alpha} s_h^{1-\alpha}}{n+g+\delta}\right)^{\frac{\beta}{1-\alpha-\beta}} A_0 e^{gt}$$

Evaluating the Model II

- Taking logs:

$$\begin{aligned} \log y &= \log A_0 + gt - \frac{\alpha+\beta}{1-\alpha-\beta}\log(n+g+\delta) + \\ &+ \frac{\alpha}{1-\alpha-\beta}\log s_k + \frac{\beta}{1-\alpha-\beta}\log s_h \end{aligned}$$

- What if we have a lot of countries $i = 1, ..., n$?

- We can assume that $\log A_0 = a + \varepsilon_i$

- Also assume that g and δ are constant across countries.

Evaluating the Model III

- Then we have:

$$\begin{aligned}\log y_i &= a + gt - \frac{\alpha+\beta}{1-\alpha-\beta}\log(n_i + g + \delta) + \\ &\quad + \frac{\alpha}{1-\alpha-\beta}\log s_{ki} + \frac{\beta}{1-\alpha-\beta}\log s_{hi} + \varepsilon_i\end{aligned}$$

- This is a functional form that can be taken to the data.

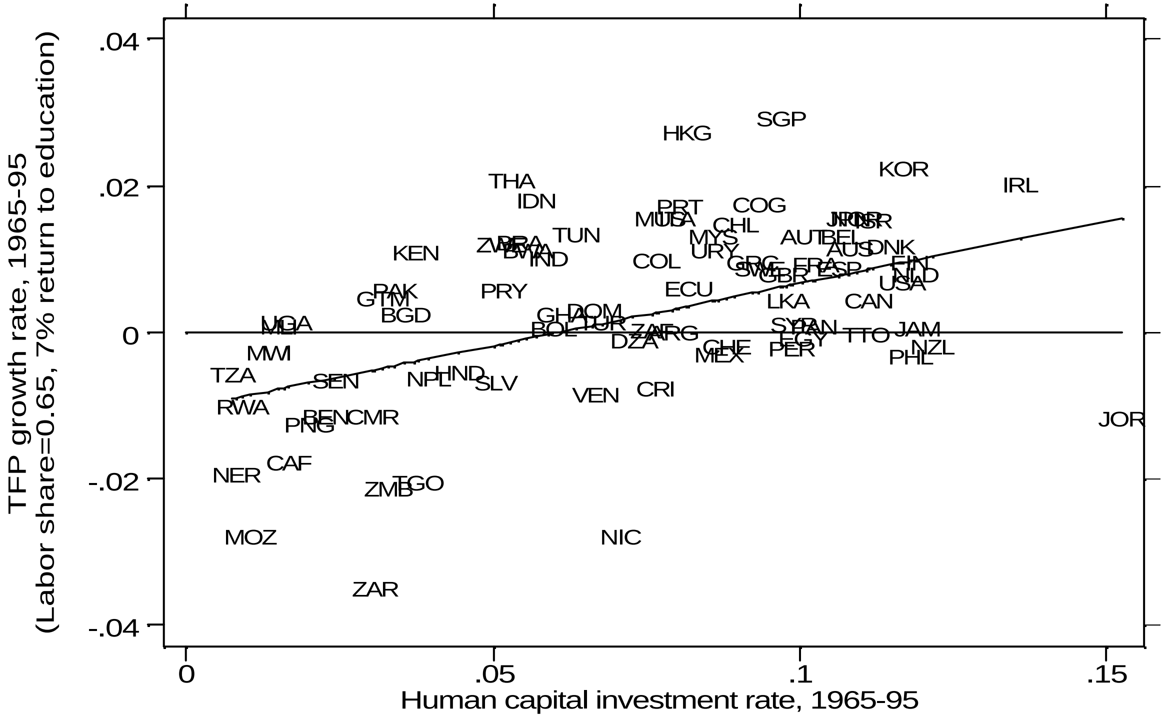


Figure 1: Relation of TFP growth to schooling rate

ESTIMATION OF THE AUGMENTED SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	6.89	7.81	8.63
	(1.17)	(1.19)	(2.19)
ln(I/GDP)	0.69	0.70	0.28
	(0.13)	(0.15)	(0.39)
$\ln(n + g + \delta)$	−1.73	−1.50	−1.07
	(0.41)	(0.40)	(0.75)
ln(SCHOOL)	0.66	0.73	0.76
	(0.07)	(0.10)	(0.29)
$\overline{R}^2$	0.78	0.77	0.24
s.e.e.	0.51	0.45	0.33
Restricted regression:			
CONSTANT	7.86	7.97	8.71
	(0.14)	(0.15)	(0.47)
$\ln(I/GDP) - \ln(n + g + \delta)$	0.73	0.71	0.29
	(0.12)	(0.14)	(0.33)
$\ln(SCHOOL) - \ln(n + g + \delta)$	0.67	0.74	0.76
	(0.07)	(0.09)	(0.28)
$\overline{R}^2$	0.78	0.77	0.28
s.e.e.	0.51	0.45	0.32
Test of restriction:			
p-value	0.41	0.89	0.97
Implied α	0.31	0.29	0.14
	(0.04)	(0.05)	(0.15)
Implied β	0.28	0.30	0.37
	(0.03)	(0.04)	(0.12)

Note. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05. SCHOOL is the average percentage of the working-age population in secondary school for the period 1960–1985.

Convergence and World Income Distribution

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The Convergence Hypothesis

- Fact: Enormous variation in incomes per worker across countries
- Question: Do poor countries eventually catch up?
- Convergence hypothesis: They do, in the right sense!
- Main prediction of convergence hypothesis: Poor countries should grow faster than rich countries.
- Let us look at the data.

Neoclassical Growth Model and Convergence

Countries with same s, n, δ, α, g

- eventually same growth rate of output per worker and same level of output per worker (*absolute* convergence).

- countries starting further below the balanced growth path (poorer countries) should grow faster than countires closer to balanced growth path.

- seems to be the case for the sample of now industrialized countries.

Countries with same g, but potentially differing s, n, δ, α

- countries have different balanced growth path.

- countries that start further below *their* balanced growth path (countires that are poor relative to their BGP) should grow faster than rich countries (relative to their BGP). This is called *conditional* convergence.

- data for full sample lend support to conditional convergence.

World Income Distribution

What is happening with the distribution of world income?

Look at the data again.

Conclusion: The Neoclassical Growth Model

- Offers a simple and elegant account of a number of growth facts.

- However:

    1. leaves unexplained factors that make countries leave (or not attain) their BGP.

    2. leaves unexplained why certain countries have higher s, n than others.

    3. leaves unexplained technological progress, the source of growth.

Figure 1.a: Growth Rate Versus Initial Per Capita GDP

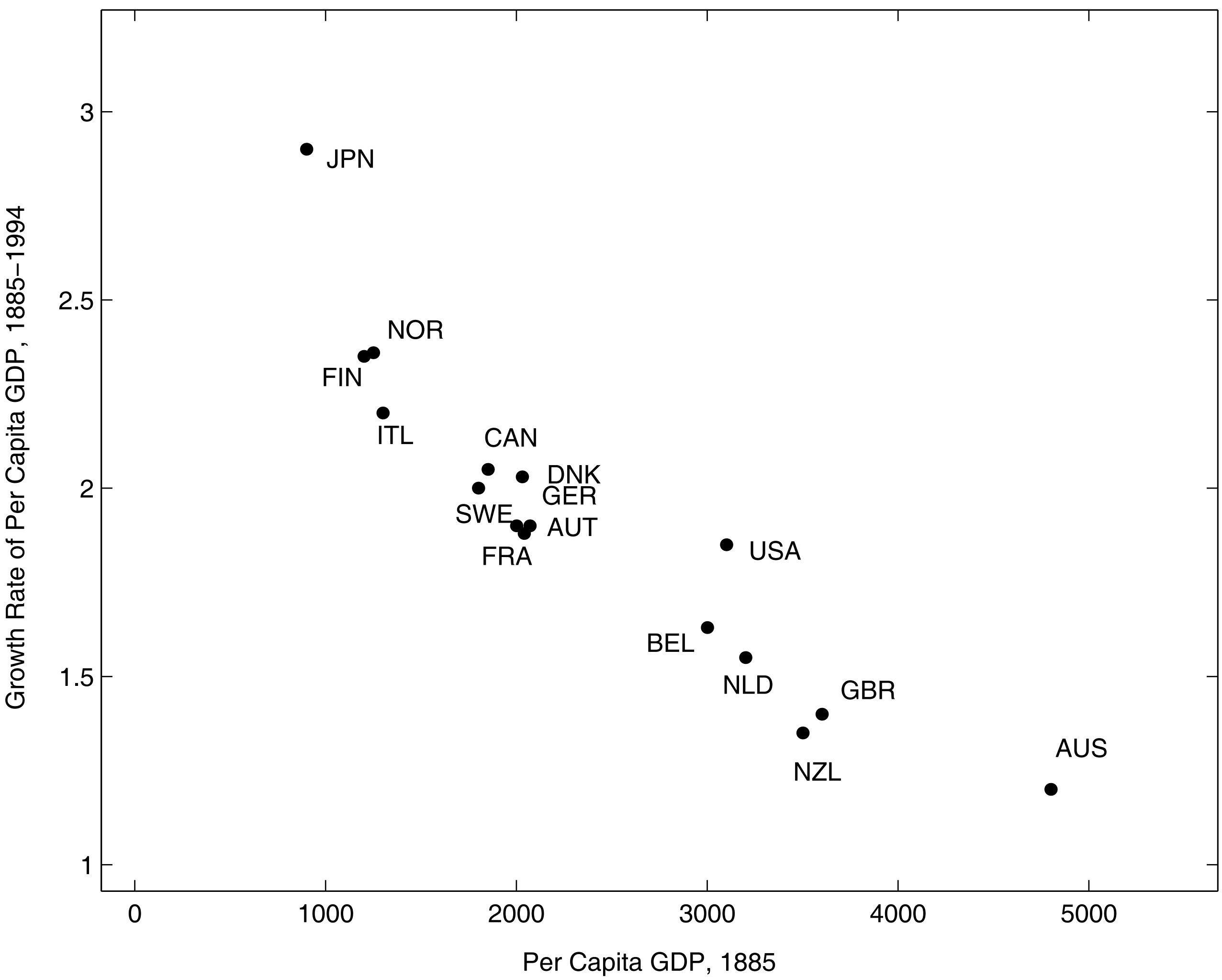


Figure 1.b: Growth Rate Versus Initial Per Capita GDP

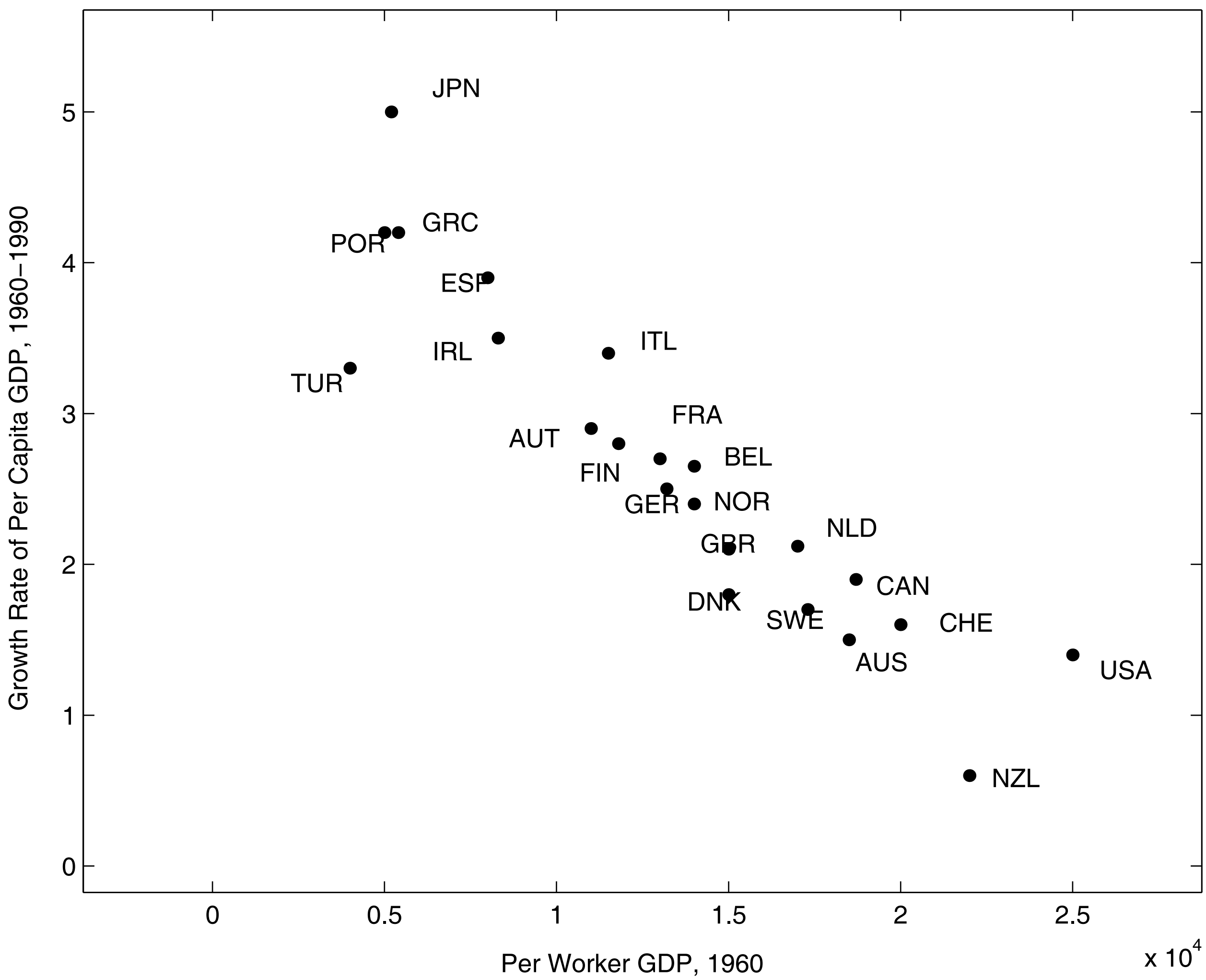


Figure 1.c: Growth Rate Versus Initial Per Capita GDP

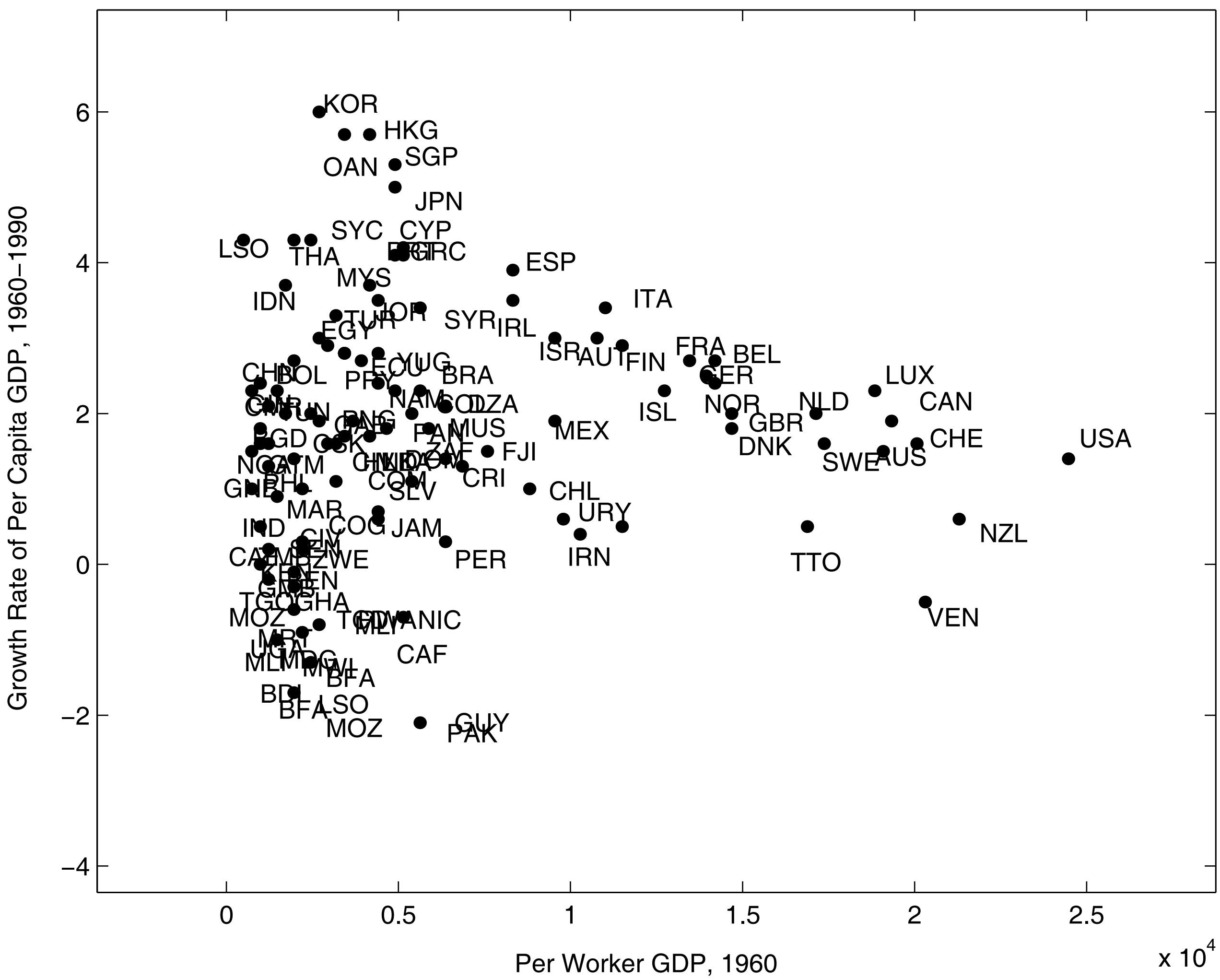
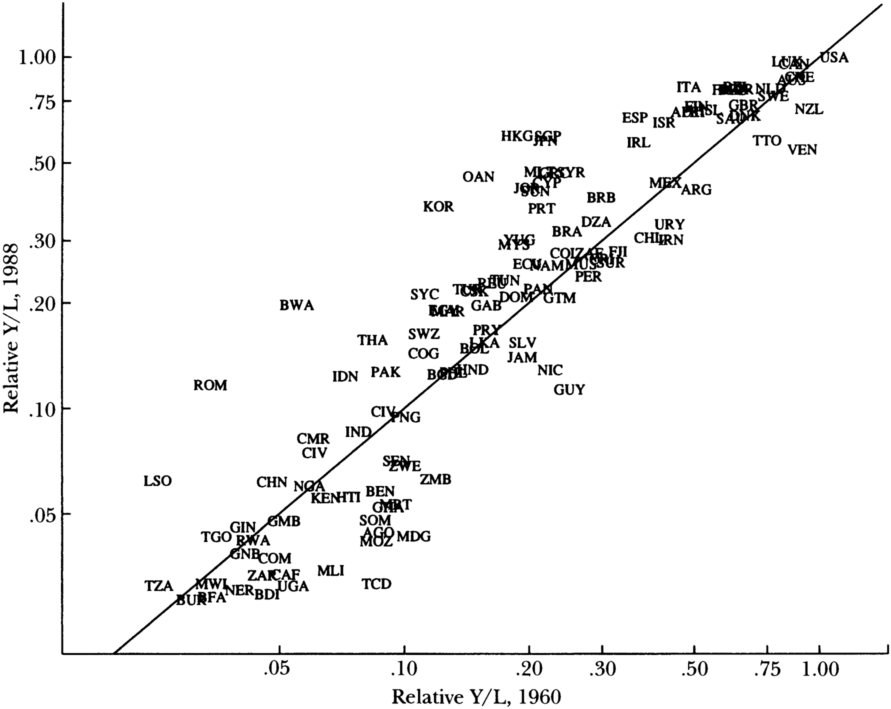


Figure 2

Relative Y/L, 1960 vs. 1988

(log scale)



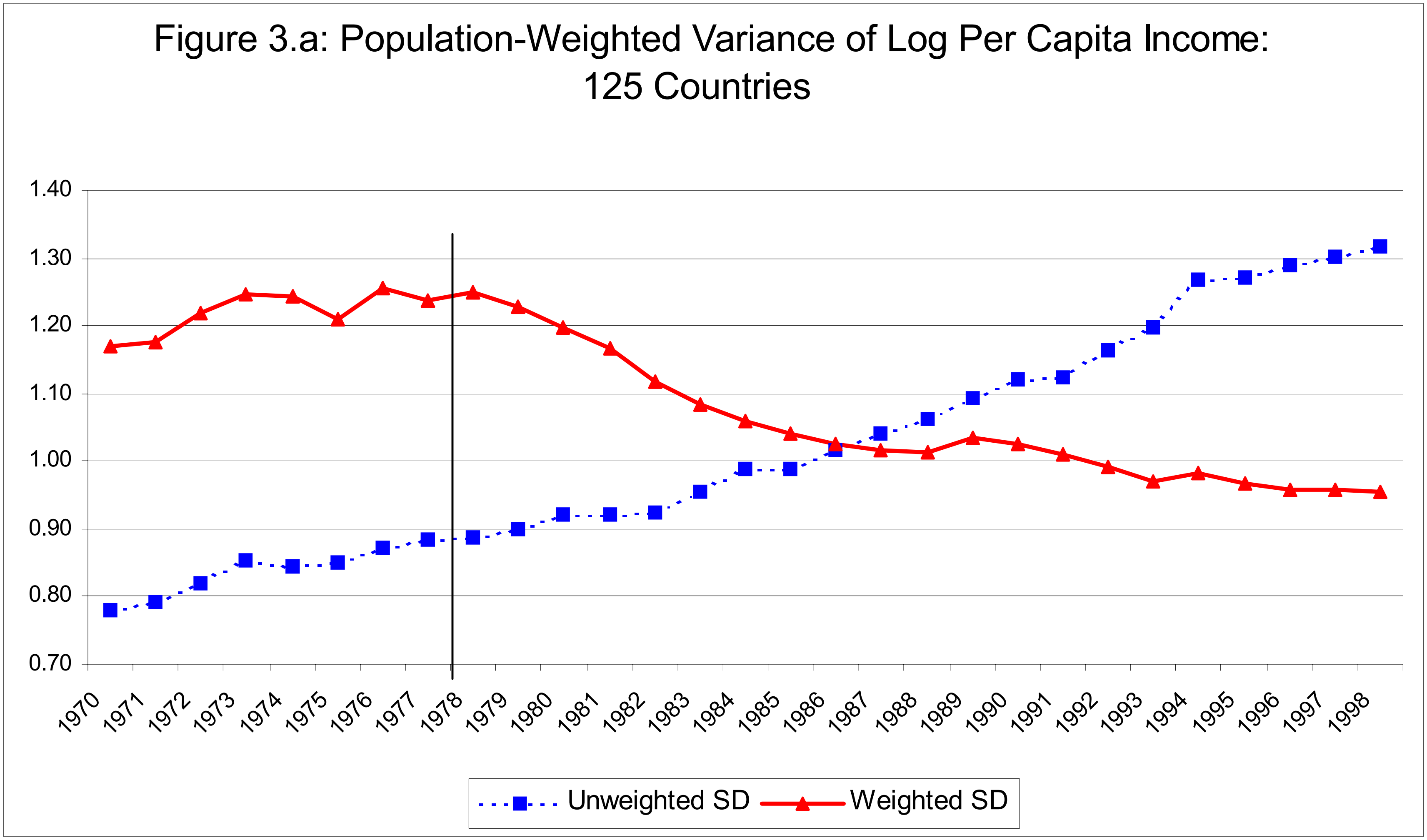


Figure 3.a: Population-Weighted Variance of Log Per Capita Income:
125 Countries
1.40
1.30
1.20
1.10
1.00
0.90
0.80
0.70
1970
1971
1972
1973
1974
1975
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986
1987
1988
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
Unweighted SD
Weighted SD

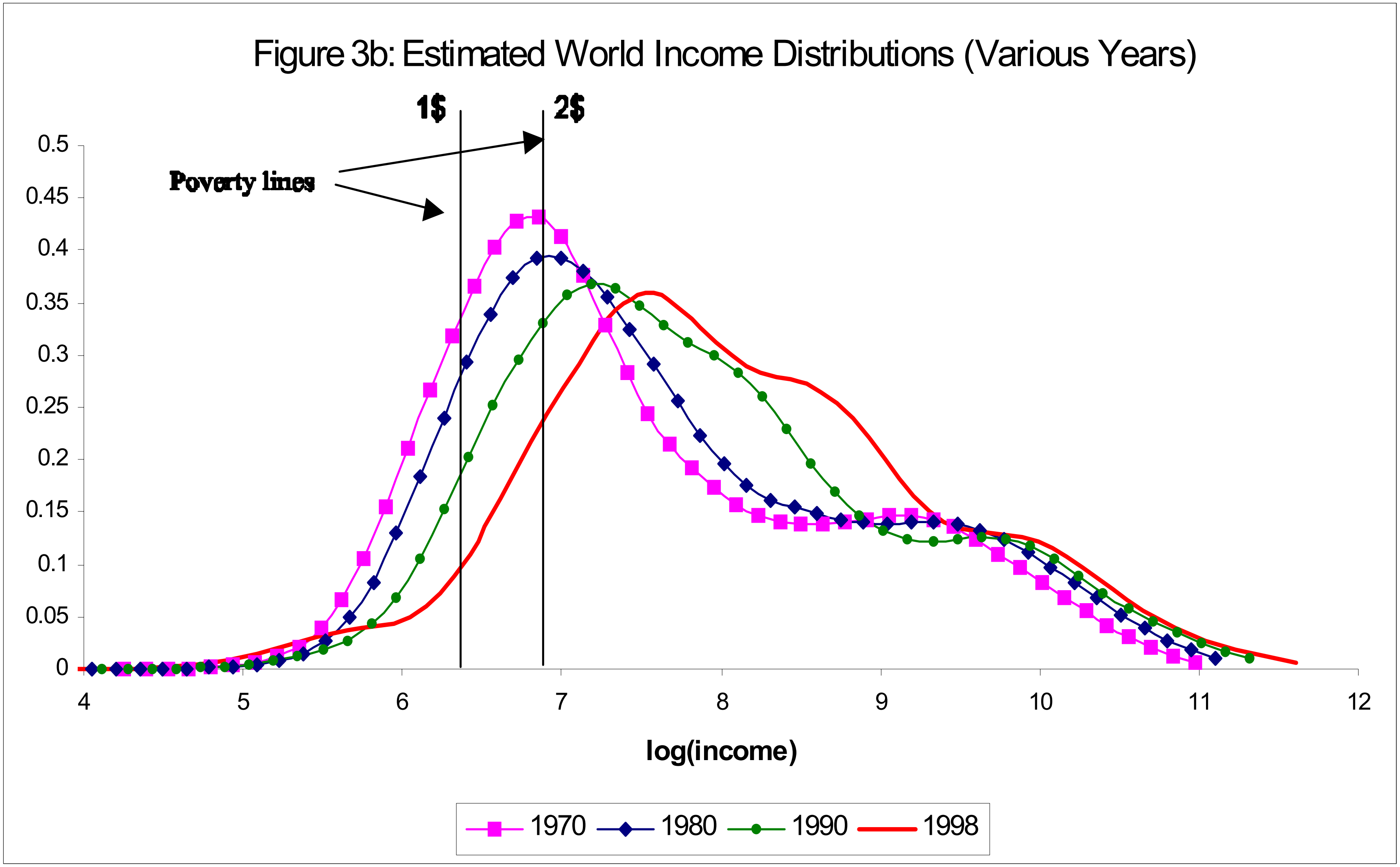


Figure 3b: Estimated World Income Distributions (Various Years)
1$
2$
Poverty lines
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
4
5
6
7
8
9
10
11
12
log(income)
1970
1980
1990
1998

Figure 3b1: Individual-Country and Global Distributions: 1970

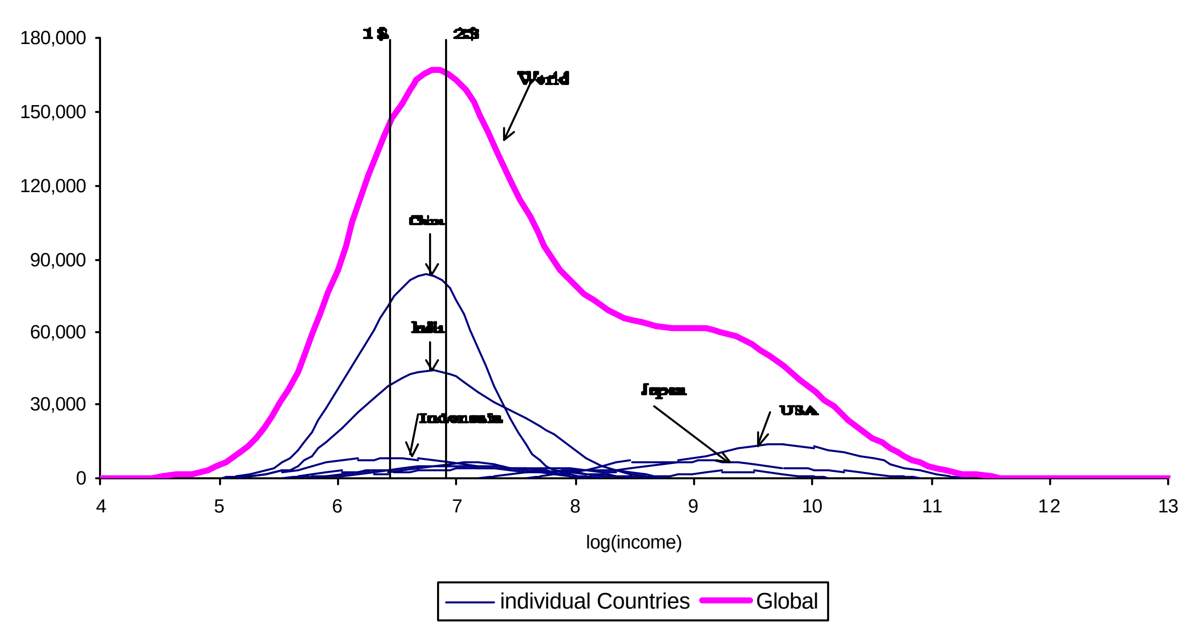


Figure 3b2: Individual-Country and Global Distributions: 1980

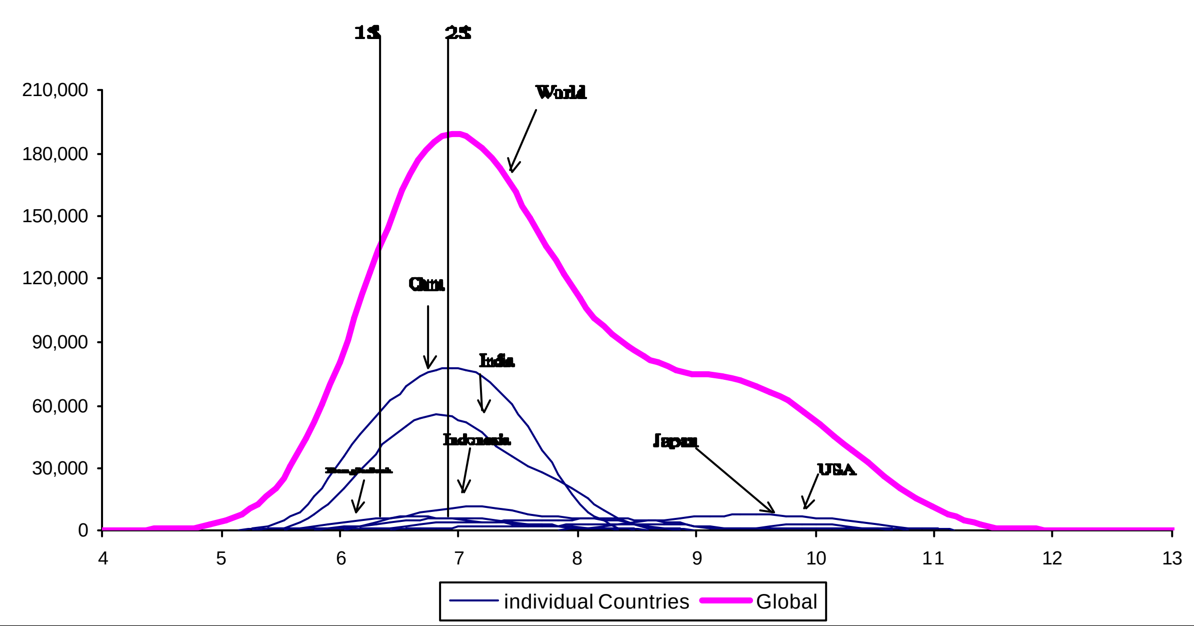


Figure 3b3: Individual-Country and Global Distributions: 1990

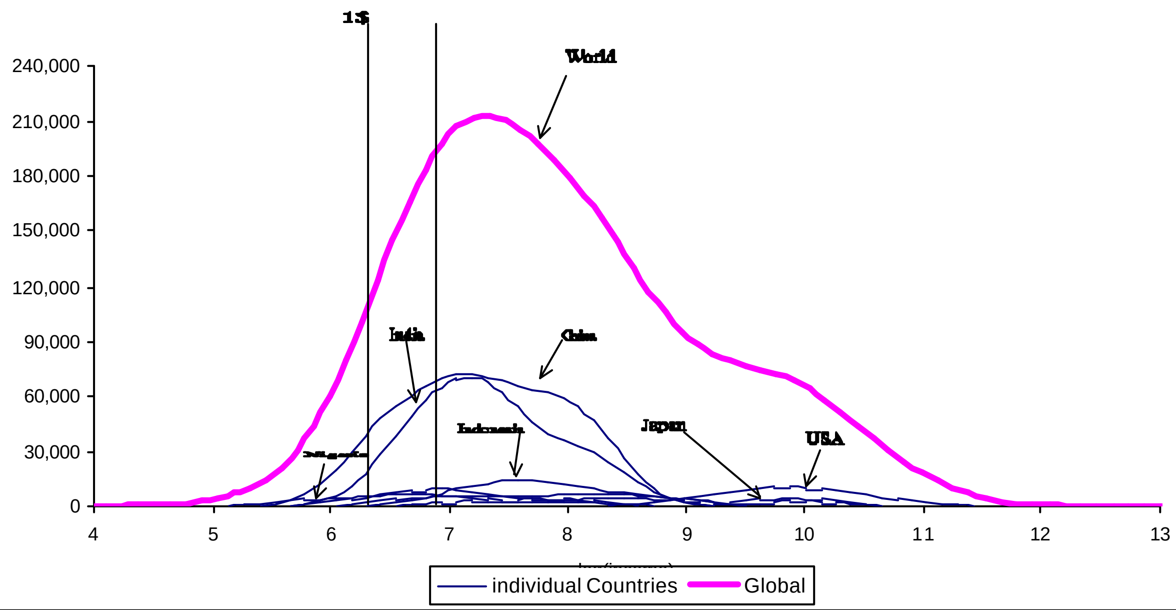


Figure 3b4: Individual-Country and Global Distributions: 1998

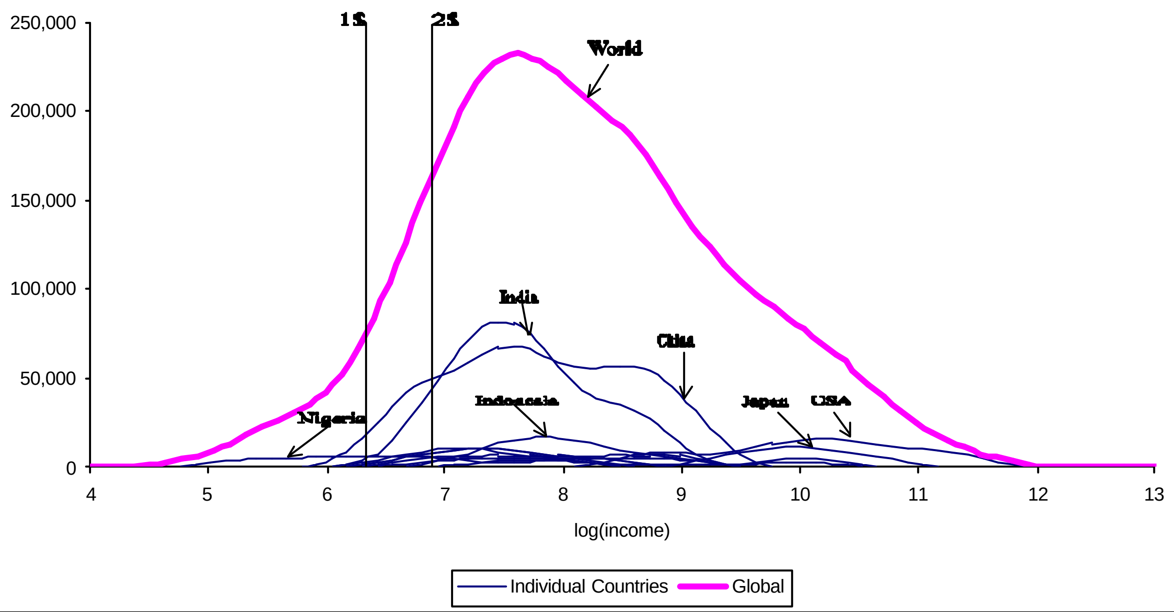


Figure 3b5: Income Distribution: China

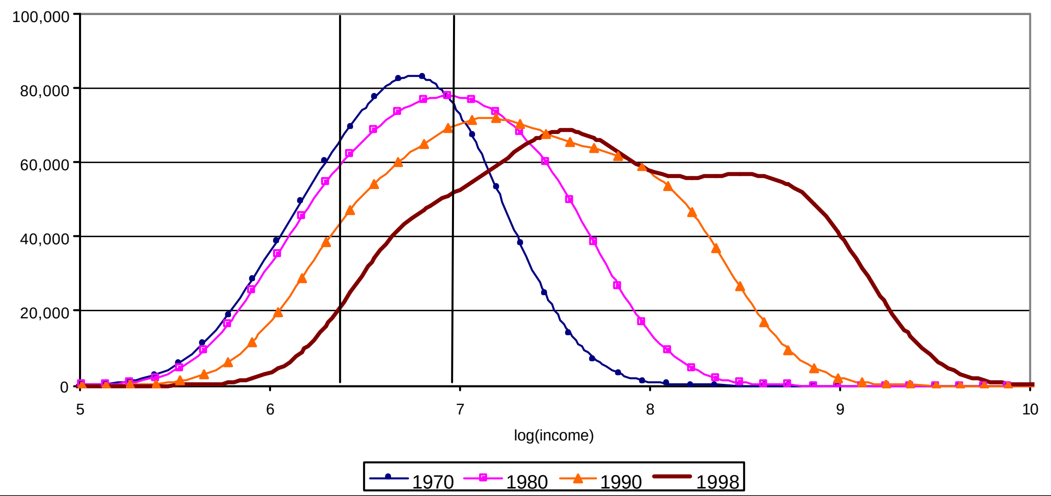


Figure 3b6: Income Distribution: India

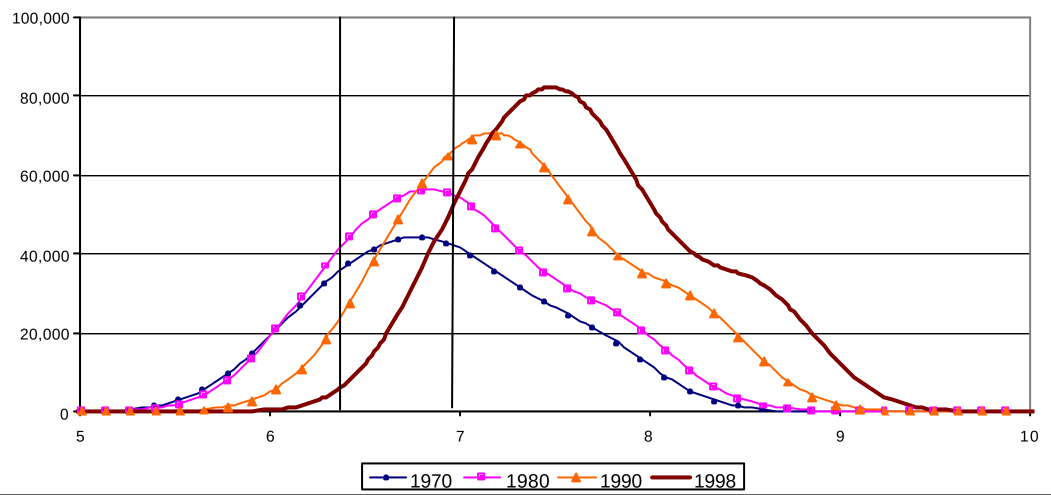
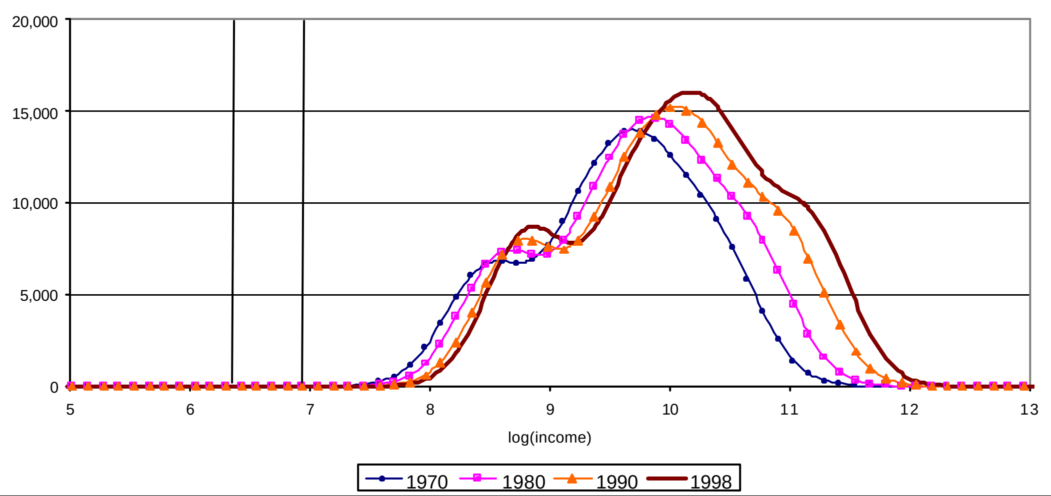


Figure 3b7: Income Distribution: USA



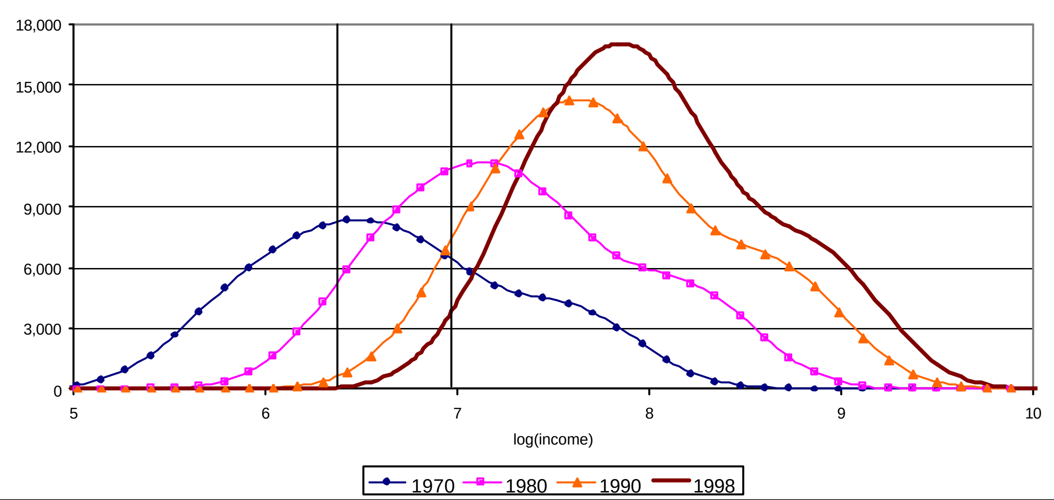


Figure 3b7: Income Distribution: Indonesia

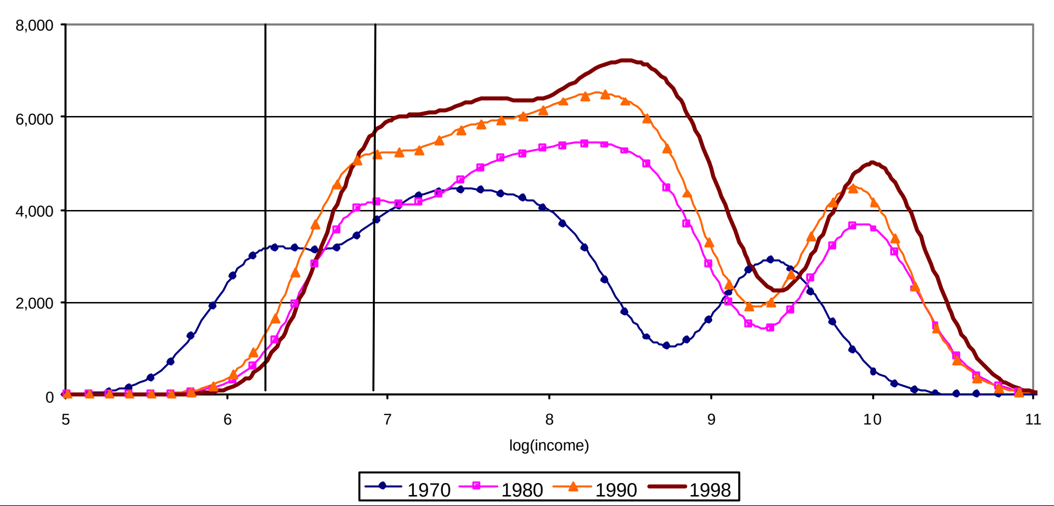


Figure 3b8: Income Distribution: Brazil

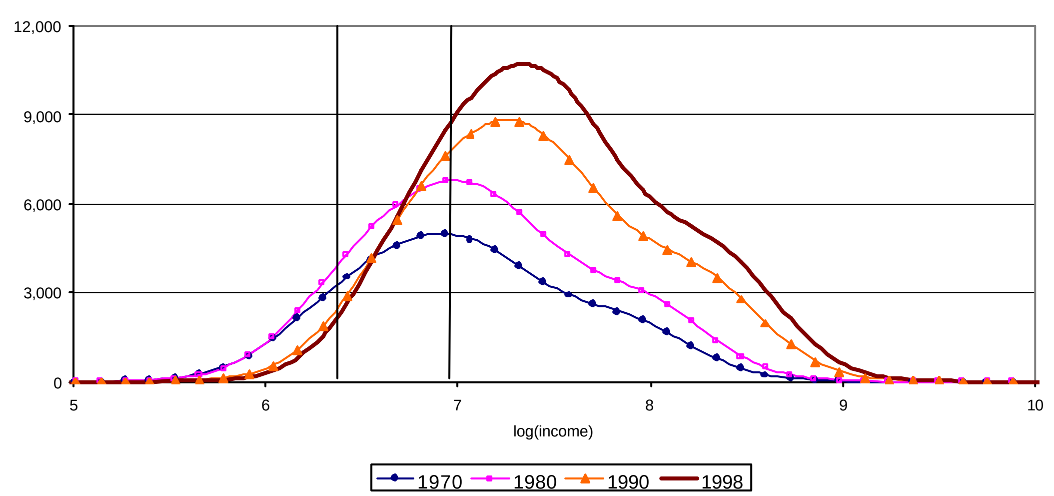


Figure 3b9: Income Distribution: Pakistan

Figure 3b10: Income Distribution: Japan

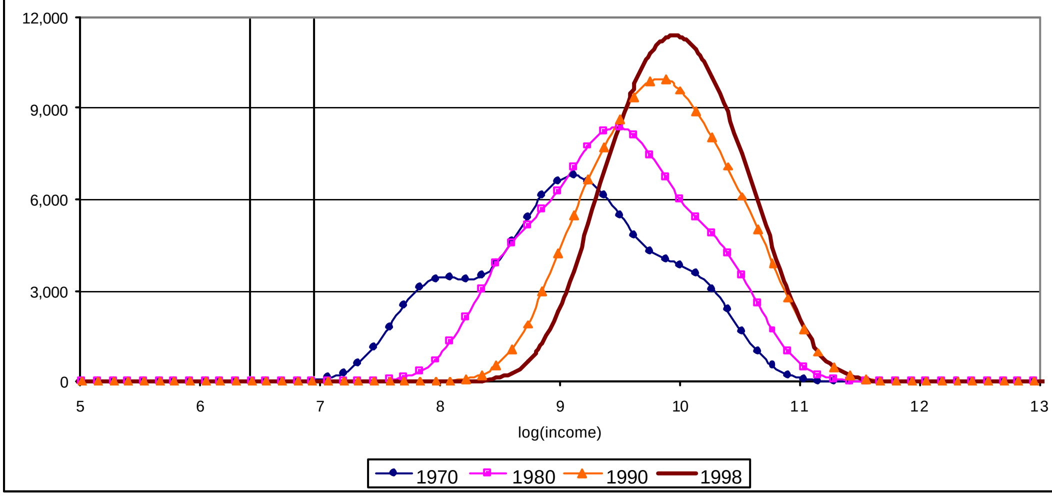


Figure 3b11: Income Distribution: Bangladesh

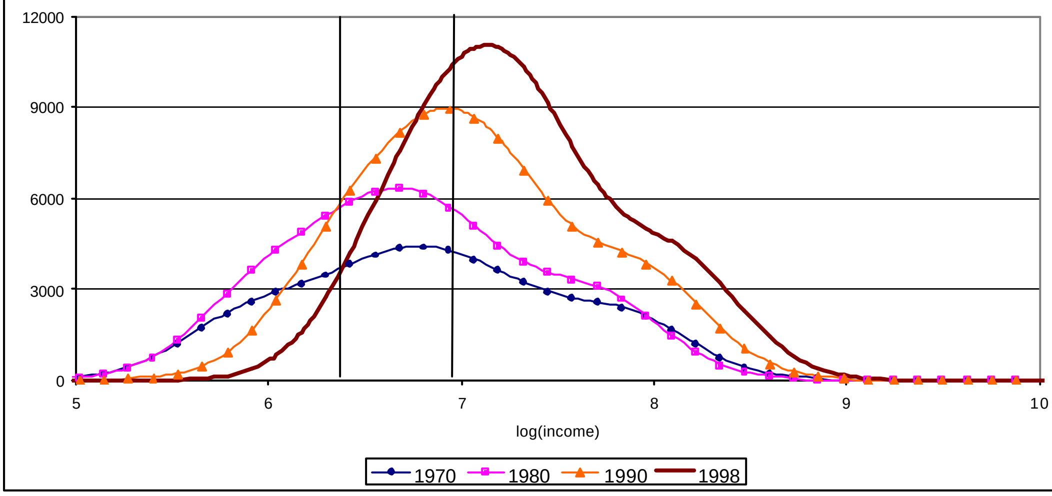
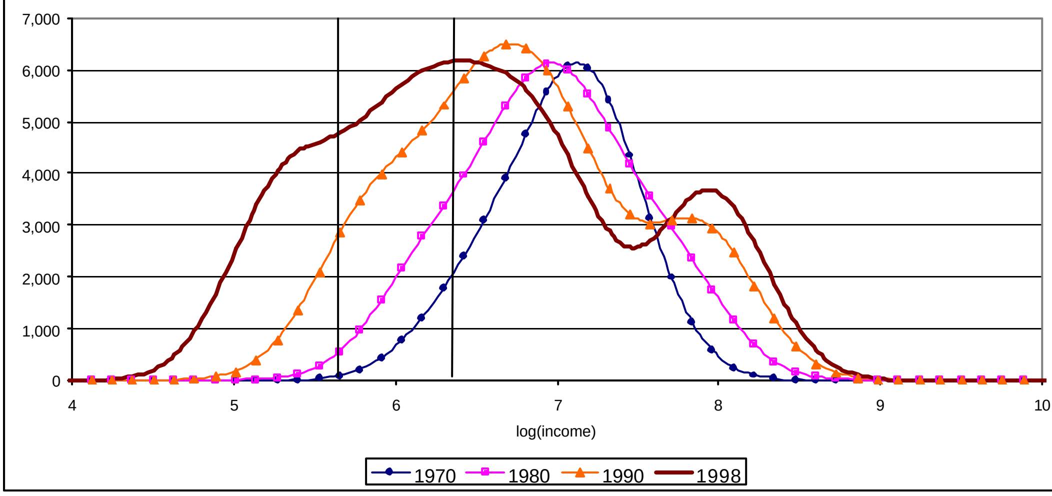


Figure 3b12: Income Distribution: Nigeria



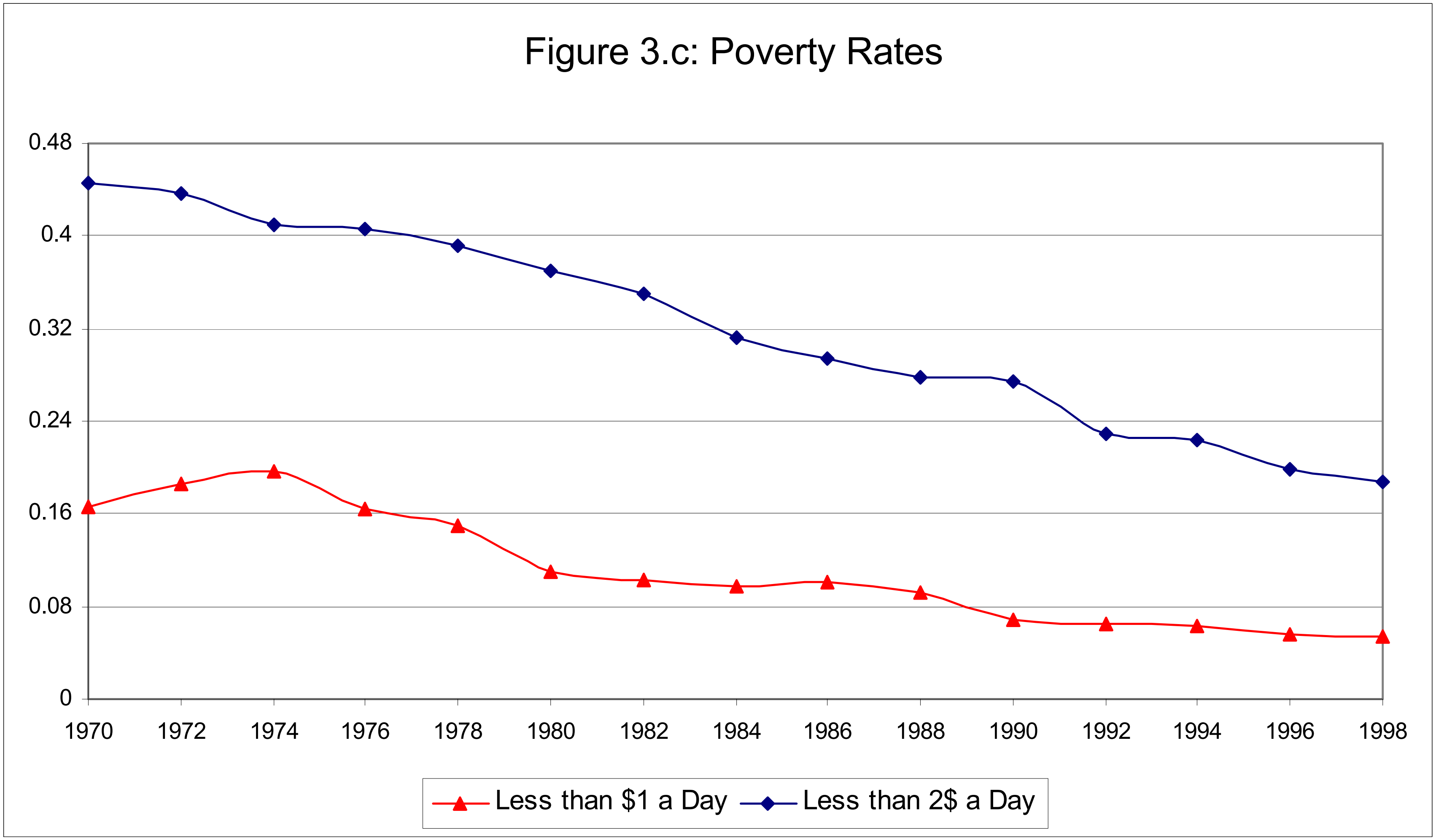


Figure 3.c: Poverty Rates
0.48
0.4
0.32
0.24
0.16
0.08
0
1970
1972
1974
1976
1978
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
Less than $1 a Day
Less than 2$ a Day

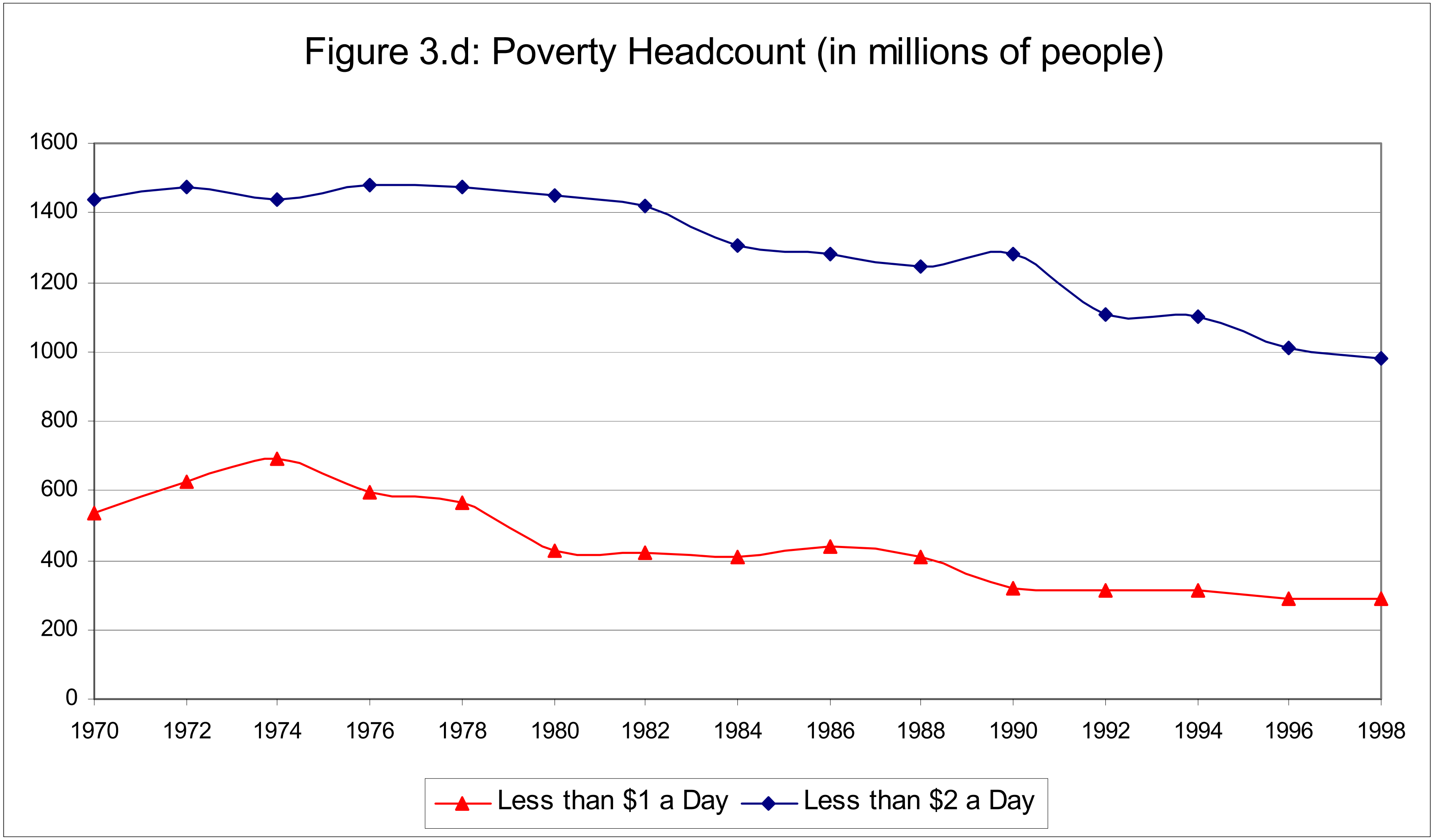


Figure 3.d: Poverty Headcount (in millions of people)
1600
1400
1200
1000
800
600
400
200
0
1970
1972
1974
1976
1978
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
Less than $1 a Day
Less than $2 a Day

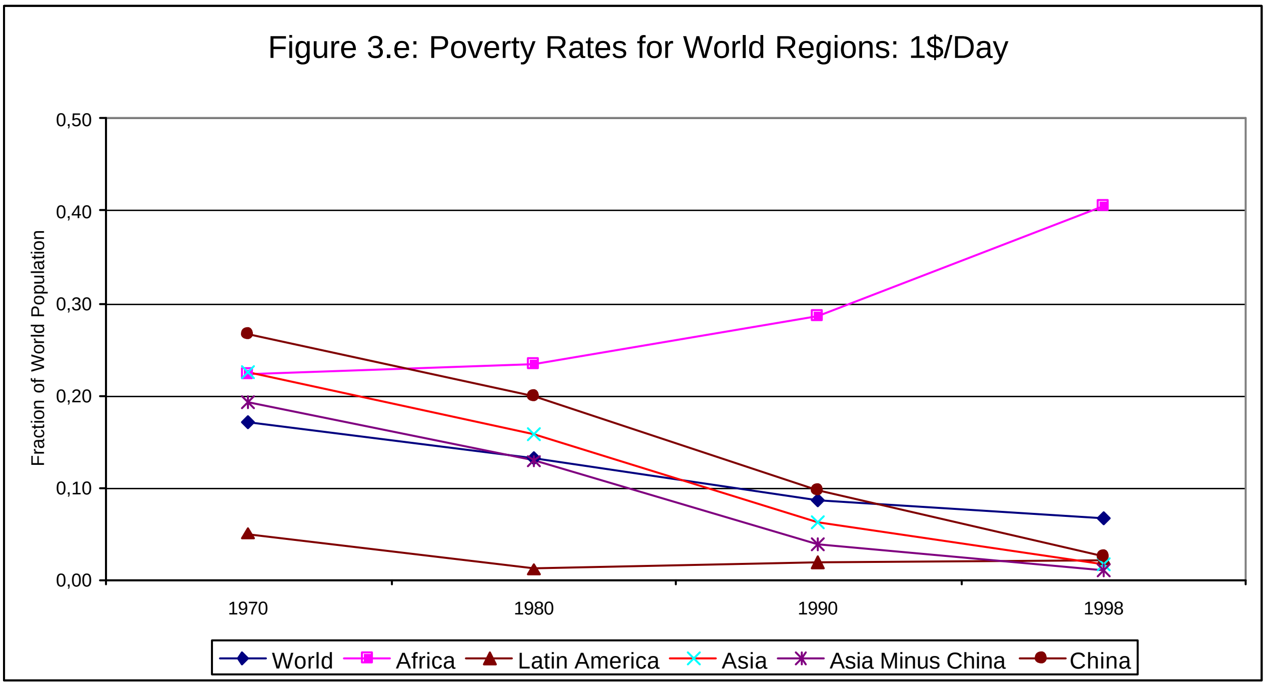


Figure 3.e: Poverty Rates for World Regions: 1$/Day
Fraction of World Population
0,50
0,40
0,30
0,20
0,10
0,00
1970
1980
1990
1998
World
Africa
Latin America
Asia
Asia Minus China
China

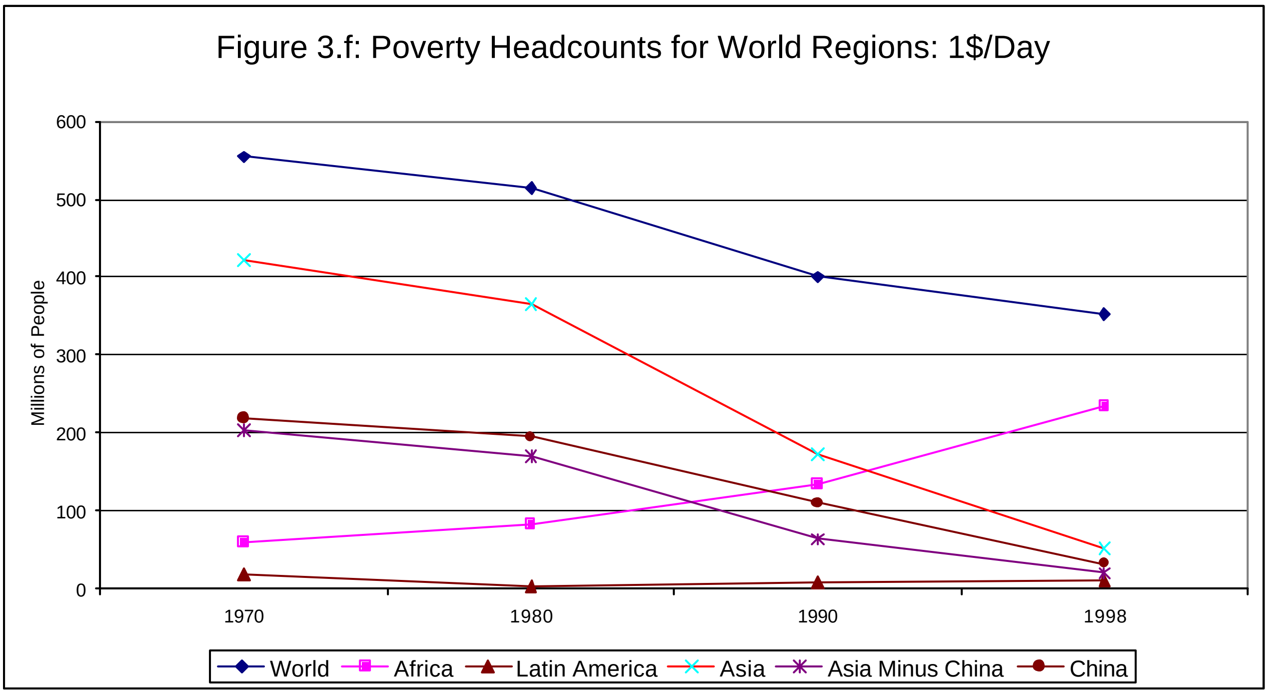


Figure 3.f: Poverty Headcounts for World Regions: 1$/Day
Millions of People
600
500
400
300
200
100
0
1970
1980
1990
1998
World
Africa
Latin America
Asia
Asia Minus China
China

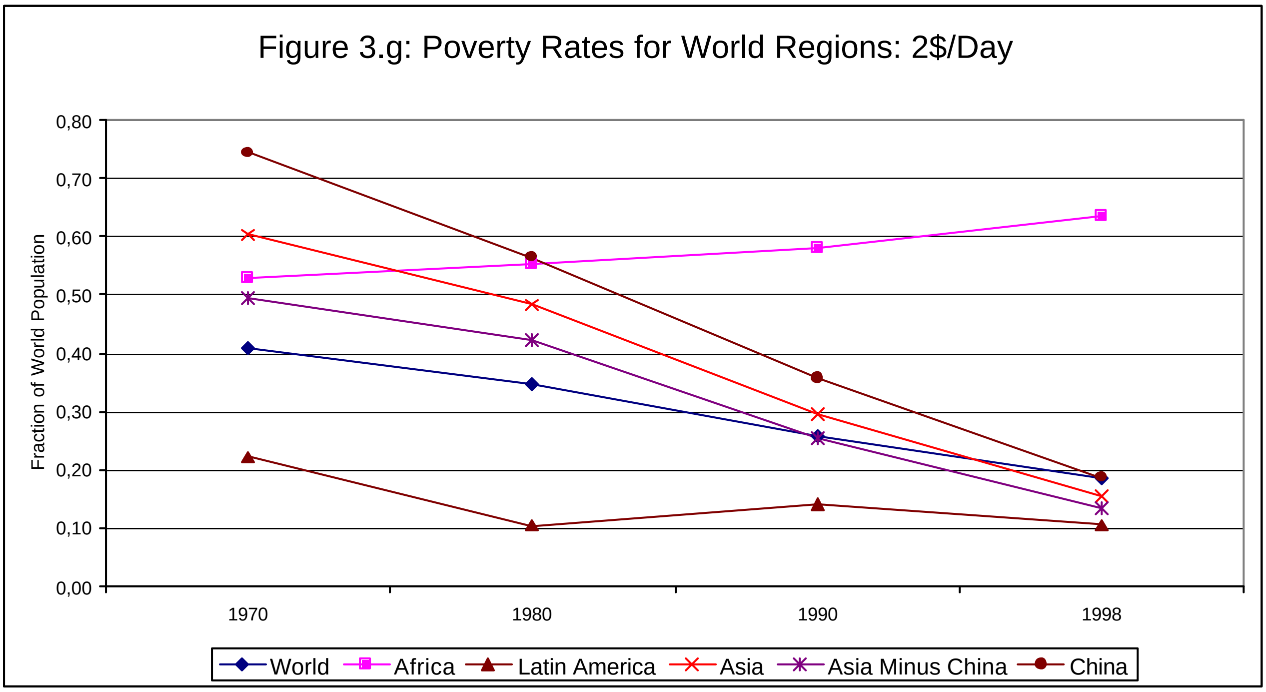


Figure 3.g: Poverty Rates for World Regions: 2$/Day
Fraction of World Population
0,80
0,70
0,60
0,50
0,40
0,30
0,20
0,10
0,00
1970
1980
1990
1998
World
Africa
Latin America
Asia
Asia Minus China
China

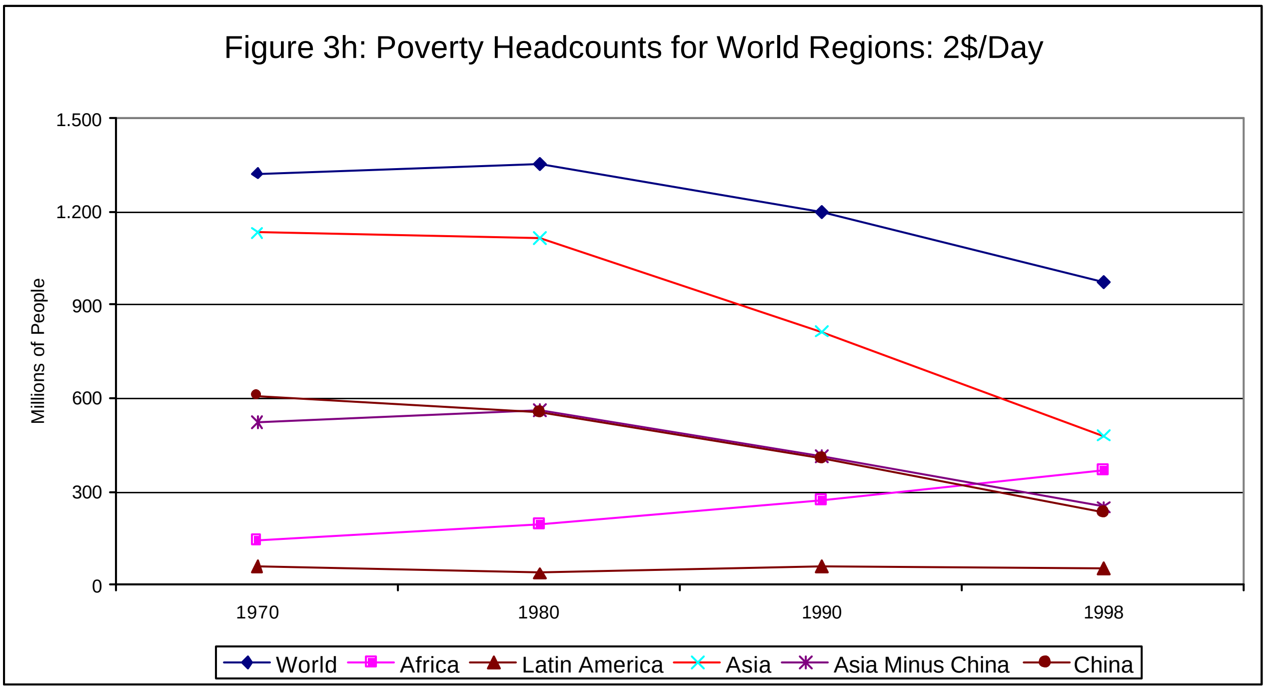


Figure 3h: Poverty Headcounts for World Regions: 2$/Day
Millions of People
1.500
1.200
900
600
300
0
1970
1980
1990
1998
World
Africa
Latin America
Asia
Asia Minus China
China

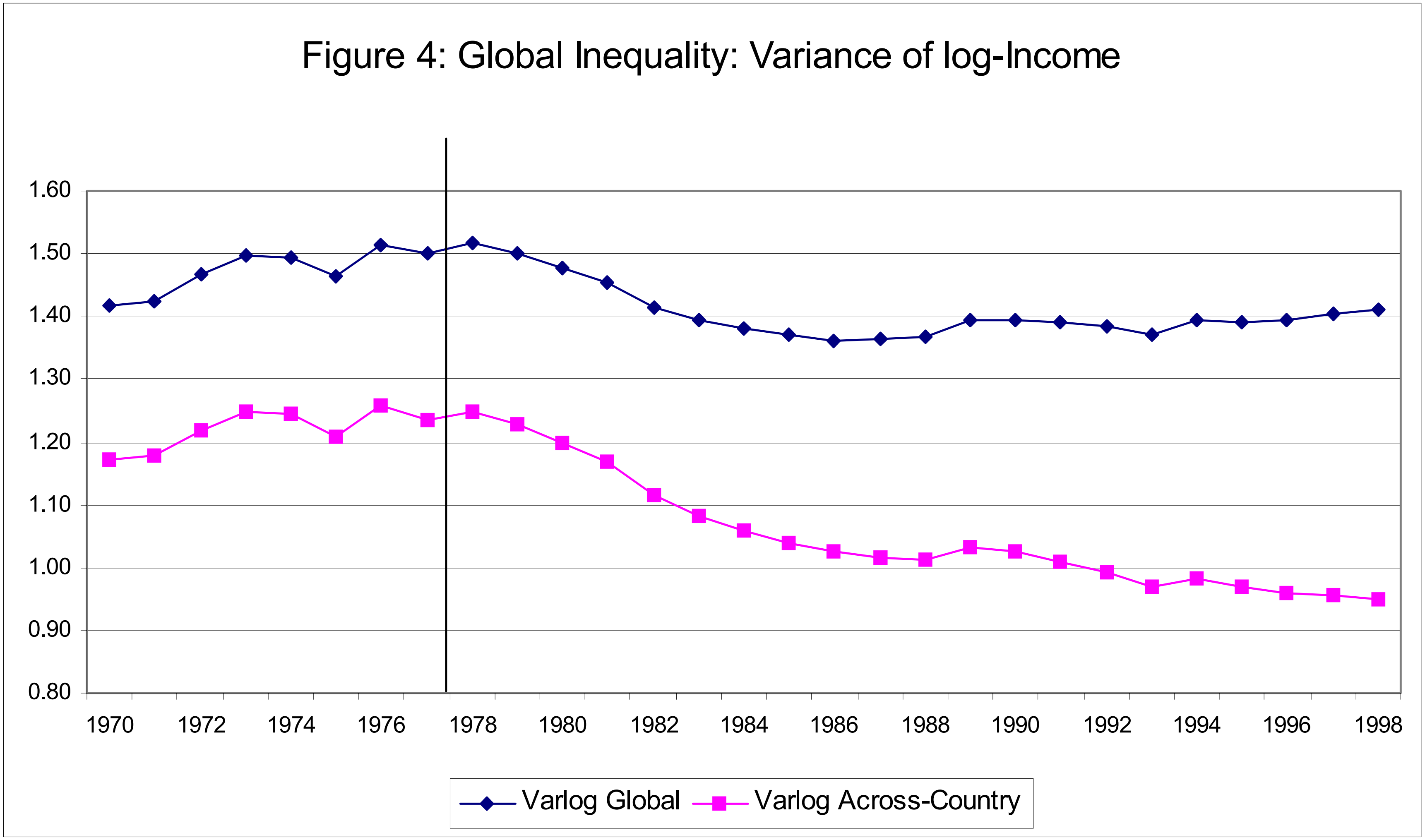


Figure 4: Global Inequality: Variance of log-Income
1.60
1.50
1.40
1.30
1.20
1.10
1.00
0.90
0.80
1970
1972
1974
1976
1978
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
Varlog Global
Varlog Across-Country

Figure 5

Steady State Incomes, Based on Current Policies

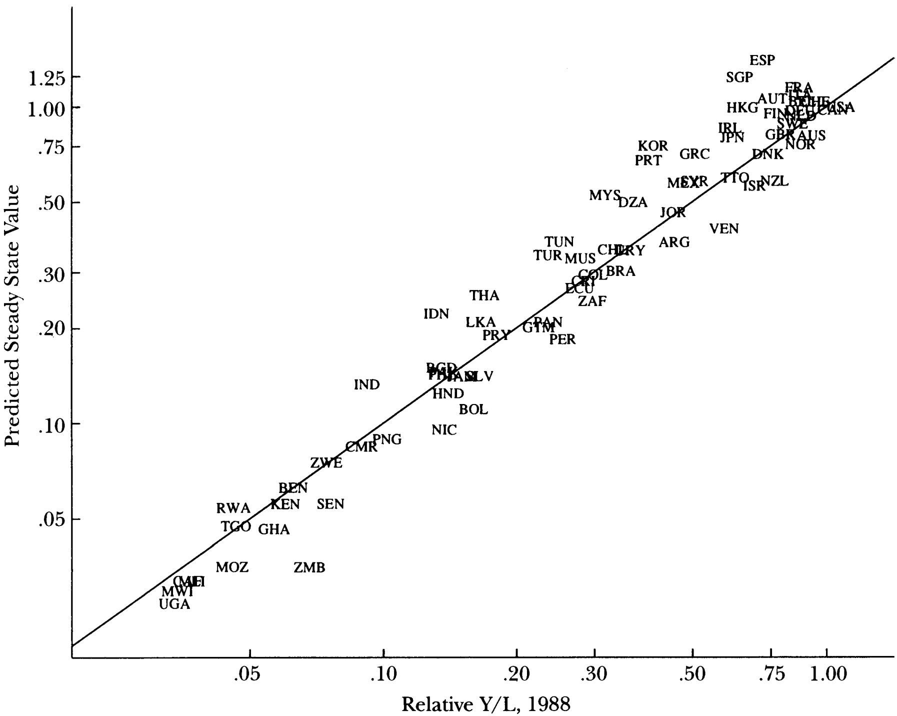


Figure 6

Steady States Implied by Transition Dynamics

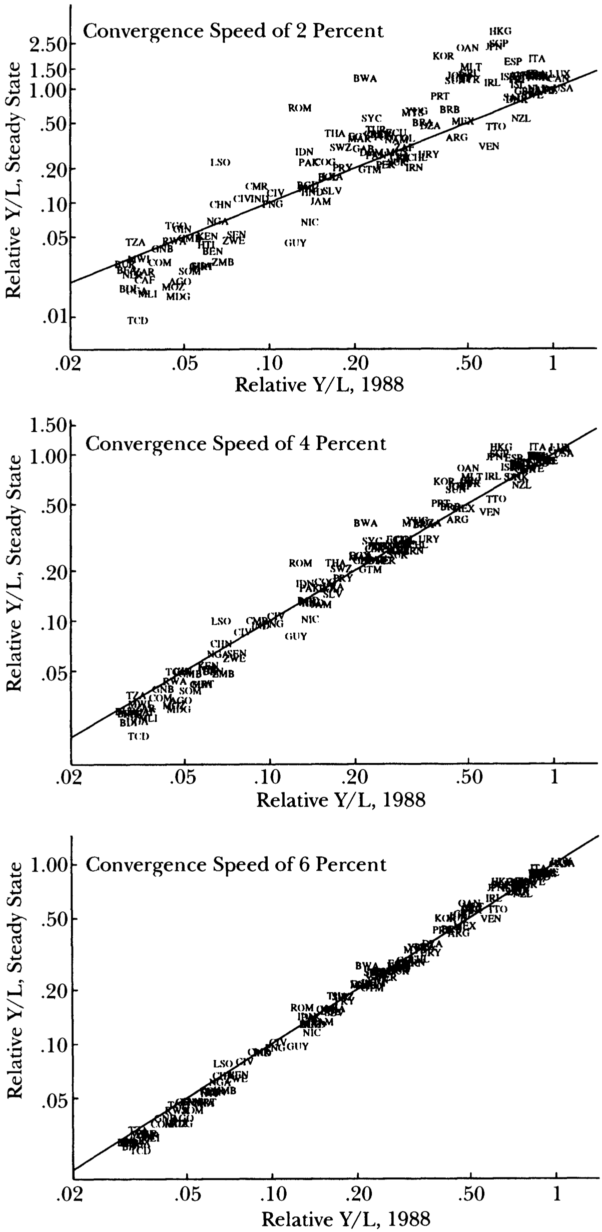


Table 1
Frequency of Growth Miracles and Growth Disasters

Interval	*Number of Countries*	*Fast Growth*	*Intermediate Growth*	*Slow Growth*
All Countries	(121)	40	45	15
$\tilde{y} \leq .05$	(18)	22	61	17
$.05 < \tilde{y} \leq .10$	(23)	22	35	43
$.10 < \tilde{y} \leq .20$	(31)	65	32	3
$.20 < \tilde{y} \leq .40$	(24)	42	50	8
$.40 < \tilde{y} \leq .80$	(21)	43	52	5
$\tilde{y} > .80$	(4)	0	75	25

Notes: Entries in the main part of the table reflect the percentage of countries in each interval exhibiting fast, intermediate and slow growth. Fast growth is defined to be one percentage point faster than U.S. growth (1.4 percent), and slow growth is defined to be one percentage point slower.

Table 2
World Income Distributions, Using Markov Transition Method

			Predicted		
Interval	*1960*	*1988*	*2010*	*2050*	*Long-Run*
$\tilde{y} \leq .05$	15	17	15	12	8
$.05 < \tilde{y} \leq .10$	19	13	13	11	8
$.10 < \tilde{y} \leq .20$	26	17	14	13	11
$.20 < \tilde{y} \leq .40$	20	22	23	23	24
$.40 < \tilde{y} \leq .80$	17	22	23	26	30
$\tilde{y} > .80$	3	9	12	15	19

Note. Entries in the table reflect the percentage of countries with relative incomes in each interval.

Endogenous Growth Theory

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New Growth Theory

- Remember from Solow Model:

$$g = \frac{\dot{A}}{A} = \frac{\dot{y}}{y}$$

  Growth depends on technological progress.

- Good thing: now we know where to look at.

- Challenge: we need a theory of technological progress.

- Paul Romer's big contribution to economics.

Ideas as Engine of Growth

- Technology: the way inputs to the production process are transformed into output.

- Without technological progress:

$$Y = K^{\alpha} L^{1-\alpha}$$

  With technological progress:

$$Y = K^{\alpha} (AL)^{1-\alpha}$$

- Technological progress due to new ideas: very different examples.

- Why (and under what circumstances) are resources are spent on the development of new ideas?

Ideas

- What is an idea?

- What are the basic characteristics of an idea?

  1. Ideas are *nonrivalrous* goods.

  2. Ideas are, at least partially, *excludable*.

Different Types of Goods

1. Rivalrous goods that are excludable: almost all private consumption goods, such as food, apparel, consumer durables fall into this group.

2. Rivalrous goods that have a low degree of excludability: tragedy of the commons.

3. Nonrivalrous goods that are excludable: most of what we call ideas falls under this point.

4. Nonrivalrous and nonexcludable goods: these goods are often called public goods.

Nonrivalrousness and Excludability of Ideas

- Nonrivalrousness: implies that cost of providing the good to one more consumer, the *marginal cost* of this good, is constant at zero. Production process for ideas is usually characterized by substantial fixed costs and low marginal costs. Think about software.

- Excludability: required so that firm can recover fixed costs of development. Existence of intellectual property rights like patent or copyright laws are crucial for the private development of new ideas.

Intellectual Property Rights and the Industrial Revolution

- Ideas engine of growth.

- Intellectual property rights needed for development of ideas.

- Sustained growth recent phenomenon.

- Coincides with establishment of intellectual property rights.

Data on Ideas

- Measure technological progress directly through ideas

- Measure ideas via measuring patents

- Measure ideas indirectly by measuring resources devoted to development of ideas

Important Facts from Data

- Number of patents issued has increased: in 1880 roughly 13,000 patents issued in the US, in 1999 150,000
- More and more patents issued in the US are issued to foreigners. The number of patents issued to US firms or individuals constant at 40,000 per year between 1915 and 1991.
- Number of researchers engaged in research and development (R&D) in the US increased from 200,000 in 1950 to 1,000,000 in 1990.
- Fraction of the labor force in R&D increased from 0.25% in 1950 to 0.75% in 1990.

Infrastructure or Institutions

- Question: why does investment rate s differ across countries?

- Answer: some countries have political institutions that make investing more profitable than others.

- Investment has costs and benefits: some countries invest more than others because either the costs of investment are lower or the benefits are higher.

Cost of Investment

- Cost of investment: resources to develop idea, purchase of buildings and equipment.
- Cost of obtaining all legal permissions.
- Hernando de Soto "The Other Path" (1989).
- Deficient or corrupt bureaucracy can impede profitable investment activities.

Benefits of Investment

1. The size of the market. Depends on openness of the economy

2. The extent to which the benefits from the investment accrue to the investor. Diversion of benefits due to high taxes, theft, corruption, the need to bribe government officials or the payment of protection fees to the Mafia or Mafia-like organizations.

3. Rapid changes in the economic environment in which firms and individuals operate: increase uncertainty of investors.

4. Data show that these considerations may be important

A Basic Model of Endogenous Growth

- Can we built a model that puts all this ideas together?

- Yes, Romer 1990. You can get a copy of the paper at the Class Web Page.

- A bit of work but we can deal with it.

Basic Set-up of the Model I

- Model of Research and Growth.

- Three sectors: final-goods sector, intermediate-goods sector and research sector.

- Those that invent a new product and those that sell it do not need to be the same: Holmes and Schmitz, Jr. (1990).

- Why? comparative advantage.

Basic Set-up of the Model II

- Total labor: L.

- Use for production of final goods, L_Y, or to undertake research, $L_A = L - L_Y$.

- Total capital: K.

- Used for production of intermediate goods.

Final-Goods Sector I

- Competitive producers.

- Production Function:

$$Y = L_Y^{1-\alpha} \int_0^A x(i)^\alpha \, di$$

- Optimization problem:

$$\Pi = L_Y^{1-\alpha} \int_0^A x(i)^\alpha \, di - w_Y L_Y - \int_0^A p(i)\, x(i)\, di$$

Final-Goods Sector II

- First Order Conditions:

$$w_Y = (1-\alpha) L_Y^{-\alpha} \int_0^A x(i)^\alpha \, di = (1-\alpha) \frac{Y}{L_Y}$$

$$p(i) = \alpha L_Y^{1-\alpha} x(i)^{\alpha-1} \text{ for } \forall \; i \in [0, A]$$

- Interpretation.

Intermediate-Goods Sector I

- Continuum of monopolist.
- Only use capital for production.
- Optimization problem:

$$\pi(i) = \max_{x(i)} p(i)\,x(i) - rx(i)$$

- Since $p(i) = \alpha L_Y^{1-\alpha} x(i)^{\alpha-1}$ we have:

$$\pi(i) = \max_{x(i)} \alpha L_Y^{1-\alpha} x(i)^{\alpha} - rx(i)$$

Intermediate-Goods Sector II

- First Order Conditions:

$$\alpha^2 L_Y^{1-\alpha} x\,(i)^{\alpha-1} = r \Rightarrow \alpha L_Y^{1-\alpha} x\,(i)^{\alpha-1} = \frac{1}{\alpha} r$$

$$p\,(i) = \frac{1}{\alpha} r$$

- Interpretation: mark-up of a monopolist.

Intermediate-Goods Sector III

- Total demand:

$$p(i) = \frac{1}{\alpha} r = \alpha L_Y^{1-\alpha} x(i)^{\alpha-1} \Rightarrow x(i) = \left(\frac{1}{\alpha^2} r\right)^{\frac{1}{\alpha-1}} L_Y$$

- The profit of the monopolist:

$$\pi(i) = \frac{1}{\alpha} r x(i) - r x(i) = \left(\frac{1}{\alpha} - 1\right) r x(i)$$

Aggregation I

- Solution of monopolist is independent of i:

$$x(i) = x \text{ and } \pi(i) = \pi \text{ for } \forall\ i \in [0, A]$$

- Then:

$$Y = L_Y^{1-\alpha} \int_0^A x(i)^\alpha \, di = L_Y^{1-\alpha} \int_0^A x^\alpha di = L_Y^{1-\alpha} x^\alpha \int_0^A di = A x^\alpha L_Y^{1-\alpha}$$

Aggregation II

- Since the total amount of capital in the economy is given:

$$\int_0^A x(i)\,di = K$$

- Then:

$$\int_0^A x(i)\,di = x\int_0^A di = Ax = K \Rightarrow x = \frac{K}{A}$$

- Plugging it back:

$$Y = Ax^{\alpha}L_Y^{1-\alpha} = A\left(\frac{K}{A}\right)^{\alpha}L_Y^{1-\alpha} = K^{\alpha}(AL_Y)^{1-\alpha}$$

Aggregation III

- Taking logs and derivatives:

$$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \alpha \frac{\dot{x}}{x} + (1-\alpha)\frac{\dot{L_Y}}{L_Y}$$

- Then, in a balance growth path, since $\frac{\dot{L_Y}}{L_Y} = 0$ and $x = \left(\frac{1}{\alpha^2} r\right)^{\frac{1}{\alpha-1}} L_Y$ are constant:

$$g = \frac{\dot{Y}}{Y} = \frac{\dot{A}}{A}$$

Research Sector I

- Production function for ideas:

$$\dot{A} = BAL_A$$

- Then:

$$g = \frac{\dot{A}}{A} = BL_A$$

Research Sector II

- P_A is the price of the new design A.
- Arbitrage idea.
- By arbitrage:

$$rP_A = \pi + \dot{P}_A$$

Research Sector III

- Then $r = \frac{\pi}{P_A} + \frac{\dot{P_A}}{P_A}$

- In a BGP, r and $\frac{\dot{P_A}}{P_A}$ are constant and then also $\frac{\pi}{P_A}$. But since π is also constant:

$$P_A \text{ is constant} \Rightarrow \frac{\dot{P_A}}{P_A} = 0$$

- And:

$$r = \frac{\pi}{P_A} = \frac{\left(\frac{1}{\alpha} - 1\right) rx}{P_A} \Rightarrow P_A = \left(\frac{1}{\alpha} - 1\right) x$$

Research Sector IV

- Each unit of labor in the research sector then gets:

$$w_R = BAP_A = BA\left(\frac{1}{\alpha} - 1\right)x$$

- Remember that the wage in the final-goods sector was:

$$w_Y = (1-\alpha)\frac{Y}{L_Y}$$

- By free entry into the research sector both wages must be equal:

$$w = w_R = w_Y$$

Research Sector V

- Then:

$$BA\left(\frac{1}{\alpha}-1\right)x=(1-\alpha)\frac{Y}{L_Y}$$

- and with some algebra:

$$\frac{BA}{\alpha}=\frac{Y}{xL_Y}=\frac{Ax^{\alpha}L_Y^{1-\alpha}}{xL_Y}=Ax^{\alpha-1}L_Y^{-\alpha}\Rightarrow\frac{B}{\alpha}=x^{\alpha-1}L_Y^{-\alpha}$$

Balanced Growth Path I

- As in the Solow's model:

$$\dot{K} = sY - \delta K = sK^{\alpha} (AL_Y)^{1-\alpha} - \delta K$$

- Dividing by K:

$$g = \frac{\dot{K}}{K} = s\left(\frac{K}{A}\right)^{\alpha-1} L_{Y}^{1-\alpha} - \delta = sx^{\alpha-1} L_{Y}^{1-\alpha} - \delta$$

Balanced Growth Path II

- Then:

$$g = BL_A = B(L - L_Y) = s\frac{B}{\alpha}L_Y - \delta \Rightarrow$$
$$BL + \delta = \left(1 + \frac{s}{\alpha}\right) BL_Y \Rightarrow$$
$$L_Y = \frac{BL + \delta}{\left(1 + \frac{s}{\alpha}\right) B} = \frac{1}{1 + \frac{s}{\alpha}}L + \frac{\delta}{\left(1 + \frac{s}{\alpha}\right) B}$$

- And we can compute all the remaining variables in the model.

Is the Level of R&D Optimal?

- Sources of inefficiency:

  1. Monopoly power.
  2. Externalities

- Possible remedies.

- Implications for Antitrust policy.

The Very Long Run

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The Very Long Run

- Economist want to understand the growth experience of ALL human history (*Big History Movement*).

- What are the big puzzles:

  1. Why are there so big differences in income today?

  2. Why did the West develop first? or Why not China? or India?

- Data Considerations.

Where Does Data Come From?

- Statistics: Customs, Tax Collection, Census, Parish Records.

- Archeological Remains: Farms, Skeletons.

- Literary Sources: Memories, Diaries, Travel Books.

Some Basic Facts I

- For most human history, income per capita growth was glacially slow.

- Before 1500 little or no economic growth.

  Paul Bairoch (*Economics and World History: Myths and Paradoxes*)

  1. Living standards were roughly equivalent in Rome (1st century A.D.), Arab Caliphates (10th Century), China (11th Century), India (17th century), Western Europe (early 18th century).

  2. Cross-sectional differences in income were a factor of 1.5 or 2.

Some Basic Facts II

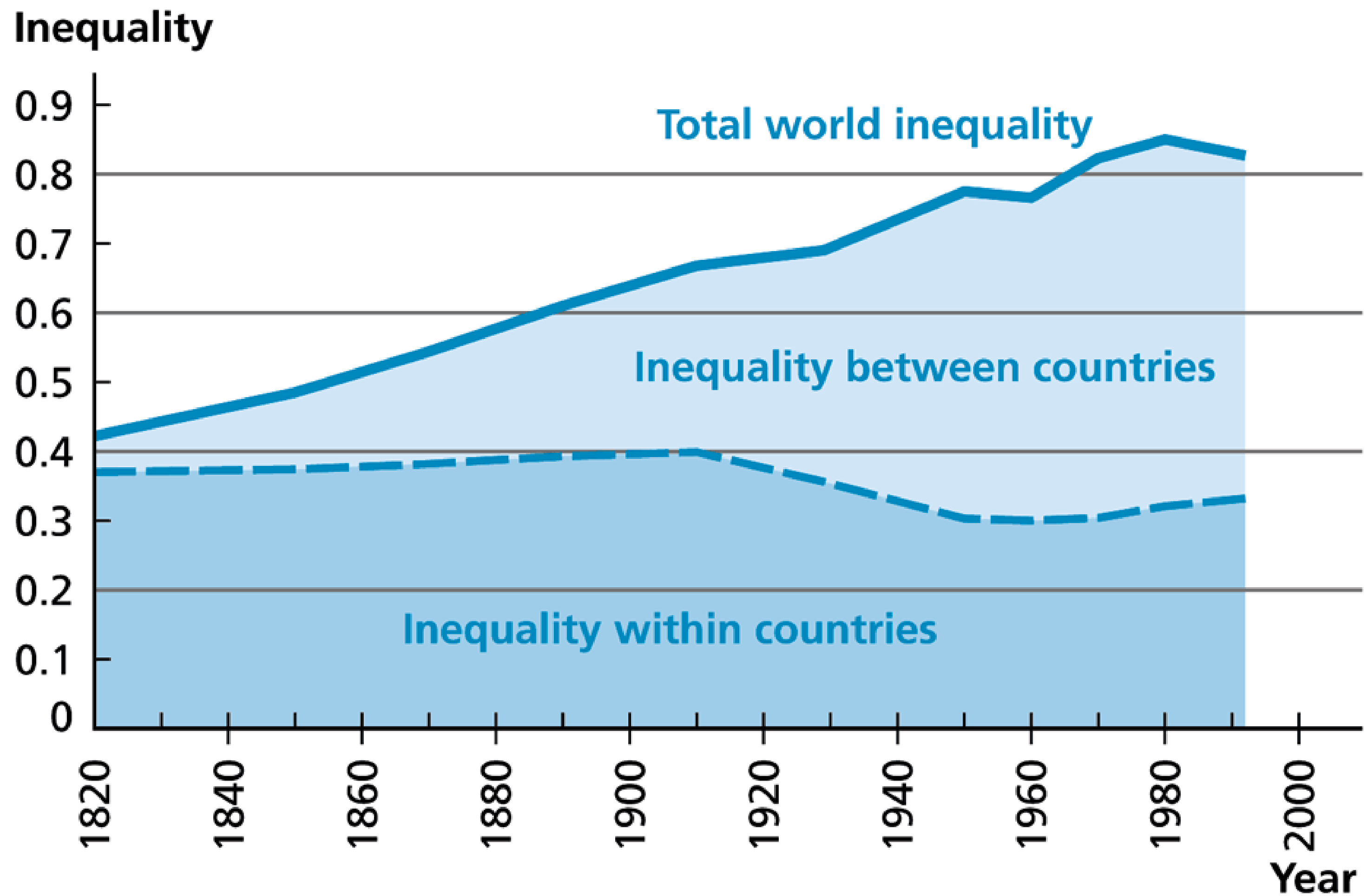
- Angus Maddison (*The World Economy: A Millenial Perspective*) calculates 1500-1820 growth rates:

    1. World GDP per capita: 0.05%.

    2. Europe GDP per capita: 0.14%.

- After 1820: great divergence in income per capita.

FIGURE 1.8
World Inequality and Its Components, 1820–1992



Source: Bourguignon and Morrison (2002).

Europe Becomes Dominant

- From 1492 to 1770, different human populations come into contact. European countries expand until early 20th century:

  1. American and Australia: previous cultures were nearly wiped out.

  2. Asia: partial control.

  3. Africa: somehow in the middle.

- Proximate causes: weaponry and social organization of Europeans was more complex.

- Ultimate causes: why?

Possible Explanations

- Geography.
- Colonies.
- Culture.

Geography

- How can Geography be important?

- Examples:

  1. Europe is 1/8 of the size of Africa but coastline is 50% longer.

  2. Wheat versus Rice, Braudel (*The Structures of Everyday Life: Civilization and Capitalism, 15th-18th Century*).

- Let's look at a map.

Figure 9.
Populations remote from coastline or major navigable river

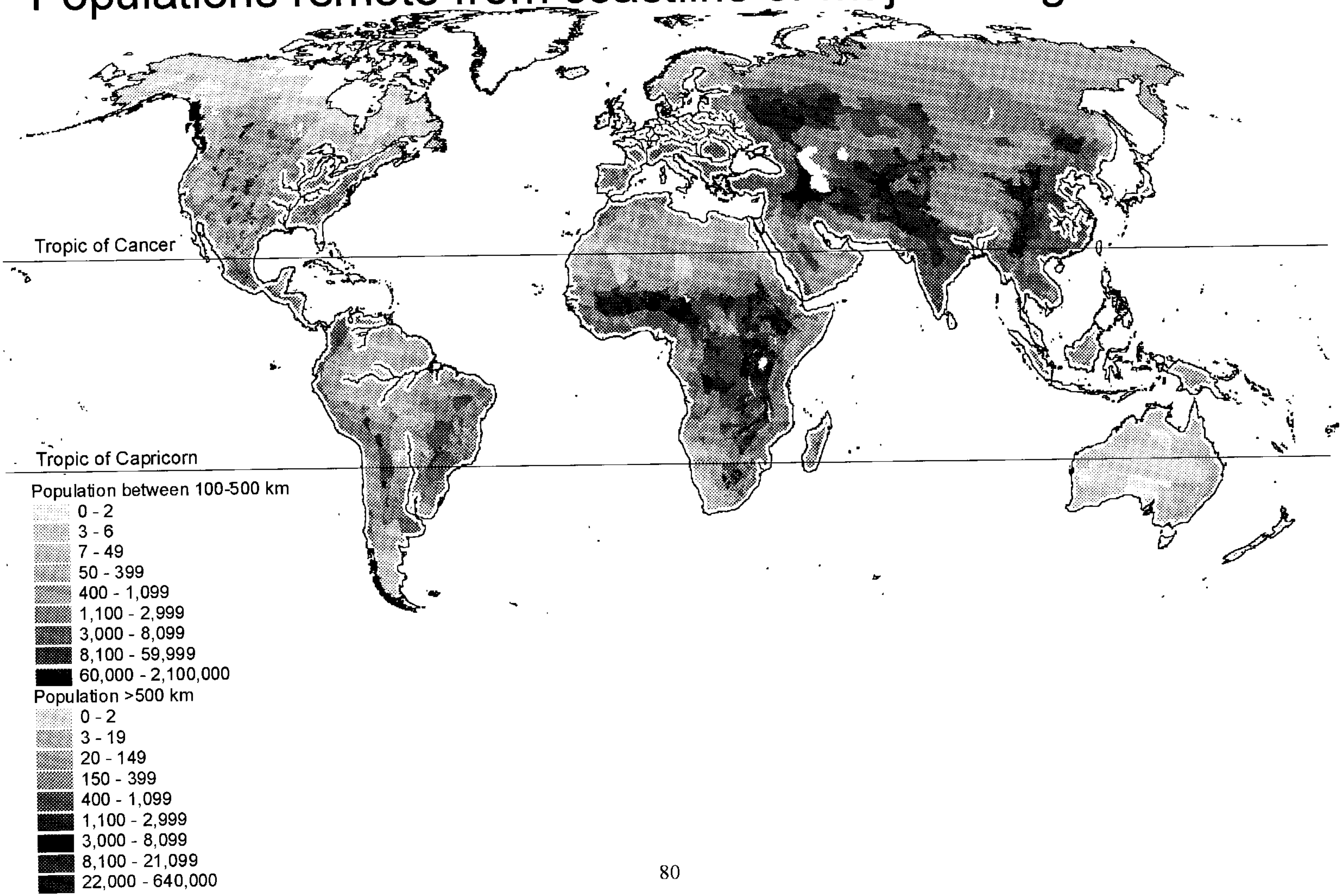
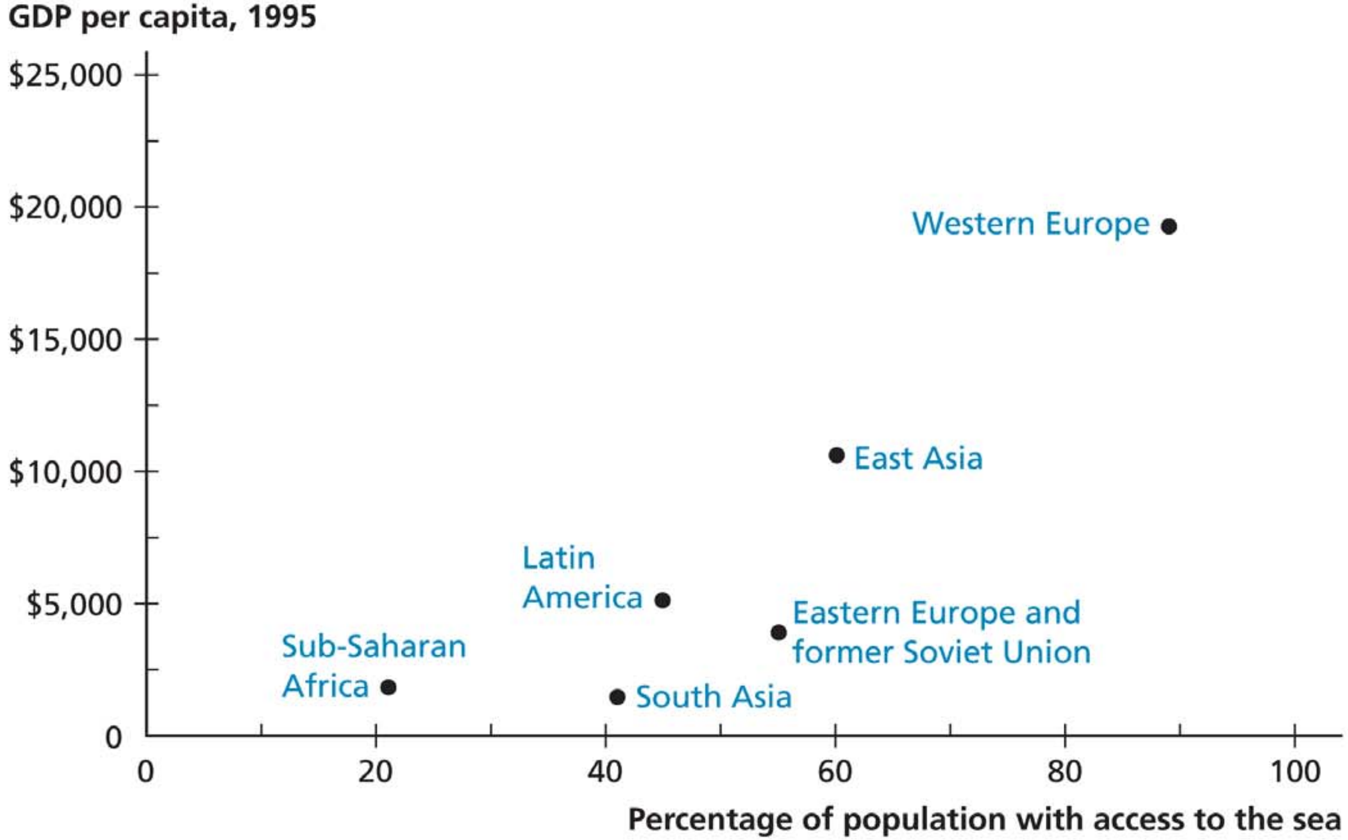


FIGURE 15.2

Regional Variation in Income and Access to the Sea



Source: Gallup, Sachs, and Mellinger (1998).

Jared Diamond (*Guns, Germs, and Steel*): geography.

- Euroasia is bigger (50% than America, 250% as Sub-saharian Africa, 800% than Australia):

  1. More plants i.e. out of 56 food grains, 39 are native to Euroasia, 11 to America, 4 to Sub-saharian Africa, and 2 to Australia.

  2. More animals to domesticate: cows/pigs/horses/sheeps/goats versus llamas and alpacas.

- Euroasia is horizontal: transmission of technology, plants, and animals.

- Consequence: higher population density$\rightarrow$guns and germs.

Why Not China?

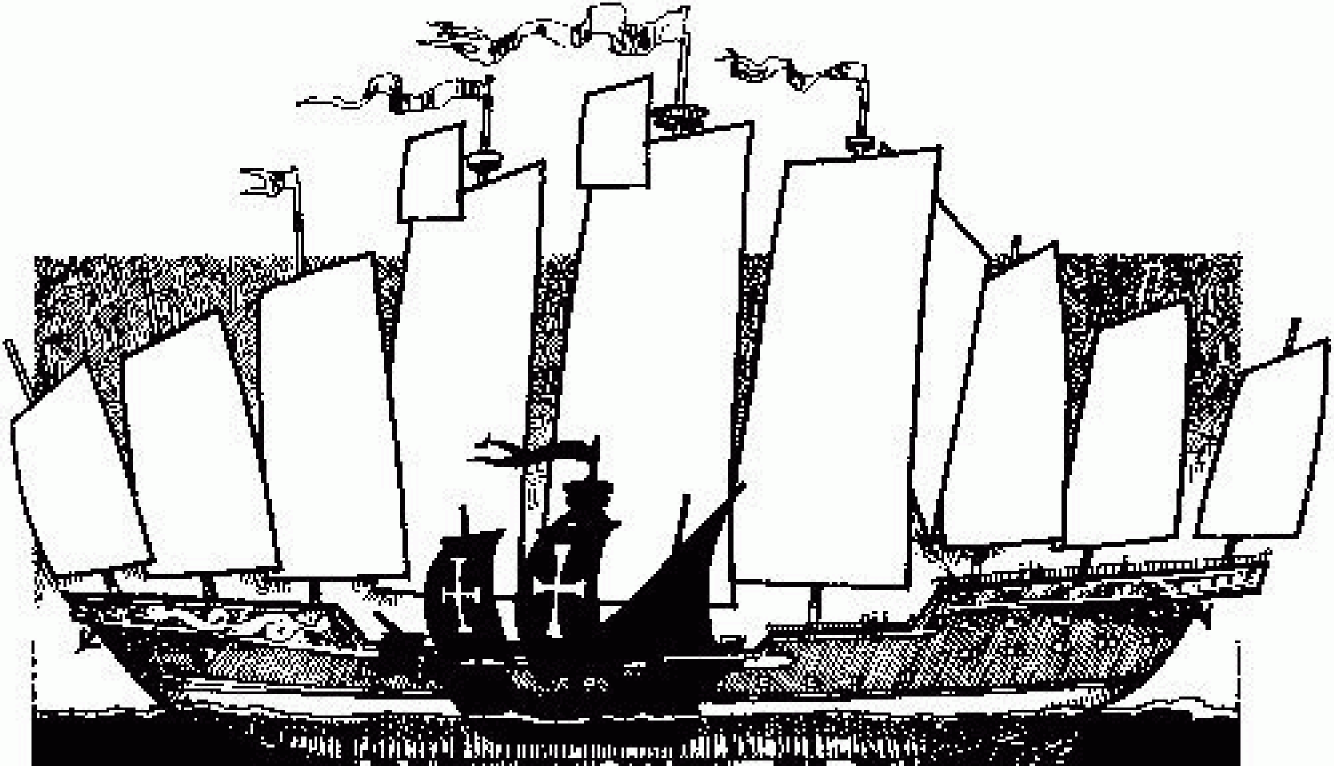
- But, how can Diamond explain China?

- Between the 8th and the 12th century, China experienced a burst of economic activity: gunpowder, printing, water-powered spinning wheel

- Voyages of exploration by admiral Zheng: Louise Levathes (*When China Ruled the Seas: The Treasure Fleet of the Dragon Throne, 1405-1433*).

- With the arrival of the Ming dynasty (1368), China stagnates.

- Europe gets ahead.



Eric Jones (*The European Miracle*): geography, hypothesis 1.

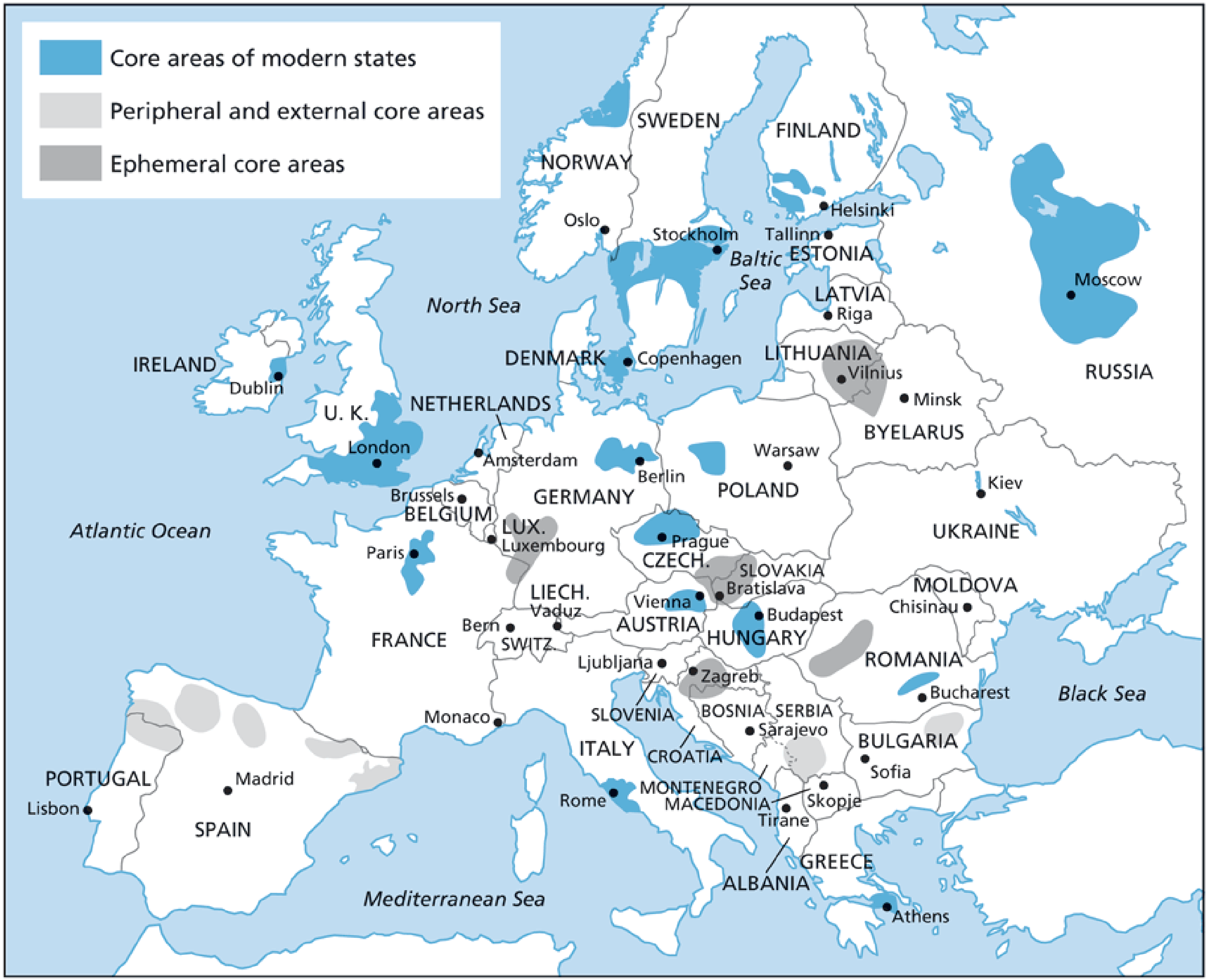
- China was first unified around 221 B.C. Since then, except for relatively short periods, unified state (last partition ended with arrival of Mongols in 13th century).

- Europe has never been unified since the fall of Roman Empire (476 a.d.).

- Why? Dispersion of core areas.

Figure 15.3

Core Areas in Preindustrial Europe



Source: Pounds and Ball (1964).

FIGURE 15.4
Core Areas in PreIndustrial China

Source: Stover (1974).

Kenneth Pomeranz (*The Great Divergence: China, Europe, and the Making of the Modern World Economy*): geography, hypothesis 2.

- Coal:

  1. Far away from production centers

  2. Steam engine versus ventilations.

- Environmental limits.

Colonies

Immanuel Wallerstein (*The Modern World System*).

- Small initial differences in income.
- Patterns of labor control and trade policies created "plantation" economies.
- Trade: primary goods for manufacturing.
- Forward and Backward linkages.

Differences across Colonies

Daron Acemoglu, Simon Johnson, and James Robinson (*The Colonial Origins of Comparative Development: An Empirical Investigation*).

- Differences in settlers mortality.

- Differences in outcomes:

  1. British America: 9 universities for 2.5 million people.

  2. Spanish and Portuguese America: 2 universities for 17 million people.

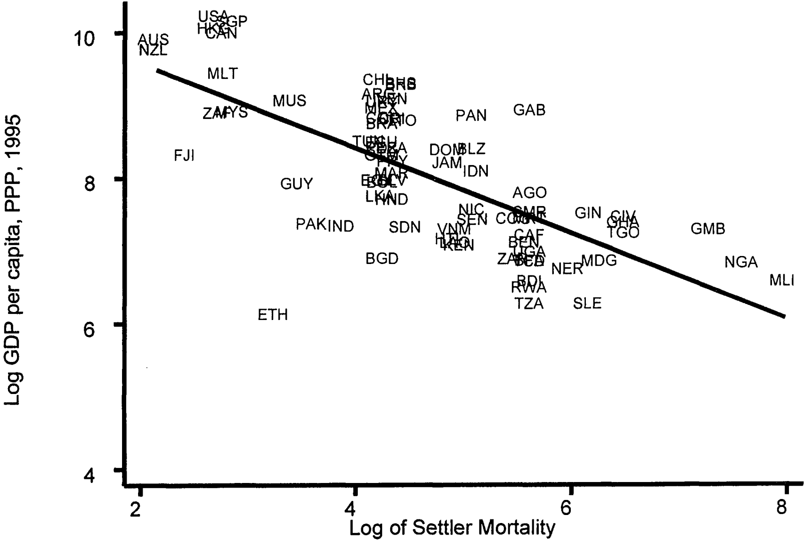


FIGURE 1. REDUCED-FORM RELATIONSHIP BETWEEN INCOME AND SETTLER MORTALITY

Cultural Differences: Yes

- Max Weber (*The Protestant Ethic and the Spirit of Capitalism*)

- Letter from the Chinese emperor Qian Long to King George III of England:

  "Our dynasty's majestic virtue has penetrated unto every country under Heaven...As your Ambassador can see for himself, we possess all things. I set no value on objects strange or ingenious, and have no use for your country's manufactures".

- Leibniz's Instructions to a European traveler to China:

  "Not too worry so much about getting things European to the Chinese, but rather about getting remarkable Chinese inventions to us".

Cultural Differences: No

- A Western traveler, 1881

  "The Japanese are a happy race, and being content with little, are not likely to achieve much".

- Karen Kupperman (*Providence Island, 1630-1641 : The Other Puritan Colony*):

  Documents differences between Providence Island and New England.

- Philip Benedict (*The Faith and Fortunes of France's Huguenots*):

  Differences between Catholics and Huguenots in France.

An Empirical Application:

Population Growth and Technological Change since 1 Million B.C.

- Basic lesson so far: growth depends on technology progress.

- Intuition: more people probably must imply higher knowledge accumulation.

- Growth and population may be closely link.

- Empirical evidence.

A Simple Model

- Production function:

$$Y = T^{\alpha} (AL)^{1-\alpha}$$

- Technology progress:

$$\dot{A} = BAL$$

- Malthusian assumption:

$$\frac{Y}{L} = y^*$$

Solving the Model

- We find the level of population allowed by a technology:

$$\frac{T^{\alpha}(AL)^{1-\alpha}}{L} = y^* \Rightarrow L^* = \left(\frac{1}{y^*}\right)^{\frac{1}{\alpha}} A^{\frac{1-\alpha}{\alpha}} T$$

- Growth rate of population:

$$\frac{\dot{L^*}}{L^*} = \frac{1-\alpha}{\alpha}\frac{\dot{A}}{A}$$

- Then:

$$n_t = \frac{\dot{L^*}}{L^*} = \frac{1-\alpha}{\alpha}\frac{BAL}{A} = \frac{1-\alpha}{\alpha}BL$$

Time Series Evidence

- A first look at the data.

- Regression:

$$
\begin{aligned}
n_t &= \underset{(0.0355)}{-0.0026} + \underset{(0.0258)}{0.524}\, L_t \\
R^2 &= 0.92,\ D.W = 1.10
\end{aligned}
$$

- Robust to different data sets and specifications.

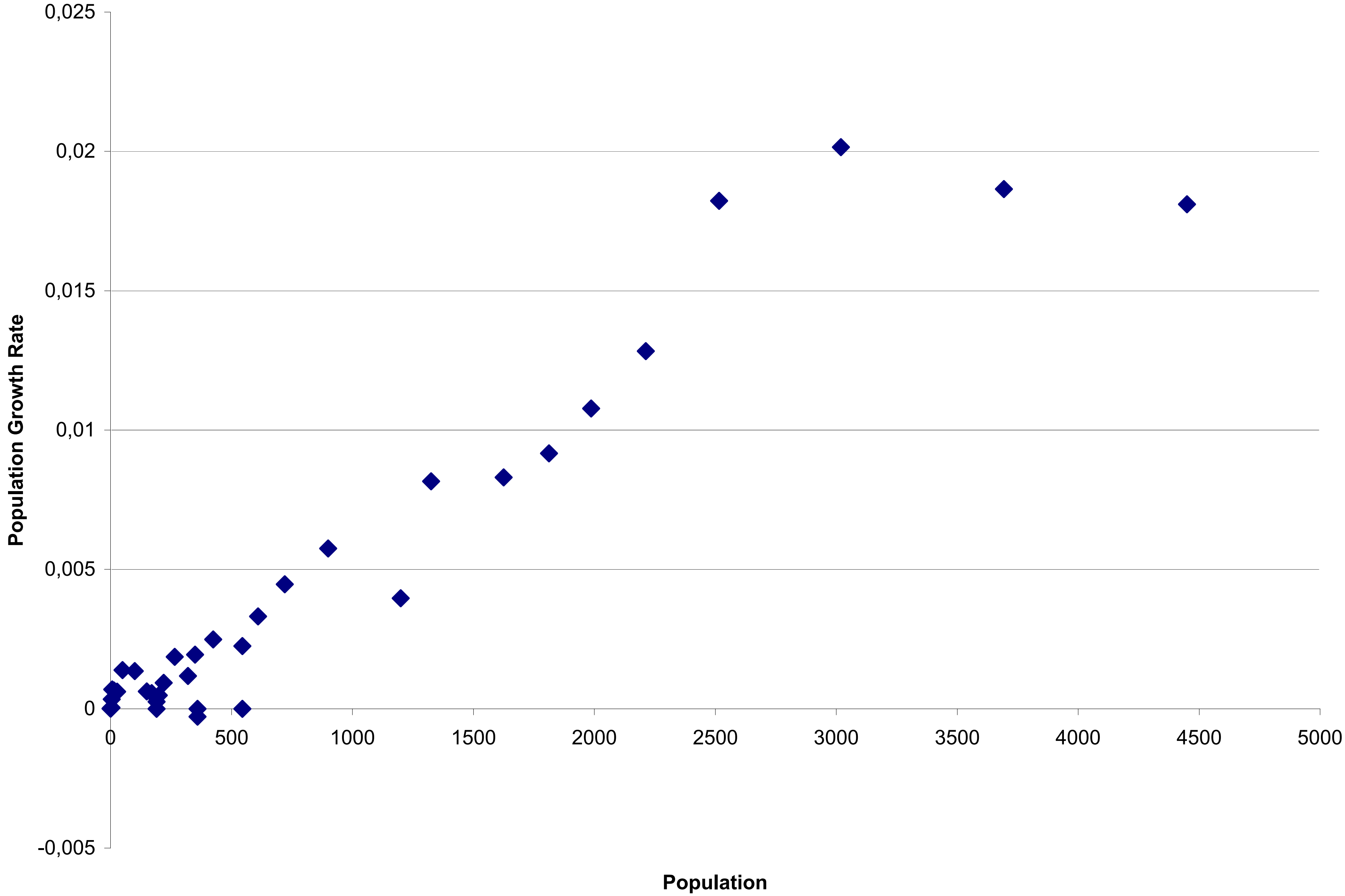
Cross-Section Evidence

- World population was separated from 10,000 BC to circa 1500 AD

- Population and Population Density circa 1500:

	Land Area	Population	Pop/km^2
"Old World"	83.98	407	4.85
Americas	38.43	14	0.36
Australia	7.69	0.2	0.026
Tasmania	0.068	0.0012-0.005	0.018-0.074
Flinders Islands	0.0068	0.0	0.0

- England *vs.* Europe and Japan *vs.* Asia.


0,025
0,02
0,015
0,01
0,005
0
-0,005
Population Growth Rate
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
Population

Introduction to General Equilibrium I: Households

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The Representative Household

- Who is the representative household?

- Robinson Crusoe in a desert island.

- Justification: aggregation.

- When does aggregation work and when does it not?

What are we going to do?

- Think about the goods existing in the economy.
- Think about what does Robinson prefer.
- Think about his constraints.
- Think about what will Robinson do given his preferences and his constraints

Commodity Space

- 2 goods, consumption c and leisure $1 - l$.

- Each goods set:

  1. $c \in \Re_+$

  2. $l \in [0, 1]$

- Then $(c, l) \in \Re_+ \times [0, 1]$.

Preferences

- Preferences: binary relation $\succeq$ defined over pairs (c, l):

$$(c_i, l_i) \succeq \left(c_j, l_j\right)$$

- Assumptions on preferences:

  1. Complete: for $\forall\ (c_i, l_i), \left(c_j, l_j\right) \in \Re_+ \times [0, 1]$ either $(c_i, l_i) \succeq \left(c_j, l_j\right)$ or $\left(c_j, l_j\right) \succeq (c_i, l_i)$.

  2. Reflexive: for $\forall\ (c_i, l_i) \in \Re_+ \times [0, 1]$ $(c_i, l_i) \succeq (c_i, l_i)$.

  3. Transitive: for $\forall\ (c_i, l_i), \left(c_j, l_j\right), (c_k, l_k) \in \Re_+ \times [0, 1]$, if $(c_i, l_i) \succeq \left(c_j, l_j\right)$ and $\left(c_j, l_j\right) \succeq (c_k, l_k) \Rightarrow (c_i, l_i) \succeq (c_k, l_k)$.

Indifference Curves

- Loci of pairs such that:

$$\begin{aligned} (c_i, l_i) &\succeq \left(c_j, l_j\right) \\ \left(c_j, l_j\right) &\succeq (c_i, l_i) \end{aligned}$$

- If we assume that preferences are strictly monotonic, convex and normal, the indifference curves are:

  1. Negative sloped in $1 - l$.

  2. Convex.

Utility Function I

- Working directly with binary relations difficult.
- Can we transform them into a function?
- Why is this useful?

Utility Function II

- Definition: a real-value function $u : \Re^2 \rightarrow \Re$ is called a utility function representing the binary relation $\succeq$ defined over pairs (c, l) if for $\forall\ (c_i, l_i), \left(c_j, l_j\right) \in \Re_+ \times [0,1]$, $(c_i, l_i) \succeq \left(c_j, l_j\right) \Leftrightarrow u(c_i, l_i) \geq u\left(c_j, l_j\right)$.

- Theorem: if the binary relation $\succeq$ is complete, reflexive, transitive, strictly monotone and continuous, there exist a continuous real-value function u that represents $\succeq$.

- Proof (Debreu, 1954): intuition.

Utility Function III

- Utility function and monotone transformations.

- Interpretation of u.

- Differentiability of u.

Budget Constraint

- Leisure $1 - l \Rightarrow$labor supply l.

- Wage w.

- Then

$$c = lw$$

- Interpretation for Robinson.

Household's Problem

- Problem for Robinson is then

$$\max_{c,l} u\left(c, 1-l\right)$$
$$s.t.\ c = lw$$

- First order condition:

$$-\frac{u_l}{u_c} = w$$

- Interpretation: marginal rate of substitution equal to relative price of leisure.

A Parametric Example

- $u(c, l) = \log c + \gamma \log (1 - l)$

- FOC+Budget constraint:

$$\gamma \frac{c^*}{1 - l^*} = w$$
$$c^* = l^* w$$

- Then:

$$l^* = \frac{1}{1 + \gamma}$$

Income and Substitution Effect

- We will follow the Hicksian decomposition.
- Substitution Effect: changes in w make leisure change its relative price with total utility constant.
- Income Effect: changes in w induce changes in total income even if l^* stays constant.
- For $u(c,l) = \log c + \gamma \log(1-l)$ income and substitution effect cancel each other!

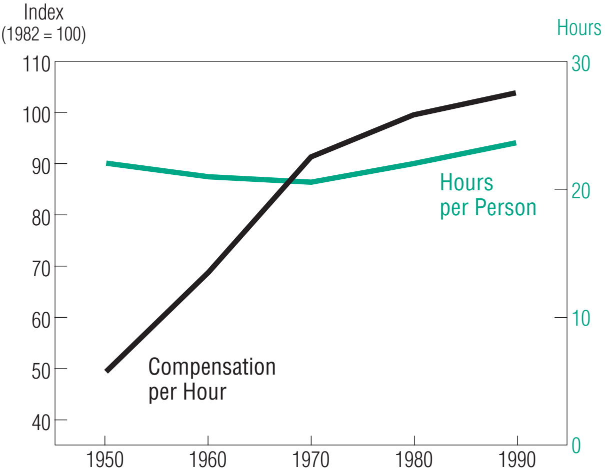
Theory and Data

- Can we use the theory to account for the data?
- What are the trends in labor supply?

Chart 1

Two Aggregate Facts

Average Weekly Hours Worked per Person
and Real Compensation per Hour Worked*
in the United States, 1950–90



*The compensation series is an index of hourly compensation in the business sector, deflated by the consumer price index for all urban consumers.

Sources: Tables 1 and 14

Charts 2–4

Possible Shifts in Hours Worked

Extrapolated Average Weekly Hours Worked per Person
by Cohorts at Various Ages in the United States

Chart 2 **Males**

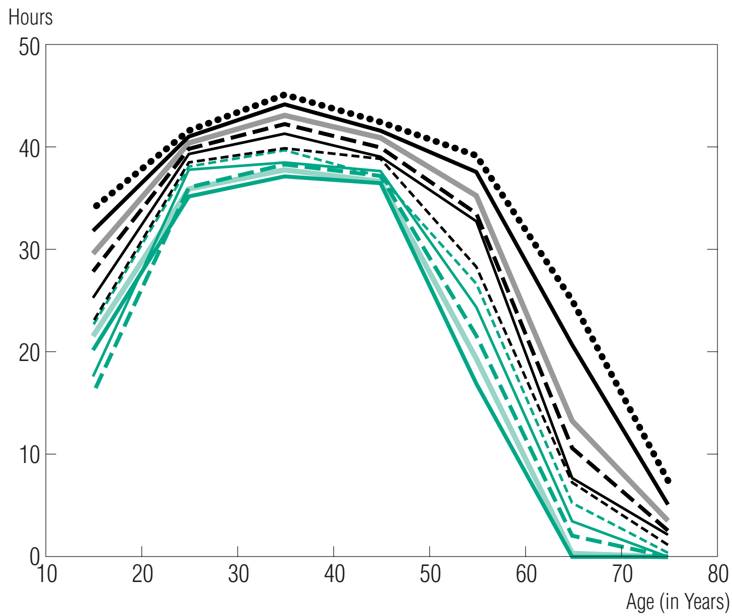


Chart 3 **Females**

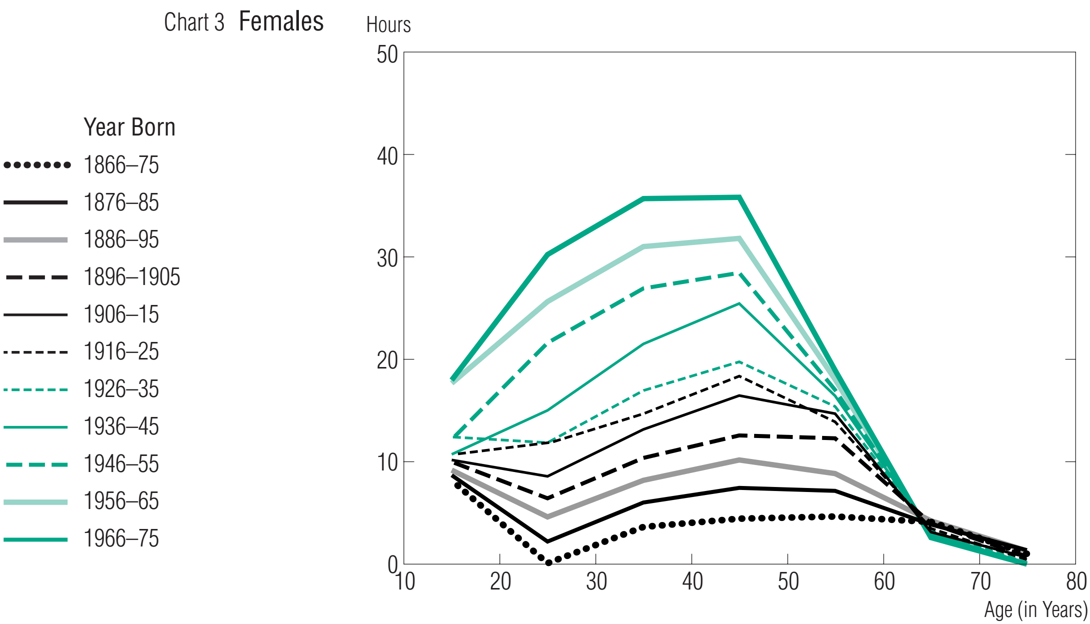
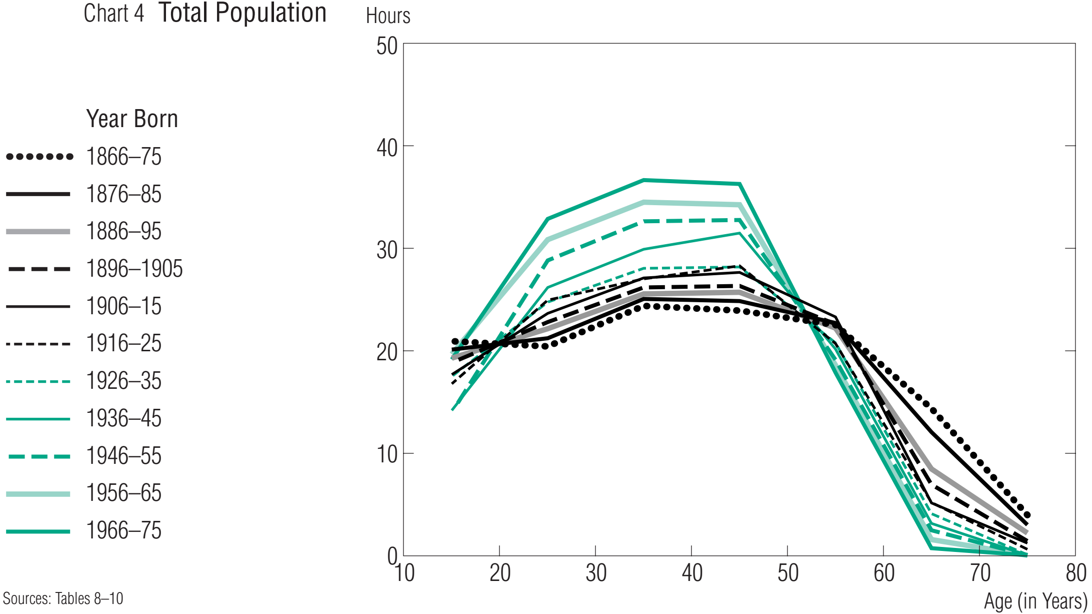


Chart 4 **Total Population**



Sources: Tables 8–10

Table 1

A Look Behind an Aggregate Fact

In the United States, 1950–90

Year	Average Weekly Hours Worked: Per Person	Average Weekly Hours Worked: Per Worker	Employment-to-Population Ratio
1950	22.03	40.71	.52
1960	20.97	37.83	.52
1970	20.55	36.37	.53
1980	22.00	35.97	.58
1990	23.62	36.64	.61
% Change 1950–90	7.2	–10.0	17.3

Source: U.S. Department of Commerce, Bureau of the Census

Tables 2–4

A Distribution of Hours Worked

Average Weekly Hours Worked per Person
for Demographic Categories in the United States, 1950–90

Table 2 By Sex

Year	Weekly Hours Worked per Person by: Total Population	Sex: Males	Sex: Females
1950	22.03	33.46	10.95
1960	20.97	30.70	11.82
1970	20.55	28.54	13.29
1980	22.00	28.30	16.24
1990	23.62	28.53	19.09
% Change 1950–90	7.2	–14.7	74.3

Table 4 By Marital Status*

Year	Married With Spouse: Present	Married With Spouse: Absent	Single	Widowed	Divorced
	Weekly Hours Worked per Person by Marital Status				
1950	23.89	23.11	28.10	11.82	28.65
1960	23.86	20.43	25.72	10.37	26.31
1970	24.31	20.50	24.19	9.41	26.17
1980	24.15	22.71	25.42	6.86	27.22
1990	26.26	22.22	27.73	5.98	28.41
% Change 1950–90	9.9	–3.9	–1.3	–49.4	–.8

*This excludes individuals less than 25 years old.

Table 3 By Age

Year	15–24	25–34	35–44	45–54	55–64	65–74	75–84
	Weekly Hours Worked per Person by Age (in Years)						
1950	17.47	24.92	27.09	26.31	22.19	12.03	3.93
1960	14.15	24.73	27.00	27.63	22.58	8.43	2.97
1970	14.05	26.16	28.03	28.27	23.28	6.91	2.17
1980	19.64	28.80	29.89	28.16	20.68	5.11	1.39
1990	19.13	30.83	32.62	31.47	20.75	5.15	1.18
% Change 1950–90	9.5	23.7	20.4	19.6	–6.5	–57.2	–70.0

Source: U.S. Department of Commerce, Bureau of the Census

Tables 5–6

A More Comprehensive Distribution of Hours Worked

Average Weekly Hours Worked per Person for Sets of Demographic Categories in the United States, 1950–90

Table 5 Married . . .

Status		Sex	Year	15–24	25–34	35–44	45–54	55–64	65–74	75–84
				Weekly Hours Worked per Person by Age (in Years)						
Spouse Present	*Total*	Males	1950	38.69	41.14	43.06	41.95	37.58	23.39	9.77
			1960	36.58	40.67	41.79	40.99	35.74	14.74	6.22
			1970	34.19	40.30	41.52	40.65	34.74	11.51	4.08
			1980	33.63	38.70	40.22	38.89	29.83	8.20	2.95
			1990	34.18	40.25	41.34	40.03	28.39	7.71	2.50
		% Change 1950–90		–11.7	–2.2	–4.0	–4.6	–24.5	–67.0	–74.4
		Females	1950	9.17	8.09	9.60	8.61	4.60	1.79	.56
			1960	10.00	9.10	12.35	13.55	8.66	2.27	.94
			1970	14.65	12.21	14.95	16.18	11.75	2.55	1.03
			1980	18.36	18.77	19.64	18.16	11.95	2.48	.70
			1990	21.13	23.90	25.41	24.04	13.83	2.79	.65
		% Change 1950–90		130.4	195.4	164.7	179.2	200.7	55.9	16.1
	Youngest Child Under 6 Years Old	Females	1950	3.40	4.60	6.49	6.41	4.24	3.93	6.82
			1960	5.71	5.75	6.36	9.17	7.25	2.10	2.25
			1970	9.08	8.33	9.04	12.11	10.13	3.70	6.02
			1980	11.72	13.47	13.00	11.77	9.32	1.34	.30
			1990	15.49	19.48	19.62	18.55	13.11	6.61	7.86
		% Change 1950–90		355.6	323.5	202.3	189.4	209.2	68.2	15.2
	Youngest Child 6–17 Years Old	Females	1950	3.89	5.57	7.64	6.81	4.50	2.28	10.08
			1960	13.27	13.44	13.75	11.99	8.75	2.53	1.45
			1970	16.23	15.90	15.85	14.49	11.41	4.15	6.81
			1980	15.46	20.79	20.01	16.76	11.91	3.90	3.41
			1990	23.43	24.85	25.70	23.01	15.09	5.98	11.01
		% Change 1950–90		502.3	346.1	236.4	237.9	235.3	162.3	9.2
Spouse Absent	*Total*	Males	1950	24.17	27.54	31.56	30.48	26.62	16.54	5.93
			1960	17.13	25.80	27.83	29.49	24.69	9.66	3.47
			1970	16.49	27.12	29.67	30.48	25.06	9.13	2.94
			1980	25.27	30.64	31.99	29.18	20.58	5.99	2.40
			1990	21.03	27.31	28.80	29.84	21.63	6.23	1.48
		% Change 1950–90		–13.0	–.8	–8.7	–2.1	–18.7	–62.3	–75.0
		Females	1950	15.37	20.00	22.26	19.74	13.82	4.42	1.04
			1960	14.24	17.52	20.51	20.74	15.58	4.30	1.57
			1970	16.05	18.03	20.17	21.43	17.25	5.43	2.16
			1980	17.12	21.77	22.78	21.32	15.79	3.75	1.50
			1990	15.89	21.95	25.26	24.22	15.72	4.01	.84
		% Change 1950–90		3.4	9.8	13.5	22.7	13.7	–9.3	–19.2

Table 6 . . . And Not Married

Status	Sex	Year	15–24	25–34	35–44	45–54	55–64	65–74	75–84
			Weekly Hours Worked per Person by Age (in Years)						
Single	Males	1950	18.29	31.58	33.82	31.97	27.18	15.47	6.12
		1960	12.67	30.61	30.35	28.98	24.30	9.74	5.01
		1970	11.37	29.78	29.82	28.03	22.60	8.58	4.17
		1980	19.23	30.55	29.01	26.24	19.60	6.26	2.06
		1990	18.76	31.50	30.17	26.64	17.87	5.83	2.03
		% Change 1950–90	2.6	–.3	–10.8	–16.7	–34.3	–62.3	–66.8
	Females	1950	14.33	30.58	30.51	28.61	22.77	10.36	3.14
		1960	10.70	29.33	29.37	28.94	24.40	10.63	3.35
		1970	10.43	28.82	27.65	27.62	24.23	8.41	3.07
		1980	17.23	29.15	28.24	25.76	20.68	4.93	1.19
		1990	17.35	29.73	30.21	27.59	18.55	4.98	1.02
		% Change 1950–90	21.1	–2.8	–1.0	–3.6	–18.5	–51.9	–67.5
Widowed	Males	1950	19.65	33.50	35.76	34.12	29.15	14.99	4.67
		1960	19.74	32.00	31.33	31.97	25.95	9.24	3.56
		1970	19.68	29.63	32.08	31.93	25.36	7.24	2.34
		1980	18.64	28.31	29.66	29.10	20.89	5.24	1.70
		1990	15.20	26.62	28.70	29.06	18.32	4.90	1.38
		% Change 1950–90	–22.6	–20.5	–19.7	–14.8	–37.2	–67.3	–70.4
	Females	1950	17.02	21.75	23.90	20.11	12.96	4.31	.83
		1960	15.64	17.61	22.82	23.35	15.71	4.72	1.18
		1970	17.66	21.00	21.85	23.52	17.82	4.20	1.04
		1980	17.12	17.25	21.13	20.71	15.68	3.30	.66
		1990	10.56	18.50	24.06	24.41	15.16	3.59	.57
		% Change 1950–90	–38.0	–14.9	.7	21.4	17.0	–16.7	–31.3
Divorced	Males	1950	29.53	32.82	34.93	32.71	28.77	15.76	11.67
		1960	24.08	30.54	31.51	29.50	25.75	9.75	4.95
		1970	25.56	33.14	33.35	31.16	24.62	8.63	4.12
		1980	29.16	33.73	34.39	30.90	22.34	6.09	2.65
		1990	29.17	33.94	34.23	32.90	22.23	6.76	2.46
		% Change 1950–90	–1.2	3.4	–2.0	.6	–22.7	–57.1	–78.9
	Females	1950	25.27	28.72	30.68	27.32	21.99	10.07	1.96
		1960	24.01	27.69	29.87	29.51	24.05	9.19	2.48
		1970	25.16	27.59	29.73	29.71	25.04	7.99	3.26
		1980	24.42	29.38	30.38	28.73	22.65	5.53	1.48
		1990	23.26	29.13	32.82	31.86	23.73	6.68	1.49
		% Change 1950–90	–8.0	1.4	7.0	16.6	7.9	–33.7	–24.0

Source: U.S. Department of Commerce, Bureau of the Census

Table 7

Partial Life-Cycle Profiles of Hours Worked by Males

Based on U.S. Census Data

Year Born	Average Weekly Hours Worked per Person at Age (in Years)						
	15–24	25–34	35–44	45–54	55–64	65–74	75–84
1866–75	—	—	—	—	—	—	7.46
1876–85	—	—	—	—	—	20.75	5.12
1886–95	—	—	—	—	35.34	13.31	3.53
1896–1905	—	—	—	40.06	33.60	10.65	2.57
1906–15	—	—	41.41	39.15	32.84	7.71	2.16
1916–25	—	38.60	39.98	38.95	28.38	7.28	—
1926–35	22.65	38.20	39.79	37.20	26.73	—	—
1936–45	17.65	37.89	38.59	37.75	—	—	—
1946–55	15.96	36.15	38.40	—	—	—	—
1956–65	21.59	36.00	—	—	—	—	—
1966–75	20.23	—	—	—	—	—	—

Source: U.S. Department of Commerce, Bureau of the Census

Tables 8–10

Extrapolated Life-Cycle Profiles of Hours Worked

U.S. Census Data Extrapolated as Explained in Appendix C*

Table 8 By Males

	Average Weekly Hours Worked per Person at Age (in Years)						
Year Born	15–24	25–34	35–44	45–54	55–64	65–74	75–84
1866–75	34.08	41.71	45.23	42.55	39.27	25.10	7.46
1876–85	31.90	41.13	44.28	41.70	37.66	20.75	5.12
1886–95	29.67	40.50	43.20	41.01	35.34	13.31	3.53
1896–1905	27.92	39.93	42.35	40.06	33.60	10.65	2.57
1906–15	25.35	39.42	41.41	39.15	32.84	7.71	2.16
1916–25	23.00	38.60	39.98	38.95	28.38	7.28	1.17
1926–35	22.65	38.20	39.79	37.20	26.73	5.27	.39
1936–45	17.65	37.89	38.59	37.75	24.44	3.48	.00
1946–55	15.96	36.15	38.40	37.28	21.64	2.07	.00
1956–65	21.59	36.00	37.87	36.73	19.39	.33	.00
1966–75	20.23	35.27	37.23	36.57	16.95	.00	.00

Table 9 By Females

	Average Weekly Hours Worked per Person at Age (in Years)						
Year Born	15–24	25–34	35–44	45–54	55–64	65–74	75–84
1866–75	8.19	.08	3.63	4.44	4.65	4.12	1.01
1876–85	8.69	2.20	6.02	7.45	7.16	3.87	1.36
1886–95	9.19	4.61	8.19	10.18	8.85	4.23	1.29
1896–1905	10.00	6.43	10.38	12.58	12.30	3.98	.74
1906–15	10.18	8.58	13.17	16.48	14.71	3.13	.66
1916–25	10.68	11.84	14.70	18.38	13.91	3.48	.43
1926–35	12.43	11.87	16.97	19.78	15.41	3.23	.14
1936–45	10.73	15.03	21.53	25.48	16.45	2.98	.00
1946–55	12.18	21.63	26.96	28.48	17.03	2.93	.00
1956–65	17.68	25.67	31.05	31.85	18.06	2.75	.00
1966–75	17.99	30.27	35.74	35.87	18.95	2.59	.00

*Highlighted areas indicate actual U.S. census data. The other data are extrapolations.

Table 10 By Total Population

Year Born	Average Weekly Hours Worked per Person at Age (in Years) 15–24	25–34	35–44	45–54	55–64	65–74	75–84
1866–75	20.91	20.40	24.36	23.90	22.50	14.34	3.93
1876–85	20.09	21.20	25.04	24.83	22.69	12.03	2.97
1886–95	19.26	22.13	25.53	25.69	22.19	8.43	2.17
1896–1905	18.81	22.78	26.16	26.31	22.58	6.91	1.39
1906–15	17.65	23.63	27.09	27.63	23.28	5.11	1.18
1916–25	16.75	24.92	27.00	28.27	20.68	5.15	.58
1926–35	17.47	24.73	28.03	28.16	20.75	4.06	.05
1936–45	14.15	26.16	29.89	31.47	20.14	3.11	.00
1946–55	14.05	28.80	32.62	32.75	19.09	2.44	.00
1956–65	19.64	30.83	34.49	34.24	18.56	1.53	.00
1966–75	19.13	32.86	36.65	36.27	17.84	.69	.00

Source: U.S. Department of Commerce, Bureau of the Census

Table 11

Lifetime Hours Worked

Average Weekly Hours Worked Between Ages 15 and 84
by Cohorts Born Between 1896 and 1945 in the United States

	Weekly Hours Worked per Person by		
		Sex	
Year Born	Total Population	Males	Females
1896–1905	17.85	28.15	8.06
1906–15	17.94	26.86	9.56
1916–25	17.62	25.34	10.49
1926–35	17.61	24.32	11.40
1936–45	17.85	22.83	13.17
% Change 1896–1945	0	–18.9	63.4

Sources: Tables 8–10

Table 12

Partial Life-Cycle Profiles for the Portion of the Population Employed . . .

Sex	Year Born	Employment-to-Population Ratio at Age (in Years)						
		15–24	25–34	35–44	45–54	55–64	65–74	75–84
Males	1866–75	—	—	—	—	—	—	.19
	1876–85	—	—	—	—	—	.49	.15
	1886–95	—	—	—	—	.80	.36	.11
	1896–1905	—	—	—	.89	.79	.31	.09
	1906–15	—	—	.91	.89	.78	.23	.08
	1916–25	—	.87	.90	.90	.68	.22	—
	1926–35	.55	.87	.90	.86	.64	—	—
	1936–45	.50	.87	.88	.86	—	—	—
	1946–55	.47	.85	.87	—	—	—	—
	1956–65	.61	.83	—	—	—	—	—
	1966–75	.60	—	—	—	—	—	—
	% Change	9.1	–4.6	–4.4	–3.4	–20.0	–55.1	–57.9
Females	1866–75	—	—	—	—	—	—	.03
	1876–85	—	—	—	—	—	.10	.04
	1886–95	—	—	—	—	.23	.13	.04
	1896–1905	—	—	—	.32	.34	.13	.03
	1906–15	—	—	.34	.45	.41	.11	.03
	1916–25	—	.31	.41	.51	.40	.12	—
	1926–35	.33	.33	.48	.56	.44	—	—
	1936–45	.32	.43	.62	.68	—	—	—
	1946–55	.38	.61	.73	—	—	—	—
	1956–65	.56	.69	—	—	—	—	—
	1966–75	.59	—	—	—	—	—	—
	% Change	78.8	122.6	114.7	112.5	91.3	20.0	0

Source: U.S. Department of Commerce, Bureau of the Census

Table 13

. . . And for the Hours Worked per Worker

		Average Weekly Hours Worked per Worker at Age (in Years)						
Sex	Year Born	15–24	25–34	35–44	45–54	55–64	65–74	75–84
Males	1866–75	—	—	—	—	—	—	38.62
	1876–85	—	—	—	—	—	42.07	34.13
	1886–95	—	—	—	—	43.95	37.35	32.56
	1896–1905	—	—	—	44.96	42.53	34.88	29.59
	1906–15	—	—	45.22	43.92	42.02	33.06	28.64
	1916–25	—	44.47	44.44	43.43	41.50	32.89	—
	1926–35	40.49	43.84	44.02	43.06	41.69	—	—
	1936–45	33.94	43.19	43.58	43.93	—	—	—
	1946–55	32.10	42.46	44.20	—	—	—	—
	1956–65	34.80	43.09	—	—	—	—	—
	1966–75	33.46	—	—	—	—	—	—
	% Change	–17.4	–3.1	–2.3	–2.3	–5.1	–21.8	–25.8
Females	1866–75	—	—	—	—	—	—	36.34
	1876–85	—	—	—	—	—	37.56	32.37
	1886–95	—	—	—	—	38.10	32.17	31.08
	1896–1905	—	—	—	38.58	36.04	30.36	25.08
	1906–15	—	—	38.32	36.62	35.77	27.64	24.51
	1916–25	—	38.14	35.79	36.00	34.73	27.94	—
	1926–35	37.71	35.45	34.94	35.30	34.98	—	—
	1936–45	33.25	34.72	34.79	37.12	—	—	—
	1946–55	31.48	35.47	36.91	—	—	—	—
	1956–65	31.57	37.14	—	—	—	—	—
	1966–75	30.53	—	—	—	—	—	—
	% Change	–19.0	–2.6	–3.7	–3.8	–8.2	–25.6	–32.6

Source: U.S. Department of Commerce, Bureau of the Census

Table 14

Possible Factors Behind Work Reallocations

In the United States, 1950–90

Year	Index of Real Compensation* (1982=100)	Average Monthly Social Security Benefit (1990 $)	Total Fertility Rate**	% of Population in Each Marital Status Category: Married With Spouse Present	Married With Spouse Absent	Single	Widowed	Divorced
1950	49.4	238	3,337	64.45	4.01	21.10	8.25	2.24
1960	68.8	327	3,449	65.45	3.87	20.12	8.01	2.55
1970	91.3	397	2,480	61.33	3.86	23.23	8.17	3.41
1980	99.5	541	1,840	57.95	2.25	25.98	7.62	6.20
1990	103.8	603	2,081	53.56	4.29	26.42	7.37	8.35
% Change 1950–90	110.1	153.4	–37.6	–16.9	7.0	25.2	–10.7	272.8

*This is an index of hourly compensation in the business sector, deflated by the consumer price index for all urban consumers.

**The *fertility rate* for any year is the number of births that 1,000 females would have in their lifetime if, at each age, they experienced that year's birthrate.

Sources: See Appendix A.

A Decomposition of Average Weekly Hours Worked per Person

Year	Actual Hours per Person	Hours per Person Recalculated With: 1950 Weights	Hours per Person Recalculated With: 1950 Hours
1950	22.03	22.03	22.03
1960	20.97	21.40	21.57
1970	20.55	21.93	20.92
1980	22.00	23.30	20.99
1990	23.62	25.00	21.50
% Change 1950–90	7.2	13.5	–2.4

Source of basic data: See Appendix A.

Introduction to General Equilibrium II: Firms

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What is a firm?

- A technology:

$$y = F(k,l) = Ak^{\alpha}l^{1-\alpha}$$

  for $\alpha \in (0,1)$.

- Operational definition.

- We are in a static world: we will assume k to be constant.

Properties of the Technology I

From lectures in growth we know that:

1. Constant returns to scale.

2. Inputs are essential.

3. Marginal productivities are positive and decreasing.

4. Inada Conditions.

Problem of the Firm I

- Wants to maximize profits given r and w

  (we are taking the consumption good as the numeraire!):

$$\pi = Ak^{\alpha}l^{1-\alpha} - rk - wl$$

- We take first order conditions:

$$\begin{aligned} \alpha Ak^{\alpha-1}l^{1-\alpha} &= r && (1) \\ (1-\alpha)\,Ak^{\alpha}l^{-\alpha} &= w && (2) \end{aligned}$$

- We want to solve for k and l.

Problem of the Firm II

- We begin dividing (1) by (2):

$$\frac{\alpha A k^{\alpha-1} l^{1-\alpha}}{(1-\alpha) A k^{\alpha} l^{-\alpha}} = \frac{r}{w}$$

or

$$\frac{\alpha}{1-\alpha}\frac{l}{k} = \frac{r}{w}$$

or

$$k = \frac{w}{r}\frac{\alpha}{1-\alpha} l \tag{3}$$

Problem of the Firm III

- but if we substitute (3) in (2):

$$
\begin{aligned}
(1-\alpha) A k^{\alpha} l^{-\alpha} &= w \\
(1-\alpha) A \left(\frac{w}{r}\frac{\alpha}{1-\alpha} l\right)^{\alpha} l^{-\alpha} &= w \\
(1-\alpha) A \left(\frac{w}{r}\frac{\alpha}{1-\alpha}\right)^{\alpha} &= w
\end{aligned}
$$

  l disappears!

- You can check that the same happens with k if we substitute (3) in (1).

- What is wrong?

Problem of the Firm IV

- We have constant returns to scale.

- The size of the firm is indeterminate: we can have just one!

- To see that remember that profits are always zero if firm maximizes:

$$\begin{aligned} \pi &= Ak^{\alpha}l^{1-\alpha} - rk - wl \\ &= Ak^{\alpha}l^{1-\alpha} - \alpha Ak^{\alpha-1}l^{1-\alpha}k - (1-\alpha)\, Ak^{\alpha}l^{-\alpha}l = 0 \end{aligned}$$

- So the firms really only pick the labor-capital ratio given relative prices:

$$\frac{l}{k} = \frac{\alpha}{1-\alpha}\frac{r}{w}$$

Problem of the Firm V

- In equilibrium, markets clear so:

$$l(r,w) = l^s(w)$$
$$k(r,w) = k^f$$
$$r = \alpha A k(r,w)^{\alpha-1} l(r,w)^{1-\alpha}$$
$$w = (1-\alpha) A k(r,w)^{\alpha} l(r,w)^{-\alpha}$$

- We have a system of four equations in four unknowns.

What are we missing?

- A lot.

- Wages are a lot of time different from marginal productivites.

- Reasons for that:

  1. Efficiency Wages: Shapiro-Stiglitz (1984).

  2. Bargaining: Nash (1950).

  3. Monopoly rents: Holmes and Schmitz (2001).

  4. Sticky wages: Taylor (1980).

Table 7
Annual Turnover Rates, 1913–15

	1913	1914	1915
Average force employed	13,623	12,115	18,028
Total leaving	50,448	6,508	2,931
Turnover rate (%)	370	54	16
Resignations	39,575	5,199	2,871
Layoffs	2,383	385	23
Discharges	8,490	926	27

SOURCE.—Slichter (1921, p. 244).

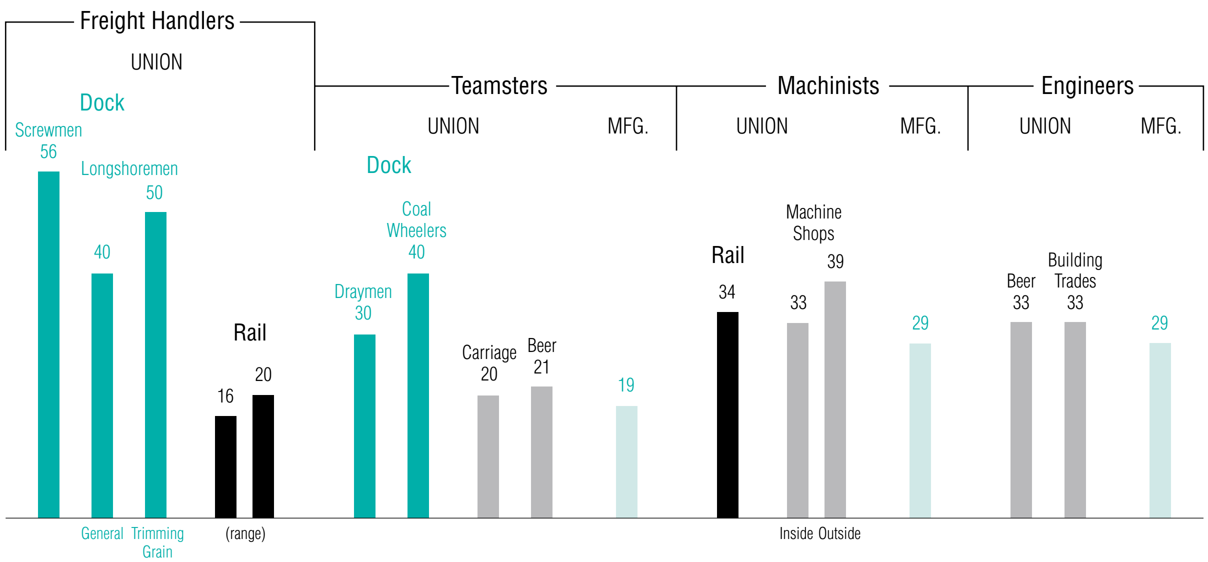
Table 8
Absenteeism at Ford

	Total Workers	Number Absent	Percent Absent
October 6, 1913	12,548	1,250	10
October 6, 1914	12,645	311	2.5

SOURCE.—Abell (1915, p. 37).

Dock Workers Were Paid More Than Most Other Workers in New Orleans . . .

Hourly Wage Rates (Cents) in Union Agreements and in Manufacturing Establishments, 1904–5



Source: Lee 1906

Introduction to General Equilibrium III: Government

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What is the Government?

- Operational definition: takes taxes and spends them.

$$\begin{aligned} G &= T \\ T &= \tau^l w \end{aligned}$$

- No public debt.

- Why?

New Problem of the Household

- Problem for Robinson is now

$$\max_{c,l} u(c, 1-l)$$
$$s.t.\ c = \left(1-\tau^l\right) wl + rk$$

- Why the new last term?

- FOC:

$$-\frac{u_l}{u_c} = \left(1-\tau^l\right) w$$

A Parametric Example

- $u(c,l) = \log c + \gamma \log(1-l)$

- FOC+Budget constraint:

$$\gamma \frac{c^*}{1-l^*} = \left(1-\tau^l\right) w$$
$$c^* = \left(1-\tau^l\right) w l^* + rk$$

- Then:

$$l^* = \frac{\left(1-\tau^l\right) w - rk}{(1+\gamma)\left(1-\tau^l\right) w}$$

- Taxes affect labor supply!!!

- How important is the effect?

- Two Examples:

  1. Tax reform of 1986.

  2. Why do Americans work so much more than Europeans?

Table 1
Average and Marginal Federal Tax Rates (Percent) at Alternative Levels of Family Income, 1980–88[a]

		Combined tax rate[b]					
		One-half Median income		Median income		Twice Median income	
Year	Median income (dollars)	Avg.	Mrg.	Avg.	Mrg.	Avg.	Mrg.
1980	24,332	18.3	30.3	23.7	36.3	24.8	43.0
1982	27,619	19.9	29.4	24.5	38.4	25.9	39.0
1984	31,097	19.9	27.4	23.7	35.4	24.8	38.0
1986	34,716	20.9	28.3	24.8	36.3	25.7	38.0
1988	37,482[c]	19.8	30.0	24.2	30.0	23.9	28.0

Source: U.S. Department of the Treasury.
[a]Tax rates for a four-person family. It is assumed that each family contains only a single–earner and that all taxable income consists of wage or self-employment earnings.
[b]Combined tax rate includes federal income tax and FICA payroll tax.
[c]Estimate.

Table 3
Change in Annual Hours of Work by Income Quintile, 1989 (Percent[a])

Demographic group	*Bottom quintile*	*Second quintile*	*Middle quintile*	*Fourth quintile*	*Top quintile*	*All quintiles*
Men						
Aged 25–64	31.0*	3.6	4.1*	2.5*	3.2*	6.0*
Aged 25–44	10.5*	1.5	2.5	2.7	1.8	3.3*
Aged 45–64	96.4*	6.3	4.0*	3.5	4.3*	9.2*
Married, aged 25–64	19.4*	2.2	2.3	3.9*	3.6*	5.1*
Women						
Aged 25–64	16.7*	−6.9*	6.4	10.5*	11.8*	5.4
Aged 25–44	−14.0*	−11.1*	4.5	4.8	2.2	−1.6
Aged 45–64	141.4*	0.1	6.3	19.3*	27.4*	18.1*
Married, aged 25–64	20.9*	1.2	4.9	3.9	13.8*	7.1*

Table 1

Output, Labor Supply, and Productivity

In Selected Countries in 1993–96 and 1970–74

		Relative to United States (U.S. = 100)		
Period	Country	Output per Person*	Hours Worked per Person*	Output per Hour Worked
1993–96	Germany	74	75	99
	France	74	68	110
	Italy	57	64	90
	Canada	79	88	89
	United Kingdom	67	88	76
	Japan	78	104	74
	United States	100	100	100
1970–74	Germany	75	105	72
	France	77	105	74
	Italy	53	82	65
	Canada	86	94	91
	United Kingdom	68	110	62
	Japan	62	127	49
	United States	100	100	100

*These data are for persons aged 15–64.
Sources: See Appendix.

Table 2

Actual and Predicted Labor Supply

In Selected Countries in 1993–96 and 1970–74

Period	Country	Labor Supply*		Differences (Predicted Less Actual)	Prediction Factors	
		Actual	Predicted		Tax Rate τ	Consumption/ Output (c/y)
1993–96	Germany	19.3	19.5	.2	.59	.74
	France	17.5	19.5	2.0	.59	.74
	Italy	16.5	18.8	2.3	.64	.69
	Canada	22.9	21.3	–1.6	.52	.77
	United Kingdom	22.8	22.8	0	.44	.83
	Japan	27.0	29.0	2.0	.37	.68
	United States	25.9	24.6	–1.3	.40	.81
1970–74	Germany	24.6	24.6	0	.52	.66
	France	24.4	25.4	1.0	.49	.66
	Italy	19.2	28.3	9.1	.41	.66
	Canada	22.2	25.6	3.4	.44	.72
	United Kingdom	25.9	24.0	–1.9	.45	.77
	Japan	29.8	35.8	6.0	.25	.60
	United States	23.5	26.4	2.9	.40	.74

*Labor supply is measured in hours worked per person aged 15–64 per week.
Sources: See Appendix.

How Does the Government Behave?

- Economist use their tools to understand how governments behave.

- *Political Economics* (different than *Political Economy*).

- Elements:

  1. Rational Agents.

  2. Optimization.

  3. Equilibrium outcomes.

Overall Questions

- How do we explain differences and similarities in observed economic policy over time?

  1. Why do countries limit free trade?

  2. Why do countries levy inefficient taxes?

- Can we predict the effects of changing political arrangements:

  1. What would happen if we abandon the electoral college?

  2. What would happen if we introduce proportional representation?

Some Basic Results

- Arrow's Impossibility Theorem.
- Median Voter's Theorem.
- Probabilistic Voting.

Political Economics in Macro

- How taxes are fixed?

- Time-Consistency Problems.

- Redistribution.

General Equilibrium

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Now we are going to put everything together

- We have a household that decides how much to work, l, and how much to consume, c to maximize utility. It takes as given the wage, w and the interest rate r.

- We have a firm that decides how much to produce, y and how much capital, k, and labor, l to hire. It takes as given the wage, w and the interest rate r.

- We have a government (maybe not) that raises taxes, T, and spends G.

- We are in a static world: we will assume k to be constant.

Allocations, Feasible Allocations, Government Policy and Price Systems

- An allocation is a set of value for production, y, work, l, capital, k and consumption c.

- A feasible allocation is an allocation that is possible:

$$c + G = y = Ak^{\alpha} l^{1-\alpha}$$

- A government policy is a set of taxes τ^l and government spending G.

- A price system is a set of prices w and r.

A Competitive Equilibrium

A *Competitive Equilibrium* is an allocation $\{y, l, k, c\}$, a price system $\{w, r\}$ and a government policy $\left\{\tau^l, T\right\}$ such that:

1. Given the price system and the government policy, households choose l and c to maximize their utility.

2. Given the price system and the government policy, firms maximize profits, i.e. they $\alpha A k^{\alpha-1} l^{1-\alpha} = r$ and $(1-\alpha) A k^{\alpha} l^{-\alpha} = w$.

3. Markets clear:

$$c + G = y = A k^{\alpha} l^{1-\alpha}$$

Existence of an Equilibrium

- Does it exist an equilibrium?

- Tough problem.

- Shown formally by Arrow (Nobel Prize Winner 1972) and Debreu (Nobel Prize Winner 1983).

- Uniqueness?

What do we get out of the concept of an Equilibrium?

- Consistency: we are sure that everyone is doing things that are compatible. Economics is only social science that is fully aware of this big issue.

- We talk about Competitive Equilibrium but there are other concepts of equilibrium: Ramsey Equilibrium, Nash Equilibrium, etc...

- It is a prediction about the behavior of the model. Theory CAN and SHOULD be tested against the data. Some theories are thrown away.

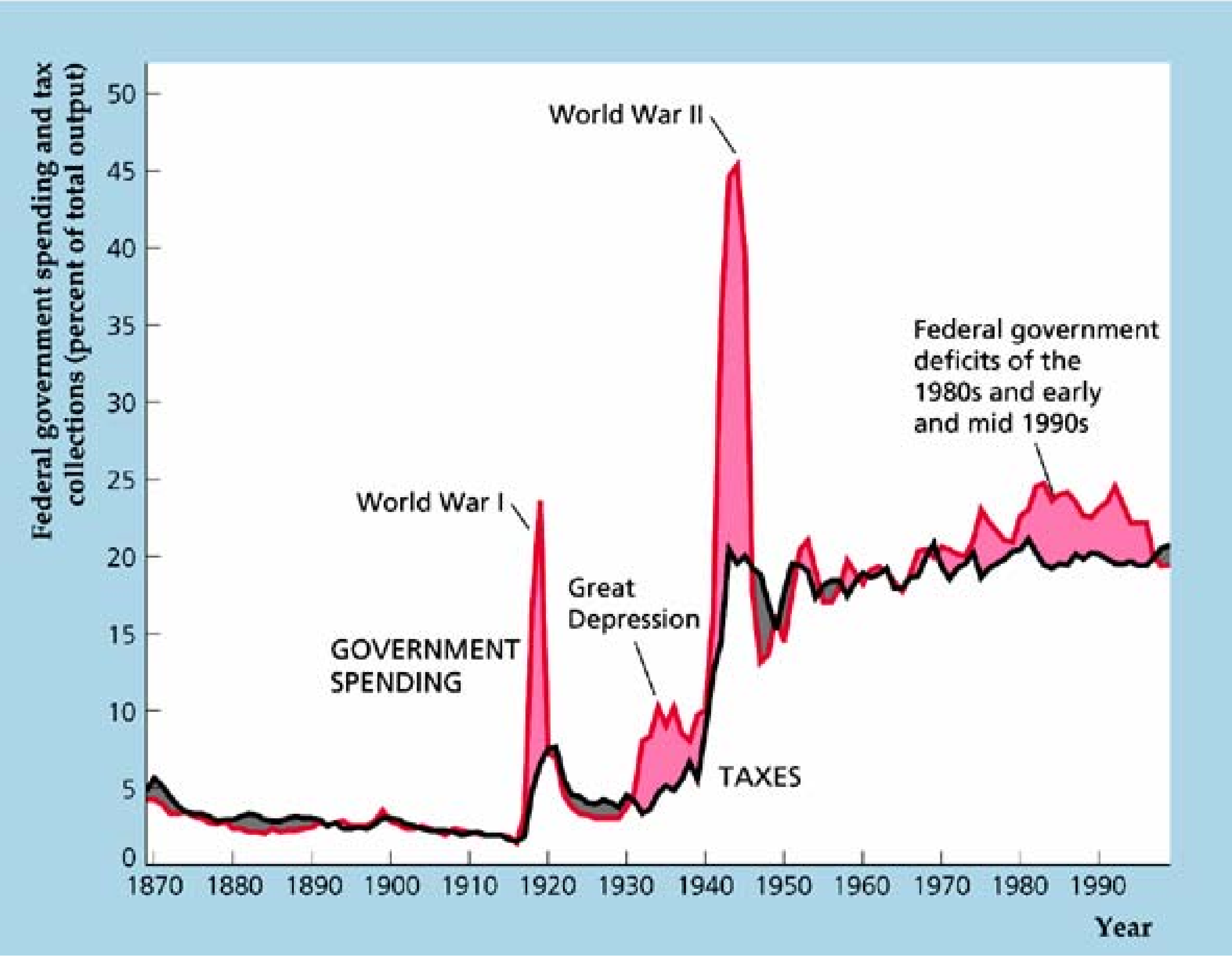
Application I: WWII and the Increase in G

- During WWII government spending to finance the war effort increased to levels unseen previously in the US.

- What are the predictions of the model for this increase in spending?

- The assumption that government spending is a pure loss of output arguably makes sense here. Pure spending/diversion of resources in short run. Positive effects more long-run and harder to measure.

Figure 1.06 U.S. Federal government spending and tax collections, 1869-1999



- Preferences $u(c, l) = \log c + \gamma\,(1 - l)$.

- Cobb-Douglas technology.

- Production possibilities (goods market):

$$c = y - G = Ak^{\alpha}l^{1-\alpha} - G$$

- To simplify $g = G/y$. So:

$$c = (1 - g)Ak^{\alpha}l^{1-\alpha}.$$

- Household utility maximization:

$$\frac{u_l(c, l)}{u_c(c, l)} = \frac{\gamma}{1/c} = w \Rightarrow \gamma c = w$$

- Firm profit maximization:

$$(1-\alpha)\,Ak^{\alpha}l^{-\alpha} = w$$

- Equate and impose goods market clearing:

$$\begin{aligned}
\gamma c &= (1-\alpha)\,Ak^{\alpha}l^{-\alpha} \\
\Rightarrow \gamma(1-g)Ak^{\alpha}l^{1-\alpha} &= (1-\alpha)\,Ak^{\alpha}l^{-\alpha} \\
\Rightarrow l &= \frac{1-\alpha}{\gamma(1-g)}
\end{aligned}$$

- Government spending has a pure income effect here (since financed by lump sum taxes). Increases labor supply.

- Solve for rest of allocation:

$$
\begin{aligned}
y &= Ak^{\alpha}\left[\frac{1-\alpha}{\gamma(1-g)}\right]^{1-\alpha} \\
c &= (1-g)y = (1-g)^{\alpha}k^{\alpha}\left[\frac{1-\alpha}{\gamma}\right]^{1-\alpha}
\end{aligned}
$$

- Output increases with g, consumption decreases.

- Solve for wages and interest rates:

$$
\begin{aligned}
w &= (1-\alpha)\,Ak^{\alpha}\left[\frac{1-\alpha}{\gamma(1-g)}\right]^{-\alpha} \\
r &= \alpha Ak^{\alpha-1}\left[\frac{1-\alpha}{\gamma(1-g)}\right]^{1-\alpha}
\end{aligned}
$$

- Wages decrease with g, interest rates increase.

Summing Up

- Following increase in $g = G/Y$, the model predicts an increase in (y, l, r), decrease in (c, w).

- Private consumption spending is "crowded out" by increased government spending.

- Output increases but loss of welfare as both $c, 1 - l$ fall.

- These predictions match US experience of WWII.

Figure 5.7 GDP, Consumption, and Government Expenditures

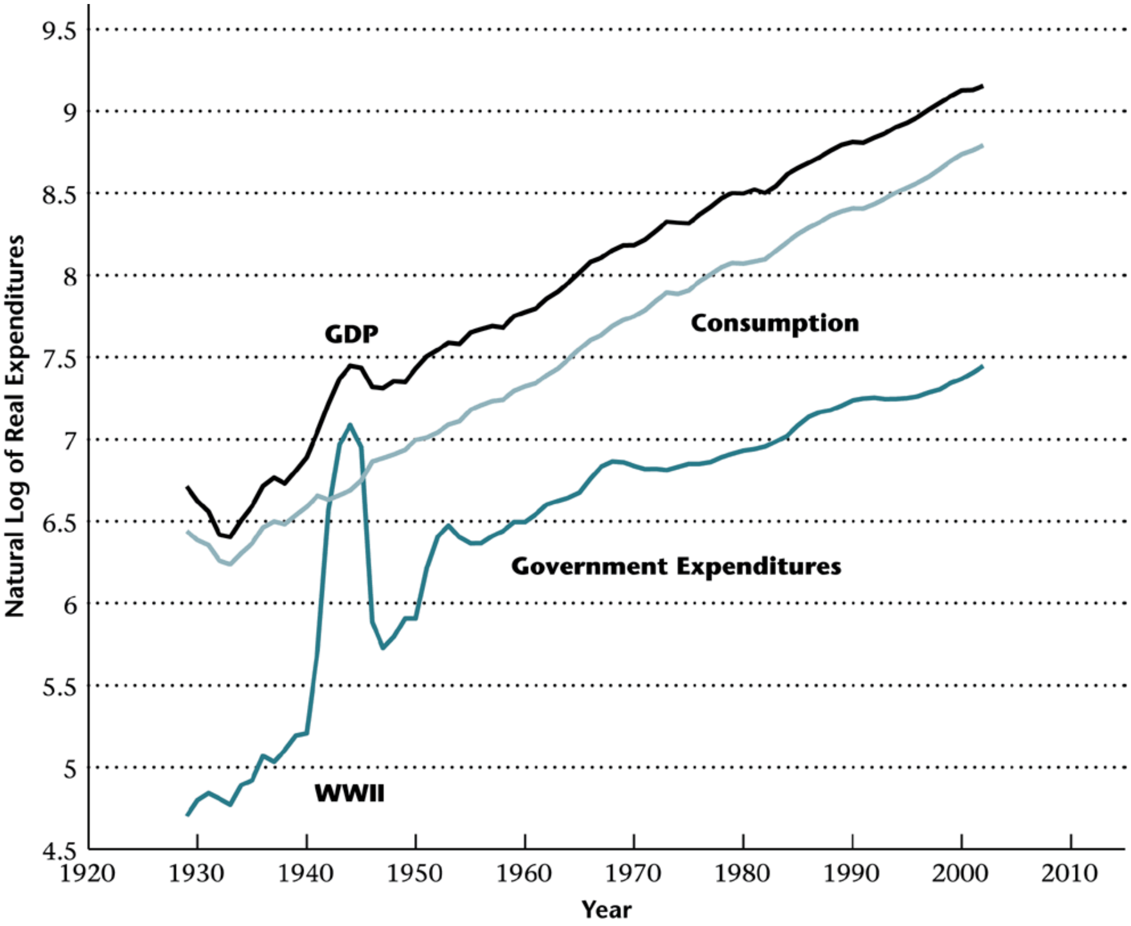


Figure 15.04 Deficits and primary deficits: Federal, state, and local, 1940-1998

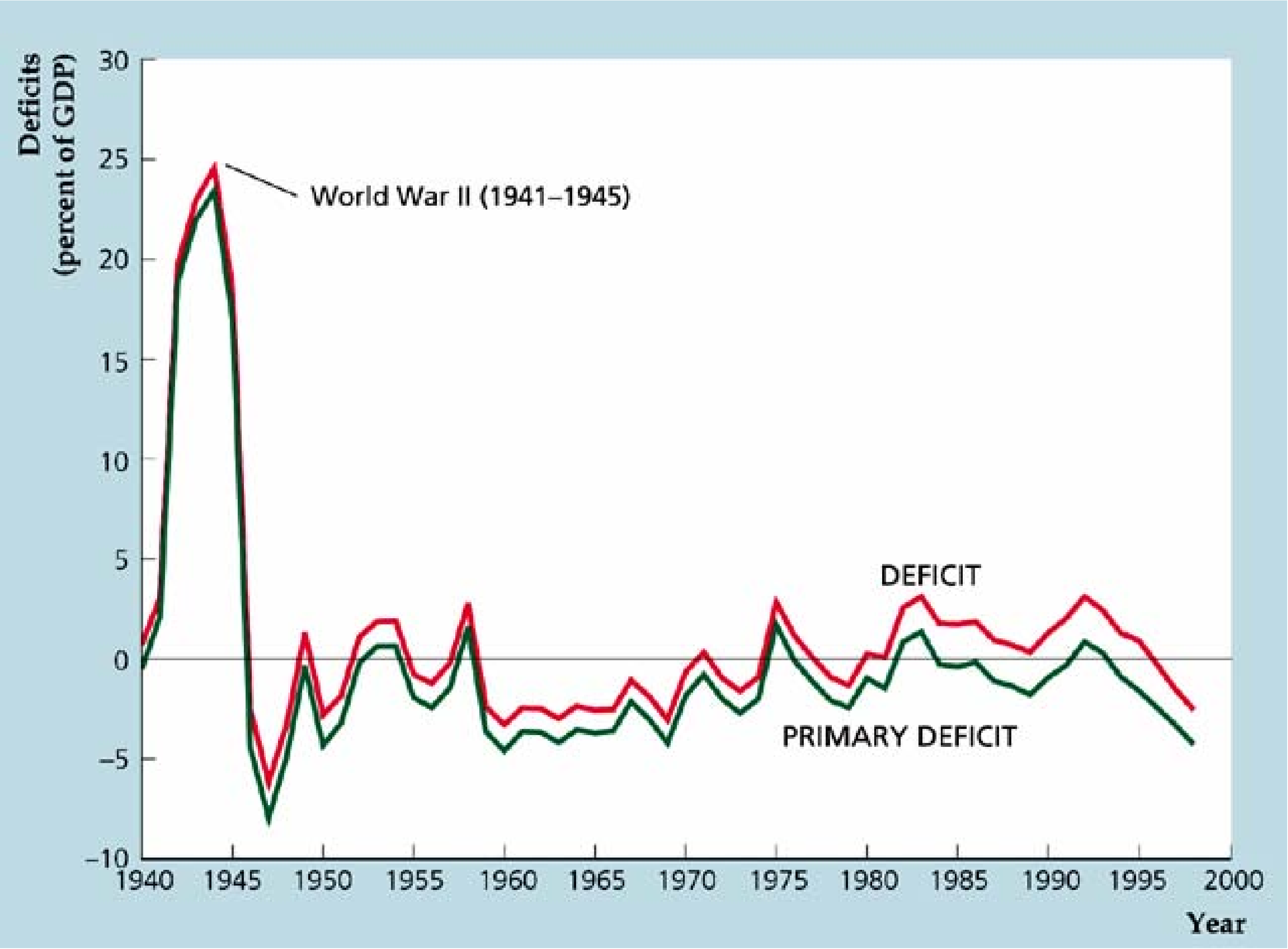
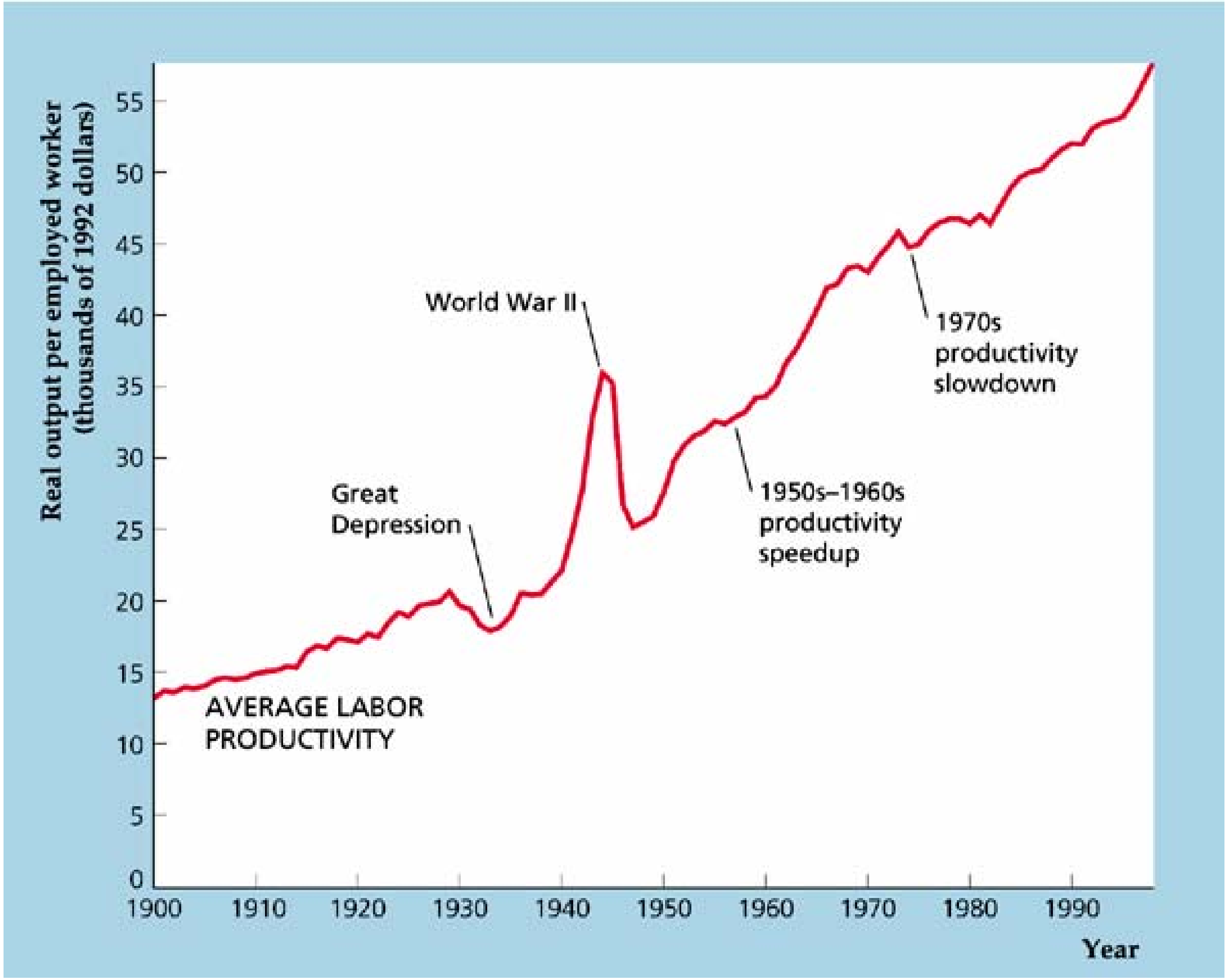


Figure 1.02 Average labor productivity in the United States, 1900-1998



What Does This Analysis Miss?

- Government debt. A large fraction of the wartime spending was financed by government debt.
  Deficit/GDP ratio hit 24% by 1944.

- Debt allows for intertemporal substitution of resources and smoothing burden of taxation. If needed to increase (distortionary) taxes to finance full war spending, production would have been less.

- Increased productivity. Wartime mobilization of production increased labor productivity dramatically.

- Led to larger increase in production than our model suggests.

Application II: Skill Biased Technical Change and Inequality

- Large literature documenting increase in income and wealth inequality in the US.

- Started in the 1970s and continues today.

- At same time, has been a large increase in the returns to education:

  1. Average wages of college graduates from increased by 60% for males and 90% for females from 1963 to 2002.

  2. Average wages of high school graduates only increased by 20% for males and 50% for females over same period.

Figure 4 Men's Earnings by Quantiles

Index of Real Wages of Full-Time Full-Year Men
Ages 22–65 by Specific Percentiles

Index, 1961=100

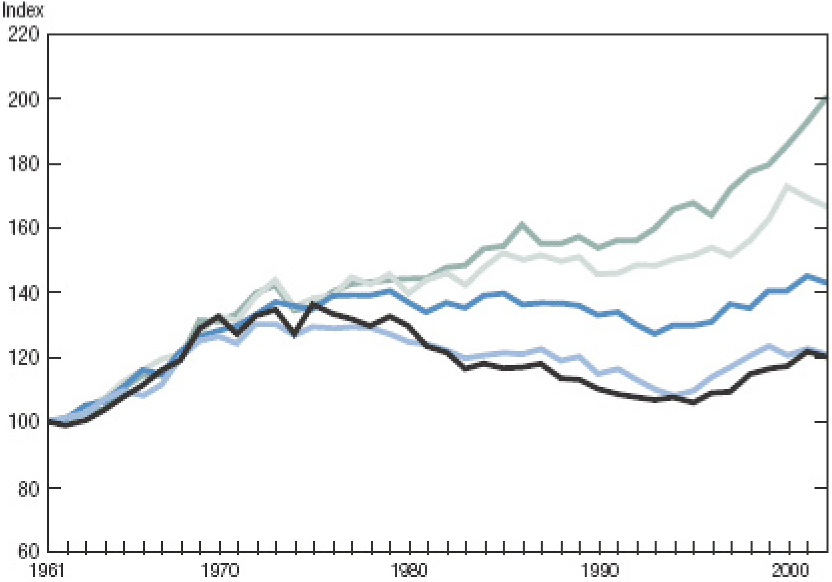


Figure 5 Women's Earnings by Quantiles

Index of Real Wages of Full-Time Full-Year Women
Ages 22–65 by Specific Precentiles

Index, 1981=100

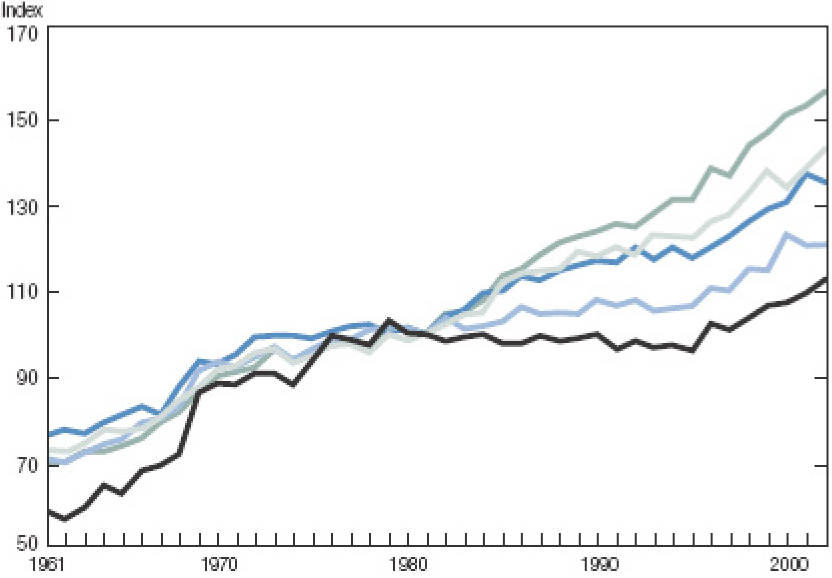


Figure 6 Men's Earnings by Education

Index of Mean of Real Wages of Full-Time Full-Year Men
Ages 22–65 by Education Group

Index, 1963=100

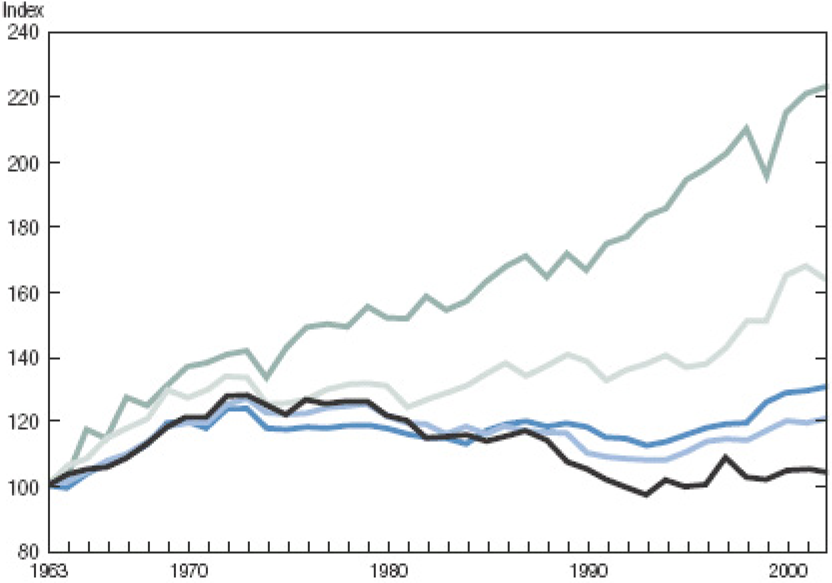
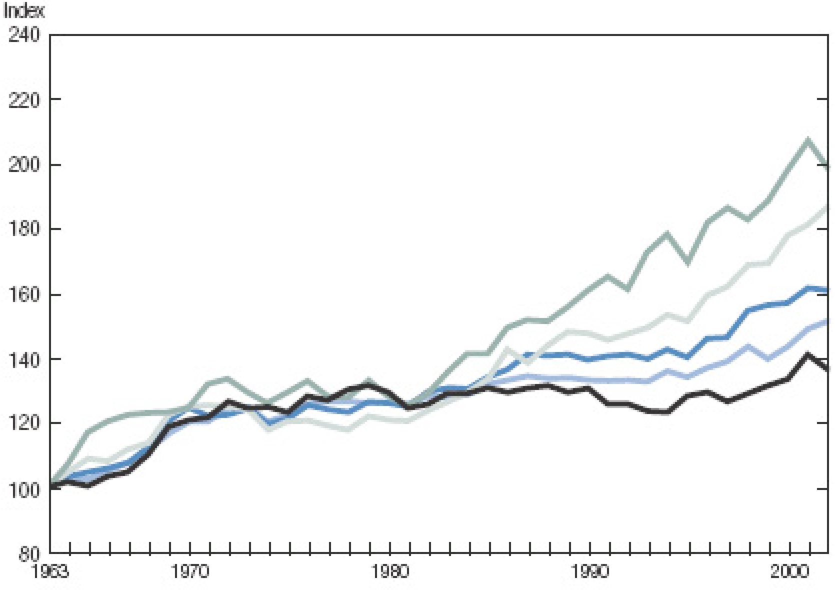


Figure 7 Women's Earnings by Education

Index of Mean of Real Wages of Full-Time Full-Year Women Ages 22–65 by Education Group

Index, 1963=100



Main explanation:

- Skill-biased technical change.
- Skilled and unskilled labor are effectively different labor markets.
- Productivity changes have increased the relative demand for skilled labor.

Analysis of Skill-Biased Change

- Extend the previous to two types of households: skilled and unskilled.

- Households: Assume both skilled and unskilled workers have same preferences:

$$u(c, l) = c - \frac{l^2}{2}$$

- Assume skilled workers own a share β of the capital stock, unskilled a share $(1 - \beta)$.

- Wages w_s for skilled w_u for unskilled.

Skilled Household Problem

$$\max_{l_s}\left\{l_s w_s + \beta r K - \frac{{l_s}^2}{2}\right\}$$

Optimality conditions:

$$-\frac{u_l}{u_c} = l_s = w_s$$

- So $l_s = w_s$, $c_s = w_s^2 + \beta r K$.
- Unskilled household problem is equivalent:

  $l_u = w_u$, $c_u = w_u^2 + (1-\beta)\, rK$.

Firms

- Assume representative firm hires both skilled and unskilled labor. Each has different productivity (z_s, z_u).

- Firm substitutes between skilled and unskilled for total labor input.

$$l = (z_s l_s^\rho + z_u l_u^\rho)^{1/\rho}$$

  where $0 < \rho < 1$.

- Thus production is:

$$y = k^\alpha l^{1-\alpha}$$

- Firms maximize profits:

$$k^{\alpha} l(l_s, l_u)^{1-\alpha} - rk - w_s l_s - w_u l_u$$

- FOC's – each type paid its marginal product:

$$\begin{aligned}(1-\alpha)k^{\alpha} l^{-\alpha} \left(z_s l_s^{\rho} + z_u l_u^{\rho}\right)^{1/\rho - 1} z_s l_s^{\rho-1} &= w_s \\ (1-\alpha)k^{\alpha} l^{-\alpha} \left(z_s l_s^{\rho} + z_u l_u^{\rho}\right)^{1/\rho - 1} z_u l_u^{\rho-1} &= w_u\end{aligned}$$

Characterize Equilibrium

- Divide firm FOC's:

$$\begin{aligned}\frac{w_s}{w_u} &= \frac{z_s l_s^{\rho-1}}{z_u l_u^{\rho-1}} \Rightarrow \\ \log\frac{w_s}{w_u} &= \log\frac{z_s}{z_u} + (\rho-1)\log\frac{l_s}{l_u}\end{aligned}$$

- $\frac{w_s}{w_u}$, the skilled premium depends on:

1. Relative productivities: $\frac{zs}{zu}$.
2. Relative supplies: $\frac{l_s}{l_u}$

- Then:

$$\gamma_{\frac{w_s}{w_u}} = \gamma_{\frac{z_s}{z_u}} + (\rho - 1)\,\gamma_{\frac{l_s}{l_u}}$$

- Changes in skill-premium depend on:

  1. Relative changes in productivities.

  2. Relative changes in abundance of factors.

Why Did it Happen?

- Nature of modern science.
- Size of the Market.
- Economics of Superstars.

What Does This Analysis Miss?

- Captures broad aggregate facts. Misses on some dimensions.

- Consumption Inequality. Evidence that inequality in consumption was less than inequality in income (Krueger and Perri, 2003). Here we have greater consumption inequality (since $\beta \approx 1$) :

$$\frac{c_s}{c_u} = \frac{w_s^2 + \beta r K}{w_u^2 + (1-\beta) r K} \approx \left(\frac{w_s}{w_u}\right)^2 + \frac{rK}{w_u^2}$$

- Changes by Gender. Most dramatic effects have been increase in female labor supply, especially in skilled labor. Hard to argue this was all from skill-biased technical change. Composition effects may be more important. (Eckstein and Nagypal, 2004)

Welfare Theorems

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Pareto Optimality

- An allocation is *Pareto Optimal* if there is no way to rearrange production or reallocate goods so that someone is made better off without making someone else worse off.

- Pareto Optimality $\neq$ perfect state of the world or any concept like that.

The Social Planner

- Let us imagine we have a powerful dictator, the Social Planner, that can decide how much the households consume and work and how much the firms produce.

- The Social Planner does not follow prices. But it understands opportunity cost.

- The Social Planner is benevolent. It searches for the best possible allocation.

Social Planner's Problem I

- Maximizes utility household given a level of government purchases G^*

$$\max_{c,l} u\left(c, 1-l\right)$$

such that

$$\begin{aligned} c + G &= Ak^{\alpha}l^{1-\alpha} \\ G &= G^* \\ k &= k^* \end{aligned}$$

- Note: we do not have prices in the budget constraint!!!

- Standard Maximization problem.

Social Planner's Problem II

- We can rewrite the problem as:

$$\max_{l} u\left(Ak^{*\alpha}l^{1-\alpha} - G^{*}, 1 - l\right)$$

- First Order Condition with respect to l:

$$u'_{c}\left(Ak^{*\alpha}l^{1-\alpha} - G^{*}, 1 - l\right)(1-\alpha)\, Ak^{*\alpha}l^{-\alpha}$$

$$-u'_{1-l}\left(Ak^{*\alpha}l^{1-\alpha} - G^{*}, 1 - l\right) = 0$$

Social Planner's Problem III

- We rearrange as:

$$\frac{u'_{1-l}\left(Ak^{*\alpha}l^{1-\alpha} - G^*, 1-l\right)}{u'_c\left(Ak^{*\alpha}l^{1-\alpha} - G^*, 1-l\right)} = (1-\alpha)\, Ak^{*\alpha}l^{-\alpha}$$

- The lhs is the Marginal Rate of Substitution, MRS while the rhs is the Marginal Rate of Transformation, MRT.

- Thus, optimality implies:

$$MRS = MRT$$

- Let's look at it graphically.

The Big Question

- What is the relation between the solution to the Planners Problem and the Competitive Equilibrium?

- Or equivalently, is the Competitive Equilibrium Pareto-Optimal?

- Why do we care about this question?

  1. Positive reasons

  2. Normative reasons.

The Intuition

- First think about the case when $G^* = \tau^l = 0$

- Look again at the Social Planner's optimality condition

$$\frac{u'_{1-l}\left(Ak^{*\alpha}l^{1-\alpha}, 1-l\right)}{u'_c\left(Ak^{*\alpha}l^{1-\alpha}, 1-l\right)} = (1-\alpha)\, Ak^{*\alpha}l^{-\alpha}$$

- Remember that the Household first order condition was:

$$\frac{u'_{1-l}\left(Ak^{*\alpha}l^{1-\alpha} - G^*, 1-l\right)}{u'_c\left(Ak^{*\alpha}l^{1-\alpha} - G^*, 1-l\right)} = w$$

- And that firms profit maximization implied:

$$w = (1-\alpha)\, A k^{*\alpha} l^{-\alpha}$$

- First order conditions are equivalent!!!

The Formal Statement

- First Fundamental Welfare Theorem: under certain conditions, the Competitive Equilibrium is Pareto Optimal.

- We have the converse.

- Second Fundamental Welfare Theorem: under certain conditions, a Pareto optimum is a Competitive Equilibrium.

Some consequences

- First Fundamental Welfare Theorem states that, under certain conditions, an allocation achieved by a market economy is Pareto-Optimal.

- Formalization of Adam Smith's "invisible hand" idea.

- Strong theoretical point in favour of decentralized allocation mechanisms: prices direct agents to do what is needed to get a Pareto optimum.

- Second Fundamental Welfare Theorem states what is the best way to change allocations: redistribute income. Do not mess with prices!!!

How robust is the First Welfare theorem?

- Not too much.

- Plenty of reasons that deviate the allocation from a Pareto optimum:

  1. Taxes.
  2. Externalities.
  3. Asymmetric Information.
  4. Market Incompleteness.
  5. Bounded Rationality of Agents.

What if taxes are not zero?

- Now think about the case when $G^* \neq 0, \tau^l \neq 0$

- Look again at the Social Planner's optimality condition

$$\frac{u'_{1-l}\left(Ak^{*\alpha}l^{1-\alpha}, 1-l\right)}{u'_c\left(Ak^{*\alpha}l^{1-\alpha}, 1-l\right)} = (1-\alpha)\, Ak^{*\alpha}l^{-\alpha}$$

- But now the Household first order condition is:

$$\frac{u'_{1-l}\left(Ak^{*\alpha}l^{1-\alpha} - G^*, 1-l\right)}{u'_c\left(Ak^{*\alpha}l^{1-\alpha} - G^*, 1-l\right)} = \left(1-\tau^l\right) w$$

- And since that firms profit maximization implied:

$$w = (1 - \alpha) A k^{*\alpha} l^{-\alpha}$$

- First order conditions are NOT equivalent!!!

Externalities

- What is an externality? When an agents consumption or production decision changes the production or consumption possibilities of other agents.

- Externalities can be good or bad.

- Example:

  1. Cities

  2. Environment

Asymmetric Information

- Information is dispersed in society.

- We may want to change our behavior based on the information we have.

- Akerlof-Spence-Stiglitz, Nobel Prize Winners 2001.

- Townsend and Prescott (1985).