Macroeconometric Modeling

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- **Has Macro Progressed?**. Page 3.
- **Nonlinear Optimization Algorithms**. Page 5.
- **Single Equation Estimation**. Page 6.
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- **FIML and 3SLS Estimates of the US Model**. Page 126.
- **The Fair-Parke Program**. Page 54.
- **References in Fair (1984)**. Page 457.

Overview

This document, including the links in it, is a discussion of macroeconometric modeling. There are seven parts. The first five are: 1) methodology, 2) econometric techniques, 3) a particular application—the MC model, 4) properties of the MC model, and 5) the use of the MC model to analyze the economy. The sixth part presents the equations for the U.S. part of the MC model (Appendix A), and the seventh part presents the equations for the rest-of-the-world part (Appendix B). A complement to this document is the **User's Guide to the Fair-Parke Program**. Many of the results in this document can be duplicated using the Fair-Parke (FP) program and related files.

This document encompasses much of my research in macroeconometrics. I have taken some discussion word for word, with footnotes on where the discussion is from. In some cases I have simply linked to a past article or pages in a book with no added discussion, where the links are meant to be part of this document. For example, most of Part 1, Macroeconomic Methodology, is simply two links. Regarding the references in the links, if the link consists of an article in its entirety, the references are at the end of the article rather than at the end of this document. Many of the links are to pages in Fair (1984), and the references in this material are at the end of this document in a separate link. For links that are neither complete articles nor pages in Fair (1984) the references are just part of the overall references at the end of this document.

You will see that the following discussion is as much about analyzing the economy as it is about discussing macroeconometric techniques. The end result of macroeconometric modeling is to use the techniques to understand how the economy works, and this has been an important part of my research.

The notation regarding sections is the following. Within each part there are sections and subsections. For example, within Part 2 there is a section 2.3, with subsections 2.3.1 and 2.3.2. Within each section the equations are numbered (1), (2), etc., and the tables are numbered 1, 2, etc. When a new section (but not subsection) begins, the numbering of the equations and tables starts over. When links to previous material are used, the numbering is whatever is used in the material.

1 Macroeconomic Methodology

1.1 The Cowles Commission Approach

The methodology followed in the construction of macroeconometric models is what will be called here the "Cowles Commission approach." This approach began with Tinbergen's (1939) model building in the late 1930s. Theory is used to guide the choice of left-hand-side and right-hand-side variables for the stochastic equations in a model, and the resulting equations are estimated using a consistent estimation technique—for example, two-stage least squares (2SLS). Sometimes restrictions are imposed on the coefficients in an equation, and the equation is then estimated with these restrictions imposed. It is generally not the case that all the coefficients in a stochastic equation are chosen ahead of time and thus no estimation done. In this sense the methodology is empirically driven and the data rule.

Typical theories for macro models are that households behave by maximizing expected utility and that firms behave by maximizing expected profits. In the process of using a theory to guide the specification of an equation to be estimated, there can be much back and forth movement between specification and estimation. If, for example, a variable or set of variables is not significant or a coefficient estimate is of the wrong expected sign, one may go back to the specification for possible changes. Because of this, there is always a danger of data mining—of finding a statistically significant relationship that is in fact spurious. Testing for misspecification is thus (or should be) an important component of the methodology.

There are generally from the theory many exclusion restrictions for each stochastic equation, and so identification is rarely a problem—at least based on the theory used.

The transition from theory to empirical specifications is also not always straightforward. The quality of the data is never as good as one might like, so compromises have to be made. Also, extra assumptions usually have to be made for the empirical specifications, in particular about unobserved variables like expectations and about dynamics. There usually is, in other words, considerable "theorizing" involved in this transition process. In many cases future expectations of a variable are assumed to be adaptive—to depend on a few lagged values of the variable itself, and in many cases this is handled by simply adding lagged variables to the equation being estimated. When this is done, it is generally not possible to distinguish partial adjustment effects from expectation effects—both lead to lagged variables being part of the set of explanatory variables.

The first of the following two links is the paper Fair (1992). It discuses the Cowles Commission approach and how it relates to real business cycle theories and new-Keynesian economics. The second is the paper Fair (2012a). It discusses how the Cowles Commission approach relates to the dynamic stochastic general equilibrium (DSGE) methodology.

1.1.1 The Cowles Commission Approach, Real Business Cycle Theories, and New-Keynesian Economics

The first link is: **The Cowles Commission Approach, Real Business Cycle Theories, and New-Keynesian Economics**.

1.2 Has Macro Progressed?

The second link is: **Has Macro Progressed?**

The results on page 10 of this paper are for the January 30, 2010, version of the MC model. For the current version (November 11, 2013) the results are:

1. Increase in federal purchases of goods: Real GDP: old (2.0, 1.8, 1.0), new (1.6, 1.4, 1.2). GDP deflator: old (0.5, 1.0, 1.0), new (0.4, 0.8, 1.2). Bill rate: old (0.8, 0.9, 0.4), new (0.7, 0.7, 0.6). Unemployment rate: old (1.0, 1.0, 0.3), new (0.7, 0.6, 0.4).

2. Increase in transfer payments: Real GDP: old (1.0, 1.1, 0.4), new (0.7, 0.8, 0.6).

3. Increase in bill rate: Real GDP: old (0.4, 0.7, 0.2), new (0.4, 0.6, 0.3).

4. Increase in capital gains: Nominal GDP: old (0.26, 0.50, 0.29), new (0.38, 0.65, 0.38). Real GDP: old (0.21, 0.32, -0.03), new (0.31, 0.43, 0.02).

5. Price shock: Real GDP: old (0.18, 0.42, 0.85), new (0.17, 0.53, 1.11).

6. Dollar depreciation: Real GDP: old (0.39, 0.39, 0.12), new (0.24, 0.41, 0.23). GDP Deflator: old (1.35, 2.11, 3.20), new (0.80, 1.32, 2.59).

2 Econometric Techniques

2.1 The General Model

The general non rational expectations model considered in this document is dynamic, nonlinear, and simultaneous:

$$f_i(y_t, y_{t-1}, \ldots, y_{t-p}, x_t, \alpha_i) = u_{it}, \quad i = 1, \ldots, n, \quad t = 1, \ldots, T, \tag{1}$$

where y_t is an n–dimensional vector of endogenous variables, x_t is a vector of exogenous variables, and α_i is a vector of coefficients. The first m equations are assumed to be stochastic, with the remaining equations identities. The vector of error terms, $u_t = (u_{1t}, \ldots, u_{mt})'$, is assumed to be *iid*. The function f_i may be nonlinear in variables and coefficients. u_i will be used to denote the T–dimensional vector $(u_{i1}, \ldots, u_{iT})'$.

This specification is fairly general. It includes as a special case the VAR model. It also incorporates autoregressive errors. If the original error term in equation i follows a rth order autoregressive process, say $w_{it} = \rho_{1i}w_{it-1} + \ldots + \rho_{ri}w_{it-r} + u_{it}$, then equation i in the model in (1) can be assumed to have been transformed into one with u_{it} on the right hand side. The autoregressive coefficients $\rho_{1i}, \ldots, \rho_{ri}$ are incorporated into the α_i coefficient vector, and additional lagged variable values are introduced. This transformation makes the equation nonlinear in coefficients if it were not otherwise, but this adds no further complications because the model is already allowed to be nonlinear. The assumption that u_t is *iid* is thus not as restrictive as it would be if the model were required to be linear in coefficients.

The general rational expectations (RE) version of the model is

$$\begin{array}{c} f_i(y_t, y_{t-1}, \ldots, y_{t-p}, E_{t-1}y_t, E_{t-1}y_{t+1}, \ldots, E_{t-1}y_{t+h}, x_t, \alpha_i) = u_{it} \\ i = 1, \ldots, n, \quad t = 1, \ldots, T, \end{array} \tag{2}$$

where E_{t-1} is the conditional expectations operator based on the model and on information through period $t-1$. The function f_i may be nonlinear in variables, parameters, and expectations. The model in (2) will be called the "RE model." In the following discussion the non RE model is considered first. The RE model is discussed in Section 2.12.

For the non RE model in (1) the 2SLS estimate of α_i is obtained by minimizing

$$S_i = u_i' Z_i (Z_i' Z_i)^{-1} Z_i' u_i \tag{3}$$

with respect to α_i, where Z_i is a $T \times K_i$ matrix of first stage regressors. The first stage regressors are assumed to be correlated with the right-hand-side endogenous variables in the equation but not with the error term.

2.2 Nonlinear Optimization Algorithms

A number of econometric techniques require the use of numerical nonlinear optimization algorithms. The following link, which is part of Chapter 2 in Fair (1984), discusses some of these algorithms, particularly the DFP algorithm. The link is **Nonlinear Optimization Algorithms**.

2.3 Single Equation Estimation

2.3.1 Non Time Varying Coefficients

The following link is part of Chapter 6 in Fair (1984). The estimators discussed are ordinary least squares (OLS), two-stage least squares (2SLS), least absolute deviations (LAD), and two-stage least absolute deviations (2SLAD). Serial correlation of the error terms is considered, as is the case of nonlinearity in the coefficients. The notation in Chapter 6 is the same as the notation above with one exception. In Chapter 6 the vector x_t is taken to include x_t above as well as the lagged endogenous variables, $y_{t-1}, \ldots, y_{t-p}$. The model in (1) in Section 2.1 is thus written:

$$f_i(y_t, x_t, \alpha_i) = u_{it}, \quad i = 1, \ldots, n, \quad t = 1, \ldots, T.$$

Reference in this discussion is made to the DFP algorithm, which is discussed in the link above. The 2SLS method with serial correlation discussed in this link was originally proposed in Fair (1970). The LAD and 2SLAD methods were originally discussed in Fair (1974c).

The link is **Single Equation Estimation**.

2.3.2 Time Varying Coefficients

The above discussion of single equation estimation does not consider the case of time varying coefficients. It is hard to deal with this case when using macro data because the variation in the data is generally not large enough to allow more than a few coefficients to be estimated per equation with any precision. Postulating time varying coefficients introduces more coefficients to estimate per equation, which can be a problem. A method is proposed in this subsection for dealing with one type of time varying coefficients that may be common in macro equations. The method is used in Section 3.6 for some of the U.S. equations in the MC model.

A common assumption in the time varying literature is that coefficients follow random walks—see, for example, Stock and Watson (1998). This assumption is problematic in macro work since it does not seem likely that macroeconomic relationships change via random walk coefficients or similar assumptions. It seems more likely that they change in slower, perhaps trend like, ways. Also, it seems unlikely that changes take place over the entire sample period. If there is a change, it may begin after the beginning of the sample period and end before the end of the sample period. The assumption used here postulates no change for a while, then smooth trend change for a while, and then no change after that. The assumption can be applied to any number of coefficients in an equation, although it is probably not practical with macro data to deal with more than one or two coefficients per equation.

In the discussion in this subsection the notation will depart from the notation used for the general model. Assume that the equation to be estimated is:

$$y_t = \beta_t + X_t\alpha + u_t, \qquad t = 1, ..., T \tag{4}$$

β_t is a time varying scalar, α is a vector, and the vector X_t can include endogenous and lagged endogenous variables. Define T_1 to be $\pi_1 T$ and T_2 to be $\pi_2 T$, where $0 < \pi_1 < \pi_2 < 1$. It is assumed that

$$\beta_t = \begin{cases} \gamma & : \; 1 \le t < T_1 \\ \gamma + \frac{\delta}{T_2 - T_1}(t - T_1) & : \; T_1 \le t \le T_2 \\ \gamma + \delta & : \; t > T_2 \end{cases} \tag{5}$$

$\delta/(T_2 - T_1)$ is the amount that β_t changes per period between T_1 and T_2. Before T_1, β_t is constant and equal to γ, and after T_2, it is constant and equal to $\gamma + \delta$. The parameters to estimate are α, γ, δ, π_1, and π_2. There are thus two parameters to estimate per changing coefficient, γ and δ, plus π_1 and π_2. This specification is flexible in that it allows the point at which β_t begins to change and the point at which it ceases to change to be estimated. One could do this for any of the coefficients in α, at a cost of two rather than one parameter estimated and assuming that π_1 and π_2 are the same for all coefficients.

Assume that equation (4) is to be estimated by 2SLS using a $T \times K$ matrix Z as first stage regressors. This is simply a nonlinear 2SLS estimation problem. Given values of α, γ, δ, π_1, and π_2, u_t can be computed given data on y_t and X_t, $t = 1, \ldots, T$. The minimand is

$$S = u'Z(Z'Z)^{-1}Z'u \tag{6}$$

where $u = (u_1, \ldots, u_T)'$. The problem can thus be turned over to a nonlinear minimization algorithm like DFP. The estimated variance-covariance matrix of the coefficient estimates (including the estimates of π_1 and π_2) is the standard matrix for nonlinear 2SLS—see the discussion in the link in Subsection 2.3.1.

For the estimation of the U.S. equations in Section 3.6, I experimented with this technique using the constant term in the equation as the changing coefficient (as β_t above). In the end seven equations appeared to have time varying constant terms as judged by the significance of the estimate of δ. It also turned out that the estimates of π_1 and π_2 were fairly similar across the seven equations. For the final estimates of the model, π_1 and π_2 were taken to be the same for all seven equations. The overall sample period was 1954:1–2013:3, and π_1 was taken to be a T_1 at 1968:4 and π_2 was taken to be a T_2 at 1988.4. These periods were then taken to be fixed for all the estimates. (In the present case T_1 is 60 and T_2 is 140, where the sample period is 1 through 239.)

If T_1 and T_2 are fixed, the estimation is simple. In equation (4) γ is the coefficient of the constant term (the vector of one's) and δ is the coefficient of

$$C_{2t} = D_{2t}\frac{t - T_1}{T_2 - T_1} + D_{3t} \tag{7}$$

where D_{2t} is 1 between T_1 and T_2 and zero otherwise and D_{3t} is 1 after T_2 and 0 otherwise.

Finally, note that if β_t is the constant term and is changing over the whole sample period in the manner specified above, this is handled by simply adding the constant term and t as explanatory variables to the equation.

2.4 Full Information Estimation

The following link, which concerns full information estimation, is part of Chapter 6 in Fair (1984). The model and notation are the same as in the discussion of single equation estimation. The estimators discussed are three-stage least squares (3SLS) and full-information maximum likelihood (FIML). Serial correlation of the error terms and nonlinearity in the coefficients are covered. Sample size requirements and computational issues are discussed. The link is **Full Information Estimation**.

2.5 Solution

Once the α_i coefficients in the model in (1) in Section 2.1 have been estimated, the model can be solved. For a deterministic simulation the error terms u_{it} are set to zero. A dynamic simulation is one in which the predicted values of the endogenous variables for past periods are used as values for the lagged endogenous variables when solving for the current period. A solution also requires values of the exogenous variables for the solution period and values of the lagged endogenous variables up to the first period of the overall solution period.

The following link is part of Chapter 7 in Fair (1984). It discusses the solution of models and the use of the Gauss-Seidel technique. The link is **Solution and Gauss-Seidel Technique**.

2.6 Stochastic Simulation

Stochastic simulation differs from deterministic simulation in that the error terms are drawn from some distribution rather than simply set to zero. The following link is a discussion of stochastic simulation from Chapter 7 in Fair (1984). This includes a discussion of various numerical ways that error terms and coefficients can be drawn from distributions. The link is **Stochastic Simulation**.

The following are three additional points regarding the discussion in the above link. First, given the data from the repetitions, it is possible to compute the variances of the stochastic simulation estimates and thus to examine the precision of the estimates. The variance of $\tilde{\tilde{y}}_{itk}$ in equation (7.7) in the link is simply $\tilde{\sigma}^2_{itk}/J$, where $\tilde{\sigma}^2_{itk}$ is defined in equation (7.8). The variance of $\tilde{\sigma}^2_{itk}$, denoted $var(\tilde{\sigma}^2_{itk})$, is

$$var(\tilde{\sigma}^2_{itk}) = \left(\frac{1}{J}\right)^2 \sum_{j=1}^{J} [(\tilde{y}^j_{itk} - \tilde{\tilde{y}}_{itk})^2 - \tilde{\sigma}^2_{itk}]^2. \qquad (1)$$

Second, assumptions other than normality can be used in the analysis. Alternative assumptions about the distributions simply change the way the errors are drawn.

Third, it is possible to draw errors from estimated residuals rather than from estimated distributions. In a theoretical paper Brown and Mariano (1984) analyzed the procedure of drawing errors from the residuals for a static nonlinear econometric model with fixed coefficient estimates. For the stochastic simulation results in Fair (1998) errors were drawn from estimated residuals for a dynamic, nonlinear, simultaneous equations model with fixed coefficient estimates, and this may have been the first time this approach was used for such models. An advantage of drawing from estimated residuals is that no assumption has to be made about the distribution of the error terms. Drawing errors in this way is sometimes called "bootstrapping," to which we now turn.

2.7 Bootstrapping[1]

Drawing errors to analyze the properties of econometric models in macroeconomics was introduced in the seminal paper by Adelman and Adelman (1959). This procedure came to be called "stochastic simulation."

The bootstrap was introduced in statistics by Efron (1979).[2] Although the bootstrap procedure is obviously related to stochastic simulation, the literature that followed Efron's paper stressed the use of the bootstrap for estimation and the evaluation of estimators, not for evaluating models' properties. While there is by now a large literature on the use of the bootstrap in economics (as well as statistics), most of it has focused on small time series models. Good recent reviews are Li and Maddala (1996), Horowitz (1997), Berkowitz and Kilian (2000), and Härdle, Horowitz, and Kreiss (2001).

The main purpose of the discussion in this section is to integrate for the model in (1) in Section 2.1, namely a dynamic, nonlinear, simultaneous equations model, the bootstrap approach to evaluating estimators and the stochastic simulation approach to evaluating models' properties. The procedure in section 2.7.3 below for treating coefficient uncertainty has not been used before for this kind of a model. The model and notation used in this section refer to the model in (1) in Section 2.1.

The paper closest to the present discussion is Freedman (1984), who considered the bootstrapping of the 2SLS estimator in a dynamic, linear, simultaneous equations model. Runkle (1987) used the bootstrap to examine impulse response functions in VAR models, and Kilian (1998) extended this work to correct for bias. There is also work on bootstrapping GMM estimators (see, for example, Hall and Horowitz (1996)), but this work is of limited relevance here because it does not assume knowledge of a complete model.

In his review of bootstrapping MacKinnon (2002) analyzes an example of a linear simultaneous equations model consisting of one structural equation and one reduced form equation. He points out (p. 14) that "Bootstrapping even one equation of a simultaneous equations model is a good deal more complicated that bootstrapping an equation in which all the explanatory variables are exogenous or predetermined. The problem is that the bootstrap DGP must provide a way to generate all of the endogenous variables, not just one of them." In this section the process generating the endogenous variables is the complete model in (1) in Section 2.1.

This section does not provide the theoretical restrictions on the model in (1) in

[1] The discussion in this section is taken from Fair (2003a).

[2] See Hall (1992) for the history of resampling ideas in statistics prior to Efron's paper.

Section 2.1 that are needed for the bootstrap procedure to be valid. Assumptions beyond *iid* errors and the existence of a consistent estimator are needed, but these have not been worked out in the literature for the model considered here. This section simply assumes that the model meets whatever restrictions are sufficient for the bootstrap procedure to be valid. It remains to be seen what restrictions are needed beyond *iid* errors and a consistent estimator.

2.7.1 Distribution of the Coefficient Estimates

Initial Estimation

It is assumed that a consistent estimate of α is available, denoted $\hat{\alpha}$. This could be, for example, the 2SLS or 3SLS estimate of α. Given this estimate and the actual data, the vector of all the errors in the model, u, can be estimated. Let $\hat{u}$ denote the estimate of u after the residuals have been centered at zero.[3] Statistics of interest can be analyzed using the bootstrap procedure. These can include t-statistics of the coefficient estimates and possible χ^2 statistics for various hypotheses. τ will be used to denote the vector of estimated statistics of interest.

The Bootstrap Procedure

The bootstrap procedure for evaluating estimators for the model in (1) in Section 2.1 is:

1. For a given trial j, draw u_t^{*j} from $\hat{u}$ with replacement for $t = 1, \ldots, T$. Use these errors and $\hat{\alpha}$ to solve the model dynamically for $t = 1, \ldots, T$.[4] Treat the solution values as actual values and estimate α by the consistent estimator (2SLS, 3SLS, or whatever). Let $\hat{\alpha}^{*j}$ denote this estimate. Compute also the test statistics of interest, and let τ^{*j} denote the vector of these values.

2. Repeat step 1 for $j = 1, \ldots, J$.

Step 2 gives J estimates of each element of $\hat{\alpha}^{*j}$ and τ^{*j}. Using these values, confidence intervals for the coefficient estimates can be computed (see Subsection

[3]Freedman (1981) has shown that the bootstrap can fail for an equation with no constant term if the residuals are not centered at zero. If the residuals are centered at zero, $\hat{u}_{it}$, an element of $\hat{u}$, is $f_i(y_t, y_{t-1}, \ldots, y_{t-p}, x_t, \hat{\alpha}_i)$ except for the adjustment that centers the residuals at zero.

[4]This is just a standard dynamic simulation, where instead of using zero values for the error terms the drawn values are used.

2.7.2). Also, for the originally estimated value of any test statistic, one can see where it lies on the distribution of the J values.

Note that each trial generates a new data set. Each data set is generated using the same coefficient vector, $\hat{\alpha}$, but in general the data set has different errors for a period from those that existed historically. Note also that since the drawing is with replacement, the same error vector may be drawn more than once in a given trial, while others may not be drawn at all. All data sets are conditional on the actual values of the endogenous variables prior to period 1 and on the actual values of the exogenous variables for all periods.

2.7.2 Estimating Coverage Accuracy

Three confidence intervals are considered here.[5] Let β denote a particular coefficient in α. Let $\hat{\beta}$ denote the base estimate (2SLS, 3SLS, or whatever) of β, and let $\hat{\sigma}$ denote its estimated asymptotic standard error. Let $\hat{\beta}^{*j}$ denote the estimate of β on the jth trial, and let $\hat{\sigma}^{*j}$ denote the estimated asymptotic standard error of $\hat{\beta}^{*j}$. Let t^{*j} equal the t-statistic $(\hat{\beta}^{*j} - \hat{\beta})/\hat{\sigma}^{*j}$. Assume that the J values of t^{*j} have been ranked, and let t^*_r denote the value below which r percent of the values of t^{*j} lie. Finally, let $|t^{*j}|$ denote the absolute value of t^{*j}. Assume that the J values of $|t^{*j}|$ have been ranked, and let $|t^*|_r$ denote the value below which r percent of the values of $|t^{*j}|$ lie. The first confidence interval is simply $\hat{\beta} \pm 1.96\hat{\sigma}$, which is the 95 percent confidence interval from the asymptotic normal distribution. The second is $(\hat{\beta} - t^*_{.975}\hat{\sigma}, \hat{\beta} - t^*_{.025}\hat{\sigma})$, which is the equal-tailed percentile-t interval. The third is $\hat{\beta} \pm |t^*|_{.950}\hat{\sigma}$, which is the symmetric percentile-t interval.

The following Monte Carlo procedure is used to examine the accuracy of the three intervals. This procedure assume that the data generating process is the model in (1) in Section 2.1 with true coefficients $\hat{\alpha}$.

a. For a given repetition k, draw u^{**k}_t from $\hat{u}$ with replacement for $t = 1, \ldots, T$. Use these errors and $\hat{\alpha}$ to solve the model dynamically for $t = 1, \ldots, T$. Treat the solution values as actual values and estimate α by the consistent estimator (2SLS, 3SLS, or whatever). Let $\hat{\alpha}^{**k}$ denote this estimate. Use this estimate and the solution values from the dynamic simulation to compute the residuals, u, and center them at zero. Let $\hat{u}^{**k}$ denote the estimate of u after the residuals have been centered at zero.[6]

[5] See Li and Maddala (1996), pp. 118-121, for a review of the number of ways confidence intervals can be computed using the bootstrap. See also Hall (1988).

[6] From the model in (1) in Section 2.1, $\hat{u}^{**k}_{it}$, an element of $\hat{u}^{**k}$, is

b. Perform steps 1 and 2 in Subsection 2.7.1, where $\hat{u}^{**k}$ replaces $\hat{u}$ and $\hat{\alpha}^{**k}$ replaces $\hat{\alpha}$. Compute from these J trials the three confidence intervals discussed above, where $\hat{\beta}^{**k}$ replaces $\hat{\beta}$ and $\hat{\sigma}^{**k}$ replaces $\hat{\sigma}$. Record for each interval whether or not $\hat{\beta}$ is outside of the interval.

c. Repeat steps a and b for $k = 1, \ldots, K$.

After completion of the K repetitions, one can compute for each coefficient and each interval the percent of the repetitions that $\hat{\beta}$ was outside the interval. For, say, a 95 percent confidence interval, the difference between the computed percent and 5 percent is the error in coverage probability.

2.7.3 Analysis of Models' Properties

The bootstrap procedure is extended in this section to evaluating properties of models like the model in (1) in Section 2.1. The errors are drawn from the estimated residuals, which is contrary to what has been done in the previous literature except for Fair (1998). Also, the coefficients are estimated on each trial. In the previous literature the coefficient estimates either have been taken to be fixed or have been drawn from estimated distributions.

When examining the properties of models, one is usually interested in a period smaller than the estimation period. Assume that the period of interest is s through S, where $s \geq 1$ and $S \leq T$. The bootstrap procedure for analyzing properties is:

1. For a given trial j, draw u_t^{*j} from $\hat{u}$ with replacement for $t = 1, \ldots, T$. Use these errors and $\hat{\alpha}$ to solve the model in (1) in Section 2.1 dynamically for $t = 1, \ldots, T$. Treat the solution values as actual values and estimate α by the consistent estimator (2SLS, 3SLS, or whatever). Let $\hat{\alpha}^{*j}$ denote this estimate. Discard the solution values; they are not used again.

2. Draw u_t^{*j} from $\hat{u}$ with replacement for $t = s, \ldots, S$.[7] Use these errors and $\hat{\alpha}^{*j}$ to solve the model in (1) in Section 2.1 dynamically for $t = s, \ldots, S$. Record the solution value of each endogenous variable for each period. This simulation and the next one use the actual (historical) values of the variables prior to period s, not the values used in computing $\hat{\alpha}^{*j}$.

$f_i(y_t^{**k}, y_{t-1}^{**k}, \ldots, y_{t-p}^{**k}, x_t, \hat{\alpha}_i^{**k})$ except for the adjustment that centers the residuals at zero, where y_{t-h}^{**k} is the solution value of y_{t-h} from the dynamic simulation ($h = 0, 1, \ldots, p$).

[7] If desired, these errors can be the same errors drawn in step 1 for the s through S period. With a large enough number of trials, whether one does this or instead draws new errors makes a trivial difference. It is assumed here that new errors are drawn.

3. Multiplier experiments can be performed. The solution from step 2 is the base path. For a multiplier experiment one or more exogenous variables are changed and the model is solved again. The difference between the second solution value and the base value for a given endogenous variable and period is the model's estimated effect of the change. Record these differences.

4. Repeat steps 1, 2, and 3 for $j = 1, \ldots, J$.

5. Step 4 gives J values of each endogenous variable for each period. It also gives J values of each difference for each period if a multiplier experiment has been performed.

A distribution of J predicted values of each endogenous variable for each period is now available to examine. One can compute, for example, various measures of dispersion, which are estimates of the accuracy of the model. Probabilities of specific events happening can also be computed. If, say, one is interested in the event of two or more consecutive periods of negative growth in real output in the s through S period, one can compute the number of times this happened in the J trials. If a multiplier experiment has been performed, a distribution of J differences for each endogenous variable for each period is also available to examine. This allows the uncertainty of policy effects in the model to be examined.[8]

If the coefficient estimates are taken to be fixed, then step 1 above is skipped. The same coefficient vector ($\hat{\alpha}$) is used for all the solutions. Although in much of the stochastic simulation literature coefficient estimates have been taken to be fixed, this is not in the spirit of the bootstrap literature. From a bootstrapping perspective, the obvious procedure to follow after the errors have been drawn is to first estimate the model and then examine its properties, which is what the above procedure does. For estimating event probabilities, however, one may want to take the coefficient estimates to be fixed. In this case step 1 above is skipped. If step 1 is skipped, the question being asked is: conditional on the model, including the coefficient estimates, what is the probability of the particular event occurring?

[8]The use of stochastic simulation to estimate event probabilities was first discussed in Fair (1993b), where the coefficient estimates were taken to be fixed and errors were drawn from estimated distributions. Estimating the uncertainty of multiplier or policy effects in nonlinear models was first discussed in Fair (1980b), where both errors and coefficients were drawn from estimated distributions.

2.7.4 More on Estimating Event Probabilities[9]

The use of the procedure in the previous section to estimate event probabilities can be used for testing purposes. It is possible for a given event to compute a *series* of probability estimates and compare these estimates to the actual outcomes. Consider an event A_t, such as two consecutive quarters of negative growth out of five for the period beginning in quarter t. Let P_t denote a model's estimate of the probability of A_t occurring, and let R_t denote the actual outcome of A_t, which is 1 if A_t occurred and 0 otherwise. If one computes these probabilities for $t = 1, \ldots, T$, there are T values of P_t and R_t available, where each value of P_t is derived from a separate stochastic simulation.

To see how good a model is at estimating probabilities, P_t can be compared to R_t for $t = 1, \ldots, T$. Two common measures of the accuracy of probabilities are the quadratic probability score (QPS):

$$QPS = (1/T)\sum_{t=1}^{T} 2(P_t - R_t)^2 \tag{1}$$

and the log probability score (LPS):

$$LPS = -(1/T)\sum_{t=1}^{T}[(1 - R_t)\log(1 - P_t) + R_t \log P_t] \tag{2}$$

where T is the total number of observations.[10] It is also possible simply to compute the mean of P_t (say $\bar{P}$) and the mean of R_t (say $\bar{R}$) and compare the two means. QPS ranges from 0 to 2, with 0 being perfect accuracy, and LPS ranges from 0 to infinity, with 0 being perfect accuracy. Larger errors are penalized more under LPS than under QPS.

The testing procedure is thus simply to define various events and compute QPS and LPS for alternative models for each event. If model 1 has lower values than model 2, this is evidence in favor of model 1.

2.7.5 Bias Correction

Since 2SLS and 3SLS estimates are biased, it may be useful to use the bootstrap procedure to correct for bias. This is especially true for estimates of lagged dependent variable coefficients. It has been known since the work of Orcutt (1948)

[9] Some of the discussion in this subsection is taken from Fair (1993b).

[10] See, for example, Diebold and Rudebusch (1989).

and Hurwicz (1950) that least squares estimates of these coefficients are biased downward even when there are no right hand side endogenous variables.

In the present context a bias-correction procedure using the bootstrap is as follows.

1. From step 2 in Subsection 2.7.1 there are J values of each coefficient available. Compute the mean value for each coefficient, and let $\bar{\alpha}$ denote the vector of the mean values. Let $\gamma = \bar{\alpha} - \hat{\alpha}$, the estimated bias. Compute the coefficient vector $\hat{\alpha} - \gamma$ and use the coefficients in this vector to adjust the constant term in each equation so that the mean of the error terms is zero. Let $\tilde{\alpha}$ denote $\hat{\alpha} - \gamma$ except for the constant terms, which are as adjusted. $\tilde{\alpha}$ is then taken to be the unbiased estimate of α. Let θ denote the vector of estimated biases: $\theta = \hat{\alpha} - \tilde{\alpha}$.

2. Using $\tilde{\alpha}$ and the actual data, compute the errors. Denote the error vector as $\tilde{u}$. ($\tilde{u}$ is centered at zero because of the constant term adjustment in step 1.)

3. The steps in Subsection 2.7.3 can now be performed where $\tilde{\alpha}$ replaces $\hat{\alpha}$ and $\tilde{u}$ replaces $\hat{u}$. The only difference is that after the coefficient vector is estimated by 2SLS, 3SLS, or whatever, it has θ subtracted from it to correct for bias. In other words, subtract θ from $\hat{\alpha}^{*j}$ on each trial.[11]

[11] One could for each trial do a bootstrap to estimate the bias—a bootstrap within a bootstrap. The base coefficients would be $\hat{\alpha}^{*j}$ and the base data would be the generated data on trial j. This is expensive, and an approximation is simply to use θ on each trial. This is the procedure used by Kilian (1998) in estimating confidence intervals for impulse responses in VAR models. Kilian (1998) also does, when necessary, a stationary correction to the bias correction to avoid pushing stationary impulse response estimates into the nonstationary region. This type of adjustment is not pursued here.

2.8 Testing Single Equations

In Sections 3.6 and 3.7 a number of tests are performed on each estimated equation in the MC model. The tables in Appendices A and B present the results of these tests. The tests are as follows.

2.8.1 Chi-Square Tests

Many single equation tests are simply of the form of adding a variable or a set of variables to an equation and testing whether the addition is statistically significant. Let S_i^{**} denote the value of the minimand before the addition, let S_i^* denote the value after the addition, and let $\hat{\sigma}_{ii}$ denote the estimated variance of the error term after the addition. Under fairly general conditions, as discussed in Andrews and Fair (1988), $(S_i^{**} - S_i^*)/\hat{\sigma}_{ii}$ is distributed as χ^2 with k degrees of freedom, where k is the number of variables added. For the 2SLS estimator the minimand is defined in equation (3) in Section 2.1. Possible applications of the χ^2 test are the following.

Dynamic Specification (lags test)

Many macroeconomic equations include the lagged dependent variable and other lagged endogenous variables among the explanatory variables. A test of the dynamic specification of a particular equation is to add *further* lagged values to the equation and see if they are significant. If, for example, in equation 1 y_{1t} is explained by y_{2t}, y_{3t-1}, and x_{1t-2}, then the variables added are y_{1t-1}, y_{2t-1}, y_{3t-2}, and x_{1t-3}. If in addition y_{1t-1} is an explanatory variable, then y_{1t-2} is added. Hendry, Pagan, and Sargan (1984) show that adding these lagged values is quite general in that it encompasses many different types of dynamic specifications. Therefore, adding the lagged values and testing for their significance is a test against a fairly general dynamic specification. This test will be called the "lags" test.

The lags test also concerns the acceleration principle.[12] If, for example, the level of income is specified as an explanatory variable in an expenditure equation, but the correct specification is the change in income, then when lagged income is added as an explanatory variable with the current level of income included, the lagged value should be significant. If the lagged value is not significant, this is evidence against the use of the change in income.

[12] See Chow (1968) for an early analysis of the acceleration principle.

Time Trend (T test)

Long before unit roots and cointegration became popular, model builders worried about picking up spurious correlation from common trending variables. One check on whether the correlation might be spurious is to add the time trend to the equation. If adding the time trend to the equation substantially changes some of the coefficient estimates, this is cause for concern. A simple test is to add the time trend to the equation and test if this addition is significant. This test will be called the "T" test.

Serial Correlation of the Error Term (RHO test)

As noted in Section 2.1, if the error term in an equation follows an autoregressive process, the equation can be transformed and the coefficients of the autoregressive process can be estimated along with the structural coefficients. Even if, say, a first order process has been assumed and the first order coefficient estimated, it is still of interest to see if there is serial correlation of the (transformed) error term. This can be done by assuming a more general process for the error term and testing its significance. If, for example, the addition of a second order process over a first order process results in a significant increase in explanatory power, this is evidence that the serial correlation properties of the error term have not been properly accounted for. This test will be called the "RHO" test.

2.8.2 AP Stability Test

A useful stability test is the Andrews and Ploberger (AP) (1994) test. It does not require that the date of the structural change be chosen *a priori*. If the overall sample period is 1 through T, the hypothesis tested is that a structural change occurred between observations T_1 and T_2, where T_1 is an observation close to 1 and T_2 is an observation close to T.

The particular AP test considered here is as follows.

1. Compute the χ^2 value for the hypothesis that the change occurred at observation T_1. This requires estimating the equation three times—once each for the estimation periods 1 through $T_1 - 1$, T_1 through T, and 1 through T. Denote this value as $\chi^{2(1)}$. [13]

[13] When the 2SLS estimator is used, this χ^2 value is computed as follows. Let $S_i^{(1)}$ be the value of the minimand in equation (3) in Section 2.1 for the first estimation period, and let $S_i^{(2)}$ be the value for the second estimation period. Define $S_i^* = S_i^{(1)} + S_i^{(2)}$. Let S_i^{**} be the value of the

2. Repeat step 1 for the hypothesis that the change occurred at observation T_1+1. Denote this χ^2 value as $\chi^{2(2)}$. Keep doing this through the hypothesis that the change occurred at observation T_2. This results in $N = T_2 - T_1 + 1$ χ^2 values being computed—$\chi^{2(1)}, \ldots, \chi^{2(N)}$.

3. The Andrews-Ploberger test statistic (denoted AP) is

$$AP = \log[(e^{\frac{1}{2}\chi^{2(1)}} + \ldots + e^{\frac{1}{2}\chi^{2(N)}})/N].$$

   In words, the AP statistic is a weighted average of the χ^2 values, where there is one χ^2 value for each possible split in the sample period between observations T_1 and T_2.

Asymptotic critical values for AP are presented in Tables I and II in Andrews and Ploberger (1994). The critical values depend on the number of coefficients in the equation and on a parameter λ, where in the present context $\lambda = [\pi_2(1 - \pi_1)]/[\pi_1(1 - \pi_2)]$, where $\pi_1 = (T_1 - .5)/T$ and $\pi_2 = (T_2 - .5)/T$.

If the AP value is significant, it may be of interest to examine the individual χ^2 values to see where the maximum value occurred. This is likely to give one a general idea of where the structural change occurred even though the AP test does not reveal this in any rigorous way.

2.8.3 End-of-Sample Stability Test

In discussing the end-of-sample stability test in Andrews (2003) it will be useful to consider a specific example. Consider an equation that is estimated by 2SLS for the 1954:1–2013:3 period, 239 observations, observations 1 through 239. Say that one wants to test the null hypothesis that the coefficients in the equation are the same over the entire 1954:1–2013:3 period versus the alternative hypothesis that the coefficients are different before and after 2004:1, which is observation 201. There are thus 39 observations after the potential break point. If the potential break

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minimand in (3) when the equation is estimated over the full estimation period. When estimating over the full period, the Z_i matrix used for the full period must be the union of the matrices used for the two subperiods in order to make S_i^{**} comparable to S_i^*. This means that for each first stage regressor z_{it} two variables must be used in Z_i for the full estimation period, one that is equal to z_{it} for the first subperiod and zero otherwise and one that is equal to z_{it} for the second subperiod and zero otherwise. The χ^2 value is then $(S_i^{**} - S_i^*)/\hat{\sigma}_{ii}$, where $\hat{\sigma}_{ii}$ is equal to the sum of the sums of squared residuals from the first and second estimation periods divided by $T - 2k_i$, where k_i is the number of estimated coefficients in the equation.

point were earlier in the sample period, the methods in Andrews and Fair (1988) could be used to test the hypothesis. These methods cover the 2SLS estimator. However, given that there are only 39 observations after the potential break point, these methods are not practical because the number of first stage regressors is likely to be close to the number of observations. In other words, it is not practical to estimate the equation using only observations for the 2004:1–2013:3 period, which the test requires. The end-of-sample stability test can be used when there are fewer observations after the potential break point than regressors. In the present example this test is as follows:

1. Estimate the equation by 2SLS over the entire period 1954:1–2013:3 (239 observations). If the equation has autoregressive errors, assume that the autoregressive coefficients have been estimated along with the structural coefficients, as discussed in Section 2.1. The subperiod of interest in the present example is 2004:1–2013:3, 39 observations. If the number of first stage regressors is less than 39, let d denote S_i in equation (3) in Section 2.1, where S_i is computed for the 2004:1–2013:3 subperiod. If the number of first stage regressors is greater than or equal to 39, let d denote the sum of squared residuals computed for the 2004:1–2013:3 subperiod. In Andrew's notation, m is 39 and T is the total number of observations (239) minus m, or 200.

2. Andrews considers $T - m + 1$, or 162, subsets of the period 1-T, which is 1–200. Take half of m and round up, which is 20. For the first subset estimate the equation using observations 21–200, and use these coefficient estimates to compute S_i or the sum of squared residuals for the 1–39 period. (Remember that S_i is used if the number of first stage regressors is less than m and that the sum of squared residuals is used if the number is greater than or equal to m.) Let d_1 denote this value. For the second subset estimate the equation using observations 1 and 22–200, and use these coefficient estimates to compute S_i or the sum of squared residuals for the 2–40 period. Let d_2 denote this value. For the third sebset estimate the equation using observations 1–2 and 23–200, and use these coefficient estimates to compute S_i or the sum of squared residuals for the 3–41 period. Let d_3 denote this value. For the last (162) subset estimate the equation using observations 1–161 and 182–200, and use these coefficient estimates to compute S_i or the sum of squared residuals for the 162-200 period. Let d_{162} denote this value. Then sort d_i by size ($i = 1, \ldots, 162$). (Note that each of the 162 sample

periods used to estimate the coefficients includes half (rounded up) of the observations for which S_i or the sum of squared residuals is computed. This choice is ad hoc, but a fairly natural finite sample adjustment. The adjustment works well in Andrews' simulations.)

3. Observe where d falls within the distribution of d_i. If, say, d exceeds 95 percent of the d_i values and a 95 percent confidence level is being used, then the hypothesis of stability is rejected. The p-value is simply the percent of the d_i values that lie above d.

This test is easy to implement, since it just requires running a number of 2SLS regressions.

This test can also be used for beginning-of-sample stability. Say the overall period is 1–239 and one wants to test period 1–39 for stability. The equation is first estimated for the entire 1–239 period and d is computed for the subperiod 1–39. For the first subset the equation is estimated for 40–219 and d_1 is computed for 201–239. For the second subset the equation is estimated for 40–215 and 239 and d_2 is computed for 200–238. For the third subset the equation is estimated for 40–217 and 238–239 and d_3 is computed for 199–237. For the last subset (162) the equation is estimated for 40–58 and 79–239 and d_{162} is computed for 40–78+ The rest is the same as above for the end-of-sample test.

2.8.4 Test of Overidentifying Restrictions

A common test of overidentifying restrictions when using 2SLS is to regress the 2SLS residuals, denoted $\hat{u}_i$, on Z_i and compute the R^2. Then $T \cdot R^2$ is distributed as χ^2_q, where q is the number of variables in Z_i minus the number of explanatory variables in the equation being estimated.[14] The null hypothesis is that all the first stage regressors are uncorrelated with u_i. If $T \cdot R^2$ exceeds the specified critical value, the null hypothesis is rejected, and one would conclude that at least some of the first stage regressors are not predetermined. This test will be denoted "overid."

2.8.5 Testing the RE Assumption (leads test)

The RE model is discussed in Section 2.12, and so the discussion in this subsection is jumping ahead. A "leads" test will be briefly discussed here, but Section 2.12

[14]See Wooldridge (2000), pp. 484–485, for a clear discussion of this.

provides the details, including the case in which u_{it} in equation (2) in Section 2.1 is serially correlated.

A test of the RE hypothesis is to add variable values *led* one or more periods to an equation and estimate the resulting equation using Hansen's (1982) method. If the led values are not significant, this is evidence against the RE hypothesis.

For example, say that $E_{t-1}y_{2t+1}$ and $E_{t-1}y_{2t+2}$ are postulated to be explanatory variables in an equation i, where the expectations are assumed to be rational. If it is assumed that variables in a matrix Z_i are used in part by agents in forming their (rational) expectations, then Hansen's method in this context is simply 2SLS with adjustment for the moving average process of the error term. The expectations variables are replaced by the actual values y_{2t+1} and y_{2t+2}, and the first stage regressors are the variables in Z_i. Consistent estimation does not require that Z_i include all the variables used by agents in forming their expectations. The requirement for consistency is that Z_i be uncorrelated with the expectation errors, which is true if expectations are rational and Z_i is at least a subset of the variables used by the agents. If the coefficient estimates of y_{2t+1} and y_{2t+2} are insignificant, this is evidence against the RE hypothesis.

2.9 Testing Complete Models

2.9.1 Evaluating Predictive Accuracy

The following link is part of Chapter 8 in Fair (1984). It discusses methods for evaluating the accuracy of complete models, including a method for estimating how much a model is misspecified. The main method discussed in this link was originally proposed in Fair (1980a). The link is **Evaluating Predictive Accuracy**.

2.9.2 Comparing Information in Forecasts[15]

Introduction

The above link discusses the estimation of forecast error variances. If one is interested in comparing alternative models, the estimated variances discussed in the link can be compared across models. One might choose, for example, the model with the lowest variances. This subsection discusses an alternative way of comparing models, which is to examine whether their forecasts have independent information. The method is presented in Fair and Shiller (1990) and will be denoted the "FS method" here.

This subsection focuses on the information contained in each model's forecast. Models obviously differ in structure and in the data used, and so their forecasts are not perfectly correlated with each other. How should one interpret the differences in forecasts? Does each model have a strength of its own, so that each forecast represents useful information unique to it, or does one model dominate in the sense of incorporating all the information in the other models plus some?

Structural econometric models make use of large information sets in forecasting a given variable. The information set used in a large scale macroeconometric model is typically so large that the number of predetermined variables exceeds the number of observations available for estimating the model. Estimation can proceed effectively only because of the large number of *a priori* restrictions imposed on the model, restrictions that do not work out to be simple exclusion restrictions on the reduced form equation for the variable forecasted.

VAR models are typically much smaller than structural models and in this sense use less information. The above question with respect to VAR models versus structural models is thus whether the information not contained in VAR models (but contained in structural models) is useful for forecasting purposes. In other words, are the *a priori* restrictions of large scale models useful in producing derived

[15]The discussion in this subsection is taken from Fair and Shiller (1990).

reduced forms that depend on so much information, or is most of the information extraneous?

One cannot answer this question by doing conventional tests of the restrictions in a structural model. These restrictions might be wrong in important ways and yet the model contain useful information. Even ignoring this point, however, one cannot perform such tests with most large scale models because, as noted above, there are not enough observations to estimate unrestricted reduced forms.

The question whether one model's forecast of a variable, for example, real GDP, carries different information from another's can be examined by regressing the actual change in the variable on the forecasted changes from the two models. This procedure, which is discussed below, is related to the literature on encompassing tests[16] and the literature on the optimal combination of forecasts.[17] The procedure proposed here has two advantages over the standard procedure of computing root mean squared errors (RMSEs) to compare alternative forecasts. First, if the RMSEs are close for two forecasts, little can be concluded about the relative merits of the two. With the current procedure one can sometimes discriminate more. Second, even if one RMSE is much smaller than the other, it may still be that the forecast with the higher RMSE contains information not in the other forecast. There is no way to test for this using the RMSE framework.

It should be stressed that the current procedure does not allow one to discover whether all the variables in a model contribute useful information for forecasting. If, say, the regression results reveal that a large model contains all the information in smaller models plus some, it may be that the good results for the large model are due to a small subset of it. It can only be said that the large model contains all the information in the smaller models that it has been tested against, not that it contains no extraneous variables.

The procedure requires that forecasts be based only on information available prior to the forecast period. Assume that the beginning of the forecast period is t, so that only information through period $t-1$ should be used for the forecasts. There are four ways in which future information can creep into a current forecast. The first is if actual values of the exogenous variables for periods after $t-1$ are used in the forecast. The second is if the coefficients of the model have been estimated over a sample period that includes observations beyond $t-1$. The third is if information beyond $t-1$ has been used in the specification of the model even

[16]See, for example, Davidson and MacKinnon (1981), Hendry and Richard (1982), Chong and Hendry (1986), and Mizon and Richard (1986). See also Nelson (1972) and Cooper and Nelson (1975) for an early use of encompassing like tests.

[17]See, for example, Granger and Newbold (1986).

though for purposes of the tests the model is only estimated through period $t-1$. The fourth is if information beyond period $t-1$ has been used in the revisions of the data for periods $t-1$ and back, such as revised seasonal factors and revised benchmark figures.

One way to handle the exogenous variable problem is to estimate, say, an autoregressive equation for each exogenous variable in the model and add these equations to the model. The expanded model effectively has no exogenous variables in it. This method of dealing with exogenous variables in structural models was advocated by Cooper and Nelson (1975) and McNees (1981). McNees, however, noted that the method handicaps the model: "It is easy to think of exogenous variables (policy variables) whose future values can be anticipated or controlled with complete certainty even if the historical values can be represented by covariance stationary processes; to do so introduces superfluous errors into the model solution." (McNees, 1981, p. 404).

The coefficient problem can be handled by doing rolling estimations for each model. For the forecast for period t, for example, the model can be estimated through period $t-1$; for the forecast for period $t+1$, the model can be estimated through period t; and so on. By "model" in this case is meant the model inclusive of any exogenous variable equations. If the beginning observation is held fixed for all the regressions, the sample expands by one observation each time a time period elapses.

The third problem—the possibility of using information beyond period $t-1$ in the specification of the model—is more difficult to handle. Models are typically changed through time, and model builders seldom go back to or are interested in "old" versions. For the work in Fair and Shiller (1989), however, a version of the US model was used that existed as of the second quarter of 1976, and all the predictions were for the period after this.

The data revision problem is very hard to handle. It is extremely difficult to try to purge the data of the possible use of future information. It is not enough simply to use data that existed at any point in time, say period $t-1$, because data for period t are needed to compare the predicted values to the actual values. To handle the data revision problem one would have to try to construct data for period t that are consistent with the old data for period $t-1$, and this is not straightforward.

Forecasts that are based only on information prior to the forecast period will be called "quasi ex ante" forecasts. They are not true ex ante forecasts if they were not issued at the time, but they are forecasts that could in principle have been issued had one been making forecasts at the time.

Quasi ex ante forecasts may, of course, have different properties from forecasts made with a model estimated with future data. If the model is misspecified (e.g., parameters change through time), then the rolling estimation forecasts (where estimated parameters vary through time) may carry rather different information from forecasts estimated over the entire sample.[18] The focus here is on quasi ex ante forecasts.

It should also be noted that some models may use up more degrees of freedom in estimation than others, and with varied estimation procedures it is often very difficult to take formal account of the number of degrees of freedom used up. In the extreme case where there were so many parameters in a model that the degrees of freedom were completely used up when it was estimated (an obviously over parameterized model), it would be the case that the forecast value equals the actual value and there would be a spurious perfect correspondence between the variable forecasted and the forecast. One can guard against this degrees of freedom problem by requiring that no forecasts be within sample forecasts, which is true of quasi ex ante forecasts proposed here.[19]

The Procedure

The notation in this subsection deviates from the notation used for the general model in Section 2.1. It is unique to this subsection. Let ${}_{t-s}\hat{Y}_{1t}$ denote a forecast of Y_t made from model 1 using information available at time $t - s$ and using the model's estimation procedure and forecasting method each period. Let ${}_{t-s}\hat{Y}_{2t}$ denote the same thing for model 2. (In the notation above, these two forecasts should be quasi ex ante forecasts.) The parameter s is the length ahead of the forecast, $s > 0$. Note that the estimation procedure used to estimate a model and the model's forecasting method are considered as part of the model; no account is taken of these procedures here.

The procedure is based on the following regression equation:

$$Y_t - Y_{t-s} = \alpha + \beta({}_{t-s}\hat{Y}_{1t} - Y_{t-s}) + \gamma({}_{t-s}\hat{Y}_{2t} - Y_{t-s}) + u_t \qquad (1)$$

[18]Even if the model is not misspecified, estimated parameters will change through time due to sampling error. If the purpose were to evaluate the forecasting ability of the true model (i.e., the model with the true coefficients), there would be a generated regressor problem. However, the interest here is in the performance of the model *and* its associated estimation procedure. If one were interested in adjusting for generated regressors, the correction discussed in Murphy and Topel (1985) could not be directly applied here because the covariance matrix of the coefficient estimates used to generate the forecasts changes through time because of the use of the rolling regressions. Murphy and Topel require a single covariance matrix.

[19]Nelson (1972) and Cooper and Nelson (1975) do not stipulate that the forecasts be based only on information through the previous period.

If neither model 1 nor model 2 contains any information useful for s period ahead forecasting of Y_t, then the estimates of β and γ should both be zero. In this case the estimate of the constant term α would be the average s period change in Y. If both models contain independent information[20] for s period ahead forecasting, then β and γ should both be nonzero. If both models contain information, but the information in, say, model 2 is completely contained in model 1 and model 1 contains further relevant information as well, then β but not γ should be nonzero.[21]

The procedure is to estimate equation (1) for different models' forecasts and test the hypothesis H_1 that $\beta = 0$ and the hypothesis H_2 that $\gamma = 0$. H_1 is the hypothesis that model 1's forecasts contain no information relevant to forecasting s periods ahead not in the constant term and in model 2, and H_2 is the hypothesis that model 2's forecasts contain no information not in the constant term and in model 1.

As noted above, this procedure bears some relation to encompassing tests, but the setup and interests are somewhat different. For example, it does not make sense in the current setup to constrain β and γ to sum to one, as is usually the case for encompassing tests. If both models' forecasts are just noise, the estimates of both β and γ should be zero. Also, say that the true process generating Y_t is $Y_t = X_t + Z_t$, where X_t and Z_t are independently distributed. Say that model 1 specifies that Y_t is a function of X_t only and that model 2 specifies that Y_t is a function of Z_t only. Both forecasts should thus have coefficients of one in equation (1), and so in this case β and γ would sum to two. It also does not make sense in the current setup to constrain the constant term α to be zero. If, for example, both models' forecasts were noise and equation (1) were estimated without a constant term, then the estimates of β and γ would not generally be zero when the mean of the dependent variable is nonzero.

It is also not sensible in the current setup to assume that u_t is identically distributed. It is likely that u_t is heteroskedastic. If, for example, $\alpha = 0, \beta = 1$, and $\gamma = 0$, u_t is simply the forecast error from model 1, and in general forecast errors are heteroskedastic. Also, if k period ahead forecasts are considered, where $k > 1$, this introduces a $k - 1$ order moving average process to the error term in equation (1).[22]

[20] If both models contain "independent information" in the present terminology, their forecasts will not be perfectly correlated. Lack of perfect correlation can arise either because the models use different data or because they use the same data but impose different restrictions on the reduced form.

[21] If both models contain the same information, then the forecasts are perfectly correlated, and β and γ are not separately identified.

[22] The error term in equation (1) could, of course, be serially correlated even for the one period

Both heteroskedasticity and the moving average process can be corrected for in the estimation of the standard errors of the coefficient estimates. This can be done using the procedure given by Hansen (1982), Cumby, Huizinga, and Obstfeld (1983), and White and Domowitz (1984) for the estimation of asymptotic covariance matrices. Let $\theta = (\alpha \;\; \beta \;\; \gamma)'$. Also, define X as the $T \times 3$ matrix of variables, whose row t is $X_t = (1 \;\; {}_{t-s}\hat{Y}_{1t} - Y_{t-s} \;\; {}_{t-s}\hat{Y}_{2t} - Y_{t-s})$, and let $\hat{u}_t = Y_t - Y_{t-s} - X_t\hat{\theta}$. The covariance matrix of $\hat{\theta}$, $V(\hat{\theta})$, is

$$V(\hat{\theta}) = (X'X)^{-1}S(X'X)^{-1} \tag{2}$$

where

$$S = \Omega_0 + \sum_{j=1}^{s-1}(\Omega_j + \Omega_j') \tag{3}$$

$$\Omega_j = \sum_{t=j+1}^{T}(u_t u_{t-j})\hat{X}_t'\hat{X}_{t-j} \tag{4}$$

where $\hat{\theta}$ is the ordinary least squares estimate of θ and s is the forecast horizon. When s equals 1, the second term on the right hand side of (3) is zero, and the covariance matrix is simply White's (1980) correction for heteroskedasticity.

Note that as an alternative to equation (1) the *level* of Y could be regressed on the forecasted *levels* and a constant. If Y is an integrated process, then any sensible forecast of Y will be cointegrated with Y itself. In the level regression, the sum of β and γ will thus be constrained in effect to one, and one would in effect be estimating one less parameter. If Y is an integrated process, running the levels regression with an additional independent variable Y_{t-1} (thereby estimating β and γ without constraining their sum to one) is essentially equivalent to the differenced regression (1). For variables that are not integrated, the levels version of (1) can be used.

It should finally be noted that there are cases in which an optimal forecast does not tend to be singled out as best in regressions of the form (1), even with many observations. Say the truth is $Y_t - Y_{t-1} = aX_{t-1} + e_t$. Say that model 1 does rolling regressions of $Y_t - Y_{t-1}$ on X_{t-1} and uses these regressions to forecast. Say that model 2 always takes the forecast to be bX_{t-1} where b is some number other than a, so that model 2 remains forever an incorrect model. In equation (1) regressions the two forecasts tend to be increasingly collinear as time goes on;

ahead forecasts. Such serial correlation, however, does not appear to be a problem for the work in the next chapter, and so it has been assumed to be zero here.

essentially they are collinear after the first part of the sample. Thus, the estimates of β and γ tend to be erratic. Adding a large number of observations does not cause the regressions to single out the first model; it only has the effect of enforcing that $\hat{\beta} + (\hat{\gamma} b)/a = 1$.

2.10 Optimal Control Analysis

Some interesting questions in macroeconomics can be examined using optimal control techniques. The following link is from Chapter 10 in Fair (1984). It discusses the numerical solution of optimal control problems for the model in (1) in Section 2.1. The method discussed in this link was originally proposed in Fair (1974b). The link is **Optimal Control Analysis**.

2.11 Certainty Equivalence[23]

2.11.1 Introduction

The assumption of certainty equivalence (CE) is often used in solving optimal control problems in macroeconomics, which was the case in the link in the previous section. The advantage of using CE is that if the error terms are set to their expected values (usually zero), the computational work is simply to solve an unconstrained nonlinear optimization problem, and there are many algorithms available for doing this. This section examines in specific cases how much is lost when using CE for nonlinear models.

2.11.2 Analytic Results

It is difficult to find in the literature analytic comparisons of truly optimal and CE solutions. One example is in Binder, Pesaran, and Samiei (2000), who examine the finite horizon life cycle model of consumption under uncertainty. They consider the simple case of a negative exponential utility function, a constant rate of interest, and labor income following an arithmetic random walk. In this case it is possible to compute both the truly optimal and CE solutions analytically.

Using their solution code,[24] I computed for different horizons both the truly optimal and certainty equivalence solutions. These computations are based on the following values: interest rate = .04, discount factor = .98, negative exponential utility parameter = .01, initial and terminal values of wealth = 500, initial value of income = 200, standard deviation of random walk error = 5.

Let c_1^* denote the truly optimal first-period value of consumption, and let c_1^{**} denote the value computed under the assumption of certainty equivalence. For a life cycle horizon of 12 years, c_1^* was 0.30 percent below c_1^{**}. For 24 years it was 0.60 percent below; for 36 years it was 0.87 percent below, and for 48 years it was 1.09 percent below. Although these differences seem modest, it is not clear how much they can be generalized, given the specialized nature of the model. The following considers a more general case.

2.11.3 Relaxing the CE Assumption

Recall from the link in the previous section that the control problem is to maximize the expected value of W with respect to the control values, subject to the model

[23] The discussion in this section is taken from Chapter 10 in Fair (1994).

[24] I am indebted to Michael Binder for providing me with the code.

in (1) in Section 2.1. The equation for W, equation (10.3) in the above link, is repeated here:

$$W = \sum_{t=1}^{T} h_t(y_t, x_t, z_t) \tag{1}$$

The vector of control variables is denoted z_t, where z_t is a subset of x_t, and z is the vector of all the control values: $z = (z_1, \ldots, z_T)$. The problem under CE is to choose z to maximize W subject to model in (1) in Section 2.1 with the error terms for $t = 1, \ldots, T$ set to zero. For each value of z a value of W can be computed, which is all an optimization algorithm like DFP needs.

If the model is nonlinear or the function h_t is not quadratic, the computed value of W for a given value of z and zero error terms is not equal to the expected value. The optimum, therefore, does not correspond to the expected value of W being maximized other than in the linear/quadratic case.

An alternative, as discussed in the link in the previous section, is to compute for each choice of z the expected value of W using stochastic simulation. This increases the cost by a factor of J, where J is the number of trials.

To see how accurate the CE assumption is, one can compute the optimal value of z using CE and the optimal value using stochastic simulation and compare the two. Let z^* denote the optimal value of z using CE and let z^{**} denote the optimal value using stochastic simulation. However this is done, only the value for period 1 would be implemented. After period 1 passes and the values for period 1 are known, the whole process would be repeated beginning with period 2. The main interest for comparison purposes is thus to compare z_1^* to z_1^{**}. It is not necessary to compare solution values beyond 1 because these are never implemented. This comparison is done in Section 3.11 below.

2.12 Additional Work for the RE Model

The RE model is equation (2) in Section 2.1. This section discusses the estimation and analysis of models like this. The restriction on the expectations of the future variable values is that they are rational, or "model consistent." Agents are assumed to use the model to solve for the expectations.

2.12.1 Single Equation Estimation of RE Models[25]

With only slight modifications, the 2SLS estimator can be used to estimate equations that contain expectational variables in which the expectations are formed rationally. It will be useful to begin with an example. Assume that the equation to be estimated is

$$y_{it} = X_{1it}\alpha_{1i} + E_{t-1}X_{2it+j}\alpha_{2i} + u_{it}, \quad t = 1, \ldots, T, \tag{1}$$

where X_{1it} is a vector of explanatory variables and $E_{t-1}X_{2it+j}$ is the expectation of X_{2it+j} based on information through period $t-1$. j is some fixed positive integer. This example assumes that there is only one expectational variable and only one value of j, but this is only for illustration. The more general case will be considered shortly.

A traditional assumption about expectations is that the expected future values of a variable are a function of its current and past values. One might postulate, for example, that $E_{t-1}X_{2it+j}$ depends on X_{2it} and X_{2it-1}, where it assumed that X_{2it} (as well as X_{2it-1}) is known at the time the expectation is made. The equation could then be estimated with X_{2it} and X_{2it-1} replacing $E_{t-1}X_{2it+j}$ in (1). Note that this treatment, which is common to the Cowles Commission approach, is not inconsistent with the view that agents are "forward looking." Expected future values do affect current behavior. It's just that the expectations are formed in fairly simply ways—say by looking only at the current and lagged values of the variable itself.

Assume instead that $E_{t-1}X_{2it+j}$ is rational and assume that there is an observed vector of variables (observed by the econometrician), denoted here as Z_{it}, that is used in part by agents in forming their (rational) expectations. The following method does not require for consistent estimates that Z_{it} include all the variables used by agents in forming their expectations.

Let the expectation error for $E_{t-1}X_{2it+j}$ be

$$_{t-1}\epsilon_{it+j} = X_{2it+j} - E_{t-1}X_{2it+j} \quad t = 1, \ldots, T, \tag{2}$$

[25]The material in this subsection is taken from Fair (1993a).

where X_{2it+j} is the actual value of the variable. Substituting (2) into (1) yields

$$\begin{aligned} y_{it} &= X_{1it}\alpha_{1i} + X_{2it+j}\alpha_{2i} + u_{it} -_{t-1} \epsilon_{it+j}\alpha_{2i} \\ &= X_{it}\alpha_i + v_{it} \end{aligned} \qquad t = 1, \ldots, T, \tag{3}$$

where $X_{it} = (X_{1it}\ X_{2it+j})$, $\alpha_i = (\alpha_{1i}\ \alpha_{2i})'$, and $v_{it} = u_{it} -_{t-1} \epsilon_{it+j}\alpha_{2i}$.

Consider now the 2SLS estimation of (3), where the vector of first stage regressors is the vector Z_{it} used by agents in forming their expectations. A necessary condition for consistency is that Z_{it} and v_{it} be uncorrelated. This will be true if both u_{it} and ${}_{t-1}\epsilon_{it+j}$ are uncorrelated with Z_{it}. The assumption that Z_{it} and u_{it} are uncorrelated is the usual 2SLS assumption. The assumption that Z_{it} and ${}_{t-1}\epsilon_{it+j}$ are uncorrelated is the rational expectations assumption. If expectations are formed rationally and if the variables in Z_{it} are used (perhaps along with others) in forming the expectation of X_{2it+j}, then Z_{it} and ${}_{t-1}\epsilon_{it+j}$ are uncorrelated. Given this assumption (and the other standard assumptions that are necessary for consistency), the 2SLS estimator of α_i in equation (3) is consistent.

The 2SLS estimator does not, however, account for the fact that v_{it} in (3) is a moving average error of order $j-1$, and so it loses some efficiency for values of j greater than 1. The modification of the 2SLS estimator to account for the moving average process of v_{it} is Hansen's (1982) generalized method of moments (GMM) estimator, which will now be described.

Write (3) in matrix notation as

$$y_i = X_i\alpha_i + v_i \tag{4}$$

where X_i is $T \times k_i$, α_i is $k_i \times 1$, and y_i and v_i are $T \times 1$. Also, let Z_i denote, as above, the $T \times K_i$ matrix of first stage regressors. The assumption in (3) that there is only one expectational variable and only one value of j can now be relaxed. The matrix X_i can include more than one expectational variable and more than one value of j per variable. In other words, there can be more than one led value in this matrix.

The 2SLS estimate of α_i in (4) is

$$\hat{\alpha}_i = [X_i'Z_i(Z_i'Z_i)^{-1}Z_i'X_i]^{-1}X_i'Z_i(Z_i'Z_i)^{-1}Z_i'y_i \tag{5}$$

This use of the 2SLS estimator for models with rational expectations is due to McCallum (1976).

As just noted, this use of the 2SLS estimator does not account for the moving average process of v_{it}, and so it loses efficiency if there is at least one value of j

greater than 1. Also, the standard formula for the covariance matrix of $\hat{\alpha}_i$ is not correct when at least one value of j is greater than 1. If, for example, j is 3 in (3), an unanticipated shock in period $t+1$ will affect ${}_{t-1}\epsilon_{it+3}$, ${}_{t-2}\epsilon_{it+2}$, and ${}_{t-3}\epsilon_{it+1}$, and so v_{it} will be a second order moving average. Hansen's GMM estimator accounts for this moving average process. The GMM estimate in the present case (denoted $\tilde{\alpha}_i$) is

$$\tilde{\alpha}_i = (X_i' Z_i M_i^{-1} Z_i' X_i)^{-1} X_i' Z_i M_i^{-1} Z_i' y_i \tag{6}$$

where M_i is some consistent estimate of $\lim T^{-1} E(Z_i' v_i v_i' Z_i)$. The estimated covariance matrix of $\tilde{\alpha}_i$ is

$$T(X_i' Z_i M_i^{-1} Z_i' X_i)^{-1} \tag{7}$$

There are different versions of $\tilde{\alpha}_i$ depending on how M_i is computed. To compute M_i, one first needs an estimate of the residual vector v_i. The residuals can be estimated using the 2SLS estimate $\hat{\alpha}_i$:

$$\hat{v}_i = y_i - X_i \hat{\alpha}_i \tag{8}$$

A general way of computing M_i is as follows. Let $f_{it} = \hat{v}_{it} \otimes Z_{it}$, where $\hat{v}_{it}$ is the tth element of $\hat{v}_i$. Let $R_{ip} = (T-p)^{-1} \sum_{t=p}^{T} f_{it} f_{it-p}'$, $p = 0, 1, \ldots, P$, where P is the order of the moving average. M_i is then $(R_{i0} + R_{i1} + R_{i1}' + \ldots + R_{iP} + R_{iP}')$. In many cases computing M_i in this way does not result in a positive definite matrix, and so $\tilde{\alpha}_i$ cannot be computed. I have never had much success in obtaining a positive definite matrix for M_i computed in this way.

There are, however, other ways of computing M_i. One way, which is discussed in Hansen (1982) and Cumby, Huizinga, and Obstfeld (1983) but is not pursued here, is to compute M_i based on an estimate of the spectral density matrix of $Z_{it}' v_{it}$ evaluated at frequency zero. An alternative way is to compute M_i under the following assumption:

$$E(v_{it} v_{is} \mid Z_{it}, Z_{it-1}, \ldots) = E(v_{is} v_{is}) \quad , \quad t \geq s \tag{9}$$

which says that the contemporaneous and serial correlations in v_i do not depend on Z_i. This assumption is implied by the assumption that $E(v_{it} v_{is}) = 0, t \geq s$, if normality is also assumed. Under this assumption M_i can be computed as follows. Let $a_{ip} = (T-p)^{-1} \sum_{t=p}^{T} \hat{v}_{it} \hat{v}_{it-p}$ and $B_{ip} = (T-p)^{-1} \sum_{t=p}^{T} Z_{it} Z_{it-p}'$, $p = 0, 1, \ldots, P$. M_i is then $(a_{i0} B_{i0} + a_{i1} B_{i1} + a_{i1} B_{i1}' + \ldots + a_{iP} B_{iP} + a_{iP} B_{iP}')$. In practice, this way of computing M_i usually results in a positive definite matrix.

The Case of an Autoregressive Structural Error

Since many macroeconometric equations have autoregressive error terms, it is useful to consider how the above estimator is modified to cover this case. Return for the moment to the example in (1) and assume that the error term u_{it} in the equation follows a first order autoregressive process:

$$u_{it} = \rho_{1i} u_{it-1} + \eta_{it} \tag{10}$$

Lagging equation (1) one period, multiplying through by ρ_{1i}, and subtracting the resulting expression from (1) yields

$$\begin{aligned} y_{it} = \rho_{1i} y_{it-1} + X_{1it}\alpha_{1i} - X_{1it-1}\alpha_{1i}\rho_{1i} + E_{t-1}X_{2it+j}\alpha_{2i} \\ -E_{t-2}X_{2it+j-1}\alpha_{2i}\rho_{1i} + \eta_{it} \end{aligned} \tag{11}$$

Note that this transformation yields a new viewpoint date, $t-2$. Let the expectation error for $E_{t-2}X_{2it+j-1}$ be

$$_{t-2}\epsilon_{it+j-1} = X_{2it+j-1} - E_{t-2}X_{2it+j-1} \tag{12}$$

Substituting (2) and (12) into (11) yields

$$\begin{aligned} y_{it} = \rho_{1i} y_{it-1} + X_{1it}\alpha_{1i} - X_{1it-1}\alpha_{1i}\rho_{1i} + X_{2it+j}\alpha_{2i} - X_{2it+j-1}\alpha_{2i}\rho_{1i} \\ +\eta_{it} -_{t-1} \epsilon_{it+j}\alpha_{2i} +_{t-2} \epsilon_{it+j-1}\alpha_{2i}\rho_{1i} \\ = \rho_{1i} y_{it-1} + X_{it}\alpha_i - X_{it-1}\alpha_i\rho_{1i} + v_{it} \end{aligned} \tag{13}$$

where X_{it} and α_i are defined after (3) and now $v_{it} = \eta_{it} -_{t-1}\epsilon_{it+j}\alpha_{2i} +_{t-2}\epsilon_{it+j-1}\alpha_{2i}\rho_{1i}$. Equation (13) is nonlinear in coefficients because of the introduction of ρ_{1i}. Again, X_{it} can in general include more than one expectational variable and more than one value of j per variable.

Given a set of first stage regressors, equation (13) can be estimated by 2SLS. The estimates are obtained by minimizing

$$S_i = v_i' Z_i (Z_i' Z_i)^{-1} Z_i' v_i = v_i' D_i v_i \tag{14}$$

(14) is just equation (3) in Section 2.1 rewritten for the error term in (13). A necessary condition for consistency is that Z_{it} and v_{it} be uncorrelated, which means that Z_{it} must be uncorrelated with η_{it}, $_{t-1}\epsilon_{it+j}$, and $_{t-2}\epsilon_{it+j-1}$. In order to insure that Z_{it} and $_{t-2}\epsilon_{it+j-1}$ are uncorrelated, Z_{it} must not include any variables that are

not known as of the beginning of period $t-1$. This is an important additional restriction in the autoregressive case.[26]

In the general nonlinear case (14) (or (3) in Section 2.1) can be minimized using a general purpose optimization algorithm. In the particular case considered here, however, a simple iterative procedure can be used, where one iterates between estimates of α_i and ρ_{1i}. Minimizing $v_i'D_iv_i$ with respect to α_i and ρ_{1i} results in the following first order conditions:

$$\hat{\alpha}_i = [(X_i - X_{i-1}\hat{\rho}_{1i})'D_i(X_i - X_{i-1}\hat{\rho}_{1i})]^{-1}(X_i - X_{i-1}\hat{\rho}_{1i})'D_i(y_i - y_{i-1}\hat{\rho}_{1i}) \quad (15)$$

$$\hat{\rho}_{1i} = \frac{(y_{i-1} - X_{i-1}\hat{\alpha}_i)'D_i(y_i - X_i\hat{\alpha}_i)}{(y_{i-1} - X_{i-1}\hat{\alpha}_i)'D_i(y_{i-1} - X_{i-1}\hat{\alpha}_i)} \quad (16)$$

where the -1 subscript denotes the vector or matrix of observations lagged one period. Equations (15) and (16) can easily be solved iteratively. Given the estimates $\hat{\alpha}_i$ and $\hat{\rho}_{1i}$ that solve (15) and (16), one can compute the 2SLS estimate of v_i, which is

$$\hat{v}_i = y_i - y_{i-1}\hat{\rho}_{1i} - X_i\hat{\alpha}_i + X_{i-1}\hat{\alpha}_i\hat{\rho}_{1i} \quad (17)$$

Regarding Hansen's estimator, given $\hat{v}_i$, one can compute M_i in one of the number of possible ways. These calculations simply involve $\hat{v}_i$ and Z_i. Given M_i, Hansen's estimates of α_i and ρ_{1i} are obtained by minimizing[27]

$$SS_i = v_i'Z_iM_i^{-1}Z_i'v_i = v_i'C_iv_i \quad (18)$$

Minimizing (18) with respect to α_i and ρ_{1i} results in the first order conditions (15) and (16) with C_i replacing D_i. The estimated covariance matrix is

$$T(G_i'C_iG_i)^{-1} \quad (19)$$

where $G = (X_i - X_{i-1}\hat{\rho}_{1i} \quad y_{i-1} - X_{i-1}\hat{\alpha}_i)$.

To summarize, Hansen's method in the case of a first order autoregressive structural error consists of: 1) choosing Z_{it} so that it does not include any variables not known as of the beginning of period $t-1$, 2) solving (15) and (16), 3) computing $\hat{v}_i$ from (17), 4) computing M_i in one of the number of possible ways using $\hat{v}_i$ and Z_i, and 5) solving (15) and (16) with C_i replacing D_i.

[26]There is a possibly confusing statement in Cumby, Huizinga, and Obstfeld (1983), p. 341, regarding the movement of the instrument set backward in time. The instrument set must be moved backward in time as the order of the autoregressive process increases. It need not be moved backward as the order of the moving average process increases due to an increase in j.

[27]The estimator that is based on the minimization of (18) is also the 2S2SLS estimator of Cumby, Huizinga, and Obstfeld (1983).

2.12.2 Solution of RE Models[28]

The "extended path" (EP) method for solving RE models, which is discussed in this subsection, is presented in Fair and Taylor (1983). It is an extension of the iterative technique used in Fair (1979b) for solving a model with rational expectations in the bond and stock markets, which is itself based on an idea in Poole (1976). The EP method has come to be widely used for deterministic simulations of rational expectations models. The EP method has been programmed as part of the TROLL computer package and is routinely used to solve large scale rational expectations models at the IMF, the Federal Reserve, the Canadian Financial Ministry, and other government agencies. It has also been used for simulation studies such as DeLong and Summers (1986) and King (1988). Other solution methods for rational expectations models are summarized in Taylor and Uhlig (1990).

The RE model (2) in Section 2.1 is rewritten here with first order autoregressive errors explicitly added.

$$f_i(y_t, y_{t-1}, \ldots, y_{t-p}, E_{t-1}y_t, E_{t-1}y_{t+1}, \ldots, E_{t-1}y_{t+h}, x_t, \alpha_i) = u_{it} \tag{20}$$

$$u_{it} = \rho_i u_{it-1} + \epsilon_{it}, \quad (i = 1, \ldots, n) \tag{21}$$

The EP method will now be described.

Case 1: $\rho_i = 0$

Consider solving the model for period s. It is assumed that estimates of α_i are available, that current and expected future values of the exogenous variables are available, and that the current and future values of the error terms have been set to their expected values (which will always be taken to be zero here). If the expectations $E_{s-1}y_s$, $E_{s-1}y_{s+1}$, ..., $E_{s-1}y_{s+h}$ were known, (20) could be solved in the usual ways (usually by the Gauss-Seidel technique). The model would be simultaneous, but future predicted values would not affect current predicted values. The EP method iterates over solution *paths*. Values of the expectations through period $s+h+k+h$ are first guessed, where k is a fairly large number relative to h.[29] Given these guesses, the model can be solved for periods s through $s+h+k$ in the usual ways. This solution provides new values for the expectations through

[28]Some of the discussion in this subsection is taken from Fair and Taylor (1990).

[29]Guessed values are usually taken to be the actual values if the solution is within the period for which data exist. Otherwise, the last observed value of a variable can be used for the future values or the variable can be extrapolated in some simple way. Sometimes information on the steady state solution (if there is one) can be used to help form the guesses.

period $s+h+k$—the new expectations values are the solution values. Given these new values, the model can be solved again for periods s through $s+h+k$, which provides new expectations values, and so on. This process stops (if it does) when the solution values for one iteration are within a prescribed tolerance criterion of the solution values for the previous iteration for all periods s through $s+h+k$.

So far the guessed values of the expectations for periods $s+h+k+1$ through $s+h+k+h$ (the h periods beyond the last period solved) have not been changed. If the solution values for periods s through $s+h$ depend in a nontrivial way on these guesses, then overall convergence has not been achieved. To check for this, the entire process above is repeated for k one larger. If increasing k by one has a trivial effect (based on a tolerance criterion) on the solution values for s through $s+h$, then overall convergence has been achieved; otherwise k must continue to be increased until the criterion is met. In practice what is usually done is to experiment to find the value of k that is large enough to make it likely that further increases are unnecessary for any experiment that might be run and then do no further checking using larger values of k.

The expected future values of the exogenous variables (which are needed for the solution) can either be assumed to be the actual values (if available and known by agents) or be projected from an assumed stochastic process. If the expected future values of the exogenous variables are not the actual values, one extra step is needed at the end of the overall solution. In the above process the expected values of the exogenous variables would be used for all the solutions, the expected values of the exogenous variables being chosen ahead of time. This yields values for $E_{s-1}y_s$, $E_{s-1}y_{s+1}, \ldots, E_{s-1}y_{s+h}$. Given these values, (20) is then solved for period s using the *actual* value of x_s, which yields the final solution value $\hat{y}_s$. To the extent that the expected value of x_s differs from the actual value, $E_{s-1}y_s$ will differ from $\hat{y}_s$.

Two points about this method should be mentioned. First, no general convergence proofs are available. If convergence is a problem, one can sometimes "damp" the solution values to obtain convergence. In practice convergence is usually not a problem. There may, of course, be more than one set of solution values, and so there is no guarantee that the particular set found is unique. If there is more than one set, the set that the method finds may depend on the guesses used for the expectations for the h periods beyond $s+h+k$.

Second, the method relies on the certainty equivalence assumption even though the model is nonlinear. Since expectations of functions are treated as functions of the expectations in future periods in equation 7.18, the solution is only approximate unless f_i is linear. This assumption is like the linear quadratic approximation to rational expectations models that has been proposed, for example, by Kydland and Prescott (1982). Although the certainty equivalence assumption is widely used, including in the engineering literature, it is, of course, not always a good approximation.

Case 2: $\rho_i \neq 0$ and Data Before $s-1$ Available

The existence of serial correlation complicates the problem considerably. The error terms for period $t-1$ ($u_{it-1}, i = 1, \ldots, n$) depend on expectations that were formed at the end of period $t-2$, and so a new viewpoint date is introduced. This case is discussed in Section 2.2 in Fair and Taylor (1983), but an error was made in the treatment of the second viewpoint date. The following method replaces the method in Section 2.2 of this paper.[30]

Consider again solving for period s. If the values of u_{is-1} were known, one could solve the model as above. The only difference is that the value of an error term like u_{is+r-1}would be $\rho_i^r u_{is-1}$ instead of zero. The overall solution method first uses the EP method to solve for period $s-j$, where $j > 0$, based on the assumption that $u_{is-j-1} = 0$. Once the expectations are solved for, (20) is used to solve for u_{is-j}. The actual values of y_{s-j} and x_{s-j} are used for this purpose (although the solution values are used for the expectations) because these are structural errors being estimated, not reduced form errors. Given the values for u_{is-j}, the model is solved for period $s-j+1$ using the EP method, where an error term like u_{is-j+r} is computed as $\rho_i^r u_{is-j}$. Once the expectations are solved for, (20) is used to solve for u_{is-j+1}, which can be used in the solution for period $s-j+2$, and so on through the solution for period s.

The solution for period s is based on the assumption that the error terms for period $s-j-1$ are zero. To see if the solution values for period s are sensitive to this assumption, the entire process is repeated with j increased by 1. If going back one more period has effects on the solution values for period s that are within a prescribed tolerance criterion, then overall convergence has been achieved; otherwise j must continue to be increased. Again, in practice one usually finds a value of j that is large enough to make it likely that further increases are unnecessary for any experiment that might be run and then do no further checking using larger values of j.

It should be noted that once period s is solved for, period $s+1$ can be solved for without going back again. From the solution for period s, the values of u_{is} can

[30]The material in Fair and Taylor (1983) is also presented in Fair (1984), Chapter 11, and so the corrections discussed in this subsection pertain to both sources.

be computed, which can then be used in the solution for period $s+1$ using the EP method.

Case 3: $\rho_i \neq 0$ and Data Before Period $s-1$ not Available

This case is based on the assumption that $\epsilon_{is-1} = 0$ when solving for period s. This type of an assumption is usually made when estimating multiple equation models with moving average residuals. The solution problem is to find the values of u_{is-1} that are consistent with this assumption. The overall method begins by guessing values for u_{is-2}. Given these values, the model can be solved for period $s-1$ using the EP method and the fact that $u_{is+r-2} = \rho_i^r u_{is-2}$. From the solution values for the expectations, (20) and (21) can be used to solve for ϵ_{is-1}.[31] If the absolute values of these errors are within a prescribed tolerance criterion, convergence has been achieved. Otherwise, the new guess for u_{is-2} is computed as the old guess plus ϵ_{is-1}/ρ_i. The model is solved again for period $s-1$ using the new guess and the EP method, and so on until convergence is reached.

At the point of convergence u_{is-1} can be computed as $\rho_i u_{is-2}$, where u_{is-2} is the estimated value on the last iteration (the value consistent with ϵ_{is-1} being within a prescribed tolerance criterion of zero). Given the values of u_{is-1}, one can solve for period s using the EP method, and the solution is finished.

Computational Costs of the EP Method

The easiest way to think about the computational costs of the solution method is to consider how many times the equations of a model must be "passed" through. Let N be the number of passes through the model that it takes to solve the model for one period, given the expectations. N is usually some number less than 10 when the Gauss-Seidel technique is used. The EP method requires solving the model for $h+k+1$ periods. Let M be the number of iterations it takes to achieve convergence over these periods. Then the total number of passes for convergence is $N \cdot M(h+k+1)$. If, say, h is 5, k is 30, M is 15, and N is 5, then the total number of passes needed to solve the model for one period is 11,250, which compares to only 5 when there are no expectations. If k is increased by one to check for overall convergence, the total number of passes is slightly more than doubled, although,

[31] These are again estimates of the structural error terms, not the reduced form error terms. Step (iii) on page 1176 in Fair and Taylor (1983) is in error in this respect. The errors computed in step (iii) should be the structural error terms.

as noted above, this check is not always done. In the discussion of computational costs in the rest of this section, it will be assumed that this check is not done.

For Case 2 above the number of passes is increased by roughly a factor of j if overall convergence is not checked. Checking for overall convergence slightly more than doubles the number of passes. j is usually a number between 5 and 10. If q is the number of iterations it takes to achieve convergence for Case 3 above, the number of passes is increased by a factor of $q + 1$. In practice q seems to be between about 5 and 10. Note for both Cases 2 and 3 that the number of passes is increased relative to the non serial correlation case only for the solution for the first period (period s). If period $s + 1$ is to be solved for, no additional passes are needed over those for the regular case.

2.12.3 FIML Estimation of RE Models[32]

Assume that the estimation period is 1 through T. The objective function that FIML maximizes (assuming normality) is presented in equation (6.33) in the link in Section 2.4. It is repeated here for convenience

$$L = -\frac{T}{2}\log|\Sigma| + \sum_{t=1}^{T}\log|J_t| \tag{22}$$

Σ is the covariance matrix of the error terms and J_t is the Jacobian matrix for period t. Σ is of the dimension of the number of stochastic equations in the model, and J_t is of the dimension of the total number of equations in the model. The ij element of Σ is $(1/T)\Sigma_{t=1}^{T}\epsilon_{it}\epsilon_{jt}$. Since the expectations have viewpoint date $t-1$, they are predetermined from the point of view of taking derivatives for the Jacobian, and so no additional problems are involved for the Jacobian in the rational expectations case. In what follows α will be used to denote the vector of all the coefficients in the model. In the serial correlation case α also includes the ρ_i coefficients.

FIML estimation of moderate to large models is expensive even in the standard case, and some tricks are needed to make the problem computationally feasible. An algorithm that can be used for large scale applications is discussed in Parke (1982), and this algorithm will not be discussed here. Suffice it to say that FIML estimation of large scale models is computationally feasible—see Section 3.8 and also Fair and Parke (1980). What any algorithm needs to do is to evaluate L many times for alternative values of α in the search for the value that maximizes L.

[32]Some of the discussion in this subsection is also taken from Fair and Taylor (1990).

In the standard case computing Σ for a given value of α is fairly inexpensive. One simply solves (20) and (21) for the ϵ_{it} error terms given the data and the value of α. This is only one pass through the model since it is the structural error terms that are being computed. In the rational expectations case, however, computing the error terms requires knowing the values of the expectations, which themselves depend on α. Therefore, to compute Σ for a given value of α one has to solve for the expectations for each of the T periods. If, say, 11,250 passes through the model are needed to solve the model for one period and if T is 100, then 1,125,000 passes are needed for one evaluation of Σ and thus one evaluation of L.[33]

It should be clear that the straightforward combination of the EP solution method and FIML estimation procedures is not likely to be computationally feasible for most applications. There is, however, a way of cutting the number of times the model has to be solved over the estimation period to roughly the number of estimated coefficients. The trick is to compute numerical derivatives of the expectations with respect to the parameters and use these derivatives to compute Σ (and thus L) each time the algorithm requires a value of L for a given value of α.

Consider the derivative of $E_{t-1}y_{t+r}$ with respect to the first element of α. One can first solve the model for a given value of α and then solve it again for the first element of α changed by a certain percent, both solutions using the EP method. The computed derivative is then the difference in the two solution values of $E_{t-1}y_{t+r}$ divided by the change in the first element of α. To compute all the derivatives requires $K+1$ solutions of the model over the T number of observations, where K is the dimension of α.[34] One solution is for the base values, and the K solutions are for the K changes in α, one coefficient change per solution. From these $K+1$ solutions, $K \cdot T(h+1)$ derivatives are computed and stored for each expectations variable, one derivative for each length ahead for each period for each coefficient.[35] Once these derivatives are computed, they can be used in the computation of Σ for a given change in α, and no further solutions of the model are needed. In other

[33]Note that these solutions of the error term ϵ_{it} are only approximations when f_i is nonlinear. Hence, the method gives an approximation of the likelihood function.

[34]In the notation presented in the link Subsection 2.3.1, k rather than K is used to denote the dimension of α. K, however, is used in this subsection for the dimension of α since k has already been used in the description of the EP method.

[35]Derivatives computed this way are "one sided." "Two sided" derivatives would require an extra K solutions, where each coefficient would be both increased and decreased by the given percentage. For the work here two sided derivatives seemed unnecessary. For the results below each coefficient was increased by five percent from its base value when computing the derivatives. Five percent seemed to give slightly better results than one percent, although no systematic procedure of trying to find the optimal percentage size was undertaken.

words, when the maximization algorithm changes α and wants the corresponding value of L, the derivatives are first used to compute the expectations, which are then used in the computation of Σ. Since one has (from the derivatives) an estimate of how the expectations change when α changes, one does not have to solve the model any more to get the expectations.

Assuming that the solution method in Case 3 above is used for the FIML estimates, derivatives of u_{it-1} with respect to the coefficients are also needed when the errors are serially correlated. These derivatives can also be computed from the $K+1$ solutions, and so no extra solutions are needed in the serial correlation case.

Once the $K+1$ solutions of the model have been done and the maximization algorithm has found what it considers to be the optimum, the model can be solved again for the T periods using the optimal coefficient values and then L computed. This value of L will in general differ from the value of L computed using the derivatives for the same coefficient values, since the derivatives are only approximations. At this point the new solution values (not computed using the derivatives) can be used as new base values and the problem turned over to the maximization algorithm again. This is the second "iteration" of the overall process. Once the maximization algorithm has found the new optimum, new base values can be computed, a new iteration performed, and so on. Convergence is achieved when the coefficient estimates from one iteration to the next are within a prescribed tolerance criterion of each other. This procedure can be modified by recomputing the derivatives at the end of each iteration. This may improve convergence, but it obviously adds considerably to the expense. At a minimum, one might want to recompute the derivatives at the end of overall convergence and then do one more iteration. If the coefficients change substantially on this iteration, then overall convergence has not in fact been achieved.

Examples of using this method for the FIML estimation of RE models are presented in Fair and Taylor (1990), and this material is not repeated here. The reader is referred to the original paper.

2.12.4 Stochastic Simulation of RE Models[36]

For models with rational expectations one must state very carefully what is meant by a stochastic simulation of the model and what stochastic simulation is to be used for. In the present case stochastic simulation is *not* used to improve on the accuracy of the solutions of the expected values. The expected values are computed exactly

[36]Some of the discussion in this subsection is also taken from Fair and Taylor (1990).

as described above—using the EP method. This way of solving for the expected values can be interpreted as assuming that agents at the beginning of period s form their expectations of the endogenous variables for periods s and beyond by 1) forming expectations of the exogenous variables for periods s and beyond, 2) setting the error terms equal to their expected values (say zero) for periods s and beyond, 3) using the existing set of coefficient estimates of the model, and then 4) solving the model for periods s and beyond. These solution values are the agents' expectations.

For present purposes stochastic simulation begins once the expected values have been solved for. Given the expected values for periods s through $s+h$, stochastic simulation is performed for period s. The problem is now no different from the problem for a standard model because the expectations are predetermined. If it is assumed that the errors are distributed $N(0, \hat{\Sigma})$, where $\hat{\Sigma}$ is the FIML estimate of Σ from the last subsection. then errors from this distribution can be drawn for period s. Alternatively, errors can be drawn from estimated (historic) residuals. Given these draws (and the expectations), the model can be solved for period s in the usual ways. This is one repetition. Another repetition can be done using a new draw of the vector of error terms, and so on. The means and variances of the forecast values can be computed using equations (7.7) and (7.8) in the link in Section 2.6. Note in this setup that agents are assumed *not* to know the error draws when forming their expectations. Their expectations are based on the assumption that the errors for periods s and beyond are zero. Their expectations are not the same as the solution of the model with the drawn errors for period s because they used zero errors for period s. Note that if there is, say, an interest rate rule in the model—a monetary policy reaction function—agents know this rule in that it is used in the solution for their expectations. The rule is part of the structure of the model.

One can also use this approach to analyze the effects of uncertainty in the coefficients by assuming that the coefficients are distributed $N(\hat{\alpha}, \hat{V}_4)$, where $\hat{\alpha}$ is the FIML estimate of α and $\hat{V}_4$ is the estimated covariance matrix of $\hat{\alpha}$. In this case each draw also involves the vector of coefficients.[37]

If u_{it} is serially correlated as in (21), then an estimate of u_{is-1} is needed for the solution for period s. This estimate is, however, available from the solution of the model to get the expectations (see Case 2 in the previous subsection), and so no further work is needed. The estimate of u_{is-1} is simply taken as predetermined

[37]In principle one could reestimate the model to get coefficients rather than draw from $N(\hat{\alpha}, \hat{V}_4)$, as discussed in Section 2.7, but in practice this is unlikely to be computationally feasible.

for all the repetitions, and u_{is} is computed as $\rho_i u_{is-1}$ plus the draw for ϵ_{is}. (Note that the ϵ errors are drawn, not the u errors.)

Stochastic simulation is quite inexpensive if only results for period s are needed because the model only needs to be solved once using the EP method. Once the expectations are obtained, each repetition merely requires solving the model for period s. The EP method is not needed because the expectations are predetermined. If, on the other hand, results for more than one period are needed and the simulation is dynamic, the EP method must be used p times for each repetition, where p is the length of the period.

Consider the multiperiod problem. As above, the expectations with viewpoint date $s-1$ can be solved for and then a vector of error terms drawn for period s and (perhaps) a vector of coefficients also drawn to compute the predicted value of y_{is} for each i. This is the first step.

Now go to period $s+1$, where the viewpoint date is s. An agent's expectation of, say, y_{is+2} is different with viewpoint date s than with viewpoint date $s-1$. In particular, the value of y_{is} is in general different from what the agent at the end of period $s-1$ expected it to be (because of the error terms that were drawn for period s).[38] A new set of expectations must thus be computed with viewpoint date s. Agents are assumed to use the original set of coefficients (not the set that was drawn if in fact coefficients were drawn) and to set the values of the error terms for periods $s+1$ and beyond equal to zero. Then given the solution values for period s and the actual value of x_s, agents are assumed to solve the model for their expectations for periods $s+1$ and beyond. This requires a second use of the EP method. These expectations are then predetermined for viewpoint date s. Given these expectations, a vector of error terms for period $s+1$ is drawn and the model is solved for period $s+1$. If equation i has a serially correlated error, then u_{is+1} is equal to $\rho_i^2 u_{is-1}$ plus the draw for ϵ_{is+1}. Now go to period $s+2$ and repeat the process, where another use of the EP method is needed to compute the new expectations. The process is repeated through the end of the period of interest. At the end, this is one repetition. If the length of the period is p, then the EP method is used p times per repetition. The overall process is then repeated for the second repetition, and so on. Note that if coefficients are drawn, only one coefficient draw is used per repetition, i.e., per dynamic simulation. After J repetitions one can compute means and variances just as above, where there are now means and variances for each period ahead of the prediction. Also note that agents are always

[38]It may also be that the actual value of x_s differs from what the agent expected it to be at the end of $s-1$.

assumed to use the original set of coefficients and for each viewpoint date to set the current and future error terms to zero. They do not perform stochastic simulation themselves.

Stochastic simulation results for a RE model are presented in Fair and Taylor (1990), and this material is not repeated here. The reader is again referred to the original paper. These results and others suggest that stochastic simulation as defined above is computationally feasible for models with rational expectations. Stochastic simulation is in fact likely to be cheaper than even FIML estimation using the derivatives. If, for example, the FIML estimation period is 100 observations and there are 25 coefficients to estimate, FIML estimation requires that the model be solved 2600 times using the EP method to get the derivatives. For a stochastic simulation of 8 periods and 100 repetitions, on the other hand, the model has to be solved using the EP method only 800 times.

2.12.5 Optimal Control of RE Models: Deterministic Case[39]

The optimal control procedure outlined in Section 2.10 can be used for RE models under the CE assumption. The procedure simply requires that the model be capable of being solved for a given set of control values. The solution can be done using the EP method discussed above.[40]

To set up the problem, assume that the period of interest is t through $t+S-1$ (a horizon of length S) and that the objective is to maximize the expected value of W, where W is

$$W = \sum_{s=t}^{t+S-1} g_s(y_s, x_s). \tag{23}$$

Let z_t be a q–dimensional vector of control variables, where z_t is a subset of x_t, and let z be the $q \cdot (S+h+k)$–dimensional vector of all the control values: $z = (z_t, \ldots, z_{t+S+h+k-1})$, where k is taken to be large enough for solution convergence through period $t+S-1$.[41] If all the error terms are set to zero, then for each value

[39] Some of the discussion in this subsection is taken from Fair (2003b).

[40] This subsection and the next are based on the assumption of known coefficients—$\hat{\alpha}$ is taken to be fixed. This analysis does not consider, for example, the possibility of unknown coefficients and learning. Amman and Kendrick (1999) consider this case within the context of the linear quadratic optimization problem for models with rational expectations. It would be interesting in future work to consider the case of unknown coefficients with learning in the more general setting here.

[41] Remember that the guessed values of the expectations for periods $t+S+h+k$ through $t+S+h+k+h-1$ are never changed in the solution. k has to be large enough so that increasing it by one has a trivial effect on the relevant solution values.

of z one can compute a value of W by first solving the model for $y_t, \ldots, y_{t+S-1}$ and then using these values along with the values for $x_t, \ldots, x_{t+S-1}$ to compute W in equation (23). The problem can then be turned over to an optimization algorithm like DFP. Each evaluation of W requires only one use of the EP method. Think about the problem this way. The DFP method announces a z vector to try. (Note that z has more than S elements per control variable because extra periods are needed at the end for the expectations and solution.) Agents are assumed to know this vector and solve for their expectations (using zero errors). Conditional on these expectations the model is solved for periods t through $t + S - 1$ (also using zero errors), which allows W to be computed. The vector z that is found by the DFP algorithm to maximize W is the optimal vector, and this vector is consistent with the expectations of the agents.

Once the problem is solved, z_t^*, the optimal vector of control values for period t, is implemented. If, for example, the Fed is solving the control problem and there is one control variable—the interest rate—then the Fed would implement through open market operations the optimal value of the interest rate for period t. In the process of computing z_t^* the optimal values for periods $t + 1$ through $t + S + h + k - 1$ are also computed. Agents are assumed to know these values when they solve the model to form their expectations. For the Fed example, one can think of the Fed announcing and implementing the period t value of the interest rate and at the same time announcing the planned future values.

After z_t^* is implemented and period t passes, the entire process can be repeated beginning in $t+1$. In the present deterministic case, however, the optimal value of z_{t+1} chosen at the beginning of $t + 1$ would be the same as the value chosen at the beginning of t, and so there is no need to reoptimize. Reoptimization is needed in the stochastic case, which is discussed in the next subsection.

Each evaluation of W requires $N \cdot M \cdot (S + h + k)$ passes, since the path is of length $S + h + k$, where from Subsection 2.12.2 N is the number of passes through the model needed to solve for one period and M is the number of EP iterations needed for convergence. Each iteration of the DFP algorithm requires $2q \cdot (S+h+k)$ evaluations of W to compute the derivatives numerically, assuming that two function evaluations are used per derivative calculation, and then a few more evaluations to do line searching, where q is the number of control variables. Let L denote the number of evaluations that are needed for the line searching after the derivatives have been computed, and let I denote the total number of iterations of the DFP algorithm that are needed for convergence to the optimum. The total number of evaluations of W is thus $I \cdot (2q \cdot (S+h+k)+L)$. Since each evaluation of W requires $N \cdot M \cdot (S + h + k)$ passes, the total number of passes needed to compute z_t^* is $N \cdot M \cdot (S + h + k) \cdot I \cdot (2q \cdot (S + h + k) + L)$.

2.12.6 Stochastic Simulation and Optimal Control of RE Models[42]

Subsection 2.12.4 discussed stochastic simulation without optimal control. If there is, say, a Fed interest rate rule in the model, the Fed does not solve a formal optimal control problem but simply follows the rule. Agents know the rule because it is part of the model. Stochastic simulation is used to compute measures of dispersion, like forecast-error variances. In this subsection the control authority (like the Fed) is assumed to choose the values of its control variables by solving an optimal control problem. Given this setup, stochastic simulation is used to compute measures of dispersion.

In the previous subsection computing one value of W required one use of the EP method. When stochastic simulation is introduced, the number jumps to S, where S is the length of the control period. This will now be discussed in steps. Continue to assume that the control period is t through $t + S - 1$. The steps are:

1. Solve the optimal control problem in the previous subsection using zero errors for all periods. This solution produces z_t^*, the optimal value of the control vector for period t. In the process of doing this the expectations with viewpoint date $t - 1$ are computed. The final set computed is compatible with z_t^*, where zero errors are used. Then draw u_t^*, errors for period t, from an estimated distribution or from estimated residuals, as discussed in Subsection 2.12.2. Solve the model for period t using these errors, z_t^*, and the computed expectations at the optimum (which don't change).

2. Go to period $t + 1$. Using the predicted values for period t solve the optimal control problem for the control period $t + 1$ through $t + S$ using zero errors for periods $t + 1$ and beyond.[43] This produces z_{t+1}^*. Then draw u_{t+1}^*, and solve the model for period $t + 1$ using these errors, z_{t+1}^*, and the computed expectations at the optimum of the second control problem.

3. Go to period $t + 2$ and repeat step 2 with $t + 2$ replacing $t + 1$. Continue doing this through period $t + S - 1$.

[42]Some of the discussion in this subsection is also taken from Fair (2003b).

[43]Note that the end of the control horizon has been increased by one period. It is assumed here that all control problems are of the same length, namely S.

4. At the end of steps 1–3 there are predicted values of each endogenous variable for periods t through $t + S - 1$. Record these values. These steps required the solution of S deterministic optimal control problems. The total number of passes through the model needed for the solution of one control problem is presented at the end of the previous subsection. The total number of passes needed here is thus S times this number. This is one repetition. Repeat steps 1–3 J times, where J is the number of repetitions. This gives J values of each endogenous variable for each period, from which measures of dispersion can be computed. The computational cost of J repetitions is, of course, just J times the cost for one repetition.

The measures of dispersion computed in this subsection can be compared to measures computed in Subsection 2.12.4 using estimated policy rules instead of explicitly computed optimal control problems to see how much gain there is from explicitly solving control problems.

The above procedure is obviously computationally expensive, but it can be made less expensive if one is only interested in measures of dispersion for the first few periods. Consider step 1 and assume that one is only interested in measures of dispersion for period t. What step 1 needs are only the solution values for period t (including z_t^*), and the horizon only needs to be taken long enough so that increasing it further has a trivial effect (based on a tolerance criterion) on the solution values for period t. One can initially experiment with different values of the horizon to see how large it has to be to meet the tolerance criterion. Let R denote this length. If then step 2 is to be performed because measures of dispersion are needed for period $t + 1$, a horizon of length R may be all that is needed.

In term of speed it is obviously important that efficient code be written for passing through the model, since most of the time is spent passing through. A practical way to proceed after the code is written is to set limits on N, M, I, and J that are small enough to make the problem computationally feasible. Once the bugs are out and the (preliminary) results seem sensible, the limits can be gradually increased to gain more accuracy. If two cases are being compared using stochastic simulation, such as an estimated rule versus an optimal control procedure, the same draws of the errors should be used for both cases. This can considerably lessen stochastic simulation error for the comparisons.

Finally, it is useful to consider what is lost in the present treatment of stochastic simulation and optimal control. The above procedure is open loop and uses reoptimization over time. Agents know the current period values of the control variables that are implemented and the announced planned future values when they solve the model to form their expectations. They take the planned future values as deterministic rather than stochastic, and they take the error terms to be deterministic, namely zero. Agents do not take into account the fact that everything will be redone at the beginning of each period after the error terms for that period are realized and known. The overall procedure is thus not fully optimal. Also, the deterministic optimal control problems that are solved (many times) are not fully optimal, although the results in Subsection 3.11 below suggest that this is not a serious problem.

2.13 The FP Program

All the calculations in this document have been done using the Fair-Parke (FP) program. The first version of this program was available in 1980, and it has been expanded over time. The following link is Appendix C in Fair (1984). It discusses the logic of the program. One of the advantages of the program is that it allows the user to move easily from the estimation of individual equations to the solution and analysis of the entire model. The link is: **The Fair-Parke Program**.

3 The MC Model

3.1 Introduction

The theoretical framework that has been used to guide the specification of the MC model was first presented in Fair (1974a), followed by the empirical version in Fair (1976). This work stresses three ideas: 1) basing macroeconomics on solid microeconomic foundations, 2) allowing for the possibility of disequilibrium in some markets, and 3) accounting for all balance-sheet and flow of funds constraints. Households and firms make decisions by solving maximization problems. Households' decision variables include consumption, labor supply, and the demand for money. Firms' decision variables include production, investment, employment, and the demand for money. Firms are assumed to behave in a monopolistically competitive environment, and prices and wages are also decision variables of firms. The values of prices and wages that firms set are not necessarily market clearing. Disequilibrium in the goods markets takes the form of unintended changes in inventories. Disequilibrium in the labor market takes the form of unemployment, where households are constrained by firms from working as much as the solutions of their unconstrained maximization problems say they want to.

Disequilibrium comes about because of expectation errors. In order for a firm to form rational expectations, it would have to know the maximization problems of all the other firms and of the households. Firms are not assumed to have this much knowledge (i.e., they do not know the complete model), and so they can make expectation errors.

Tax rates and most government spending variables are exogenous in the model. Regarding monetary policy, in the initial specification of the theoretical model—Fair (1974a)—the amount of government securities outstanding was taken as exogenous, i.e., as a policy variable of the monetary authority. In 1978 an estimated interest rate rule was added to the empirical version of the model—Fair (1978b)—which was then added to the discussion of the theoretical model in Fair (1984), Chapter 3. The rule is one in which the Fed "leans against the wind," where the nominal interest rate depends positively on the rate of inflation and on output or the unemployment rate.

Interest rate rules, commonly referred to as "Taylor rules" from Taylor (1993), have a long history in macroeconomics. The first rule is in Dewald and Johnson (1963), who regressed the Treasury bill rate on the constant, the Treasury bill rate lagged once, real GNP, the unemployment rate, the balance-of-payments deficit, and the consumer price index. The next example can be found in Christian (1968), followed by many others. These rules should thus probably be called Dewald-Johnson rules, since Dewald and Johnson preceded Taylor by about 30 years!

Because the model accounts for all flow-of-fund and balance-sheet constraints, there is no natural distinction between stock market and flow market determination of exchange rates. This distinction played an important role in exchange rate modeling in the 1970s. In the model an exchange rate is merely one endogenous variable out of many, and in no rigorous sense can it be said to be *the* variable that clears a particular market.

3.2 Theory: One Country

The modeling procedure that was used in Fair (1974a) and continued in later work is to specify a theoretical model, choose values of the parameters in the model, and then analyze the model via numerical techniques. Some changes were made between the theoretical model in Fair (1974a) and that in Fair (1984). The following link is Chapter 3 in Fair (1984). It discusses the single country theoretical model. This model and the two country theoretical model discussed in the next section have been used to guide the specification of the (empirical) MC model. The link to the single country theoretical model is **Single Country Model**.

3.3 Theory: Two Countries

The latest complete discussion of the two country theoretical model is in Fair (1994, Chapter 2), and this discussion is repeated in this section.

3.3.1 Background

The theoretical two country model that has guided the specification of the MC model was first presented in Fair (1979a). This model was in part a response to the considerable discussion in the literature that had taken place in the 1970s as to whether the exchange rate is determined in a stock market or in a flow market. [See, for example, Frenkel and Rodriguez (1975), Frenkel and Johnson (1976), Dornbusch (1976), Kouri (1976), and the survey by Myhrman (1976).] The monetary approach to the balance of payments stressed the stock market determination of the exchange rate, which was contrasted with "the popular notion that the exchange rate is determined in the flow market so as to assure a balanced balance of payments" [Frenkel and Rodriguez (1975, p. 686)]. In the model in Fair (1979a), on the other hand, there is no natural distinction between stock market and flow market determination of the exchange rate. The exchange rate is merely one endogenous variable out of many, and in no rigorous sense can it be said to be *the* variable that clears a particular market. In other words, there is no need for a stock-flow distinction in the model; stock and flow effects are completely integrated. [Other studies in the 1970s in which the stock-flow distinction was important included Allen (1973), Black (1973), Branson (1974), and Girton and Henderson (1976).] The reason there is no stock-flow distinction in the model is the accounting for all flow of funds and balance-sheet constraints. These constraints are accounted for in the single country model, and they are also accounted for when two single country models are put together to form a two country model.

The main features of the model in Fair (1979a) that are relevant for the construction of the MC model were discussed in Fair (1984), Section 3.2. Contrary to the case for the single country theoretical model, however, the two country theoretical model was not analyzed by simulation techniques in Fair (1984). In this section a version of the two country model is presented that will be analyzed by simulation techniques. This should help in understanding the properties of the theoretical model before it is used to guide the specification of the MC model. Again, the simulation of the theoretical model is not meant to be a test of the model in any sense.

3.3.2 Notation

In what follows capital letters denote variables for country 1, lower case letters denote variables for country 2, and an asterisk (*) on a variable denotes the other country's holdings or purchase of the variable. There are three sectors per country: private non financial (h), financial (b), and government (g). The private non financial sector includes both households and firms. It will be called the "private sector." Members of the financial sector will be called "banks." Each country specializes in the production of one good (X, x). Each country has its own money (M, m) and its own bond (B, b). Only the private sector of the given country holds the money of the country. The bonds are one period securities. If a sector is a debtor with respect to a bond (i.e., a supplier of the bond), then the value of B or b for that sector is negative. The interest rate on B is R and on b is r. The price of X is P and of x is p. e is the price of country 1's currency in terms of country 2's currency, so that, for example, and increase in e is a depreciation of country 1's currency. The government of each country holds a positive amount of the international reserve (Q, q), which is denominated in the units of country 1's currency, and collects taxes (T, t) as a proportion of income (Y, y). The government of a country does not hold the bond of the other country and does not buy the good of the other country. f_{ij} is the derivative of f_i with respect to argument j.

3.3.3 Equations

There are 17 equations per country and one redundant equation. The equations for country 1 are as follows. (The derivative indicates the expected effect of the particular variable on the left hand side variable.) The demands for the two goods by the private sector of country 1 are

$$X_h = f_1(P, e \cdot p, R', Y - T), \quad f_{11} < 0, f_{12} > 0, f_{13} < 0, f_{14} > 0 \tag{1}$$

$$x_h^* = f_2(P, e \cdot p, R', Y - T), \quad f_{21} > 0, f_{22} < 0, f_{23} < 0, f_{24} > 0 \tag{2}$$

R' is the real interest rate, $R - (EP_{+1} - P)$, where EP_{+1} is the expected value of P for the next period based on current period information. The equations state that the demands are a function of the two prices, the real interest rate, and after tax income. X_h is the purchase of country 1's good by the private sector of country 1, and x_h^* is the purchase of country 2's good by the private sector of country 1. The domestic price level is assumed to be a function of demand pressure as measured by Y and of the level of import prices, $e \cdot p$:

$$P = f_3(Y, e \cdot p), \quad f_{31} > 0, f_{32} > 0 \tag{3}$$

There is assumed to be no inventory investment, so that production is equal to sales:

$$Y = X_h + X_g + X_h^* \tag{4}$$

where X_g is the purchase of country 1's good by its government and X_h^* is the purchase of country 1's good by country 2. Taxes paid to the government are

$$T = TX \cdot Y \tag{5}$$

where TX is the tax rate.

The demand for real balances is assumed to be a function of the interest rate and income:

$$\frac{M_h}{P} = f_6(R, Y), \quad f_{61} < 0, f_{62} > 0 \tag{6}$$

Borrowing by the banks from the monetary authority (BO) is assumed to be a function of R and of the discount rate RD:

$$BO = f_7(R, RD), \quad f_{71} > 0, f_{72} < 0 \tag{7}$$

Since the private sector is assumed to be the only sector holding money,

$$M_b = M_h \tag{8}$$

where M_b is the money held in banks. Equation (8) simply says that all money is held in banks. Banks are assumed to hold no excess reserves, so that

$$BR = RR \cdot M_b \tag{9}$$

where BR is the level of bank reserves and RR is the reserve requirement rate.

Let Ee_{+1} be the expected exchange rate for the next period based on information available in the current period. Then from country 1's perspective, the expected (one period) return on the bond of country 2, denoted Er, is $\frac{Ee_{+1}}{e}(1+r)-1$, where r is the interest rate on the bond of country 2. The demand for country 2's bond is assumed to be a function of R and Er :

$$b_h^* = f_{10}(R, Er), \quad f_{10,1} < 0, f_{10,2} > 0 \tag{10}$$

b_h^* is the amount of country 2's bond held by country 1. Equation (10) and the equivalent equation for country 2 are important in the model. If capital mobility is such as to lead to uncovered interest parity almost holding (i.e., R almost equal to Er), then large changes in b_h^* will result from small changes in the difference between R and Er. If uncovered interest parity holds exactly, which is not assumed here,[44] then equation (10) and the equivalent equation for country 2 drop out, and there is effectively only one interest rate in the model.

The next three equations determine the financial saving of each sector:

$$S_h = P \cdot X_g + P \cdot X_h^* - e \cdot p \cdot x_h^* - T + R \cdot B_h + e \cdot r \cdot b_h^* \tag{11}$$

$$S_b = R \cdot B_b - RD \cdot BO \tag{12}$$

$$S_g = T - P \cdot X_g + R \cdot B_g + RD \cdot BO \tag{13}$$

Equation (11) states that the saving of the private sector is equal to revenue from the sale of goods to the government, plus export revenue, minus import costs, minus taxes paid, plus interest received (or minus interest paid) on the holdings of country 1's bond, and plus interest received on the holdings of country 2's bond. If the private sector is a net debtor with respect to the bond of country 1, then B_h is negative and $R \cdot B_h$ measures interest payments. Remember that the private sector (h) is a combination of households and firms, and so transactions between households and firms net out of equation (11). Equation (12) states that the saving of banks is equal to interest revenue on bond holdings (assuming B_b is positive) minus interest payments on borrowings from the monetary authority. Equation (13) determines the government's surplus or deficit. It states that the saving of the government is equal to tax revenue, minus expenditures on goods, minus interest costs (assuming B_g is negative), and plus interest received on loans to banks.

The next three equations are the budget constraints facing each sector:

$$0 = S_h - \Delta M_h - \Delta B_h - e \cdot \Delta b_h^* \tag{14}$$

$$0 = S_b - \Delta B_b + \Delta M_b - \Delta(BR - BO) \tag{15}$$

[44] It was incorrectly stated in Fair (1984), pp. 154–155, that the version of the model that is used to guide the specification of the MC model is based on the assumption of perfect substitution of the two bonds. The correct assumption is that uncovered interest parity does not hold. As will be seen in Section 3.7, the MC model consists of estimated interest rate and exchange rate equations (reaction functions) for a number of countries (all exchange rates are relative to the U.S. dollar). If there were uncovered interest parity between, say, the bonds of countries A and B, it would not be possible to estimate interest rate equations for countries A and B plus an exchange rate equation. There is an exact relationship between the expected future exchange rate, the two interest rates, and the spot exchange rate if uncovered interest parity holds, and so given a value of the expected future exchange rate, only two of the other three values are left to be determined. It would not make sense in this case to estimate three equations. Covered interest parity, on the other hand, does roughly hold in the data used here. This will be seen in Section 3.7 in the estimation of the forward rate equations.

$$0 = S_g - \Delta B_g + \Delta(BR - BO) - \Delta Q \tag{16}$$

Equation (14) states that any nonzero value of saving of the private sector must result in the change in its money or bond holdings. Equation (15) states that any nonzero value of saving of the financial sector must result in the change in bond holdings, money deposits (which are a liability to banks), or nonborrowed reserves. Equation (16) states that any nonzero value of saving of the government must result in the change in bond holdings, nonborrowed reserves (which are a liability to the government), or international reserve holdings.

There is also a constraint across all sectors, which says that someone's asset is someone else's liability with respect to the bond of country 1:

$$0 = B_h + B_b + B_g + B_h^* \tag{17}$$

These same 17 equations are assumed to hold for country 2, with lower case and upper case letters reversed except for Q and with $1/e$ replacing e. Q is replaced by q/e. (Remember that Q and q are in the units of country 1's currency.) The last equation of the model is

$$0 = \Delta Q + \Delta q$$

which says that the change in reserves across countries is zero. This equation is implied by equations (11)–(17) and the equivalent equations for country 2, and so it is redundant. There are thus 34 independent equations in the model.

It will be useful in what follows to consider two equations that can be derived from the others. First, let S denote the financial saving of country 1, which is the sum of the saving of the three sectors:

$$S = S_h + S_b + S_g$$

S is the balance of payments on current account of country 1. Summing equations (14)–(15) and using (17) yields the first derived equation:

$$0 = S + \Delta B_h^* - e \cdot \Delta b_h^* - \Delta Q \tag{i}$$

This equation simply says that any nonzero value of saving of country 1 must result in the change in at least one of the following three: country 2's holdings of country 1's bond, country 1's holding of country 2's bond, and country 1's holding of the international reserve. The second derived equation is obtained by summing equations (11)–(13) and using (17):

$$S = P \cdot X_h^* - e \cdot p \cdot x_h^* - R \cdot B_h^* + e \cdot r \cdot b_h^* \tag{ii}$$

This equation says that the saving of country 1 is equal to export revenue, minus import costs, minus interest paid to country 2, and plus interest received from country 2.

3.3.4 Closing the Model

The exogenous government policy variables are: X_g, government purchases of goods; TX, the tax rate; RD, the discount rate; RR, the reserve requirement rate; and the same variables for country 2. Not counting these variables, there are 40 variables in the model: B_b, B_g, B_h, B^*_h, BO, BR, M_b, M_h, P, Q, R ,S_b, S_g, S_h, T, X_h, X^*_h, Y, these same 18 variables for country 2, e, Ee_{+1}, EP_{+1}, and Ep_{+1}. In order to close the model one needs to make an assumption about how the three expectations are determined and to take three other variables as exogenous. (Remember there are 34 independent equations in the model.)

Assume for now that exchange rate expectations are static in the sense that $Ee_{+1} = e$ always. (This implies that $Er = r$ and $ER = R$. Remember that R does not necessarily equal r since uncovered interest parity is not necessarily assumed to hold.) Assume also that the two price expectations are static, $EP_{+1} = P$ and $Ep_{+1} = p$. The model can then be closed by taking B_g, b_g, and Q as exogenous. These are the three main tools of the monetary authorities. Taking these three tools of the monetary authorities as exogenous thus closes the model.

Instead of taking the three tools to be exogenous, however, one can assume that the monetary authorities use the tools to manipulate R, r, and e. If reaction functions for these three variables are used (or the three variables are taken to be exogenous), then B_g, b_g, and Q must be taken to be endogenous. The solution values of B_g, b_g, and Q are whatever is needed to have the target values of R, r, and e met.

Note that in closing the model no mention was made of stock versus flow effects. The exchange rate e is just one of the many endogenous variables, and it is determined, along with the other endogenous variables, by the overall solution of the model.

3.3.5 Links in the Model

The trade links in the model are standard. Country 1 buys country 2's good (x^*_h), and country 2 buys country 1's good (X^*_h). The price links come through equation (3) and the equivalent equation for country 2. Country 2's price affects country 1's price, and vice versa. The interest rate and exchange rate links are less straightforward, and these will be discussed next in the context of the overall properties of the model.

3.3.6 Properties of the Model

As will be discussed in the next subsection, the exchange rate and interest rate equations in the MC model are based on the assumption that the monetary authorities manipulate R, r, and e. (Thus, from above, B_g, b_g, and Q are endogenous in the MC model.) The interest rate and exchange rate equations are interpreted as reaction functions, where the explanatory variables in the equations are assumed to be variables that affect the monetary authorities' decisions. The key question in this work is what variables affect the monetary authorities' decisions. If capital mobility is high in the sense that uncovered interest parity almost holds, it will take large changes in the three tools to achieve values of R, r, and e much different from what the market would otherwise achieve. Since the monetary authorities are likely to want to avoid large changes in the tools, they are likely to be sensitive to and influenced by market forces. In other words, they are likely to take market forces into account in setting their target values of R, r, and e. Therefore, one needs to know the market forces that affect R, r, and e in the theoretical model in order to guide the choice of explanatory variables in the estimated reaction functions in the MC model.

In order to examine the market forces on R, r, and e in the theoretical model, a simulation version has been analyzed. Particular functional forms and coefficients have been chosen for equations (1), (2), (3), (6), (7), and (10) and the equivalent equations for country 2. The five equations for country 1 are:

$$\log X_h = a_1 - .25 \cdot \log P + .25 \cdot \log e \cdot p - 1.0 \cdot R' + .75 \cdot \log(Y - T) \quad (1)'$$

$$\log x_h^* = a_2 + 1.0 \cdot \log P - 1.0 \cdot \log e \cdot p - 1.0 \cdot R' + .75 \cdot \log(Y - T) \quad (2)'$$

$$\log P = a_3 + .1 \cdot \log e \cdot p + .1 \cdot \log Y \quad (3)'$$

$$\log \frac{M_h}{P} = a_6 - 1.0 \cdot R + .5 \cdot \log Y \quad (6)'$$

$$BO = a_7 + 50 \cdot R - 50 \cdot RD \quad (7)'$$

$$b_h^* = a_{10} - 100 \cdot R + 100 \cdot Er \quad (10)'$$

The same functional forms and coefficients were used for country 2. The a_i coefficients were chosen so that when the model was solved using the base values of all the variables, the solution values were the base values.[45] The model was solved using the Gauss-Seidel technique.

The properties of the model can be examined by changing one or more exogenous variables, solving the model, and comparing the solution values to the base values. The following experiments were chosen with the aim of learning about the market forces affecting R, r, and e in the model. Unless otherwise noted, the experiments are based on the assumption that $Ee_{+1} = e$. This means from equation 2.10 and the equivalent equation for country 2 that b_h^* and B_h^* are simply a function of R and r. The experiments are also based on the assumptions that $EP_{+1} = P$ and $Ep_{+1} = p$.

In all but the last experiment, e is endogenous and Q is exogenous. Taking Q to be exogenous means that the monetary authorities are not manipulating e. This is a way of examining the market forces on e without intervention. The solution value of e for each experiment is the value that would pertain if the monetary authorities did not intervene at all in the foreign exchange market in response to whatever change was made for the experiment. B_g and b_g are always endogenous for the experiments because all the experiments either have R and r exogenous or M_b and m_b exogenous. In other words, it is always assumed that the monetary authorities either keep interest rates or money supplies unchanged in response to whatever change was made for the experiment. When R and r are exogenous, M_b and m_b are endogenous, and vice versa. All shocks in the experiments are for country 1.

The results of all the experiments are reported in Table 1, and the following discussion of the experiments relies on this table. Only signs are presented in the table because the magnitudes mean very little given that the coefficients and base values are not empirically based. The simulation experiments are simply meant to be used to help in understanding the qualitative effects on various variables. Even the qualitative results, however, are not necessarily robust to alternative choices of the coefficients. At least some of the signs in Table 1 may be reversed with different coefficients. The simulation work is meant to help in understanding the theoretical model, but the results from this work should not be taken as evidence that all the signs in the table hold for all possible coefficient values. In two cases it is necessary to know which interest rate (R or r) changed the most, and these cases are noted in Table 1 and discussed below.

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[45] The base values were $X_h = x_h = 60$, $X_h^* = x_h^* = 20$, $X_g = x_g = 20$, $Y = y = 100$, $TX = tx = .2$, $T = t = 20$, $M_h = M_b = m_h = m_b = 100$, $RR = rr = .2$, $BR = br = 20$, $e = 1$, all prices = 1, all interest rates = .07, and all other variables, including lagged values when appropriate, equal to zero.

Table 1
Simulation Results for the Two Country Model

	Experiment					
	1	2	3	4	5	6
	$R(-)$	$M_b(+)$	Eq2.3(+)	Eq2.3(+)	Eq2.2(+)	$R(-)$
e	+	+	+	+	+	0
R	–	$-^a$	0	$+^b$	+	–
r	0	–	0	+	–	0
S	+	+	0	–	–	–
s	–	–	0	+	+	+
b_h^*	+	+	0	–	–	+
B_h^*	–	–	0	+	+	–
x_h^*	–	–	0	+	–	+
X_h^*	+	+	0	–	+	+
Y	+	+	0	–	+	+
y	–	–	0	+	–	+
P	+	+	+	+	+	+
p	–	–	0	+	–	+
M_h	+	+	+	0	0	+
m_h	–	0	0	0	0	+
Q	0	0	0	0	0	–
q	0	0	0	0	0	+
B_g	+	+	0	–	+	+
b_g	–	–	0	+	–	–

Q is exogenous exept for experiment 6.
Size of changes:
1. R lowered by .001, r exogenous
2. M_b raised by 1.0, m_b exogenous
3. Equation 2.3 shocked by .10, R and r exogenous
4. Equation 2.3 shocked by .10, M_b and m_b exogenous
5. Equation 2.2 shocked by .10, M_b and m_b exogenous
6. R lowered by .001, r and e exogenous

[a] R decreased more than did r.
[b] R increased more than did r.

Experiment 1: R decreased, r unchanged

For this experiment the interest rate for country 1 was lowered (from its base value) and the interest rate for country 2 was assumed to remain unchanged. (Both interest rates are exogenous in this experiment.) This change resulted in a depreciation of country 1's currency.[46] The fall in R relative to r led to an increase in the demand for the bond of country 2 by country 1 (b^*_h increased) and a decrease in the demand for the bond of country 1 by country 2 (B^*_h decreased). From equation (i) above it can be seen that this must result in an increase in S, country 1's balance of payments, since Q is exogenous and unchanged. S is increased by increasing country 1's exports and decreasing its imports—equation (ii)—which is accomplished by a depreciation. Another way of looking at this is that the fall in R relative to r led to a decreased demand for country 1's currency because of the capital outflow, which resulted in a depreciation of country 1's currency. Output for country 1 (Y) increased because of the lower interest rate and the depreciation, and the demand for money increased because of the lower interest rate and the higher level of income. The monetary authority of country 1 bought bonds to achieve the reduction in R (B_g increased).

Although not shown in Table 1, experiments with alternative coefficients in the equations explaining b^*_h and B^*_h—equation (10) and the equivalent equation for country 2—showed that the more sensitive are the demands for the foreign bonds to the interest rate differential, the larger is the depreciation of the exchange rate and the larger is the increase in B_g for the same drop in R. In other words, the higher is the degree of capital mobility, the larger is the size of open market operations that is needed to achieve a given target value of the interest rate.

Remember that the above experiment is for the case in which exchange rate expectations are static, i.e. where $Ee_{+1} = e$. If instead expectations are formed in such a way that Ee_{+1} turns out to be less than e, which means that the exchange rate is expected to appreciate in the next period relative to the value in the current period (i.e., reverse at least some of the depreciation in the current period), then the depreciation in the current period is less. This is because if Ee_{+1} is less than e, the expected return on country 2's bond (Er) falls. The differential between R and Er thus falls less as a result of the decrease in R, which leads to a smaller increase in b^*_h and a smaller decrease in B^*_h. There is thus less downward pressure on country 1's currency and thus a smaller depreciation. If expectations are formed in such a way that Ee_{+1} turns out to be greater than e, which means that the exchange rate is expected to depreciate further in the next period, there is more of a depreciation in the current period. The expected return on country 2's bond rises, which leads to greater downward pressure on country 1's exchange rate.

[46] Remember that a rise in e is a depreciation of country 1's currency. The $+$ in Table 1 for e for experiment 1 thus means that country 1's currency depreciated.

Experiment 2: M_b increased, m_b unchanged

For this experiment the monetary authorities are assumed to target the money supplies (M_b and m_b are exogenous), and the money supply of country 1 was increased. The increase in M_b led to a decrease in R, both absolutely and relative to r, which led to a depreciation of country 1's currency. The results of this experiment are similar to those of experiment 1. The monetary authority of country 1 bought bonds to increase the money supply (B_g increased). Country 1's output increased as a result of the depreciation and the fall in R. Note that the effect of a change in the money supply on the exchange rate works through the change in relative interest rates. The interest rate of country 1 falls relative to that of country 2, which decreases the demand for country 1's bond and increases the demand for country 2's bond, which leads to a depreciation of country 1's exchange rate.

Experiment 3: Positive price shock, R and r unchanged

For this experiment the price equation for country 1 was shocked positively. The monetary authorities were assumed to respond to this by keeping interest rates unchanged. The positive price shock resulted in a depreciation of country 1's currency. Given the coefficients and base values that are used for the simulation model, the exchange rate depreciated by the same percent that P increased, and there was no change in any real magnitudes. The reason for the exchange rate depreciation is the following. Other things being equal, a positive price shock leads to a decrease in the demand for exports and an increase in the demand for imports, which puts downward pressure on S. If, however, interest rates are unchanged, then b_h^* and B_h^* do not change, which means from equation i that S cannot change. Therefore, a depreciation must take place to decrease export demand and increase import demand enough to offset the effects of the price shock.

Experiment 4: Positive price shock, M_b and m_b unchanged

This experiment is the same as experiment 3 except that the money supplies rather than the interest rates are kept unchanged. The positive price shock with the money supplies unchanged led to an increase in R. Even though R increased relative to r, country 1's currency depreciated. The negative effects of the price shock offset the positive effects of the interest rate changes.

Experiment 5: Positive import demand shock, M_b and m_b unchanged

For this experiment the import demand equation of country 1 was shocked positively. The increased demand for imports led to a depreciation of country 1's currency, since there was an increased demand for country 2's currency. The depreciation led to an increase in Y and P, which with an unchanged money supply, led to an increase in R. R also increased relative to r, which increased B_h^* and decreased b_h^*. The balance of payments, S, worsened. It may at first glance seem odd that a positive import shock would lead to an increase in Y, but remember that the shock does not correspond to any shock to the demand for the domestic good. The experiment is not a substitution away from the domestic good to the imported good, but merely an increase in demand for the imported good. The latter results in an increase in Y because of the stimulus from the depreciation.

Experiment 6: R decreased, r unchanged, e unchanged

This experiment is the same as experiment 1 except that e rather than Q is exogenous. In this case the monetary authorities choose B_g, b_g, and Q so as to lower R and keep r and e unchanged. One of the key differences between the results for this experiment and the results for experiment 1 is that the balance of payments, S, decreases rather than increases. In experiment 1 S had to increase because of the increase in the demand for country 2's bond by country 1 and the decrease in the demand for country 1's bond by country 2. In experiment 1 S must increase because Q is exogenous—equation i. The increase in S is accomplished by a depreciation. In the present experiment there is still an increase in the demand for country 2's bond and a decrease in the demand for country 1's bond—because R falls relative to r—but S does not necessarily have to increase because Q can change. The net effect is that S decreases (and thus Q decreases). The reason for the decrease in S is fairly simple. The decrease in R is an expansionary action in country 1, and among other things it increases the country's demand for imports. This then worsens the balance of payments. There is no offsetting effect from a depreciation of the currency to reverse this movement.

This completes the discussion of the experiments. They should give one a fairly good idea of the properties of the model. Of main concern here are the effects of the various changes on the domestic interest rate and the exchange rate. Table 2 presents a summary of these effects in the model (experiment 6 is not included in the table because both R and e are exogenous in it).

Table 2
Summary of the Experiments

Experiment	Effect on: Domestic Interest Rate	Effect on: Exchange Rate
1. Interest rate lowered	—	Depreciation
2. Money supply raised	Lowered	Depreciation
3. Positive price shock; interest rates unchanged	—	Depreciation
4. Positive price shock; money supply unchanged	Raised	Depreciation
5. Positive import shock; money supply unchanged	Raised	Depreciation

3.3.7 The Use of Reaction Functions

As noted in the previous subsection, reaction functions for interest rates and exchange rates have been estimated in the MC model. To put this approach in perspective, it will help to consider an alternative approach that in principle could have been followed. If equations (1), (2), (3), (6), (7), (10), and the equivalent equations for country 2 were estimated, one could solve the model for R, r, and e (and the other endogenous variables) by taking B_g, b_g, and Q as exogenous. R, r, and e would thus be determined without having to estimate any direct equations for them. Their values would be whatever is needed to clear the two bond markets and the market for foreign exchange. In doing this, however, one would be making the rather extreme assumption that the monetary authorities' choices of B_g, b_g, and Q are never influenced by the state of the economy, i.e. are always exogenous.

If one believes that monetary authorities intervene at least somewhat, there are essentially two options open. One is to estimate equations with B_g, b_g, and Q on the left hand side, and the other is to estimate equations with R, r, and e on the left hand side. If the first option is followed, then the B_g, b_g, and Q equations are added to the model and the model is solved for R, r, and e. If the second option is followed, the R, r, and e equations are added to the model and the model is solved for B_g, b_g, and Q. The first option is awkward because one does not typically think of the monetary authorities having target values of the instruments themselves. It is more natural to think of them having target values of interest rates (or money supplies[47]) and exchange rates, and this is the assumption made for the MC model.

There is also a practical reason for taking the present approach. If B_g, b_g, and Q are taken to be exogenous or equations estimated for them, equations like (10), which determine the bilateral demands for securities, must be estimated. In practice it is very difficult to estimate such equations. One of the main problems is that data on bilateral holdings of securities either do not exist or are not very good. If instead equations for interest rates and exchange rates are estimated, one can avoid estimating equations like (10) in order to determine interest rates and exchange rates if one is willing to give up determining B_g, b_g, and Q. For many applications one can get by without knowing the amounts of government bonds outstanding and government reserve holdings. One can simply keep in mind that the values of these variables are whatever is needed to have the interest rate and exchange rate values be met.

3.3.8 Further Aggregation

Data on bilateral security holdings were not collected for the MC model, and so data on variables like B_h^* and b_h^* are not available. Instead, a net asset variable, denoted A in the MC model, was constructed for each country. In terms of the variables in the theoretical model, $\Delta A = -\Delta B_h^* + e \cdot \Delta b_h^* + \Delta Q$. Equation (i) thus becomes

$$0 = S - \Delta A \qquad (i)'$$

Data on S are available for each country, and A was constructed as $A_{-1} + S$, where an initial value for A for each country was first chosen.

This aggregation is very convenient because it allows A to be easily constructed. The cost of doing this is that capital gains and losses on bonds from exchange rate changes are not accounted for. Given the current data, there is little that can be done about this limitation.

[47] It is in the spirit of the present approach to estimate money supply reaction functions rather than interest rate reaction functions. In either case B_g is endogenous. No attempt has been made in the construction of the MC model to try to estimate money supply reaction functions. The present work is based on the implicit assumption that interest rate reaction functions provide a better approximation of the way monetary authorities behave than do money supply reaction functions.

3.4 Transition to the MC Model

As discussed in Section 1.1, the transition from theory to empirical work in macroeconomics is not always straightforward. Compromises and extra assumptions generally have to be made in moving from theory to empirical specifications.

The first step in the transition is to choose the data and variables. The discussion of this is in Appendices A and B. Appendix A pertains to the United States part of the model, called the "US model," and Appendix B pertain to the rest of the world, called the "ROW model." The second step is to choose which variables are to be treated as exogenous, which are to be determined by stochastic (estimated) equations, and which are to be determined by identities. This is also covered in Appendices A and B. The third step, which is where most of the theory is used, is to choose the explanatory variables in the stochastic equations and the functional forms of the equations.

In the discussion of the empirical results, a hypothesis will be said to be rejected if the p-value for the test is less than .01. If a hypothesis is not rejected, the test will be said to have been "passed." A coefficient estimate will be said to be significant if its t-statistic is greater than 2.0 in absolute value. A variable will be said to be significant if its coefficient estimate is significant.

It will be seen in the discussion of the results that a number of tests are not passed. If an equation does not pass a test, it is not always clear what should be done. If, for example, the hypothesis of structural stability is rejected, one possibility is to divide the sample period into two parts and estimate two separate equations. If this is done, however, the resulting coefficient estimates are not always sensible in terms of what one would expect from theory. Similarly, when the additional lagged values are significant, the equation with the additional lagged values does not always have what one would consider sensible dynamic properties. In other words, when an equation fails a test, the change in the equation that the test results suggest may not produce what seem to be sensible results. In many cases, the best choice seems to be to stay with the original equation even though it failed the test. Some of this difficulty may be due to small sample problems, which will lessen over time as sample sizes increase. This is an important area for future work and is what makes macroeconomics interesting. Obviously less confidence should be placed on equations that fail a number of the tests than on those that do not.

3.5 Overview of the MC Model

There are 39 countries in the MC model for which stochastic equations are estimated. The countries are listed in Table B.1 in Appendix B. There are 25 stochastic equations for the United States and up to 13 each for the other countries. The total number of stochastic equations is 310, and the total number of estimated coefficients is about 1,300. In addition, there are 1,379 bilateral trade share equations estimated, so the total number of stochastic equations is 1,689. The total number of endogenous and exogenous variables, not counting various transformations of the variables and the trade share variables, is about 2,000. Trade share data were collected for 59 countries, and so the trade share matrix is 59×59.

The estimation periods begin in 1954 for the United States and as soon after 1960 as data permit for the other countries. Data permitting, they end as late as 2013:2 for the ROW model and 2013:3 for the US model. The estimation technique is 2SLS except when there are too few observations to make the technique practical, where ordinary least squares is used. The estimation accounts for possible serial correlation of the error terms. The variables used for the first stage regressors for a country are the main predetermined variables in the model for the country.

There is a mixture of quarterly and annual data in the model. Quarterly equations are estimated for 14 countries, and annual equations are estimated for the remaining 25. However, all the trade share equations are quarterly. There are quarterly data on all the variables that feed into the trade share equations, namely the exchange rate, the local currency price of exports, and the total value of imports per country. When the model is solved, the predicted annual values of these variables for the annual countries are converted to predicted quarterly values using a simple distribution assumption. The quarterly predicted values from the trade share equations are converted to annual values by summation or averaging when this is needed. The solution of the MC model is explained in Section B.6 in Appendix B.

As noted above, the United States part of the overall MC model is denoted the US model and the remaining part is denoted the ROW model. The ROW model consists of the individual models of all the other countries. Also, all the equations that pertain to the links among countries, such as the trade share equations, are put in the ROW model. There are 26 stochastic equations for the US model alone and one additional equation when the US model is embedded in the overall MC model.

The discussion of the model below relies heavily on the tables in Appendices A and B. All the variables and equations in the US model are presented in Appendix A. Table A.1 lists the six sectors of the model, and Table A.2 lists all the variables in alphabetical order. Table A.2 also shows which variables appear in which equations, which is useful for tracking the effects of various variables. All the equations, both the stochastic equations and the identities, are listed in Table A.3, but not the coefficient estimates. The coefficient estimates and test results are presented in Table A.4 for the 26 stochastic equations. Within Table A.4, Table A1 refers to equation 1, Table A2 refers to equation 2, and so on through Table A30.[48]

The remaining tables in Appendix A are for completeness. They allow the model be reproduced by someone else. These tables can be skipped if desired. Table A.5 lists the "raw data" variables, i.e., the variables for which data were collected. Table A.6 shows the links using the raw data variables between the national income and product accounts (NIPA) and the flow of funds accounts (FFA). Table A.7 shows how the variables in the model were constructed from the raw data variables. Table A.9 lists the first stage regressors used for each equation for the 2SLS estimator. (There is no Table A.8.)

Appendix B does for the ROW model what Appendix A does for the US model. Table B.1 lists the countries in the model, and Table B.2 lists all the variables for a given country in alphabetical order. Table B.2 also shows how each variable in the model is constructed from the data. All the equations, both the stochastic equations and the identities, are listed in Table B.3, but not the coefficient estimates. The coefficient estimates and test results are presented in Table B.4 for the stochastic equations. There are up to 13 equations per country, and within Table B.4, Table B1 refers to equation 1, Table B2 refers to equation 2, and so on through Table B14.[49] Table B.5 shows the links between the US and ROW models, and Table B.6 shows how the balance of payments data were used.

Regarding the treatment of expectations, it will be seen that lagged dependent variables are used as explanatory variables in many of the equations. They are generally highly significant even after accounting for any autoregressive properties of the error terms. It is well known that lagged dependent variables can be accounting for either partial adjustment effects or expectational effects and that it is difficult to identify the two effects separately. For the most part no attempt is made in the empirical work to separate the two effects. The rational expectations assumption is, however, tested in the manner discussed in Subsection 2.8.5. Also, since most of the equations are estimated by 2SLS, one can think of the predicted values from the first stage regressions as representing the predictions of the agents if it is assumed that agents know the values of the first stage regressors at the time

[48]There are no equations 9, 20, 21, and 22 and so no Tables A9, A20, A21, and A22.

[49]There is no equation 12 and so no Table B12.

Table 1
Determination of Some Variables per Country in the ROW Model

	Output or Income	Interest Rates Short & Long	Domestic Price Level	Import Price Level	World Price Level
Estimated Equations					
1 Consumption	+	−			
2 Investment	+	−			
3 Imports	+[a]	−	+	−	
4 Domestic Price Level	+			+	
5 Interest Rate (Short)	+		+[b]		
6 Exchange Rate[c]	−	−	+		
7 Export Price Level			+		+

	Export Price Level	Exchange Rate[c]	Export Prices Other Countries
When Countries are Linked Together			
8 Import Price Level		+	+
9 Exports	−	+	+

Identities

10 Output = Consumption + Investment + Government Spending + Exports − Imports

11 Current Account = Export Price Level × Exports − Import Price Level ×Imports

12 World Price Level= Weighted average of all countries' Export Prices

The signs are the expected signs of the coefficient estimates or effects.
[a]Explanatory variable is consumption plus investment plus government spending.
[b]Rate of Inflation.
[c]Exchange rate is local currency per dollar, so an increase is a depreciation.

they make their decisions.

Because of the MC model's size, it is difficult to get a big picture of how it works. In this subsection an attempt is made to give an overview of the model for a given country without getting bogged down in details and notation. The model for the United States is more detailed than the models for the other countries, and the discussion in this section pertains only to the models for the other countries. Table 1 is used as a framework for discussion. The table outlines for a given country how thirteen variables are determined. The first seven (consumption, investment, imports, domestic price level, short term interest rate, exchange rate, and export price level) are determined by estimated equations; the next two (import price level and exports) are determined when all the countries are linked together; and the last three (output, current account, and world price level) are determined by identities.

Unless otherwise stated, the price levels are prices in local currency. Consumption, investment, imports, exports, and output are in real (local currency) terms.

The exchange rate is local currency per US dollar, so an increase in the exchange rate is a depreciation of the currency relative to the dollar.

The following discussion ignores dynamic issues. In most estimated equations there is a lagged dependent variable among the explanatory variables to pick up partial adjustment and/or expectational effects, but these variables are not listed in the table. Inventory investment is not discussed; the labor sector is not discussed; and the relationship between the short term and long term interest rate is not discussed. Finally, in terms of what is not discussed, it should be kept in mind that not every effect exists for every country.

The seven variables determined by estimated equations in Table 1 are:

1. **Consumption** depends on income and an interest rate. The interest rate is either the short rate or the long rate. Monetary policy thus has a direct effect on consumption through the interest rate variables.

2. **Investment** depends on output and an interest rate. As with consumption, monetary policy has a direct effect on investment through the interest rate variables.

3. The level of **imports** depends on consumption plus investment plus government spending, on the domestic price level, and on the import price level. The price variables are important in this equation. If, for example, the import price level rises relative to the domestic price level, this has a negative effect on import demand. A depreciation of the country's currency thus lowers the demand for imports because it increases the import price level.

4. The **domestic price level** depends on output and the import price level, where output is meant to represent some measure of demand pressure. The import price level is a key variable in this equation. It is significant for almost all countries. When the import price level rises, this has a positive effect on the prices of domestically produced goods. This is the main channel through which a depreciation of the country's currency affects the domestic price level.

5. The **short term interest rate** depends on output and the rate of inflation. The estimated equation for the interest rate is interpreted as an interest rate rule of the monetary authority. The estimated interest rate rules for the various countries are "leaning against the wind" equations. Other things being equal, an increase in output or an increase in the rate of inflation leads to an increase in the interest rate.

6. The **exchange rate** depends on the short term interest rate and the domestic price level. All the explanatory variables are relative to the respective U.S. variables if the exchange rate is relative to the dollar and are relative to the respective German variables if the exchange rate is relative to the DM (or euro beginning in 1999). A depreciation of a country's currency occurs if there is a relative decrease in the country's interest rate or a relative increase in the country's price level.

7. The **export price level** in local currency is determined as a weighted average of the domestic price level and a world price level converted to local currency, where the weight is estimated. If the weight on the world price level converted to local currency is one (and thus the weight on the domestic price level zero), the country is a complete price taker on world markets. In this case, if the world price level in dollars is little affected by the individual country, then a depreciation of a country's currency of a given percent increases the export price level in local currency by roughly the same percent (since the world price level converted to local currency increases by roughly the same percent), leaving the export price level in dollars roughly unchanged. Otherwise, the export price level in dollars falls with a depreciation, where the size of the fall depends on the estimated weight in the equation.

The next two variables in Table 1 are determined when the countries are linked together.

8 The **import price level** in local currency for a given country i depends on its dollar exchange rate and other countries' export prices in dollars. The import price level is a weighted average of all other countries' export prices converted to local currency, with a weight for a particular country j being the amount imported by i from j as a fraction of i's total imports. If there is a depreciation of i's currency and no change in the other countries' export prices in their own local currency, then the import price level in local currency will rise by the full percent of the depreciation.

9 The total level of **exports** for a given country i is the sum of its exports to all the other countries. The amount that country i exports to country j is determined by the trade share equations. The share of j's total imports imported from i depends on i's export price level in dollars relative to a weighted average of all the other countries' export price levels in dollars. The higher is i's relative export price level, the lower is i's share of j's total

imports. There are 1,333 estimated trade share equations. Many estimated equations are thus involved in determining the response of a country's total exports to a change in its export price level.

The three identities in Table 1 are straightforward. They determine, respectively, **output**, the **current account**, and the **world price level**.

Effects of a Depreciation

Table 1 can be used to trace through the effects of a depreciation of a country's currency. Assume that there is an exogenous depreciation of a country's currency. The depreciation raises the import price level in local currency. The increase in the import price level then has two main effects, other things being equal. The first is that the demand for imports falls (equation 3), and the second is that the domestic price level rises (equation 4). (All the equation references in the rest of this section are to the equations in Table 1.) The depreciation also reduces the price of exports in dollars unless the country is a complete price taker (equation 7). The decrease in the price of exports in dollars leads to an increase in the demand for the country's exports (equation 9). The depreciation is thus expansionary and inflationary: the level of imports falls, the level of exports rises, and the domestic price level increases. The effect on the current account is ambiguous because of the usual "J-curve" reasons.

Effects of an Interest Rate Decrease

Table 1 can also be used to trace through the effects of a decrease in a country's interest rate. Assume that there is an exogenous decrease in a country's interest rate. This leads, other things being equal, to an increase in consumption and investment (equations 1 and 2). It also leads to a depreciation of the country's currency (equation 6), which has the effects discussed above. In particular, exports increase (equation 9). The effect on aggregate demand in the country from the interest rate decrease is thus positive from the increase in consumption, investment, and exports.

There are two main effects on imports, one positive and one negative (equation 3). The positive effect is that consumption and investment are higher, some of which is imported. The negative effect is that the price of imports in higher because of the depreciation, which has a negative effect on the demand for imports. The net effect on imports can thus go either way.

There is also a positive effect on the price level. As noted above, the depreciation leads to an increase in the price of imports (equation 8). This in turn has a positive effect on the domestic price level (equation 4). In addition, if aggregate demand increases, this increases demand pressure, which has a positive effect on the domestic price level (also equation 4).

There are other effects that follow from these, including effects back on the short-term interest rate itself through the interest rate rule (equation 5), but these are typically second order in nature, especially in the short run. The main effects are as just described. The decrease in a country's interest rate should thus stimulate the economy, depreciate the currency, and lead to a rise in its price level.

This completes the general overview. The next two sections discuss the exact specifications.

3.6 The US Stochastic Equations

3.6.1 Introduction

The Cowles Commission methodology that was followed in the specification and estimation of the stochastic equations is discussed in Section 1.1. The estimates that are presented in Tables A1 through A30 (within Table A.4 in Appendix A)[50] are those of the "final" specifications. Lagged dependent variables are generally used as explanatory variables to account for expectational and/or partial adjustment effects. Explanatory variables were dropped if they had highly insignificant coefficient estimates or estimates of the wrong expected sign. Most of the equations are estimated by 2SLS. The equations were first estimated under the assumption of a first order autoregressive error term, and the assumption was retained if the estimate of the autoregressive coefficient was significant. In a few cases higher order processes are used.

The χ^2 tests per equation, which are reported in the tables, are 1) adding lagged values of all the variables in the equation—the lags test, 2) estimating the equation under the assumption of a *fourth* order autoregressive process for the error term—the RHO test, 3) adding the time trend—the T test, and 4) adding values *led* one or more quarters—the leads tests. The other tests are 5) testing for structural stability using the AP test, 6) testing for structural stability using the end-of-sample test, and 7) testing the overidentifying restrictions. The basic estimation period is 1954:1-2013:3, for a total of 239 observations.

For the leads tests, three sets of led values are tried per equation. For the first set the values of the relevant variables led once are added; for the second set the values led one through four quarters are added; and for the third set the values led one through eight quarters are added, where the coefficients for each variable are constrained to lie on a second degree polynomial with an end point constraint of zero. The test in each case is a χ^2 test that the additional variables are significant. The three tests are called "Leads +1," "Leads +4," and "Leads +8."

The "theoretical model " referred to below is the model discussed in Section 3.2. The notation for the six sectors in the US model is presented in Table A.1. It is h for households, f for firms, b for financial, r for foreign, g for federal government, and s for state and local governments. Before discussing the individual equations, the age distribution variables will be discussed.

[50]As noted in Section 3.5, there are no Tables A9, A20, A21, and A22. There were originally 30 equations in the US model, and equations 9, 20, 21, and 22 have been dropped. The original numbering has been retained.

3.6.2 Tests of Age Distribution Effects[51]

A striking feature of post war U.S. society has been the baby boom of the late 1940s and the 1950s and the subsequent falling off of the birth rate in the 1960s. The number of births in the United States rose from 2.5 million in 1945 to 4.2 million in 1961 and then fell back to 3.1 million in 1974. This birth pattern implies large changes in the percentage of prime age (25–54) people in the working age (16+) population. In 1952 this percentage was 57.9, whereas by 1977 it had fallen to 49.5. Since 1980 the percentage of prime aged workers has risen sharply as the baby boomers have begun to pass the age of 25.

An important issue in macroeconomics is whether the coefficients of macroeconomic equations change over time as other things change. The Lucas (1976) critique focuses on policy changes, but other possible changes are changes in the age distribution of the population. This subsection discusses a procedure for examining the effects of the changes in the U.S. population age distribution on macroeconomic equations. The procedure is as follows.

Divide the population into J age groups. Let $D1_{ht}$ be 1 if individual h is in age group 1 in period t and 0 otherwise; let $D2_{ht}$ be 1 if individual h is in age group 2 in period t and 0 otherwise; and so on through DJ_{ht}. Consider an equation that is linear in coefficients, and let equation i for individual h be:

$$\begin{gathered} y_{hit} = X_{hit}\alpha_i + \beta_{0i} + \beta_{1i}D1_{ht} + \ldots + \beta_{Ji}DJ_{ht} + u_{hit} \\ h = 1, \ldots, N_t, \quad t = 1, \ldots, T \end{gathered} \tag{1}$$

where y_{hit} is the value of variable i in period t for individual h (e.g., consumption of individual h in period t), X_{hit} is a vector of explanatory variables excluding the constant, α_i is a vector of coefficients, and u_{hit} is the error term. The constant term in the equation is $\beta_{0i} + \beta_{ji}$ for an individual in age group j in period t. N_t is the total number of people in the population in period t.

Equation (1) is restrictive because it assumes that α_i is the same across all individuals, but it is less restrictive than a typical macroeconomic equation, which also assumes that the constant term is the same across individuals. Given X_{hit}, y_{hit} is allowed to vary across age groups in equation (1). Because most macroeconomic variables are not disaggregated by age groups, one cannot test for age sensitive α_i's. For example, suppose that one of the variables in X_{hit} is Y_{ht}, the income of individual h in period t. If the coefficient of Y_{ht} is the same across individuals, say γ_{1i}, then $\gamma_{1i}Y_{ht}$ enters the equation, and it can be summed in the manner discussed in

[51] The discussion in this section is taken from Fair and Dominguez (1991).

the next paragraph. If, on the other hand, the coefficient differs across age groups, then the term entering the equation is $\gamma_{11i}D1_{ht}Y_{ht} + \ldots + \gamma_{1Ji}DJ_{ht}Y_{ht}$. The sum of a variable like $D1_{ht}Y_{ht}$ across individuals is the total income of individuals in age group 1, for which data are not generally available. One is thus restricted to assuming that age group differences are reflected in different constant terms in equation (1).

Let N_{jt} be the total number of people in age group j in period t, let y_{it} be the sum of y_{hit}, let X_{it} be the vector whose elements are the sums of the corresponding elements in X_{hit}, and let u_{it} be the sum of u_{hit}. (All sums are for $h = 1, \ldots, N_t$.) Given this notation, summing equation (1) yields:

$$y_{it} = X_{it}\alpha_i + \beta_{0i}N_t + \beta_{1i}N_{1t} + \ldots + \beta_{Ji}N_{Jt} + u_{it}, \quad (t = 1, \ldots, T) \qquad (2)$$

If equation (2) is divided through by N_t, it is converted into an equation in per capita terms. Let $p_{jt} = N_{jt}/N_t$, and reinterpret y_{it}, the variables in X_{it}, and u_{it} as being the original values divided by N_t. Equation (2) in per capita terms can then be written:

$$y_{it} = X_{it}\alpha_i + \beta_{0i} + \beta_{1i}p_{1t} + \ldots + \beta_{Ji}p_{Jt} + u_{it}, \quad (t = 1, \ldots, T) \qquad (3)$$

A test of whether age distribution matters is simply a test of whether the $\beta_{1i}, \ldots, \beta_{Ji}$ coefficients in equation (3) are significantly different from zero.[52] If the coefficients are zero, one is back to a standard macroeconomic equation. Otherwise, given X_{it}, y_{it} varies as the age distribution varies. Since the sum of p_{jt} across j is one and there is a constant in the equation, a restriction on the β_{ji} coefficients must be imposed for estimation. In the estimation work below, the age group coefficients are restricted to sum to zero: $\sum_{j=1}^{J} \beta_{ji} = 0$. This means that if the distributional variables do not matter, then adding them to the equation will not affect the constant term.

The Age Distribution Data

The age distribution data that are used in the estimation of the US model are from the U.S. Census Bureau, monthly population estimates. Estimates are available monthly for ages 0 through 100. Fifty five age groups are considered here: ages

[52] Stoker (1986) characterizes this test (that all proportion coefficients are zero) as a test of microeconomic linearity or homogeneity (that all marginal reactions of individual agents are identical). He shows that individual differences or more general behavioral nonlinearities will coincide with the presence of distributional effects in macroeconomic equations.

$16, 17, \ldots, 69$, and 70+. The "total" population, N_t, is taken to be the population 16+. In terms of the above notation, 55 p_{jt} variables $(j = 1, \ldots, 55)$ have been constructed, where the 55 variables sum to one for a given t.

Constraints on the Age Coefficients

Since there are 55 β_{ji} coefficients to estimate, some constraints must be imposed on them if there is any hope of obtaining sensible estimates. One constraint is that the coefficients sum to zero. Another constraint, which was used in Fair and Dominguez (1991), is that the coefficients lie on a second degree polynomial. The second degree polynomial constraint allows enough flexibility to see if the prime age groups behave differently from the young and old groups while keeping the number of unconstrained coefficients small. A second degree polynomial in which the coefficients sum to zero is determined by two coefficients, and so there are two unconstrained coefficients to estimate per equation. The two variables that are associated with two unconstrained coefficients will be denoted AGE_{1t} and AGE_{2t}.

The variables AGE_{1t} and AGE_{2t} are as follows. First, the age variables enter equation i as $\sum_{j=1}^{55} \beta_{ji} p_{jt}$, where $\sum_{j=1}^{55} \beta_{ji} = 0$. The polynomial constraint is

$$\beta_{ji} = \gamma_0 + \gamma_1 j + \gamma_2 j^2 \quad , \quad (j = 1, \ldots, 55) \tag{4}$$

where γ_0, γ_1, and γ_2 are coefficients to be determined.[53] The zero sum constraint on the β_{ji}'s implies that

$$\gamma_0 = -\, \gamma_1 \frac{1}{55} \sum_{j=1}^{55} j \; - \; \gamma_2 \frac{1}{55} \sum_{j=1}^{55} j^2 \tag{5}$$

The way in which the age variables enter the estimated equation is then

$$\gamma_1 AGE_{1t} + \gamma_2 AGE_{2t}$$

where

$$AGE_{1t} = \sum_{j=1}^{55} j p_{jt} \; - \; \frac{1}{55} (\sum_{j=1}^{55} j)(\sum_{j=1}^{55} p_{jt}) \tag{6}$$

and

$$AGE_{2t} = \sum_{j=1}^{55} j^2 p_{jt} \; - \; \frac{1}{55} (\sum_{j=1}^{55} j^2)(\sum_{j=1}^{55} p_{jt}) \tag{7}$$

[53] For ease of notation, no i subscripts are used for the γ coefficients.

Given the estimates of γ_1 and γ_2, the 55 β_{ji} coefficients can be computed. This technique is simply Almon's (1965) polynomial distributed lag technique, where the coefficients that are constrained are the coefficients of the p_{jt} variables $(j = 1, \ldots, 55)$ rather than coefficients of the lagged values of some variable.

One test of whether age distribution matters is thus to add AGE_{1t} and AGE_{2t} to the equation and test if the two variables are jointly significant.

For the estimation of the equations in Section 3.6.3 a different set of constraints was imposed on the β_{ji} coefficients. The population 16+ was divided into four groups (16–25, 26–55, 56–65, and 66+) and it was assumed that the coefficients are the same within each group. Given the constraint that the coefficients sum to zero, this leaves three unconstrained coefficients to estimate. Let $P1625$ denote the percent of the 16+ population aged 16–25, and similarly for $P2655$, $P5665$, and $P66+$. Let γ_0 denote the coefficient of $P1625$ in the estimated equation, γ_1 the coefficient of $P2655$, γ_2 the coefficient of $P5665$, and γ_3 the coefficient of $P66+$, where $\gamma_0 + \gamma_1 + \gamma_2 + \gamma_3 = 0$. The summation constraint can be imposed by entering three variables in the estimated equation:

$$\begin{aligned} AG1 &= P2655 - P1625 \\ AG2 &= P5665 - P1625 \\ AG3 &= (P66+) - P1625 \end{aligned}$$

$AG1$, $AG2$, and $AG3$ are variables in the US model. The coefficient of $AG1$ in an equation is $\gamma_1 - \gamma_0$, the coefficient of $AG2$ is $\gamma_2 - \gamma_0$, and the coefficient of $AG3$ is $\gamma_3 - \gamma_0$. From the estimated coefficients for $AG1$, $AG2$, and $AG3$ and the summation constraint, one can calculate the four γ coefficients.

Imposing the constraints in the manner just described has an advantage over imposing the quadratic constraint of allowing more flexibility in the sense that three unconstrained coefficients are estimated instead of two. Also, I have found that the quadratic constraint sometimes leads to extreme values of β_{ji} for the very young and very old ages. The disadvantage of the present approach over the quadratic approach is that the coefficients are not allowed to change within the four age ranges.

3.6.3 Household Expenditure and Labor Supply Equations

The two main decision variables of a household in the theoretical model are consumption and labor supply. The determinants of these variables include the initial value of wealth and the current and expected future values of the wage rate, the price level, the interest rate, the tax rate, the level of transfer payments, and a possible labor constraint.

In the econometric model the expenditures of the household sector are disaggregated into four types: consumption of services, CS, consumption of nondurable goods, CN, consumption of durable goods, CD, and residential investment, IHH. Four labor supply variables are used: the labor force of men 25-54, $L1$, the labor force of women 25-54, $L2$, the labor force of all others 16+, $L3$, and the number of people holding more than one job, called "moonlighters," LM. These eight variables are determined by eight estimated equations.

Real after-tax income, YD/PH, is used as an explanatory variable in the expenditure equations, which implicitly assumes that the labor constraint is always binding on the household sector. In an earlier version of the model—Fair (1984)—a real wage rate variable and a labor constraint variable were used instead of YD/PH. The labor constraint variable was constructed to be zero or nearly zero in tight labor markets and to increase as labor markets loosen. The "classical" case is when the labor constraint is zero, where expenditures depend on the real wage rate. The "Keynesian" case is when labor markets are loose and the labor constraint variable is not zero. In this case the labor constraint variable is correlated with hours paid for, and so having both the real wage rate and the labor constraint variable in the equation is similar to having a real labor income variable in the equation. Tests of these two specifications generally support the use of YD/PH over the real wage rate and the labor constraint variable, and so YD/PH has been used. This does not necessarily mean, however, that the classical case never holds in practice. It may be that the use of the labor constraint variable is not an adequate way to try to account for the classical case. This is an area for future research.

The household real wealth variable is AA. The household after-tax interest rate variables in the model are RSA, a short term rate, and RMA, a long term rate. These interest rates are nominal rates. Section 3.12 is concerned with testing for nominal versus real interest rate effects, and it will be seen that in most cases the data support the use of nominal over real interest rates.

Variable $cnst2$ in the tables is variable C_{2t} in equation (7) in Subsection 2.3.2. In the estimation quarter 1968:4 corresponds to T_1 and quarter 1988:4 corresponds to T_2. The coefficient estimate for C_{2t} is the estimate of $/delta$. If the estimate is significant, this suggests that the constant term changed between 1968:4 and 1988:4.

Table A1: Equation 1. CS, consumer expenditures: services

Equation 1 is in real, per capita terms and is in log form. The explanatory variables include income, an interest rate, wealth, and the age variables. The age variables are highly jointly significant, and all the other variables are significant. Regarding the various tests, for the leads tests income is the variable for which led values were tried—in the form $\log[YD/(POP \cdot PH)]$. For the lags test the lagged values of the age variables were not included. The equation fails all the tests except the End test. The specification of this equation is thus problematic. The equation is sensitive to the use of alternative lags and to the treatment of serial correlation.

Table A2: Equation 2. CN, consumer expenditures: nondurables

Equation 2 is also in real, per capita, and log terms. The explanatory variables include income, an interest rate, wealth, and the age variables. The age variables are jointly significant at the 5 percent level, but not the 1 percent level. The other variables are significant except for $cnst2$, which has a t-statistic of -1.87, and the interest rate, which has a t-statistic of -1.78. Both the level and change of the lagged dependent variable are significant in the equation, and so the dynamic specification is more complicated than that of equation 1. Again, income is the variable for which led values were tried, and for the lags test the lagged values of the age variables were not included. The equation fails the lags, RHO, Leads+1, overid, and two of the three AP stability tests. The failure of the lags and RHO tests suggests that the dynamics have not been completely captured.

Table A3: Equation 3. CD, consumer expenditures: durables

Equation 3 is in real, per capital terms. The explanatory variables include income, an interest rate, wealth, the age variables, $DELD(KD/POP)_{-1} - (CD/POP)_{-1}$, and $(KD/POP)_{-1}$. KD is the stock of durable goods, and $DELD$ is the depreciation rate of the stock. The construction of these two variables is explained in Appendix A.

The justification for including the stock variable in the equation is as follows. Let KD^{**} denote the stock of durable goods that would be desired if there were no adjustment costs of any kind. If durable consumption is proportional to the stock of durables, then the determinants of consumption can be assumed to be the determinants of KD^{**}:

$$KD^{**} = f(...), \tag{8}$$

where the arguments of f are the determinants of consumption. Two types of partial adjustments are then postulated. The first is an adjustment of the durable stock:

$$KD^* - KD_{-1} = \lambda(KD^{**} - KD_{-1}), \tag{9}$$

where KD^* is the stock of durable goods that would be desired if there were no costs of changing durable expenditures. Given KD^*, desired durable expenditures, CD^*, is postulated to be

$$CD^* = KD^* - (1 - DELD)KD_{-1}, \tag{10}$$

where $DELD$ is the depreciation rate. By definition $CD = KD - (1 - DELD)KD_{-1}$, and equation (10) is merely the same equation for the desired values. The second type of adjustment is an adjustment of durable expenditures, CD, to its desired value:

$$CD - CD_{-1} = \gamma(CD^* - CD_{-1}) + \epsilon. \tag{11}$$

This equation is assumed to reflect costs of changing durable expenditures. Combining equations (8)–(11) yields:

$$\begin{aligned} CD - CD_{-1} = \gamma(DELD \cdot KD_{-1} - CD_{-1}) - \gamma\lambda KD_{-1} \\ + \gamma\lambda f(\ldots) + \epsilon. \end{aligned} \tag{12}$$

This specification of the two types of adjustment is a way of adding to the durable expenditure equation both the lagged dependent variable and the lagged stock of durables. Otherwise, the explanatory variables are the same as they are in the other expenditure equations.[54]

The interest rate used in equation 3, RMA, is multiplied by a scale variable, CDA. CDA is exogenous in the model. It is constructed from a peak to peak interpolation of CD/POP.

The age variables are jointly significant, and all the other variables are significant. The estimate of γ, the coefficient of $DELD(KD/POP)_{-1}$ −

[54]Note in Table A3 that CD is divided by POP and CD_{-1} and KD_{-1} are divided by POP_{-1}, where POP is population. If equations (8)–(11) are defined in per capita terms, where the current values are divided by POP and the lagged values are divided by POP_{-1}, then the present per capita treatment of equation (11) follows. The only problem with this is that the definition used to justify equation (9) does not hold if the lagged stock is divided by POP_{-1}. All variables must be divided by the same population variable for the definition to hold. This is, however, a minor problem, and it has been ignored here. The same holds for equation 4.

$(CD/POP)_{-1}$, is .232. This is the partial adjustment coefficient for CD. The estimate of $\gamma\lambda$, the coefficient of $(KD/POP)_{-1}$, is .028, which gives an implied value of λ, the partial adjustment coefficient for KD^*, of .121. KD^* is thus estimated to adjust to KD^{**} at a rate of .121 per quarter. Income is the variable for which led values were tried, and for the lags test the lagged values of the age variables were not included. The equation fails the T test, the Leads+4 test, the overid test, and two of the three AP stability tests. It passes the rest.

Table A4: Equation 4. IHH, residential investment—h

The same partial adjustment model is used for residential investment than was used above for durable expenditures, which adds $DELH(KH/POP)_{-1} - (IHH/POP)_{-1}$, and $(KH/POP)_{-1}$ to the residential investment equation. KH is the stock of housing, and $DELH$ is the depreciation rate of the stock. The construction of these two variables is explained in Appendix A. Equation 4 does not include the wealth variable because the variable was not significant. Likewise, it does not include the age variables because they were not significant. It is estimated under the assumption of a second order autoregressive process for the error term. The interest rate used in equation 4, RMA_{-1}, is multiplied by a scale variable, $IHHA$. $IHHA$ is exogenous in the model. It is constructed from a peak to peak interpolation of IHH/POP.

All the variables are significant in equation 4 except for $cnst2$ and the income variable, which have t-statistics of 1.77 and 1.28 respectively. Income is the variable for which led values were tried. The equation passes all but the overid test and two of the three AP stability tests. The estimate of γ, the partial adjustment coefficient for IHH, is .300. The estimate of $\gamma\lambda$ is .020, which gives an implied value of λ, the partial adjustment coefficient for KH^*, of .067.

Table A5: Equation 5. $L1$, labor force—men 25-54

Equation 5 explains the labor force participation rate of men 25-54. It is in log form and includes as explanatory variables the wealth variable and the unemployment rate. The unemployment rate is meant to pick up the effect of the labor constraint on labor supply (a discouraged worker effect). The wealth variable has a negative coefficient estimate, as expected, as does the unemployment rate. The equation passes the lags test and fails the RHO, T, and overid tests. It passes the AP tests, but fails the End test.

Table A6: Equation 6. $L2$, labor force—women 25-54

Equation 6 explains the labor force participation rate of women 25-54. It is in log form and includes as explanatory variables the real wage and the wealth variable. Again, the wealth variable has a negative coefficient estimate, as expected. The real wage variable has a positive coefficient estimate, implying that the substitution effect dominates the income effect. Neither of these coefficient estimates is significant, however. The variable for which led values were tried is the real wage, $\log(WA/PH)$. The equation passes all but the AP tests. One of the χ^2 tests has $\log PH$ added as an explanatory variable. This is a test of the use of the real wage in the equation. If $\log PH$ is significant, this is a rejection of the hypothesis that the coefficient of $\log WA$ is equal to the negative of the coefficient of $\log PH$, which is implied by the use of the real wage. As can be seen, $\log PH$ is not significant. There is no overid test because only four first stage regressors were used.

Table A7: Equation 7. $L3$, labor force—all others 16+

Equation 7 explains the labor force participation rate of all others 16+. It is also in log form and includes as explanatory variables the real wage, the wealth variable, and the unemployment rate. All the coefficient estimates are significant. The coefficient estimate of the real wage is positive and the coefficient estimates of the wealth variable and the unemployment rate are negative. The variable for which led values were tried is the real wage. The equation passes all the tests except for the third AP test.

Table A8: Equation 8. LM, number of moonlighters

Equation 8 determines the number of moonlighters. It is in log form and includes as explanatory variables the real wage and the unemployment rate. The coefficient estimate of the real wage is positive, suggesting that the substitution effect dominates for moonlighters, although it is not significant, with a t-statistic of 1.73. The coefficient estimate of the unemployment rate is negative and significant, which is the discouraged worker effect applied to moonlighters. The variable for which led values were tried is the real wage. The equation passes the lags, RHO, T, and leads tests. It fails the test of adding $\log PH$ ($\log PH$ is significant), which is evidence against the real wage constraint. It fails the three AP tests and passes the overid and End tests.

This completes the discussion of the household expenditure and labor supply equations. A summary of some of the general results across the equations is in Section 3.6.11.

3.6.4 The Firm Sector Equations

In the maximization problem of a firm in the theoretical model there are five main decision variables: the firm's price, production, investment, demand for employment, and wage rate. These five decision variables are determined jointly in that they are the result of solving one maximization problem. The variables that affect this solution include 1) the initial stocks of excess capital, excess labor, and inventories, 2) the current and expected future values of the interest rate, 3) the current and expected future demand schedules for the firm's output, 4) the current and expected future supply schedules of labor facing the firm, and 5) the firm's expectations of other firms' future price and wage decisions.

In the econometric model seven variables are chosen to represent the five decisions: 1) the price level for the firm sector, PF, 2) production, Y, 3) investment in nonresidential plant and equipment, IKF, 4) the number of jobs in the firm sector, JF, 5) the average number of hours paid per job, HF, 6) the average number of overtime hours paid per job, HO, and 7) the wage rate of the firm sector, WF. Each of these variables is determined by a stochastic equation, and these are the main stochastic equations of the firm sector.

Moving from the theoretical model of firm behavior to the econometric specifications is not straightforward, and a number of approximations have been made. One of the key approximations is to assume that the five decisions of a firm are made sequentially rather than jointly. The sequence is from the price decision, to the production decision, to the investment and employment decisions, and to the wage rate decision. In this way of looking at the problem, the firm first chooses its optimal price path. This path implies a certain expected sales path, from which the optimal production path is chosen. Given the optimal production path, the optimal paths of investment and employment are chosen. Finally, given the optimal employment path, the optimal wage path is chosen.

Table A10: Equation 10. PF**, price deflator for** $X - FA$

Equation 10 is the key price equation in the model. The equation is in log form. The price level is a function of the lagged price level, the wage rate inclusive of the employer social security tax rate, the price of imports, the unemployment rate, and the time trend. The unemployment rate is taken as a measure of demand pressure.

The lagged price level is meant to pick up expectational effects, and the wage rate and import price variables are meant to pick up cost effects. The log of the wage rate variable has subtracted from it $\log LAM$, where LAM is a measure of potential labor productivity. The construction of LAM is explained in Appendix A; it is computed from a peak to peak interpolation of measured productivity.

An important feature of the price equation is that the price *level* is explained by the equation, not the price *change*. This treatment is contrary to the standard Phillips-curve treatment, where the price (or wage) change is explained by the equation. It is also contrary to the standard NAIRU specification, where the change in the change in the price level (i.e., the change in the inflation rate) is explained. In the theoretical model the natural decision variables of a firm are the levels of prices and wages. For example, the market share equations in the theoretical model have a firm's market share as a function of the ratio of the firm's price to the average price of other firms. These are price levels, and the objective of the firm is to choose the price level path (along with the paths of the other decision variables) that maximizes the multiperiod objective function. A firm decides what its price *level* should be relative to the price *levels* of other firms. This thus argues for a specification in levels, which is used here. The issue of the best functional form for the price equation is the subject matter of Section 3.13, where the NAIRU model is tested.

The time trend, T, in equation 10 is meant to pick up any trend effects on the price level not captured by the other variables. Adding the time trend to an equation like 10 is similar to adding the constant term to an equation specified in terms of changes rather than levels. The time trend will also pick up any trend mistakes made in constructing LAM. If, for example, $LAM_t = LAM_t^a + \alpha_1 t$, where LAM_t^a is the correct variable to subtract from the wage rate variable to adjust for potential productivity, then the time trend will absorb this error.

The constant term in equation 10 is assumed to be time varying, and so $cnst2$ is added as an explanatory variable. In addition, the coefficient of T is assumed to be time varying, with the same T_1 and T_2 as for $cnst2$. The additional variable added in this case is $cnst2 \times T$, which is denoted TB in Table A10. All the variables in equation 10 are significant except the constant term and $cnst2$. The variable for which led values were tried is the wage rate variable. The lags, RHO, and leads tests are passed. The last two χ^2 tests have output gap variables added. When each of these variables is added, it is not significant and (not shown) the unemployment rate retains its significance. The unemployment rate thus dominates the output gap variables. The equation fails the AP tests, with the break in the late 1970s. It passes the overid and End tests.

Equation 11. Y, production—f

The specification of the production equation is where the assumption that a firm's decisions are made sequentially begins to be used. The equation is based on the assumption that the firm sector first sets it price, then knows what its sales for the current period will be, and from this latter information decides on what its production for the current period will be.

In the theoretical model production is smoothed relative to sales. The reason for this is various costs of adjustment, which include costs of changing employment, costs of changing the capital stock, and costs of having the stock of inventories deviate from some proportion of sales. If a firm were only interested in minimizing inventory costs, it would produce according to the following equation (assuming that sales for the current period are known):

$$Y = X + \beta X - V_{-1}, \tag{13}$$

where Y is the level of production, X is the level of sales, V_{-1} is the stock of inventories at the end of the previous period, and β is the inventory-sales ratio that minimizes inventory costs. The construction of V is explained in Appendix A. Since by definition $V - V_{-1} = Y - X$, producing according to equation (13) would ensure that $V = \beta X$. Because of the other adjustment costs, it is generally not optimal for a firm to produce according to equation (13). In the theoretical model there was no need to postulate explicitly how a firm's production plan deviated from equation (13) because its optimal production plan just resulted, along with the other optimal paths, from the direct solution of its maximization problem. For the empirical work, however, it is necessary to make further assumptions.

The estimated production equation is based on the following three assumptions:

$$\log V^* = \beta \log X, \tag{14}$$

$$\log Y^* = \log X + \alpha(\log V^* - \log V_{-1}), \tag{15}$$

$$\log Y - \log Y_{-1} = \lambda(\log Y^* - \log Y_{-1}) + \epsilon, \tag{16}$$

where * denotes a desired value. (In the following discussion all variables are assumed to be in logs.) Equation (14) states that the desired stock of inventories is proportional to current sales. Equation (15) states that the desired level of production is equal to sales plus some fraction of the difference between the desired stock of inventories and the stock on hand at the end of the previous period. Equation (16) states that actual production partially adjusts to desired production each period.

Combining equations (14)–(16) yields

$$\log Y = (1-\lambda)\log Y_{-1} + \lambda(1+\alpha\beta)\log X - \lambda\alpha\log V_{-1} + \epsilon. \qquad (17)$$

Equation 11 is the estimated version of equation (17). The equation is estimated under the assumption of a third order autoregressive process of the error term, and three dummy variables are added to account for the effects of a steel strike in the last half of 1959.

The estimate of $1-\lambda$ is .359, and so the implied value of λ is .641, which means that actual production adjusts 64.1 percent of the way to desired production in the current quarter. The estimate of $\lambda\alpha$ is .213, and so the implied value of α is .332. This means that (in logs) desired production is equal to sales plus 33.2 percent of the desired change in inventories. The estimate of $\lambda(1+\alpha\beta)$ is .801, and so the implied value of β is .752. The variable for which led values were used is the log level of sales, $\log X$. Equation 11 passes all the tests. The passing of the leads tests, which means that the led values are not significant, is evidence against the hypothesis that firms have rational expectations regarding future values of sales.

The estimates of equation 11 are consistent with the view that firms smooth production relative to sales. The view that production is smoothed relative to sales was challenged by Blinder (1981) and others. This work was in turn challenged in Fair (1989) as being based on faulty data. The results in Fair (1989), which use data in physical units, suggest that production is smoothed relative to sales. The results using the physical units data thus provide some support for the current aggregate estimates.

Table A12: Equation 12. KK, stock of capital—f

Equation 12 explains the stock of capital of the firm sector, KK. Given KK, the nonresidential fixed investment of the firm sector, IKF, is determined by identity 92:

$$IKF = KK - (1-DELK)KK_{-1}, \qquad 92$$

where $DELK$ is the depreciation rate. The construction of KK and $DELK$ is explained in Appendix A. Equation 12 will sometimes be referred to as an "investment" equation, since IKF is determined once KK is.

Equation 12 is based on the assumption that the production decision has already been made. In the theoretical model, because of costs of changing the capital stock, it may sometimes be optimal for a firm to hold excess capital. If there were no such costs, investment each period would merely be the amount needed to have enough capital to produce the output of the period. In the theoretical model there was no need to postulate explicitly how investment deviates from this amount, but for the empirical work this must be done.

The estimated equation for KK is based on the following two equations:

$$\begin{aligned}\log(KK^*/KK_{-1}) = \alpha_0 \log(KK_{-1}/KKMIN_{-1}) + \alpha_1 \Delta \log Y \\ +\alpha_2 \Delta \log Y_{-1} + \alpha_3 \Delta \log Y_{-2} + \alpha_4 \Delta \log Y_{-3} \\ +\alpha_5 \Delta \log Y_{-4} + \alpha_6 r,\end{aligned} \tag{18}$$

$$\begin{aligned}\log(KK/KK_{-1}) - \log(KK_{-1}/KK_{-2}) = \lambda[\log(KK^*/KK_{-1}) - \\ - \log(KK_{-1}/KK_{-2})] + \epsilon,\end{aligned} \tag{19}$$

where r is some measure of the cost of capital, α_0 and α_6 are negative, and the other coefficients are positive. The construction of $KKMIN$ is explained in Appendix A. It is, under the assumption of a putty-clay technology, an estimate of the minimum amount of capital required to produce the current level of output, Y. $KK_{-1}/KKMIN_{-1}$ is thus the ratio of the actual capital stock on hand at the end of the previous period to the minimum required to produce the output of that period. $\log(KK_{-1}/KKMIN_{-1})$ will be referred to as the amount of "excess capital" on hand.

KK^* in equation (18) is the value of the capital stock the firm would desire to have on hand in the current period if there were no costs of changing the capital stock. The desired change, $\log(KK^*/KK_{-1})$, depends on 1) the amount of excess capital on hand, 2) five change-in-output terms, and 3) the cost of capital. The lagged output changes are meant to be proxies for expected future output changes. Other things equal, the firm desires to increase the capital stock if the output changes are positive. Equation (19) is a partial adjustment equation of the actual capital stock to the desired stock. It states that the actual percentage change in the capital stock is a fraction of the desired percentage change.

Ignoring the cost of capital term in equation (18), the equation says that the desired capital stock approaches $KKMIN$ in the long run if output is not changing. How can the cost of capital term be justified? In the theoretical model the cost of capital affects the capital stock by affecting the kinds of machines that are purchased. If the cost of capital falls, machines with lower labor requirements are purchased, other things being equal. For the empirical work, data are not available by types of machines, and approximations have to be made. The key approximation that is made in Appendix A is the postulation of a putty-clay technology in the construction of $KKMIN$. If there is in fact some substitution of capital for labor in the short run, the cost of capital is likely to affect the firm's desired capital stock, and this is the reason for including a cost of capital term in equation (18).

Combining equations (18) and (19) yields:

$$\begin{aligned}\Delta \log KK = \lambda\alpha_0 \log(KK_{-1}/KKMIN_{-1}) + (1-\lambda)\Delta \log KK_{-1} \\ +\lambda\alpha_1 \Delta \log Y + \lambda\alpha_2 \Delta \log Y_{-1} + \lambda\alpha_3 \Delta \log Y_{-2} \\ +\lambda\alpha_4 \Delta \log Y_{-3} + \lambda\alpha_5 \Delta \log Y_{-4} + \lambda\alpha_6 r + \epsilon.\end{aligned} \tag{20}$$

Equation 12 is the estimated version of equation (20).

The estimate of $1-\lambda$ is .905, and so the implied value of λ is .095. The estimate of $\lambda\alpha_0$ is $-.0070$, and so the implied value of α_0 is $-.074$. This is the estimate of the size of the effect of excess capital on the desired stock of capital. The variable for which led values were tried is the log change in output. Equation 12 passes all the tests except the last two AP tests. The passing of the leads tests is evidence against the hypothesis that firms have rational expectations with respect to future values of output.

There are two cost of capital variables in equation 12. Both are lagged two quarters. One is an estimate of the real AAA bond rate, which is the nominal AAA bond rate, RB, less the four-quarter rate of inflation. The four-quarter rate of inflation is taken as a proxy for the expected rate of inflation over the horizon relevant for RB. The cost of capital variable is a function of stock price changes. It is the ratio of capital gains or losses on the financial assets of the household sector (mostly from corporate stocks) over three quarters to nominal potential output. This ratio is a measure of how well or poorly the stock market is doing. If the stock market is doing well, for example, the ratio is high, which should in general lower the cost of capital to firms. The interest rate variable has a coefficient estimate of the expected sign, although it is not significant, with a t-statistic of only -0.29. The capital gains variable is significant, with a t-statistic of 3.98.

Table A13: Equation 13. JF, number of jobs—f

The employment equation 13 and the hours equation 14 are similar in spirit to the capital stock equation 12. They are also based on the assumption that the production decision is made first. Because of adjustment costs, it is sometimes optimal in the theoretical model for firms to hold excess labor. Were it not for the costs of changing employment, the optimal level of employment would merely be the amount needed to produce the output of the period. In the theoretical model there was no need to postulate explicitly how employment deviates from this amount, but this must be done for the empirical work.

The estimated employment equation is based on the following two equations:

$$\log(JF^*/JF_{-1}) = \alpha_0 \log[JF_{-1}/(JHMIN_{-1}/HFS_{-1})] + \alpha_1 \Delta \log Y, \quad (21)$$

$$\log(JF/JF_{-1}) - \log(JF_{-1}/JF_{-2}) = \lambda[\log(JF^*/JF_{-1}) - \log(JF_{-1}/JF_{-2})] + \epsilon, \quad (22)$$

where α_0 is negative and the other coefficients are positive. The construction of $JHMIN$ and HFS is explained in Appendix A. $JHMIN$ is, under the assumption of a putty-clay technology, an estimate of the minimum number of worker hours required to produce the current level of output, Y. HFS is an estimate of the desired number of hours worked per worker. $JF_{-1}/(JHMIN_{-1}/HFS_{-1})$ is the ratio of the actual number of workers on hand at the end of the previous period to the minimum number required to produce the output of that period if the average number of hours worked were HFS_{-1}. $\log[JF_{-1}/JHMIN_{-1}/HFS_{-1})]$ will be referred to as the amount of "excess labor" on hand.

JF^* in equation (21) is the number of workers the firm would desire to have on hand in the current period if there were no costs of changing employment. The desired change, $\log(JF^*/JF_{-1})$, depends on the amount of excess labor on hand and the change in output. This equation says that the desired number of workers approaches $JHMIN/HFS$ in the long run if output is not changing. Equation (22) is a partial adjustment equation of the actual number of workers to the desired number.

Combining equations (21) and (22) yields:

$$\Delta \log JF = \lambda\alpha_0 \log[JF_{-1}/(JHMIN_{-1}/HFS_{-1})] + (1-\lambda)\Delta \log JF_{-1} + \lambda\alpha_1 \Delta \log Y + \epsilon. \quad (23)$$

Equation 13 is the estimated version of equation (23). It has a dummy variable, $D593$, added to pick up the effects of a steel strike.

The estimate of $1-\lambda$ is .606, and so the implied value of λ is .394. The estimate of $\lambda\alpha_0$ is -.036, and so the implied value of α_0 is -.091. This is the estimate of the size of the effect of excess labor on the desired number of workers. The variable for which led values were tried is the change in the log of output. The equation fails the lags, RHO, and AP tests and passes the rest. Again, the passing of the leads tests is evidence against the hypothesis that firms have rational expectations with respect to future values of output.

The ideas behind the employment demand equation 13 and the hours demand equation 14 discussed next go back to my Ph.D. dissertation, Fair (1969). See also Fair (1985), which shows that the aggregate equations are consistent with the survey results of Fay and Medoff (1985). These two equations have held up remarkably well over the years.

Table A14: Equation 14. HF, average number of hours paid per job—f

The estimated hours equation is:

$$\begin{gathered}\Delta \log HF = \lambda \log(HF_{-1}/HFS_{-1}) \\ +\alpha_0 \log[JF_{-1}/(JHMIN{-1}/HFS_{-1})] + \alpha_1 \Delta \log Y + \epsilon.\end{gathered} \tag{24}$$

The first term on the right hand side of equation (24) is the (logarithmic) difference between the actual number of hours paid for in the previous period and the desired number. The reason for the inclusion of this term in the hours equation but not in the employment equation is that, unlike JF, HF fluctuates around a slowly trending level of hours. This restriction is captured by the first term in (24). The other two terms are the amount of excess labor on hand and the current change in output. Both of these terms affect the employment decision, and they should also affect the hours decision since the two are closely related. Equation 14 is the estimated version of equation (24).

The estimate of λ is $-.174$, and the estimate of α_0 is $-.027$. All the coefficient estimates are significant in the equation. The variable for which led values were tried is the change in the log of output. The equation passes the RHO, leads, and End tests. It fails the others.

Table A15: Equation 15. HO, average number of overtime hours paid per job—f

Equation 15 explains overtime hours, HO. Let $HFF = HF - HFS$, which is the deviation of actual hours per worker from desired hours. One would expect HO to be close to zero for low values of HFF (i.e., when actual hours are much below desired hours), and to increase roughly one for one for high values of HFF. An approximation to this relationship is

$$HO = e^{\alpha_1 + \alpha_2 HFF + \epsilon}, \tag{25}$$

which in log form is

$$\log HO = \alpha_1 + \alpha_2 HFF + \epsilon. \tag{26}$$

Equation 15 is the estimated version of equation (26). Both HFF and HFF_{-1} are included in the equation, which appears to capture the dynamics better. The equation is estimated under the assumption of a first order autoregressive error term.

All the coefficient estimates in equation 15 are significant, and the equation passes all the tests.

Table A16: Equation 16. WF, average hourly earnings excluding overtime—f

Equation 16 is the wage rate equation. It is in log form. In the final specification, the wage rate was simply taken to be a function of the constant term, the time trend, the current value of the price level, the lagged value of the price level, and the lagged value of the wage rate. Labor market tightness variables like the unemployment rate were not significant in the equation. The time trend is added to account for trend changes in the wage rate relative to the price level. The potential productivity variable, LAM, is subtracted from the wage rate in equation 16. The price equation, equation 10, is identified because the wage rate equation includes the lagged wage rate, which the price equation does not. The wage rate equation is identified because the price equation includes the price of imports and the unemployment rate, which the wage rate equation does not.

A constraint was imposed on the coefficients in the wage equation to ensure that the determination of the real wage implied by equations 10 and 16 is sensible. Let $p = \log PF$ and $w = \log WF$. The relevant parts of the price and wage equations regarding the constraints are

$$p = \beta_1 p_{-1} + \beta_2 w + \ldots, \quad (27)$$

$$w = \gamma_1 w_{-1} + \gamma_2 p + \gamma_3 p_{-1} + \ldots. \quad (28)$$

The implied real wage equation from these two equations should not have $w - p$ as a function of either w or p separately, since one does not expect the real wage to grow simply because the levels of w and p are growing. The desired form of the real wage equation is thus

$$w - p = \delta_1 (w_{-1} - p_{-1}) + \ldots, \quad (29)$$

which says that the real wage is a function of its own lagged value plus other terms. The real wage in equation (29) is *not* a function of the level of w or p separately. The constraint on the coefficients in equations (27) and (28) that imposes this restriction is:

$$\gamma_3 = [\beta_1/(1 - \beta_2)](1 - \gamma_2) - \gamma_1. \quad (30)$$

This constraint is imposed in the estimation by first estimating the price equation to get estimates of β_1 and β_2 and then using these estimates to impose the constraint on γ_3 in the wage equation.

The coefficient estimates in equation 16 are all significant. The equation passes all but the End test. One of the χ^2 tests is a test of the real wage restriction, and this restriction is not rejected by the data. The final χ^2 test in the table has the unemployment rate added as an explanatory variable, and it is not significant. As noted above, no demand pressure variables were found to be significant in the wage equation.

Table A18: Equation 18. DF, dividends paid—f

Let Π denote after-tax profits. If in the long run firms desire to pay out all of their after-tax profits in dividends, one can write $DF^* = \Pi$, where DF^* is the long run desired value of dividends for profit level Π. If it is assumed that actual dividends are partially adjusted to desired dividends each period as

$$DF/DF_{-1} = (DF^*/DF_{-1})^{\lambda} e^{\epsilon}, \tag{31}$$

then the equation to be estimated is

$$\Delta \log DF = \lambda \log(\Pi/DF_{-1}) + \epsilon. \tag{32}$$

Equation 18 is the estimated version of equation (32). The level of after-tax profits in the notation of the model is $PIEF - TFG - TFS - TFR$.

The estimate of λ is .025, which implies a slow adjustment of actual to desired dividends. The equation passes all the tests except the End test. The last χ^2 test in Table A18 shows that the constant term is not significant. The above specification does not call for the constant term, and this is supported by the data. Regarding the first χ^2 test in the table, because of the assumption that $DF^* = \Pi$, the coefficient of $\log(PIEF - TFG - TFS - TFR)$ is restricted to be the negative of the coefficient of $\log DF_{-1}$. If instead $DF^* = \Pi^{\gamma}$, where γ is not equal to one, then the restriction does not hold. The first test in the table is a test of the restriction (i.e., a test that $\gamma = 1$), and the hypothesis that $\gamma = 1$ is not rejected.

3.6.5 Money Demand Equations

In earlier versions of the US model a demand for money equation of the household sector was estimated (old equation 9). The data became unreliable, and this equation is no longer in the model. The model contains two demand for money equations: a demand for money equation for the firm sector and a demand for currency equation. These two equations are not in fact important in the model because of the use of the interest rate rule (equation 30 below). They are included more for completeness than anything else. When the interest rate rule is used, the short term interest rate is determined by the rule and the overall money supply is whatever is needed to have the demand for money equations be met.

Before presenting these two equations, it is necessary to discuss how the dynamics are handled. The key question about the dynamics is whether the adjustment of actual to desired values is in nominal or real terms. Let M_t^*/P_t denote the desired level of real money balances, let y_t denote a measure of real transactions, and let r_t denote a short term interest rate. Assume that the equation determining desired money balances is in log form and write

$$\log(M_t^*/P_t) = \alpha + \beta \log y_t + \gamma r_t. \tag{33}$$

Note that the log form has not been used for the interest rate. Interest rates can at times be quite low, and it may not be sensible to take the log of the interest rate. If, for example, the interest rate rises from .02 to .03, the log of the rate rises from -3.91 to -3.51, a change of .40. If, on the other hand, the interest rate rises from .10 to .11, the log of the rate rises from -2.30 to -2.21, a change of only .09. One does not necessarily expect a one percentage point rise in the interest rate to have four times the effect on the log of desired money holdings when the change is from a base of .02 rather than .10. In practice the results of estimating money demand equations do not seem to be very sensitive to whether the level or the log of the interest rate is used. For the work here the level of the interest rate has been used.

If the adjustment of actual to desired money holdings is in real terms, the adjustment equation is

$$\log(M_t/P_t) - \log(M_{t-1}/P_{t-1}) = \lambda[\log(M_t^*/P_t) - \log(M_{t-1}/P_{t-1})] + \epsilon. \tag{34}$$

If the adjustment is in nominal terms, the adjustment equation is

$$\log M_t - \log M_{t-1} = \lambda(\log M_t^* - \log M_{t-1}) + \mu. \tag{35}$$

Combining (33)and (34) yields

$$\log(M_t/P_t) = \lambda\alpha + \lambda\beta \log y_t + \lambda\gamma r_t + (1-\lambda)\log(M_{t-1}/P_{t-1}) + \epsilon. \tag{36}$$

Combining (33)and (35) yields

$$\log(M_t/P_t) = \lambda\alpha + \lambda\beta \log y_t + \lambda\gamma r_t + (1-\lambda)\log(M_{t-1}/P_t) + \mu. \tag{37}$$

Equations (36) and (37) differ in the lagged money term. In (36), which is the real adjustment specification, M_{t-1} is divided by P_{t-1}, whereas in (37), which is the nominal adjustment specification, M_{t-1} is divided by P_t.

A test of the two hypotheses is simply to put both lagged money variables in the equation and see which one dominates. If the real adjustment specification is correct, $\log(M_{t-1}/P_{t-1})$ should be significant and $\log(M_{t-1}/P_t)$ should not, and vice versa if the nominal adjustment specification is correct. This test may, of course, be inconclusive in that both terms may be significant or insignificant, but I have found that this is rarely the case. This test was performed on the two demand for money equations, and in each case the nominal adjustment specification won. The nominal adjustment specification has thus been used for the equations.

Table A17: Equation 17. MF, demand deposits and currency—f

Equation 17 is the demand for money equation of the firm sector. The equation is in log form. The transactions variable is the level of nonfarm firm sales, $X - FA$, and the interest rate variable is the after-tax three-month Treasury bill rate. The tax rates used in this equation are the corporate tax rates, $D2G$ and $D2S$.

All the variables are significant in the equation. The test results show that the lagged dependent variable that pertains to the real adjustment specification, $\log(MF/PF)_{-1}$, is insignificant. The equation passes all the other tests except the End test.

Table A26: Equation 26. CUR, currency held outside banks

Equation 26 is the demand for currency equation. It is in per capita terms and is in log form. The transactions variable that is used is the level of nonfarm firm sales. The interest rate variable used is RSA, and the equation is estimated under the assumption of a first order autoregressive error term.

All the variables in the equation are significant. The test results show that the lagged dependent variable that pertains to the real adjustment specification, $\log[CUR/(POP \cdot PF)]_{-1}$, is not significant, which supports the nominal adjustment specification. The equation passes the lags, T, overid, and End tests and fails the RHO and AP tests.

3.6.6 Other Financial Equations

The stochastic equations for the financial sector consist of two term structure equations, and an equation explaining the change in stock prices.

Table A23: Equation 23. RB, bond rate;
Table A24: Equation 24. RM, mortgage rate

The expectations theory of the term structure of interest rates states that long term rates are a function of the current and expected future short term rates. The two long term interest rates in the model are the bond rate, RB, and the mortgage rate, RM. These rates are assumed to be determined according to the expectations theory, where the current and past values of the short term interest rate (the three-month Treasury bill rate, RS) are used as proxies for expected future values. Equations 23 and 24 are the two estimated equations. The lagged dependent variable is used in each of these equations, which implies a fairly complicated lag structure relating each long term rate to the past values of the short term rate. In addition, a constraint has been imposed on the coefficient estimates. The sum of the coefficients of the current and lagged values of the short term rate has been constrained to be equal to one minus the coefficient of the lagged long term rate. This means that, for example, a sustained one percentage point increase in the short term rate eventually results in a one percentage point increase in the long term rate. (This restriction is imposed by subtracting RS_{-2} from each of the other interest rates in the equations.) Equation 23 (but not 24) is estimated under the assumption of a first order autoregressive error term.

The overall results for the two equations are quite good. The short term interest rates are significant in the two estimated equations except for RS_{-1} in equation 24. The first test result for each equation shows that the coefficient restriction is not rejected for either equation. Both equations pass the lags and RHO tests. Equation 24 passes the T test. Equation 23 passes the three AP tests, and equation 24 passes one of the three. The variable for which led values were tried is the short term interest rate, RS, and the χ^2 tests show that the led values are not significant except for the Leads+4 test for equation 23. (The Leads+8 test for equation 23 failed.) Two inflation expectations variables, $\dot{p}^e_{4t}$ and $\dot{p}^e_{8t}$, were added to the equations, and the test results also show that these variables are not significant.

Table A25: Equation 25. CG, capital gains or losses on the financial assets of h

The variable CG is the change in the market value of financial assets held by the household sector, almost all of which is the change in the market value of corporate stocks held by the household sector. In the theoretical model the aggregate value of stocks is determined as the present discounted value of expected future after-tax cash flow, the discount rates being the current and expected future short term interest rates. The theoretical model thus implies that CG should be a function of changes in expected future after-tax cash flow and of changes in the current and expected future interest rates. In the empirical work the change in the bond rate, ΔRB, is used as a proxy for changes in expected future interest rates, and the change in after-tax profits, $\Delta(PIEF - TFG - TFS - TFR)$, is used as a proxy for changes in expected future after-tax cash flow. In the estimated equation CG and the change in after-tax profits are normalized by $PX_{-1}YS_{-1}$, which is a measure of potential output in nominal terms. Equation 25 is the estimated equation, where $CG/(PX_{-1}YS_{-1})$ is regressed on the constant term, ΔRB, and $\Delta[(PIEF - TFG - TFS - TFR)]/(PX_{-1}YS_{-1})$.

The fit of equation 25 is poor. The coefficient estimates have the right sign but are not significant except for the estimate of the constant term. The equation passes the lags, RHO, T, and AP tests. The variables for which led values were tried are the change in the bond rate and the change in after-tax profits. The led values are not significant. For the final χ^2 test ΔRS, the change in the short term rate, was added under the view that it might also be a proxy for expected future interest rate changes, and it is not significant. The equation fails the End test. The fact that the equation passes all the tests (except the End test) suggests that it is unlikely that it can be improved much.

It will be seen in Sections 4.2, 5.3, and 5.4 that the effects of CG on the economy are large. They account, for example, for most of the unusual features of the U.S. economy in the last half of the 1990s. Although fluctuations in CG have large effects, the results of estimating equation 25 show that most of these fluctuations are not explained.

3.6.7 Interest Payments Equation

Table A29: Equation 29. $INTG$, interest payments—g

$INTG$ is the level of net interest payments of the federal government. Data on this variable are NIPA data. AG is the level of net financial assets of the federeal government. Data on this variable are FFA data. AG is negative because the federal government is a net debtor. It consistgs of both short term and long term securities.

The current level of interest payments of the federal government depends on the amount of existing securities issued at each date in the past and on the relevant interest rate prevailing at each date. The link from AG to $INTG$ is thus complicated. It depends on past issues and the interest rates paid on these issues. A number of approximations have to be made in trying to model this link, and the procedure used here is a follows.

Let RQG denote a weighted average of the current value of the short term interest rate, RS, and current and past values of 0.75 times the long term bond rate, RB, with weights of .4 and .6:[55] RB is multiplied by 0.75, since the federal government pays a lower interest rate than the AAA corporate bond rate, which is RB. RQG is

$$\begin{aligned} RQG = [.4RS + .75(.6)(RB + RB_{-1} + RB_{-2} + RB_{-3} + RB_{-4} + RB_{-5} \\ + RB_{-6} + RB_{-7})/8]/400. \end{aligned} \tag{38}$$

In this equation RS and RB are divided by 400 to put RQG at a quarterly rate in percent units. The variable $INTG/(-AG)$ is the ratio of interest payments of the federal government to the net financial debt of the federal government. This ratio is a function of current and past interest rates, among other things. In the empirical specification $INTG/(-AG)$ is taken to depend on the constant term, RQG, and $INTG_{-1}/(-AG_{-1})$. This equation, which is equation 29, is estimated under the assumption of a first order autoregressive error term.

The results are in Table A29. The coefficient estimate for RQG is positive and significant. The equation fails all the tests by wide margins. It is clearly only a rough approximation.

Equation 29 is important in the model because when interest rates change, federal interest payments change, which changes household income and the deficit and debt of the federal government.

[55]These weights were chosen after some experimentation. The results are not sensitive to slightly different choices.

3.6.8 The Import Equation

Table A27: Equation 27. IM, Imports

The import equation is in per capita terms and is in log form. The explanatory variables include per capita expenditures on consumption and investment, a price deflator for domestically produced goods, PF, relative to the import price deflator, PIM, and four dummy variables to account for two dock strikes.

The coefficient estimates are significant and of the expected signs. The equation passes all but the RHO and overid tests. The variable for which led values were tried is the per capita expenditure variable, and the led values are not significant. The last χ^2 test in Table A27 adds $\log PF$ to the equation, which is a test of the restriction that the coefficient of $\log PF$ is equal to the negative of the coefficient of $\log PIM$. The $\log PF$ variable is not significant, and so the restriction is not rejected.

3.6.9 Unemployment Benefits

Table A28: Equation 28. UB, unemployment insurance benefits

Equation 28 explains unemployment insurance benefits, UB. It is in log form and contains as explanatory variables the level of unemployment, the nominal wage rate, and the lagged dependent variable. The inclusion of the nominal wage rate is designed to pick up the effects of increases in wages and prices on legislated benefits per unemployed worker. The equation is estimated under the assumption of a first order autoregressive error term. All the coefficient estimates are significant. The equation passes all but the AP tests.

3.6.10 Interest Rate Rule

Table A30: Equation 30. RS, three-month Treasury bill rate

A key question in any macro model is what one assumes about monetary policy. In the theoretical model monetary policy is determined by an interest rate reaction function or rule, and in the empirical work an equation like this is estimated. This equation is interpreted as an equation explaining the behavior of the Federal Reserve (Fed).

In one respect trying to explain Fed behavior is more difficult than, say, trying to explain the behavior of the household or firm sectors. Since the Fed is run by a relatively small number of people, there can be fairly abrupt changes in behavior if the people with influence change their minds or are replaced by others with different views. Abrupt changes are less likely to happen for the household and firm sectors because of the large number of decision makers in each sector. Having said this, however, only one abrupt change in behavior appears evident in the data, which is between 1979:4 and 1982:3. This period, 1979:4–1982:3, will be called the "early Volcker" period.[56] The stated policy of the Fed during this period was that it was focusing more on monetary aggregates than it had done before.

Equation 30 is the estimated interest rate reaction function. It has on the left hand side RS. This treatment is based on the assumption that the Fed has a target bill rate each quarter and achieves this target through manipulation of its policy instruments. Although in practice the Fed controls the federal funds rate, the quarterly average of the federal funds rate and the quarterly average of the three-month Treasury bill rate are so highly correlated that it makes little difference which rate is used in estimated interest rate rules using quarterly data. The right hand side variables in the equation are variables that seem likely to affect the target rate. The variables that were chosen are 1) the rate of inflation, 2) the unemployment rate, 3) the change in the unemployment rate, and 4) the percentage change in the money supply lagged one quarter. The break between 1979:4 and 1982:3 was modeled by adding the variable $D794823 \cdot PCM1_{-1}$ to the equation, where $D794823$ is a dummy variable that is 1 between 1979:4 and 1982:3 and 0 otherwise. The estimated equation also includes the lagged dependent variable and two lagged bill rate changes to pick up the dynamics.

Beginning in 2008:4 and continuing beyond the end of the sample period used here (2013:3), the nominal short term interest rate used here (RS) effectively hit the zero lower bound. To handle this in the estimation, the estimation period for equation 30 was taken to end in 2008:3. The data beyond 2008:3 are not appropriate to use for estimation since Fed behavior in this period is constrained by the zero lower bound. (In the solution of the model the predicted value of RS is set to zero if otherwise it would be negative.)

The coefficient estimates in equation 30 are all significant. Equation 30 is a "leaning against the wind" equation. RS is estimated to depend positively on the inflation rate and the lagged growth of the money supply and negatively on the unemployment rate and the change in the unemployment rate. Adjustment and smoothing effects are captured by the lagged values of RS. The coefficient on lagged money supply growth is nearly twenty times larger for the early Volcker

[56]Paul Volcker was chair of the Fed between 1979:3 and 1987:2, but the period in question is only 1979:4–1982:3.

period than either before or after, which is consistent with the Fed's stated policy of focusing more on monetary aggregates during this period. This way of accounting for the Fed policy shift does not, of course, capture the richness of the change in behavior, but at least it seems to capture some of the change.

Equation 30 passes all the tests. The variables for which led values were tried are inflation and the unemployment rate, and the led values are not significant. The inflation expectations variables, $\dot{p}^e_{4t}$ and $\dot{p}^e_{8t}$, were added to the equation, and these variables are also not significant. Regarding the leads tests, these are tests of whether the Fed's expectations of future values of inflation and the unemployment rate are rational. The fact that the led values are not significant is evidence against the Fed having rational expectations.

Regarding stability tests for equation 30, any interesting test must exclude the early Volcker period since any hypothesis of stability that includes it is likely to be rejected. The Fed announced that its behavior was different during this period. One obvious hypothesis to test is that the equation's coefficients are the same before 1979:4 as they are after 1982:3. This was done using a Wald test. The Wald statistic is presented in equation 3.6 in Andrews and Fair (1988). It has the advantage that it works under very general assumptions about the properties of the error terms and can be used when the estimator is 2SLS, which it is here. The Wald statistic is distributed as χ^2 with (in the present case) 8 degrees of freedom. The hypothesis of stability is not rejected. As reported in Table A30, the Wald statistic is 15.22, which has a p-value of .0550. The hypothesis is thus not rejected at the 5 percent level.

As noted in Section 3.1, the first example of an estimated interest rate rule is in Dewald and Johnson (1963), followed by Christian (1968). An equation like equation 30 was first estimated in Fair (1978b). After this, McNees (1986, 1992) estimated rules in which some of the explanatory variables were the Fed's internal forecasts of various variables. Khoury (1990) provides an extensive list of estimated rules through 1986. Two more recent studies are Judd and Rudebusch (1998), where rules are estimated for various subsets of the 1970–1997 period, and Clarida, Galí, and Gertler (2000), where rules are estimated for the different Fed chairmen.

There seems to be a general view in the recent literature that estimated interest rate rules do not have stable coefficient estimates over time. For example, Judd and Rudebusch (1998, p. 3) state "Overall, it appears that there have not been any great successes in modeling Fed behavior with a single, stable reaction function." The passing of the stability test for equation 30 is thus contrary this view. One likely reason that the stability hypothesis has generally been rejected in the literature is that most tests have included the early Volcker period, which is clearly different from the periods both before and after. The tests in Judd and Rudebusch (1998), for example, include the early Volcker period.

3.6.11 Additional Comments

The following are general comments about the results in Tables A1–A30, usually pertaining to groups of equations.

Lags, RHO, T, and Stability Tests

For the χ^2 tests, 20 of 25 equations pass the lags test, 18 of 25 pass the RHO test, and 17 of 21 pass the T test. Of the 72 AP stability tests, 34 are passed. For the end-of-sample stability test, 18 of 25 are passed. 14 of the 21 overidentifying restrictions tests are passed. The overall results thus suggest that the specifications of the equations are fairly accurate regarding dynamic and trend effects. The results are less strong for the AP test, where for some of the equations there are signs of a changed structure in the 1970s.

Rational Expectations Tests

The led values are significant at the one percent level in only 6 of the 50 cases. They are significant at the five percent level in only 12 of the 50 cases. Overall, the results are thus not supportive of the hypothesis that expectations are rational.

The present negative results about the RE hypothesis are consistent with Chow's (1989) results, where he finds that the use of adaptive expectations performs much better than the use of rational expectations in explaining present value models.

Age Distribution Effects

The age variables, $AG1$, $AG2$, and $AG3$, are jointly significant at the one percent level in two of the household expenditure equations, explaining CS and CD. This is thus some evidence that the U.S. age distribution has an effect on U.S. macroeconomic equations.[57]

[57] This same conclusion was also reached in Fair and Dominguez (1991). In this earlier study, contrary to the case here, the age variables were also significant in the equations explaining CN and IHH.

Excess Labor, Excess Capital, and Other Physical Stock Effects

The excess capital variable is significant in the investment equation, 12, and the excess labor variable is significant in the employment and hours equations, 13 and 14. Regarding other stock effects, the stock of inventories has a negative effect on production (equation 11), the stock of durable goods has a negative effect on durable expenditures (equation 3), and the stock of housing has a negative effect on residential investment (equation 4).

The existence of these physical stock effects in the model means that there are endogenous features in the model that mitigate business cycles. As physical stocks are drawn down, this has a positive effect on new expenditures, which is expansionary. For example, the smaller is the stock of housing (variable KH), the larger will be housing investment (variable IHH), other things being equal.

Stock Market Effects

The real wealth variable, AA, appears in three of the four household expenditure equations. AA is affected by CG, which is mostly the change in the value of stocks held by the household sector, and so changes in stock prices affect expenditures in the model through their effect on household wealth. The wealth variable also appears in three of the four labor supply equations, where the estimated effect is negative, and so changes in stock prices also affect labor supply. Finally, one of the cost of capital variables in the investment equation 12 is a function of lagged values of CG, and so stock prices have an effect on plant and equipment investment through this variable.

The way in which financial crises affect the real economy in the model is through AA. A stock market crash, for example, leads to a large decrease in AA, which has large negative effects on household expenditures through the wealth effect. The size of the wealth effect is explored in Section 5.7.

Interest Rate Effects

Either the short term or long term interest rate is significant in the four household expenditure equations. Also, interest income is part of disposable personal income, YD, which is significant in the four equations. Therefore, an increase in interest rates has a negative effect on household expenditures through the interest rate variables and a positive effect through the disposable personal income variable. In addition, the change in the long term interest rate, RB, has a negative effect on the change in the value of stocks (equation 25), and so interest rates have a negative effect on household expenditures through their effect on household wealth. A measure of a real long term interest rate has a negative effect on investment in equation 12, although the variable is not significant. The short term interest rate also appears in the two demand for money equations.

Money Demand Adjustment

In the two money demand equations the nominal adjustment specification dominates the real adjustment specification. The nominal adjustment specification is equation (37).

Unemployment Rate

The unemployment rate is significant in three of the four labor supply equations. There is thus some evidence that a discouraged worker effect is in operation. The unemployment rate is the demand pressure variable in the price equation 10 and is highly significant. The unemployment rate and the change in the unemployment rate are significant in equation 30, the estimated interest rate rule.

Price of Imports

The price of imports, PIM, is an explanatory variable in the price equation 10, where it has a positive effect on the domestic price level. It also appears in the import equation 27, where it has a negative effect on imports, other things being equal.

Potential Productivity

Potential productivity, LAM, is exogenous in the model. It is constructed from a peak to peak interpolation of measured productivity. It appears in the price and wage equations 10 and 16. It is also used in the definition of $JHMIN$, which appears in the employment and hours equations 13 and 14, and it is in the definition of potential output, YS.

Dummy Variables

Three dummy variables appear in equation 11 to account for a steel strike; one dummy variable appears in equation 13 to account for the same steel strike; and four dummy variables appear in equation 27 to account for two dock strikes. A dummy variable appears in equation 30 to account for the announced change in Fed behavior in the early Volcker period.

Time Varying Coefficients

$cnst2$ is significant in equations 1, 3, 12, and 27. It has a t-statistic of -1.87 in equation 2 and 1.77 in equation 4. TB in equation 10, which is $cnst2 \times T$, is significant. There is thus some evidence of time varying coefficients between the two chosen quarters, 1968.4 and 1988.4.

3.7 The ROW Stochastic Equations

3.7.1 Introduction

Stochastic equations are estimated for 38 countries aside from the United States, with up to 13 equations estimated per country. The estimates and test results are presented in Tables B1 through B14 in Table B.4 in Appendix B. (As noted in Section 3.5, there is no equation 12 and Table B12.) The 2SLS technique was used for the quarterly countries and for equations 1, 2, and 3 for the annual countries. Ordinary least squares was used for the other equations for the annual countries. The 2SLS technique had to be used sparingly for the annual countries because of the limited number of observations. The first stage regressors for each equation are available from the FP program MC input files.

The estimation periods were chosen based on data availability. With three exceptions, the periods were chosen to use all the available data. The three exceptions are the interest rate, exchange rate, and forward rate equations, where the estimation periods were chosen to begin after the advent of floating exchange rates. The earliest starting quarter (year) for these periods was 1972:2 (1972). For the EMU countries the estimation periods for the interest rate, exchange rate, and forward rate equations end in 1998:4. Because the EMU countries have had a common monetary policy since 1999:1, there are no longer individual interest rate, exchange rate, and forward rate equations for these countries. The end-of-sample stability test was not performed for these equations for the EMU countries.

The tests per equation are similar to those done for the US equations. For the the AP test T_1 is taken to be roughly 40 quarters or 10 years after the first observation and T_2 is taken to be roughly 40 quarters or 10 years before the last observation. For the end-of-sample stability test the end period begins 12 quarters or 3 years before the last observation. For the serial correlation test the order of the autoregressive process was two for the quarterly countries and one for the annual countries. (For the test for the United States the order was four.) The led values were one-quarter-ahead values for the quarterly countries and one-year-ahead values for the annual countries. Subject to data limitations, the specification of the ROW equations follows fairly closely the specification of the US equations. Data limitations prevented all 13 equations from being estimated for all 38 countries. Also, some equations for some countries were initially estimated and then rejected for giving what seemed to be poor results.

Because much of the specification of the ROW equations is close to that of the US equations, the specification discussion in this section is brief. Only the differences are emphasized.

A † after a coefficient estimate in Tables B1–B14 indicates that the variable is lagged one period. To save space, only the p-values are presented for each test in the tables except for the AP stability test. As for the US equations, an equation will be said to pass a test if the p-value is greater than .01. For the AP stability test the AP value is presented along with the degrees of freedom and the value of lambda. No tests are performed for countries AR, BR, and PE because of very short estimation periods. Also, stability tests are not performed for countries with very short estimation periods.

There are obviously a lot of estimates and test results in the tables, and it is not feasible to discuss each estimate and test result in detail. The following discussion tries to give a general idea of the results.

3.7.2 The Equations and Tests

Table B1: Equation 1. IM: Total Imports

Equation 1 explains the total real per capita imports of the country. The explanatory variables include the price of domestic goods relative to the price of imports, per capital expenditures on consumption plus investment plus government spending, and the lagged dependent variable. The variables are in logs. Equation 1 is similar to equation 27 in the US model. The main difference is that the expenditure variable includes government spending, which it does not in equation 27.

The coefficient estimate for the expenditure variable is of the expected sign for all countries, and most of the estimates are significant. Many of the estimates of the coefficient of the relative price variable are significant. One of the tests in Table B1 is where the log of the domestic price level is added to test the relative price constraint. The constraint is rejected (i.e., $\log PY$ is significant) for p-values less than 0.01, which is 5 of the 24 cases.

Table B2: Equation 2: C: Consumption

Equation 2 explains real per capita consumption. The explanatory variables include the short term or long term interest rate, per capita income, and the lagged dependent variable. The variables are in logs except for the interest rates. Equation 2 is similar to the consumption equations in the US model. The three main differences are 1) there is only one category of consumption in the ROW model compared to three in the US model 2) the income variable is total GDP instead of disposable personal income, and 3) there is no wealth variable.

The income variable is significant for most countries, and the interest rate variable is significant for many countries. The interest rate in these equations provide a key link from monetary policy changes to changes in real demand.

Table B3: Equation 3: I: Fixed Investment

Equation 3 explains real fixed investment. It includes as explanatory variables the lagged value of investment, the current value of output, and the short term or long term interest rate. The variables are in logs except for the interest rates. Equation 3 differs from the investment equation 12 for the US, which uses a capital stock series. Sufficient data are not available to allow good capital stock series to be constructed for most of the other countries, and so no capital stock series were constructed for the ROW model. The simpler equation just mentioned was estimated for each country.

The output variable is significant for most countries, and an interest rate variable is significant for many. Again, the interest rates in these equations provide a key link from monetary policy changes to changes in real demand, in this case investment demand.

Table B4: Equation 4: Y: Production

Equation 4 explains the level of production. It is the same as equation 11 for the US model—see equation (17) in Section 3.6.4. It includes as explanatory variables the lagged level of production, the current level of sales, and the lagged stock of inventories.

The value of λ presented in Table B4 is one minus the coefficient estimate of lagged production. Also presented in the table are the implied values of α and β in equation (17) in Section 3.6.4. For the quarterly countries λ ranges from .529 to .860 and α ranges from .006 to .095. For the United States λ was .641 and α was .332.

As was the case for equation 11 in the US model, the coefficient estimates of equation 4 are consistent with the view that firms smooth production relative to sales, and so these results add support to the production smoothing hypothesis. The equation is estimated for only a small number of countries because the data on inventory investment do not exist or are not very good for many countries.

Equation 5: PY: Price Deflator

Equation 5 explains the GDP price deflator. It is the same as equation 10 for the US model except for the use of a different demand pressure variable. It includes as explanatory variables the lagged price level, the price of imports, a demand pressure variable, and the time trend. The demand pressure variable is the output gap variable, ZZ, which equals $\log Y - \log YS$, where Y is actual output and YS is a measure of potential output. The construction of YS is discussed in Appendix B.

The demand pressure variable is significant for many countries. The price of imports appears in the equation for all but two countries, and in most cases it is significant. Import prices thus appear to have important effects on domestic prices for most countries.

Table B6: Equation 6: $M1$: Money

Equation 6 explains the per capita demand for money.[58] The same nominal versus real adjustment specifications were tested here as were tested for US equations 17 and 26. Equation 6 includes as explanatory variables one of the two lagged money variables, depending on which adjustment specification won, the short term interest rate, and income. The estimates in Table B6 show that the nominal adjustment specification (the coefficient α_3) was chosen in 11 of the 18 cases.

As was the case for the United States, the demand for money equations for the other countries are presented for sake of completeness only. The short term interest rate in a country is determined by the interest rate rule (equation 7 next), and the money supply is whatever is needed to have the money demand equation met.

Table B7: Equation 7: RS: Short Term Interest Rate

Equation 7 explains the short term (three month) interest rate. It is interpreted as the interest rate rule of each country's monetary authority, and it is similar to equation 30 in the US model. For the EMU countries the equation is only relevant for the period through 1998:4. The explanatory variables that were tried (as possibly influencing the monetary authority's interest rate decision) are 1) the rate of inflation, 2) the output gap variable ZZ, 3) the German short term interest

[58] Money demand equations of this type were first estimated in Fair (1987) for a number of countries.

rate (for the European countries only), and 4) the U.S. short term interest rate. The U.S. interest rate was included on the view that some monetary authorities' decisions may be influenced by the Fed's decisions. Similarly, the German interest rate was included in the (non German) European equations on the view that the (non German) European monetary authorities' decisions may be influenced by the decisions of the German central bank.

Table B7 shows that the inflation rate is included in 18 of the 24 cases, ZZ in 17 cases, the German rate in 6 cases, and the U.S. rate in 14 cases. There is thus evidence that monetary authorities are influenced by inflation and demand pressure, as well as possibly German and U.S. behavior.

Equation 7 for EU is explained at the end of this section. It is only relevant from 1999:1 on.

Table B8: Equation 8: RB: Long Term Interest Rate

Equation 8 explains the long term interest rate. It is the same as equations 23 and 24 in the US model. For the EMU countries the equation is only relevant for the period through 1998:4. For the quarterly countries the explanatory variables include the lagged dependent variable and the current and two lagged short rates. For the annual countries the explanatory variables include the lagged dependent variable and the current and one lagged short rates. The same restriction was imposed on equation 8 as was imposed on equations 23 and 24, namely that the coefficients on the short rate sum to one in the long run. The first test in Table B8 shows that the restriction that the coefficients sum to one is only rejected in 2 of the 20 cases.

Equation 8 for EU is explained at the end of this section. It is only relevant from 1999:1 on.

Table B9: Equation 9 E or H: Exchange Rate

Equation 9 explains the country's exchange rate: E for the non European countries plus Germany and H for the non German European countries. E is a country's exchange rate is relative to the U.S. dollar, and H is a country's exchange rate relative to the Deutsche mark (DM). An increase in E is a *depreciation* of the country's currency relative to the dollar, and an increase in H is a *depreciation* of the country's currency relative to the DM. For the EMU countries the equation is only relevant for the period through 1998:4.

The theory behind the specification of equation 9 is discussed in Section 3.3. Equation 9 is interpreted as an exchange rate reaction function. The equations for E and H have the same general specification except that U.S. variables are the base variables for the E equations and German variables are the base variables for the H equations. The following discussion will focus on E.

It will first be useful to define two variables:

$$r = [(1 + RS/100)/(1 + RS_{US}/100)]^{.25}, \tag{1}$$

$$p = PY/PY_{US}. \tag{2}$$

r is a relative interest rate measure. RS is the country's short term interest rate, and RS_{US} is the U.S. short term interest rate (denoted simply RS in the US model). RS and RS_{US} are divided by 100 in the definition of r because they are in percentage points rather than percents. Also, the interest rates are at annual rates, and so the term in brackets in the definition of r is raised to the .25 power to put r at a quarterly rate. For the annual countries .25 is not used. p is the relative price level, where PY is the country's GDP price deflator and PY_{US} is the U.S. GDP price deflator (denoted $GDPD$ in the US model).[59]

The equation for E is based on the following two equations.

$$E^* = \alpha p r^{\beta}, \tag{3}$$

$$E/E_{-1} = (E^*/E_{-1})^{\lambda} e^{\epsilon}. \tag{4}$$

Equation (3) states that the long run exchange rate, E^*, depends on the relative price level, p, and the relative interest rate, r. The coefficient on the relative price level is constrained to be one, which means that in the long run the real exchange rate is assumed merely to fluctuate as the relative interest rate fluctuates. Equation (4) is a partial adjustment equation, which says that the actual exchange rate adjusts λ percent of the way to the long run exchange rate each period.

Equations (3) and (4) imply that

$$\log(E/E_{-1}) = \lambda \log \alpha + \lambda(\log p - \log E_{-1}) + \lambda\beta \log r + \epsilon. \tag{5}$$

The restriction that the coefficient of the relative price term is one can be tested by adding $\log E_{-1}$ to equation (5). If the coefficient is other than one, this variable should have a nonzero coefficient. This is one of the tests performed in Table B9.

[59]The relative interest rate is defined the way it is so that logs can be used in the specification below. This treatment relies on the fact that the log of $1 + x$ is approximately x for small values of x.

The equations for the European countries (except Germany) are the same as above with H replacing E, RS_{GE} replacing RS_{US}, and PY_{GE} replacing PY_{US}.

Exchange rate equations were estimated for 23 countries. For a number of countries the estimate of the coefficient of the relative interest rate variable was of the wrong expected sign, and in these cases the relative interest rate variable was dropped from the equation. Also, for 10 countries—CA, JA, AU, IT, NE, ST, UK, DE, NO, NZ—the estimate of λ in equation (5) was very small ("very small" defined to be less than .049), and for these countries the equation was reestimated with λ constrained to be .050.

The unconstrained estimates of λ in the equation vary from .049 to .172 for the quarterly countries and from .141 to .353 for the annual countries. A small value for λ means that it takes considerable time for the exchange rate to adjust to a relative price level change. The relative interest rate variable appears in 8 equations. It is only significant in two (NE and NZ), however, and so there is only limited support for the hypothesis that relative interest rates affect exchange rates.

The first test in Table B9 is of the restriction discussed above. The restriction is tested by adding $\log E_{-1}$ or $\log H_{-1}$ to the equation. It is rejected in 7 of the 23 cases.

Since equation 9 is in log form, the standard errors are roughly in percentage terms. The standard errors for a number of the European countries are quite low, but remember that these are standard errors for H, not E. The variance of H is much smaller than the variance of E for the European countries.

The relative interest rate variable appears in the equations for Japan, Germany, and the United Kingdom, and so relative interest rates have an effect on the exchange rates of these three key countries in the model. As noted above, however, they are not significant, and so the relative interest rate effects are at best weak.

Equation 9 for EU is explained at the end of this section. It is only relevant from 1999:1 on.

Table B10: Equation 10 F: Forward Rate

Equation 10 explains the country's forward exchange rate, F. This equation is the estimated arbitrage condition, and although it plays no role in the model, it is of interest to see how closely the quarterly data on EE, F, RS, and RS_{US} match the arbitrage condition. (EE differs from E in that it is the exchange rate at the end of the period, not the average for the period.) The arbitrage condition in this notation is

$$F/EE = [(1 + RS/100)/(1 + RS_{US}/100)]^{.25} e^{\epsilon}. \qquad (6)$$

In equation 10, $\log F$ is regressed on $\log EE$ and $.25 \log(1 + RS/100)/(1 + RS_{US}/100)$. If the arbitrage condition were met exactly, the coefficient estimates for both explanatory variables would be one and the fit would be perfect.

The results in Table B10 show that the data are generally consistent with the arbitrage condition, especially considering that some of the interest rate data are not exactly the right data to use. Note the t-statistic for Switzerland of 11,391.90! Equation 10 plays no role in the model because F does not appear in any other equation.

Table B11: Equation 11 PX: Export Price Index

Equation 11 explains the export price index, PX. It provides a link from the GDP price deflator, PY, to the export price index. Export prices are needed when the countries are linked together. If a country produced only one good, then the export price would be the domestic price and only one price equation would be needed. In practice, of course, a country produces many goods, only some of which are exported. If a country is a price taker with respect to its exports, then its export prices would just be the world prices of the export goods. To try to capture the in between case where a country has some effect on its export prices but not complete control over every price, the following equation is postulated:

$$PX = PY^{\lambda}[PW\$(E/E00)]^{1-\lambda}e^{\epsilon}. \tag{7}$$

$PW\$$ is the world price index in dollars, and so $PW\$(E/E00)$ is the world price index in local currency. Equation (7) thus takes PX to be a weighted average of PY and the world price index in local currency, where the weights sum to one. Equation 11 was not estimated for any of the major oil exporting countries, and so $PW\$$ was constructed to be net of oil prices. (See equations L-5 in Table B.3.)

Equation (7) was estimated in the following form:

$$\log PX - \log[PW\$(E/E00)] = \lambda[\log PY - \log[PW\$(E/E00)] + \epsilon. \tag{8}$$

The restriction that the weights sum to one and that $PW\$$ and E have the same coefficient (i.e, that their product enters the equation) can be tested by adding $\log PY$ and $\log E$ to equation (8). If this restriction is not met, these variables should be significant. This is one of the tests performed in Table B11.

Equation 11 was estimated for 26 countries. For 2 of the countries—CH and ME—the estimate of λ was very small, and in these two cases the equation was reestimated with λ constrained to be 0.5.

The results in Table B11 show that the estimates of the autoregressive parameters are generally large. The estimates of λ vary from .394 to .868 for the quarterly countries and from .196 to .789 for the annual countries. The first test in Table B11 is of the restriction discussed above. The restriction is rejected in 13 of the 24 cases.

It should be kept in mind that equation 11 is meant only as a rough approximation. If more disaggregated data were available, one would want to estimate separate price equations for each good, where some goods' prices would be strongly influenced by world prices and some would not. This type of disaggregation is beyond the scope of the model.

Table B13: Equation 13: J: Employment

Equation 13 explains the change in employment. It is in log form, and it is similar to equation 13 for the US model. It includes as explanatory variables the amount of excess labor on hand, the change in output, and the time trend. It also includes the lagged change in output for CA. It does not include the lagged change in employment, which US equation 13 does.

Most of the coefficient estimates for the excess labor variable are significant in Table B13, which is support for the theory that firms at times hold excess labor and that the amount of excess labor on hand affects current employment decisions. Most of the change in output terms are also significant.

Table B14: Equation 14: $L1$: Labor Force

Equation 14 explains the labor force participation rate. It is in log form and is similar to equations 5, 6, and 7 in the US model. The explanatory variables include the time trend, the unemployment rate, and the lagged dependent variable. The unemployment rate is used to try to pick up discouraged worker effects.

Tables B7, B8, B9: EU Specifications

The 11 countries that make up the EU in the model are listed at the bottom of Table B.1 in Appendix B. The EU variables that are used in the model are listed near the bottom of Table B.2. The EU variables that are needed are RS, RB, E, Y, YS, and PY. Any other EU variables that are used are functions of these six variables. Data on the first three variables are available from the IFS. Y for EU is taken to be the sum of Y for the six quarterly EU countries: GE, AU, FR, IT, NE, and FI. The annual countries that are excluded are BE, IR, PO, SP, and GR. Similarly, YS for EU is taken to be the sum of YS for the six quarterly EU countries. PY for EU is the ratio of nominal output to real output for the six countries.

There are three estimated EU equations, explaining RS, RB, and E. These are equations 7, 8, and 9. The estimates are presented at the top of Tables B7, B8, and B9. The estimation period is 1972:2–2012:4 for equation 7, 1970:3–2012:4 for equation 8, and 1972:2–2012:4 for equation 9. German data are used prior to 1999:1. PY for EU appears in equations 7 and 9. The EU output gap variable, ZZ, appears in equation 7. It is equal to $(YS - Y)/YS$, where Y and YS are the EU variables discussed above.

Remember that equation 7 for Germany is the estimated interest rate rule of the Bundesbank when it determined German monetary policy (through 1998:4). The use of German data prior to 1999:1 to estimate equation 7 for the EU means that the behavior of the European Central Bank (ECB) is assumed to be the same as the behavior of the Bundesbank except that the right hand side variables are EU variables rather than German ones. Likewise, the structure of the EU exchange rate equation 9 is assumed to be the same as the German equation except that the right hand side variables are changed from German ones to EU ones. The same is also true of the long run interest rate equation 8.

Using only the six quarterly EU countries to construct Y, YS, and PY means that implicit in equation 7 is the assumption that the ECB only takes these six countries into account when setting its monetary policy. Although most of EU output is from the six quarterly countries, in future work the other countries should be included. This was not one here because of the lack of good quarterly data for the other countries.

The estimates in the three tables show that the estimates for EU are close to the estimates for Germany alone. This is, of course, not surprising since the German and EU equations share part of the estimation period. The three EU equations are relevant from 1999:1 on; they play no role in the model prior to this time. When these three equations are relevant, equations 7, 8, and 9 for the individual EU countries are not part of the model. See Table B.3 for more detail.

3.7.3 Additional Comments

Lags, RHO, T, Stability Tests

The equations do moderately well for the lags, RHO, and T tests. For the lags test there are 80 failures out of 264 cases (30.3 percent); for the RHO test there are 89 failures out of 235 (37.9 percent); and for the T test there are 50 failures out of 182 (27.5 percent). These results suggest that the dynamic specifications of the equations are reasonably good. The results are not strong for the AP stability test, where there are 134 failures out of 250 (53.6 percent). More observations are probably needed before much can be done about this problem. The end-of-sample stability test results, on the other hand, are quite good, with only 21 failures out of 215 (9.8 percent). For the overid test there are 59 failures out of 119 (49.6 percent).

Rational Expectations Tests

There is little support for the use of the led values and thus little support for the rational expectations hypothesis. The led values are significant in only 15 out of 96 cases (15.6 percent).

Excess Labor and Other Stock Effects

The excess labor variable is significant in most of the employment equations 13. The stock of inventories is significant in most of the production equations 4.

Interest Rate Effects

Either the short term or long term interest rate appears in most of the consumption and investment equations 2 and 3. The short term interest rate also appears in the demand for money equations 6. The relative interest rate appears in 7 of the exchange rate equations 9. The U.S. short term interest rate appears in 17 of the interest rate rules 7, and the German short term interest rate appears in 7 of the rules.

Money Demand Adjustment

The nominal adjustment specification dominates the real adjustment specification in 11 of the 18 cases for the money demand equations 6.

Demand Pressure Variables

The demand pressure variable, ZZ, appears in nearly all the price equations 5. it appears in many of the estimated interest rate rules 7.

Price of Imports

The price of imports, PM, appears in all but two of the 30 price equations 5. It also appears in all but one of the 13 quarterly import equations 1 and in 12 of the 24 annual import equations. .

Potential Productivity

Potential productivity, LAM, is exogenous in the model. It is constructed from a peak to peak interpolation of measured productivity, Y/J. It appears in the price equations 5. It is also used in the definition of $JMIN$, which appears in the employment equations 13.

3.7.4 The Trade Share Equations

a_{ijt} is the fraction of country i's exports imported by j in period t, where i runs from 1 to 58 and j runs from 1 to 59. The data on a_{ij} are quarterly, with observations for most i, j pairs beginning in 1960:1.

One would expect a_{ijt} to depend on country i's export price relative to an index of export prices of all the other countries. The empirical work consisted of trying to estimate the effects of relative prices on a_{ijt}. A separate equation was estimated for each i, j pair. The equation is the following:

$$\begin{gathered} a_{ijt} = \beta_{ij1} + \beta_{ij2} a_{ijt-1} + \beta_{ij3}(PX\$_{it}/(\textstyle\sum_{k=1}^{58} a_{kjt} PX\$_{kt}) + u_{ijt}, \\ t = 1, \dots, T. \end{gathered} \tag{9}$$

$PX\$_{it}$ is the price index of country i's exports, and $\sum_{k=1}^{58} a_{kjt} PX\$_{kt}$ is an index of all countries' export prices, where the weight for a given country k is the share of k's exports to j in the total imports of j. (In this summation $k = i$ is skipped.)

With i running from 1 to 58, j running from 1 to 59, and not counting $i = j$, there are 3,364 ($= 58 \times 58$) i, j pairs. There are thus 3,364 potential trade share equations to estimate. In fact, only 1,856 trade share equations were estimated. Data did not exist for all pairs and all quarters, and if fewer than 26 observations were available for a given pair, the equation was not estimated for that pair. A few other pairs were excluded because at least some of the observations seemed extreme and likely suffering from measurement error. Almost all of these cases were for the smaller countries.

Each of the 1,856 equations was estimated by ordinary least squares. The results are summarized in Table 1. The main coefficient of interest is β_{ij3}, the coefficient of the relative price variable. Of the 1,856 estimates of this coefficient, 74.3 percent (1,379) were of the expected negative sign. 35.8 percent had the correct sign and a t-statistic greater than two in absolute value, and 55.5 percent had the correct sign and a t-statistic greater than one in absolute value. 4.5 percent had the wrong sign and a t-statistic greater than two, and 12.7 percent had the wrong sign and a t-statistic greater than one. The overall results are thus quite supportive of the view that relative prices affect trade shares. The results for only countries 1–15 are similar, as Table 1 shows.

The average of the 1,379 estimates of β_{ij3} that were of the right sign is -0.395. β_{ij3} measures the short run effect of a relative price change on the trade share. The long run effect is $\beta_{ij3}/(1-\beta_{ij2})$, and the average of the 1,379 values of this is -1.880. For only countries 1–15, the two estimates are -0.261 and -1.671.

The trade share equations with the wrong sign for β_{ij3} were not used in the solution of the model. The trade shares for these i, j pairs were taken to be exogenous.

In the solution of the model the predicted values of α_{ijt}, say, $\hat{\alpha}_{ijt}$, do not obey the property that $\sum_{i=1}^{58} \hat{\alpha}_{ijt} = 1$. Unless this property is obeyed, the sum of total world exports will not equal the sum of total world imports. For solution purposes each $\hat{\alpha}_{ijt}$ was divided by $\sum_{i=1}^{58} \hat{\alpha}_{ijt}$, and this adjusted figure was used as the predicted trade share. In other words, the values predicted by the equations in (9) were adjusted to satisfy the requirement that the trade shares sum to one.

Table 1
Summary Results for the 1,856 Trade Share Equations

	Percentage of Correct and Incorrect Signs for $\hat{\beta}_{ij3}$	
	All Countries	Countries 1–15
Correct Sign	74.3	75.0
Correct Sign, $t > 2.0$	35.8	36.4
Correct Sign, $t > 1.0$	55.5	55.5
Incorrect Sign	25.7	25.0
Incorrect Sign, $t > 2.0$	4.5	4.7
Incorrect Sign, $t > 1.0$	12.7	12.4

	Average Size of the Coefficient Estimates that were of the Right Sign	
	All Countries	Countries 1–15
$\hat{\beta}_{ij3}$	-0.395	-0.261
$\hat{\beta}_{ij3}/(1-\hat{\beta}_{ij2})$	-1.880	-1.671

3.8 FIML and 3SLS Estimates of the US Model

The following link is from Chapter 6 in Fair (1984). FIML and 3SLS estimates of the US model are presented. This material has not been updated, and so the estimates are for an earlier version of the US model. This material shows that FIML and 3SLS estimates are computationally feasible for a model as large as the US model. The link is: **FIML and 3SLS Estimates of the US Model**.

3.9 Bootstrapping Results for the US Model

3.9.1 Estimating Coverage Accuracy

The procedure for estimating coverage accuracy in Subsection 2.7.2 is applied in this subsection to the US model. You should review this earlier material if necessary. The estimation period is the basic estimation period of the model—1954:1–2013:3 for all equations except equation 15, which is 1956:1–2013:3, and equation 30, which is 1954:1–2008:3. Equation 4, the housing investment equation, has the characteristic that the sum of the two serial correlation coefficient estimates is close to one for the base set of estimates (the estimates in Table A4 in Appendix A). On some trials this sum exceeds one and sometimes leads to explosive situations. For this reason equation 4 was dropped from the analysis in this chapter. Housing investment, IHH, has been taken to be exogenous. There are thus a total of 25 stochastic equations.

Both the number of trials, J, and the number of repetitions, K, were taken to be 350, for a total of 122,500 times the model was estimated (by 2SLS). There were 116 solution failures out of the 122,500 trials, and these failures were skipped. Selected results are presented in Table 1 for the 95 percent confidence intervals. Rejection rates are presented for 12 of the coefficients in the model, and the average for the 12 coefficients is presented.

The average rejection rate over the 12 coefficients is .110 for the asymptotic interval, which compares to .065 and .059 for the two bootstrap intervals. The asymptotic distribution thus rejects too often, and the bootstrap distributions are fairly accurate. Although not shown in Table 1, the results are similar if 90 percent confidence intervals are used. In this case the asymptotic rejection rate averaged across the 12 coefficients is .186. The corresponding values for the two bootstrap intervals are .118 and .111. Given these fairly good bootstrap results it seems likely that the US model falls within the required conditions for validity of the bootstrap. As mentioned in Section 2.7, it has not been proven that the bootstrap procedure is valid for the US model.

The results in Table 1 suggest that confidence intervals using the asymptotic distribution are too narrow. For a 95 percent confidence interval, for example, a little over 10 percent of the estimates tend to lie outside of the interval, rather than the ideal 5 percent. This is not a huge inaccuracy, but the bootstrap confidence intervals are better.

Table 1
Estimated Coverage Accuracy for the US Model

	Percent of Rejections using 95 Percent Confidence Intervals		
	a	b	c
Equation 1: Consumption of services (CS)			
ldv	0.117	0.051	0.057
income	0.106	0.054	0.049
Equation 2: Consumption of nondurables (CN)			
ldv	0.197	0.071	0.083
income	0.177	0.089	0.094
Equation 3: Consumption of durables (CD)			
ldv	0.089	0.069	0.051
income	0.114	0.080	0.046
Equation 10: Price deflator for the firm sector (PF)			
ldv	0.120	0.046	0.054
import price deflator	0.069	0.046	0.057
unemployment rate	0.063	0.051	0.046
Equation 30: Three-month Treasury bill rate (RS)			
ldv	0.117	0.077	0.060
inflation	0.080	0.080	0.069
unemployment rate	0.071	0.071	0.040
Average (12)	0.110	0.065	0.059

a: Asymptotic confidence interval.
b: Bootstrap equal-tailed percentile-t interval.
c: Bootstrap symmetric percentile-t interval.
• Average (12) = Average for the 12 coefficients.
• ldv: lagged dependent variable.

3.9.2 Bootstrap Results Using the US Model

In this subsection the overall bootstrap procedure discussed in Section 2.7 is applied to the US model. The estimation period is the same as that used above for the coverage accuracy results. This estimation period will be called 1954:1–2013:3, although, as noted above, two of the equations have slightly different estimation periods. The estimation method is 2SLS. The calculations were run in one large batch job, and it is easiest just to discuss what the job did. The steps are:

1. Estimate the 25 equations by 2SLS for 1954:1–2013:3. Compute standard errors of the coefficient estimates, and perform the Andrews-Ploberger (1994)

(AP) test on selected equations. Using the 2SLS estimates and zero values for the errors, solve the model dynamically for 2000:4-2002:3 and perform a multiplier experiment for this period. Using the actual data and the 2SLS estimates, compute the 25-dimensional error vectors centered at zero for the 1954:1–2013:3 (239 vectors).

2. Do the following 2000 times: 1) draw with replacement 239 error vectors from the residual vectors for 1954:1–2013:3, 2) using the drawn errors and the 2SLS estimates from step 1, solve the model dynamically for 1954:1–2013:3 to get new data, 3) using the new data, estimate the model by 2SLS, compute t-statistics for the coefficient estimates, and perform the AP tests, 4) reset the data prior to 2000:4 to the actual data, 5) draw with replacement 8 error vectors from the residual vectors for 2000:4–2002:3, 6) using the new 2SLS estimates and the drawn errors, solve the model dynamically for 2000:4–2002:3 and perform the multiplier experiment for this period.

3. Step 2 gives for each equation 2000 values of each coefficient estimate, t-statistic, and AP statistic. It also gives 2000 predicted values of each endogenous variable for each quarter within 2000:4–2002:3 and 2000 differences for each endogenous variable and each quarter from the multiplier experiment. These values can be analyzed as desired. Some examples are given below. Steps 4-6 that follow are the bias-correction calculations.

4. From the 2000 values for each coefficient, compute the mean and then subtract the mean from twice the 2SLS coefficient estimate from step 1. Use these values to adjust the constant term in each equation so that the mean of the error terms is zero. Using these coefficients (including the adjusted constant terms), record the differences between the 2SLS coefficient estimates from step 1 and these coefficients. Call the vector of these values the "bias-correction vector." Using the new coefficients and zero values for the errors, solve the model dynamically for 2000:4–2002:3 and perform the multiplier experiment for this period. Using the actual data and the new coefficients, compute the 25-dimensional error vectors centered at zero for the 1954:1–2013:3 (239 vectors).

5. Do the following 2000 times: 1) draw with replacement 239 error vectors from the residual vectors from step 4 for 1954:1–2013:3, 2) using the drawn errors and the coefficients from step 4, solve the model dynamically for 1954:1–2013:3 to get new data, 3) using the new data, estimate the model

by 2SLS and adjust the estimates for bias using the bias-correction vector from step 4, 4) reset the data prior to 2000:4 to the actual data, 5) draw with replacement 8 error vectors from the residual vectors from step 4 for 2000:4–2002:3, 6) using the new coefficient estimates and the drawn errors, solve the model dynamically for 2000:4–2002:3 and perform the multiplier experiment for this period.

6. Step 5 gives 2000 predicted values of each endogenous variable for each quarter within 2000:4–2002:3 and 2000 differences for each endogenous variable and each quarter from the multiplier experiment.

The same sequence of random numbers was used for the regular calculations (steps 1-3) as was used for the bias-correction calculations (steps 4-6). This lessens stochastic simulation error in comparisons between the two sets of results. There was one model failure for the no-bias, coefficient-uncertainty case, and there were seven model failures for the bias, coefficient-uncertainty case. There were no failures for the two no-coefficient-uncertainty cases.

Table 2 presents some results from step 2 for the coefficient estimates. Results for 12 coefficients from 5 equations are presented. The 5 equations are the three consumption equations 1–3, the price equation 10, and the interest rate rule 30. The coefficients are for the lagged dependent variable in each equation, income in each consumption equation, the price of imports and the unemployment rate in the price equation, and inflation and the unemployment rate in the interest rate rule. These are some of the main coefficients in the model. The first three columns show the 2SLS estimate, the mean from the 1999 trials, and the ratio of the two. For the lagged dependent variable coefficients the ratio is less than one for 4 of the 5 cases. This is as expected since the 2SLS estimates of these coefficients are biased downward. The smallest ratio is 0.952, a bias of 4.8 percent.

Column 4 gives the asymptotic confidence intervals; column 5 gives the confidence intervals using the equal-tailed percentile-t interval; and column 6 gives the symmetric percentile-t interval using the absolute values of the t-statistics. The differences across the three intervals are modest. It is interesting (and encouraging) that the asymptotic confidence intervals seem fairly accurate in this respect.

Table 2
Confidence Intervals for Selected Coefficients

	(1) $\hat{\beta}$	(2) $\bar{\beta}$	(3) (2)/(1)	(4) a	(5) b	(6) c
Equation 1: Consumption of services (CS)						
ldv	0.8175	0.8086	0.989	0.7833	0.7951	0.7835
				0.8468	0.8562	0.8516
income	0.1202	0.1284	1.068	0.0957	0.0873	0.0909
				0.1448	0.1383	0.1495
Equation 2: Consumption of nondurables (CN)						
ldv	0.7405	0.7046	0.952	0.6674	0.6876	0.6404
				0.8135	0.8539	0.8405
income	0.1192	0.1424	1.194	0.0712	0.0452	0.0548
				0.1672	0.1468	0.1836
Equation 3: Consumption of durables (CD)						
ldv	0.2317	0.2537	1.095	0.1586	0.1356	0.1467
				0.3048	0.2894	0.3167
income	0.0639	0.0714	1.117	0.0494	0.0436	0.0453
				0.0784	0.0721	0.0825
Equation 10: Price deflator for the firm sector (PF)						
ldv	0.9052	0.8911	0.984	0.8858	0.8940	0.8793
				0.9247	0.9341	0.9312
PIM	0.0392	0.0393	1.003	0.0346	0.0341	0.0340
				0.0438	0.0444	0.0444
UR	-0.1768	-0.1775	1.004	-0.2032	-0.2040	-0.2052
				-0.1505	-0.1459	-0.1484
Equation 30: Three-month Treasury bill rate (RS)						
ldv	0.9186	0.9074	0.988	0.9013	0.9077	0.8975
				0.9360	0.9423	0.9398
inflation	0.0667	0.0706	1.058	0.0422	0.0388	0.0400
				0.0913	0.0911	0.0934
$100 \cdot UR$	-0.1085	-0.1039	0.958	-0.1403	-0.1459	-0.1433
				-0.0766	-0.0784	-0.0736

a: $\hat{\beta} - 1.96\hat{\sigma}$, $\hat{\beta} + 1.96\hat{\sigma}$ b: $\hat{\beta} - t^*_{.975}\hat{\sigma}$, $\hat{\beta} - t^*_{.025}\hat{\sigma}$ c: $\hat{\beta} - |t^*|_{.950}\hat{\sigma}$, $\hat{\beta} + |t^*|_{.950}\hat{\sigma}$

- $\hat{\beta}$ = 2SLS estimate; $\hat{\sigma}$ = estimated asymptotic standard error of $\hat{\beta}$.
- $\bar{\beta}$ = mean of the values of $\hat{\beta}^{*j}$, where $\hat{\beta}^{*j}$ is the estimate of β on the jth trial.
- t^*_r = value below which r percent of the values of t^{*j} lie, where $t^{*j} = (\hat{\beta}^{*j} - \hat{\beta})/\hat{\sigma}^{*j}$, where $\hat{\sigma}^{*j}$ is the estimated asymptotic standard error of $\hat{\beta}^{*j}$.
- $|t^*|_r$ = value below which r percent of the values of $|t^{*j}|$ lie.
- ldv: lagged dependent variable.
- PIM = price of imports, UR = unemployment rate.

Table 3 presents results for the AP test for the two long term interest rate equations, explaining the bond rate RB and the mortgage rate RM.[60] The overall estimation period is 1954:1–2013:3, and the period for a possible break was taken to be 1970:1-1979:4. These are the same periods as are used in the estimation of the US model in Table A.4 in Appendix A. Table 3 gives for each equation the computed AP value, the bootstrap confidence values, and the asymptotic confidence values. The asymptotic confidence values are taken from Table 1 in Andrews and Ploberger (1994). The value of λ in the AP notation for the present results is 2.07. The bootstrap confidence values for an equation are computed using the 1999 values of the AP statistic. The 5 percent value, for example, is the value above which 100 of the AP values lie.

Table 3 shows that the bootstrap values are similar to the respective asymptotic values. This is again encouraging regarding the use of the asymptotic distribution for hypothesis testing.

Table 4 presents results for the simulations for 2000:4–2002:3. Results for four variables are presented: the log of real GDP, the log of the GDP price deflator, the unemployment rate, and the three-month Treasury bill rate. Four sets of results are presented: with and without coefficient uncertainty and with and without bias correction.[61] Consider the first set of results (upper left corner) in Table 4. The first column gives the deterministic prediction (based on setting the error terms to zero and solving once), and the second gives the median value of the 1999 predictions. These two values are close to each other, which means there is little bias in the deterministic prediction. The third column gives the difference between the median predicted value and the predicted value below which 15.87 percent of the values lie, and the fourth column gives the difference between the predicted value above which 15.87 percent of the values lie and the median value. For a normal distribution these two differences are the same and equal one standard error. Computing these differences is one possible way of measuring predictive uncertainty in the model. The same differences are presented for the other three sets of results in Table 4.

[60] The AP test was not performed for the interest rate rule because the equation is already estimated under the assumption of a change in Fed behavior in the 1979:4–1982:3 period.

[61] The results without coefficient uncertainty were obtained in a separate batch job. This batch job differed from the one outlined above in that in part 6) of step 2 the 2SLS estimates from step 1 are used, not the new 2SLS estimates. Also, in part 6) of step 5 the coefficients from step 4 are used, not the new coefficient estimates.

Table 3
Results for the AP Tests

Equation	# of coefs.	AP	Bootstrap 1%	5%	10%	Asymptotic 1%	5%	10%
23 RB	5	4.32	8.28	6.08	5.16	7.57	5.73	4.85
24 RM	4	5.75	7.18	5.43	4.35	6.94	4.90	4.08

- Sample period: 1954:1–2012:1.
- Period for possible break: 1970:1–1979:4.
- Value of λ = 2.07.
- Asymptotic values from Andrews and Ploberger (1994), Table I.
- RB = bond rate
- RM = mortgage rate

Three conclusions can be drawn from the results in Table 4. First, the left and right differences are fairly close to each other. Second, the differences with no coefficient uncertainty are only slightly smaller than those with coefficient uncertainty, and so most of the predictive uncertainty is due to the additive errors. Third, the bias-correction results are fairly similar to the non bias-correction ones, which suggests that bias is not a major problem in the model. In most cases the uncertainty estimates are larger for the bias-correction results.

Table 5 presents results for the multiplier experiment. The experiment was an increase in real government purchases of goods of one percent of real GDP for 2000:4–2002:3. The format of Table 5 is similar to that of Table 4, where the values are multipliers[62] rather than predicted values. The first column gives the multiplier computed from deterministic simulations, and the second gives the median value of the 1999 multipliers (1993 in the bias-correction case). As in Table 3, these two values are close to each other. The third column gives the difference between the median multiplier and the multiplier below which 15.87 percent of the values lie, and the fourth column gives the difference between the multiplier above which 15.87 percent of the values lie and the median multiplier. These two columns are measures of the uncertainty of the government spending effect in the model.

[62]The word ‘multiplier’ is used here to refer to the difference between the predicted value of a variable after the policy change and the predicted value of the variable before the change. This difference is not strictly speaking a multiplier because it is not divided by the government spending change.

Table 4
Simulation Results for 2000:4–2002:3

Var.	h	$\hat{Y}$	$Y_{.5}$	left	right	$\hat{Y}$	$Y_{.5}$	left	right
			Coefficient Uncertainty				**No Coefficient Uncertainty**		
				No Bias Correction					
$\log GDPR$	1	8.060	8.059	0.00491	0.00493	8.060	8.060	0.00433	0.00423
	4	8.066	8.063	0.01140	0.01078	8.066	8.065	0.00987	0.01062
	8	8.083	8.080	0.01476	0.01485	8.083	8.083	0.01342	0.01399
$\log 100 \cdot GDPD$	1	4.416	4.417	0.00343	0.00345	4.416	4.416	0.00325	0.00288
	4	4.435	4.436	0.00717	0.00780	4.435	4.435	0.00661	0.00622
	8	4.447	4.448	0.01253	0.01291	4.447	4.447	0.01066	0.00987
$100 \cdot UR$	1	4.118	4.120	0.281	0.237	4.118	4.134	0.270	0.230
	4	4.139	4.210	0.705	0.654	4.139	4.129	0.616	0.680
	8	4.495	4.596	0.899	0.975	4.495	4.514	0.923	0.943
RS	1	5.859	5.878	0.482	0.421	5.859	5.841	0.443	0.415
	4	6.065	5.994	1.161	1.078	6.065	6.117	1.160	1.091
	8	5.299	5.169	1.521	1.400	5.299	5.299	1.394	1.403
				Bias Correction					
$\log GDPR$	1	8.061	8.060	0.00520	0.00568	8.061	8.061	0.00448	0.00446
	4	8.069	8.680	0.01276	0.01319	8.065	8.068	0.01001	0.01094
	8	8.087	8.087	0.01731	0.01716	8.087	8.087	0.01395	0.01447
$\log 100 \cdot GDPD$	1	4.416	4.416	0.00357	0.00342	4.416	4.156	0.00343	0.00288
	4	4.434	4.436	0.00760	0.00814	4.434	4.434	0.00665	0.00610
	8	4.447	4.448	0.01317	0.01393	4.447	4.447	0.01084	0.00998
$100 \cdot UR$	1	4.128	4.136	0.306	0.254	4.130	4.140	0.277	0.253
	4	4.097	4.119	0.787	0.764	4.105	4.103	0.645	0.694
	8	4.378	4.353	1.058	1.077	4.394	4.441	0.965	0.991
RS	1	5.858	5.877	0.489	0.458	5.858	5.832	0.396	0.438
	4	6.186	6.259	1.274	1.203	6.191	6.261	1.224	1.101
	8	5.554	5.621	1.672	1.781	5.551	5.548	1.463	1.505

- h = number of quarters ahead.
- $\hat{Y}$ = predicted value from deterministic simulation.
- Y_r = value below which r percent of the values of Y^j lie, where Y^j is the predicted value on the jth trial.
- left = $Y_{.5} - Y_{.1587}$, right = $Y_{.8413} - Y_{.5}$, units are percentage points.
- $GDPR$ = real GDP, $GDPD$ = GDP deflator, UR = unemployment rate, RS = three-month Treasury bill rate.

Table 5
Multiplier Results for 2000:4–2002:3

Var.	h	$\hat{d}$	$d_{.5}$	left	right	$\hat{d}$	$d_{.5}$	left	right
		No Bias Correction				**Bias Correction**			
$\log GDPR$	1	0.00856	0.00905	0.00061	0.00066	0.00810	0.00816	0.00071	0.00077
	4	0.01578	0.01641	0.00072	0.00077	0.01514	0.01524	0.00074	0.00077
	8	0.01443	0.01486	0.00093	0.00094	0.01402	0.01417	0.00100	0.00096
$\log 100 \cdot GDPD$	1	0.00093	0.00083	0.00008	0.00009	0.00086	0.00086	0.00009	0.00010
	4	0.00385	0.00342	0.00043	0.00048	0.00363	0.00361	0.00048	0.00056
	8	0.00781	0.00690	0.00087	0.00098	0.00750	0.00747	0.00099	0.00111
$100 \cdot UR$	1	-0.192	-0.201	0.035	0.032	-0.183	-0.183	0.034	0.032
	4	-0.673	-0.693	0.084	0.077	-0.650	-0.651	0.088	0.080
	8	-0.682	-0.705	0.075	0.071	-0.660	-0.663	0.086	0.075
RS	1	0.163	0.168	0.034	0.043	0.154	0.152	0.035	0.039
	4	0.681	0.687	0.112	0.116	0.673	0.666	0.116	0.123
	8	0.787	0.767	0.115	0.120	0.808	0.794	0.127	0.143

- h = number of quarters ahead.
- $\hat{Y}^a$ = predicted value from deterministic simulation, no policy change.
- $\hat{Y}^b$ = predicted value from deterministic simulation, policy change.
- $\hat{d}$ = $\hat{Y}^b - \hat{Y}^a$
- Y^{aj} = predicted value on the jth trial, no policy change.
- Y^{bj} = predicted value on the jth trial, policy change.
- d^j = $Y^{bj} - Y^{aj}$
- d_r = value below which r percent of the values of d^j lie.
- left = $d_{.5} - d_{.1587}$, right = $d_{.8413} - d_{.5}$, units are percentage points.
- $GDPR$ = real GDP, $GDPD$ = GDP deflator, UR = unemployment rate, RS = three-month Treasury bill rate.

Three conclusions can be drawn from the results in Table 5. First, the left and right differences are fairly close to each other. Second, the differences are fairly small relative to the size of the multipliers, and so the estimated policy uncertainty is fairly small for a government spending change. Third, the bias-correction results are similar to the non bias-correction ones, which again suggests that bias is not a major problem in the model.

The results in this subsection are suggestive of the usefulness of the bootstrapping procedure for models like model in (1) in Section 2.1 Computations like those in Table 3 can be done for many different statistics. Computations like those in Table 4 can be used to compare different models, where various measures of dispersion can be considered. These measures account for both uncertainty from the additive error terms and coefficient estimates, which puts models on an equal footing if they have similar sets of exogenous variables. Computations like those in Table 5 can be done for a wide variety of policy experiments. Finally, the results in Table 1 in the previous subsection show that the bootstrap works well for the US model regarding coverage accuracy.

3.10 Uncertainty and Misspecification Estimates for the US Model

The procedure discussed in the link in Subsection 2.9.1 for estimating the possible misspecification of a model is used in this section on the US model. Uncertainty estimates from the additive errors and the coefficient estimates have already been presented in Table 4 in Subsection 3.9.2 for the US model for the 2000:4–2002:3 period. These are the values in the upper left part of the table. They are based on 1999 trials using the bootstrap technique discussed in Section 2.7. For one-quarter-ahead forecast for $\log GDPR$ the left and right estimates are 0.00491 and 0.00493, respectively. These are estimates of one standard error for a normal distribution. Although not reported in Table 4, the 1999 trials allow one to compute variances for each variable and each quarter ahead. For example, the square root of the variance (the standard error) for the one-quarter-ahead forecast of $\log GDPR$ is 0.0051, close to 0.00491 and 0.00493.

Standard errors are presented in row b in Table 1 in this section based on the same 1999 trials that were used for the results in the upper left part of Table 4 in Subsection 3.9.2. The standard errors in row a are for the no coefficient uncertainty case, the upper right part of Table 4. These are based on 2000 trials. All these values are for no bias correction. Row a is thus no coefficient uncertainty, and row b includes coefficient uncertainty. Just to be clear, for the a row the coefficients are not reestimated on each trial, whereas they are for the b row. Comparing rows a and b in Table 1 shows that much more of the variance of a prediction is due to the additive error terms than to the coefficient estimates.

The task of this section is to explain row d, which incorporates uncertainty from the possible misspecification of the model. It will be useful to review here the part of the method in Subsection 2.9.1 that pertains to the results in this section. One main difference between the method discussed in Subsection 2.9.1 and the method used here is that for the stochastic simulations the coefficients are estimated on each trial, rather than being drawn from estimated distributions.

Let $\tilde{\sigma}^2_{itk}$ denote the stochastic simulation estimate of the variance of the prediction error for a k period ahead prediction of variable i from a simulation beginning in period t, where additive errors are drawn and coefficients are estimated. Each number in the b row in Table 1 is the square root of $\tilde{\sigma}^2_{itk}$, where there are 1999 trials, the errors are drawn from residuals for the 1954:1–2013:3 period, and the prediction period is 2000:4–2002:3.

Table 1
Sources of Uncertainty: US Model

	$\log GDPR$			$\log GDPD$			$100 \cdot UR$			RS		
	Quarters Ahead			Quarters Ahead			Quarters Ahead			Quarters Ahead		
Model	1	4	8	1	4	8	1	4	8	1	4	8
a	0.0051	0.0108	0.0142	0.0034	0.0065	0.0104	0.26	0.67	0.92	0.53	1.15	1.43
b	0.0056	0.0115	0.0151	0.0035	0.0076	0.0129	0.28	0.71	0.96	0.54	1.16	1.47
d	0.0038	0.0115	0.0183	0.0028	0.0089	0.0185	0.22	0.64	1.01	0.54	1.74	2.33

• Prediction period: 2000:4–2002:3.
a: uncertainty from structural errors only.
b: uncertainty from structural errors and coefficient estimates.
d: uncertainty from structural errors, coefficient estimates, and possible misspecification of the model.
• Errors are in percentage points.

The misspecification estimates are based on successive stochastic simulations. It will be easiest to explain the method by focusing on the actual sample periods that were used for the present results. Altogether, 123 stochastic simulations were run, each using 100 trials. The overall procedure is as follows, where the estimation technique is 2SLS:

1. Estimate the model for the 1954:1–1982:4 period.[63] Call these coefficient estimates the "base" estimates. Using the base coefficient estimates, compute residuals centered at zero for the 1954:1–1982:4 period. Call these residuals the "base" residuals.

2. Draw error vectors for the 1954:1–1982:4 period with replacement from the base residuals. Use these errors and the base coefficient estimates to solve the model dynamically for the 1954:1–1982:4 period. Use the predicted values from this simulation to estimate the model for the 1954:1–1982:4 period. Call these coefficient estimates the "trial" estimates. Set the data (the predicted values) back to the actual values.[64]

3. Draw error vectors for the 1983:1–1984:4 period with replacement from the base residuals . Use these errors and the trial coefficient estimates to solve the model dynamically for the 1983:1–1984:4 period. Call these predicted values the "trial" predicted values and record them. Set the predicted values for the 1983:1–1984:4 period back to the actual values.

[63]The beginning quarter for equation 15 is always 1956:1. Also, the ending quarter for equation 30 is never greater than 2008:3.

[64]This is needed because there are lagged endogenous variables in the model, and for the solution described next, actual values are used for quarters before the first quarter of the prediction period.

4. Steps 2 and 3 constitute one trial. Do steps 2 and 3 100 times. After the 100 trials, compute for each endogenous variable and each quarter within the 1983:1–1984:4 period the mean of the predicted values and the variance of the prediction error. Denote the mean as $\tilde{\mu}_{isk}$ and the variance as $\tilde{\sigma}^2_{isk}$, where i is endogenous variable i, k is the length ahead of the prediction, and s is the beginning quarter of the prediction period. The difference between the mean value and the actual value, y_{is+k-1}, is the mean prediction error, denoted $\hat{\epsilon}_{isk}$:
$$\hat{\epsilon}_{isk} = y_{is+k-1} - \tilde{\mu}_{isk} \quad (1)$$
Let d_{isk} denote the difference between the square of the mean prediction error and $\tilde{\sigma}^2_{isk}$:
$$d_{isk} = \hat{\epsilon}^2_{isk} - \tilde{\sigma}^2_{isk} \quad (2)$$
This notation will be used in the discussion below.

5. Go back to step 1 and increase the last quarter by one. Do steps 1 and 2 with the last quarter increased by one. Then do step 3 with both the first and last quarters increased by one. Then do step 4 with both the first and last quarters increased by one.

6. Repeat step 5 until the last quarter is 2013:2. This is 123 uses of step 5. The prediction period in step 3 cannot end after 2013:3, the last quarter of data, and so for use 123, the prediction period is just one quarter: 2013:3. For use 122, the prediction period is 2013:2–2013:3, and so on.

7. After step 6 there are for each endogenous variable i 123 d_{isk} values for $k = 1$, 122 for $k = 2$, and so on. For each i and k compute the mean of the d_{isk} values. Denote the means as $\bar{d}_{ik}$.

If it is assumed that $\tilde{\mu}_{isk}$ in step 4 exactly equals the true expected value, then $\hat{\epsilon}_{isk}$ in equation (1) is a sample draw from a distribution with a known mean of zero and variance σ^2_{isk}, where σ^2_{isk} is the true variance. The square of this error, $\hat{\epsilon}^2_{isk}$, is thus under this assumption an unbiased estimate of σ^2_{isk}. One therefore has two estimates of σ^2_{isk}, one computed from the mean prediction error and one computed by stochastic simulation. d_{isk} in equation (2) is the difference between these two estimates. If it is further assumed that $\tilde{\sigma}^2_{isk}$ exactly equals the true value (i.e., $\tilde{\sigma}^2_{isk} = \sigma^2_{isk}$), then d_{isk} is the difference between the estimated variance based on the mean prediction error and the true variance. Therefore, under the two assumptions of no error in the stochastic simulation estimates, the expected value of d_{isk} is zero for a correctly specified model.

If a model is misspecified, it is not in general true that the expected value of d_{isk} is zero. If the model is misspecified, the estimated residuals that are used for the draws are inconsistent estimates of the true errors and the coefficient estimates obtained on each trial are inconsistent estimates of the true coefficients. The effect of misspecification on d_{isk} is ambiguous, although if data mining has occurred in that the estimated residuals are on average too small in absolute value, the mean of d_{isk} is likely to be positive. In other words, if data mining has occurred, the stochastic simulation estimates of the variances are likely to be too small because they are based on draws from estimated residuals that are too small in absolute value. In addition, if the model is misspecified, the outside sample prediction errors are likely to be large on average, which suggests a positive mean for the d_{isk} values.

Consider the 123[65] values of d_{isk} in step 6 for a given variable i and a given k. If the expected value of d_{isk} is constant across time, then $\bar{d}_{ik}$ in step 7 is an estimate of the expected value. The assumption that the expected value is constant across time is used here. Other possible assumptions are discussed in Subsection 2.9.1. The assumption of a constant expected value means that misspecification affects the expected value in the same way for all s.

Finally, given $\bar{d}_{ik}$, an estimate of the total variance of the prediction error period t, denoted $\hat{\sigma}^2_{itk}$, is:

$$\hat{\sigma}^2_{itk} = \tilde{\sigma}^2_{itk} + \bar{d}_{ik} \tag{3}$$

Values of the square root of $\hat{\sigma}^2_{itk}$ are presented in the d row in Table 1. Each value in the d row is the square root of the sum of the square of the value in the b row and $\bar{d}_{ik}$. Table 1 shows that the differences between the d and b rows are fairly small. The largest differences occur for $k = 8$, where, for example, for $\log GDPR$ the b-row value is 0.0151 and the d-row value is 0.0183. For $\log GDPD$ the two values are 0.0129 and 0.0185. For $100/cdotUR$ they are 0.96 and 1.01, and for RS they are 1.47 and 2.33. Overall, the results suggest that the US model is not seriously misspecified.

[65]Fewer for k greater than 1.

3.11 Examining the CE Assumption Using the US Model

This section uses the procedure discussed in Subsection 2.11.3 to examine the accuracy of the CE assumption for the US model. The welfare function, W, is taken to be a loss function:

$$W = \sum_{t=s}^{S}[10000 \cdot (Y_t - Y_t^*)/Y_t^*]^2 + 10000 \cdot (UR_t - UR_t^*)^2 + 10000 \cdot (\dot{PF}_t - \dot{PF}_t^*)^2 \quad (1)$$

where Y is output (variable Y in the US model), UR is the unemployment rate (variable UR in the US model), and $\dot{PF}$ is the rate of inflation (percentage change at an annual rate in variable PF in the US model). The superscript * denotes the actual (historical) value of the variable. Consider the case in which the estimated residuals are added to the equations and taken to be exogenous. This means that when the model is solved using the actual values of the exogenous variables, a perfect tracking solution results—the predicted values are just the actual values. If in this case W in equation (1) is minimized using CE for some given set of control variables, the optimal z values are just the actual z values. The optimal value of W is zero, which occurs when the control values equal the actual values.

In the non CE case the bootstrap procedure can be used to compute the expected value of W, denoted $\bar{W}$. For the work here the US model was estimated for the basic 1954:1–2013:3 period and the residuals for the 1956:1–2007:4 period were used for the draws (as in Section 3.9). The control period was taken to be 1994:1–1998:4. The estimated residuals for this period were first added to the model and taken to be exogenous (so the base run is the perfect tracking solution). The drawn errors were then added to the equations with the estimated residuals already added. The number of trials was 2000.

The DFP algorithm was used to find the optimal value of z. To be clear on what is involved, for each value of z tried by the algorithm, the model was solved 2000 times for the 1994:1–1998:4 period. Each time a new draw of the errors is made with replacement from the estimated residuals. The 2000 solutions result in 2000 values of W, and $\bar{W}$ is computed as the mean of these values. This procedure is thus 2000 times more expensive than the CE case, since in the CE case the model is just solved once per evaluation of the objective function. The main point here is that it is possible to go from a value of z to a value of $\bar{W}$, which is all that the DFP algorithm needs.

The advantage of this setup is that one can compare the CE and non CE cases by simply comparing the "truly optimal" control value to the actual value, since the actual value is the optimal value in the CE case. One thus needs to compute only the truly optimal value. Remember that the only value of z that matters is the value for the first quarter, since reoptimization can be done each quarter.

The control variable was taken to be COG, federal government purchases of goods. The results are easy to describe. The truly optimal value of COG for the first quarter was 116.500, which compares to the actual value of 117.017. This difference of 0.44 percent is quite small, and so the truly optimal solution is quite close to the CE solution. (Remember that the actual value is the optimal value under CE.) There is thus little loss from using CE for models like (1) in Section 2.1.

The value of $\bar{W}$ at the optimum was 132.62 in the non CE case. (In the CE case the value of the objective function at the optimum is, of course, zero.) To get a sense of magnitudes, if the absolute value of $(Y - Y^*)/Y^*$ were .015 per quarter, the absolute value of $UR - UR^*$ were .015 per quarter, and the absolute value of $\dot{PF} - \dot{PF}^*$ were .015 per quarter, the value of $\bar{W}$ would be 135.0 ($= 10000 \times 20 \times 3 \times .015^2$). The average quarterly deviation (brought about by the stochastic simulation) is thus fairly large—on the order of 1.5 percent for each of the three variables. What the present results show is that even though this deviation is fairly large, little is lost by ignoring it and using CE when solving optimal control problems.

3.12 Testing the Use of Nominal versus Real Interest Rates[66]

3.12.1 Introduction

This section contains an important set of empirical results. It will be seen that the data rather strongly support the use of nominal over real interest rates in most expenditure equations. The consumption and investment equations in the MC model are used to test for nominal versus real interest rate effects. The aim of the tests is to see if the interest rates that households and firms use in their decision making processes are better approximated by nominal or real rates.

3.12.2 The Test

The test is as follows. Let for period t i_t denote the nominal interest rate, r_t the real interest rate, and $\dot{p}_t^e$ the expected future rate of inflation, where the horizon for $\dot{p}_t^e$ matches the horizon for i_t. By definition $r_t = i_t - \dot{p}_t^e$. Consider the specification of a consumption or investment equation in which the following appears on the right hand side:

$$\alpha i_t + \beta \dot{p}_t^e.$$

For the real interest rate specification $\alpha = -\beta$, and for the nominal interest rate specification $\beta = 0$. The real interest rate specification can be tested by adding $\dot{p}_t^e$ to an equation with $i_t - \dot{p}_t^e$ included, and the nominal interest rate specification can be tested by adding $\dot{p}_t^e$ to an equation with i_t included. The added variable should have a coefficient of zero if the specification is correct, and one can test for this.

Four measures of $\dot{p}_t^e$ were tried for countries with quarterly data (all at annual rates). Two of these have already been used for the tests in Tables A23, A24, and A30 in Appendix A, namely $\dot{p}_{4t}^e$, which is $P_t/P_{t-4} - 1$, and $\dot{p}_{8t}^e$, which is $(P_t/P_{t-8})^{.5} - 1$, where P_t denotes the price level for quarter t. The other two measures used in this chapter are the one quarter change, $(P_t/P_{t-1})^4 - 1$, and the two quarter change led once, $(P_{t+1}/P_{t-1})^2 - 1$. Three measures were tried for countries with only annual data: the one year change, $P_t/P_{t-1} - 1$, the two year change, $(P_t/P_{t-2})^{.5} - 1$, and the two year change led once, $(P_{t+1}/P_{t-1})^{.5} - 1$, where P_t denotes the price level for year t.

[66]The results in this section are updates of those in Fair (2002).

Table 1
Nominal versus Real Interest Rates: $\alpha i_t + \beta \dot{p}_t^e$

	Variable	Real Test ($\alpha = -\beta$) p-value a	b	c	d	Nominal Test ($\beta = 0$) p-value a	b	c	d	Sample Period
					Countries with Quarterly Data					
1	US: CS	.000	.000	.000	.000	.035	.265	.166	.102	1954:1-2013:3
2	US: CN	.001	.001	.001	.001	.001	.000	.000	.002	1954:1-2013:3
3	US: CD	.000	.000	.000	.004	.216	.021	.186	.179	1954:1-2013:3
4	US: IHH	.000	.000	.000	.000	.208	.008	.082	.002	1954:1-2013:3
5	CA: C	.000	.000	.000	.000	.231	.345	.000	.070	1961:2-2013:2
6	CA: I	.283	.072	.066	.426	.000	.157	.320	.000	1961:2-2013:2
7	JA: C	.000	.001	.001	.000	.003	.028	.039	.004	1966:1-2013:2
8	AU: I	.000	.001	.001	.001	.006	.055	.071	.029	1970:1-2013:1
9	FR: C	.039	.254	.453	.074	.119	.574	.266	.401	1961:1-2012:4
10	FR: I	.000	.000	.000	.000	.004	.831	.804	.036	1961:1-2012:4
11	IT: I	.030	.073	.133	.031	.736	.212	.089	.855	1961:1-2003:1
12	NE: I	.130	.138	.029	.256	.003	.118	.386	.004	1961:1-2013:1
13	ST: C	.000	.000	.000	.000	.159	.325	.058	.061	1977:1-2013:2
14	ST: I	.217	.096	.022	.320	.090	.946	.059	.018	1977:1-2013:2
15	UK: C	.000	.000	.000	.010	.000	.000	.000	.000	1961:1-2013:2
16	UK: I	.149	.226	.185	.160	.135	.043	.021	.236	1961:1-2013:2
17	FI: C	.471	.698	.567	.759	.544	.073	.158	.298	1961:1-2013:2
18	AS: I	.057	.013	.034	.060	.727	.145	.510	.626	1966:1-2013:2
19	SO: C	.068	.080	.134	.013	.630	.988	.780	.192	1962:1-2012:4
20	SO: I	.000	.000	.000	.000	.054	.040	.001	.022	1962:1-2012:4
21	KO: C	.188	.152	.244	.177	.712	.788	.424	.720	1974:1-2012:4
					Countries with Annual Data					
22	BE: I		.000	.000	.000		.654	.852	.308	1962-2012
23	DE: I		.057	.059	.350		.980	.519	.098	1962-2012
24	NO: I		.044	.025	.012		.025	.003	.020	1962-2012
25	SW: I		.048	.012	.098		.933	.544	.492	1965-2012
26	GR: C		.088	.125	.104		.000	.000	.000	1962-2009
27	GR: I		.000	.000	.000		.775	.685	.228	1962-2009
28	IR: C		.004	.003	.003		.600	.284	.099	1962-2012
29	PO: C		.001	.004	.001		.001	.019	.000	1962-2012
30	SP: I		.013	.009	.004		.103	.071	.024	1962-2012
31	NZ: C		.001	.001	.001		.633	.240	.735	1962-2011
32	NZ: I		.053	.052	.038		.377	.437	.209	1962-2011
33	ID: C		.020	.275	.001		.782	.056	.073	1962-2012
34	PH: C		.000	.000	.000		.053	.094	.000	1962-2012

- Quarterly countries: P_t = price level for quarter t.
  a: $\dot{p}_t^e = (P_t/P_{t-1})^4 - 1$, b: $\dot{p}_t^e = P_t/P_{t-4} - 1$, c: $\dot{p}_t^e = (P_t/P_{t-8})^{.5} - 1$,
  d: $\dot{p}_t^e = (P_{t+1}/P_{t-1})^2 - 1$.
- Annual countries: P_t = price level for year t.
  b: $\dot{p}_t^e = P_t/P_{t-1} - 1$, c: $\dot{p}_t^e = (P_t/P_{t-2})^{.5} - 1$, d: $\dot{p}_t^e = (P_{t+1}/P_{t-1})^{.5} - 1$.
- Variables: CS = Consumption of Services, CN = Consumption of Non Durables,
  CD = Consumption of Durables, IHH = Residential Investment,
  IKF = Nonresidential Fixed Investment, C = Total Consumption, I = Total Investment.

The results of the tests are presented in Table 1. The equations that are tested are the ones in Tables A1, A2, A3, and A4 in Appendix A and in Tables B2 and B3 in Appendix B. An equation was tested if the absolute value of the t-statistic of the coefficient estimate of the nominal interest rate variable was greater than

1.5. Nominal interest rates are used in all the equations.[67] In Table 1 the p-value is presented for each equation and each measure of $\dot{p}_t^e$. Columns a, b, c, and d correspond to the four measures of $\dot{p}_t^e$.

As noted in Subsection 2.12.1, when the 2SLS estimator is used, which it is in most cases for the present results, the predicted values from the first stage regressions can be interpreted as predictions of the agents in the economy under the assumption that agents know the values of the first stage regressors at the time they form their expectations. Since both i_t and $\dot{p}_t^e$ are treated as endogenous in the 2SLS estimation, agents can be assumed to have used the first stage regressions for i_t and $\dot{p}_t^e$ for their predictions. These predictions use the information in the predetermined variables in the model. This interpretation is important when considering the use of P_{t+1} in one of the measures of $\dot{p}_t^e$. Agents in effect are assumed to form predictions of P_{t+1} by running first stage regressions.

3.12.3 The Results

The results for the real interest rate specification are in the left half of Table 1. A low p-value is evidence against the real interest rate hypothesis that $\alpha = -\beta$. The results for the nominal interest rate specification are in the right half of Table 1. A low p-value is evidence against the nominal interest rate hypothesis that $\beta = 0$.

The results are stronger for the nominal interest rate hypothesis than for the real. For the U.S. household expenditure equations (rows 1–4) all of the 16 p-values are less than .01 for the real test, whereas only 6 of the 16 are less than .01 for the nominal test. For the other quarterly countries, 28 of the 68 p-values are less than .01 for the real test and only 14 of the 68 values for the nominal test. For the annual countries 21 of 39 values are less than .01 for the real test and only 7 of 39 for the nominal test.

When both i_t and $\dot{p}_t^e$ are included separately in an equation, an interesting question is whether most of the estimates of β, the coefficient of $\dot{p}_t^e$, are positive, which the real interest rate hypothesis implies. It turns out that many of the estimates are in fact negative. Although not shown in the table, when both i_t and $\dot{p}_t^e$ are included separately, only 1 of the 16 estimates of β for the United States is positive. Of the 15 negative estimates, 7 are significant. For the other quarterly countries, 32 of 68 are negative. Of the 32 negative estimates, 12 are significant. For the annual countries 15 of 39 are negative. Of the 15 negative estimates, 4 are significant.

[67] There is a potential bias from starting with equations chosen using nominal rather than real interest rates. Some experimentation was done to see if other equations would be added if real interest rates were used first, but no further equations were found.

The fact that a number of the negative estimates of β are significant is, of course, completely at odds with the real interest rate hypothesis.

Overall, the nominal interest rate specification dominates the real interest rate specification. Why this is the case is an interesting question. One possibility is that $\dot{p}_t^e$ is simply a constant, so that the nominal interest rate specification is also the real interest rate specification (with the constant absorbed in the constant term of the equation). If, for example, agents think the monetary authority is targeting a fixed inflation rate, this might be a reason for $\dot{p}_t^e$ being constant. Whatever the case, the empirical results do not favor the use of $i_t - \dot{p}_t^e$ in aggregate expenditure equations when $\dot{p}_t^e$ depends on current and recent values of inflation.[68]

[68]It may be the case, of course, that some more complicated measure of $\dot{p}_t^e$ leads to the real interest rate specification dominating. The present conclusion is conditional on measures of $\dot{p}_t^e$ that depend either on current and past values of inflation or, in case d, on the one-period-ahead future value of inflation.

3.13 The Price and Wage Equations versus the NAIRU Model[69]

3.13.1 Introduction

The price and wage equations in the US model—equations 10 and 16—and the price equations in the ROW model—equations 5—have quite different dynamic properties from those of the NAIRU model. The purpose of this section is to test the NAIRU dynamics. It will be seen that the NAIRU dynamics are generally rejected.

Subsection 3.13.6 below discusses an alternative way of thinking about the relationship between the price level and the unemployment rate, one in which there is a highly nonlinear relationship at low values of the unemployment rate. Unfortunately, it is hard to test this view because there are so few observations of very low values of the unemployment rate.

3.13.2 The NAIRU Model

The NAIRU view of the relationship between inflation and the unemployment rate is that there is a value of the unemployment rate (the NAIRU) below which the price level forever accelerates and above which the price level forever decelerates. The simplest version of the NAIRU equation is

$$\pi_t - \pi_{t-1} = \beta(u_t - u^*) + \gamma s_t + \epsilon_t, \quad \beta < 0, \quad \gamma > 0, \tag{1}$$

where t is the time period, π_t is the rate of inflation, u_t is the unemployment rate, s_t is a cost shock variable, ϵ_t is an error term, and u^* is the NAIRU. If u_t equals u^* for all t, the rate of inflation will not change over time aside from the short-run effects of s_t and ϵ_t (assuming s_t and ϵ_t have zero means). Otherwise, the rate of inflation will increase over time (the price level will accelerate) if u_t is less than u^* for all t and will decrease over time (the price level will decelerate) if u_t is greater than u^* for all t.

A more general version of the NAIRU specification is

$$\pi_t = \alpha + \sum_{i=1}^{n} \delta_i \pi_{t-i} + \sum_{i=0}^{m} \beta_i u_{t-i} + \sum_{i=0}^{q} \gamma_i s_{t-i} + \epsilon_t, \quad \sum_{i=1}^{n} \delta_i = 1. \tag{2}$$

For this specification the NAIRU is $-\alpha / \sum_{i=0}^{m} \beta_i$. If the unemployment rate is always equal to this value, the inflation rate will be constant in the long run aside from the short-run effects of s_t and ϵ_t.

[69]The results for the United States in this section are updates of those in Fair (2000). The results for the other countries are updates from those in Chapter 4 in Fair (2004a).

A key restriction in equation (2) is that the δ_i coefficients sum to one (or in equation (1) that the coefficient of π_{t-1} is one). This restriction is used in much of the literature. See, for example, the equations in Akerlof, Dickens, and Perry (1996), p. 38, Fuhrer (1995), p. 46, Gordon (1997), p. 14, Layard, Nickell, and Jackman (1991), p. 379, and Staiger, Stock, and Watson (1997), p. 35. The specification has even entered the macro textbook literature—see, for example, Mankiw (1994), p. 305. Also, there seems to be considerable support for the NAIRU view in the policy literature. For example, Krugman (1996, p. 37) in an article in the *New York Times Magazine* writes "The theory of the Nairu has been highly successful in tracking inflation over the last 20 years. Alan Blinder, the departing vice chairman of the Fed, has described this as the 'clean little secret of macroeconomics.' "

An important question is thus whether equations like (2) with the summation restriction imposed are good approximations of the actual dynamics of the inflation process. The basic test that is performed in this section is the following. Let p_t be the log of the price level for period t, and let π_t be measured as $p_t - p_{t-1}$. Using this notation, equations (1) and (2) can be written in terms of p rather than π. Equation (1), for example, becomes

$$p_t = 2p_{t-1} - p_{t-2} + \beta(u_t - u^*) + \gamma s_t + \epsilon_t. \qquad (3)$$

In other words, equation (1) can be written in terms of the current and past two price levels,[70] with restrictions on the coefficients of the past two price levels. Similarly, if in equation (2) n is, say, 4, the equation can be written in terms of the current and past five price levels, with two restrictions on the coefficients of the five past price levels. (Denoting the coefficients on the past five price levels as a_1 through a_5, the two restrictions are $a_4 = 5 - 4a_1 - 3a_2 - 2a_3$ and $a_5 = -4 + 3a_1 + 2a_2 + a_3$.) The main test in this section is of these two restrictions. The restrictions are easy to test by simply adding p_{t-1} and p_{t-2} to the NAIRU equation and testing whether they are jointly significant.

An equivalent test is to add π_{t-1} (i.e., $p_{t-1} - p_{t-2}$) and p_{t-1} to equation (2). Adding π_{t-1} breaks the restriction that the δ_i coefficients sum to one, and adding both π_{t-1} and p_{t-1} breaks the summation restriction and the restriction that each price level is subtracted from the previous price level before entering the equation. This latter restriction can be thought of as a first derivative restriction, and the summation restriction can be thought of as a second derivative restriction.

[70] "Price level" will be used to describe p even though p is actually the log of the price level.

Equation (2) was used for the tests, where s_t in the equation is postulated to be $pm_t - \tau_0 - \tau_1 t$, the deviation of pm from a trend line. pm is the log of the price of imports, which is taken here to be the cost shock variable. In the empirical work for the United States n is taken to be 12 and m and q are taken to be 2. For the other quarterly countries n is taken to be 8, with m and q taken to be 2. For the annual countries n is taken to be 3, with m and q taken to be 1. This fairly general specification regarding the number of lagged values is used to lessen the chances of the results being due to a particular choice of lags.

Equation (2) was estimated in the following form:

$$\Delta\pi_t = \lambda_0 + \lambda_1 t + \sum_{i=1}^{n-1}\theta_i\Delta\pi_{t-i} + \sum_{i=0}^{m}\beta_i u_{t-i} + \sum_{i=0}^{q}\gamma_i pm_{t-i} + \epsilon_t, \qquad (4)$$

where $\lambda_0 = \alpha + (\gamma_0+\gamma_1+\gamma_2)\tau_0 + (\gamma_0+2\gamma_1+3\gamma_2)\tau_1$ and $\lambda_1 = (\gamma_0+\gamma_1+\gamma_2)\tau_1$. α and τ_0 are not identified in equation (4), but for purposes of the tests this does not matter. If, however, one wanted to compute the NAIRU (i.e., $-\alpha/\sum_{i=1}^{m}\beta_i$), one would need a separate estimate of τ_0 in order to estimate α.[71]

For reference it will be useful to write equation (4) with π_{t-1} and p_{t-1} added:

$$\begin{aligned}\Delta\pi_t = \lambda_0 + \lambda_1 t + \textstyle\sum_{i=1}^{n-1}\theta_i\Delta\pi_{t-i} + \sum_{i=0}^{m}\beta_i u_{t-i} + \sum_{i=0}^{q}\gamma_i pm_{t-i} \\ +\phi_1\pi_{t-1} + \phi_2 p_{t-1} + \epsilon_t.\end{aligned} \qquad (5)$$

3.13.3 Tests for the United States

χ^2 Tests

The estimation period for the tests for the United States is 1955:3–2013:3. The results of estimating equations (4) and (5) are presented in Table 1. In terms of the variables in the US model, $p = \log PF$, $u = UR$, and $pm = \log PIM$. Regarding the estimation technique, the possible endogeneity of u_t and pm_t is ignored and ordinary least squares is used. Ordinary least squares is the standard technique used for estimating NAIRU models.

[71]The present specification assumes that the NAIRU is constant, although if the NAIRU had a trend, this would be absorbed in the estimate of the coefficient of the time trend in equation (4) (and would change the interpretation of λ_1). Gordon (1997) has argued that the NAIRU may be time varying.

Table 1
Estimates of Equations (4) and (5)
for the United States

	Equation (4)		Equation (5)	
Variable	Estimate	t-stat.	Estimate	t-stat.
cnst	-.00107	-0.39	-.02551	-3.62
t	.000015	1.43	.000138	4.13
u_t	-.225	-2.19	-.241	-2.46
u_{t-1}	.058	0.31	.107	0.61
u_{t-2}	.127	1.19	.014	0.14
pm_t	.030	2.10	.045	3.24
pm_{t-1}	.018	0.66	.007	0.26
pm_{t-2}	-.049	-3.24	-.027	-1.75
$\Delta\pi_{t-1}$	-.783	-11.80	-.406	-4.24
$\Delta\pi_{t-2}$	-.530	-6.85	-.257	-2.80
$\Delta\pi_{t-3}$	-.339	-4.24	-.154	-1.77
$\Delta\pi_{t-4}$	-.221	-2.76	-.091	-1.08
$\Delta\pi_{t-5}$	-.278	-3.55	-.183	-2.27
$\Delta\pi_{t-6}$	-.286	-3.69	-.207	-2.63
$\Delta\pi_{t-7}$	-.193	-2.59	-.130	-1.73
$\Delta\pi_{t-8}$	-.069	-0.93	-.026	-0.35
$\Delta\pi_{t-9}$	-.097	-1.32	-.055	-0.75
$\Delta\pi_{t-10}$	-.254	-3.63	-.206	-3.01
$\Delta\pi_{t-11}$	-.104	-1.80	-.086	-1.56
π_{t-1}			-.483	-5.28
p_{t-1}			-.039	-4.68
SE	.00388		.00365	
χ^2			29.67	

- p_t = log of price level, $\pi_t = p_t - p_{t-1}$, u_t = unemployment rate, pm_t = log of the price of imports.
- Estimation method: ordinary least squares.
- Estimation period: 1955:3–2013:3.
- When p_{t-1} and p_{t-2} are added in place of π_{t-1} and p_{t-1}, the respective coefficient estimates are -.522 and .483 with t-statistics of -5.35 and 5.28. All else is the same.
- Five percent χ^2 critical value = 5.99; one percent χ^2 critical value = 9.21.

Table 1 shows that when π_{t-1} and p_{t-1} are added, the standard error of the equation falls from .00388 to .00365. The t-statistics for the two variables are -5.28 and -4.68, respectively, and the χ^2 value for the hypothesis that the coefficients of both variables are zero is 29.67.[72]

The 5 percent critical χ^2 value for two degrees of freedom is 5.99 and the 1 percent critical value is 9.21. If the χ^2 distribution is a good approximation to the actual distribution of the "χ^2" values, the two variables are highly significant and thus the NAIRU dynamics strongly rejected. If, however, equation (4) is in fact the way the price data are generated, the χ^2 distribution may not be a good approximation for the test.[73] To check this, the actual distribution was computed using the following procedure.

First, estimate equation (4), and record the coefficient estimates and the estimated variance of the error term. Call this the "base" equation. Assume that the error term is normally distributed with mean zero and variance equal to the estimated variance. Then:

1. Draw a value of the error term for each quarter. Add these error terms to the base equation and solve it dynamically to generate new data for p. Given the new data for p and the data for u and pm (which have not changed), compute the χ^2 value as in Table 1. Record this value.

2. Do step 1 1000 times, which gives 1000 χ^2 values. This gives a distribution of 1000 values.

3. Sort the χ^2 values by size, choose the value above which 5 percent of the values lie and the value above which 1 percent of the values lie. These are the 5 percent and 1 percent critical values, respectively.

These calculations were done, and the 5 percent critical value was 20.07 and the 1 percent critical value was 26.81. These values are considerably larger than the critical values from the actual χ^2 distribution (5.99 and 9.21), but they are still smaller than the computed value of 29.67. The two price variables are thus significant at the 99 percent confidence level even using the alternative critical values.

The above procedure treats u and pm as exogenous, and it may be that the estimated critical values are sensitive to this treatment. To check for this, the

[72]Note that there is a large change in the estimate of the coefficient of the time trend when π_{t-1} and p_{t-1} are added. The time trend is serving a similar role in equation (5) as the constant term is in equation (4).

[73]If the χ^2 distribution is not a good approximation, then the t-distribution will not be either, and so standard tests using the t-statistics in Table 1 will not be reliable. The following analysis focuses on correcting the χ^2 critical values, and no use of the t-statistics is made.

following two equations were postulated for u and pm:

$$pm_t = a_1 + a_2 t + a_3 pm_{t-1} + a_4 pm_{t-2} + a_5 pm_{t-3} + a_6 pm_{t-4} + \nu_t, \qquad (6)$$

$$\begin{aligned} u_t = b_1 + b_2 t + b_3 u_{t-1} + b_4 u_{t-2} + b_5 u_{t-3} + b_6 u_{t-4} + b_7 pm_{t-1} \\ + b_8 pm_{t-2} + b_9 pm_{t-3} + b_{10} pm_{t-4} + \eta_t. \end{aligned} \qquad (7)$$

These two equations along with equation (4) were taken to be the "model," and they were estimated by ordinary least squares along with equation (4) to get the "base" model. The error terms ϵ_t, ν_t, and η_t were then assumed to be multivariate normal with mean zero and covariance matrix equal to the estimated covariance matrix (obtained from the estimated residuals). Each trial then consisted of draws of the three error terms for each quarter and a dynamic simulation of the model to generate new data for p, pm, and u, from which the χ^2 value was computed. The computed critical values were not very sensitive to this treatment of pm and u, and they actually fell slightly. The 5 percent value was 16.37 compared to 20.07 above, and the 1 percent value was 23.13 compared to 26.81 above.

The U.S. data thus reject the dynamics implied by the NAIRU specification: π_{t-1} and p_{t-1} are significant when added to equation (4). This rejection may help explain two results in the literature. Staiger, Stock, and Watson (1996), using a standard NAIRU specification, estimate variances of NAIRU estimates and find them to be very large. This is not surprising if the NAIRU specification is misspecified. Similarly, Eisner (1997) finds the results of estimating NAIRU equations sensitive to various assumptions, particularly assumptions about whether the behavior of inflation is symmetric for unemployment rates above and below the assumed NAIRU. Again, this sensitivity is not surprising if the basic equations used are misspecified.

Recursive RMSE Tests

An alternative way to examine equations (4) and (5) is to consider how well they predict outside sample. To do this, the following root mean squared error (RMSE) test was performed. Each equation was first estimated for the period ending in 1969:4 (all estimation periods begin in 1955:3), and a dynamic eight-quarter-ahead prediction was made beginning in 1970:1. The predicted values were recorded. The equation was then estimated through 1970:1, and a dynamic eight-quarter-ahead prediction was made beginning in 1970:2. This process was repeated through the estimation period ending in 2013:2. Since observations were available through 2013:3, this procedure generated 175 one-quarter-ahead predictions, 174 two-quarter-ahead predictions, through 168 eight-quarter-ahead predictions, where all the predictions are outside sample. RMSEs were computed using these predictions and the actual values.

The actual values of u and pm were used for all these predictions, which would not have been known at the time of the predictions. The aim here is not to generate predictions that could have in principle been made in real time, but to see how good the dynamic predictions from each equation are conditional on the actual values of u and pm.

The RMSEs are presented in the first two rows of Table 2 for the four- and eight-quarter-ahead predictions for p, π, and $\Delta\pi$. Comparing the two rows (equation (4) versus (5)), the RMSEs for $\Delta\pi$ are similar, but they are much smaller for p and π for equation (5). The NAIRU restrictions clearly lead to a loss of predictive power for the price level and the rate of inflation. It is thus the case that the addition of π_{t-1} and p_{t-1} to the NAIRU equation (4) has considerably increased the accuracy of the predictions, and so these variables are not only statistically significant but also important in a predictive sense.

Equation (5) is not the equation that determines the price level in the US model. The price level is determined by equation 10, and this equation includes the wage rate as an explanatory variable. Equation 10 also includes the unemployment rate, the price of imports, the lagged price level, the time trend, and the constant term. The wage rate is determined by equation 16, and this equation includes the price level and the lagged price level as explanatory variables. Equation 16 also includes the lagged wage rate, the time trend, and the constant term. As discussed in Subsection 3.6.4, a restriction, equation (30), is imposed on the coefficients in the wage rate equation to insure that the properties of the implied real wage equation are sensible. The two equations are estimated by 2SLS.

An interesting question is how accurate equations 10 and 16 are relative to equation (5) in terms of predicting p, π, and $\Delta\pi$. In terms of the present notation equations 10 and 16 are:

$$p_t = \beta_0 + \beta_1 p_{t-1} + \beta_2 w_t + \beta_3 pm_t + \beta_4 u_t + \beta_5 t + \epsilon_t, \qquad 10$$

$$w_t = \gamma_0 + \gamma_1 w_{t-1} + \gamma_2 p_t + \gamma_3 p_{t-1} + \gamma_5 t + \mu_t, \qquad 16$$

where

$$\gamma_3 = [\beta_1/(1-\beta_2)](1-\gamma_2) - \gamma_1.$$

In terms of the notation in the US model $w = \log(WF/LAM)$. The estimates of equations 10 and 16 are in Tables A10 and A16 in Appendix A.

Table 2
Recursive RMSE Results

	p		π		$\Delta\pi$	
	Quarters Ahead					
	4	8	4	8	4	8
Eq. (4)	1.82	4.24	2.60	3.26	2.12	2.16
Eq. (5)	1.66	3.22	2.27	2.30	2.06	2.15
Eqs. 10 & 16	1.56	3.14	2.08	2.40	1.86	1.87

- p = log of the price level, $\pi = \Delta p$.
- Prediction period: 1970:1–2013:3.
- Errors are in percentage points.

The basic procedure followed for computing the RMSEs for equations 10 and 16 was the same as that followed for equation (4) and equation (5). The beginning estimation quarter was 1954.1, and the first end estimation quarter was 1969.4. Each of the 175 sets of estimates used the 2SLS technique with the coefficient restriction imposed, where the values used for β_1 and β_2 in the restriction were the estimated values from equation 10. The same first stage regressors were used for these estimates as were used in the basic estimation of the equations. The predictions of p and w from equations 10 and 16 were generated using the actual values of u and pm, just as was done for equations (4) and (5).

The RMSEs are presented in the third row in Table 2. The results show that the RMSEs using equations 10 and 16 are smaller than those using even equation (5) except for the eight-quarter-ahead prediction of π. For the eight-quarter-ahead predictions, the RMSE for p is 3.14 versus 3.22 for equation (5), and the RMSE for π is 2.40 versus 2.30 for equation (5). For $\Delta\pi$ the RMSE using equations 10 and 16 is smaller: 1.87 versus 2.15 for equation (5). The structural price and wage equations clearly do better than even the price equation with the NAIRU restrictions relaxed.

In the early 1980s there began a movement away from the estimation of structural price and wage equations to the estimation of reduced-form price equations like equation (4).[74] The current results call into question this practice in that considerable predictive accuracy seems to be lost when this is done.

[74]See, for example, Gordon (1980) and Gordon and King (1982).

Table 3
Results for Equations (4) and (5) for the ROW Countries

	Coef. Ests. (t-statistics) π_{-1}	p_{-1}	χ^2	Estimated Critical $\chi^2_{.05}$	$\chi^2_{.01}$	RMSEs (quarters ahead) p 4	p 8	π 4	π 8	$\Delta\pi$ 4	$\Delta\pi$ 8
Quarterly											
CA	-.261	-.002	8.24	18.88	23.15	2.68	5.20	3.74	4.22	3.16	3.21
	(-2.80)	(-0.28)				2.81	5.94	3.98	5.09	3.22	3.32
AU	-.773	-.024	21.18	17.58	23.41	1.15	2.86	1.44	2.26	1.31	1.33
	(-4.63)	(-3.35)				0.94	1.97	1.29	1.60	1.41	1.45
FR	-.149	-.016	3.74	17.10	22.39	1.48	3.62	1.99	2.90	1.35	1.36
	(-1.98)	(-1.47)				1.76	4.29	2.29	3.14	1.36	1.35
GE	-.394	-.008	12.11	16.50	21.20	1.22	2.49	2.02	2.29	2.63	2.62
	(-2.69)	(-1.61)				1.55	3.26	2.23	2.64	2.65	2.61
IT	-.338	-.013	6.44	14.97	22.02	2.43	4.88	3.76	4.37	4.61	5.12
	(-2.67)	(-2.07)				2.41	4.37	3.77	4.12	4.79	5.33
NE	-.436	-.026	31.50	19.23	26.44	2.05	5.12	2.96	4.39	3.23	3.24
	(-3.81)	(-2.63)				1.77	3.90	2.71	3.31	3.28	3.20
ST	-.658	-.012	34.89	18.45	26.36	1.28	2.78	1.78	2.06	1.22	1.16
	(-6.22)	(-2.90)				1.44	2.85	1.88	1.94	1.18	1.16
UK	-.368	-.051	35.72	16.41	21.46	2.57	6.48	3.80	5.19	3.49	3.39
	(-4.70)	(-5.39)				2.45	5.02	3.61	3.60	3.56	3.47
AS	-.222	-.015	8.60	18.05	23.85	3.17	6.00	4.35	4.52	3.44	3.35
	(-2.13)	(-1.24)				3.17	6.13	4.34	4.68	3.48	3.39
KO	-.952	-.018	39.15	15.42	21.19	4.24	9.91	6.64	8.59	7.84	7.23
	(-6.62)	(-2.81)				2.96	4.71	5.61	5.40	8.16	7.22

	Coef. Ests. (t-statistics) π_{-1}	p_{-1}	χ^2	Estimated Critical $\chi^2_{.05}$	$\chi^2_{.01}$	RMSEs (years ahead) p 2	p 3	π 2	π 3	$\Delta\pi$ 2	$\Delta\pi$ 3
Annual											
SW	-.502	-.045	9.96	17.78	25.66	6.63	11.89	4.20	5.52	2.44	2.52
	(-3.55)	(-1.76)				5.68	9.07	3.50	3.85	2.44	2.45
IR	-.639	-.175	19.15	23.77	31.50	8.24	14.97	5.24	7.23	3.22	3.13
	(-3.86)	(-3.40)				8.60	12.92	4.94	4.65	2.85	2.43
PO	-.506	-.130	10.87	17.11	21.48	5.87	10.24	3.60	4.55	2.45	2.23
	(-3.71)	(-3.20)				5.01	7.40	2.96	3.11	2.34	2.18
SP	-.229	-.032	6.52	19.08	26.97	3.29	6.53	2.23	3.33	1.31	1.38
	(-2.39)	(-1.74)				5.90	11.50	3.84	5.52	1.85	1.76
TH	-.984	-.103	33.46	18.78	26.28	5.72	8.44	3.53	3.97	3.77	3.71
	(-6.39)	(-1.74)				6.15	7.90	3.37	2.86	3.27	3.14

• p = log of the price level, $\pi = \Delta p$.
• Five percent χ^2 critical value = 5.99; one percent χ^2 critical value = 9.21.
• For the RMSE results the first row for each country contains the RMSEs for equation (4) and the second row contains the RMSEs for equation (5).

3.13.4 Tests for the ROW Countries

Test results for the ROW countries are reported in this subsection. All the results are in Table 3. For each country the results of adding π_{t-1} and p_{t-1} are presented first, and then the RMSE results are presented. For the RMSE results the first row for each country contains the RMSEs for equation (4) and the second row contains the RMSEs for equation (5). The procedure used to compute the χ^2 critical values is the same as that used for the United States. All critical values were computed using equations (6) and (7). The demand pressure variable for the ROW countries is variable ZZ (corresponding to u above), and the price of imports variable is variable PM (corresponding to pm above).

For the annual countries the maximum lag length in each equation was 2, not 4. With two exceptions, a country was included in Table 3 if equation 5 for it in Table B5 included a significant demand pressure variable, where "significant" was taken to be a t-statistic greater than 2.0. The exceptions ares countries CH and CE, where the sample periods are fairly small. Results for 15 countries are presented in Table 3, 10 quarterly countries and 5 annual countries.

The estimation period for a country was the same as that in Table B5 except when the beginning quarter or year had to be increased to account for lags. For the recursive RMSEs, the first estimation period ended in 1979:3 for the quarterly countries except for ST, AS, and KO, where the first estimation period ended in 1989:3 (because of a shorter overall sample period). For the annual countries the first estimation period ended in 1988.

The computed critical values in Table 3 (denoted $\chi^2_{.05}$ and $\chi^2_{.01}$) are considerably larger than the χ^2 critical values of 5.99 for 5 percent and 9.21 for 1 percent. Using the χ^2 critical values, the two added variables are jointly significant (i.e., the NAIRU restrictions are rejected) at the 5 percent level in all but 1 of the 15 cases and at the 1 percent level in all but 6 of the 15 cases. On the other hand, using the computed critical values the two added variables are jointly significant at the 5 percent level in only 5 of the 15 cases and at the 1 percent level in only 4 of the 15 cases. The results thus depend importantly on which critical values are used.

The RMSE results are also mixed. Consider the 8-quarter-ahead RMSEs for the quarterly countries for π. For 5 of the 10 cases the RMSEs are smaller for equation (5), the equation without the NAIRU restrictions imposed. For the three-year-ahead RMSEs for the annual countries for π the RMSEs are smaller for equation (5) for 4 of the 5 cases.

Overall the ROW results are thus mixed regarding equation (5) versus equation (4).

3.13.5 Dynamics

This subsection examines using the U.S. estimates the dynamic properties of various equations. No tests are performed; this section is just an analysis of properties. The question considered is the following: if the unemployment rate were permanently lowered by one percentage point, what would the price consequences of this be?

To answer this question, the following experiment was performed for each equation. A dynamic simulation was run beginning in 2013:4 using the actual values of all the variables from 2013:3 back. The values u and of pm from 2013:4 on were taken to be the actual value for 2013:3. Call this simulation the "base" simulation. A second dynamic simulation was then run where the only change was that the unemployment rate was decreased permanently by one percentage point from 2013:4 on. The difference between the predicted value of p from this simulation and that from the base simulation for a given quarter is the estimated effect of the change in u on p.[75]

The results for four equations are presented in Table 4. The equations are 1) equation (4), 2) equation (4) with π_{t-1} added, 3) equation (5), which is equation (4) with both π_{t-1} and p_{t-1} added, and 4) equations 10 and 16 together. When equation (4) is estimated with π_{t-1} added, the summation (second derivative) restriction is broken but the first derivative restriction is not. For this estimated equation the δ_i coefficients summed to .888.[76]

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[75]Because the equations are linear, it does not matter what values are used for pm as long as the same values are used for both simulations. Similarly, it does not matter what values are used for u as long as each value for the second simulation is one percentage point higher than the corresponding value for the base simulation.

[76]When π_{t-1} is added to equation (4), the χ^2 value is 7.09 with computed 5 and 1 percent critical values of 11.78 and 16.10, respectively. π_{t-1} is thus not significant at even the 5 percent level when added to equation (4) even though the sum of .888 seems substantially less than one. (When p_{t-1} is added to the equation with π_{t-1} already added, the χ^2 value is 21.89 with computed 5 and 1 percent critical values of 13.49 and 20.54, respectively. p_{t-1} is thus significant when added to the equation with π_{t-1} already added.) Recursive RMSE results as in Table 2 were also obtained for the equation with only π_{t-1} added. The six RMSEs corresponding to those in Table 2 are 1.74, 3.63, 2.39, 2.65, 2.10, and 2.17. These values are in between those for equation (4) and equation (5).

Table 4
Effects of a One Percentage Point Fall in u

	Equation (4)		Equation (4) π_{t-1} added		Equation (5)		Eqs. 10, 16	
Quar.	$P^{new} \div P^{base}$	$\pi^{new} - \pi^{base}$	$P^{new} \div P^{base}$	$\pi^{new} - \pi^{base}$	$P^{new} \div P^{base}$	$\pi^{new} - \pi^{base}$	$P^{new} \div P^{base}$	$\pi^{new} - \pi^{base}$
1	1.0022	0.90	1.0019	0.76	1.0024	0.96	1.0020	0.81
2	1.0044	0.86	1.0037	0.72	1.0039	0.60	1.0039	0.74
3	1.0058	0.58	1.0048	0.44	1.0055	0.63	1.0056	0.68
4	1.0075	0.68	1.0061	0.51	1.0071	0.65	1.0072	0.63
5	1.0093	0.72	1.0074	0.53	1.0087	0.66	1.0086	0.57
6	1.0110	0.65	1.0085	0.45	1.0101	0.53	1.0099	0.53
7	1.0125	0.62	1.0096	0.41	1.0113	0.51	1.0111	0.48
8	1.0143	0.72	1.0108	0.48	1.0127	0.55	1.0122	0.44
9	1.0164	0.83	1.0121	0.56	1.0141	0.58	1.0132	0.40
10	1.0184	0.81	1.0134	0.51	1.0154	0.50	1.0142	0.37
11	1.0202	0.68	1.0144	0.38	1.0163	0.49	1.0150	0.34
12	1.0223	0.85	1.0156	0.49	1.0175	0.42	1.0158	0.31
40	1.1238	1.98	1.0582	0.66	1.0315	0.02	1.0234	0.03
∞	∞	∞	∞	0.70	1.0308	0.00	1.0245	0.00

• P = price level, $\pi = \Delta \log P$.

Before discussing results, it should be stressed that these experiments are not meant to be realistic. For example, it is unlikely that the Fed would allow a permanent fall in u to take place as p rose. The experiments are simply meant to help illustrate how the equations differ in a particular dimension.

Consider the very long run properties in Table 4 first. For equation (4), the new price level grows without bounds relative to the base price level and the new inflation rate grows without bounds relative to the base inflation rate. For equation (4) with π_{t-1} added, the new price level grows without bounds relative to the base, but the inflation rate does not. It is 0.70 percentage points higher in the long run. For equation (5) (which again is equation (4) with both π_{t-1} and p_{t-1} added), the new price level is higher by 3.08 percent in the limit and the new inflation rate is back to the base. For equations 10 and 16, the new price level is higher by 2.41 percent in the limit and the new inflation rate is back to the base.

The long run properties are thus vastly different, as is, of course, obvious from the specifications. What is interesting, however, is that the effects are fairly close for the first few quarters. One would be hard pressed to choose among the equations on the basis of which short-run implications (say the results out to 8 quarters) seem more "reasonable." Instead, tests as in this chapter are needed to try to choose.

3.13.6 Nonlinearities

If the NAIRU specification is rejected, this changes the way one thinks about the relationship between inflation and unemployment. One should not think that there is some unemployment rate below which the price level forever accelerates and above which it forever decelerates. It is not the case, however, that equation (5) (or equations 10 and 16) is a sensible alternative regarding long run properties. Equation (5) implies that a lowering of the unemployment rate has only a modest long run effect on the price level regardless of how low the initial value of the unemployment rate is. For example, the results in Table 4 for equation (5) are independent of the initial value of the unemployment rate.

A key weakness of equation (5) is (in my view) the linearity assumption regarding the effects of u on p. It seems likely that there is a nonlinear relationship between the price level and the unemployment rate at low levels of the unemployment rate. One possible specification, for example, would be to replace u in equation (5) with $1/(u-.02)$. In this case as u approaches .02, the estimated effects on p become larger and larger. I have experimented with a variety of functional forms like this in estimating price equations like equation 10 in the US model and equations 5 in the ROW model to see if the data can pick up a nonlinear relationship. Unfortunately, there are so few observations of very low unemployment rates that the data do not appear capable of discriminating among functional forms. A variety of functional forms, including the linear form, lead to very similar results. In the end I simply chose the linear form for lack of a better alternative for both the US equation 10 and the ROW equations 5. This does not mean, however, that the true functional form is linear, only that the data are insufficient for estimating the true functional form. It does mean, however, that one should not run experiments using the MC model in which unemployment rates or output gaps are driven to historically low levels. The price equations are unlikely to be reliable in these cases.

The argument here about the relationship between inflation and the unemployment rate can thus be summarized by the following two points. First, the NAIRU dynamics, namely the first and second derivative restrictions, are not accurate. Second, the relationship between the price level and the unemployment rate is nonlinear at low values of the unemployment rate. The results in this section generally support the first point, but they have nothing to say about the second point.

Conditional on this argument, the main message for policy makers is that they should not think there is some value of the unemployment rate below which the price level accelerates and above which it decelerates. They should think instead that the price level is a negative function of the unemployment rate (or other measure of demand slack), where at some point the function begins to become nonlinear. How bold a policy maker is in pushing the unemployment rate into uncharted waters will depend on how fast he or she thinks the nonlinearity becomes severe.

4 Properties of the MC Model

4.1 Effects of Inflation Shocks[77]

4.1.1 Introduction

It will be seen in this section that a positive inflation shock for the United States in the MC model is contractionary even when the nominal interest rate is held constant. There is a class of macro models in the literature that have the opposite property, and it is interesting to see why. As a rough approximation, models in this class include the following three equations:

1. Interest Rate Rule: The Fed adjusts the nominal interest rate in response to inflation and the output gap (deviation of output from potential).[78] The nominal interest rate responds positively to inflation and the output gap. The coefficient on inflation is greater than one, and so the real interest rate rises when inflation rises.

2. Price Equation: Inflation depends on the output gap, cost shocks, and expected future inflation.

3. Aggregate Demand Equation: Aggregate demand (real) depends on the real interest rate, expected future demand, and exogenous shocks. The real interest rate effect is negative.

Models in this class are nicely summarized in Clarida, Galí, and Gertler (1999), and they are used in Clarida, Galí, and Gertler (2000) to examine monetary policy rules. Taylor (2000, p. 91) points out that virtually all the papers in Taylor (1999a) use these models and that the models are widely used for policy evaluation in many central banks. In both the backward-looking model and the forward-looking model in Svensson (2003) aggregate demand depends negatively on the real interest rate, as in the aggregate demand equation above. Romer (2000) proposes a way of teaching these models at the introductory level.

The effects of an inflation shock in this basic model are easy to see. The aggregate demand equation implies that an increase in inflation with the nominal

[77] Some of the discussion in this section is taken from Fair (2002). The results in Table 1 are updates of those in Table 3 in Fair (2002).

[78] In empirical work the lagged interest rate is often included as an explanatory variable in the interest rate rule. This picks up possible interest rate smoothing behavior of the Fed.

interest rate held constant is expansionary (because the real interest rate falls). The model is in fact not stable in this case because an increase in output increases inflation through the price equation, which further increases output through the aggregate demand equation, and so on. In order for the model to be stable, the nominal interest rate must rise more than inflation, which means that the coefficient on inflation in the interest rate rule must be greater than one. Because of this feature, some have criticized Fed behavior in the 1960s and 1970s as following in effect a rule with a coefficient on inflation less than one—see, for example, Clarida, Galí, and Gertler (1999) and Taylor (1999c).

There are three main reasons the MC model has the opposite property. First, except for the U.S. investment equation 12, nominal interest rates rather than real interest rates are used in the consumption and investment equations. See Section 3.12 for results supporting this. Second, for the United States the percentage increase in nominal household wealth from a positive inflation shock is less than the percentage increase in the price level, and so there is a fall in real household wealth from a positive inflation shock. This has, other things being equal, a negative effect on real household expenditures. Third, in the price and wage equations for the United States nominal wages lag prices, and so a positive inflation shock results in an initial fall in the real wage rate and thus real labor income. A fall in real labor income has, other things being equal, a negative effect on real household expenditures.

If these three features are true, they imply that a positive inflation shock has a negative effect on aggregate demand even if the nominal interest rate is held constant. The fall in real wealth and real labor income is contractionary, and there is no offsetting rise in demand from the fall in the real interest rate. Not only does the Fed not have to increase the nominal interest rate more than the increase in inflation for there to be a contraction, it does not have to increase the nominal rate at all! The inflation shock itself will contract the economy through the real wealth and real income effects.

The omission of wages from the above class of models can be traced back to the late 1970s, where there began a movement away from the estimation of structural price and wage equations to the estimation of reduced form price equations (i.e., price equations that do not include wage rates as explanatory variables). (See the discussion in Section 3.13.) This line of research evolved to the estimation of NAIRU equations, as in the above class of models.

4.1.2 Estimated Effects of a Positive Inflation Shock

A simple experiment is performed in this subsection that shows that for the United States in the MC model a positive inflation shock is contractionary. The period used is 1994:1–1998:4, 20 quarters. The first step is to add the estimated residuals to the stochastic equations and take them to be exogenous. This means that when the model is solved using the actual values of all the exogenous variables, a perfect tracking solution results. The base path for the experiment is thus just the historical path. Then the constant term in the U.S. price equation 10 is increased by .005 (.50 percentage points) from its estimated value.[79] Also, the estimated interest rate rule for the Fed, equation 30, is dropped, and the nominal short term interest rate, RS, is taken to be exogenous for the United States. The model is then solved. The difference between the predicted value of each variable and each period from this solution and its base (actual) value is the estimated effect of the price-equation shock. Remember that this is an experiment in which there is no change in the U.S. short term nominal interest rate because the US interest rate rule is dropped. There is also no effect on U.S. long term nominal interest rates because they depend only on current and past U.S. short term nominal interest rates.

Selected results from this experiment are presented in Table 1. The main point for present purposes is in row 1, which shows that real GDP falls: the inflation shock is contractionary. The rest of this section is simply a discussion of some of the details.

Row 2 shows the effects of the change in the constant term in the price equation on the price level. The price level is .52 percent higher than its base value in the first quarter, 1.02 percent higher in the second quarter, and so on through the twentieth quarter, where it is 6.33 percent higher. (The shock to the price equation accumulates over time because of the lagged dependent variable in the equation.) Row 3 versus row 2 shows that the nominal wage rate rises less than the price level, and so there is a fall in the real wage rate, WF/PF. Row 4 shows that real disposable income falls. (Although not shown, nominal disposable income increases.) Real disposable income falls because of the fall in the real wage rate and because some nonlabor nominal income, such as interest income, rises less in

[79]Note that this is a shock to the price equation, not to the wage equation. It is similar to an increase in the price of oil. In the MC model an increase in the price of oil (which is exogenous) increases the U.S. price of imports, which is an explanatory variable in the U.S. price equation. Either an increase in the constant term in the price equation or an increase in the price of oil leads to an initial fall in the real wage because wages lag prices. If the shock were instead to the wage equation, there would be an initial rise in the real wage, which would have much different effects.

Table 1
Effects of a Positive Shock to the U.S. Price Equation 10
Nominal Interest Rate, RS, Unchanged from Base Values

		Changes from Base Values							
		Quarters Ahead							
	Variable	1	2	3	4	8	12	16	20
1	Real GDP ($GDPR$)	-0.01	-0.04	-0.09	-0.15	-0.46	-0.76	-0.96	-1.08
2	Price level (PF)	0.52	1.02	1.49	1.93	3.44	4.62	5.54	6.33
3	Wage rate (WF)	0.42	0.77	1.13	1.47	2.66	3.64	4.35	4.96
4	Real DPI (YD/PH)	-0.05	-0.10	-0.15	-0.21	-0.48	-0.70	-0.88	-1.05
5	$\Delta\Pi$	1.4	1.1	0.7	0.5	0.3	0.5	0.3	1.2
6	CG	16.6	12.0	9.2	7.8	19.1	15.7	20.3	153.0
7	Real wealth (AA)	-0.23	-0.45	-0.66	-0.86	-1.51	-1.95	-2.10	-2.17
8	CS	-0.01	-0.03	-0.06	-0.10	-0.32	-0.58	-0.80	-0.98
9	CN	-0.01	-0.03	-0.07	-0.11	-0.35	-0.57	-0.73	-0.85
10	CD	-0.05	-0.17	-0.33	-0.52	-1.56	-2.42	-2.85	-2.94
11	IHH	-0.03	-0.08	-0.15	-0.25	-0.73	-1.09	-1.28	-1.35
12	IKF	-0.01	-0.04	-0.09	-0.15	-0.73	-1.58	-2.24	-2.59
13	yen/$ rate (E_{JA})	-0.03	-0.08	-0.14	-0.22	-0.69	-1.29	-1.94	-2.60
14	DM/$ rate (E_{GE})	-0.04	-0.13	-0.24	-0.38	-1.11	-1.96	-2.75	-3.41
15	Price of imports (PIM)	0.09	0.14	0.20	0.27	0.74	1.26	1.84	2.39
16	Price of exports (PEX)	0.45	0.86	1.25	1.62	2.91	3.96	4.81	5.56
17	Real imports (IM)	0.03	0.09	0.15	0.21	0.25	0.00	-0.39	-0.74
18	Real exports (EX)	-0.04	-0.07	-0.11	-0.15	-0.38	-0.62	-0.87	-1.08
19	Current account	0.03	0.06	0.09	0.12	0.18	0.24	0.30	0.33

- All variables but 13 and 14 are for the United States.
- DPI = disposable personal income.
- $\Delta\Pi$ = Change in nominal after-tax corporate profits. $\Pi = PIEF - TFG - TFS - TFR$.
- Current Account = U.S. nominal current account as a percent of nominal GDP. The U.S. current account is $PEX \cdot EX - PIM \cdot IM$.
- Changes are in percentage points except for $\Delta\Pi$ and CG, which are in billions of dollars.
- Simulation period is 1994.1–1998.4.

percentage terms than the price level.

The change in nominal corporate after-tax profits is higher (row 5), and this in turn leads to a small increase in capital gains (CG) for the household sector (row 6). (This is U.S. equation 25 at work.) For example, the increase in capital gains in the first quarter is $16.6 billion. ($CG$ is not affected by any nominal interest rate changes because there are none.) The increase in CG leads to an increase in nominal household wealth (not shown), but row 7 shows that real household wealth is lower. This means that the percentage increase in nominal household wealth is smaller than the percentage increase in the price level. Put another way, U.S. equation 25 does not lead to a large enough increase in CG to have real household wealth rise.

The fall in real income and real wealth leads to a fall in the four categories of household expenditures (rows 8–11). Nonresidential fixed investment is lower (row 12), which is a response to the lower values of output, although this is partly offset by the fall in the real interest rate. (Remember that U.S. equation 12 is the one demand equation in the model that uses the real interest rate.)

Rows 13 and 14 present the Japanese and German nominal exchange rates relative to the U.S. dollar. (An increase in a rate is a depreciation of the currency.) The two currencies appreciate relative to the dollar. This is because the U.S. price level rises relative to the Japanese and German price levels, which leads, other things being equal, to an appreciation of the yen and DM through the estimated equations for the two exchange rates (see Table B9 in Appendix B).

Row 15 shows that the U.S. import price level rises, which is due to the depreciation of the dollar, and row 16 shows that the U.S. export price level rises, which is due to the increase in the overall U.S. price level.

The real value of imports in the model responds positively to a decrease in the import price level relative to the domestic price level and negatively to a decrease in real income. Row 17 shows that the net effect is small, with increases in the beginning and decreases at the end. The real value of U.S. exports is lower (row 18), which is due to a higher relative US export price level. (The export price level increases more than the dollar depreciates, and so U.S. export prices in other countries' currencies increase.) Even though the real value of U.S. exports is lower, there is a slight improvement in the nominal U.S. current account (row 19). This improvement is in part due to the higher U.S. export price level (a J curve type of effect).

Regarding long run effects, the present experiment is somewhat artificial because of the dropping of the estimated interest rate rule of the Fed. The rule has the property that, other things being equal, the Fed will lower the nominal interest rate when the U.S. economy contracts. This will then help bring the economy out of the contraction. The present experiment is merely meant to show what would be the case if the rule were dropped. In practice, of course, the Fed would react.

4.1.3 The FRB/US Model

The FRB/US model—Federal Reserve Board (2000)—is sometimes cited as a macroeconometric model that is consistent with the class of models discussed above (see, for example, Taylor (2000), p. 91). This model has strong real interest rate effects. In fact, if government spending is increased in the FRB/US model with the nominal interest rate held constant, real output eventually expands so much that the model will no longer solve.[80] The increase in government spending raises inflation, which with nominal interest rates held constant lowers real interest rates, which leads to an unlimited expansion. The model is not stable unless there is a nominal interest rate rule that leads to an increase in the real interest rate when inflation increases.

It may seem puzzling that two macroeconometric models could have such different properties. Given the empirical results in Section 3.12, how can it be that the FRB/US model finds such strong real interest rate effects? The answer is that many restrictions have been imposed on the model that have the effect of imposing large real interest rate effects. In most of the expenditure equations real interest rate effects are imposed rather than estimated. Direct tests of nominal versus real interest rates like the one used in Section 3.12 are not done, and so there is no way of knowing what the data actually support in the FRB/US expenditure equations.

Large stock market effects are also imposed in the FRB/US model. Contrary to the estimate of U.S. equation 25, which shows fairly small effects of nominal interest rates and nominal earnings on CG, the FRB/US model has extremely large effects. A one percentage point decrease in the real interest rate leads to a 20 percent increase in the value of corporate equity (Reifschneider, Tetlow, and Williams (1999), p. 5). At the end of 1999 the value of corporate equity was about $20 trillion (using data from the U.S. Flow of Funds accounts), and 20 percent of this is $4 trillion. There is thus a huge increase in nominal household wealth for even a one percentage point decrease in the real interest rate. A positive inflation shock with the nominal interest rate held constant, which lowers the real interest rate, thus results in a large increase in both nominal and real wealth in the model. The increase in real wealth then leads through the wealth effect in the household expenditure equations to a large increase in real expenditures. This channel is an important contributor to the model not being stable when there is an increase in inflation greater than the nominal interest rate. Again, this stock price effect is imposed rather than estimated, and so it is not necessarily the case that the data are consistent with this restriction.

[80]Private correspondence with Andrew Levin and David Reifschneider.

There is thus no puzzle about the vastly different properties of the two models. It is simply that important real interest rate restrictions have been imposed in the FRB/US model and not in the MC model.

4.1.4 Conclusion

If a positive inflation shock with the nominal interest rate held constant is in fact contractionary, this has important implications for monetary policy. The coefficient on inflation in the nominal interest rate rule need not be greater than one for the economy to be stable. Also, if one is concerned with optimal policies, the optimal response by the Fed to an inflation shock is likely to be much smaller if inflation shocks are contractionary than if they are expansionary. The use of the above class of models for monetary policy is thus risky. If they are wrong about the effects of inflation shocks, they may lead to poor monetary policy recommendations.

4.2 Analysis of the Capital Gains Variable, CG[81]

4.2.1 Introduction

The size of the wealth effect for the United States is explored in Section 5.7. The variable that drives financial wealth in the model is the capital gains variable, CG, and this variable is analyzed in this section. Changes in stock prices change CG, which changes the financial wealth of the household sector, which in turn affects household consumption expenditures.

4.2.2 Analysis of CG

The variable AH in the US model is the nominal value of net financial assets of the household sector. It is determined by the identity 66 in Table A.3 in Appendix A:

$$AH = AH_{-1} + SH - \Delta MH + CG - DISH, \qquad 66$$

where SH is the financial saving of the household sector, MH is its holdings of demand deposits and currency, CG is the value of capital gains (+) or losses (-) on the financial assets held by the household sector (almost all of which is the change in the market value corporate stocks held by the household sector), and $DISH$ is a discrepancy term.

A change in the stock market affects AH through CG. The variable CG is constructed from data from the U.S. Flow of Funds accounts. Not surprisingly, it is highly correlated with the change in the S&P 500 stock price index. When $CG/(PX_{-1}YS_{-1})$ is regressed on $(SP - SP_{-1})/(PX_{-1}YS_{-1})$, where SP is the value of the S&P 500 index at the end of the quarter and $PX_{-1}YS_{-1}$ is the value of potential nominal output in the previous quarter, the results are:

$$\frac{CG}{PX_{-1}YS_{-1}} = \underset{(6.55)}{.0580} + \underset{(32.16)}{9.28}\,\frac{SP - SP_{-1}}{PX_{-1}YS_{-1}},$$

$$R^2 = .812, 1954.1 - 2013.3 \qquad (1)$$

$PX_{-1}YS_{-1}$ is used for scale purposes in this regression to lessen the chances of heteroskedasticity. The fit of this equation is very good, reflecting the high correlation of CG and the change in the S&P 500 index. A coefficient of 9.28

[81]Th e discussion in this section is taken from Fair (2004a, Chapter 5). The results in Subsection 4.2.2 are updates of those in Fair (2004a, Chapter 5).

means that a 100 point change in the S&P 500 index results in a $928 billion dollar change in the value of stocks held by the household sector.

CG is determined by equation 25, which is repeated here:

$$\frac{CG}{PX_{-1}YS_{-1}} = \underset{(4.77)}{.106} - \underset{(-1.34)}{.145}\ \Delta RB + \underset{(1.13)}{11.79}\frac{\Delta\Pi}{PX_{-1}YS_{-1}},$$
$$R^2 = .022, 1954.1 - 2013.3 \qquad 25$$

If $SP - SP_{-1}$ is used in place of CG, the results are:

$$\frac{SP - SP_{-1}}{PX_{-1}YS_{-1}} = \underset{(2.73)}{.00589} - \underset{(-1.98)}{.0208}\ \Delta RB + \underset{(0.59)}{.598}\frac{\Delta\Pi}{PX_{-1}YS_{-1}},$$
$$R^2 = .020, 1954.1 - 2013.3 \qquad (2)$$

It is clear that equation 25 and the equivalent equation for the change in the S&P 500 index are telling the same story. The change in the bond rate (ΔRB) has a negative effect on the change in stock prices and the change in profits ($\Delta\Pi$) has a positive effect. None of the estimates are, however, statistically significant, and very little of the variation of the change in stock prices has been explained. The change in stock prices is roughly a random walk with drift, but equation 25 does at least provide a small link from interest rates to stock prices in the MC model.

4.3 Analyzing Macroeconomic Forecastability[82]

4.3.1 Introduction

This section uses the MC model to examine the limits to macroeconomic forecasting. The basic idea is that if changes in asset prices affect the macroeconomy and if these changes are unpredictable, then fluctuations in the macroeconomy due to changes in asset prices are unpredictable. Stochastic simulation is used to estimate the fraction of the forecast-error variance of output changes and the fraction of the forecast-error variance of inflation that are due to unpredictable asset-price changes. The results suggest that about 30 percent of the forecast-error variance of output growth over 8 quarters is due to asset-price changes and about 35 percent of the forecast-error variance of inflation over 8 quarters is due to asset-price changes.

There is a large literature analyzing the ability of models to forecast the probability that a recession will occur in some future quarter, in particular using the yield curve to forecast such probabilities. Two recent papers are Chauvet and Potter (2005) and Rudebusch and Williams (2008). For example, Rudebusch and Williams define a recession as a quarter with negative real growth and examine horizons of zero to four quarters ahead. They find that the yield curve has some predictive power relative to predictions from professional forecasters.

There is also a large literature, recently surveyed by Stock and Watson (2003), examining whether asset prices are useful predictors of future output growth and inflation. Stock and Watson examine data on many possible predictor variables for seven countries. Using bivariate and trivariate equations, they get mixed results. For some countries and some periods some asset prices are useful predictors, but the predictive relations are far from stable.

This section is not an examination of possible single-equation predictive relationships. Instead, a structural model of the economy, which has already been estimated, is used. This paper also does not single out recessions as special cases. The structure of the economy—the coefficients in the structural equations—are assumed to be stable over the business cycle.

This study is conditional on the estimated structure of the MC model. Using the model allows questions to be considered that cannot be using single-equation relationships. More economic theory is used than in the use of single equations. A disadvantage of this approach is that it requires a particular model. If the model is a poor approximation of the economy, the results will not be trustworthy.

[82]Some of the discussion in this section is taken from Fair (2012b). The results in Tables 1 and 2 are updated from those in Tables I and II in Fair (2012b).

4.3.2 Asset-Price Effects

The key asset-price variables in the MC model are 1) CG, the nominal value of capital gains or losses on the equity holdings of the U.S. household sector, 2) $PSI14$, the ratio of U.S. housing prices to an aggregate price deflator, 3) oil prices, and 4) exchange rates.

The equation explaining CG is discussed in Subsections 3.6.6 and 4.2.3. Very little of the variation of CG is explained by this equation, and the equation primarily just sets $CG/(PX_{-1}YS_{-1})$ equal to the estimated constant term.

Regarding exchange rates, there are 23 estimated exchange rate equations, and these are discussed in Subsection 3.7.2. Two explanatory variables in these equations are a relative interest rate variable and a relative price level variable. The equations are in logs and also include the lagged dependent variable. The lagged dependent variable has a coefficient estimate close to one in the equations, and the equations explain very little of the variance of the change in the log of the exchange rate. The exchange rate equations are thus not too different from estimated random walks with drift.

Regarding oil prices, there are 10 oil exporting countries in the model.[83] Let $POIL_i$ denote the price of exports in dollars of one of these 10 countries i, which is roughly the dollar price of oil. For some uses of the MC model $POIL_i$ is taken to be exogenous, but for this section, 10 equations have been estimated, with $\log POIL_i - \log POIL_{i-1}$ on the left hand side and a constant on the right hand side. In other words, for each of the 10 countries $\log POIL_i$ has been modeled as a random walk with drift.

$PSI14$ is also taken to be exogenous for some uses of the MC model, but for this section an equation has been estimated for it. An equation is estimated with $\log PSI14 - \log PSI14_{-1}$ on the left hand side and a constant on the right hand side. $PSI14$ has thus also been modeled as a random walk with drift.

There are thus 35 estimated asset-price equations: the CG equation, the $PSI14$ equation, 23 exchange rate equations, and 10 oil-price equations. In the stochastic-simulation experiments, which are discussed next, the amount that the variation in these asset prices affect the overall forecast-error variation is estimated.

[83] Saudi Arabia, Venezuela, Nigeria, Algeria, Indonesia, Iran, Iraq, Kuwait, Libya, and United Arab Emirates.

4.3.3 Stochastic-Simulation Experiments and Results

There are 1,700 estimated equations in the MC model, counting the estimated equation for $PSI14$ and the 10 estimated equations for $POIL_i$, of which 1,379 are trade share equations. The estimation period for the United States is 1954:1–2013:3. The estimation periods for the other countries begin as early as 1962:1 and end as late as 2013:2. The estimation period for most of the trade share equations is 1966:1–2012:4. For each estimated equation there are estimated residuals over the estimation period. Let $\hat{u}_t$ denote the 1700-dimension vector of the estimated residuals for quarter t.[84] Most of the estimation periods have the 1972:1–2007:4 period—144 quarters– in common, and this period is taken to be the "base" period. These 144 observations on $\hat{u}_t$ are used for the draws in the stochastic-simulation procedure discussed below.[85]

Twelve sets of non overlapping 8-quarter periods are analyzed. The first is 1986:1–1987:4, and the last is 2008:1–2009:4. Consider the first period. Each trial of the stochastic-simulation procedure is as follows. First, one year and then another are randomly drawn with replacement between 1972 and 2007. For each of the two years the 4 quarterly error vectors are chosen, where the quarterly order for each year is kept. Each vector consists of 1,700 errors.[86] Using these errors, the model is solved dynamically for the 1986:1–1987:4 period. In this solution the actual values of the exogenous variables are used. In addition, before solution the estimated errors (the residuals) are added to each of the 1,700 equations and taken to be exogenous. The drawn errors are then added on top of these errors. This means that if the model is solved with no drawn errors used, a perfect tracking solution is obtained. The drawn errors are thus off of the perfect tracking solution. The solution values are recorded, which completes the trial.

If the above procedure is repeated, say, N times, there are N solution values for each endogenous variable, from which forecast-error variances can be com-

[84]For equations estimated using annual data, the error is put in the first quarter of the year with zeros in the other three quarters (which are never used). If the initial estimate of an equation suggests that the error term is serially correlated, the equation is reestimated under the assumption that the error term follows an autoregressive process (usually first order). The structural coefficients in the equation and the autoregressive coefficient or coefficients are jointly estimated (by 2SLS). The $\hat{u}_t$ error terms are after adjustment for any autoregressive properties, and they are taken to be iid for purposes of the draws. As discussed below, the draws are by year—four quarters at a time.

[85]If an estimation period does not include all of the 1972:1–2007:4 period, zero errors are used for the missing quarters.

[86]Remember that for annual equations the errors for the last three quarters are zero (and never used).

puted. For the results below 250 trials were used for each stochastic-simulation experiment.

Let g be the growth rate of U.S. real GDP over 8 quarters at an annual rate, and let π be the growth rate of the U.S. GDP deflator over 8 quarters at an annual rate. The focus here is on the forecast-error variances of g and π. Let σ^2 denote the estimated forecast-error variance of g or π for one of the twelve 8-quarter periods. The twelve values of σ^2 for g and for π are presented in the first column of Table 1.

Forecast-error variances like those in Table 1 are not constant across time because the model is nonlinear. In Table 1 the range is from 0.863 to 1.024 for g and from 0.495 to 0.720 for π. (All numbers are in percentage points.) These differences are not due to stochastic-simulation error because the same draws were used for each of the twelve periods. In other words, the same 3,400,000 (= 1,700 $\times$ 8 $\times$ 250) errors were used for each stochastic-simulation experiment.

It was mentioned in the Introduction to this section that recessions are not singled out as special cases. The estimated coefficients are taken to be constant over the entire estimation period. Recessions do, however, affect the forecast-error variances in Table 1 in the sense that the model is nonlinear.

It is important to be clear on what is being estimated in Table 1. Estimated historical errors between 1972 and 2007 are being used for the draws for all equations, including the 35 asset-price equations. The variation in asset prices that is being used, for example, is the variation implicit in the historical errors of the asset-price equations. Similarly, the variation in all the other equations that is being used is that implicit in the historical errors of the equations. The historic correlation of the error terms is accounted for since the actual, historic errors are used. Also, the procedure is not based on any assumptions about error distributions. Drawing is from the estimated errors, not from some distribution.

It is also important to realize that the estimated variances in Table 1 do not depend on the actual values of the endogenous variables. For example, the estimated variance for g is computed from the 250 solution values of g; the actual value of g is never used. Also, it would not make much difference if the historical errors were not added to the equations before solution. If the errors were not added, the draws would not be off the perfect tracking solution, but instead off the predicted path using zero errors. This would give different solution values of g and a different mean, but the variance computed from the 250 solution values would be similar.

In the second column of Table 1 estimated variances, denoted σ_a^2, are presented in which no errors are used for the 35 asset-price equations. This means for the $PSI14$ equation and the 10 oil price equations, which are just estimated random

Table 1
Estimated Variances

Period	(1) σ^2	(2) σ_a^2	(3) σ^2-σ_a^2	(4) $\frac{\sigma^2-\sigma_a^2}{\sigma^2}$
Output Growth over Eight Quarters, annual rate				
1986:1–1987:4	1.024	0.646	0.378	0.369
1988:1–1989:4	0.980	0.620	0.360	0.367
1990:1–1991:4	1.014	0.649	0.365	0.360
1992:1–1993:4	0.988	0.635	0.353	0.357
1994:1–1995:4	0.934	0.617	0.317	0.339
1996:1–1997:4	0.900	0.615	0.285	0.317
1998:1–1988:4	0.863	0.596	0.267	0.309
2000:1–2001:4	0.886	0.589	0.297	0.335
2002:1–2003:4	0.896	0.601	0.295	0.329
2004:1–2005:4	0.868	0.618	0.250	0.288
2006:1–2007:4	0.897	0.645	0.252	0.281
2008:1–2009:4	0.937	0.619	0.318	0.339
Average				0.332
Inflation over Eight Quarters, annual rate				
1986:1–1987:4	0.702	0.449	0.253	0.360
1988:1–1989:4	0.720	0.459	0.261	0.363
1990:1–1991:4	0.657	0.426	0.231	0.352
1992:1–1993:4	0.613	0.413	0.200	0.326
1994:1–1995:4	0.594	0.425	0.169	0.285
1996:1–1997:4	0.521	0.359	0.162	0.311
1998:1–1988:4	0.495	0.352	0.143	0.289
2000:1–2001:4	0.509	0.338	0.171	0.336
2002:1–2003:4	0.498	0.336	0.162	0.325
2004:1–2005:4	0.526	0.332	0.194	0.369
2006:1–2007:4	0.526	0.323	0.203	0.386
2008;1–2009:4	0.510	0.331	0.179	0.351
Average				0.338

- σ^2 = total forecast-error variance.
- σ_a^2 = forecast-error variance, asset-price errors not used.
- 250 trials each experiment.
- Same draws for each experiment.
- 1,700 equations, of which 134 are asset-price equations.
- Historical errors between 1972:1 and 2007:4 drawn.
- Values are in percentage points.

walk equations with drift, the solution values are the same across all trials—there is no variation in the variables. For the other equations there is some variation because there are right hand side endogenous variables, but most of the variation has been eliminated by not using errors. Again, there is no stochastic-simulation error comparing across estimated variances because the same draws are used for all the experiments.[87] The third column presents the difference in the two variances for each period, and the fourth column presents the percent difference.

For output growth, g, between 28.1 and 36.9 percent of the total forecast-error variance is reduced when errors for the asset-price equations are not used. The average over the 12 subperiods is 33.2 percent. For inflation, π, between 28.5 and 38.6 percent is reduced, with an average of 33.8 percent. The results in Table 1 thus show that asset-price variation has important effects on overall forecast-error variation in the MC model—about a third for both g and π.

Table 2 presents for the last two periods results for four categories of asset prices. These results are based on four extra stochastic-simulation experiments per period. For the first all errors are used except those for the CG equation; for the second all errors are used except those for the $PSI14$ equation; for the third all errors are used except those for the 10 oil price equations; and for the fourth all errors are used except those for the 23 exchange rate equations. Let σ_1^2, σ_2^2, σ_3^2, and σ_4^2 denote the respective estimated variances. The difference between σ^2 and σ_1^2 is the estimate of how much σ^2 is changed when errors are not used for the CG equation, and similarly for the other three. These four differences are presented in Table 2.

The last column in Table 2 is the sum of the four differences. Because of the correlation of the error terms, this sum is not the same as σ^2 - σ_a^2, where σ_a^2 is based on not using errors for all the asset-price equations at once. Table 2 shows that for g the sum is 0.255 versus 0.252 for σ^2 - σ_a^2 for the first period and 0.319 versus 0.318 for the second. These differences are very small, and so the correlation of the error terms does not appear to be an important issue in interpreting the results. For π the differences are: 0.236 versus 0.203 and 0.213 versus 0.179.

For g the contribution from the exchange rates is very small. For the first period the other three (CG, $PSI14$, and oil prices) are 0.120, 0.057, and 0.062, and for the second period they are 0.202, 0.059, and 0.034. Stock prices thus contribute the most, with the other two about equal. The main point, however, is that all three contribute. For π neither stock prices nor housing prices contribute. Oil prices are most important: 0.148 for the first period and 0.133 for the second. For

[87]The draws for the asset-price equations are, of course, just discarded.

Table 2
Variance Components

Period	σ^2	σ^2-σ_a^2	σ^2-σ_1^2	σ^2-σ_2^2	σ^2-σ_3^2	σ^2-σ_4^2	Sum
Output Growth over Eight Quarters, annual rate							
2006:1–2007:4	0.897	0.252	0.120	0.057	0.062	0.016	0.255
2008:1–2009:4	0.937	0.318	0.202	0.059	0.034	0.024	0.319
Inflation over Eight Quarters, annual rate							
2006:1–2007:4	0.526	0.203	0.010	-0.005	0.148	0.083	0.236
2008:1–2009:4	0.510	0.179	0.017	-0.004	0.133	0.067	0.213

• See notes to Table 1.
• σ_1^2 = forecast-error variance, CG errors not used.
• σ_2^2 = forecast-error variance, $PSI14$ errors not used.
• σ_3^2 = forecast-error variance, oil-price errors not used.
• σ_4^2 = forecast-error variance, exchange-rate errors not used.

exchange rates the respective values are 0.083 and 0.067. Cost shocks (oil prices and exchange rates) thus drive the results for inflation. For output both demand shocks (stock prices and housing prices) and cost shocks (oil prices) contribute.

4.3.4 Conclusion

Results like those in Tables 1 and 2 require 1) a model that estimates the effects of asset-price changes on the economy, 2) estimates of the variation of asset-price changes, and 3) estimates of the variation of other errors. In the MC model stock prices and housing prices affect U.S. household wealth, which affects U.S. consumption. Oil prices affect import prices of all the oil importing countries, which affect domestic prices through domestic price equations. Exchange rates affect relative prices of imports and exports, which have many effects across countries. Variation is estimated by drawing from vectors of historical errors for the 1972:1–2007:4 period. The historical errors for the asset-price changes either are or are close to being errors in a random walk equation with drift. The use of these errors reflects the assumption that asset-price changes are not predictable except for a possible drift.

The results suggest that about a third of the forecast-error variances of output growth and inflation over 8 quarters are due to asset-price changes. The inflation results are due to cost shocks from oil prices and exchange rates. The output results are due to stock prices, housing prices, and oil prices. The results thus suggest that the degree of uncertainty of any particular forecast of the macroeconomy *that one can never eliminate* is large. Any forecast is based implicitly or explicitly on assumptions about asset-price changes, which one has no ability to forecast.

The forecast-error variances that are estimated here are based on historically estimated errors in the structural equations of the MC model. One can think about these errors as being either random shocks that can never be eliminated or errors that can be eliminated by better specifications. If one could develop a model with very small structural errors, then σ_a^2 in Table 1 would be close to zero. Almost all the forecast-error variation would be from asset-price changes. The percents in column 4 of Table 1 thus depend on the accuracy of the structural equations of the MC model. A more accurate model, other things being equal, would lead to higher percents, although column 3 would not be affected. The values in column 3 are simply the forecast-error variances from asset-price changes.

Since the stochastic-simulation estimates reflect historically average behavior, they do not say anything about any one particular 8-quarter period. If in a specific period asset prices change very little, macroeconomic forecasts may be quite good, and conversely if asset prices change a lot. After the fact one could take a model like the MC model and examine how well the model predicted a specific period knowing and then not knowing the asset-price changes. The difference in forecasting accuracy between knowing and not knowing the asset-price changes would obviously vary across periods, in many cases by a large amount. The estimates in this paper are less tied to specific periods. They weight equally the historical variation between 1972 and 2007. On the other hand, the results do vary somewhat across time because of the non linearity of the model.

4.4 Evaluating Monetary Policy and Fiscal Policy Rules[88]

4.4.1 Introduction

This section examines various interest rate rules, as well as policies derived by solving optimal control problems, for their ability to dampen economic fluctuations caused by random shocks. A tax rate rule is also considered. The MC and US models are used for the experiments. The results differ sharply from those that would be obtained using the class of models discussed in Section 4.1, where in these models the coefficient on inflation in the nominal interest rate rule must be greater than one in order for the economy to be stable.

Subsection 4.4.2 discusses a simple experiment in which the interest rate rule of the Fed (equation 30) is dropped from the model and RS is decreased by one percentage point. This shows the size of the effects of monetary policy changes on the economy.

Subsection 4.4.3 examines the stabilization features of four interest rate rules for the United States. The first is simply the estimated rule, equation 30, which has an estimated long run coefficient on inflation of approximately one. The other three rules are modifications of the estimated rule, with imposed long run coefficients on inflation of 0.0, 1.5, and 2.5 respectively. It will be seen that as the inflation coefficient increases there is a reduction in price variability at a cost of an increase in interest rate variability. Even the rule with a zero inflation coefficient is stabilizing, which is contrary to what would be obtained using the class of models discussed in Section 4.1.

Section 4.4.4 then computes optimal rules for particular loss functions. These solutions require a combination of stochastic simulation and solving deterministic optimal control problems. It will be seen that the optimal control results are similar to those obtained using the equation-30 estimated rule for a loss function with a higher weight on inflation than on output.

Another feature of the results in Subsections 4.4.3 and 4.4.4 is that considerable variance of the endogenous variables is left using even the best interest rate rule. Section 4.4.5 adds a fiscal policy rule—a tax rate rule—to see how much help it can be to monetary policy in trying to stabilize the economy. The results show that the tax rate rule provides some help.

[88]The results in this section are updates of those in Fair (2005a).

4.4.2 The Effects of a Decrease in RS

It will first be useful to review the effects of a change in the U.S. short term interest rate, RS, in the MC model. To examine these effects, the following experiment was run. The period used is 1994:1–1998:4, 20 quarters. The first step is to add the estimated residuals to the stochastic equations and take them to be exogenous. This means that when the model is solved using the actual values of all the exogenous variables, a perfect tracking solution results. The base path for the experiment is thus just the historical path. Then the estimated interest rate rule for the Fed, equation 30, was dropped from the model, and RS was decreased by one percentage point from its historical value for each quarter. The model was then solved. The difference between the predicted value of each variable and each period from this solution and its base (actual) value is the estimated effect of the interest rate change.

Selected results from this experiment are presented in Table 1. Row 3 shows that real output, Y, increases: the nominal interest rate decrease is expansionary. The peak response is .68 percent after 8 quarters. Row 1 shows the exogenous fall in RS of one percentage point, and row 2 shows the response of the long term bond rate, RB, to this change. After 12 quarters the bond rate has fallen .73 percentage points. This reflects the properties of the estimated term structure equation 22, where RB responds to current and past values of RS. The unemployment rate is lower (row 4), and the price level is higher (row 5). The peak unemployment response is -.28 percentage points after 8 quarters.

The change in nominal after-tax corporate profits (row 6) is higher because of the higher level of real output and higher price level. The nominal value of household capital gains, CG, is larger because of the lower bond rate and higher value of profits (equation 25). An increase in CG is an increase in nominal household wealth, and row 8 shows that real wealth, AA, also increases initially. By quarter 12, however, real wealth is slightly below the base value. This means that by quarter 12 the negative effect on real wealth from the higher price level has offset the positive effect from the higher nominal wealth.

Row 10 shows that real disposal personal income, YD/PH, decreases. An important feature of the model is that when interest rates fall, interest payments of the firm and government sectors fall, and this in turn lowers interest income of the household sector. A decrease in household interest income is a decrease in YD. The household sector is a large creditor, and this interest income effect is fairly large. Row 10 shows that the overall effect on real disposable personal income is negative. The overall effect is also negative for nominal disposable income (YD) for the first four quarters (row 9). Another factor contributing to the fall in real

Table 1
Effects of a Decrease in RS of 1.0 Percentage Points

		Changes from Base Values Quarters Ahead							
	Variable	1	2	3	4	8	12	16	20
1	Bill rate (RS)	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00
2	Bond rate (RB)	-0.33	-0.35	-0.41	-0.46	-0.62	-0.73	-0.81	-0.87
3	Real output (Y)	0.05	0.15	0.27	0.40	0.68	0.64	0.50	0.36
4	Unemployment rate ($100 \cdot UR$)	-0.01	-0.04	-0.08	-0.13	-0.28	-0.28	-0.21	-0.12
5	Price deflator (PF)	0.01	0.03	0.06	0.10	0.35	0.62	0.84	0.98
6	Change in profits ($\Delta\Pi$)	0.50	0.95	1.01	0.90	0.07	-0.10	-0.17	0.06
7	Capital gains (CG)	78.76	16.95	24.56	22.34	11.01	6.57	5.47	25.79
8	Real wealth (AA)	0.30	0.33	0.38	0.41	0.30	0.04	-0.19	-0.35
9	DPI (YD)	-0.02	-0.01	0.03	0.09	0.37	0.56	0.66	0.69
10	Real DPI (YD/PH)	-0.07	-0.09	-0.09	-0.08	-0.09	-0.17	-0.28	-0.38
11	Service consumption (CS)	0.08	0.16	0.23	0.28	0.42	0.43	0.37	0.27
12	Nondurable consumption (CN)	0.02	0.05	0.09	0.13	0.22	0.20	0.14	0.06
13	Durable consumption (CD)	0.20	0.51	0.78	1.03	1.71	1.76	1.45	1.02
14	Residential inv. (IHH)	-0.04	0.58	1.36	2.14	4.04	4.21	3.75	2.95
15	Nonresidential fixed inv. (IKF)	0.03	0.14	0.42	0.74	1.95	2.17	1.79	1.22
16	JA bill rate (RS_{JA})	-0.11	-0.20	-0.28	-0.34	-0.46	-0.48	-0.46	-0.23
17	GE bill rate (RS_{GE})	-0.07	-0.14	-0.19	-0.24	-0.37	-0.43	-0.46	-0.49
18	JA exchange rate (E_{JA})	-0.29	-0.53	-0.74	-0.91	-1.45	-1.89	-2.28	-2.76
19	GE exchange rate (E_{GE})	-0.44	-0.81	-1.13	-1.39	-2.15	-2.63	-2.96	-3.19
20	Price of imports (PIM)	0.24	0.34	0.43	0.49	0.82	1.07	1.30	1.56
21	Real imports (IM)	0.01	0.06	0.15	0.27	0.86	1.29	1.36	1.14
22	Price of exports (PEX)	0.06	0.11	0.16	0.22	0.55	0.80	1.00	1.14
23	Real exports (EX)	0.01	0.02	0.03	0.04	0.08	0.12	0.19	0.29
24	Current account	-0.02	-0.03	-0.04	-0.05	-0.12	-0.17	-0.18	-0.16

- All variables but 16–19 are for the United States.
- DPI = disposable personal income.
- $\Delta\Pi$ = Change in nominal after-tax corporate profits. (In the notation in Table A.2, $\Pi = PIEF - TFG - TFS - TFR$.)
- Current account = U.S. nominal current account as a percent of nominal GDP. The U.S. current account is $PEX \cdot EX - PIM \cdot IM$.
- Changes are in percentage points except for $\Delta\Pi$ and CG, which are in billions of dollars.
- Simulation period is 1994.1–1998.4.

disposable personal income is that there is a slight fall in the real wage (not shown). Wages lag prices in the model, and the initial response is for the nominal wage rate to increase less than the price level.

Rows 11–14 show that real household expenditures are larger except for a small initial decrease in IHH. The two positive effects on expenditures are the lower interest rates (a nominal interest rate is an explanatory variable in each of the household expenditure equations) and the higher real wealth. The negative effect is the fall in real disposable personal income. There is an additional negative effect on durable expenditures and residential investment over time, which is an increase in the stocks of durables and housing. Other things being equal, an increase in the stock of durables has a negative effect on durable expenditures and an increase in the stock of housing has a negative effect on residential investment. Row 15 shows that real plant and equipment investment, IKF, rises. This is because of the fall in the real bond rate and the rise in real output.

Rows 16–24 pertain to the effect of the rest of the world on the United States and vice versa. Rows 16 and 17 show that the Japanese and German interest rates, RS_{JA} and RS_{GE}, both decrease. These are the estimated interest rate rules for Japan and Germany at work. The US interest rate is an explanatory variable in each of these equations. This means that the Japanese and German monetary authorities are estimated to respond directly to U.S. monetary policy. Rows 18 and 19 show that the yen and the DM appreciate relative to the dollar. (Remember that a decrease in E is an appreciation of the currency.) This is because there is a fall in the U.S. interest rate relative to the Japanese and German interest rates and because there is an increase in the U.S. price level relative to the Japanese and German price levels (not shown).

The depreciation of the dollar leads to an increase in the U.S. import price level, PIM (row 20). This increase is one of the reasons for the increase in the U.S. price level (row 5), since the price of imports has a positive effect on the domestic price level in U.S. price equation 10. Even though the price of imports rises relative the domestic price level, which other things being equal has a negative effect on import demand, the real value of imports, IM, rises (row 21). In this case the positive effect from the increase in real output dominates the negative relative price effect.

The rise in the overall U.S. price level leads to a rise in the U.S. export price level, PEX (row 22). The real value of U.S. exports, EX, rises (row 23), which is due to the depreciation of the dollar. (The U.S. export price level increases less that the dollar depreciates, and so U.S. export prices in other countries' currencies fall.)

Finally, the nominal U.S. current account falls (row 24). The positive effects on the current account are the increase in real exports and the increase in the price of exports. The negative effects are the increase in real imports and the increase in the price of imports. On net the negative effects win, which is primarily due to the increase in the price of imports.

The real output effects of .40 percent after 4 quarters and .68 percent after

8 quarters are much lower than in the FRB/US model, where the effects are .6 percent after 4 quarters and 1.7 percent after 8 quarters—Reifschneider, Tetlow, and Williams (1999), Table 3. The effects are even larger after that, and the model eventually blows up if the short term nominal interest rate is held below its base value.[89] This is a feature of the class of models discussed in Section 4.1, where models in this class are unstable without an inflation coefficient in the interest rate rule greater than one. In this class of models an experiment in which the interest rate rule is dropped and the interest rate lowered is explosive.

4.4.3 Stabilization Effectiveness of Four Nominal Interest Rate Rules

The Four Rules

In the estimated interest rate rule for the Fed, equation 30, the coefficient on lagged money growth is .0131, the coefficient on inflation is .0667, and the coefficient on the lagged dependent variable is .9186 (Table A30 within Table A.4 in Appendix A). If it is assumed that in the long run money growth equals the rate of inflation, then the long run coefficient on inflation in equation 30 is 0.980 [=(.0667+.0131)/(1 - .9186)]. As noted in Section 11.1, the other three rules have imposed long run coefficients of 0.0, 1.5, and 2.5 respectively. This was done for each rule by changing the coefficient for the rate of inflation in equation 30. The respective coefficients are -.0131, .1221, and .2035. None of the other coefficients in the estimated equation were changed for the three rules.[90] This process is similar to that followed for the studies in Taylor (1999a), where the five main rules tried had inflation coefficients varying from 1.2 to 3.0. No inflation coefficient less than 1.0 was tried in these studies because the models, which belong to the class of models discussed in Section 4.1, are not stable in this case.

The Stochastic Simulation Procedure

The four interest rate rules are examined using stochastic simulation. For the work here the coefficient estimates have been taken to be fixed. The results are conditional on the model and on the coefficient estimates. The focus here, as in much of the literature, is on variances, not means. The aim of monetary policy is taken to smooth the effects of shocks. In order to examine the ability of monetary

[89] Private correspondence with David Reifschneider.

[90] A footnote below explains why the constant term in the interest rate rule does not have to be changed when the inflation coefficient is changed.

policy to do this, one needs an estimate of the likely shocks that monetary policy would need to smooth, and this can be done by means of stochastic simulation. Given an econometric model, shocks can be generated by drawing errors.

Stochastic simulation is discussed in Section 2.6, and it is applied to the MC model in Section 4.3. The same setup is used here as is used in Section 4.3, although without the extra equations added in Section 4.3. The following is a slight modification of some of the discussion in Section 4.3.

There are 1,689 estimated equations in the MC model, of which 1,379 are trade share equations. The estimation period for the United States is 1954:1–2013:3. The estimation periods for the other countries begin as early as 1962:1 and end as late as 2013:2. The estimation period for most of the trade share equations is 1966:1–2012:4. For each estimated equation there are estimated residuals over the estimation period. Let $\hat{u}_t$ denote the 1689-dimension vector of the estimated residuals for quarter t.[91] Most of the estimation periods have the 1972:1–2007:4 period—144 quarters– in common, and this period is taken to be the "base" period. These 144 observations on $\hat{u}_t$ are used for the draws in the stochastic-simulation procedure discussed below.[92]

The solution period is 1994:1–1998:4. Each trial of the stochastic-simulation procedure is as follows. First, each of five years is randomly drawn with replacement between 1972 and 2007. For each of these years the 4 quarterly error vectors are chosen, where the quarterly order for each year is kept. Each vector consists of 1,689 errors.[93] Using these errors, the model is solved dynamically for the 1994:1–1998:4 period. In this solution the actual values of the exogenous variables are used. In addition, before solution the estimated errors (the residuals) are added to each of the 1,643 equations and taken to be exogenous. The drawn errors are then added on top of these errors. This means that if the model is solved with no drawn errors used, a perfect tracking solution is obtained. The drawn errors are

[91]For equations estimated using annual data, the error is put in the first quarter of the year with zeros in the other three quarters (which are never used). If the initial estimate of an equation suggests that the error term is serially correlated, the equation is reestimated under the assumption that the error term follows an autoregressive process (usually first order). The structural coefficients in the equation and the autoregressive coefficient or coefficients are jointly estimated (by 2SLS). The $\hat{u}_t$ error terms are after adjustment for any autoregressive properties, and they are taken to be iid for purposes of the draws. As discussed in the text, the draws are by year—four quarters at a time.

[92]If an estimation period does not include all of the 1972:1–2007:4 period, zero errors are used for the missing quarters.

[93]Remember that for annual equations the errors for the last three quarters are zero (and never used).

thus off of the perfect tracking solution. Since the concern here is with stabilization around base paths and not with positions of the base paths themselves, it does not matter much which path is chosen for the base path. The choice here is simply to take as the base path the historical path (by adding the estimated residuals to the equations). After the solution the solution values are recorded, which completes the trial.

If the above procedure is repeated, say, N times, there are N solution values for each endogenous variable, from which forecast-error variances can be computed. For the results below 200 trials were used.

The estimated residuals are added to the interest rate rule, but no errors are drawn for it. Adding the estimated residuals means that when the model inclusive of the rule is solved with no errors for any equation drawn, a perfect tracking solution results.[94] Not drawing errors for the rule means that the Fed does not behave randomly but simply follows the rule.

Let y_{it}^j be the predicted value of endogenous variable i for quarter t on trial j, and let y_{it}^* be the base (actual) value. How best to summarize the 1000×20 values of y_{it}^j? One possibility for a variability measure is to compute the variability of y_{it}^j around y_{it}^* for each t: $(1/J)\sum_{j=1}^{J}(y_{it}^j - y_{it}^*)^2$, where J is the total number of trials.[95] The problem with this measure, however, is that there are 20 values per variable, which makes summary difficult. A more useful measure is the following. Let L_i^j be:

$$L_i^j = \frac{1}{T}\sum_{i=1}^{T}(y_{it}^j - y_{it}^*)^2 \tag{1}$$

where T is the length of the simulation period ($T = 20$ in the present case). Then the measure is

$$L_i = \sqrt{\frac{1}{J}\sum_{j=1}^{J} L_i^j} \tag{2}$$

[94]Each of the four rules has a different set of estimated residuals associated with it because the predicted values from the rules differ due to the different inflation coefficients. This is why the constant term does not have to be changed in the rule when the inflation coefficient is changed. The estimated residuals are changed instead.

[95]If y_{it}^* were the estimated mean of y_{it}, this measure would be the estimated variance of y_{it}. Given the J values of y_{it}^j, the estimated mean of y_{it} is $(1/J)\sum_{j=1}^{J} y_{it}^j$, and for a nonlinear model it is not the case that this mean equals y_{it}^* even as J goes to infinity. As an empirical matter, however, the difference in these two values is quite small for almost all macroeconometric models, and so it is approximately the case that the above measure of variability is the estimated variance.

L_i is a measure of the deviation of variable i from its base values over the whole period.[96]

The Results

The results for this section are presented in the first five rows in Table 2. The first row ("No rule") treats RS as exogenous. This means that the value of RS in a given quarter is the historic value for all the trials: RS does not respond to the shocks. Values of L_i are presented for real output, Y, the level of the private nonfarm price deflator, PF, the percentage change in PF, $\dot{PF}$, and RS. The following discussion will focus on Y, PF, and RS. The results for $\dot{PF}$ are generally similar to those for PF, although the differences in L_i across rules are larger for PF than for $\dot{PF}$. All the experiments for the MC model use the same error draws, i.e., the same sequence of random numbers. This considerably lessens stochastic simulation error across experiments.

The results in Table 2 are easy to summarize. Consider row 1 versus row 3 first. L_i for Y falls from 2.61 for the no rule case to 2.28 for the estimated rule, and L_i for PF falls from 2.13 to 1.80. Both output and price variability are thus lowered considerably by the estimated rule. Now consider rows 2 through 5. As the long run inflation coefficient increases from 0.0 to 2.5, the variability of PF falls, the variability of RS rises, and the variability of Y is little affected. The cost of lowering PF variability is thus an increase in RS variability, not an increase in Y variability. Which rule one thinks is best depends on the weights one attaches to PF and RS variability,

How do these results compare to those in the literature? Probably the largest difference concerns row 2, where the variability in row 2 is less than the variability in row 1. This shows that even the rule with a long run inflation coefficient of zero lowers variability. In the class of models discussed in Section 4.1 the rule in row 2 would be destabilizing. Clarida, Galí, and Gertler (2000) have a clear discussion of this. They conclude that the rule used by the Fed in the pre-1979 period probably had an inflation coefficient less than one (p. 177), and they leave as an open question why the Fed followed a rule that was "clearly inferior" (p. 178) during this period. The results in Table 2 suggest that such a rule is not necessarily bad.

[96] L_i is, of course, not the square root of an estimated variance. Aside from the fact that for a nonlinear model the mean of y_{it} is not y^*_{it}, L_i^j is an average across a number of quarters or years, and variances are not in general constant across time. L_i is just a summary measure of variability.

Table 2
Variability Estimates: Values of L_i

		Y	PF	$\dot{PF}$	RS
MC Model					
1	No rule (RS exogenous)	2.61	2.13	1.64	0.00
2	Modified rule (0.0)	2.29	1.93	1.58	0.97
3	Estimated rule(0.980)—eq. 30	2.28	1.80	1.56	1.14
4	Modified rule (1.5)	2.28	1.71	1.54	1.30
5	Modified rule (2.5)	2.29	1.60	1.52	1.55
6	3 with tax rule	2.09	1.75	1.54	1.06
US(EX,PIM) Model					
7	No rule (RS exogenous)	2.20	2.42	1.83	0.00
8	Estimated rule—eq. 30	1.89	2.11	1.75	1.23
9	Optimal ($\lambda_1 = 0.5, \lambda_2 = 0.5$)	1.89	2.30	1.80	1.04
10	Optimal ($\lambda_1 = 0.5, \lambda_2 = 5.0$)	1.86	2.15	1.76	1.21
11	Optimal ($\lambda_1 = 0.5, \lambda_2 = 10.0$)	1.87	2.07	1.73	1.41

• Simulation period = 1994:1–1998:4.
• Number of trials for MC model = 200.
• Number of trials for US(EX,PIM) model = 100.
• Modified rule (0.0) = estimated rule with long run inflation coefficient = 0.0.
• Modified rule (1.5) = estimated rule with long run inflation coefficient = 1.5.
• Modified rule (2.5) = estimated rule with long run inflation coefficient = 2.5.
• Y = real output, PF = price deflator, $\dot{PF}$ = percentage change in PF, RS = three-month Treasury bill rate.

Results regarding the tradeoff between output variability and price variability as coefficients in a rule change appear to be quite dependent on the model used. This is evident in Tables 2 and 3 in Taylor (1999b), and McCallum and Nelson (1999, p. 43) point out that increasing the inflation or output coefficient in their rule leads to a tradeoff in one of their models but a reduction in *both* output and price variability in another. In Table 2 the tradeoff is between price variability and interest rate variability as the inflation coefficient is increased. There is little tradeoff between output and price variability. Because the tradeoffs are so model specific, one must have confidence in the model used to have confidence in the tradeoff results. The results in Table 2 convey useful information if the MC model is a good approximation of the economy.

4.4.4 Optimal Control

The US(EX,PIM) Model

The optimal control procedure discussed in this section is too costly in terms of computer time to be able to be used for the entire MC model, and for the work in this section a slightly expanded version of the US model has been used, denoted the "US(EX,PIM) model." The expansion relates to U.S. exports, EX, and the U.S. price of imports, PIM. These two variables change when RS changes—primarily because the value of the dollar changes—and the effects of RS on EX and PIM were approximated in the following way.

First, for given values of α_1 and α_2 $\log EX_t - \alpha_1 RS_t$ was regressed on the constant term, t, $\log EX_{t-1}$, $\log EX_{t-2}$, $\log EX_{t-3}$, and $\log EX_{t-4}$, and $\log PIM_t - \alpha_2 RS_t$ was regressed on the constant term, t, $\log PIM_{t-1}$, $\log PIM_{t-2}$, $\log PIM_{t-3}$, and $\log PIM_{t-4}$. The estimation period was 1976:1–1998:4. Second, these two equations were added to the US model, and an experiment was run in which equation 30 was dropped and RS was decreased by one percentage point. This was done for different values of α_1 and α_2. The final values of α_1 and α_2 chosen were ones whose experimental results most closely matched the results for the same experiment using the complete MC model. The final values chosen were -.0002 and -.0004, respectively. Third, the experiment in the third row of Table 2 was run for the US model with the EX and PIM equations added and with the estimated residuals from these equations being used for the drawing of the errors. (In this case equation 30 is included in the model.) When an error for the EX equation was drawn, it was multiplied by β_1, and when an error for the PIM equation was drawn, it was multiplied by β_2. The experiment was run for different values of β_1 and β_2, and the final values chosen were ones that led to results similar to those in the third row of Table 2. The values were $\beta_1 = .5$ and $\beta_2 = .5$. The results using these values are in row 8 of Table 2. The chosen values of α_1, α_2, β_1, and β_2 were then used for the experiments in rows 9–11.

The US(EX,PIM) model is thus a version of the US model in which EX and PIM have been made endogenous with respect to their reactions to changes in RS. It is an attempt to approximate the overall MC model in this regard.

The Procedure

Much of the literature on examining rules has not been concerned with deriving rules by solving optimal control problems,[97] but optimal control techniques are obvious ones to use in this context. The following procedure has been applied to the US(EX,PIM) model. A more general discussion of optimal control procedures is in Section 2.10.

The estimated residuals for the 1956:1–2007:4 period (208 quarters) are used for the draws. Each vector of quarterly residuals has a probability of 1/208 of being drawn. Not counting the estimated interest rate rule, there are 24 estimated equations in the US(EX,PIM) model plus the EX and PIM equations discussed above.

The optimal control methodology requires that a loss function be postulated for the Fed. In the loss function used here the Fed is assumed to care about output, inflation, and interest rate fluctuations. In particular, the loss for quarter t is assumed to be:

$$H_t = \lambda_1 100[(Y_t - Y_t^*)/Y_t^*]^2 + \lambda_2 100(\dot{PF}_t - \dot{PF}_t^*)^2 + \alpha(\Delta RS_t - \Delta RS_t^*)^2 \\ +1.0/(RS_t - 0.999) + 1.0/(16.001 - RS_t) \tag{3}$$

where * denotes a base value. λ_1 is the weight on output deviations, and λ_2 is the weight on inflation deviations. The last two terms in the loss function (3) insure that the optimal values of RS will be between 1.0 and 16.0. The value of α was chosen by experimentation to yield an optimal solution with a value of L_i for RS in Table 2 about the same as the value that results when the estimated rule is used. The value chosen was 9.0. The base values in equation (3) are the actual (historic) values. The base path for each variable is the actual path (since the estimated residuals have been added to the equations), and so the losses in equation (3) are deviations from the actual values.

Assume that the control period of interest is 1 through T, where in the present case 1 is 1994:1 and T is 1998:4. Although this is the control period of interest, in order not to have to assume that life ends in T, the control problem should be thought of as one of minimizing the expected value of $\sum_{t=1}^{T+n} H_t$, where n is chosen to be large enough to avoid unusual end-of-horizon effects near T. The overall control problem should thus be thought of as choosing values of RS that minimize the expected value of $\sum_{t=1}^{T+n} H_t$ subject to the model used.

If the model used is linear and the loss function quadratic, it is possible to de-

[97]Exceptions are Feldstein and Stock (1993), Fair and Howrey (1996), and Rudebusch (1999).

rive analytically optimal feedback equations for the control variables.[98] In general, however, optimal feedback equations cannot be derived for nonlinear models or for loss functions with nonlinear constraints on the instruments, and a numerical procedure like the one outlined in Section 1.7 must be used. The following procedure was used for the results in this section. It is based on a sequence of solutions of deterministic control problems, one sequence per trial, where certainty equivalence (CE) is used.

Recall what a trial for the stochastic simulation is. A trial is a set of draws of 20 vectors of error terms, one vector per quarter. Given this set, the model is solved dynamically for the 20 quarters using an interest rate rule (or no rule). This entire procedure is then repeated 100 times (the chosen number of trials), at which time the summary statistics are computed. As will now be discussed, each trial for the optimal control procedure requires that 20 deterministic control problems be solved, and so with 100 trials, 2,000 optimal control problems have to be solved.

For purposes of solving the control problems, the Fed is assumed to know the model (its structure and coefficient estimates) and the exogenous variables, both past and future. The Fed is assumed *not* to know the future values of any endogenous variable. It is also assumed not to know any current or future error draws when solving the control problems.[99]

The procedure for solving the overall control problem is as follows.

1. Take the errors for quarters 1 through k to be zero (i.e., no draws, but remember that the estimated residuals are always added), where k is defined shortly. Compute the values of RS for quarters 1 through k that minimize $\sum_{t=1}^{k} H_t$ subject to the model as just described. This is just a deterministic optimal control problem, which can be solved, for example, by the procedure discussed in Section 2.10.[100] Let RS_1^{**} denote the optimal value of RS for quarter 1 that results from this solution. The value of k should be chosen to be large enough so that making it larger has a negligible effect on RS_1^{**}. (This value can be chosen ahead of time by experimentation.) RS_1^{**} is computed at the beginning of quarter 1 under the assumptions that 1) the

[98] See, for example, Chow (1981).

[99] The main exogenous variables in the US(EX,PIM) model are fiscal policy variables. Remember that since the base is the perfect tracking solution, the estimated residuals are always added to the stochastic equations and treated as exogenous. The error draws are on top of these residuals.

[100] Almost all the computer time for the overall procedure in this section is spent solving these optimization problems. The total computer time taken to solve the 2,000 optimization problems was about 25 minutes on a laptop computer with a i7 Intel chip.

model is known, 2) the exogenous variable values are known, and 3) the error draws for quarter 1 are not known.

2. Draw a vector of errors for quarter 1, and add these errors to the equations. Record the solution values from the model for quarter 1 using RS_1^{**} and the error draws. These solution values are what the model estimates would have occurred in quarter 1 had the Fed chosen RS_1^{**} and had the error terms been as drawn.

3. Repeat steps 1 and 2 for the control problem beginning in quarter 2, then for the control problem beginning in quarter 3, and so on through the control problem beginning in quarter T. For an arbitrary beginning quarter s, use the solution values of all endogenous variables for quarters $s-1$ and back, as well as the values of RS_{s-1}^{**} and back.

4. Steps 1 through 3 constitute one trial, i.e., one set of T drawn vectors of errors. Do these steps again for another set of T drawn vectors. Keep doing this until the specified number of trials has been completed.

The solution values of the endogenous variables carried along for a given trial from quarter to quarter in the above procedure are estimates of what the economy would have been like had the Fed chosen RS_1^{**},...,RS_T^{**} and the error terms been as drawn.

By "optimal rule" in this section is meant the entire procedure just discussed. There is obviously no analytic rule computed, just a numerical value of RS^{**} for each period.

The Results

The results are presented in rows 7–11 in Table 2. The experiments in these rows use the same error draws, i.e., the same sequence of random numbers, to lessen stochastic simulation error across experiments, although these error draws are different from those used for the experiments in rows 1–6. Rows 7 and 8 are equivalent to rows 1 and 3: no rule and estimated rule, respectively. The same pattern holds for both the MC model and the US model results, namely that the estimated rule substantially lowers the variability of both Y and PF.

Row 9 presents the results for the optimal solution with equal weights (i.e., $\lambda_1 = 0.5$ and $\lambda_2 = 0.5$) on output and inflation in the loss function. Comparing rows 7 and 9, the optimal control procedure lowered the variability of Y more than it did the variability of PF. For rows 10 and 11 the weight on inflation in the loss function is increased. This lowers the variability of PF, increases the variability of RS, and has little effect on the variability of Y. Row 11, which has a weight of 10.0 on inflation, gives similar results to those in row 8, which uses the estimated rule. In this sense the estimated rule is consistent with the Fed minimizing the loss function with weights $\lambda_1 = 0.5$ and $\lambda_2 = 10.0$ in equation (3).

Again, how do these results compare to those in the literature? A common result in the Taylor (1999a) volume is that simple rules perform nearly as well as optimal rules or more complicated rules. See Taylor (1999b, p. 10), Rotemberg and Woodford (1999, p. 109), Rudebusch and Svensson (1999, p. 238), and Levin, Wieland, and Williams (1999, p. 294). The results in rows 8 and 11 are consistent with this theme, where the estimated rule and the optimal control procedure perform about the same. The optimal control procedure in this case is one in which the Fed puts a considerably higher weight on inflation than on output in the loss function.

4.4.5 Adding a Tax Rate Rule

Turning back to the MC model, it is clear in Table 2 that considerable overall variability is left in rows 2–5. In this subsection a tax rate rule is analyzed to see how much help it can be to monetary policy in stabilizing the economy. The idea is that a particular tax rate or set of rates would be automatically adjusted each quarter as a function of the state of the economy. Congress would vote on the parameters of the tax rate rule as it was voting on the general budget plan, and the tax rate or set of rates would then become an added automatic stabilizer.

Consider, for example, the federal gasoline tax rate. If the short run demand for gasoline is fairly price inelastic, a change in the after-tax price at the pump will have only a small effect on the number of gallons purchased. In this case a change in the gasoline tax rate is like a change in after-tax income. Another possibility would be a national sales tax if such a tax existed. If the sales tax were broad enough, a change in the sales tax rate would also be like a change in after-tax income.

For the results in this section $D3G$ is used as the tax rate for the tax rate rule. It is the constructed federal indirect business tax rate in the US model—see Tables A.2 and A.7. In practice a specific tax rate or rates, such as the gasoline tax rate, would have to be used, and this would be decided by the political process. In the regular version of the US model $D3G$ is exogenous.

The following equation is used for the tax rate rule:

$$D3G_t = D3G_t^* + 0.125[.5((Y_{t-1} - Y_{t-1}^*)/Y_{t-1}^*) + .5((Y_{t-2} - Y_{t-2}^*)/Y_{t-2}^*)] \\ +0.125 * [.5(\dot{PF}_{t-1} - \dot{PF}_{t-1}^*) + .5(\dot{PF}_{t-2} - \dot{PF}_{t-2}^*)] \quad (2)$$

where, as before, * denotes a base value. It is not realistic to have tax rates respond contemporaneously to the economy, and so lags have been used in equation (2). Lags of both one and two quarters have been used to smooth tax rate changes somewhat. The rule says that the tax rate exceeds its base value as output and the inflation rate exceed their base values. Note that unlike the basic interest rate rule, equation 30, the rule (2) has not been estimated. It would not make sense to try to estimate such a rule since it is clear that the government has never followed a tax rule policy.

Results using this rule along with the estimated interest rate rule are reported in row 6 in Table 2. The use of the rule lowers L_i for Y from 2.28 when only the estimated interest rate rule is used to 2.09 when both rules are used. The respective numbers for PF are 1.80 and 1.75. The tax rate rule is thus of some help in lowering output and price variability, with a little more effect on output variability than on price variability. The variability of RS falls slightly when the tax rate rule is added, since there is less for monetary policy to do when fiscal policy is helping.

4.4.6 Conclusion

The main conclusions about monetary policy from the results in Table 2 are the following:

1. The estimated rule explaining Fed behavior, equation 30, substantially reduces output and price variability (row 3 versus row 1).

2. Variability is reduced even when the long run coefficient on inflation in the interest rate rule is set to zero (row 2 versus row 1). This is contrary to what would be the case for the class of models discussed in Section 4.1, where such a rule would be destabilizing.

3. Increasing the long run coefficient on inflation in the interest rate rule lowers price variability, but it comes at a cost of increased interest rate variability (for example, row 5 versus row 3).

4. A tax rate rule is a help to monetary policy in its stabilization effort (row 6 versus row 3).

5. The optimal control procedure with $\lambda_1 = 0.5$ and $\lambda_2 = 10.0$, which means a higher weight on inflation than on output in the loss function, gives results that are similar to the use of the estimated rule (row 11 versus row 8). The fact that the estimated rule does about as well as the optimal control procedure is consistent with many results in the literature, where simple rules tend to do fairly well.

6. Even when both the estimated interest rate rule and the tax rate rule are used, the values of L_i in Table 2 are not close to zero (row 6). Monetary policy even with the help of a fiscal policy rule does not come close to eliminating the effects of typical historical shocks.

4.5 Estimated Macroeconomic Effects of a Chinese Yuan Appreciation[101]

4.5.1 Introduction

Many argued in 2009–2010 that the U.S. economy was being hurt by the Chinese policy of essentially pegging the yuan to the dollar. For example, Krugman (2010) stated that his "back-of-the-envelope" calculations suggest that if there were no appreciation of the yuan then over the next couple of years what he calls "Chinese mercantilism" "may end up reducing U.S. employment by around 1.4 million jobs." He noted that the standard arguments against protectionism do not hold in a world of less than full employment.

The question of what a Chinese appreciation of the yuan would do to the world economy is complicated. There are many economic links among countries, and these links need to be accounted for in analyzing the effects of exchange rate changes. This section uses the MC model to estimate the effects of a yuan appreciation. It will be seen that when all links are taken into account, the effects on U.S. output and employment are modest. Krugman's job loss estimate does not appear accurate.

The main message from analyzing the model's results regarding the overall effect on U.S. output from a yuan appreciation is that there are two negative effects that turn out to be quantitatively important and roughly offset the positive effects. The first negative effect is that the yuan appreciation leads to a decrease in Chinese output, which has a negative effect on Chinese imports, some of which are from the United States. The second negative effect is that the rise in U.S. import prices (from the rise in Chinese export prices) leads to an increase in U.S. domestic prices. The increase in U.S. domestic prices results in a decrease in real wealth and real wages and an increase in the short term interest rate, all of which have, other things being equal, a negative effect on U.S. aggregate demand and output. (This second effect is discussed in Section 4.1, where it is seen that the effect of a price shock, like an increase in the price of imports, on output is negative even if there is no increase in the short term interest rate.) It will be seen that because of these two negative effects the net effect of the yuan appreciation on U.S. output and employment is close to zero—in fact slightly negative.

[101]The results in this section are updates of those in Fair (2010a).

4.5.2 Equations for China and Robustness Checks

The structural equations for China are estimated using annual data, for the period 1984–2010. Because the data are not as good and the estimation period is smaller, less confidence can be placed on the Chinese estimated equations than on the U.S. estimated equations. Because of this, some robustness checks are reported in Section 4.5.4 using alternative specifications for the Chinese model.

The first check concerns the estimated import equation for China. In this equation the price of imports relative to the domestic price level is not a significant variable, and its coefficient estimate has the wrong sign. This is contrary to the import equation for the United States and for many other countries. In the regular version of the MC model the relative price of imports variable is excluded from the Chinese import equation, which means that an increase in the relative price of imports in China does not affect Chinese imports. This is what the data say, but this, of course, could be wrong. For the first robustness check the relative price of imports variable was added to the equation, and its coefficient was constrained to be similar to coefficient estimates for other countries. The equation was reestimated with this constraint imposed. The first robustness check is to rerun the experiment using this constrained equation.

The second check concerns the response of Chinese export prices to the appreciation. Direct data on a price index of exports for China are not available, and a series was constructed using U.S. export prices and the yuan/dollar exchange rate. Because of this, in the price of exports equation for China the weight on the domestic price level was not estimated. It was simply imposed to be 0.5, which is in line with estimated weights for other countries. For the second robustness check, the weight was change to 0.8.

The third check concerns the effect of the price of imports on the domestic price level. The price of imports is an explanatory variable in the domestic price equation, and it will be seen that the estimated effect is large. The Chinese appreciation leads to a fairly large fall in the Chinese domestic price level. For the third check this effect was turned off by simply dropping the Chinese domestic price equation and taking the domestic price level to be exogenous.

The fourth check concerns the effect of a change in the domestic price level on real output. For the United States, as discussed above, an increase in the domestic price level is contractionary, other things being equal, because of the fall in real wealth and real wages. Similarly, a decrease in the domestic price level is expansionary, other things being equal. This effect is not in the Chinese model because there are no data on wealth and wages in the model. If China is in fact like the United States in this respect, the fall in Chinese output from the appreciation is overestimated in the basic experiment because the expansionary effects from the fall in the Chinese domestic price level are not taken into account. In the basic experiment Chinese output simply falls because of the decrease in exports. For the fourth check it was assumed that Chinese government spending, which is exogenous in the basic case, is changed enough to completely offset the fall in output. In other words, it is assumed that the appreciation has no effect on Chinese output.

4.5.3 The Basic Experiment and Results

The simulation period was taken to be 1999:1–2008:4. There are a total of 1,689 estimated equations in the model counting the trade share equations, and the first step was to add the estimated residuals to these equations and take them as exogenous. This means that when the model is solved, a perfect tracking solution is obtained. The second step was to decrease the yuan/dollar exchange rate by 25 percent from its actual value for each quarter. For example, the actual yuan/dollar exchange rate in 1999:1 was 8.2787, and the new value was taken to be 0.75 times this, or 6.6090. This was done for each of the 40 quarters.

The model was then solved with this change imposed. No other changes were made. For example, all the estimated exchange rate equations were left in. To the extent that the predicted values from these equations are not affected much, the exchange rates relative to the dollar do not change much, which means there is also an appreciation of the yuan relative to other currencies. For exchange rates that are exogenous, there is an exact 25 percent appreciation of the yuan relative to these currencies since the exchange rates are relative to the dollar.

Because of the many links among countries, the results are not easy to explain. The following is a step by step discussion, but the actual story is in fact more complicated because of the simultaneity. The results referred to below are presented in Table 1. The variables are defined at the bottom of the table and are defined in the text in the order they are listed in Table 1. When a variable is said to increase or decrease, this always refers to the new solution value relative to the base value. Results are presented in Table 1 for the fourth quarter of each year. When the variable is only annual, the results are for the year.

The appreciation of the yuan leads to a decrease in Chinese import prices (PM_{ch}), which through the domestic price equation leads to a decrease in Chinese domestic prices (PY_{ch}). After four years domestic prices are down 14.55 percent, which is a large change. The decrease in domestic prices and the decrease in the world price of exports in yuan (because of the appreciation) leads through the export price equation to a decrease in Chinese export prices in yuan (PX_{ch}). After four years export prices are down 19.77 percent, which is also a large change. The dollar price of Chinese exports ($PX\$_{ch}$) increases, but by less than it would have had Chinese export prices in yuan not fallen. The initial increase is 15.08 percent, and after four years the increase is down to 6.97 percent.

The higher dollar price of Chinese exports relative to the dollar price of other countries' exports leads through the trade share equations to a decrease in the demand for Chinese exports. For example, exports to the United States ($X_{ch,us}$) are down 14.52 percent initially and 29.02 percent after four years. Total Chinese exports (EX_{ch}) are down 5.34 percent initially and 12.92 percent after four years. The fall in exports has a negative effect on Chinese GDP (Y_{ch}), which in turn has a negative effect on total Chinese imports (IM_{ch}).

Turning to the United States, the import price deflator (PM_{us}) is higher because of the higher price of Chinese imports. This leads to an increase in U.S. domestic prices (PY_{us}) through the domestic price equation. This in turn leads to an increase in the price of U.S. exports (PX_{us}) through the export price equation. The increase in the U.S. price level leads to a decrease in real wealth (AA_{us}) and a decrease in real disposable income (YD_{us}). There is a slight increase in the short term interest rate (RS_{us}). According to the U.S. estimated interest rate rule, RS_{us} responds positively to an increase in inflation and negatively to a fall in output. The fall in output is small (discussed below), and the inflation effect dominates in that the short term interest rate is up slightly.

There are both positive and negative effects on U.S. GDP. Total U.S. imports (IM_{us}) are down, in large part because of the fall in imports from China, which is a positive effect. U.S. exports to China ($X_{us,ch}$) are down because of the decreased demand from China due to the contraction of the Chinese economy. Total U.S. exports (EX_{us}) are, however, up, which is a positive effect on U.S. output. U.S. consumption (C_{us}) is down because of the fall in real wealth and real income, which is a negative effect on U.S. output. The increase in the short term interest rate also has a negative effect on U.S. output, although this effect is small because the change in the interest rate is small.

The net effect on U.S. output is very small, negative at first and then positive. The decrease is 0.03 percent after one year and 0.00 percent after four years. The net effect on U.S. jobs is correspondingly small: a decrease of 0.01 percent (18,000 jobs) after one year and 0.01 percent (14,600 jobs) after four years.

To summarize, the main expansionary effect on U.S. output from the appreciation of the yuan is the fall in U.S. imports from China. The main contractionary effect is through higher U.S. prices and the fall in exports to China. The net effect on U.S. output could go either way, and it is in fact slightly negative. The net effect is, however, very small, and as a rough approximation one might say that the Chinese appreciation is a wash relative to U.S. output and employment.

The present results are certainly at odds with Krugman's estimate of 1.4 million fewer jobs if the yuan does not appreciate. (This may show the danger of back-of-the-envelope calculations when it comes to exchange rate effects!) They suggest that even if the United States convinced China to appreciate the yuan, there would be little effect on U.S. output and employment.

4.5.4 Robustness Checks

As discussed at the end of Section 2, four robustness checks were made. For the first the relative import price variable was added to the Chinese import equation.[102] No other changes were made. The results are presented in Table 2. In this case Chinese imports, IM_{ch}, are initially higher as the substitution toward imports dominates the negative income effect. Chinese output falls more than it does in Table 1 because of the substitution into imports. U.S. exports to China, $X_{us,ch}$, are now initially higher rather than lower, as they are in Table 1. The price effect on the United States is slightly smaller in Table 2 than in Table 1. This is because the lower Chinese output in Table 2 versus Table 1 leads to a larger fall in the Chinese price level and thus a smaller increase in the Chinese export price in dollars. The net difference on U.S. output and jobs is modest, comparing Tables 1 and 2. U.S. output and employment are down slightly less in Table 2, but the main conclusions from Table 1 are not changed.

For the second check the weight on the domestic price level in the Chinese export price equation was changed from 0.5 to 0.8, and the equation was reestimated with this constraint imposed. No other changes were made from the Table 1 experiment. This The results are presented in Table 3. In this case the price of exports in yuan falls less and so the price of exports in dollars rises more. The initial increase in $PX\$_{ch}$ is now 24.54 percent compared to 15.08 percent in Table 1. This results in Chinese exports, output, and imports all falling more. Also, U.S. import prices rise more due to the larger increase in Chinese export prices, which leads to U.S. domestic prices rising more. U.S. imports from China are down more because of the higher Chinese export price. U.S. output and employment are down slightly more in this case, but again the output and employment effects are modest.

[102] The coefficient on the relative import price variable was constrained to be 0.2, and the equation was reestimated with this constraint imposed.

For the third check, reported in Table 4, the Chinese domestic price equation is dropped. No other changes were made from the Table 1 experiment. This leads to a smaller decrease in the Chinese export prices in yuan because, unlike in Table 1, there is no effect from a fall in the domestic price level on export prices. The increase in Chinese export prices in dollars is thus larger. Tables 3 and 4 are thus similar relative to Table 1 in that Chinese export prices in dollars are higher. The increase is larger in Table 4 from the fourth year on. The story for Table 4 is thus similar to that for Table 3, only the differences between Tables 4 and 1 are larger than those between Tables 3 and 1. U.S. output falls by 0.02 percent after four years, and employment falls by 40,900 jobs. These effects are still quite small.

For the fourth check, reported in Table 5, the output effect on China was turned off by having government spending offset any contractionary effects. No other changes were made from the Table 1 experiment. In this case Chinese domestic prices do not fall as much as in Table 1 because there is no negative demand effect from lower output. This leads to a smaller fall in Chinese export prices in yuan and so a larger rise in export prices in dollars. The price effect on the United States is thus somewhat larger. Chinese imports do not fall, and so U.S. exports are larger in Table 5 versus Table 1. The positive effect from higher U.S. exports is roughly offset by the negative effect from higher U.S. prices, and the effects on U.S. output and employment are similar in Table 5 versus Table 1. The estimated effects thus continue to be small.

The results are thus all similar in showing small effects on U.S. output and employment. Remember that the results in Table 1 are the ones most supported by the data, although the Chinese model is based on a short sample period. Fortunately, the results are not sensitive to various changes in the Chinese model. One other check that is interesting to make is to combine the changes made for Tables 2 and 5—relative import price variable in the Chinese import equation and no change in Chinese output. These results are presented in Table 6. In this case the effects on U.S. output and employment are still small, although it is now the case U.S. output and employment are somwwhat higher at the end of the period. Comparing Tables 1 and 6, one might ask why, given that U.S. exports to China, $X_{us,ch}$, are so much larger in Table 6 than in Table 1, the U.S. output differences are so small? The main reason is the negative price effect on U.S. output. It is larger in Table 6 because the more expansive Chinese economy has led to a smaller fall in the Chinese price level and thus a larger rise in the Chinese price of exports in dollars. The price effect on U.S. output is clearly an important property of the MC model.

Table 7 is the same as Table 6 except that the U.S. interest rate rule, equation 30, is dropped and RS is taken to be exogenous. The results between the two tables are similar because the increases in RS in Table 6 are quite small. U.S. output is somewhat higher in Table 7 compared to Table 6 because there are no negative effects from a higher interest rate.

Table 1
Chinese Appreciation of 25 Percent
Regular Version of MC Model
Percentage Deviations from Base in Percentage Points

qtr	PM_{ch}	PY_{ch}	PX_{ch}	$PX\$_{ch}$	$X_{ch,us}$	EX_{ch}	Y_{ch}	IM_{ch}
1999.4	-24.88	-0.92	-13.69	15.08	-14.52	-5.34	-1.79	-0.40
2000.4	-24.88	-7.69	-16.69	11.08	-22.44	-9.52	-4.03	-1.34
2001.4	-24.85	-11.80	-18.54	8.61	-27.00	-11.81	-5.33	-2.53
2002.4	-24.80	-14.55	-19.77	6.97	-29.02	-12.92	-6.61	-3.87
2003.4	-24.75	-16.55	-20.65	5.79	-29.19	-13.09	-7.79	-5.31
2004.4	-24.70	-17.91	-21.25	5.00	-28.54	-12.99	-8.46	-6.65
2005.4	-24.67	-18.69	-21.57	4.57	-27.47	-12.30	-8.59	-7.66
2006.4	-24.64	-19.16	-21.75	4.34	-26.21	-12.01	-8.69	-8.42
2007.4	-24.60	-19.20	-21.72	4.37	-25.12	-11.88	-8.24	-8.84
2008.4	-24.59	-19.19	-21.70	4.40	-23.97	-12.30	-8.19	-9.10

qtr	PM_{us}	PY_{us}	PX_{us}	AA_{us}	YD_{us}	RS_{us}	IM_{us}	$X_{us,ch}$	EX_{us}	C_{us}
1999.4	0.84	0.12	0.23	-0.17	-0.12	0.04	-0.27	-0.70	0.13	-0.09
2000.4	0.56	0.16	0.23	-0.16	-0.09	0.03	-0.35	-2.29	0.13	-0.11
2001.4	0.49	0.18	0.25	-0.14	-0.08	0.02	-0.34	-3.96	0.19	-0.10
2002.4	0.56	0.21	0.29	-0.16	-0.08	0.03	-0.32	-5.70	0.27	-0.10
2003.4	0.78	0.26	0.37	-0.19	-0.11	0.04	-0.36	-7.94	0.27	-0.11
2004.4	0.96	0.33	0.45	-0.22	-0.13	0.05	-0.43	-10.27	0.45	-0.14
2005.4	0.96	0.40	0.52	-0.24	-0.13	0.05	-0.49	-12.01	0.40	-0.16
2006.4	1.16	0.47	0.61	-0.27	-0.15	0.07	-0.54	-12.69	0.55	-0.18
2007.4	1.30	0.56	0.71	-0.32	-0.16	0.08	-0.59	-12.63	0.57	-0.20
2008.4	1.27	0.63	0.77	-0.37	-0.15	0.08	-0.63	-12.38	0.80	-0.22

qtr	Y_{us}	J_{us}	J^a_{us}
1999.4	-0.03	-0.01	-18.0
2000.4	-0.03	-0.03	-36.3
2001.4	-0.02	-0.03	-33.2
2002.4	0.00	-0.01	-14.6
2003.4	0.00	-0.01	-8.9
2004.4	0.01	0.00	-5.7
2005.4	0.00	-0.01	-7.3
2006.4	0.02	0.01	10.2
2007.4	0.03	0.02	31.7
2008.4	0.04	0.03	42.5

[a]units in thousands of jobs
Simulation period 1999:1–2008:4.
PM = import price level, PY = domestic price level, PX = export price level,
$PX\$$ = export price level in dollars, $X_{i,j}$ = exports from i to j,
EX = total exports, Y = real output, IM = total imports,
AA = real wealth, YD = real disposable income, RS = short term interest rate,
C = consumption, J = employment.

Table 2
Chinese Appreciation of 25 Percent
Relative Import Price added to Chinese Import Equation
Percentage Deviations from Base in Percentage Points

qtr	PM_{ch}	PY_{ch}	PX_{ch}	$PX\$_{ch}$	$X_{ch,us}$	EX_{ch}	Y_{ch}	IM_{ch}
1999.4	-24.88	-1.73	-14.03	14.62	-14.10	-5.18	-3.35	4.72
2000.4	-24.86	-9.37	-17.42	10.10	-21.26	-9.07	-6.60	4.60
2001.4	-24.83	-13.90	-19.48	7.35	-24.96	-10.96	-7.90	2.54
2002.4	-24.79	-16.73	-20.78	5.63	-26.23	-11.67	-8.92	-0.09
2003.4	-24.74	-18.60	-21.63	4.50	-25.86	-11.55	-9.73	-2.70
2004.4	-24.70	-19.65	-22.08	3.89	-24.91	-11.26	-9.85	-4.81
2005.4	-24.69	-20.02	-22.23	3.70	-23.75	-10.55	-9.44	-6.14
2006.4	-24.66	-20.10	-22.24	3.68	-22.59	-10.29	-9.20	-6.92
2007.4	-24.64	-19.87	-22.10	3.87	-21.69	-10.22	-8.59	-7.17
2008.4	-24.64	-19.72	-22.02	3.98	-20.79	-10.68	-8.52	-7.23

qtr	PM_{us}	PY_{us}	PX_{us}	AA_{us}	YD_{us}	RS_{us}	IM_{us}	$X_{us,ch}$	EX_{us}	C_{us}
1999.4	0.84	0.12	0.23	-0.17	-0.11	0.05	-0.26	7.28	0.21	-0.08
2000.4	0.56	0.16	0.25	-0.16	-0.08	0.04	-0.33	7.24	0.27	-0.10
2001.4	0.48	0.19	0.26	-0.15	-0.07	0.03	-0.31	3.69	0.34	-0.10
2002.4	0.54	0.22	0.30	-0.16	-0.07	0.04	-0.30	-0.24	0.38	-0.09
2003.4	0.72	0.26	0.36	-0.18	-0.09	0.04	-0.33	-4.07	0.37	-0.11
2004.4	0.87	0.32	0.43	-0.20	-0.11	0.05	-0.39	-7.45	0.48	-0.13
2005.4	0.87	0.38	0.49	-0.22	-0.12	0.05	-0.44	-9.64	0.42	-0.15
2006.4	1.04	0.44	0.56	-0.24	-0.13	0.06	-0.48	-10.45	0.54	-0.16
2007.4	1.16	0.52	0.65	-0.28	-0.14	0.07	-0.52	-10.29	0.56	-0.18
2008.4	1.14	0.58	0.70	-0.33	-0.13	0.08	-0.55	-9.90	0.77	-0.20

qtr	Y_{us}	J_{us}	J^a_{us}
1999.4	-0.01	0.00	-4.6
2000.4	-0.01	-0.01	-13.4
2001.4	0.00	-0.01	-10.9
2002.4	0.01	0.00	1.2
2003.4	0.00	0.00	2.8
2004.4	0.01	0.00	4.1
2005.4	0.00	0.00	1.8
2006.4	0.02	0.01	18.0
2007.4	0.03	0.03	38.4
2008.4	0.05	0.04	49.4

See notes to Table 1

Table 3
Chinese Appreciation of 25 Percent
PY_{ch} **Weight of 0.8 for** PX_{ch}
Percentage Deviations from Base in Percentage Points

qtr	PM_{ch}	PY_{ch}	PX_{ch}	$PX\$_{ch}$	$X_{ch,us}$	EX_{ch}	Y_{ch}	IM_{ch}
1999.4	-24.82	-1.42	-6.60	24.54	-22.50	-8.22	-2.76	-0.62
2000.4	-24.82	-8.89	-12.31	16.93	-33.16	-14.04	-5.97	-2.02
2001.4	-24.79	-13.51	-15.87	12.18	-38.41	-16.75	-7.63	-3.72
2002.4	-24.74	-16.64	-18.29	8.95	-40.05	-17.70	-9.17	-5.55
2003.4	-24.67	-18.89	-20.02	6.64	-39.25	-17.41	-10.47	-7.41
2004.4	-24.62	-20.31	-21.12	5.17	-37.43	-16.73	-11.01	-9.03
2005.4	-24.60	-20.99	-21.64	4.48	-35.20	-15.41	-10.84	-10.15
2006.4	-24.57	-21.26	-21.84	4.22	-32.89	-14.62	-10.63	-10.87
2007.4	-24.54	-21.05	-21.65	4.47	-31.00	-14.15	-9.84	-11.15
2008.4	-24.53	-20.84	-21.48	4.69	-29.24	-14.47	-9.65	-11.23

qtr	PM_{us}	PY_{us}	PX_{us}	AA_{us}	YD_{us}	RS_{us}	IM_{us}	$X_{us,ch}$	EX_{us}	C_{us}
1999.4	1.26	0.18	0.35	-0.26	-0.18	0.06	-0.41	-1.09	0.18	-0.13
2000.4	0.75	0.23	0.34	-0.22	-0.13	0.03	-0.50	-3.44	0.16	-0.16
2001.4	0.60	0.25	0.34	-0.19	-0.10	0.02	-0.45	-5.80	0.22	-0.13
2002.4	0.65	0.27	0.38	-0.19	-0.09	0.03	-0.40	-8.15	0.32	-0.12
2003.4	0.90	0.33	0.46	-0.23	-0.12	0.05	-0.41	-11.05	0.31	-0.13
2004.4	1.08	0.40	0.55	-0.25	-0.14	0.06	-0.47	-13.91	0.53	-0.16
2005.4	1.05	0.47	0.61	-0.26	-0.14	0.05	-0.53	-15.85	0.48	-0.18
2006.4	1.26	0.54	0.70	-0.29	-0.16	0.08	-0.57	-16.31	0.65	-0.19
2007.4	1.43	0.64	0.81	-0.35	-0.17	0.09	-0.63	-15.85	0.68	-0.22
2008.4	1.38	0.72	0.87	-0.40	-0.15	0.09	-0.67	-15.22	0.96	-0.24

qtr	Y_{us}	J_{us}	J^a_{us}
1999.4	-0.04	-0.02	-29.1
2000.4	-0.05	-0.04	-56.8
2001.4	-0.03	-0.04	-49.1
2002.4	0.00	-0.02	-20.2
2003.4	0.00	-0.01	-8.3
2004.4	0.01	0.00	-2.3
2005.4	0.00	0.00	-0.7
2006.4	0.03	0.02	22.0
2007.4	0.04	0.04	49.0
2008.4	0.06	0.05	61.8

See notes to Table 1

Table 4
Chinese Appreciation of 25 Percent
Chinese PY Equation Dropped
Percentage Deviations from Base in Percentage Points

qtr	PM_{ch}	PY_{ch}	PX_{ch}	$PX\$_{ch}$	$X_{ch,us}$	EX_{ch}	Y_{ch}	IM_{ch}
1999.4	-24.88	0.00	-13.29	15.61	-14.99	-5.51	-1.85	-0.41
2000.4	-24.84	0.00	-13.26	15.66	-26.48	-10.96	-4.60	-1.48
2001.4	-24.78	0.00	-13.20	15.73	-35.57	-15.27	-6.75	-3.00
2002.4	-24.69	0.00	-13.11	15.85	-42.05	-18.61	-9.25	-4.95
2003.4	-24.57	0.00	-12.99	16.02	-46.21	-20.85	-12.06	-7.35
2004.4	-24.44	0.00	-12.84	16.22	-49.14	-22.68	-14.47	-9.97
2005.4	-24.34	0.00	-12.68	16.42	-51.20	-23.40	-16.14	-12.41
2006.4	-24.20	0.00	-12.52	16.64	-52.68	-24.65	-17.71	-14.69
2007.4	-24.07	0.00	-12.33	16.89	-53.87	-26.10	-18.10	-16.50
2008.4	-23.96	0.00	-12.20	17.07	-54.50	-28.71	-19.08	-18.11

qtr	PM_{us}	PY_{us}	PX_{us}	AA_{us}	YD_{us}	RS_{us}	IM_{us}	$X_{us,ch}$	EX_{us}	C_{us}
1999.4	0.87	0.12	0.23	-0.18	-0.12	0.04	-0.28	-0.73	0.13	-0.09
2000.4	0.76	0.19	0.30	-0.21	-0.12	0.04	-0.42	-2.54	0.15	-0.13
2001.4	0.79	0.24	0.36	-0.22	-0.12	0.03	-0.48	-4.72	0.23	-0.15
2002.4	1.01	0.32	0.47	-0.27	-0.15	0.05	-0.55	-7.33	0.34	-0.17
2003.4	1.50	0.43	0.66	-0.35	-0.22	0.07	-0.69	-11.04	0.35	-0.22
2004.4	1.97	0.58	0.86	-0.45	-0.30	0.10	-0.91	-15.46	0.58	-0.29
2005.4	2.15	0.75	1.06	-0.52	-0.34	0.10	-1.12	-19.50	0.51	-0.36
2006.4	2.72	0.94	1.31	-0.62	-0.42	0.13	-1.33	-22.14	0.73	-0.43
2007.4	3.14	1.16	1.59	-0.76	-0.47	0.16	-1.54	-23.54	0.75	-0.51
2008.4	3.20	1.36	1.80	-0.93	-0.46	0.15	-1.72	-24.54	1.17	-0.59

qtr	Y_{us}	J_{us}	J^a_{us}
1999.4	-0.03	-0.01	-18.6
2000.4	-0.04	-0.03	-43.5
2001.4	-0.04	-0.04	-51.5
2002.4	-0.02	-0.03	-40.9
2003.4	-0.04	-0.04	-47.0
2004.4	-0.04	-0.05	-60.0
2005.4	-0.07	-0.06	-82.5
2006.4	-0.04	-0.06	-76.8
2007.4	-0.04	-0.05	-62.0
2008.4	-0.03	-0.05	-59.2

See notes to Table 1

Table 5
Chinese Appreciation of 25 Percent
No Change in Chinese Output
Percentage Deviations from Base in Percentage Points

qtr	PM_{ch}	PY_{ch}	PX_{ch}	$PX\$_{ch}$	$X_{ch,us}$	EX_{ch}	Y_{ch}	IM_{ch}
1999.4	-24.88	0.00	-13.29	15.61	-14.99	-5.51	0.00	0.00
2000.4	-24.86	-5.26	-15.58	12.56	-24.02	-10.11	0.00	0.00
2001.4	-24.82	-8.00	-16.76	10.98	-30.12	-13.07	0.00	0.00
2002.4	-24.74	-9.44	-17.35	10.20	-33.93	-15.04	0.00	0.00
2003.4	-24.64	-10.21	-17.60	9.87	-35.97	-16.14	0.00	0.00
2004.4	-24.53	-10.59	-17.67	9.77	-37.19	-17.02	0.00	0.00
2005.4	-24.44	-10.78	-17.64	9.81	-37.86	-17.11	0.00	0.00
2006.4	-24.31	-10.86	-17.56	9.92	-38.17	-17.68	0.00	0.00
2007.4	-24.20	-10.88	-17.45	10.07	-38.37	-18.39	0.00	0.00
2008.4	-24.11	-10.86	-17.36	10.19	-38.22	-19.87	0.00	0.00

qtr	PM_{us}	PY_{us}	PX_{us}	AA_{us}	YD_{us}	RS_{us}	IM_{us}	$X_{us,ch}$	EX_{us}	C_{us}
1999.4	0.87	0.12	0.24	-0.18	-0.12	0.04	-0.28	-0.08	0.14	-0.09
2000.4	0.63	0.17	0.26	-0.18	-0.10	0.03	-0.37	-0.15	0.17	-0.12
2001.4	0.61	0.21	0.30	-0.17	-0.09	0.03	-0.39	-0.16	0.29	-0.12
2002.4	0.77	0.26	0.38	-0.21	-0.10	0.05	-0.41	-0.16	0.45	-0.13
2003.4	1.16	0.36	0.52	-0.27	-0.15	0.07	-0.51	-0.15	0.61	-0.16
2004.4	1.53	0.48	0.69	-0.35	-0.20	0.10	-0.66	-0.19	0.89	-0.22
2005.4	1.69	0.62	0.84	-0.40	-0.22	0.11	-0.81	-0.25	1.01	-0.27
2006.4	2.13	0.79	1.05	-0.49	-0.26	0.15	-0.96	-0.28	1.33	-0.32
2007.4	2.44	0.97	1.27	-0.59	-0.29	0.18	-1.10	-0.35	1.44	-0.38
2008.4	2.53	1.16	1.45	-0.74	-0.29	0.19	-1.22	-0.41	1.74	-0.44

qtr	Y_{us}	J_{us}	J^a_{us}
1999.4	-0.03	-0.01	-17.6
2000.4	-0.03	-0.03	-34.6
2001.4	-0.02	-0.02	-29.5
2002.4	0.01	-0.01	-7.2
2003.4	0.01	0.00	4.9
2004.4	0.03	0.01	17.9
2005.4	0.03	0.02	25.2
2006.4	0.07	0.05	65.2
2007.4	0.09	0.08	102.9
2008.4	0.10	0.09	117.9

See notes to Table 1

Table 6
Chinese Appreciation of 25 Percent
Experiments in Tables 2 and 5 Combined
Percentage Deviations from Base in Percentage Points

qtr	PM_{ch}	PY_{ch}	PX_{ch}	$PX\$_{ch}$	$X_{ch,us}$	EX_{ch}	Y_{ch}	IM_{ch}
1999.4	-24.87	0.00	-13.27	15.64	-15.00	-5.51	0.00	5.88
2000.4	-24.83	-5.25	-15.53	12.63	-24.03	-10.12	0.00	8.24
2001.4	-24.77	-7.99	-16.70	11.07	-30.14	-13.07	0.00	8.95
2002.4	-24.67	-9.43	-17.27	10.31	-33.96	-15.03	0.00	8.99
2003.4	-24.55	-10.18	-17.51	9.99	-35.99	-16.11	0.00	8.80
2004.4	-24.42	-10.56	-17.57	9.91	-37.20	-16.96	0.00	8.56
2005.4	-24.31	-10.74	-17.53	9.96	-37.85	-17.02	0.00	8.34
2006.4	-24.18	-10.81	-17.44	10.07	-38.14	-17.57	0.00	8.17
2007.4	-24.07	-10.82	-17.34	10.21	-38.32	-18.24	0.00	8.03
2008.4	-23.99	-10.80	-17.26	10.33	-38.13	-19.67	0.00	7.93

qtr	PM_{us}	PY_{us}	PX_{us}	AA_{us}	YD_{us}	RS_{us}	IM_{us}	$X_{us,ch}$	EX_{us}	C_{us}
1999.4	0.89	0.13	0.25	-0.18	-0.12	0.05	-0.28	9.09	0.24	-0.09
2000.4	0.69	0.19	0.30	-0.19	-0.09	0.05	-0.38	13.05	0.37	-0.12
2001.4	0.70	0.24	0.35	-0.20	-0.09	0.05	-0.40	13.33	0.57	-0.12
2002.4	0.89	0.31	0.45	-0.25	-0.10	0.07	-0.44	12.78	0.76	-0.14
2003.4	1.30	0.42	0.60	-0.31	-0.15	0.10	-0.55	12.82	1.06	-0.18
2004.4	1.69	0.56	0.79	-0.39	-0.20	0.13	-0.71	12.87	1.29	-0.24
2005.4	1.87	0.72	0.96	-0.44	-0.21	0.14	-0.87	12.65	1.49	-0.29
2006.4	2.33	0.90	1.18	-0.53	-0.25	0.19	-1.02	11.83	1.83	-0.35
2007.4	2.63	1.10	1.41	-0.63	-0.27	0.22	-1.15	10.87	1.94	-0.40
2008.4	2.72	1.30	1.61	-0.79	-0.28	0.23	-1.25	10.10	2.19	-0.46

qtr	Y_{us}	J_{us}	J^a_{us}
1999.4	-0.01	0.00	-2.8
2000.4	0.00	0.00	-6.3
2001.4	0.01	0.00	3.8
2002.4	0.03	0.02	25.4
2003.4	0.05	0.03	42.3
2004.4	0.06	0.05	64.6
2005.4	0.07	0.06	80.3
2006.4	0.13	0.10	134.7
2007.4	0.15	0.14	179.5
2008.4	0.15	0.15	194.7

See notes to Table 1

Table 7
Chinese Appreciation of 25 Percent
Experiments in Tables 2 and 5 Combined
plus RS Exogenous
Percentage Deviations from Base in Percentage Points

qtr	PM_{ch}	PY_{ch}	PX_{ch}	$PX\$_{ch}$	$X_{ch,us}$	EX_{ch}	Y_{ch}	IM_{ch}
1999.4	-24.86	0.00	-13.26	15.65	-14.98	-5.51	0.00	5.88
2000.4	-24.82	-5.25	-15.52	12.65	-23.98	-10.10	0.00	8.23
2001.4	-24.75	-7.98	-16.68	11.10	-30.06	-13.05	0.00	8.94
2002.4	-24.65	-9.42	-17.24	10.35	-33.86	-15.00	0.00	8.98
2003.4	-24.52	-10.17	-17.47	10.04	-35.88	-16.07	0.00	8.78
2004.4	-24.37	-10.55	-17.51	9.98	-37.06	-16.90	0.00	8.54
2005.4	-24.25	-10.72	-17.46	10.05	-37.66	-16.95	0.00	8.32
2006.4	-24.10	-10.79	-17.36	10.19	-37.91	-17.47	0.00	8.14
2007.4	-23.97	-10.79	-17.24	10.35	-38.06	-18.11	0.00	8.00
2008.4	-23.88	-10.76	-17.13	10.49	-37.85	-19.52	0.00	7.89

qtr	PM_{us}	PY_{us}	PX_{us}	AA_{us}	YD_{us}	RS_{us}	IM_{us}	$X_{us,ch}$	EX_{us}	C_{us}
1999.4	0.91	0.13	0.26	-0.17	-0.12	0.00	-0.26	9.09	0.24	-0.07
2000.4	0.72	0.21	0.32	-0.18	-0.09	0.00	-0.33	13.04	0.38	-0.10
2001.4	0.74	0.27	0.39	-0.20	-0.09	0.00	-0.34	13.32	0.58	-0.10
2002.4	0.95	0.35	0.49	-0.24	-0.11	0.00	-0.37	12.75	0.77	-0.11
2003.4	1.38	0.47	0.66	-0.30	-0.16	0.00	-0.47	12.78	1.08	-0.14
2004.4	1.80	0.63	0.86	-0.38	-0.21	0.00	-0.60	12.81	1.31	-0.19
2005.4	2.00	0.81	1.06	-0.44	-0.24	0.00	-0.73	12.57	1.52	-0.23
2006.4	2.49	1.01	1.31	-0.53	-0.28	0.00	-0.85	11.74	1.88	-0.27
2007.4	2.81	1.24	1.56	-0.65	-0.32	0.00	-0.96	10.76	2.00	-0.32
2008.4	2.93	1.45	1.78	-0.83	-0.33	0.00	-1.05	9.99	2.26	-0.37

qtr	Y_{us}	J_{us}	J^a_{us}
1999.4	0.01	0.01	8.6
2000.4	0.03	0.02	22.0
2001.4	0.04	0.03	37.8
2002.4	0.06	0.05	61.1
2003.4	0.08	0.07	85.6
2004.4	0.11	0.10	122.9
2005.4	0.13	0.12	155.9
2006.4	0.19	0.17	223.4
2007.4	0.22	0.21	276.9
2008.4	0.22	0.23	291.4

See notes to Table 1

4.6 Is Fiscal Stimulus a Good Idea?[103]

4.6.1 Introduction

The U.S. stimulus bill passed in February 2009 (the American Recovery and Reinvestment Act of 2009) was large by historical standards. The bill totaled about $750 billion over four years, with most of the stimulus in the form of increased transfer payments and decreased taxes. Only about 15 percent was in the form of increased government purchases of goods and services. There are considerable differences of opinion as to how effective the stimulus was, and the bill has stimulated research on estimating the size of government spending multipliers. Obviously, the larger the multipliers, the larger is the short-run gain in output.

This section is concerned with a more general question than simply the size of government spending multipliers or the effects of the stimulus bill. The question is whether fiscal stimulus is ever a good idea. The MC model is used to analyze this question. The experiments use federal transfer payments (variable $TRGHQ$) as the government spending variable. Since the MC model has positive government spending multipliers, one might think that the answer to the question posed here is obviously yes. It will be seen, however, that there is very little gain, if any, from an increase in transfer payments *if the increased spending must eventually be paid for*. The gain in output and employment on the way up is roughly offset by the loss in output and employment on the way down as the debt from the initial stimulus is paid off.

A property of the MC model—see Section 4.4—is that monetary policy is not powerful enough to stabilize the economy. If it were, then full employment could always be achieved through monetary policy and there would be no need for fiscal stimulus. In the experiments below different assumptions about monetary policy are used, and it will be seen that the results are not sensitive to the different assumptions.

The use of transfer payments as the government spending variable covers many tax policies as well. Many tax changes are changes in what are sometimes called "tax expenditures"—changing loopholes, deductions, etc.—rather than changes in tax rates. Changes like these are essentially changes in transfer payments. Also, federal grants-in-aid to state and local governments can be considered transfer payments to the extent that state and local governments in turn transfer the money to households. The experiments in this section thus encompass a fairly wide

[103]The discussion in this section is taken from Fair (2014a). The results are the same as in the paper.

range of policy variables. This analysis does not, however, consider government purchases of goods and services, which may have investment components. If government spending on, say, transportation pays for itself in the future through increased government revenue of various forms, there is no increase in the long run debt and so no need to reverse anything in the future.

An experiment consists of increasing transfer payments from a baseline run for 8 quarters, then either decreasing them immediately for 8 quarters or waiting 16 quarters and decreasing them for 8 quarters. The decreases are chosen to get the debt/GDP ratio back to baseline by 56 quarters after the initial quarter of the increase. The horizon is thus 14 years. Within this horizon, using a discount rate of 2 percent (versus zero) makes little difference to the conclusions, as will be seen. Discounting would, of course, make a difference if one waited, say, 30 or 40 years before contracting. Waiting this long is close to just never paying the debt back. This section is concerned with the case in which the debt must be paid back in a shorter amount of time.

4.6.2 Previous Literature

Ramey (2011) reviews the literature on estimating the size of the government spending multiplier, where government spending is purchases of goods (not transfer payments) and there are no spending decreases or tax increases later. She concludes that the multiplier is probably between 0.8 and 1.5, although the range is considerably higher than this.

Fair (2010b)[104] also compares multipliers from a few studies, both regarding an increase in government purchases of goods and an increase in transfer payments or a decrease in taxes. After four quarters for an increase in purchases of goods the multiplier is 1.44 for Romer and Bernstein (2009), 0.44 for Barro and Redlick (2011), 0.55 for Hall (2009), 1.96 for the MC model, and a range of 1.0 to 2.5 for the CBO (2010). After four quarters for an increase in transfer payments or decrease in taxes, the multiplier is 0.66 for Romer and Bernstein (2009), 1.10 for Romer and Romer (2010), 1.1 for Barro and Redlick (2011), 0.99 for the MC model, and a range of 0.8 to 2.1 for the CBO (2010). The Romer and Bernstein multiplier peaks at 0.99 after 8 quarters, the Romer and Romer multiplier peaks at 3.08 after 10 quarters, and the MC multiplier peaks at 1.10 after 6 quarters. Again, there are no future spending decreases or tax increases for these results.

The CBO (2010) uses results from two commercial forecasting models and the

[104]This paper is updated in Section 5.5 below

FRB-US model of the Federal Reserve Board to choose ranges for a number of government spending and tax multipliers on output. Romer and Bernstein (2009) follow a similar methodology. They use a commercial forecasting model and the FRB-US model to choose government spending and tax multipliers on output.

Hall (2009), Barro and Redlick (2011), and Romer and Romer (2010) follow a reduced form approach. The change in real GDP is regressed on the change in the policy variable of interest and a number of other variables. The equation estimated is not, however, a true reduced form equation because many variables are omitted, and so the coefficient estimate of the policy variable will be biased if the policy variable is correlated with omitted variables. The aim using this approach is to choose a policy variable that seems unlikely to be correlated with the omitted variables. Hall (2009) and Barro and Redlick (2011) are concerned with government spending multipliers and focus on defense spending during wars.[105] Romer and Romer (2010) are concerned with tax multipliers and use narrative records to choose what they consider exogenous tax policy actions, i.e, actions that are uncorrelated with the omitted variables.

Auerbach and Gorodnichenko (2012) use a structural VAR approach that allows for different multipliers in expansions and recessions to estimate government spending (on goods and services) multipliers. Their general result is that multipliers are larger in recessions than in expansions. Their experiments on ones with no future tax increases or spending decreases.

Coenen et al. (2012) estimate government spending multipliers for nine DSGE models. The experiments consist of government spending or tax shocks from a steady state, where each model has a fiscal-policy rule that eventually returns the economy to the steady state, so there is no long run increase in the debt/GDP ratio. The models have rational expectations, and so everyone knows that the initial increase in debt will be paid off eventually. The experiments are run under various assumptions about monetary accommodation. The experiments with these models differ from those reported above in that the debt/GDP ratio is forced back to the baseline (the steady state) in the long run. One might think that the fiscal multipliers would be small in these models because agents know that the extra spending will eventually be paid for. In fact, the short-run multipliers are fairly large in most cases and the sums of the output gaps over the entire period are generally positive. For government purchases of goods the short-run multipliers are between about 0.7 and 1.0 for the United States with no monetary accommodation and between about 1.2 and 2.2 with two years of monetary accommodation. The short-run multipliers

[105]Barro and Redlick (2011) also estimate a tax multiplier.

are also fairly large for increases in transfer payments that are targeted to liquidity-constrained households, ranging from about 1.0 to 1.5 with two years of monetary accommodation. The tone of the Coenen et al. (2012) article is that temporary fiscal stimulus can be very helpful, especially if there is monetary accommodation.

The general features of the DSGE models that lead to the above conclusion are the following. A government spending shock (or decrease in taxes) stimulates liquidity-constrained households to consume more. Given this increased demand, firms that are allowed to change their prices raise them, but firms that are not allowed to change their prices are committed to sell all that is demanded at their current (unchanged) prices. The overall price level goes up, but there is also an output effect. All this happens even though agents in the model know that the increased government debt will eventually be paid back through lower future government spending or higher taxes. The initial (essentially constrained) output effect dominates. It is also the case that the mark-up falls for those firms that cannot change their prices. The increased inflation that is generated may lead the monetary authority to raise the interest rate, and so the results are sensitive to what is assumed about monetary policy.

There is finally a recent paper by DeLong and Summers (2012), which argues that there may be times in which fiscal expansions are self-financing—no long run increase in the debt/GDP ratio. There are no estimated equations in this paper, no lagged effects of government spending on output, and some calibrated parameters that seem unrealistic or for which there is little empirical support. For example, the marginal tax-and-transfer rate is taken to be 0.33, which seems too high. In 2011 the ratio of federal government tax receipts (including social security taxes) and unemployment benefits to GDP was 0.17. This is an average rate and the marginal rate may be higher, but 37 percent of tax receipts are social security taxes, where the tax rate is flat and then zero at some income level. There is also a key hysteresis parameter in the model, also calibrated, which reflects the assumption that potential output depends on current output in depressed states of the economy. If current fiscal stimulus increases future potential output, there is obviously some effect large enough to generate enough extra future government revenue to pay for the stimulus.

4.6.3 Reduced Form Equations

The methodology behind the MC model, the Cowles Commission approach, does not have the problem of possible omitted variable bias in reduced form equations because reduced form equations are not directly estimated. What is required is that the structural equations be consistently estimated. Take, for example, a consumption or investment equation. If there are right hand side endogenous variables, like current income or a current interest rate, and thus correlation between these variables and the error term in the equation, this has to be accounted for. Two-stage least squares (2SLS) is one option. First stage regressors must be found that are correlated with the endogenous variables and uncorrelated with the error term. If one suspects that a current government spending or tax rate variable depends on current endogenous variables, the variable would need to be lagged one period before being used as a first stage regressor. The estimation is slightly more complicated if the error term in the structural equation is serially correlated. In this case the 2SLS estimator can be modified to jointly estimate the serial correlation coefficient and the structural coefficients—see Section 2.3.1. The aim in structural modeling is to find good structural equations—good approximations to reality—and to estimate them consistently.[106] Reduced form equations are not estimated but derived, and there are many nonlinear restrictions on the reduced form equations.

This structural approach uses much more information on the economy than does the reduced form approach discussed above. For example, the implicit reduced form equation for U.S. output in the MC model is nonlinear and includes hundreds of exogenous and lagged endogenous variables. There are also hundreds of nonlinear restrictions on the reduced form coefficients. Given the complexity of the economy, it seems unlikely that estimating reduced form equations with many omitted variables and no restrictions from theory on the coefficients will produce trustworthy results even if an attempt is made to account for omitted variable bias.

4.6.4 Transfer Payment Multipliers

To review some of the properties of the MC model, Table 1 presents transfer payment multipliers for the period 1992:1–2005:4. The level of real transfer payments, $TRGHQ$, is permanently increased by 1.0 percent of potential real GDP from its baseline values.[107] This is an experiment in which nothing is paid for: no changes to any exogenous variable were made except for transfer payments. When transfer payments are increased in the model, disposable income is increased, which positively affects the three consumption categories and housing investment. The

[106]Commercial forecasting models like the ones used by the CBO (2010) and Romer and Bernstein (2009) are not in the academic literature, and so it is hard to evaluate them. It does not appear, however, that the structural equations in these models are consistently estimated.

[107]Potential real GDP is taken to be $YS + PSI13(JG \cdot HG + JM \cdot HM + JS \cdot HS) + STATP$, which is equation 83 in Table A.3 with YS replacing Y.

Table 1
Transfer Payment Multipliers using the MC Model
Deviations from Baseline in Percentage Points

qtr	GDPR	UR	GDPD	RS	AGZGDP
1992.1	0.17	-0.04	0.03	0.03	0.14
1992.2	0.40	-0.11	0.10	0.11	0.22
1992.3	0.60	-0.20	0.18	0.21	0.28
1992.4	0.76	-0.29	0.26	0.30	0.34
1993.1	0.87	-0.37	0.34	0.38	0.44
1993.2	0.93	-0.42	0.42	0.44	0.55
1993.3	0.96	-0.46	0.50	0.49	0.69
1993.4	0.95	-0.47	0.58	0.52	0.85
1994.1	0.92	-0.47	0.62	0.54	1.05
1994.2	0.89	-0.45	0.67	0.54	1.26
1994.3	0.84	-0.42	0.71	0.53	1.48
1994.4	0.80	-0.41	0.74	0.53	1.72
1995.1	0.77	-0.37	0.77	0.52	1.97
1995.2	0.75	-0.33	0.79	0.49	2.22
1995.3	0.73	-0.32	0.79	0.48	2.47
1995.4	0.73	-0.31	0.81	0.47	2.72
1996.4	0.71	-0.29	0.87	0.46	3.68
1997.4	0.70	-0.29	0.95	0.46	4.67
1998.4	0.72	-0.29	1.02	0.46	5.61
1999.4	0.73	-0.29	1.11	0.46	6.52
2000.4	0.73	-0.30	1.17	0.46	7.48
2001.4	0.79	-0.31	1.24	0.47	8.64
2002.4	0.74	-0.32	1.31	0.48	9.69
2003.4	0.73	-0.32	1.34	0.47	10.43
2004.4	0.71	-0.31	1.37	0.45	11.17
2005.4	0.77	-0.35	1.40	0.48	11.85

$GDPR$ = real GDP
UR = unemployment rate
$GDPD$ = GDP deflator
RS = three-month Treasury bill rate
$AGZGDP$ = nominal federal debt/nominal GDP

- percent deviations for $GDPR$ and $GDPD$, absolute deviations for UR, RS, and $AGZGDP$.
- Experiment is a sustained increase in real transfer payments of 1.0 percent of potential real GDP

table shows that the peak multiplier for output is 0.96 after 7 quarters. The multiplier settles down to about 0.7 after about 16 quarters. Physical stock effects and interest rate effects are the main reasons for the decline in the multipliers after the peak. By 2005:4 the debt/GDP ratio has risen by 11.85 percentage points.

4.6.5 The Experiments

The results here are based on actual data through 2013:3 (data available as of November 11,2013). Values for the 2013:4–2022:4 period are used for some of the experiments, and these values are from a forecast I made on November 11, 2013 using the MC model. These values are on my website.[108]

Three 56-quarter periods are considered, beginning respectively in 1975:1, 1992:1, and 2009:1. When the MC model is solved for a given period with all the residuals set to their estimated values and the actual values of all the exogenous variables used, a perfect tracking solution is obtained. The baseline run for each of the three periods is taken to be this solution, namely just the actual values of all the variables. The estimated residuals are added to the equations and treated as exogenous for all the experiments. The experiments thus run off the perfect tracking solution. Each experiment consists of increasing real transfer payments for the first eight quarters of the period from their baseline values. As a percentage of a measure of potential output in the model, the increases are per quarter 0.5, 1.0, 1.5, 2.0, 2.0, 2.0, 2.0, and 2.0. The increases are thus phased in for the first year and then held at 2.0 percent for the second year. The first quarter of each period was chosen to be a quarter of high unemployment.

For experiment "NOWAIT" the decreases begin in the ninth quarter, where as a percentage of potential output they are per quarter 0.5λ, 1.0λ, 1.5λ, 2.0λ, 2.0λ, 2.0λ, 2.0λ, and 2.0λ. λ is chosen to be the smallest value that results in the debt/GDP ratio returning to its baseline value sometime before the end of the 56-quarter period.[109] Experiment "WAIT" is the same as experiment NOWAIT except that the decreases begin in quarter 25. For quarters 9 through 24 (and after quarter 32) the transfer payment values are the actual values. Experiment WAIT thus has a 4-year gap before the decreases begin.

Regarding monetary policy, the estimated U.S. interest rate reaction function,

[108]For countries other than the United States data were not available as late as 2013:3, and the overall forecast began earlier than 2013:4, with actual values used for the United States until 2013:4.

[109]As will be seen, the MC model cycles somewhat, including values of the debt/GDP ratio, and the stopping value of λ was taken to be the first time the debt/GDP ratio came within 0.0005 of its baseline value (0.05 percentage points).

equation 30, is used for one set of experiments, denoted "RULE." Remember that this equation has the property that the interest rate generally rises during the stimulus stage and falls during the de-stimulus stage. The Fed is estimated to "lean against the wind." For the second set of experiments, denoted "NORULE," the interest rate reaction function is dropped and the interest rate is taken to be exogenous. Its value for each quarter is the baseline value. This is the case in which the Fed does not raise interest rates on the way up, but also does not lower them on the way down. It "accommodates" the fiscal policy changes. For the period beginning in 2009:1, RULE is not used, because for part of this period the interest rate was at a zero lower bound. The estimated rule is not necessarily reliable in this case. The NORULE case keeps the interest rate at the zero lower bound when it was in fact at the zero lower bound.

The following tables present the results for five variables: real GDP (Y), the total number of jobs in the economy (J), the total number of people unemployed (U), the GDP deflator (P), and the federal government debt/GDP ratio (D). Y is at a quarterly rate. Two values are presented for each of the first three variables and each experiment. The first is the sum of the deviations of the variable from its baseline value over the 56 quarters, denoted $\sum I$, where I is the variable. The second is the discounted sum using a discount rate of 2 percent at an annual rate, denoted $\sum \beta I$. Also presented are the values of P and D at the end of the period, where the value for P is the percent deviation from baseline and the value for D is the absolute deviation from baseline (in percentage points). The values of λ are also presented. The full MC model is solved for each experiment except that the estimated U.S. interest rate rule is dropped for the NORULE experiments.

4.6.6 Results

Summary results are presented in Tables 2, 3, and 4 for the three periods. The experiments using the interest rate rule are presented first in each table except for Table 4, where the rule is not used. The first experiment of each set of three experiments is the case where there is no future de-stimulus, denoted "NOPAY." NOPAY is always stimulative. For example, in Table 3 the sum of the output deviations from baseline over the 56 quarters is \$273.4 billion for RULE and \$285.2 billions for NORULE. The sums of the jobs deviations are 13.51 and 14.35 million workers respectively. The sums of the number of people unemployed are -6.35 and -6.40 million respectively. The debt/GDP ratio at the end of the period for RULE is larger by 2.62 percentage points. For NORULE it is larger by 1.88 percentage points. The GDP deflator is larger at the end of the period in both

Table 2
Estimated Effects for the 1975:1–1988:4 Period

Experiment	$\sum Y$	$\sum J$	$\sum U$	P end	D end	$\sum \beta Y$	$\sum \beta J$	$\sum \beta U$	λ
NOPAY, RULE	172.1	10.79	-5.19	0.18	2.26	142.7	8.98	-4.33	
NOWAIT, RULE	-11.6	-0.82	0.02	-0.08	0	-11.4	-0.86	0.04	1.00
WAIT, RULE	58.2	3.76	-1.97	0.02	0	41.5	2.68	-1.50	0.70
NOPAY, NORULE	179.9	11.34	-5.05	0.34	1.59	145.2	9.17	-4.09	
NOWAIT, NORULE	21.1	1.23	-0.84	0.06	0	16.2	0.88	-0.69	0.90
WAIT, NORULE	59.3	2.89	-1.74	0.10	0	41.9	1.71	-1.31	0.65

$Y = GDPR$ = real GDP, billions of 2009 dollars
$J = JF$ = total number of jobs, millions of jobs
U = total unemployment, millions of people
$P = GDPD$ = GDP deflator
$D = AGZGDP$ = nominal federal debt/nominal GDP
λ, see text
$\sum$ = sum of deviations from baseline over the 56 quarters
$\sum \beta$ = discounted sum of deviations from baseline over the 56 quarters, 2 percent discount rate
P end = percent deviation of P from baseline in last quarter, percentage points
D end = absolute deviation of D from baseline in last quarter, percentage points

Table 3
Estimated Effects for the 1992:1–2005:4 Period

Experiment	$\sum Y$	$\sum J$	$\sum U$	P end	D end	$\sum \beta Y$	$\sum \beta J$	$\sum \beta U$	λ
NOPAY, RULE	273.4	13.51	-6.35	0.20	2.62	223.0	11.09	-5.19	
NOWAIT, RULE	8.9	0.40	-0.43	-0.02	0	2.3	0.06	-0.25	1.00
WAIT, RULE	37.7	2.56	-2.13	-0.04	0	16.9	1.36	-1.54	1.00
NOPAY, NORULE	285.2	14.35	-6.40	0.26	1.88	228.3	11.56	-5.11	
NOWAIT, NORULE	5.1	0.04	-0.25	-0.04	0	-0.6	-0.27	-0.10	1.00
WAIT, NORULE	48.5	2.78	-2.33	-0.02	0	24.5	1.36	-1.68	0.90

See notes to Table 2.

Table 4
Estimated Effects for the 2009:1–2022:4 Period

Experiment	$\sum Y$	$\sum J$	$\sum U$	P end	D end	$\sum \beta Y$	$\sum \beta J$	$\sum \beta U$	λ
NOPAY, NORULE	346.7	12.59	-5.87	0.17	2.22	274.8	10.08	-4.65	
NOWAIT, NORULE	-12.3	-0.50	-0.07	-0.03	0	-18.3	-0.74	0.06	0.95
WAIT, NORULE	22.3	1.31	-1.53	-0.08	0	-4.9	0.14	-0.97	0.90

See notes to Table 2.

cases, but the effect is very small.

Turning to the cases where there is de-stimulus, it is always true that WAIT is more stimulative than NOWAIT. Loosely speaking, by waiting four years before de-stimulating, the economy has time to build on the initial stimulus, which lessens the cost of getting the debt/GDP ratio back to baseline. In fact, except for NOWAIT, NORULE in Table 2, NOWAIT is never stimulative. The sums are negative or close to zero, even when discounting. The sums for WAIT, while positive, are very small in Tables 3 and 4. Only in Table 2 would one say that the effort might be worth it. In Table 2 the output sum for WAIT, RULE is \$58.2 billion, compared to \$172.1 billion in the NOPAY, RULE case. The MC model is nonlinear, and this is the main reason for the differences across tables.

Comparing WAIT, RULE with WAIT, NORULE, NORULE is slightly more stimulative. When RULE is in effect, the Fed increases interest rates as the stimulus is taking place, which, among other things, increases federal interest payments and thus the federal debt. Six years after the beginning of the stimulus the debt is larger than it otherwise would be because of the increased interest rates. It thus takes a little more work to get the debt/GDP ratio back to baseline than it would if interest rates never increased (from baseline), as in the NORULE case. In the RULE case interest rates do fall during the de-stimulus, which helps lower interest payments, but the net effect is for slightly more overall expansion in the NORULE case.

None of the conclusions are changed by discounting. If anything, the argument against stimulating may be a little stronger with discounting. Since there are endogenous cycles in the MC model because of physical stock effects, this means that after de-stimulus has taken place (five to nine years out) physical stocks are sometimes lower than baseline, which, other things being equal, leads to increased investment in the future. So for the last few years of the 14-year period, the output gaps can be positive. If these gaps are discounted, the overall gain from the experiment is thus smaller than if they are not discounted, other things being equal.

The long run effects on the GDP deflator are always small, as would be expected given that the sums of the output deviations are small. λ, which measures the size of the de-stimulus needed to get the debt/GDP ratio back to baseline, is larger (with one tie) for NOWAIT versus WAIT. It is also larger for RULE versus NORULE (with one tie). The range is from 0.65 to 1.05.

Table 5 gives more detailed results for the WAIT, RULE experiment in Table 3. The values in Table 5 are deviations from baseline for each of the 56 quarters. The first column is for real transfer payments, $TRGHQ$, which is the exogenous spending variable. The remaining variables are endogenous. Two physical stock

Table 5
More Detailed Results for WAIT, RULE in Table 3
Absolute Deviations from Baseline

qtr	$TRGHQ$	$GDPR$	JF	U	RS	EXK	KH	$AGZGDP$
1992.1	10.7	1.7	0.02	-0.02	0.01	-8.3	0.4	0.06
1992.2	21.6	5.8	0.10	-0.09	0.06	-27.3	1.6	0.16
1992.3	32.6	12.1	0.24	-0.20	0.16	-56.1	3.8	0.28
1992.4	43.8	20.1	0.44	-0.37	0.29	-91.8	7.0	0.43
1993.1	44.1	27.5	0.69	-0.55	0.44	-122.7	10.9	0.56
1993.2	44.4	33.3	0.95	-0.74	0.59	-144.1	15.1	0.71
1993.3	44.9	37.3	1.19	-0.89	0.72	-155.1	19.4	0.89
1993.4	45.3	39.6	1.40	-0.99	0.82	-157.0	23.4	1.10
1994.1	0.0	33.1	1.45	-0.97	0.83	-116.0	25.3	1.17
1994.2	0.0	23.1	1.37	-0.79	0.73	-59.3	25.5	1.36
1994.3	0.0	12.9	1.17	-0.55	0.56	-3.7	24.3	1.62
1994.4	0.0	4.6	0.91	-0.33	0.41	41.0	22.2	1.88
1995.1	0.0	-1.7	0.65	-0.09	0.27	73.1	19.5	2.13
1995.2	0.0	-5.7	0.41	0.11	0.13	92.5	16.6	2.34
1995.3	0.0	-7.8	0.21	0.22	0.03	101.3	13.6	2.55
1995.4	0.0	-8.6	0.05	0.29	-0.03	102.2	10.7	2.66
1996.1	0.0	-8.2	-0.05	0.32	-0.07	97.2	8.2	2.74
1996.2	0.0	-7.2	-0.11	0.31	-0.08	88.8	6.0	2.75
1996.3	0.0	-5.8	-0.14	0.28	-0.08	78.2	4.1	2.77
1996.4	0.0	-4.2	-0.14	0.23	-0.08	67.0	2.7	2.78
1997.1	0.0	-2.6	-0.13	0.18	-0.06	56.4	1.6	2.77
1997.2	0.0	-1.3	-0.10	0.13	-0.04	47.0	0.7	2.76
1997.3	0.0	-0.2	-0.06	0.08	-0.02	38.7	0.2	2.74
1997.4	0.0	0.8	-0.03	0.04	0.00	31.7	-0.1	2.74
1998.1	-13.4	-0.7	-0.03	0.03	0.00	35.8	-0.8	2.67
1998.2	-27.1	-5.2	-0.08	0.07	-0.04	53.5	-2.3	2.56
1998.3	-41.0	-12.7	-0.22	0.18	-0.12	83.6	-5.0	2.40
1998.4	-55.3	-22.3	-0.43	0.34	-0.24	122.3	-9.0	2.17
1999.1	-55.8	-31.3	-0.69	0.54	-0.38	156.0	-13.8	1.98
1999.2	-56.3	-38.0	-0.97	0.71	-0.51	177.0	-19.0	1.75
1999.3	-57.0	-42.5	-1.21	0.84	-0.62	186.2	-24.4	1.48
1999.4	-57.6	-45.1	-1.42	0.93	-0.70	185.7	-29.5	1.17
2000.1	0.0	-35.8	-1.46	0.85	-0.68	131.0	-32.0	1.11
2000.2	0.0	-23.2	-1.33	0.65	-0.56	62.4	-32.2	0.96
2000.3	0.0	-10.4	-1.08	0.40	-0.41	-4.3	-30.9	0.78
2000.4	0.0	-0.5	-0.79	0.17	-0.27	-55.6	-28.4	0.61
2001.1	0.0	6.3	-0.51	-0.05	-0.15	-89.8	-25.2	0.47
2001.2	0.0	10.5	-0.26	-0.22	-0.04	-109.7	-21.6	0.34
2001.3	0.0	12.4	-0.05	-0.32	0.04	-117.6	-17.9	0.25
2001.4	0.0	12.9	0.09	-0.38	0.09	-117.3	-14.3	0.17

Table 5 (continued)

qtr	$TRGHQ$	$GDPR$	JF	U	RS	EXK	KH	$AGZGDP$
2002.1	0.0	12.5	0.19	-0.39	0.11	-112.4	-11.0	0.12
2002.2	0.0	11.3	0.25	-0.37	0.12	-103.5	-8.0	0.08
2002.3	0.0	9.8	0.28	-0.34	0.12	-92.8	-5.4	0.06
2002.4	0.0	8.2	0.28	-0.30	0.12	-81.6	-3.2	0.04
2003.1	0.0	6.7	0.27	-0.25	0.11	-70.9	-1.4	0.03
2003.2	0.0	5.3	0.24	-0.21	0.09	-61.1	0.0	0.03
2003.3	0.0	4.0	0.21	-0.16	0.08	-52.2	1.1	0.03
2003.4	0.0	2.9	0.18	-0.13	0.06	-44.1	1.9	0.04
2004.1	0.0	2.1	0.15	-0.10	0.05	-37.2	2.5	0.04
2004.2	0.0	1.3	0.12	-0.07	0.04	-31.2	2.9	0.05
2004.3	0.0	0.8	0.10	-0.05	0.03	-26.2	3.2	0.06
2004.4	0.0	0.3	0.07	-0.03	0.02	-22.0	3.3	0.06
2005.1	0.0	0.1	0.05	-0.02	0.02	-18.6	3.3	0.07
2005.2	0.0	-0.1	0.04	-0.01	0.02	-15.9	3.2	0.07
2005.3	0.0	-0.2	0.03	-0.01	0.02	-13.6	3.1	0.07
2005.4	0.0	-0.3	0.02	-0.01	0.02	-11.4	2.9	0.08

See notes to Table 2 for $GDPR$, JF, U, and $AGZGDP$
$TRGHQ$ = real transfer payments, billions of 2005 dollars
RS = three-month Treasury bill rate, percentage points
EXK = excess capital, billions of 2009 dollars
KH = stock of housing, billions of 2009 dollars

variables are presented, excess capital, EXK,[110] and the housing stock, KH, to give a sense of the physical stock effects in the model. The table shows that four quarters after the end of the de-stimulus, the output deviations are positive, reflecting in part the physical stock effects. The debt/GDP deviations reach a peak at 2.78 right before the de-stimulus and then gradually fall to zero. (This experiment used a value of λ of 1.00. Using a value of 0.95 did not result in any of the deviations falling below 0.05 percentage points.) The Fed raised the interest rate during and somewhat after the stimulus and then lowered it during the de-stimulus.

4.6.7 Caveats

It may be surprising that a model in Cowles Commission tradition (sometimes called "Keynesian" models) like the MC model suggests that fiscal stimulus is not very effective. Keynes famous (infamous?) statement that "in the long run we

[110] EXK is $KK - KKMIN$ in the model.

are all dead" is consistent with ignoring any increases in the debt/GDP ratio that may result from fiscal stimulus, in which case stimulus is effective. The relevant statement for the present experiments, on the other hand, is "in the long run the debt must be paid off," admittedly not quite as catchy.

A key question when considering a fiscal stimulus is thus whether the long run can be ignored. In periods of low debt/GDP ratios, like much of the post war period until about 2008, permanently raising the debt/GDP ratio may not have been much of a worry. At the present time (2012), however, the debt/GDP ratio is high and rising, and it is a worry to many people. One concern is that at some (unpredictable) time there will be negative asset market reactions. For example, extending at the end of 2011 the payroll tax cuts and increased unemployment benefits through 2012 with no plan to pay down the increased debt in the future likely increased the chance of negative market reactions in the future.

Given the constraint that any stimulus must eventually be paid for, why might the conclusion that there is little gain from fiscal stimulus be wrong? Regarding monetary policy, it has modest effects in the MC model, as noted in Section 3. The policy of NORULE, which is the accommodating policy, gives slightly better results than RULE, but the differences are not large. If the model is wrong and monetary policy is powerful enough to keep the economy at full employment, then fiscal stimulus is unnecessary, making the question posed here uninteresting.

The conclusion could be sensitive to the treatment of potential output, which is taken to be exogenous in the model. If, as in the DeLong and Summers (2012) story, potential output is positively affected by stimulus measures, this would increase the case for fiscal stimulus. The main possibility in the model would be a permanent increase in long run labor or capital productivity (upward shifts of the peak-to-peak interpolations that are used to construct LAM and MUH—see Table A17). This effect is hard to estimate and probably second order, but it has been ignored here. Remember that the fiscal stimulus tool used here is the level of transfer payments or tax expenditures. The conclusion does not pertain to government purchases of goods and services, which in many cases are partly investment and may have positive rates of return.

Another feature of the MC model is that changes in asset prices are either exogenous or only slightly affected by the economy. A stimulus, for example, does not lead to large changes in stock prices, variable CG. CG rises modestly (relative to baseline) during a stimulus and falls modestly during a de-stimulus. If it is instead the case that a stimulus leads to large and permanent increases in asset prices, which would in the model have positive effects on consumption and investment, the economy could grow fast enough to lead to only a small increase in the debt/GDP ratio, which could be paid off with a small de-stimulus. Since changes in asset prices are roughly random walks with drift, it is unlikely that effects of stimulus measures on asset prices could be estimated.

If stimulus measures permanently increase animal spirits (consumer and investor confidence and the like), this could increase consumption and investment demand beyond what the model estimates. Again, the economy might grow fast enough to lead to only a small increase in the debt/GDP ratio, which could easily be paid off. This effect is also hard to estimate. There are also possible *negative* animal spirits from a stimulus in periods where the debt/GDP is high. There is recent work (see, for example, Bloom (2009)) examining the effects of uncertainty on the economy, where an increase in uncertainty may decrease aggregate demand. If a stimulus increases uncertainty because of expected future increases in the debt/GDP ratio, this could have a negative effect on consumption and investment demand.

There is an interesting "collapse" argument in Coenen et al. (2012). They argue that in really bad times, like 2008, without stimulus measures the economy might go into a downward spiral "where collapses in different sectors start to feed on each other due to balance sheet and demand interdependencies between multiple sectors." (p. 31) They point out that their DSGE models do not capture these extreme effects. Neither does the MC model. The baseline for the present experiments is the actual economy. This baseline includes, for example, the 2009 stimulus bill, since it actually passed and was put into effect, so the stimulus experiments that begin in 2009:1 are from a baseline that has already been stimulated. The MC model does not, however, have the property that the economy would have collapsed had the bill not been passed. The estimates in Section 5.5 below (which are updates of those in Fair (2010b)), for example, show that the unemployment rate would have been 1.38 percentage points higher in 2010:4 had there been no stimulus, with 2.969 million fewer jobs. This is a large effect, but not a nonlinear collapse. It could be, of course, that the MC model is misspecified and that there would have in fact been a collapse if the stimulus bill had not passed.

The collapse argument probably does not pertain to the end of 2011, when the payroll tax cuts were extended, since the economy was growing moderately at the end of 2011. It might pertain to the 2008–2009 period, but this is not obvious. The large declines in real GDP occurred in the last two quarters of 2008 and the first quarter of 2009 (3.7, 8.9, and 6.7 percent, respectively). In the second quarter of 2009 the decline was 0.7 percent, the last quarter of the decline. The stimulus bill, which passed in February 2009, may have affected the economy immediately (or even somewhat before it passed) through expectation effects, but the actual measures did not begin to take effect until the second quarter of 2009, when the economy was turning around. Had the bill not passed, would the economy have collapsed in 2009 rather than just have grown more slowly? If so, then the MC model has underestimated the effects of fiscal stimulus in very bad times.

Finally, a general criticism of the present results is that the MC model is so badly misspecified that none of the estimates are trustworthy. A different conclusion is reached using the DSGE models in Coenen et al.(2012), and one might trust these models more. However, the MC model is more empirically based than are DSGE models, which tend to be heavily calibrated. The key property of DSGE models, namely that there are gains to short run fiscal stimulus, relies on price-setting restrictions, usually Calvo pricing, liquidity-constrained households, and rational expectations, all of which have limited empirical backing. The MC model is more empirically grounded and thus possibly more trustworthy. But at a minimum the use of a model in the Cowles Commission tradition provides an alternative way of estimating the effects of fiscal stimulus—a reality check if you will on DSGE results.

4.6.8 Conclusion

The results in this section suggest that there is at most a small gain from fiscal stimulus in the form of increased transfer payments or increased tax deductions if the increased debt generated must eventually be paid back. This conclusion is robust to different assumptions about monetary policy. To the extent that there is a gain, the longer one waits to begin paying the debt back the better.

Possible caveats regarding the MC model are that 1) monetary policy is not powerful enough to keep the economy at full employment, 2) potential output is taken to be exogenous, 3) any permanent effects on asset prices and animal spirits from a stimulus are not taken into account, and 4) the model does not have the feature that in really bad times the economy might collapse without a stimulus.

4.7 Is Monetary Policy Becoming Less Effective Over Time?

In the model when the Fed raises the interest rate (RS), interest payments of the federal government ($INTG$) increase, some of which are interest receipts of the household sector from the federal government ($INTG - INTGR$), which are part of disposable income (YD). Disposable income has a positive effect on the expenditures of the household sector. From this channel, an increase in interest rates thus has a positive effect on aggregate demand, which offsets at least part of the negative effects. This channel, which will be called the "interest payments channel," is larger the larger is the size of the federal government debt ($-AG$), since the larger is the size of the debt, the more will interest payments increase for a given increase in interest rates. Since the federal government debt as a percentage of GDP ($AGZGDP$) has been rising since 2001, it might be that the net negative effect of an increase in RS has gotten smaller since 2001.

This possible change can be examined in the model by dropping the $INTG$ equation, U.S. equation 29. When this is done, a change in interest rates has no effect on $INTG$ and thus no effect on household interest receipts from the federal government. The following experiment is thus of interest. Take a period of a relative low debt/GDP ratio, 2001:1–2002:4; drop the estimated interest rate rule, equation 30; and increase RS by one percentage point. Record the effect on real GDP. Then drop the $INTG$ equation and repeat the experiment. Examine the differences in the two effects. Now repeat the exercise for the 2011:1–2012:4 period. Are the differences larger in the second period than in the first, as would be expected with a larger debt/GDP ratio in the second period than in the first?

These two experiments were done. The estimated residuals were first added to the stochastic equations and taken to be exogenous. This results in a perfect tracking solution if no changes in any exogenous variables are made. Then equation 30 was dropped and RS was increased by one percentage point from its actual value in each quarter.[111] For the first period (2001:1–2002:4) the decrease in real GDP ($GDPR$) was 0.649 percent after 8 quarters. For the second period (2011:1–2012:4) the decrease was 0.549 after 8 quarters. Because the model is nonlinear, one would not expected these values to be the same. They differ because of different values of the exogenous variables in the two periods. The interest here, however, is not in comparing these two values. The interest is in seeing how each value changes when the interest payments channel is turned off.

The second experiment is the same as the first except that the interest payments

[111]The zero lower bound in the second period is not a problem for this experiment because RS was increased.

equation 29 is dropped. For the first period the decrease in real GDP after 8 quarters was 0.740 percent, and for the second period it was 0.663 percent. Thus, for the first period the negative effect increased by 0.091 percentage points ($0.740 - 0.649$), and for the second period the negative effect increased by 0.114 percentage points ($0.663 - 0.549$). This is as expected. Turning the interest payments channel off has a larger effect in the second period, where the debt/GDP ratio is higher. In the second period with the interest payments equation in, there is a larger increase in interest payments than in the first period because the debt is higher, which, other things being equal, is more expansionary. When this effect is turned off, the second period is affected more.

The results for other variables are as expected. For example, for the unemployment rate (UR) the difference in the increases is 0.039 percentage points in the first period and 0.051 percentage points in the second period.

The results in this section thus show that monetary policy effects depend on the size of the debt/GDP ratio. As this ratio rises, other things being equal, the effects of interest rate increases become smaller. The above estimates suggest that so far the size of the effect of the interest rate channel is fairly small, although it will get larger over time as the debt/GDP ratio continues to rise.

4.8 Other Uses of Stochastic Simulation and Optimal Control

4.8.1 Sources of Economic Fluctuations

There was an active literature in the 1980's discussing the ultimate sources of macroeconomic variability. Shiller (1987) surveyed this work, where he pointed out that a number of authors attributed most of output or unemployment variability to only a few sources, sometimes only one. The sources vary from technology shocks for Kydland and Prescott (1982), to unanticipated changes in the money stock for Barro (1977), to "unusual structural shifts," such as changes in the demand for produced goods relative to services, for Lilien (1982), to oil price shocks for Hamilton (1983), to changes in desired consumption for Hall (1986). Although it may be that there are only a few important sources of macroeconomic variability, this is far from obvious. Economies seem complicated, and it may be that there are many important sources. It is possible using stochastic simulation to estimate the quantitative importance of various sources of variability from a macroeconometric model, and this is reviewed here.

Macroeconometric models provide an obvious vehicle for estimating the sources of variability of endogenous variables. There are two types of shocks that one needs to consider: shocks to the stochastic equations and shocks to the exogenous variables. Shocks to the stochastic equations can be handled by a straightforward application of stochastic simulation. Shocks to the exogenous variables are less straightforward to handle. Since by definition exogenous variables are not modeled, it is not unambiguous what one means by an exogenous variable shock. One approach is to estimate an autoregressive equation for each exogenous variable in the model and add these equations to the model. Shocks to the exogenous variables can then be handled by stochastic simulation of the expanded model.

Assume that one has a model to work with, possibly with exogenous-variable equations added, and assume that the variable of interest is real GDP. As discussed in Section 2.6, stochastic simulation can be used to estimate variances. Let $\tilde{\sigma}^2_{it}$ denote the estimated variance of real GDP (endogenous variable i) for period t, where the estimated variance is based on draws of all the error terms in the model, including the error terms in the exogenous variable equations if such equations are added. Now consider fixing one of the error terms at its expected value (usually zero) and computing the variance of GDP again. In this case the stochastic simulation is based on draws of all but one of the error terms. Let $\tilde{\sigma}^2_{it}(k)$ denote the estimated variance of real GDP based on fixing the error term in equation k at its expected value.

The difference between $\tilde{\sigma}_{it}^2$ and $\tilde{\sigma}_{it}^2(k)$ is an estimate of how much the error term in equation k contributes to the variance of GDP.[112] If, say, the variance of GDP falls by 5 percent when the error term for equation k is not drawn, one can say that equation k contributes 5 percent to the variance of GDP.

Another way to estimate this contribution would be to draw *only* the error term for equation k, compute the variance of GDP, and compare this variance to the variance when all the error terms are drawn. If the error term in equation k is correlated with the other error terms in the model, these two procedures are not the same. There is no right or wrong way of estimating this contribution, and because of the correlation, any procedure is somewhat crude. Fortunately, one can examine how sensitive the results are to the effects of the correlation of the error terms across equations to see how to weigh the results.

In the above discussion k need not refer to just one equation. One can fix the error terms in a subset k of the equations at their expected values and draw from the remaining equations. In this way one can examine the contribution that various sectors make to the variance of GDP. If the error terms across equations are correlated, then fixing, say, two error terms one at a time and summing the two differences is not the same as fixing the two error terms at the same time and computing the one difference. Again, however, one can examine the effects of the error term correlation on the results.

It is important to realize what is and what is not being estimated by this procedure. Consider an exogenous variable shock. What is being estimated is the contribution of the error term in the exogenous variable equation to the variance of GDP. This contribution is *not* the same as the multiplier effect of the exogenous variable on GDP. Two exogenous variables can have the same multiplier effects and yet make quite different contributions to the variance of GDP. If one exogenous variable fits its autoregressive equation better than does another (in the sense that its equation has a smaller estimated variance), then, other things being equal, it will contribute less to the variance of GDP. It is possible, of course, to use measures of

[112]Regarding the use of this difference as an estimate of an error term's contribution to the variance of GDP, Robert Shiller informed me that Pigou had the idea first. In the second edition of Industrial Fluctuations, Pigou (1929), after grouping sources of fluctuations into three basic categories, gave his estimate of how much the removal of each source would reduce the amplitude (i.e. the standard deviation) of industrial fluctuations. He thought that the removal of "autonomous monetary causes" would reduce the amplitude by about half. Likewise, the removal of "psychological causes" would reduce the amplitude by about half. Removal of "real causes," such as harvest variations, would reduce the amplitude by about a quarter. See Shiller (1987) for more discussion of this.

exogenous variable shocks other than error terms from autoregressive equations, but whatever measure is used, it is not likely to be the same as the size of the multiplier.

Let $\tilde{\sigma}_{it}^2$ denote the estimated variance of endogenous variable i for period t based on draws of all the error terms in the (possibly expanded) model. Let $\tilde{\sigma}_{it}^2(k)$ denote the estimated variance when the error terms in subset k of the equations are fixed at their expected values, where subset k can simply be one equation. Finally, let $\tilde{\delta}_{it}(k)$ be the difference between the two estimated variances:

$$\tilde{\delta}_{it}(k) = \tilde{\sigma}_{it}^2 - \tilde{\sigma}_{it}^2(k) \tag{1}$$

Because of the correlation of the error terms across equations, it can turn out that $\tilde{\delta}_{it}(k)$ is negative for some choices of k. Also, as noted above, it is not in general the case that $\tilde{\delta}_{it}(k)$ for, say, k equal to the first and second equations is the same as $\tilde{\delta}_{it}(k)$ for k equal to the first equation plus $\tilde{\delta}_{it}(k)$ for k equal to the second equation.

Applications of this procedure are in Fair (1988) and in Fair (1994, Section 11.4). These results show that there are a number of important contributors to the overall variance of, say, real GDP or the GDP deflator. It is not the case that only one or two shocks dominate. There appear to be no simple stories that can be told about the sources of output and price variability.

4.8.2 Performance Measures

It is common practice in political discussions to hold policymakers accountable for the state of the economy that existed during their time in power. Policy makers are generally blamed for high unemployment, low real growth, and high inflation during their time in power and praised for the opposite. Although at first glance this may seem to be a reasonable way of evaluating the economic performances of policymakers, there are at least two serious problems with it. The first is that this kind of evaluation does not take into account possible differences in the degree of difficulty of controlling the economy in different periods. The economy may be more difficult to control at one time than another either because of more unfavorable values of the uncontrolled exogenous variables or because of a more unfavorable initial state of the economy (or both). The second problem with the evaluation is that it ignores the effects of a policymaker's actions on the state of the economy beyond its time in power. If, for example, a policymaker strongly stimulates the economy near the end of its time in power, most of the inflationary effects of this policy might not be felt until after the policy maker is out of power. Any evaluation of performance that was concerned only with the time the policymaker was in power would not, of course, pick up these effects.

Optimal control analysis can be used to handle these problems. Consider the following measure of performance. Let P denote the period that policymaker p is in power. The measure, denoted M, is as follows (low values of M are good):

M = expected loss in P given p's actual behavior
$-$ expected loss in P had p behaved optimally
$+$ expected loss beyond P given p's actual behavior and given optimal behavior of future policymakers
$-$ expected loss beyond P given that p behaved optimally and given optimal behavior of future policymakers

$$= a - b + c - d.$$

The term $a - b$ is the expected loss that could have been avoided during P had p behaved optimally. The term $c - d$ is the potential expected loss to future policymakers from the fact that p did not behave optimally.

M takes account of the two problems mentioned above. If the economy was difficult to control for p, the b will be large, which will offset more than otherwise a large value of a. The term $c - d$ measures the effects of p's policies on the economy beyond P, where these effects are measured under the assumption that future policymakers behave optimally.

The optimal control techniques discussed in Section 2.10 can be used to estimate M. This requires choice of a model, the postulation of a loss function, the choice of a control variable or variables, and assumptions about what the policymaker knows at the time the control problem is solved.

Consider the following example. The policymaker is the Fed. The loss for quarter t, H_t, is equation (3) in Subsection 4.4.4. The loss is assumed to be additive across time. The control variable is the short-term interest rate, RS_t. The first quarter of the control horizon is 1, and the Fed solves the control problem at the beginning of quarter 1 not knowing the error terms for quarter 1 nor the error terms for future quarters. Regarding the exogenous variables, assume that an autoregressive equation has been estimated for each exogenous variable and these equations added to the model. This results in an expanded model with no exogenous variables, where the errors terms in the exogenous-variable equations are treated just like the structural error terms.[113]

[113]Other assumptions about the exogenous variables could be made. The key question is what

The optimal control techniques discussed in Section 2.10 can be used to compute the optimal value of RS for quarter 1, denoted RS_1^*. This requires solving the control problem for quarters 1 through k, where k is large enough such that increasing it by one have a trivial effect on RS_1^*. Zero values would be used for all the errors, including the errors in the exogenous-variable equations.

Given RS_1^*, solve the model for quarter 1 using the actual (historical) values of all the errors for quarter 1. Let y_1^* denote the solution values of the endogenous variables for quarter 1. Now solve the control problem at the beginning of quarter 2, where the values y_1^* are used (for the lagged endogenous variable values) and RS_1^* is used (if it enters with a lag in the model). Zero values are used for the errors for quarter 2 on. The control problem would be solved for quarters 2 through $k+1$. This gives an optimal value of RS for quarter 2, denoted RS_2^*. Note that this value in general differs from the optimal value of RS for quarter 2 computed from the control problem beginning in quarter 1, where the errors for quarter 1 were not known.

Given RS_2^* (as well as RS_1^* and y_1^*), solve the model for quarter 1 using the actual (historical) values of all the errors for quarter 2. Let y_2^* denote the solution values of the endogenous variables for quarter 2. Now solve the control problem at the beginning of quarter 3, and so on. This process can be repeated for as many quarters as desired. Each solution is for a deterministic control problem of length k, which is straightforward to solve even for a large nonlinear model.

Given this work, M can then be estimated. Say one is interested in comparing two Fed chairs, one who was in power from T_1 through T_2 and the other who was in power from $T_2 + 1$ through T_3. Let H_t^a denote the loss in quarter t using RS_t and the relevant actual values in y_t (and relevant lagged actual values), and let H_t^* denote the loss in quarter t using RS_t^* and the relevant optimal values in y_t^* (and relevant lagged optimal values).

For the first Fed chair, a above is the sum of H_t^a for $t = T_1 \ldots T_2$, and b is the sum of H_t^* for $t = T_1 \ldots T_2$. d is the sum of H_t^* for $t = T_2 + 1 \ldots T_2 + 1 + q$, where the optimal control problems through quarter $T_2 + 1 + q$ are computed, where q is a chosen length for the end of the comparison horizon. To compute c a new set of optimal control problems have to be solved beginning in quarter $T_2 + 1$. For this second set of optimal control problems, actual values would be used for quarters T_2 and back. c is then the sum of H_t^{**} for $t = T_2 + 1 \ldots T_2 + 1 + q$, where the notation ** means the optimal values for the second set of control problems.

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the Fed knows about the current and future values of the exogenous variables at the beginning of quarter 1.

These calculations allow M to be computed for the first Fed chair. The calculations for the second Fed chair are similar. The calculations from the second set of control problems just mentioned would be used, and new calculations for a third set of control problems would be need to compute c. If the number of quarters between T_2 and T_1 is not the same as the number between T_3 and $T_2 + 1$, M should be divided by the number of quarters in power for the comparison.

Comparisons similar to the above were carried out in Fair (2007) for past Fed chairs, where c and d were not computed. Similar comparisons were also carried out in Fair (1978a) for past U.S. presidents.

Comparisons using M are likely to be sensitive to the choice of the loss function, which makes any particular comparison somewhat problematic. Also, it has to be assumed that whatever model is used was available to the policymaker at the time the control problems were solved, which is problematic for at least early periods. Estimates of M are thus probably of limited used, but perhaps not zero.

4.9 How Might a Central Bank Report Uncertainty?

4.9.1 Introduction

An interesting question for central banks is how they should report the uncertainty of their forecasts. This section discusses what a central bank (CB) could report in a world in which it used a single macroeconometric model to make its forecasts and guide its policies. Suggestions are then made as to what might be feasible for a CB to report given that it is unlikely to be willing to commit to a single model.

The discussion in this section is simply applying the procedures discussed in earlier sections, particularly Sections 2.5, 2.6, 2.7, 2.10, and 2.12, to the question at hand. It assumes that this earlier material has been read, although there is some repetition of earlier discussion below. The general model used is the model presented in Section 2.1. It will be assumed that this model has been estimated by some consistent technique, say 2SLS or FIML. The solution of the model is as discussed in Section 2.5 for the non RE version and Subsection 2.12.2 for the RE version. Stochastic simulation and bootstrapping are as discussed in Sections 2.6 and 2.7 and Subsection 2.12.4. All of the discussion in this section is based on the use of the certainty equivalence assumption, which is discussed in Section 2.11.

4.9.2 Computing Standard Errors of Forecasts

The aim in this section, as should be the aim of a CB, is to estimate as precisely as possible standard errors of forecasts. The complication regarding a CB is that it controls a key variable in the model, namely the short term interest rate. There are at least three assumptions that can be made about CB behavior when computing standard errors. One is that the CB simply sets a path of the interest rate and never deviates from this path. This is, of course, an unrealistic assumption since any CB responds to surprise changes in the economy. The second assumption is that the CB at the beginning of each period solves an optimal control problem to choose the interest rate path. The third is that the CB uses an interest rate rule, which then simply makes the interest rate an endogenous variable in the model. We consider first the case in which the CB is using an interest rate rule.

Using an Interest Rate Rule

Assume that the CB has made at the beginning of period $T+1$ a "baseline" forecast for $T+1$ and beyond. This forecast would be based on a set of coefficient estimates, lagged endogenous variables, and choices of current and future values of the exogenous variables and errors. For RE models values of the exogenous variables and errors would be needed beyond the end of the forecast horizon (for the solution of the model through the forecast horizon).

For a model estimated for periods 1 through T, estimates of u_t, $t = 1, \ldots, T$, are available. These are the estimated residuals from which draws can be made for stochastic simulations. Consider the solution for period $T+1$ for a non RE model. For a given draw u^j_{T+1} the model can be solved, producing solution values y^j_{T+1}. Doing this J times produces a distribution of solution values, from which measures of central tendency and dispersion can be computed. The mean for endogenous variable i is

$$\bar{y}_{iT+1} = \frac{1}{J} \sum_{j=1}^{J} y^j_{iT+1} \tag{1}$$

and the variance is

$$\sigma^2_{iT+1} = \frac{1}{J} \sum_{j=1}^{J} (y^j_{iT+1} - \bar{y}_{iT+1})^2 \tag{2}$$

These two equations are the equivalent of equations (7.7) and (7.8) in the link in Section 2.6. $\bar{y}_{iT+1}$ is an estimate of the expected value of y_{iT+1}.

For a dynamic simulation of, say, two periods, errors would be drawn for periods $T+1$ and $T+2$ and the model solved for the two periods using these draws. Solution values for the two periods would be computed for each endogenous variable, which is one trial. J trials would be done, producing J values of the predictions for each of the two periods. As many periods ahead can be done as desired. If the CB were doing this, the interest rate rule would presumably be deterministic, and so errors would not be drawn for it.

For RE models if agents do not observe the error draw and continue to form expectations using zero values for the errors for periods $T+1$ and beyond, the story for period $T+1$ is the same as for non RE models. The expectations are predetermined regarding the draws. For a dynamic simulation the story for period $T+2$ is different. Using the error draw for $T+1$, the final solution values of the endogenous variables for $T+1$ are different from what the agents expected them to be, unlike in the deterministic case. At the beginning of $T+2$ they would use the observed (solution) values of the endogenous variables for $T+1$ in forming their expectations for $T+2$ and beyond. The EP method, discussed in Subsection 2.12.2, must thus be used for each period solved, not just for period $T+1$ as in the deterministic case. For a horizon of r periods, r error vectors would be drawn and the EP method used r times. This is one trial. After J trials there would be

J solution values of each endogenous variable for each period, as in the non RE case.

Optimal Control: Deterministic Case

Now consider the case in which the CB sets the interest rate by solving an optimal control problem. For a linear non RE model and a quadratic objective function, analytic, closed-form solutions are available—see, for example, Chow (1975). In this case, if the Fed reported the feedback equation, the coefficients in the model, and the covariance matrix of the errors, users would have all the information they need to compute means and variances.

In practice models are not linear, objective functions are generally not quadratic, and there may be rational expectations. The following is a more general problem. Consider the deterministic case first. Assume that the horizon is $t = T+1, \ldots, T+r$ and that the objective is to maximize the expected value of W, where W is

$$W = g(y_{T+1}, \ldots, y_{T+r}, x_{T+1}, \ldots, x_{T+r}) \tag{3}$$

In most applications the objective function is assumed to be additive across time, which means that (3) can be written

$$W = \sum_{t=T+1}^{T+r} g_t(y_t, x_t) \tag{4}$$

These two equations are the equivalent of equations (10.2) and (10.3) in the link in Section 2.10.

Assume that the control variable of the CB, the short term interest rate, is variable x_{1t}, and let $z = (x_{1T+1}, \ldots, x_{1T+r})$. If all the errors are set to zero, then for each value of z one can compute a value of W by first solving the model for $y_{T+1}, \ldots, y_{T+r}$ and then using these values along with the values for $x_{T+1}, \ldots, x_{T+r}$ to compute W in (3) or (4). Stated this way, the optimal control problem is choosing values (the elements of z) to maximize an *unconstrained* non-linear function. By substitution, the constrained maximization problem is transformed into the problem of maximizing an unconstrained function of the control variables:

$$W = \Phi(z) \tag{5}$$

where Φ stands for the mapping $z \longrightarrow y_{T+1}, \ldots, y_{T+r}, x_{T+1}, \ldots, x_{T+r} \longrightarrow W$. (Equation (5) is equation (10.4) in the link in Section 2.10.) Given this setup, the problem can be turned over to a nonlinear maximization algorithm like DFP. For each iteration of the algorithm, the derivatives of Φ with respect to the elements of z, which are needed by the algorithm, can be computed numerically. An algorithm like DFP is generally quite good at finding the optimum for a typical control problem—see the discussion in the link in Section 2.10.

Regarding the choice of r, the end of the horizon, it is sometimes the case that unusual results are obtained near the end of the horizon because there is no tomorrow. In practice the end of the horizon should be taken to be large enough so that end-of-horizon effects have small effects on the earlier control values of interest. It will be assumed that this has been done in the following discussion. r should thus be thought of as much larger than the actual horizon of interest.

In practice the CB would solve its control problem at the beginning of $T+1$ and implement x^*_{1T+1}, the optimal value of the funds rate for period $T+1$. It could also announce its plans for periods after that: $x^*_{1T+2}, \ldots, x^*_{1T+r}$. In the deterministic case this would be it. The CB would simply implement the optimal values for $T+2$ and beyond as the time came. In practice, of course, the world is not deterministic, and the economy in $T + 1$ would not be what the CB expected it to be when it solved its control problem. After period $T + 1$ is over, the CB at the beginning of $T + 2$ could reoptimize. The optimal value for $T + 2$ would no longer be the value it computed at the beginning of $T + 1$ because the errors for $T + 1$ would not in general be zero, which was what the CB was assuming at the beginning of $T + 1$. The CB could thus behave by solving a series of open-loop optimal control problems, one at the beginning of each new period.

Turn now to the RE case and assume that the agents know what the CB is doing. It is still the case that one can compute a value of W given a value of z. The extra work in the RE case is that the EP method must be used in the solution of the model. For a given z the expectations would be computed first and then the model solved. The CB would assume zero current and future errors when solving its control problem, as would the agents in computing their expectations. The DFP algorithm could still be used to find the optimal value of z, and this value would be consistent with the expectations of the agents.

Optimal Control: Stochastic Case

Consider now the stochastic case and consider non RE models first. One trial is a draw of $u^j_{T+1}, \ldots, u^j_{T+r}$. At the beginning of $T + 1$ the computed optimal value of z is not affected by the draws because the CB assumes zero current and future errors. x^*_{1T+1} is implemented at the beginning of $T + 1$. The solution of the model for $T+1$ uses this value and the error draw for $T+1$. Then at the beginning of $T+2$ the process is repeated, where the CB uses the solution values of the endogenous variables for $T+1$ and the assumption of zero errors for $T+2$ and beyond. x^*_{1T+2} is implemented at the beginning of $T+2$, and when the model is solved the error draw for $T+2$ is used. The optimal value of x_{1T+2} is different from what the CB computed at the beginning of $T+1$ because the actual error draws for $T+1$ are not in general zero. This process is repeated r times. This is one trial, so each trial requires the solution of r optimal control problems. After J trials are preformed, the J values of each endogenous variable for each period can be used to compute variances. These variances would incorporate the optimal behavior of the CB.

Now consider the RE case. Since agents also use zero current and future errors in computing their expectations, the expectations solved at the beginning of $T+1$ are the same as in the deterministic case since the optimal value of the control variable for $T+1$ is the same. Again, for each value of z tried by the algorithm, the EP method must be used. At the beginning of $T+2$ the process is repeated, just as in the non RE case, where the agents, along with the CB, use the solution values of the endogenous variables for $T+1$, which are affected by the error draw. The process is done r times, which is then one trial.

The RE case is thus no different from the non RE case except that the EP method must be used each time W is computed. This is expensive, and various tricks would probably be needed in practice to lessen computational time.

An Example using the MC Model

The following is an example using stochastic simulation and the MC model. It assumes that the Fed behaves according to the estimated interest rate rule—equation 30. There are 1,689 estimated equations in the MC model, of which 1,379 are trade share equations. The estimation period for the United States is 1954:1–2013:3. The estimation periods for the other countries begin as early as 1962:1 and end as late as 2013:2. The estimation period for most of the trade share equations is 1966:1–2012:4. For each estimated equation there are estimated residuals over the estimation period. Let $\hat{u}_t$ denote the 1689-dimension vector of the estimated residuals for quarter t.[114] Most of the estimation periods have the 1972:1–2007:4

[114]For equations estimated using annual data, the error is put in the first quarter of the year with zeros in the other three quarters (which are never used). If the initial estimate of an equation suggests that the error term is serially correlated, the equation is reestimated under the assumption that the error term follows an autoregressive process (usually first order). The structural coefficients

period—144 quarters– in common, and this period is taken to be the "base" period. These 144 observations on $\hat{u}_t$ are used for the draws in the stochastic-simulation procedure discussed below.[115]

The present results are based on a forecast I made on November 11, 2013 using the MC model. The forecast is based on a particular set of exogenous variable values and zero error values. It yields a predicted value of each endogenous variable for each quarter. These values are available on my website.[116] The period for the present experiment is 2014:1–2022:4—36 quarters. The interest here is estimating the uncertainty around each predicted value. Each trial of the stochastic simulation procedure is as follows. First, 36 error vectors are drawn with replacement from the 144 vectors of residuals, each with probability 1/144. (Each vector consists of 1,689 residuals.) Using these error vectors, one per quarter, the model is solved dynamically for the 2014:1–2022:4 period. The same set of exogenous variable values is used as was used for the baseline forecast. The solution values are recorded. This is one trial. The procedure is then repeated, say, J times. This gives J values of each endogenous variable for each quarter, from which measures of dispersion can be computed. Results are presented in Table 1. They are based on 1000 trials. (There were no solution failures on any of the trials.)

In the MC model there is an estimated interest rate rule for each major country, including the United States, and so monetary policy is endogenous in the model. For the results in Table 1 errors were drawn for the interest rate rules except for the rule for the United States. For the United States the estimated rule without any errors was taken to be the exact rule that the Fed uses. When for a particular draw the rule called for a negative interest rate, zero was used instead.

Remember that the results in Table 1 are based on historical residuals between 1979 and 2007, i.e., historically observed uncertainty. The residuals are assumed to be iid since, as discussed above, serial correlation has been removed when necessary by estimating autoregressive error coefficients.

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in the equation and the autoregressive coefficient or coefficients are jointly estimated (by 2SLS). The $\hat{u}_t$ error terms are after adjustment for any autoregressive properties, and they are taken to be iid for purposes of the draws. As discussed below, the draws are by year—four quarters at a time.

[115]If an estimation period does not include all of the 1972:1–2007:4 period, zero errors are used for the missing quarters.

[116]For countries other than the United States data were not available as late as 2013:3, and the overall forecast begain earlier 2013:4, with actual values used for the United States until 2013:4.

Table 1
Estimated Standard Errors of Forecasts for the MC Model
Values are in Percentage Points

qtr	Y	UR	P	D	R
2014:1	0.48	0.21	0.32	0.40	0.18
2014:2	0.83	0.31	0.42	0.69	0.30
2014:3	1.11	0.44	0.54	0.99	0.34
2014:4	1.38	0.64	0.76	1.30	0.48
2015:1	1.57	0.82	0.84	1.54	0.66
2015:2	1.74	0.93	0.93	1.80	0.77
2015:3	1.85	1.00	1.06	2.05	0.82
2015:4	1.94	1.06	1.24	2.27	0.92
2016:4	2.05	1.11	1.72	2.82	1.10
2017:4	2.12	1.16	2.00	3.21	1.19
2018:4	2.15	1.18	2.25	3.53	1.26
2022:4	2.38	1.25	2.91	5.01	1.35

Y = real GDP
UR = unemployment rate
P = GDP deflator
D = nominal federal debt/nominal GDP
R = three-month Treasury bill rate
Results based on 1000 trials

The estimated standard errors in Table 1 increase with the length of the horizon, as expected. After four quarters the standard errors are 1.38 for real GDP, 0.64 for the unemployment rate, 0.76 for the GDP deflator, 1.30 for the debt/GDP ratio, and 0.48 for the interest rate. After eight quarters the respective standard errors are 1.94, 1.06, 1.24, 2.27, and 0.92. Note that even though no errors are drawn for the Fed interest rate rule, there is still considerable variation in the interest rate as the Fed reacts to the shocks.

The results in Table 1 are thus illustrative of what the Fed could report. Even though the MC model is large, the time taken per solution is modest, and the 1000 trials for Table 1 took about 6 hours on a high-end laptop. The time would, of course, be larger if the model were a RE model or if optimal control problems were solved for each trial.

4.9.3 What Could a Central Bank Do In Practice?

Interest rate rules, as estimated and used in the MC model, are likely too simple an explanation of CB behavior. The solution of formal optimal control problems is likely too complicated. The following is something in between that a CB might do that would allow uncertainty estimates to be computed.

First, the CB must begin with a baseline forecast using a model. The model could, however, be subjectively adjusted by using non zero values of various current and future errors. Error values could be chosen to have the forecast from the model be what the CB thinks is most likely. These error values would then be taken to be exogenous. This baseline forecast would include the CB's chosen interest rate path.

Second, the CB must have a way of drawing values of the errors in the model. This could be from a set of historically estimated residuals or from an assumed probability distribution with a computed covariance matrix.

The procedure could then be as follows. At the beginning of $T + 1$ draw error vectors for $T + 1$ through $T + r$ and solve the model using these errors and the baseline path of the funds rate. The error values added would be on top of any of the exogenous error values discussed above. If the model is an RE model, the EP or similar method would have to be used in the solution. This solution path will obviously differ from the baseline path, and it is likely that if this path actually occurred, the CB would not pick the baseline interest-rate path. The next step is thus for the CB to change the interest-rate path to be the one it would pick if the particular error draw occurred. This could possibly be done by hand, with a few iterations needed to find the desired path. Or possibly this could be turned over to an algorithm. At any rate, at the end the interest-rate path would be consistent (in the eyes of the CB) with the particular error draws. This is one trial. Now do this J times and compute measures of uncertainty for the J values of each endogenous variable for each period and for the interest rate.

The CB could also do this for more than one model and report more than one set of results. This is likely to be more informative than to try to average uncertainty estimates across models and report the averages.

A key advantage of a procedure like this is that there is a different interest-rate path for each set of error draws, namely a path appropriate to that particular set. The path could be chosen by a rule, by solving optimal control problems, or in the in between manner discussed above. It is obviously not appropriate to keep the baseline interest-rate path the same for each set of draws, and a key part of any reporting should be estimates of the variances of the interest rates.

The procedure discussed here differs from that of the Bank of Norway. The Bank of Norway—Aastveit, Gerdrup, and Jore (2011) and Gerdrup and Nicolaisen (2011)—reports forecast densities. It has 221 models for forecasting GDP and 171 models for forecasting the CPI. Each model produces density forecasts, and these densities are then combined into one for each of the two variables. The combined forecasts are then fed into a DSGE model (NEMO) for policy analysis. The policy analysis in NEMO is conditional on these forecasts and judgment. The density forecasts from the individual models are not affected by the policy rate, and so the procedure used by the Bank of Norway is not the same as the one recommended above. No single model is used in the analysis. The final density forecasts from NEMO are in part based on the individual model's density forecasts and in part on judgment. An alternative procedure would be to use a single model for the entire analysis, as just outlined, but then report results from many different models. There would then be consistency within each model.

4.9.4 Conclusion

Using certainty equivalence, it is feasible to compute measures of dispersion using stochastic simulation. This can be done for large nonlinear simultaneous equations models, including those with rational expectations. It is also possible to incorporate optimal control behavior into the analysis. This framework, or an approximation to it, could be used by monetary authorities in reporting uncertainty estimates. The key ingredients needed are 1) a model, possibly subjectively adjusted, 2) a set of historically estimated errors for drawing or an estimated probability distribution of errors, and 3) a way of changing the optimal path of the interest rate when errors are drawn, either an interest rate rule, optimal control, or something in between.

5 Analysis of the Economy using the MC Model

This part uses the MC model to examine various question about the economy, mostly the United States economy.

5.1 Estimated European Inflation Costs from Expansionary Policies[117]

5.1.1 Introduction

If macroeconomic policies had lowered European unemployment in the 1980s, what would have been the inflation costs? Under the NAIRU model discussed in Section 3.13, this is not an interesting question. In that model there is a value of the unemployment rate (the NAIRU) below which the price level accelerates and above which the price level decelerates. This view of the inflation process is echoed, for example, in *Unemployment: Choices for Europe*, where Alogoskoufis et al. (1995, p. 124) state "We would not want to dissent from the view that there is no long-run trade-off between activity and inflation, so that macroeconomic policies by themselves can do little to secure a lasting reduction in unemployment." Under this view it is not sensible to talk about long-run tradeoffs between unemployment and inflation.

Since the results in Section 3.13 call into question the NAIRU dynamics, it is of interest to see what an alternative model would say about the European inflation cost question. This chapter uses the MC model to estimate what would have happened to European unemployment and inflation in the 1982:1–1990:4 period had the Bundesbank followed an easier monetary policy than it in fact did.

If the true relationship between the price level and unemployment is highly nonlinear at low values of the unemployment rate, a view put forth in Subsection 3.13.6, it is problematic to consider policy experiments in which unemployment rates are pushed to very low values. Due to few observations at low unemployment rates, it is not possible to pin down the point at which the relationship becomes highly nonlinear (if it does), and so the estimated price equations are not reliable at low values of the unemployment rate. For present purposes, however, this is not likely to be a problem because the experiment is over a period in which unemployment was generally quite high.

5.1.2 The Experiment

The Setup

The experiment is a decrease in the German short-term interest rate between 1982:1 and 1990:4. To perform this experiment the interest rate rule of the Bundesbank

[117] Some of the discussion in this section is taken from Fair (1999). The results in Table 1 are updates of those in Table 3 in Fair (1999).

was dropped, and the German short-term interest rate was taken to be exogenous. The interest rate rules for all the other countries in the model were retained, which means, for example, that the fall in the German rate directly affects the interest rates of the countries whose rules have the German rate as an explanatory variable. The German interest rate was lowered by 1 percentage point for 1982:1-1983:4, by .75 percentage points for 1984:1-1985:4, by .5 percentage points for 1986:1-1987:4, and by .25 percentage points for 1988:1-1990:4.

The first step is to add the estimated residuals to the model and take them to be exogenous. Doing this and then solving the model using the actual values of all the exogenous variables results in a perfect tracking solution. The German interest rate is then lowered and the model is solved. The difference between the predicted value for each variable for each period from this solution and its actual value is the estimated effect of the monetary-policy change on the variable. Selected results of this experiment are presented in Table 1 for 6 countries: Germany, France, Italy, the United Kingdom, the United States, and Japan. Each fourth-quarter value is presented in the table.

The second column in Table 1, labeled UR, gives the actual value of the unemployment rate in percentage points, and the third column, labeled π, gives the actual value of the inflation rate (percentage change in the GDP price deflator at an annual rate) in percentage points. These values are provided for reference purposes. The values in the remaining columns are either absolute or percentage changes from the base values (remember that the base values are the actual values). Absolute changes are given for the interest rate, the unemployment rate, the inflation rate, and the current account as a fraction of GDP, while percentage changes are given for the other variables. All the values are in percentage points.

Table 1
Effects of a Decrease in the German Interest Rate in 1982:1–1990:4

Qtr. Ah.	Act. Values UR	π	Deviations from Base Values RS	E	Y	UR	PY	π	PM	PX	IM	EX	S^*
GE													
4	5.31	1.93	-1.00	1.58	0.07	-0.04	0.02	0.03	0.79	0.21	-0.03	0.06	-0.13
8	5.86	3.30	-1.00	2.76	0.14	-0.09	0.07	0.07	1.23	0.38	-0.10	0.13	-0.17
12	5.83	4.05	-0.75	3.20	0.21	-0.15	0.15	0.10	1.41	0.50	-0.19	0.21	-0.14
16	5.83	4.07	-0.75	3.58	0.28	-0.21	0.27	0.13	1.55	0.63	-0.27	0.30	-0.10
20	5.43	1.59	-0.50	3.50	0.34	-0.27	0.41	0.16	1.49	0.72	-0.34	0.40	-0.01
24	5.57	3.93	-0.50	3.53	0.40	-0.33	0.58	0.18	1.61	0.87	-0.39	0.47	0.02
28	5.41	4.20	-0.25	3.21	0.44	-0.37	0.77	0.20	1.57	1.00	-0.42	0.50	0.08
32	4.90	2.96	-0.25	3.06	0.47	-0.41	0.97	0.21	1.61	1.16	-0.43	0.45	0.10
36	4.25	0.99	-0.25	3.02	0.48	-0.43	1.19	0.21	1.68	1.33	-0.43	0.35	0.11
FR													
4	7.00	7.35	-0.52	1.62	0.04	-0.01	0.13	0.24	0.81	0.33	-0.08	0.01	-0.10
8	7.50	6.52	-0.67	2.92	0.13	-0.04	0.41	0.37	1.38	0.72	-0.15	0.02	-0.13
12	8.70	4.61	-0.57	3.57	0.21	-0.08	0.77	0.39	1.74	1.10	-0.14	0.00	-0.14
16	8.90	5.50	-0.53	4.19	0.28	-0.13	1.15	0.42	2.18	1.48	-0.09	-0.03	-0.16
20	9.00	2.87	-0.39	4.34	0.33	-0.19	1.48	0.31	2.18	1.76	0.06	-0.02	-0.10
24	9.00	3.29	-0.34	4.56	0.37	-0.24	1.79	0.31	2.52	2.06	0.19	-0.01	-0.14
28	8.60	4.58	-0.19	4.38	0.39	-0.28	2.06	0.24	2.62	2.28	0.33	-0.01	-0.15
32	8.00	4.65	-0.15	4.31	0.38	-0.30	2.27	0.18	2.76	2.46	0.46	-0.02	-0.18
36	7.90	2.75	-0.14	4.29	0.36	-0.31	2.42	0.14	2.89	2.58	0.54	-0.01	-0.21
IT													
4	6.87	23.47	0.03	1.59	0.02	0.00	0.11	0.21	0.88	0.41	-0.09	0.08	-0.09
8	7.72	16.99	0.10	2.80	0.06	-0.02	0.33	0.31	1.42	0.80	-0.23	0.20	-0.08
12	8.17	9.59	0.19	3.31	0.10	-0.04	0.60	0.32	1.66	1.09	-0.34	0.31	-0.04
16	8.63	9.37	0.26	3.78	0.13	-0.07	0.91	0.34	1.94	1.37	-0.41	0.36	-0.01
20	9.99	6.43	0.30	3.80	0.13	-0.09	1.16	0.24	1.72	1.52	-0.39	0.37	0.07
24	10.11	8.38	0.32	3.92	0.14	-0.10	1.39	0.25	1.96	1.73	-0.37	0.38	0.06
28	10.03	4.97	0.33	3.69	0.13	-0.11	1.60	0.20	1.99	1.87	-0.33	0.38	0.07
32	9.50	8.37	0.32	3.60	0.12	-0.11	1.78	0.19	2.12	2.00	-0.28	0.33	0.06
36	8.64	6.70	0.30	3.61	0.10	-0.10	1.96	0.19	2.33	2.14	-0.26	0.28	0.04
UK													
4	11.08	8.55	-0.01	1.23	0.01	0.00	0.04	0.12	0.44	0.20	-0.01	0.01	-0.08
8	11.66	5.55	-0.02	2.13	0.01	0.00	0.15	0.15	0.59	0.38	-0.02	0.02	-0.06
12	11.63	5.27	-0.02	2.44	0.02	-0.01	0.27	0.12	0.61	0.51	-0.01	0.03	-0.03
16	11.28	5.57	-0.02	2.71	0.03	-0.01	0.36	0.07	0.59	0.57	0.01	0.08	0.02
20	11.27	3.46	-0.02	2.62	0.04	-0.02	0.43	0.06	0.58	0.59	0.03	0.11	0.02
24	9.68	3.65	-0.01	2.63	0.06	-0.03	0.50	0.07	0.66	0.65	0.05	0.15	0.02
28	8.00	10.01	0.01	2.35	0.06	-0.04	0.57	0.06	0.68	0.70	0.07	0.16	0.02
32	6.99	7.45	0.02	2.21	0.07	-0.05	0.62	0.05	0.73	0.73	0.08	0.18	0.02
36	7.46	6.25	0.03	2.15	0.07	-0.06	0.68	0.05	0.78	0.77	0.09	0.17	0.01
US													
4	10.68	4.53	-0.02	-NaN	0.02	-0.01	-0.04	-0.05	-0.43	-0.15	0.11	-0.05	0.02
8	8.54	2.73	-0.03	-NaN	0.05	-0.02	-0.12	-0.08	-0.80	-0.31	0.29	-0.10	0.02
12	7.28	2.71	-0.03	-NaN	0.07	-0.03	-0.21	-0.08	-1.01	-0.41	0.47	-0.10	0.02
16	7.05	2.33	-0.03	-NaN	0.09	-0.04	-0.30	-0.08	-1.16	-0.52	0.61	-0.07	0.02
20	6.83	2.82	-0.03	-NaN	0.09	-0.04	-0.38	-0.08	-1.22	-0.59	0.69	-0.01	0.01
24	5.87	3.56	-0.02	-NaN	0.09	-0.04	-0.44	-0.06	-1.15	-0.63	0.69	0.04	0.00
28	5.35	3.23	-0.01	-NaN	0.08	-0.03	-0.47	-0.03	-0.99	-0.61	0.62	0.10	-0.01
32	5.37	2.81	0.00	-NaN	0.07	-0.03	-0.48	-0.01	-0.88	-0.61	0.53	0.12	-0.01
36	6.11	3.06	0.00	-NaN	0.06	-0.02	-0.49	0.00	-0.80	-0.60	0.44	0.14	-0.01

Table 1 (continued)

Qtr. Ah.	Act. Values UR	π	Deviations from Base Values RS	E	Y	UR	PY	π	PM	PX	IM	EX	S^*
JA													
4	2.46	-0.58	-0.01	-0.01	0.00	0.00	0.00	0.00	-0.19	-0.44	0.02	0.01	-0.04
8	2.62	1.43	-0.02	-0.01	0.00	0.00	-0.01	-0.01	-0.41	-0.79	0.06	0.06	-0.06
12	2.68	2.75	-0.02	0.01	0.01	0.00	-0.02	-0.01	-0.54	-0.90	0.12	0.15	-0.06
16	2.78	1.84	-0.02	0.04	0.02	0.00	-0.03	-0.01	-0.65	-1.03	0.17	0.21	-0.06
20	2.79	-0.48	-0.02	0.09	0.02	0.00	-0.04	-0.01	-0.77	-1.01	0.24	0.27	-0.04
24	2.67	-0.47	-0.02	0.15	0.01	0.00	-0.05	-0.01	-0.71	-0.95	0.29	0.25	-0.04
28	2.40	1.51	-0.01	0.20	0.00	0.00	-0.06	-0.01	-0.62	-0.76	0.32	0.20	-0.03
32	2.19	2.28	-0.01	0.25	-0.01	0.00	-0.07	-0.01	-0.47	-0.64	0.33	0.16	-0.04
36	2.09	4.61	0.00	0.29	-0.01	0.00	-0.07	0.00	-0.34	-0.57	0.33	0.11	-0.05

E = exchange rate, local currency per $.
EX = real level of exports.
IM = real level of imports.
PM = import price deflator.
PX = export price index.
PY = GDP price deflator.
π = percentage change in PY.
RS = three-month interest rate.
S^* = current account as a percent of nominal GDP.
UR = unemployment rate.
Y = real GDP.

Qualitative Discussion

Before discussing the numbers, it will be useful to review qualitatively what is likely to happen in the model in response to the decrease in the German interest rate.[118] Consider first the effects of an interest rate decrease in a particular country. A decrease in the short-term rate in a country leads to a decrease in the long-term rate through the term structure equation. A decrease in the short-term rate also leads to a depreciation of the country's currency (assuming that the interest rate decrease is relative to other countries' interest rates). The interest rate decreases lead to an increase in consumption and investment. The depreciation of the currency leads to an increase in exports. The effect on exports works through the trade-share equations. The dollar price of the country's exports that feeds into the trade-share equations is lower because of the depreciation, and this increases the share of the other countries' total imports imported from the particular country. The effect on aggregate demand in the country from the interest rate decrease is thus positive from the increase in consumption, investment, and exports.

There are two main effects on imports, one positive and one negative. The

[118]It may also be useful to review the qualitative discussion in Section 3.5 regarding the effects of a depreciation and an interest rate decrease in the MC model. Some of the discussion here repeats this earlier discussion.

positive effect is that consumption and investment are higher, some of which is imported. The negative effect is that the price of imports in higher because of the depreciation, which has a negative effect on the demand for imports. The net effect on imports can thus go either way.

There is also a positive effect on inflation. As just noted, the depreciation leads to an increase in the price of imports. This in turn has a positive effect on the domestic price level through the price equation. In addition, if aggregate demand increases, this increases demand pressure, which has a positive effect on the domestic price level.

There are many other effects that follow from these, including effects back on the short-term interest rate itself through the interest rate rule, but these are typically second order in nature, especially in the short run. The main effects are as just described.

The decrease in the German interest rate should thus stimulate the German economy, depreciate the DM, and lead to a rise in the German price level. How much the price level rises depends, among other things, on the size of the coefficient estimate of the demand pressure variable in the German price equation. The size of the price level increase also depends on how much the DM depreciates and on the size of the coefficient estimate of the import price variable in the price equation.

For those European countries whose interest rate rules include the German interest rate as an explanatory variable, the fall in the German rate will lead to a direct fall in their interest rates. In addition, the depreciation of the DM (relative to the dollar) will lead to a depreciation of the other European countries' currencies (relative to the dollar) because they are fairly closely tied to the DM in the short run through the exchange rate equations.

The Results

Turn now to the results in Table 1. By the end of the nine-year period the German exchange rate relative to the dollar, E, depreciated 3.02 percent, the price level, PY, was 1.19 percent higher, the inflation rate, π, was .31 percentage points higher, and the unemployment rate, UR, was 0.43 percentage points lower—all compared to the base case (the actual values). (An increase in E for a country is a depreciation of the country's currency relative to the dollar.) The current account as a percent of GDP, S^*, was initially lower and then higher.

The interest rate, RS, for France fell because French monetary policy is directly affected by German monetary policy. (The German interest rate is an explanatory variable in the French interest rate rule.) By the end of the period the French exchange rate had depreciated 4.29 percent, the price level was 2.42 percent higher, the inflation rate was .14 percentage points higher, and the unemployment rate was .31 percentage points lower.

The Italian lira is closely tied to the DM in the model, and the lira depreciated. This led to a rise in the Italian price level, which led the Italian monetary authorities to raise the interest rate. This offset much of the stimulus from the depreciation. By the end of the period the price level was 1.96 percent higher, the inflation rate .19 percentage points higher, and the unemployment rate .10 percentage points lower.

For the United Kingdom the pound depreciated relative to the dollar, but by less than did the DM. The pound thus appreciated relative to the DM (and other European currencies), and this appreciation led to only small increases in the U.K. import price deflator. The increases in the U.K. domestic price level were also small. The effects on the U.K. real variables were modest.

The main effect on the United States was a fall in the price of imports, caused by the appreciation of the dollar relative to the European currencies. This led to a slight fall in the U.S. domestic price level. U.S. imports increased because the price of imports fell relative to the domestic price level and because output was slightly higher The effect on U.S. output was small. Similarly, the Japanese price of imports fell, and there was a slight fall in the Japanese domestic price level. Japanese imports also increased slightly.

5.1.3 Conclusion

Table 2 summarizes some of the results from Table 1. Going out 36 quarters, the cost for Germany of a 0.43 percentage point fall in the unemployment rate is a 1.19 percent rise in the price level. At the end of the period inflation is still higher than the base rate by 0.21 percentage points. For France the fall in the unemployment rate is 0.31 percentage points and the rise in inflation is 0.14 percentage points. The corresponding numbers for Italy are 0.10 and 0.19, and the corresponding numbers for the United Kingdom are 0.06 and 0.05. Whether these costs are considered worth incurring depends, of course, on one's welfare function. Given the estimated costs in Table 2, some would surely argue that the Bundesbank should have been more expansionary in the 1980s.

The accuracy of the present results depends, of course, on the accuracy of the price equations in the MC model. The results in Section 3.13 support the MC equations' dynamics over the NAIRU dynamics, which thus provides some support for the present results. Remember that the present results are not governed by the

Table 2
Changes from the Base Values
after 36 Quarters

	Price Level	Inflation Rate	Unempl. Rate	Output
GE	1.19	0.21	-0.43	0.48
FR	2.42	0.14	-0.31	0.36
IT	1.96	0.19	-0.10	0.10
UK	0.68	0.05	-0.06	0.07

NAIRU dynamics. It is not the case that an experiment like this will result in accelerating price levels, so there are no horrible events lurking beyond the 36-quarter horizon of the present experiment.

Finally, remember that the MC estimates of the price equations do not pin down the point at which the relationship between the price level and unemployment becomes nonlinear. As noted above, this is not likely to be a problem for the present experiment because it is over a period in which unemployment was generally quite high. It would not be sensible, however, to, say, triple the size of the German interest rate decrease and examine the inflation consequences.

5.2 Estimated Stabilization Costs of the EMU[119]

5.2.1 Introduction

When different countries adopt a common currency, each gives up its own monetary policy. In the common-currency regime monetary policy responds to a shock in a particular country only to the extent that the common monetary authority responds to the shock. If this response is less than the response that the own country's monetary authority would have made in the pre common-currency regime, there are stabilization costs of moving to a common currency. This section uses the MC model and stochastic simulation to estimate the stabilization costs to Germany, France, Italy, and the Netherlands from joining the European Monetary Union (EMU). Costs to the United Kingdom from joining are also estimated. Variability estimates are computed for the non EMU and EMU regimes.[120]

The question that this section attempts to answer is a huge one, and the results should be interpreted with considerable caution. In order to answer this question one needs 1) an estimate of how the world economy operates in the non EMU regime, 2) an estimate of how it operates in the EMU regime, and 3) an estimate of the likely shocks to the world economy. Each of these estimates in this section is obviously only an approximation.

Prior to the beginning of the EMU in 1999, there was a large literature analyzing the economic consequences of a common European currency. Wyplosz (1997) provides a useful review. Much of this literature is in the Mundell (1961), McKinnon (1963), and Kenen (1969) framework and asks whether Europe meets the standards for an optimum currency area. The questions asked include how open the countries are, how correlated individual shocks are across countries, and the degree of labor mobility. There was also work examining real exchange rate variances. The smaller are these variances, the smaller are the likely costs of moving to a common currency. von Hagen and Neumann (1994) compared variances of price levels within West German regions with variances of real exchange rates between the regions and other European countries.

The MC model contains estimates of how open countries are in that there are estimated import demand equations and estimated trade-share equations in the model. The model also contains estimates of the correlation of individual shocks

[119]The results in this chapter are updates of those in Fair (1998). This 1998 paper was written before the euro began, so it is obviously dated. But the results are still relevant as estimates of the stabilization costs.

[120] For other results using stochastic simulation to examine the EMU, see Hallett, Minford, and Rastogi (1993), Masson and Symansky (1992), and Masson and Turtelboom (1997).

across countries through the estimated residuals in the individual stochastic equations. Real exchange rates are endogenous because there are estimated equations for nominal exchange rates and individual country price levels. The MC model thus has embedded in it estimates of a number of the features of the world economy that are needed to analyze optimum-currency-area questions. The degree of labor mobility among countries, however, is not estimated: the specification of the model is based on the assumption of no labor mobility among countries. To the extent that there is labor mobility, the present stabilization-cost estimates are likely to be too high.

A key feature of the MC model for present purposes is that there are estimated monetary-policy rules for each of the European countries prior to 1999:1. These are the estimated interest rate rules—equation 7 for a given country in the ROW model. In the EMU regime these rules for the joining European countries are replaced with one rule—one interest rate rule for the EMU. There are also estimated exchange rate equations for each of the European countries in the model—equation 9 for a given country in the ROW model. In the EMU regime these equations for the joining European countries are replaced with one equation—the exchange rate equation for the euro. Finally, there are estimated term structure equations for each of the European countries—equation 8 for a given country in the ROW model. In the EMU regime these equations for the joining European countries are replaced with one term structure equation.

It may be useful to review the experiment in Section 5.1, where the German interest rate was decreased, to get a sense of some of the relevant properties of the MC model regarding the experiments in this section.

5.2.2 The Stochastic Simulation Procedure

Stochastic simulation is discussed in Section 2.6, and it is applied to the MC model in Sections 4.3 and 4.4. The same setup is used here as is used in Section 4.4. The simulation period is 1994:1–1998:4. The period used for the estimated residuals for purposes of drawing errors is 1976:1–1998:4. The number of trials is 200, and the values of L_i are computed as in equation (2) in Section 4.4. Again, the coefficient estimates are taken as fixed for purposes of the stochastic simulations.

There are 16 European countries in the model, eight quarterly and eight annual. The first experiment pertains to four of these: Germany, France, Italy, and the Netherlands. For the second experiment the United Kingdom is added.

5.2.3 Results for the non EMU Regime

Since the simulation period considered here is before 1999:1, the non EMU regime is simply the actual regime. Results for this experiment are presented as experiments 1 and 2 in Table 1. Values of L_i are presented for six countries, GE, FR, IT, NE, UK, and US, and for three variables, real GDP, Y, the GDP deflator, PY, and the short term interest rate, RS. (For the United States, Y is real output of the firm sector and PF is the price deflator.)

Even though results for only six countries are presented in Table 1, the entire MC model is used for the experiments. The same draws (i.e., the same sequence of random numbers) were used for each experiment in order to lessen stochastic-simulation error for the comparisons between experiments. For each of the six countries drawn errors are not used for the interest rate rule, the term structure equation, and the exchange rate equation. Since moving from the current regime to the EMU regime requires changing these equations for the European countries, it seemed best for comparison purposes not to complicate matters by having to make assumptions about what errors to use in the EMU regime for these equations. The variability estimates are thus based on all types of shocks except financial ones. This difference pertains only to the six countries; for all the other countries the error draws are as in Chapter 11.[121]

For the first experiment the estimated interest rate rules for the five European countries are dropped from the model (but not the US interest rate rule), and the five short-term interest rates are taken to be exogenous. This is not meant to be a realistic case, but merely to serve as a baseline for comparison. The results are in the first column for each variable in Table 1. The second experiment differs from the first in that the five interest rate rules are added back in. Otherwise, everything else is the same. The results are presented in the second column for each variable.

Comparing columns 1 and 2 for output shows how stabilizing the estimated interest rate rules are. For all of the European countries L_i falls when the interest rate rules are added. The largest decrease is for Germany, where L_i falls from 2.06 to 1.82. The smallest decrease is for France. The estimated interest rate rule for France (see Table B7) has small and insignificant coefficient estimates for the output gap variable and the inflation variable. According to the estimated rule, the Bank of France responded mostly to the German and U.S. interest rates. The

[121] In Section 4.4 errors were not drawn for equation 30 for the US, and this is true here as well. Errors were drawn for the US term structure equations 23 and 24 in Section 4.4, but in the present case errors are not drawn for these two equations (thus treating the United States like the other five countries).

Table 1
Values of L_i for Four Experiments

	Real Output				Price Level				Short-Term Interest Rate			
	Experiment				Experiment				Experiment			
	1	2	3	4	1	2	3	4	1	2	3	4
GE	2.06	1.82	2.02	2.04	1.97	1.79	1.97	1.98	0.00	2.53	2.29	2.33
FR	2.10	2.03	2.41	2.45	4.08	3.90	4.27	4.31	0.00	1.85	2.29	2.33
IT	2.93	2.74	3.05	3.07	5.81	5.34	6.16	6.19	0.00	3.22	2.29	2.33
NE	3.61	3.42	3.84	3.88	5.33	5.02	5.20	5.20	0.00	1.84	2.29	2.33
UK	2.85	2.73	2.78	3.36	4.34	4.05	4.75	4.75	0.00	1.24	1.24	2.33
US	1.75	1.74	1.76	1.76	1.53	1.64	1.57	1.57	1.08	1.13	1.11	1.11

1 = interest rate rules for GE, FR, IT, NE, and UK dropped.
2 = interest rate rules for GE, FR, IT, NE, and UK used.
3 = EMU regime consisting of GE, FR, IT, and NE.
4 = EMU regime consisting of GE, FR, IT, NE, and UK.
L_i is defined in equation (2) in Section 4.4.

rule is thus not likely to be very stabilizing for France, which the results in Table 1 show is the case. The variability for the price level also falls in Table 1 from column 1 to 2 for the five European countries, where in this case the largest fall is for Italy.

5.2.4 Results for the EMU Regimes

The actual EMU regime began in 1999:1, and this regime is part of the MC model from 1999:1 on. For present purposes, an EMU regime needs to be constructed that is comparable to the non EMU regime regarding shocks. For the results in this section the same error draws are used as were used for the results in columns 1 and 2 in Table 1. Given these shocks, the question is how stabilization is affected by moving to a common monetary policy.

A hypothetical EMU regime must thus be created for the 1994:1–1998:4 period. In fact two EMU regimes are considered here, one including Germany, France, Italy, and the Netherlands, and the other including these four countries plus the United Kingdom. Three changes are required to do this. Consider first the regime without the United Kingdom.

First, the interest rate rules for France, Italy, and the Netherlands were dropped, and their short-term interest rates were assumed to move one for one with the German rate. The output gap variable that is included in the estimated German rule is the German output gap, and this variable was replaced by the total output gap of the four countries. In addition, the German inflation variable was replaced by a total inflation variable for the four countries.[122] The coefficient estimates in this equation were not changed, and the U.S. interest rate, which is an explanatory variable in the equation, was retained. The behavior of the European monetary authority is thus assumed to be the same as the historically estimated behavior of the Bundesbank except that the response is now to the total variables for the four countries rather than just to the German variables.

Second, the term structure equations for France, Italy, and the Netherlands were dropped, and their long-term interest rates were assumed to move one for one with the German rate. The long-term German interest rate equation was retained as is. The only explanatory variables in this equation are the lagged value of the long-term rate and the current value and lagged values of the short-term rate.

Third, the exchange rate equations for France, Italy, and the Netherlands were dropped, and their exchange rates were fixed to the German rate. The German exchange rate equation has as explanatory variables the German price level relative to the U.S. price level and the German short-term interest rate relative to the U.S. short-term interest rate. This equation was used as is except that the German price level was replaced by the total price level for the four countries. (The German short-term interest rate is now, of course, the common short-term interest rate of the four countries, as discussed above.)

No other changes were made to the model. To summarize, then, in this assumed EMU regime, the two main changes are 1) the postulation of a four-country interest rate rule that responds to the four-country output gap and the four-country inflation rate and 2) the postulation of an exchange rate equation for the four-country currency that responds to the four-country price level relative to the U.S. price level and the four-country short-term interest rate relative to the U.S. short-term interest rate.

The results for this regime are presented in column 3 in Table 1. The output variability results are as expected. Comparing columns 2 and 3, L_i increases for the four countries. In fact, for France, Italy, and the Netherlands, the values in

[122]For a given country k and period t, let Y_{kt} be its real output, PY_{kt} its domestic price level, and h_{kt} its exchange rate vis-à-vis the DM. Also, let h_{k05} be its exchange rate in 2005, the base year for real output. Then total nominal output for the four countries combined, denominated in DM, is $\sum_{k=1}^{4}(PY_{kt}Y_{kt})/h_{kt}$ and total real output, denominated in 2005 DM, is $\sum_{k=1}^{4} Y_{kt}/h_{k05}$. The price level for the four countries combined is the ratio of total nominal output to total real output. The total inflation variable is the percentage change in the price level for the four countries combined. Total potential output, denominated in 2005 DM, is $\sum_{k=1}^{4} YS_{kt}/h_{k05}$, where YS_{kt} is the potential output of country i for period t. The output-gap variable used is the percent (log) deviation of total actual output from total potential output.

column 3 are larger than those in column 1, which means that joining the EMU and using the EMU rule is worse than using no rule at all. For the price variability estimates, the values in column 3 are larger than those in column 2 for all four countries. Joining the EMU also increases price variability.

For output variability the United Kingdom is not much affected by the four countries joining together (column 3 versus column 2 for the UK). Its interest rate rule is still stabilizing (column 3 versus column 1). For the final experiment the United Kingdom was added to the four-country regime. Everything is the same in this five-country regime except that total output now includes U.K. output and the total price level now includes the U.K. price level. The U.K. interest rate rule, exchange rate equation, and term structure equation are dropped.

The five-country results are presented in column 4 in Table 1. These results show that the United Kingdom is hurt regarding both output and price variability from joining the group. For output L_i rises from 2.78 in column 3 to 3.36 in column 4, and for price it rises from 4.75 to 5.06. The effects on the other European countries are modest comparing column 4 to column 3. .

The effects on the United States are modest for all of the cases. (Remember that the U.S. interest rate rule is used in all of the experiments.)

5.2.5 Conclusion

This section has used a particular methodology for examining the stabilizations costs of the EMU, and Table 1 provides quantitative estimates of these costs for a four-country and a five-country regime. The estimated costs are largest for Germany and smallest for France, with Italy and the Netherlands in between. There are also noticeable costs for the United Kingdom from joining the group.

The estimates in Table 1 are conditional on the particular interest rate rules for each country. The rules used here are the estimated rules. If different rules were used, say a more stabilizing individual rule for France, different results would be obtained. In general, the more stabilizing a rule is for a given country, the larger are the stabilization costs of joining the EMU likely to be. The results also depend on the choice of the EMU rule. For the work here the German rule has been used with different output and inflation variables, but other choices are clearly possible.

There are possible biases in the Table 1 estimates that are difficult to examine. There is, for example, no labor mobility in the model, and to the extent that there is labor mobility between countries in Europe the real stabilization costs are likely to be smaller than those in Table 1. It would be difficult to modify the MC model to try to account for labor mobility. Also, if the change in regimes results in the shocks across countries being more highly correlated than they were historically, this is likely to bias the current cost estimates upward. The more highly correlated are the shocks, the more is the common European monetary policy rule likely to be stabilizing for the individual countries. It would be difficult to try to estimate how the historical correlations might change.

It may also be the case that the historical shocks used for the stochastic-simulation draws are too large. The shocks are estimated residuals in the stochastic equations, and they reflect both pure random shocks and possible misspecification. However, if the shocks are too large, it is not clear how the cost estimates in Table 1 would be affected since using the correct smaller shocks would lower the values of L_i for all the experiments.

Another issue to consider is whether the EMU regime increases credibility. If, for example, Italian long-term interest rates are lower after Italy joins (because Italian policy is then more credible), this could have a beneficial effect on Italian growth. Level effects of this sort are not taken into account in this study, since only stabilization costs are being estimated.

5.3 Testing for a New Economy in the 1990s[123]

5.3.1 Introduction

There was much talk in the United States in the last half of the 1990s about the existence of a new economy or a "new age." Was this talk just media hype or were there in fact large structural changes in the 1990s? One change that seems obvious is the huge increase in stock prices relative to earnings beginning in 1995. This can be seen in Figure 1, where the price-earnings (PE) ratio for the S&P 500 index is plotted for the 1952:1–1999:4 period. The increase in the PE ratio beginning in 1995 is quite large. The mean of the PE ratio is 14.6 for the 1952.1–1994.4 period and 23.7 for the 1995.1–1999:4 period. This increase appears to be a major structural change, and an important question is whether there were other such changes.

In the next subsection the end-of-sample stability test of Andrews, which is discussed in Subsection 2.8.3, is used to test the hypothesis of no structural change in the 25 equations of the US model beginning in 1995. It will be seen that the hypothesis of stability is rejected for only one equation: equation 25 explaining CG, which is capital gains or losses on stocks held by the household sector. The rejection for this equation is, of course, not surprising given Figure 1. It may be surprising, however, that there were no other major rejections, since a number of macroeconomic variables have large changes beginning about 1995. Four such variables are plotted in Figures 2–5. They are 1) the personal saving rate, SRZ, (lower after 1995), 2) the U.S. current account as a fraction of GDP, $-SR/GDP$, (lower after 1995), 3) the ratio of nonresidential fixed investment to real GDP, $(IKB + IKF + IKG + IKH)/GDPR$, (higher after 1995), and 4) the federal government budget surplus as a percent of GDP, SGP/GDP, (higher after 1995). The results in this section suggest that all four of these unusual changes are because of the stock market boom and not because of structural changes in the stochastic equations.

The fact that the stability hypothesis is not rejected for the three U.S. consumption equations means that, conditional on wealth, the behavior of consumption does not seem unusual. The wealth effect on consumption also explains the low U.S. current account because some of any increased consumption is increased consumption of imports. Similarly, conditional on the low cost of capital caused by the stock market boom, the behavior of investment does not seem unusual according to the stability test of the investment equation. Finally, the rise in the federal

[123] This section is a modification and update of Fair (2004b).

Figure 1
S&P 500 Price-Earnings Ratio
1952:1--1999:4

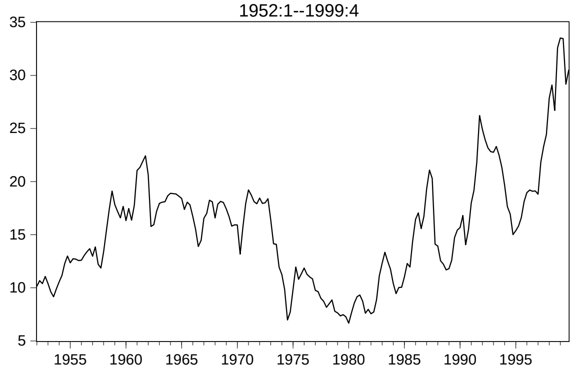


Figure 2
NIPA Personal Saving Rate
1952:1--1999:4

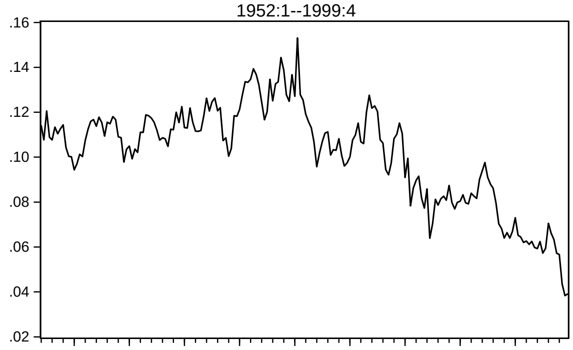


Figure 3
Ratio of U.S. Current Account to GDP
1952:1--1999:4

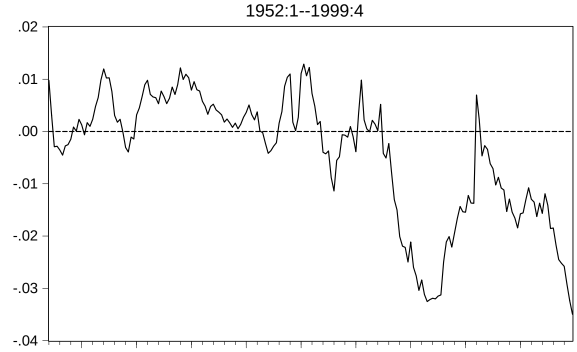


Figure 4
Investment-Output Ratio
1952:1--1999:4

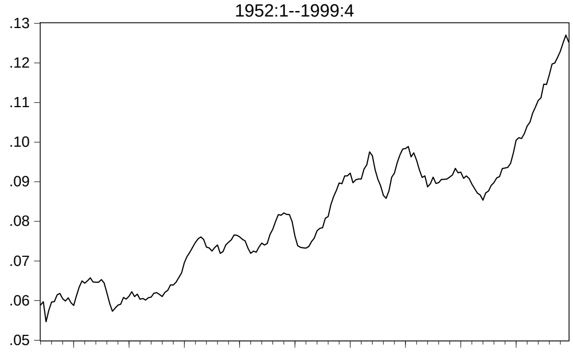
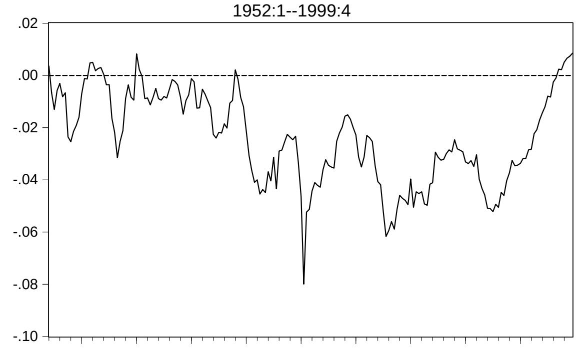


Figure 5
Ratio of Federal Government Surplus to GDP
1952:1--1999:4



government budget surplus is explained by the robust economy fueled by consumption and investment spending.

To examine the effects of the stock market boom, a counterfactual experiment is performed in this section using the MC model. The experiment is one in which the stock market boom is eliminated. The results show that had there been no stock market boom, the behavior of the four variables in Figures 2–5 would not have been unusual.

The overall story is thus quite simple: the only main structural change in the last half of the 1990s was the stock market boom. All other unusual changes can be explained by it. What is not simple, however, is finding a reason for the stock market boom in the first place. If earnings growth had been unusually high in the last half of the 1990s, this might have led investors to expect unusually high growth in the future, which would have driven up stock prices relative to current earnings. Figures 6 and 7, however, show that there was nothing unusual about earnings in the last half of the 1990s. Figure 6 plots the four-quarter growth rate of S&P 500 earnings, and Figure 7 plots the ratio of NIPA after-tax profits to GDP, $PIEF/GDP.$.

Much of the new economy talk was about productivity growth, and Subsection 5.3.4 examines productivity growth (the growth rate of $PROD$). It will be seen that using 1995 as the base year to measure productivity growth, which is commonly done, is misleading because 1995 is a cyclically low productivity year. If 1992 is used instead, productivity growth in the last half of the 1990s is only slightly higher than earlier (from 1.59 percent to 2.00 percent per year). There is thus nothing in the productivity data that would suggest a huge increase in stock prices relative to earnings. The huge increase in PE ratios beginning in 1995 thus appears to be a puzzle. This section is not an attempt to explain this puzzle. Rather, it shows that conditional on the stock market boom, the rest of the economy does not seem unusual.

5.3.2 End-of-Sample Stability Tests

For the end-of-sample stability tests in Appendix A for the US model, the sample period was 1954:1–2013:3, with the potential break at 1995:1. For the work here the sample period is 1954:1–1999:4, with again the potential break at 1995:1. In other words, what happened after 1999:4 is not considered.

Figure 6
Fourth-Quarter Growth Rate of S&P 500 Earnings
1952:1--1999:4

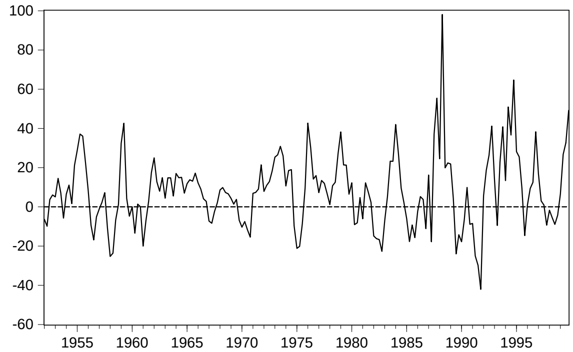
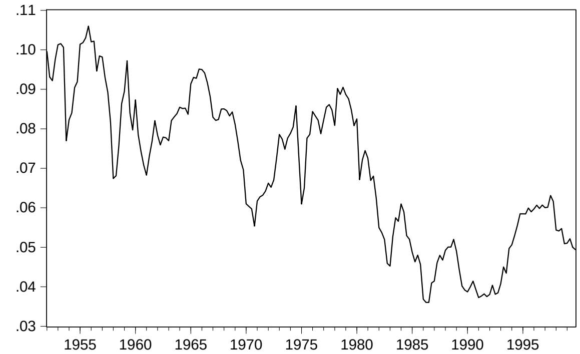


Figure 7
Ratio of NIPA Profits to GDP
1952:1--1999:4



The p-values for the 25 equations are presented in Table 1.[124] The p-values are all considerably larger than 0.05 except for equation 25, the CG equation, where the p-value is zero. The results are thus supportive of the view that there were no major structural changes beginning in 1995:1 except for the stock market boom. The next subsection estimates what the economy would have been like had there been no stock market boom.

5.3.3 Counterfactual: No Stock Market Boom

For the 10-year period prior to 1995 (1985:1–1994:4) the sum of the quarterly values of CG, which is the total capital gain on household financial assets for this period, was $5.055 trillion. This is an average of $126.4 billion per quarter. The sum for the next 5 years (1995:1–1999:4) was $11.568 trillion, an average of $578.4 billion per quarter.

The counterfactual experiment assumes that the capital gain for each quarter of the 1995:1–1999:4 period was $126.4 billion, which is the average for the prior 10-year period. This gives a total capital gain of $2.528 trillion, which is $9.040 trillion less than the actual value of $11.568 trillion.

The entire MC model is used for the experiment. The experiment is for the 1995:1–1999:4 period. The estimated residuals are first added to all the stochastic equations, including the trade share equations, and then taken to be exogenous. This means that when the model is solved using the actual values of all the

[124] Remember from the discussion of the stability tests in Subsection 3.?.? that the coefficient estimates of the dummy variables are taken as fixed when performing the tests. Also, the beginning quarter for equation 15 is 1956:1.

Table 1
End-of-Sample Test Results for the United States
Estimation Period: 1954:1–1999:4
Break Quarter Tested: 1995:1

Eq.	Dependent Variable	p-value
1	Service consumption	0.993
2	Nondurable consumption	0.883
3	Durable consumption	0.338
4	Residential investment	0.807
5	Labor force, men 25-54	0.448
6	Labor force, women 25-54	0.979
7	Labor force, all others 16+	0.669
8	Moonlighters	1.000
10	Price level	1.000
11	Inventory investment	0.952
12	Nonresidential fixed investment	0.724
13	Workers	0.800
14	Hours per worker	0.455
15	Overtime hours	1.000
16	Wage rate	0.372
17	Demand for money, f	0.186
18	Dividends	0.745
23	AAA bond rate	0.331
24	Mortgage rate	0.324
25	Capital gains or losses	0.000
26	Demand for currency	0.669
27	Imports	0.869
28	Unemployment benefits	1.000
29	Interest payments, g	0.972
30	Fed interest rate rule	0.979

• h = household sector, f = firm sector, g = federal government sector.
• Estimation technique: 2SLS.

exogenous variables, a perfect tracking solution is obtained. The actual values are thus the base values. Equation 25 is then dropped from the model, and the value of CG in each quarter is taken to be \$126.4 billion. The model is then solved. The difference between the solution value and the actual value for each endogenous variable for each quarter is the effect of the CG change. The solution values will be called values in the "no boom" case.

Figures 8–15 plot some of the results. Each figure presents the actual values of the variable and the solution values. Figure 8 shows that the personal saving rate, SRZ, is considerably higher in the no boom case. No longer are the values

Figure 8
NIPA Personal Saving Rate
1995:1--1999:4

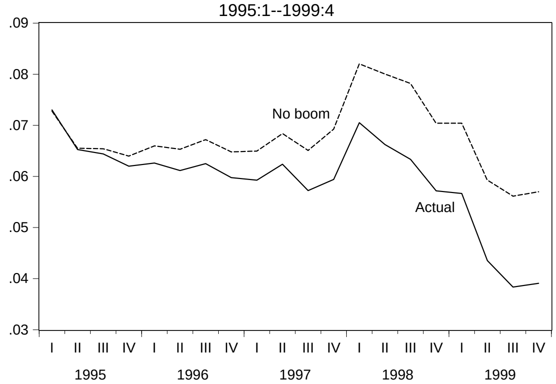


Figure 9
Ratio of U.S. Current Account to GDP
1995:1--1999:4

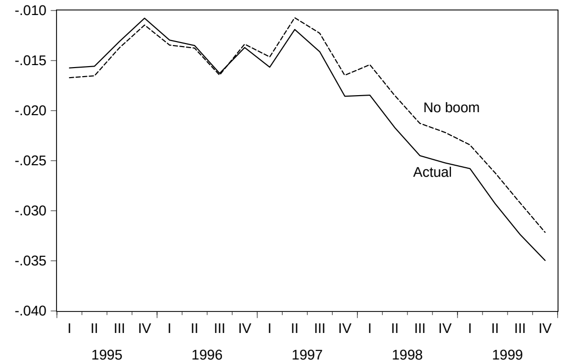


Figure 10
Investment-Output Ratio
1995:1--1999:4

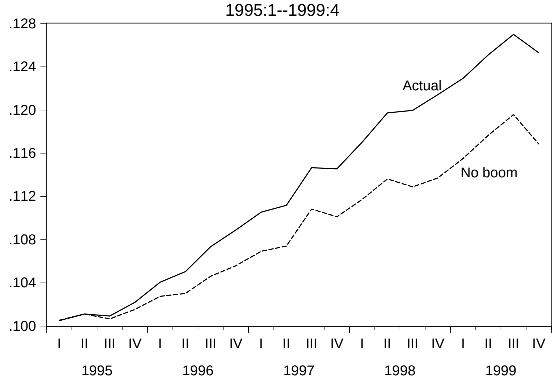


Figure 11
Ratio of Federal Government Budget Surplus to GDP
1995:1--1999:4

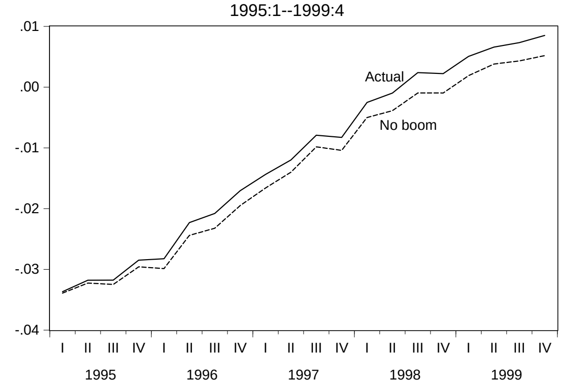


Figure 12
Four-Quarter Growth Rate of Real GDP
1995:1--1999:4

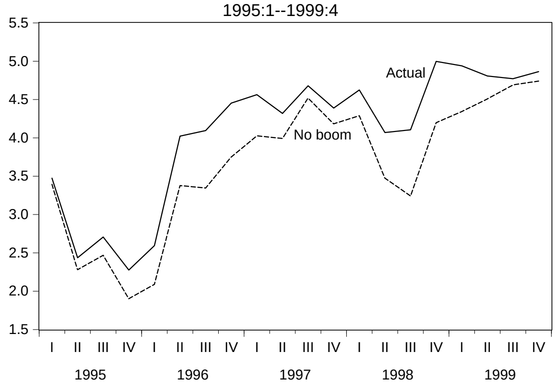


Figure 13
Unemployment Rate
1995:1--1999:4

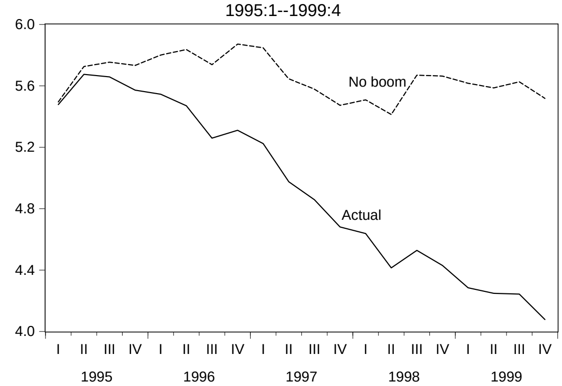


Figure 14
Four-Quarter Percentage Change in the GDP Deflator
1995:1--1999:4

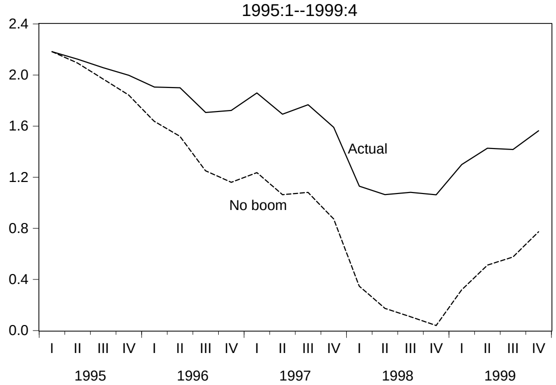
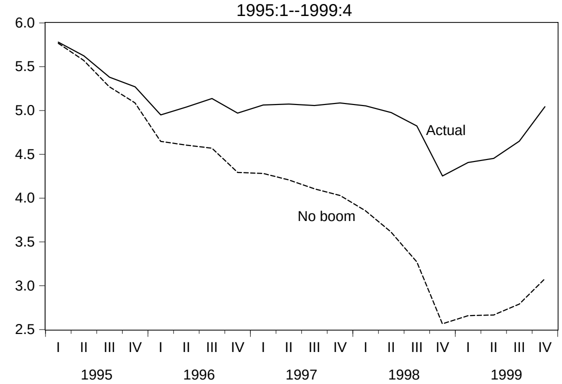


Figure 15
Three-Month Treasury Bill Rate
1995:1--1999:4



outside the range of historical experience in 1999. This is the wealth effect on consumption at work. With no stock market boom, households are predicted to consume less. Figure 9 shows that the current account deficit, $-SR/GDP$, is not as large in the no boom case: imports are lower because of the lower consumption. Figure 10 shows that there is a much smaller rise in the investment-output ratio, $(IKB+IKF+IKG+IKH)/GDPR$, in the no boom case. Investment is not as high because the cost of capital is not as low and because output is lower. Figure 11 shows that the federal government surplus, SGP/GDP, is not as large, which is due to the less robust economy.

Figure 12 plots the four-quarter percentage change in real GDP, and Figure 13 plots the unemployment rate. Both show, not surprisingly, that the real side of the economy is worse in the no boom case. In the fourth quarter of 1999, for example, the unemployment rate in the no boom case is 5.5 percent, which compares to the actual value of 4.1 percent. Figure 14 plots the four-quarter percentage change in the GDP deflator. It shows that the rate of inflation is lower in the no boom case (because of the higher unemployment rate), although in neither case would one consider inflation to be a problem.

Figure 15 plots the three-month Treasury bill rate, RS, which is the rate determined by equation 30, the estimated interest rate rule of the Fed. The figure shows that the bill rate is lower in the no boom case. The Fed is predicted to respond to the more sluggish economy by lowering rates. In the fourth quarter of 1999, the bill rate is 3.1 percent in the no boom case, which compares to the actual value of 5.0 percent. It is interesting to note that this amount of easing of the Fed is not enough to prevent the unemployment rate from rising, as was seen in Figure 13. This is consistent with the results in Section 4.4, which show that the Fed has limited ability to control the economy.

It is thus clear from these figures that according to the MC model the U.S. economic boom in the last half of the 1990s was fueled by the wealth effect and cost of capital effect from the stock market boom. Had it not been for the stock market boom, the economy would have looked more or less normal.

5.3.4 Aggregate Productivity

As noted in the introduction to this section, much of the new economy talk was about productivity growth. For the above experiment long run productivity growth is exogenous: the MC model does not explain long run productivity growth. This issue will now be addressed.

Figure 16a plots the log of output per worker hour, $PROD$, for the for the 1952:1–2013:3. Also plotted in the figure is a peak-to-peak interpolation line, with peaks in 1955:2, 1966:1, 1973:1, 1992:4, and 2010:4. These are the peaks used to construct variable LAM in Table A.7 in Appendix A. The annual growth rates between the peaks are 3.26, 2.59, 1.59, and 2.00 percent, respectively. Figure 16b is an enlarged version of Figure 16a for the period from 1985:1–2013:3.

An interesting feature of Figure 16a is the fairly modest increase in the peak-to-peak productivity growth rate after 1992:4: from 1.59 to 2.00 percent. This difference of 0.41 percentage points is certainly not large enough to classify as a movement into a new age.

It can be seen in Figure 16b why some were so optimistic about productivity growth in the last half of the 1990s. Between 1995:3 and 1999:4 productivity grew at an annual rate of 2.37 percent, which is a noticeable improvement from the 1.59 percent rate between 1973:1 and 1992:4. What this overlooks, however, is that productivity actually fell slightly between 1992:4 and 1995:3 (annual rate of -0.07 percent), so 1995 is a low year to use as a base. Some of the change in productivity growth after 1995:3 is cyclical productivity growth. How much? A rough estimate is to subtract the 2.37 percent growth between 1995:3 and 1999:4 from the 2.00 estimate of long-run productivity growth between 1992:4 and 2010:4 from the peak-to-peak interpolation line, which is 0.37 percent.

Regarding other studies of productivity growth in the 1990s, Blinder and Yellen (2001) test for a break in productivity growth beginning in 1995:4, and they find a significant break once their regression equation is estimated through 1998:3. From Figures 16b this is not surprising, given the rapid productivity growth between 1995:4 and 1998:3. Again, however, 1995:4 is a misleading base to use. Oliner and Sichel (2000) compare productivity growth in 1990–1995 to

Figure 16a
Log of Output per Worker Hour
1952:1--2013:3

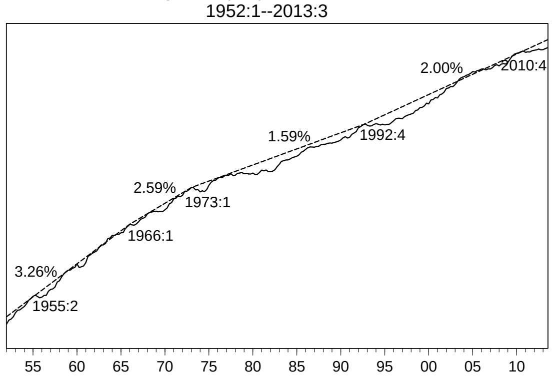
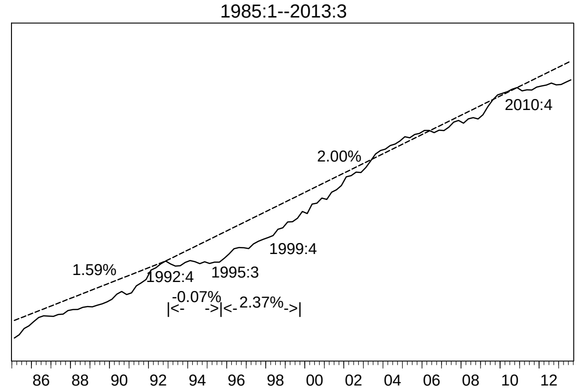


Figure 16b
Log of Output per Worker Hour
1985:1--2013:3



that in 1996–1999 and do not adjust for cyclical growth. This is also true in Nordhaus (2000), who compares productivity growth in 1990-1995 to that in 1996-1998.

Gordon (2000a, 2000b) argues that some of the actual productivity growth after 1995 is cyclical. He estimates in Gordon (2000b, p. 219) that of the actual 2.82 percent productivity growth in the nonfarm business sector between 1995:4 and 1999:4, 0.54 is cyclical and 2.28 is long run. This estimate of 0.54, which is backed out of a regression, is fairly close to the 0.37 figure estimated above for the 1995:3–1999:4 period using the interpolation line in Figure 16b. Gordon's actual number of 2.82 percent is larger than the actual number of 2.37 percent in Figure 16b. This difference is partly due to the fact that Figure 16b uses revised data. The data revisions that occurred after Gordon's work had the effect of lowering the estimates of productivity growth.

Gordon's results and the results from Figure 16b are thus supportive of each other. Although Gordon estimates long run productivity growth to be 2.28 percent, Figure 16b suggests that this number is about 2 percent based on the revised data. The message of Figure 16b is thus that productivity growth has increased in the last half of the 1990s, but only by about 0.4 percentage points.

5.3.5 Conclusion

The results in this section are consistent with the simple story that the only major structural change in the last half of the 1990s was the huge increase in stock prices relative to earnings. The only U.S. macroeconometric equation in the MC model for which the hypothesis of end-of-sample stability is rejected is the stock price equation. The counterfactual experiment using the MC model in which the stock market boom is turned off shows that were it not for the boom the behavior of variables like the saving rate, the U.S. current account, the investment output ratio, and the federal government budget would not have been historically unusual. Also, the data on aggregate productivity do not show a large increase in trend productivity growth in the last half of the 1990s: there is no evidence in the data of a new age of productivity growth.

None of the results here provide any hint as to why the stock market began to boom in 1995. In fact, they deepen the puzzle, since there appear to be no major structural changes in the economy (except the stock market) and there is no evidence of a new age of productivity growth. In addition, Figures 6 and 7 show no unusual behavior of earnings in the last half of the 1990s. In short, there is no obvious fundamental reason for the stock market boom.

5.4 Policy Effects in the Post Boom U.S. Economy[125]

5.4.1 Introduction

The section considers the question of why the U.S. economy in the 2000:4–2004:3 period was sluggish in light of the large expansionary fiscal and monetary policies that took place. The answer does not appear to be that there were large structural changes in the economy or systematic bad shocks. This section tests for such changes and shocks, and the results are generally negative. Instead, the main culprits seem to be large negative effects from declines in the stock market and exports. Although not tested here, some of the decline in exports may be the result of stock market declines in the rest of the world, in which case most of the explanation is simply the stock market declines themselves through negative wealth effects.

The United States had in the 2000:4–2004:3 period large expansionary fiscal and monetary policies and yet a recession and fairly slow recovery from the recession. The sluggish economy in this period can be seen from Figures 1–3, which contain plots for the 1985:1–2004:3 period. Figure 1 plots the log of real GDP, $\log GDPR$; Figure 2 plots the log of the total number of jobs, $\log(JF + JG + JM + JS)$; and Figure 3 plots the unemployment rate, UR. Figure 2 is striking in showing essentially no job growth for the entire 2000:4–2004:3 period. Figure 4 shows that the inflation rate, the four-quarter percentage change in $GDPD$, remained low during the 2000:4–2004:3 period: inflation was clearly not a problem. In the discussion below the total number of jobs will be called "employment."

The expansionary fiscal and monetary policies can be seen from Figures 5–8. Figure 5 plots the ratio of federal personal income taxes to taxable income, THG/YT; Figure 6 plots the ratio of federal corporate profit taxes to corporate profits, $D2G$; Figure 7 plots the ratio of real federal purchases of goods to potential real output, COG/YS;[126] and Figure 8 plots the three-month Treasury bill rate, RS. (Ignore for now the dotted horizontal lines in Figures 5, 7, and 8—and in Figure 12 below.) Taxes fell dramatically beginning in 2001, and federal spending as a share of output rose fairly consistently from 2001:1 on. The Fed began lowering interest rates in 2001:1, as Figure 8 shows. Finally, Figure 9 shows

[125]This section is a modification and update of Fair (2005b).

[126]In Figure 7, and in Figures 11 and 12 below, the variables of interest have been divided by potential rather than actual real output to avoid having the plots be influenced by cyclical fluctuations in actual real output.

Plots for 1985:1–2004:3

Figure 1
Log of Real GDP

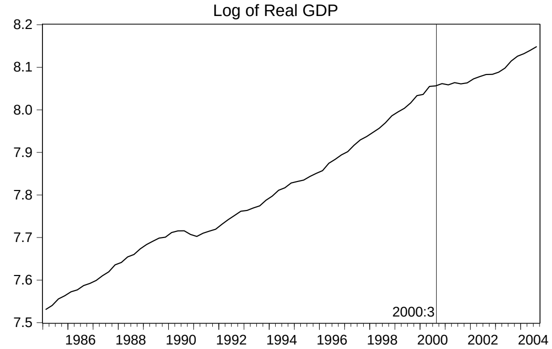


Figure 2
Log of Employment

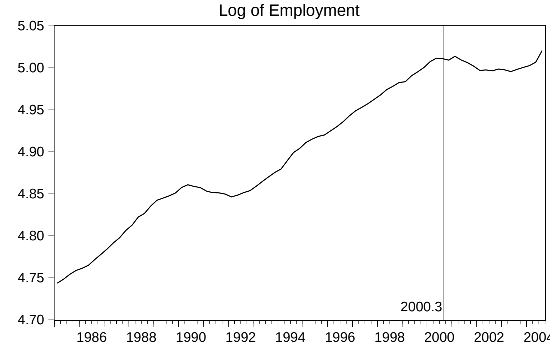


Figure 3
Unemployment Rate

Figure 4
Four-Quarter Percentage Change in the GDP Deflator

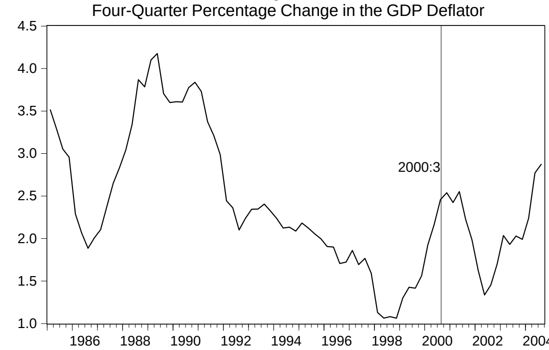


Figure 5
Ratio of Federal Personal Income Taxes to Taxable Income

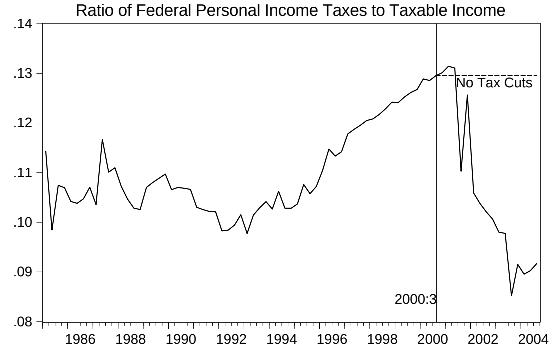
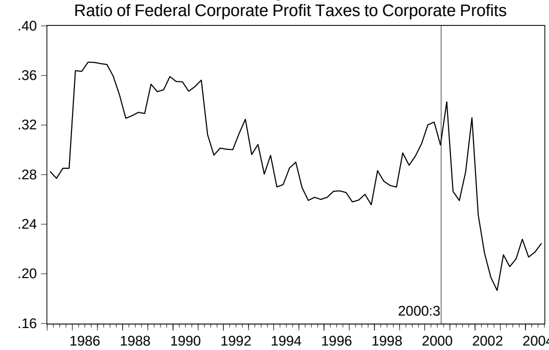


Figure 6
Ratio of Federal Corporate Profit Taxes to Corporate Profits



Plots for 1985:1–2004:3

Figure 7
Ratio of Federal Purchases of Goods to Potential Real Output

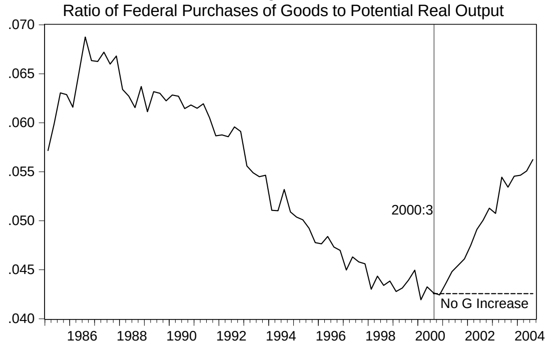


Figure 8
Three-Month Treasury Bill Rate

Figure 9
Ratio of Federal Government Surplus to GDP

Figure 10
Ratio of U.S. Current Account to GDP

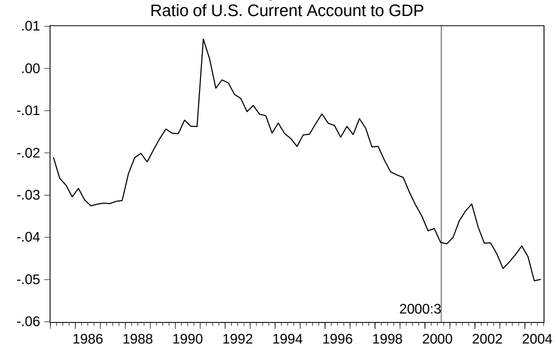


Figure 11
Ratio of Real Household Wealth to Potential Real Output

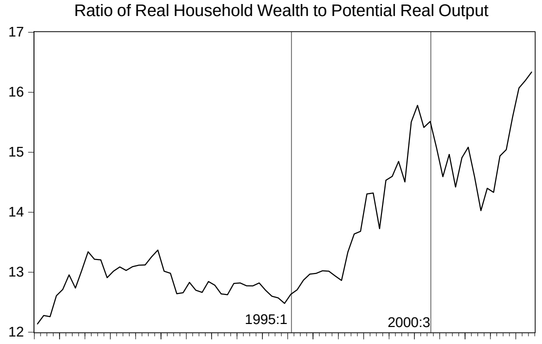


Figure 12
Ratio of U.S. Real Exports to Potential Real Output

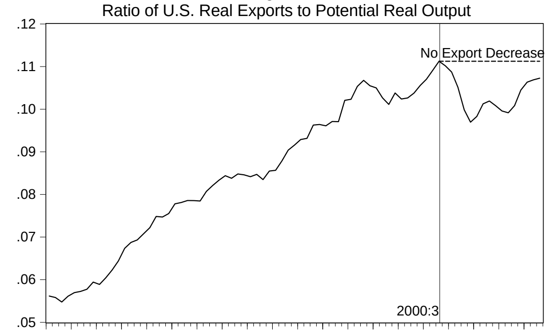
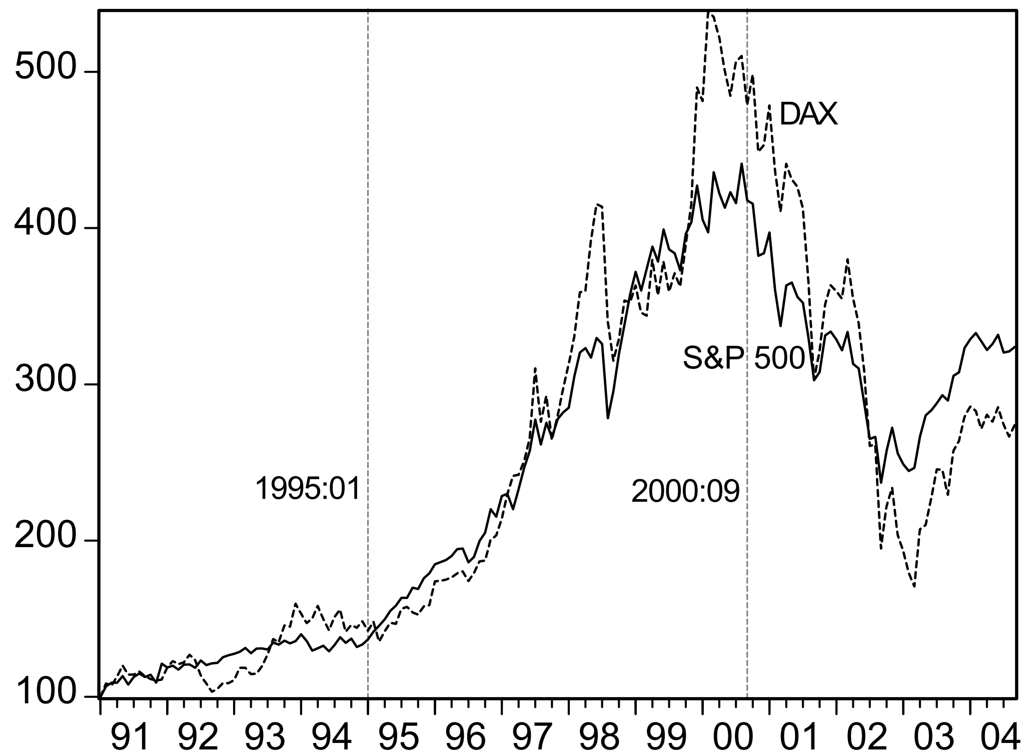


Figure 13
U.S. S&P 500 Stock Price Index and German DAX Stock Price Index
1991:01-2004:09: 1991:01 = 100



the movement of the federal government budget from large surplus to large deficit in the period after 2000 (variable SGP/GDP), and Figure 10 shows that the U.S. current account deficit remained large after 2000 (variable $-SR/GDP$). The period 2000:4–2004:3 will be called the "post boom" period. The 2000:4 quarter was chosen to begin this period because it is the first quarter following the peak of U.S. stock prices (see Figure 13 below).

A key question about this period is why with so much stimulus from 2000:4 on (Figures 5–8) did the economy not do better (Figures 1–3)? The MC model is used to try to answer this question.

In the next subsection, Subsection 5.4.2, the estimated U.S. equations are tested for structural change beginning in 2000:4. Did the U.S. economy change in structural ways in the post boom period, which might then account for its unusual behavior? The results suggest no. In Subsection 5.4.3 the post boom period is examined for possible bad shocks. Were there a series of negative demand shocks that contributed to the sluggish economy? The estimated residuals of the U.S. consumption and investment equations are examined for large systematic values. There do not appear from this exercise to be systematically bad shocks.[127] Another test in Subsection 5.4.3 is to set the U.S. consumption and investment residuals to zero (with all the other residuals set to their estimated values), solve the model, and see if the solution yields a stronger economy. This is not the case, and so the demand residuals do not appear to be the culprit.

Having ruled out structural change and bad shocks, what explanations are left? One important characteristic of the post boom period was a large fall in stock prices. The effect of the decrease in stock prices on U.S. household wealth can be seen from Figure 11, where the ratio of real U.S. household wealth to potential real output is plotted, variable AA/YS. There was a huge decrease in wealth beginning in the middle of 2000. Clearly, part of the sluggishness of the post boom period could be due to negative wealth effects. The experiments using the MC model suggest that this is the case.

Another important characteristic of the post boom period was a sharp fall in U.S. exports, which can be seen in Figure 12, where the ratio of U.S. real exports to potential real output is plotted, variable EX/YS. It is interesting that the fall in exports began almost exactly at the same time as the fall in stock prices. The fall in stock prices that began in the middle of 2000 was a worldwide phenomenon. An example of this is presented in Figure 13, where the U.S. S&P 500 stock price

[127] The word "shocks" here is *not* meant to refer to changes in stock prices and changes in exports. As will be seen, these changes were large and negative in the post boom period.

index is plotted along with the German DAX stock price index. It is clear that there is a strong positive correlation. Although not shown, the same is true of most other countries' stock price indices. It is thus possible that some of the decline in the demand for U.S. exports was due to negative wealth effects on demand in other countries. More will be said about this later.

Subsection 5.4.4 consists of a number of counterfactual experiments using the MC model. These experiments are designed to estimate various quantitative effects. The first three experiments provide estimates of the effects of the expansionary fiscal and monetary policies in the post boom period. The estimates are briefly as follows. Had there been no tax cuts, employment would have been 1.8 percent lower by 2004:3 than it actually was; had there been no large increases in federal purchases of goods, employment would have been 1.5 percent lower; and had there been no fall in short-term interest rates, employment would have been 2.1 percent lower. These effects are roughly additive in the model (fourth experiment), and the combined estimate is that employment would have been 5.5 percent lower in 2004:3 than it actually was had there been no tax cuts, no increase in government spending, and no decrease in the short term interest rate. Note from Figure 2 that what actually took place in the post boom period was essentially no employment growth, and so had there been no policy stimulus, it is estimated that employment would have fallen by about 5.5 percent rather than remaining roughly unchanged. In the fourth experiment the estimate is that the unemployment rate in 2004:3 would have been 2.8 percentage points higher than it actually was. The actual unemployment rate in 2004:3 was 5.4 percent, and so had there been no policy stimulus, the estimate is that the unemployment rate would have been 8.2 percent.

The fifth experiment in Subsection 5.4.4 provides an estimate of the size of the U.S. wealth effect. Had there been no U.S. stock market decline, it is estimated that employment by the end of the period would have been 1.7 percent higher than otherwise and the unemployment rate would have been 1.1 percentage points lower. The sixth experiment provides an estimate of the effect of the decline in U.S. exports. Had U.S. exports not declined, it is estimated that employment by the end of the period would have been 0.9 percent higher than otherwise and the unemployment rate would have been 0.3 percentage points lower.[128] Again, these effects are roughly additive (seventh experiment), and the combined estimate

[128]In the US model wealth has a negative effect on labor supply—equations 5, 6, and 7—and so, other things being equal, an increase in wealth decreases the labor force, which lowers the unemployment rate. This is the reason the unemployment rate falls more in the stock market experiment than in the export experiment even though employment rises more in the export experiment.

is that employment would have been 2.5 percent higher than otherwise and the unemployment rate would have been 1.3 percentage points lower had there been no stock market and export decline.

These results thus suggest that the policy stimulus in the post boom period offset much of the stock market and export effects. Focusing on 2004:3, where the actual unemployment rate was 5.4 percent, the estimate, as mentioned above, is that it would have been 8.2 percent without the policy stimulus. However, had there been no stimulus and no stock market and export decline, the estimate is that the unemployment rate would have been 6.9 percent (8.2 minus 1.3).

There do not appear to be other estimates of the size of the negative wealth effect in the post boom period. For example, essentially no mention is made of stock-market effects in the Council of Economic Advisers (2005), *Economic Report of the President*, the OECD Economic Outlook (2005), Weller, Bivens, and Sawicky (2004), and Zandi (2004). The stimulative fiscal and monetary policies in the post boom period have been extensively discussed in the press, and it has been argued that these policies helped mitigate the 2001 recession. But the real puzzle has not been addressed, namely why given the very large changes in policy (Figures 5–8) there was a recession and a fairly sluggish recovery from it.

The present results also suggest that some policy stimulus would have been needed even with no stock market and export decline to keep the unemployment rate from rising from its low of 3.9 percent in 2000:4. Figures 4–6 show that in 2000:3 the ratio of federal personal income taxes to taxable income was fairly high, federal government spending was fairly low, and the interest rate was fairly high. According to the model, even with no stock market and export decline, some change in at least one of these policy variables would have been needed to avoid an increase in the unemployment rate.

5.4.2 End-of-Sample Stability Tests

The first stability test is to see if there were structural changes in the post boom period. The hypothesis tested is that the coefficients in each of the 26 U.S. stochastic equations are the same both before and after 2000:4. The method in Andrews (2003), which is discussed in Subsection 2.8.3, is used for the tests.[129] The method

[129] One is never sure about the power of these kinds of tests, although the results in Andrews (2003) suggests that the test has good power properties. Also, as discussed in the next paragraph, one bad residual is enough to lead to a rejection of the stability hypothesis.

requires estimation over different subsets of the overall sample period.[130] The test produces a p-value for each equation tested. A p-value of, say, less than .05 is a rejection of the hypothesis of stability at the 95 percent confidence level.

The results for the 25 equations are presented in Table 1. There are six rejections of the hypothesis of stability at the 95 percent confidence level. The first, and most important, is for durable consumption, equation 3. In 2001:4, the first quarter after 9/11, there was a huge increase in durable consumption, due in large part to the introduction of zero percent financing for cars, and, as will be seen in the next subsection, the equation substantially underpredicted durable consumption in this period. This was enough to lead to a rejection of the stability hypothesis. More will be said about this in the next section.

Four of the other five rejections are for minor equations in the model: 5) labor force of men, 25-54, 17) demand for money of the firm sector, 18) dividend payments, and 28) unemployment benefits. The other rejection is for equation 25, which explains capital gains or losses on corporate stocks held by the household sector, denoted CG. In this equation CG depends on the change in after-tax profits and the change in the bond rate, although very little of the variance is explained. Not surprisingly, the change in stock prices is essentially unpredictable. Neither of the explanatory variables in this equation has values in the 1990's and early 2000's that would predict the huge increase in stock prices in the last half of the 1990's and the huge decrease beginning in 2000. For the experiments in Section 5, equation 25 has been dropped and CG has been taken to be exogenous.

Overall, the results in Table 1 are supportive of the view that there were no major structural changes in the post boom period. The equations for which the stability hypothesis is not rejected include all the aggregate demand equations (consumption, investment, imports) except for the durable consumption equation, the price and wage equations, the labor supply and labor demand equations except the labor supply of men 25-54, and the estimated interest rate rule of the Fed.

[130]Dummy variables appear in a few of the U.S. stochastic equations. These variables take on a value of 1.0 during certain quarters and 0.0 otherwise. For example, there are four dummy variables in the U.S. import equation that are, respectively, 1.0 in 1969:1, 1969:2, 1971:4, and 1972:1 and 0.0 otherwise. These are meant to pick up effects of two dock strikes. A dummy variable coefficient obviously cannot be estimated for sample periods in which the dummy variable is always zero. This rules out the use of the end-of-sample test if some of the sample periods that are used in the test have all zero values for at least one dummy variable. To get around this problem when performing the test, all dummy variable coefficients were taken to be fixed and equal to their estimates based on the entire sample period.

Table 1
End of Sample Stability Test Results
for the 25 U.S. Equations

Eq.	Dependent Variable	p-value
1	Service consumption	.977
2	Nondurable consumption	.733
3	Durable consumption	.000
4	Housing investment	.221
5	Labor force, men 25-54	.012
6	Labor force, women 25-54	.500
7	Labor force, all others 16+	.238
8	Moonlighters	.895
10	Price level	.901
11	Inventory investment	.820
12	Nonresidential fixed investment	.012
13	Workers	.302
14	Hours per worker	1.000
15	Overtime hours	.884
16	Wage rate	.576
17	Demand for money, f	.000
18	Dividends	.047
23	AAA bond rate	.238
24	Mortgage rate	.384
25	Capital gains or losses	.000
26	Demand for currency	.849
27	Imports	.727
28	Unemployment benefits	.000
29	Interest payments, g	.599
30	Fed interest rate rule	.407

- h = household sector, f = firm sector, g = federal government sector.
- Overall sample period: 1954:1–2004:3 except 1956:1–2004:3 for equation 15.
- Break point tested: 2000:4.
- Estimation technique: 2SLS.

5.4.3 Examination of Residuals

If there were large negative demand shocks in the post boom period, one would expect the estimated residuals from the demand equations to show this. This is examined in two ways in this subsection. The first is simply to look at the large residuals from the demand equations. Table 2 presents these residuals for seven demand equations—three consumption equations, three investment equations, and the import equation. For each equation the residuals in the post boom period were divided by the estimated standard error of the equation, and values greater than or equal to 0.75 in absolute value were chosen for Table 2. A value in Table 2 is the actual value minus the predicted value divided by the estimated standard error of the equation. For imports the sign is reversed because a positive residual is a negative domestic demand shock. If no number is presented, the ratio was less than 0.75 in absolute value.

If there were large negative demand shocks, Table 2 should show many negative values. This is not the case. The largest absolute value is 5.3 percent for 2001:4 for durable consumption, which, as noted in the previous subsection, is primarily the huge response to zero percent financing for cars, which is not explained by the equation. So this shock is in the wrong direction. The worst quarter for negative shocks is 2001:3, where the five shocks are negative. The largest negative shock in absolute value is for nonresidential fixed investment in 2004:1, which is -2.3 percent.

Table 2 examines only fairly large shocks. It could be that there are a series of smaller (negative) shocks that cumulate over time to large negative effects. To test for this, the residuals in the seven equations were set to zero for the post boom period and the MC model was solved. All the other residuals were set to their estimated values for this solution. For a given endogenous variable and quarter, the difference between the actual value and the solution value is an estimate of the effect of the residual change on the variable. (If the model is solved using estimated values for all the residuals, the solution values are just the actual values—a perfect tracking solution.) Table 3 shows the actual and solution values for real GDP and the unemployment rate. If this period were dominated by negative shocks, the actual values of real GDP, which are based on the actual demand shocks, should be smaller than the solution values, where are based on zero demand shocks. Similarly, the actual values of the unemployment rate should be greater than the solution values. The results in Table 3 show no clear pattern. In fact, In fact, the largest differences in absolute value are positive for real GDP (and negative for the unemployment rate).

Tables 2 and 3 thus say that conditional on the equations being good approximations, the post boom period does not appear to be one of unusually bad shocks. Demand shocks do not appear to explain the sluggishness of the post boom period.

Table 2
Large Absolute-Value Residuals
100(Actual - Predicted)/Standard Error

	Equation						
	1	2	3	4	12	11	27
2000:4	1.0		-0.8		-1.1		
2001:1		-1.7			-1.1	-2.1	
2001:2			-1.2				-1.3
2001:3	-1.6		-1.3	-1.7		-1.2	-1.5
2001:4			5.3	2.6		-1.7	-1.4
2002:1			-1.4	-1.3			
2002:2		-1.0			-1.2		
2002:3			1.9				
2002:4		1.0	-1.2				
2003:1			-1.1		1.4		
2003:2			1.6	0.8	1.4		
2003:3		1.4	1.9	0.8			
2003:4				2.4			
2004:1					-2.3		
2004:2		-0.8	-0.9				1.0
2004:3	-1.0	-1.1	-1.0	0.8			

Estimation period is 1954:1–2004:3
Equation 1: Service consumption
Equation 2: Nondurable consumption
Equation 3: Durable consumption
Equation 4: Housing investment
Equation 12: Nonresidential fixed investment
Equation 11: Inventory investment
Equation 27: −Imports

There is one further interesting point from Table 2. Remember that the income variable in the consumption and housing investment equations is aggregate disposable income, YD. This is an aggregate variable, and it is not affected by income distribution changes. There was much talk in the 2004 presidential election campaign and earlier about the ineffectiveness of the tax cuts passed during the Bush administration because so much of the tax savings went to very high income people.[131] A test of this ineffectiveness hypothesis is to examine the residuals from the first four equations in Table 2. Under this hypothesis there should be many negative residuals: the consumption and housing investment equations should overpredict demand because they are treating all of the tax savings

[131]Zandi (2004) argues that the tax cuts would have been more effective had they been aimed less at high income people. Weller, Bivens, and Sawicky (2004), p. 59, also make this point.

Table 3
Estimated Effects of No Demand Shocks

	Real GDP			**Unemployment Rate**		
	Actual	Solution	% Diff.	Actual	Solution	Diff.
2000.4	12682.0	12629.3	0.41	3.92	4.05	-0.13
2001.1	12645.7	12769.1	-0.98	4.23	4.01	0.22
2001.2	12712.7	12766.5	-0.42	4.41	4.19	0.22
2001.3	12674.1	12710.7	-0.29	4.82	4.63	0.19
2001.4	12705.1	12629.4	0.60	5.54	5.57	-0.04
2002.1	12824.6	12840.0	-0.12	5.71	5.68	0.03
2002.2	12894.6	12956.1	-0.48	5.85	5.72	0.12
2002.3	12956.6	13018.0	-0.47	5.73	5.56	0.17
2002.4	12962.8	13007.1	-0.34	5.85	5.68	0.17
2003.1	13028.6	12952.9	0.58	5.87	6.00	-0.13
2003.2	13151.7	12983.2	1.28	6.15	6.51	-0.36
2003.3	13374.0	13073.9	2.24	6.11	6.82	-0.71
2003.4	13525.7	13206.4	2.36	5.82	6.76	-0.94
2004.1	13606.5	13669.4	-0.46	5.69	5.59	0.10
2004.2	13710.7	13804.4	-0.68	5.60	5.39	0.21
2004.3	13830.9	13940.1	-0.79	5.44	5.15	0.30

Notes: Solution based on zero values for the residuals in equations 1, 2, 3, 4, 11, 12, and 27 and actual values for the other residuals.

flowing into YD the same. If the people receiving most of the tax savings spend less of their income than others, then the equations, which treat all income the same, should overpredict spending. Since Table 2 does not show a preponderance of large negative residuals, the results do not support the ineffectiveness hypothesis. This test, of course, relies only on aggregate data and may have low power, but the results at least suggest that the income distribution effects on aggregate demand from the tax cuts may be small.[132]

5.4.4 Counterfactual Experiments: 2000:4–2004:3

Seven experiments using the MC model are reported in this subsection. They are designed to estimate quantitative effects. In each experiment one or more exogenous variables are changed for the 2000:4–2004:3 period and the effects of these changes are analyzed. The estimated residuals are first added to all the stochastic equations. This means that when the model is solved using the actual values of all the exogenous variables, a perfect tracking solution is obtained. The actual values

[132]Note that this is just an argument about aggregate demand effects. It is not an argument in favor of the particular tax legislation that was passed.

are thus the base values. Unless otherwise noted, the variables discussed below are U.S. variables.

In the regular version of the model monetary policy is endogenous: the short-term interest rate, RS, is determined by the estimated Fed interest rate rule, equation 30. For the experiments here, equation 30 is dropped. RS is taken to be exogenous, and its values are either taken to be the actual values or particular values chosen for the experiment. Similarly, the capital gains equation determining CG, equation 25, is dropped. CG is taken to be exogenous, and its values are either taken to be the actual values or particular values chosen for the experiment.

It should be stressed that the experiments here are meant to answer "what if" questions. For example, the first experiment asks what would have happened had personal income tax rates not been lowered and RS and CG not been changed from their historical values. In practice, of course, had tax rates been lowered the Fed would have behaved differently (by following equation 30 according to the model). Also, CG would have changed. But the interest here is to examine effects conditional on RS and CG being exogenous.

Experiment 1: No Tax Cuts

The first experiment concerns personal income tax rates. Figure 5 plots the ratio of federal personal income taxes to taxable income. In the model this ratio is endogenous because the tax system is progressive. The exogenous tax-rate variable in the model is denoted $D1G$. For the first experiment $D1G$ was taken to be unchanged from its actual value in 2000:3. In Figure 5 this is roughly equivalent to taking the ratio to be the horizontal dotted line. After this change, the model is solved. The difference between the solution value and the actual value for each endogenous variable for each quarter is the effect of the $D1G$ change. The solution values will be called values in the "no tax cuts" case.

Figures 14a–14f plot results for six variables: the four-quarter percentage change in real GDP, the log of employment, the unemployment rate, the four-quarter change in the GDP deflator, the ratio of the federal government budget surplus to GDP, and the ratio of the U.S. current account to GDP. Table 4 presents results for the last quarter, 2004:3. In the no tax cuts case employment is 1.8 percent lower by 2004:3, the unemployment rate is 0.8 percentage points higher, and the government budget has improved by 2.2 percent of GDP.

Experiment 1: No Tax Cuts 2000:4–2004:3

Figure 14a
Four-Quarter Growth Rate of Real GDP

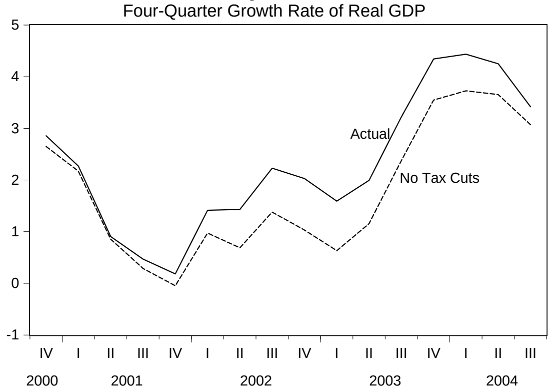


Figure 14b
Log of Employment

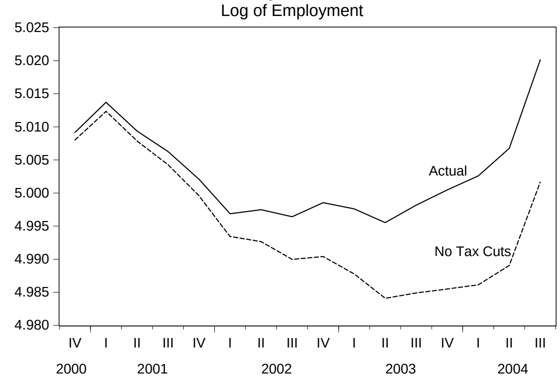


Figure 14c
Unemployment Rate

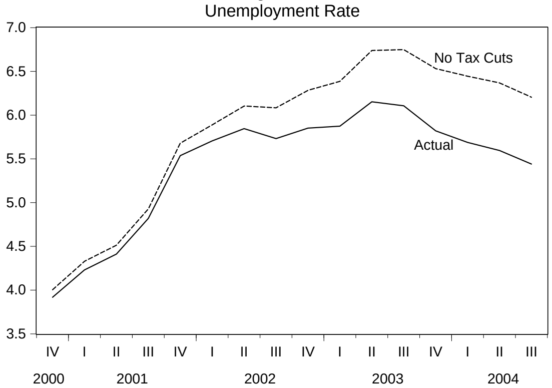


Figure 14d
Four-Quarter Percentage Change in the GDP Deflator

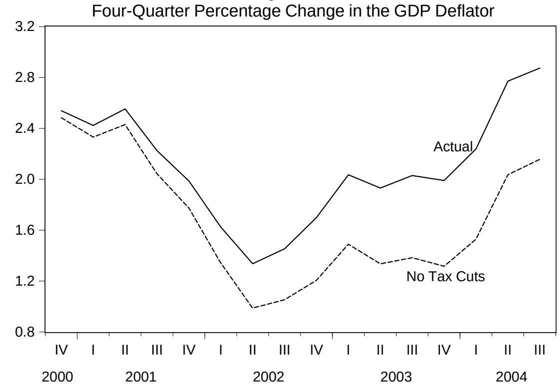


Figure 14e
Ratio of Federal Government Surplus to GDP

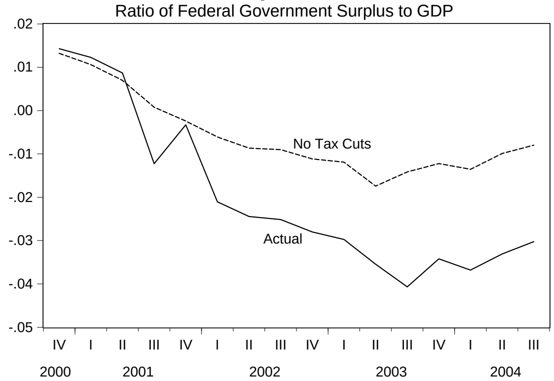
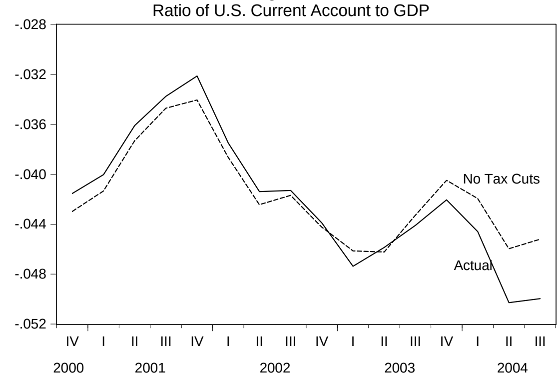


Figure 14f
Ratio of U.S. Current Account to GDP



Experiment 2: No G Increase   2000:4–2004:3

Figure 15a
Four-Quarter Growth Rate of Real GDP

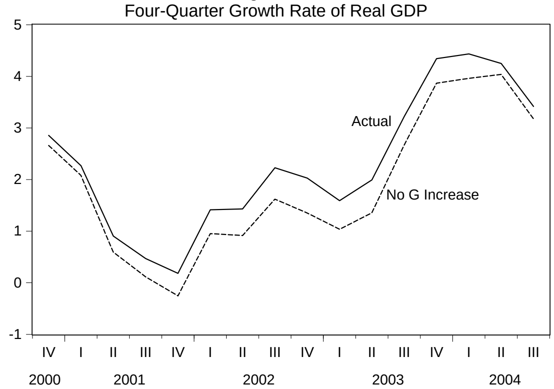


Figure 15b
Log of Employment

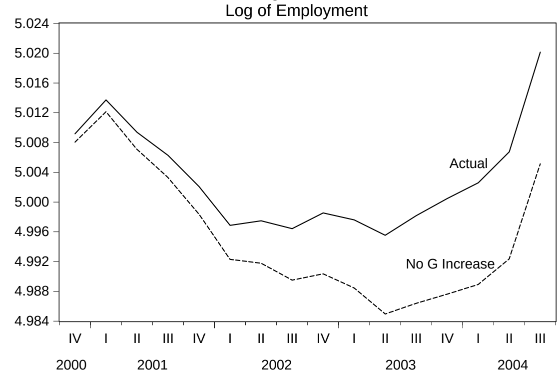


Figure 15c
Unemployment Rate

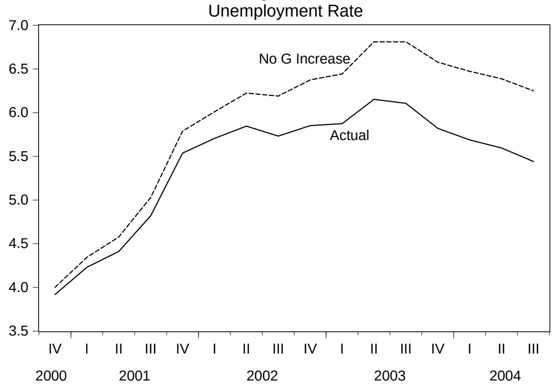


Figure 15d
Four-Quarter Percentage Change in the GDP Deflator

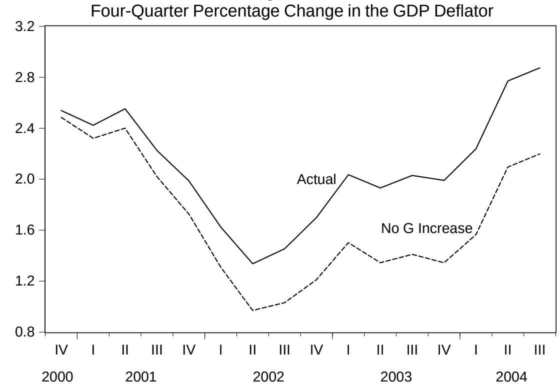


Figure 15e
Ratio of Federal Government Surplus to GDP

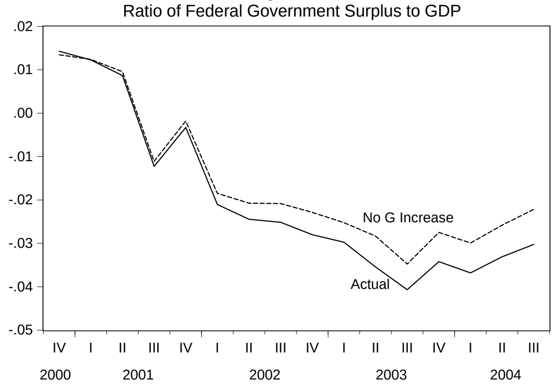
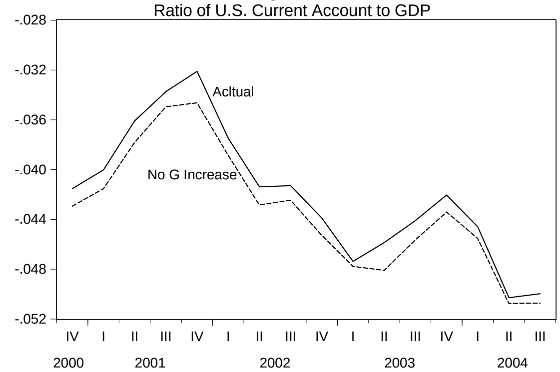
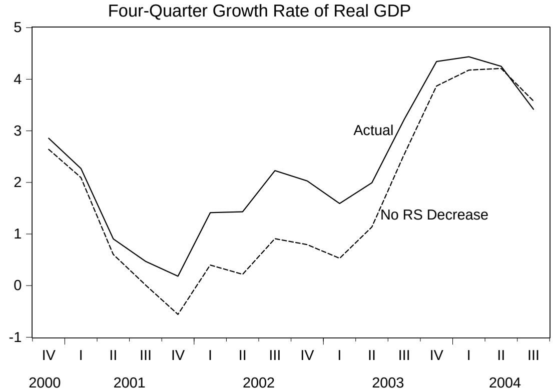


Figure 15f
Ratio of U.S. Current Account to GDP



Experiment 3: No RS Decrease 2000:4–2004:3

Figure 16a
Four-Quarter Growth Rate of Real GDP



Figlure 16b
Log of Employment

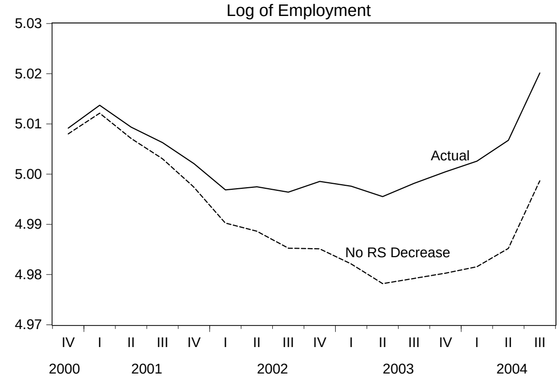


Figure 16c
Unemployment Rate

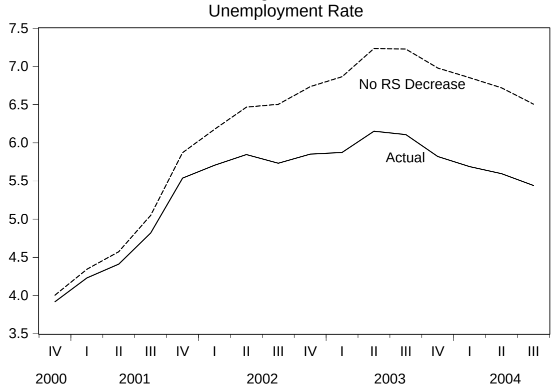


Figure 16d
Four-Quarter Percentage Change in the GDP Deflator

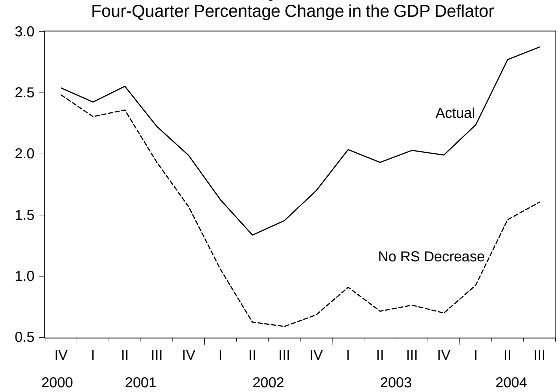


Figure 16e
Ratio of Federal Government Surplus to GDP

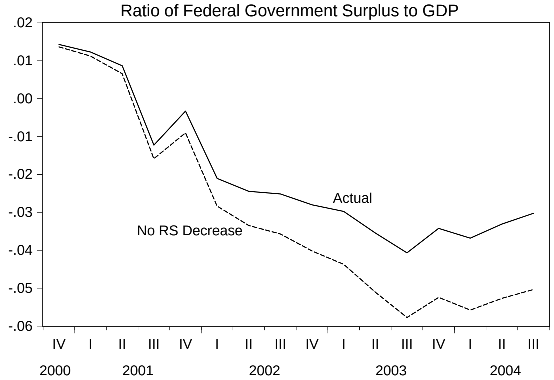
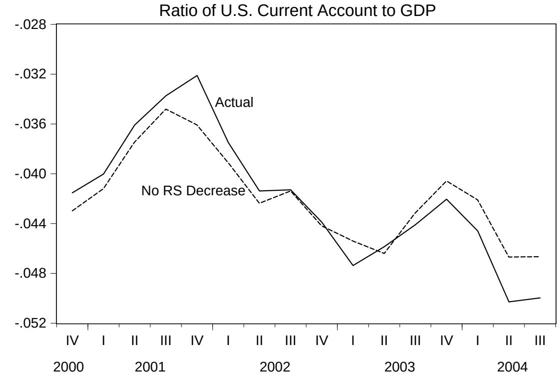


Figure 16f
Ratio of U.S. Current Account to GDP



Experiment 4: No Stimulus 2000:4–2004:3

Figure 17a
Four-Quarter Growth Rate of Real GDP

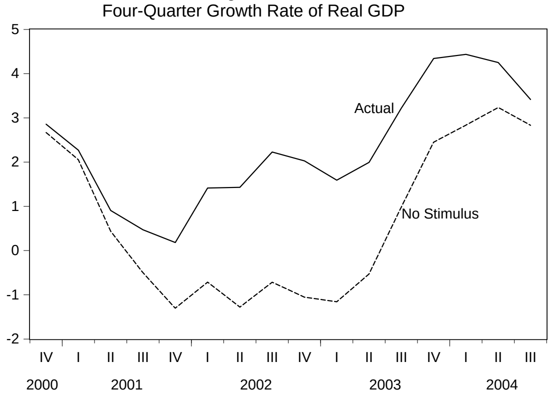


Figure 17b
Log of Employment

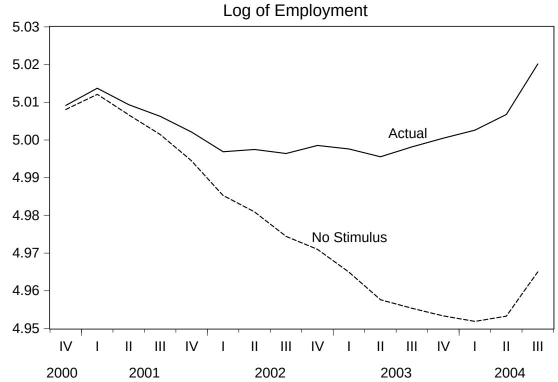


Figure 17c
Unemployment Rate

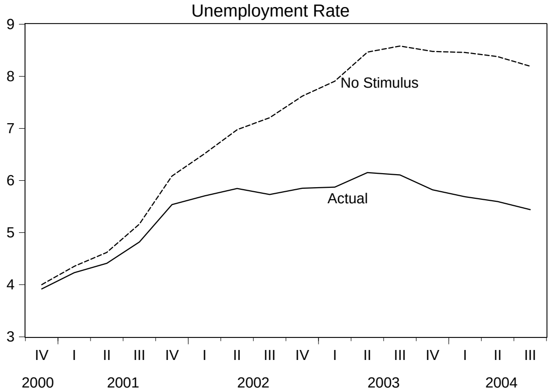


Figure 17d
Four-Quarter Percentage Change in the GDP Deflator

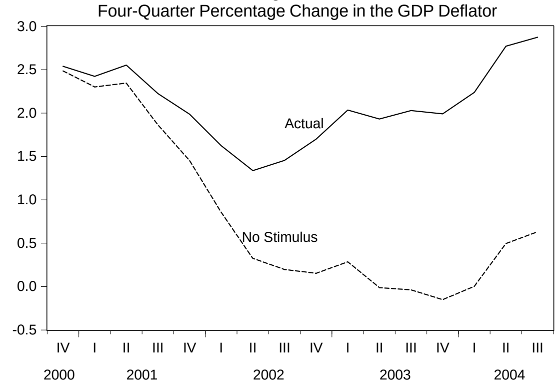


Figure 17e
Ratio of Federal Government Surplus to GDP

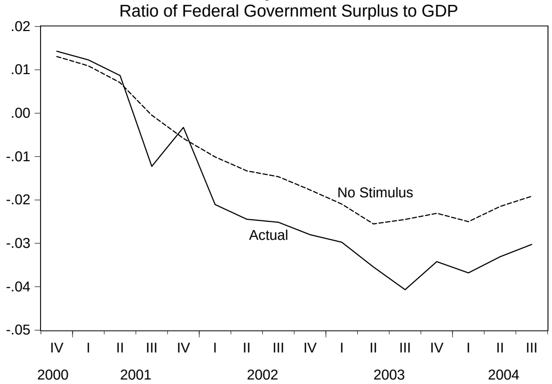
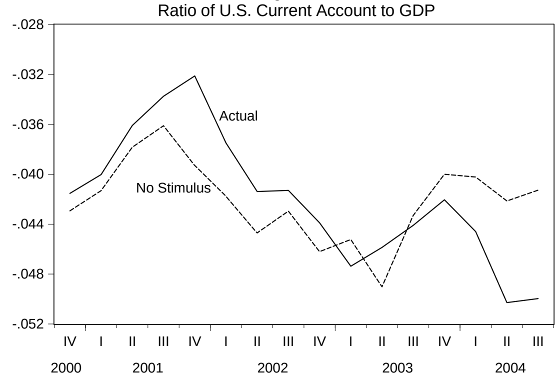


Figure 17f
Ratio of U.S. Current Account to GDP



Experiment 5: No Stimulus and No Stock Market Decline 2000:4–2004:3

Figure 18a
Four-Quarter Growth Rate of Real GDP

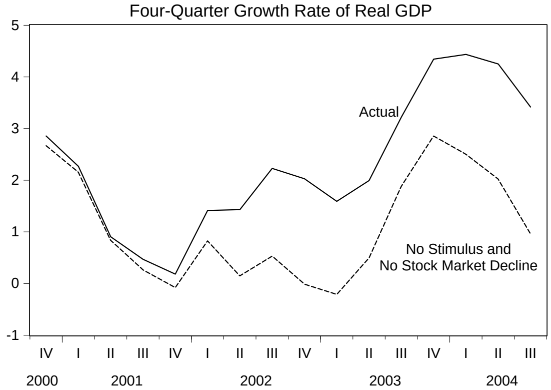


Figure 18b
Log of Employment

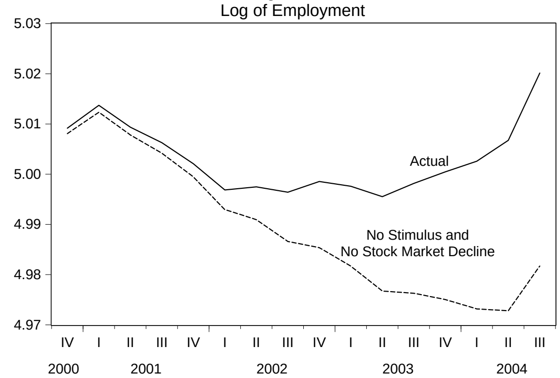


Figure 18c
Unemployment Rate

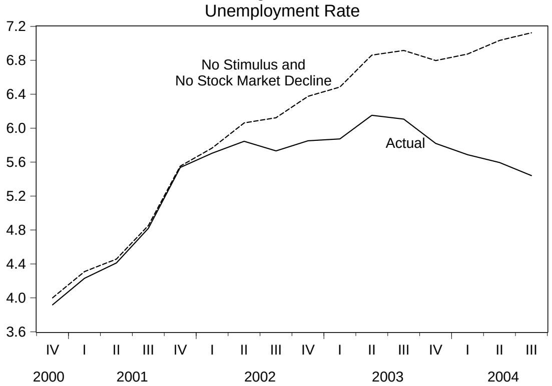


Figure 18d
Four-Quarter Percentage Change in the GDP Deflator

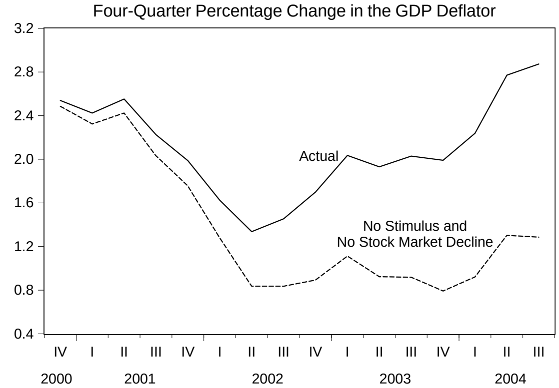


Figure 18e
Ratio of Federal Government Surplus to GDP

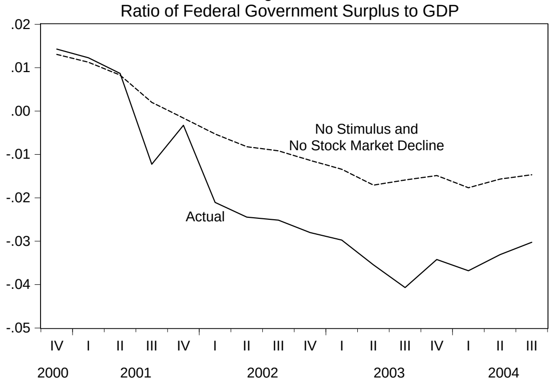
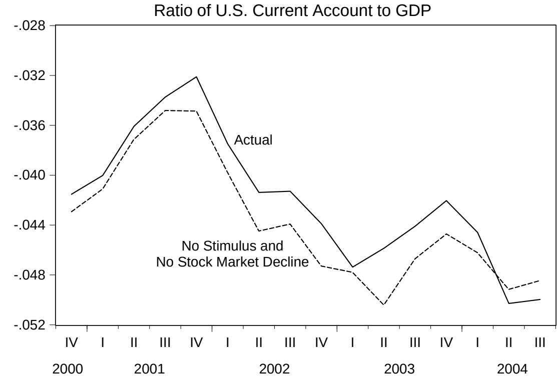


Figure 18f
Ratio of U.S. Current Account to GDP



Experiment 6: No Stimulus and No Export Decline 2000:4–2004:3

Figure 19a
Four-Quarter Growth Rate of Real GDP

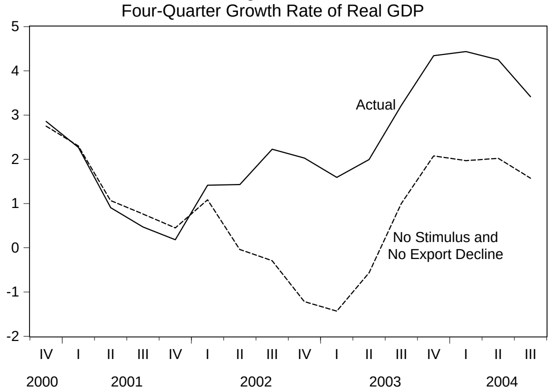


Figure 19b
Log of Employment

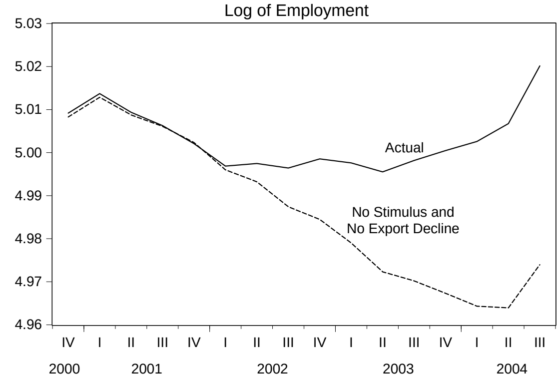


Figure 19c
Unemployment Rate

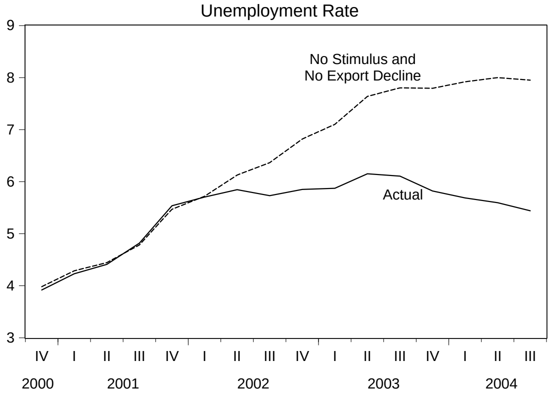


Figure 19d
Four-Quarter Percentage Change in the GDP Deflator

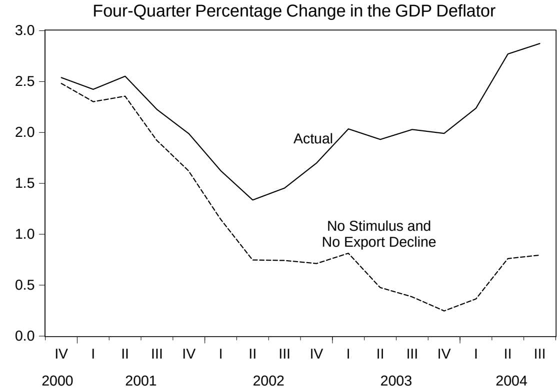


Figure 19e
Ratio of Federal Government Surplus to GDP

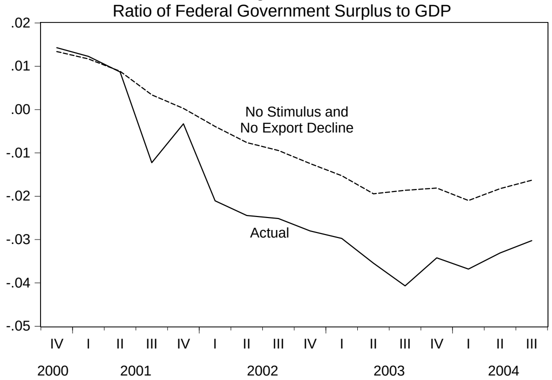
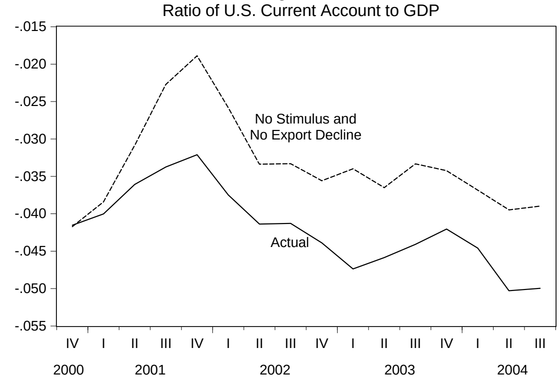


Figure 19f
Ratio of U.S. Current Account to GDP



Experiment 7: No Stimulus and No Stock Market and Export Decline
2000:4–2004:3

Figure 20a
Four-Quarter Growth Rate of Real GDP

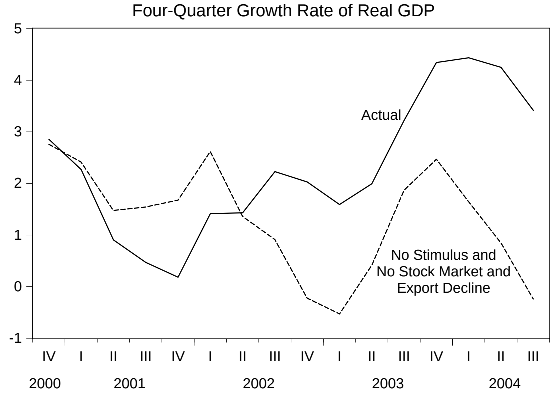


Figure 20b
Log of Employment

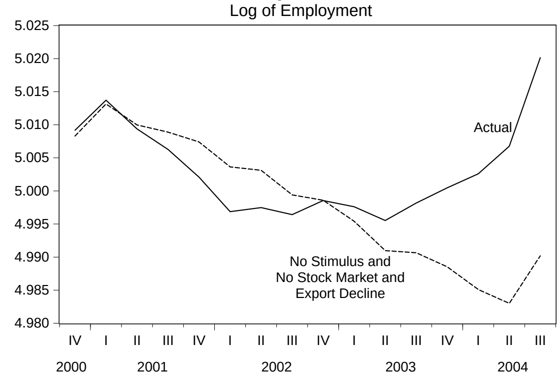


Figure 20c
Unemployment Rate

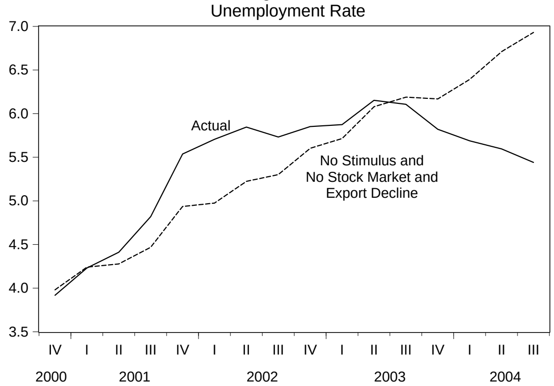


Figure 20d
Four-Quarter Percentage Change in the GDP Deflator

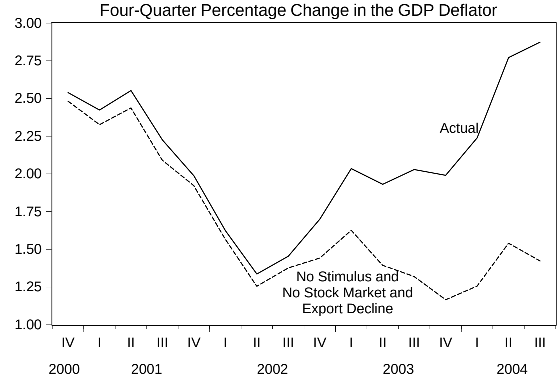


Figure 20e
Ratio of Federal Government Surplus to GDP

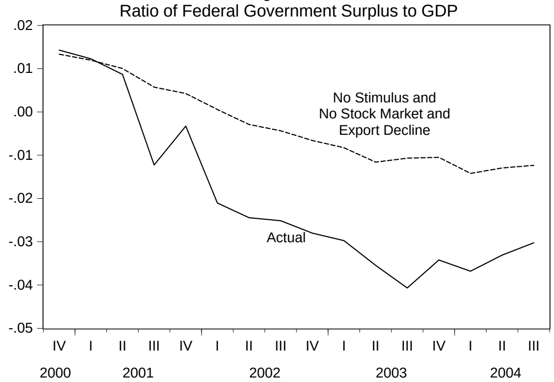
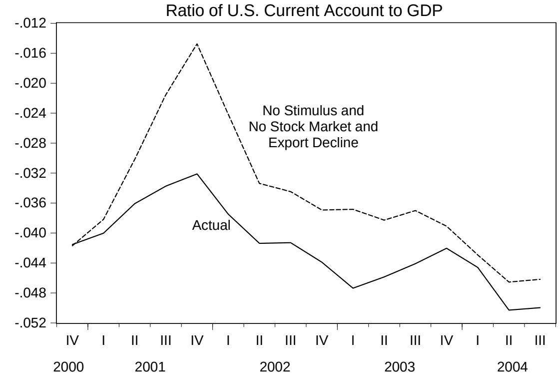


Figure 20f
Ratio of U.S. Current Account to GDP



Experiment 2: No G Increase

For the second experiment real federal government purchases of goods was taken to be 4.27 percent of potential real output, which is the actual percent in 2000:3. This case will be called the "no G increase" case.[133] Figure 7 shows a plot of this assumption. Figures 15a–15f and Table 4 present results. In this case employment is 1.5 percent lower by 2004:3, the unemployment rate is 0.8 percentage points higher, and the government budget has improved by 0.8 percent of GDP.

Experiment 3: No RS Decrease

For the third experiment the short-term interest rate, RS, was kept unchanged from its 2000:3 value, as shown in Figure 8. In this case there is no easing by the Fed; it will be called the "no RS decrease" case. Figures 16a–16f and Table 4 present results. In this case employment is 2.1 percent lower by 2004:3, the unemployment rate is 1.1 percentage points higher, and the government budget has worsened by 2.0 percent of GDP. The government budget worsens because of lower tax revenue due to the fall in taxable income and because of higher government interest payments due to the higher interest rates.

Experiment 4: No Stimulus

The fourth experiment is a combination of the first three. It will be called the "no stimulus" case. Figures 17a–17f and Table 4 present results. As noted in the Introduction, the results across the first three experiments are roughly additive, which can be seen in Table 4. In the no stimulus case employment is 5.5 percent lower by 2004:3, the unemployment rate is 2.8 percentage points higher, and the government budget has improved by 1.1 percent of GDP.

The results so far show the quantitative effects of the fiscal and monetary policy stimulus. As would be expected from looking at the size of the changes in the policy variables in Figures 5, 7, and 8, the quantitative effects on the economy are estimated to be quite large. Had there been no stimulus, the economy would have been much worse.

[133] There is, of course, some increase in government purchases of goods because potential output is increasing.

Table 4
Predicted minus Base for 2004:3
(percentage points)

Experiment	Employment		Unemployment Rate		Fed. Gov. Surplus	
1. No Tax Cuts	-1.8		0.8		2.2	
2. No G Increase	-1.5		0.8		0.8	
3. No RS Decrease	-2.1		1.1		-2.0	
4. No Stimulus: 1 + 2 + 3	-5.5		2.8		1.1	
		minus 4		minus 4		minus 4
5. 4 + no stock market fall	-2.8	1.7	1.7	-1.1	1.6	0.5
6. 4 + no export decline	-4.6	0.9	2.5	-0.3	1.4	0.3
7. 5 + 6	-3.0	2.5	1.5	-1.3	1.8	0.7

Experiment 5: No Stimulus and No Stock Market Decline

The fifth experiment estimates the effects on the economy from the fall in stock prices. So far CG, the capital gains or losses on financial assets held by the household sector, has been taken to be exogenous. CG, which is from the U.S. Flow of Funds Accounts, is a good measure of the effects of stock price changes on the household sector. The sum of CG between 1995:1 and 2000:3, the period of the stock market boom, was \$11.891 trillion, an average of \$517 billion per quarter. Then between 2000:4 and 2002:3 the sum was $-$\$5.199 trillion, an average of $-$\$650 billion per quarter. So about half of the gain during the boom was lost in this eight-quarter period. From 2002:4 on the stock market picked up, and the sum of CG between 2002:4 and 2004:3 was \$6.343 trillion, an average of \$793 billion per quarter.

The ratio of CG to nominal GDP averaged 0.104 between 1954:1 and 1994:4. Between 1980:1 and 1994:4 the average was virtually the same—0.100. For the fifth experiment the ratio of CG to nominal GDP was taken to be 0.104 in each quarter between 2000:4 and 2004:3. In other words, the stock market from 2000:4 on was taken to behave as it had on average from 1994:4 back. In this experiment there is no stock market "correction," just historically average behavior going forward.

The fifth experiment combines the CG changes and the no stimulus changes. If only the CG changes were used (with policy taken as it actually happened), the economy would be driven to values of the unemployment rate below historical experience. Macroeconometric models like the MC model are not necessarily reliable when pushed beyond the range of the historical data, and it is best to avoid doing this whenever possible. In the present case this can be done by combining the CG changes with the no stimulus changes.

Figures 18a–18f and Table 4 present results for the fifth experiment. Table 4 shows that in this case employment is 3.8 percent lower in 2004:3, which compares to 5.5 percent lower in experiment 4 using the actual stock market decrease. The fall in the stock market is thus estimated to have led employment to be 1.7 (= 5.5 - 3.8) percent lower than otherwise. Also, the fall is estimated to have led the unemployment rate to be 1.1 percentage points higher and the government budget to worsen by 0.5 percent of GDP.

Experiment 6: No Stimulus and No Export Decline

The sixth experiment estimates the effects on the economy from the fall in U.S. exports. U.S. total exports, EX, is endogenous. It is determined by the other countries' import demands for U.S. goods and services, which are endogenous in the MC model. To perform this experiment, an exogenous component of EX, U.S. exports of services, $USXS$, was increased to correspond to an increase in EX such that in the solution the ratio of EX to potential real output is roughly equal to its value in 2000:3, as shown in Figure 12. In the solution the ratio of EX to potential real output will not exactly equal the actual ratio in 2000:3 because EX is endogenous, and so this treatment is only an approximation.

The sixth experiment combines the EX changes and the no stimulus changes. Figures 19a–19f and Table 4 present results. In this case employment is 4.6 percent lower in 2004:3, which compares to 5.5 percent lower in experiment 4 where EX was not adjusted. The fall in exports is thus estimated to have led employment to be 0.9 percent lower than otherwise (5.5 - 4.6). Also, the fall is estimated to have led the unemployment rate to be 0.3 percentage points higher and the government budget to worsen by 0.3 percent of GDP.

Experiment 7: No Stimulus and No Stock Market and Export Decline

The seventh experiment is a combination of experiments 5 and 6. Figures 20a–20f and Table 4 present results. Again, the results are roughly additive, which can be seen in Table 4. In this combined case—no stimulus and no stock market and export decline—employment is 2.5 percent lower by 2004:3, the unemployment rate is 1.3 percentage points higher, and the government budget has worsened by 0.7 percent of GDP.

A useful way to summarize the overall results is to compare Figures 17c and 20c. Figure 17c shows that had there been no policy stimulus the unemployment rate would have risen to about 8.5 percent by 2003:2, whereas the actual rate was 6.2 percent. Figure 20c shows that had there been no policy stimulus and also no stock market and export decline, the unemployment rate would have only gradually risen and would have remained below the actual rate until 2003:3. Some policy stimulus would have been needed to keep the unemployment rate from rising, but much less than actually occurred. Figure 20e is also interesting in showing that the federal government budget would have been in surplus or roughly balanced over this period had there been no policy stimulus and no stock market and export decline.

5.4.5 Conclusion

The answer in this section to the question posed in the Introduction, namely why the U.S. economy in the 2000:4–2004:3 period was fairly sluggish in light of the large expansionary fiscal and monetary policies, is that there were large negative effects from the decline in the stock market and exports. The answer is not that there were large structural changes in the economy or systematic bad shocks, since none were found. There is also no evidence that the tax cuts were less stimulative than they otherwise would have been because of after-tax income distribution effects.

The present analysis has taken the decline in the stock market and exports to be exogenous. Whatever led to household wealth falling by $5.199 trillion between 2000:4 and 2002:3 is not explained.[134] The decline in U.S. exports is also not explained. It is interesting, as noted in the Introduction, that the timing of the decline in exports matches closely the timing of the stock market decline. Between 2000:4 and 2002:1, U.S. exports of goods and services declined $120.4 billion in real terms (2000 dollars). Of this, $79.6 billion was in exports of capital goods, except automotive, and $12.0 billion was in durable industrial supplies and materials. The decline in travel (mostly foreign tourism in the United States) was $15.8 billion. The events of 9/11 undoubtedly contributed to this decline in travel, although travel was not the main source of the overall decline in exports. Much of the overall decline would appear to be a decrease in capital investment abroad, and this decrease could have been affected by the generally worldwide decline in stock prices.[135] If much of the decrease in capital investment was due to the decline in

[134]Productivity grew fairly well in the post boom period, and so the fall in stock prices cannot be blamed on any productivity slowdown.

[135]The effects of the decline in stock prices in other countries on those countries' demand for imports cannot be examined using the MC model because wealth effects from stock price changes are only estimated for the United States. The MC model also does not account for the possible

stock prices, then the decline in stock prices is the main source of the sluggish post boom period. In this case only the stock market decline need be considered to be taken as exogenous.

The results in this section are similar to those in Section 5.3 except with the opposite sign. It was seen in Section 5.3 that most of the unusual features of the U.S. economy in the last half of the 1990s is due to the huge increase in stock prices. (Remember from Subsection 5.4.? that the increase in household wealth between 1995:1 and 2000:3 was $13.557 trillion.) In this section much of the unusual features of the economy in the first part of the 2000s are attributed to the huge decrease in stock prices, especially if much of the export decline was a result of the stock market decline.

The main point of this section and Section 5.3 is reflected in Figures 11 and 13. Had the stock market from 1995 on grown at its historically average rate rather than the actual rates in the figures, the MC model says that the economy would have been much different. The wealth effects both going up and going down are estimated to be quite large. No explanation is offered here as to why the stock market boomed in the last half of the 1990s and fell substantially after that. It seems highly unlikely that an econometrically estimated equation can be found that explains much of this variance. With hindsight, however, it is interesting to speculate whether monetary policy could have stopped the stock market boom in the late 1990s. Had it been able to, it appears that the quantitative effects on real output would have been large.

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correlation of U.S. exports and imports due to re-exports. For example, if exports are down because re-exports are down, then imports are down, and this correlation is not taken into account. In other words, none of any fall in imports is attributed to a fall in exports. The vulnerability of the economy to trade is thus likely to be at least slightly over stated in the MC model.

5.5 Estimated Macroeconomic Effects of the U.S. Stimulus Bill[136]

5.5.1 Introduction

This section uses the MC model to analyze the macroeconomic effects of the U.S. stimulus bill passed in February 2009. The policy changes are taken from a report issued on March 2, 2009 by the Congressional Budget Office (CBO) (2009). A baseline simulation is first run under the assumption that the stimulus bill passed (which it did), and then a simulation is run with the stimulus taken out. The difference between the predicted values from the two simulations for each variable and each quarter is an estimate of the stimulus effects on that variable. The simulation period is 2009:1–2020:4.

There is considerable controversy about the stimulus effects, and a number of methodologies have been followed to estimate them. The CBO (2010) uses results from two commercial forecasting models and the FRB-US model of the Federal Reserve Board to choose ranges for a number of government spending and tax multipliers on output. These multipliers are then used to compute stimulus effects on output. Additional equations are used to link changes in other variables, like employment and the unemployment rate, to output changes. The estimates are partial in that they are not the result of solving a complete model. Many potential endogenous effects are ignored. Also, as will be seen, the ranges chosen for the multipliers are large, which leads to large ranges for the estimated stimulus effects.

Romer and Bernstein (2009) follow a similar methodology. They use a commercial forecasting model and the FRB-US model to choose government spending and tax multipliers on output. They use these multipliers to compute stimulus effects on output, and they have an additional equation linking employment changes to output changes. They present results for 2010:4. Again, these estimates are not the result of solving a complete model.

Another procedure for estimating multipliers is what might be called a "reduced form" procedure. The change in real GDP is regressed on the change in a policy variable of interest and a number of other variables. The equation estimated is not, however, a true reduced form equation because many variables are omitted, and so the coefficient estimate of the policy variable will be biased if the policy variable is correlated with omitted variables. The aim using this approach is to choose a policy variable that seems unlikely to be correlated with the omitted variables. Hall (2009) and Barro and Redlick (2010) are concerned with government spending

[136]This section is an update of the results in Fair (2010b).

multipliers and focus on defense spending during wars.[137] Romer and Romer (2009) are concerned with tax multipliers and use narrative records to choose what they consider exogenous tax policy actions, i.e, actions that are uncorrelated with the omitted variables.

The methodology of structural macroeconometric modeling upon which the MC model is based does not have the problem of possible omitted variable bias in reduced form equations, since reduced form equations are not directly estimated. What is required is that the structural equations be consistently estimated. Take, for example, a consumption or investment equation. If there are right hand side endogenous variables, like current income or a current interest rate, and thus correlation between these variables and the error term in the equation, this has to be accounted for. Two-stage least squares (2SLS) is one option. First stage regressors must be found that are correlated with the endogenous variables and uncorrelated with the error term. If one suspects that a current government spending or tax rate variable depends on current endogenous variables, the variable would need to be lagged one period before being used as a first stage regressor. The estimation is slightly more complicated if the error term in the structural equation is serially correlated. In this case the 2SLS estimator can be modified to jointly estimate the serial correlation coefficient and the structural coefficients, as discussed in Section 2.3.1. The aim in structural modeling is to find good structural equations—good approximations to reality—and to estimate them consistently.[138] Reduced form equations are not estimated but derived, and there are many nonlinear restrictions on the reduced form equations.

This structural approach uses much more information on the economy than does the reduced form approach mentioned above. For example, the implicit reduced form equation for U.S. output in the MC model is nonlinear and includes hundreds of exogenous and lagged endogenous variables. There are also hundreds of nonlinear restrictions on the reduced form coefficients. Given the complexity of the economy, it seems unlikely that estimating reduced form equations with many omitted variables and no restrictions from theory on the coefficients will produce trustworthy results even if an attempt is made to account for omitted variable bias.

[137] Barro and Redlick (2010) also estimate a tax multiplier.

[138] Commercial forecasting models like the ones used by the CBO (2010) and Romer and Bernstein (2009) are not in the academic literature, and so it is hard to evaluate them. It does not appear, however, that the structural equations in these models are consistently estimated.

5.5.2 Multiplier Comparisons

It will be useful before discussing the stimulus experiment to show the multiplier properties of the MC model regarding U.S. government spending on goods (COG) and on transfer payments ($TRGHQ$). (Both COG and $TRGHQ$ are in real terms.) Table 1 presents results for the MC model alone, and Table 2 compares the MC multipliers for output to multipliers from the studies mentioned in Section 1.

The results from two simulations are presented in Table 1, one in which COG is permanently increased by 1.0 percent of real GDP and one in which $TRGHQ$ is permanently increased by 1.0 percent of real GDP. The simulation period is 2009:1–2020:4, and the baseline run is the one discussed in the next section. No other changes were made for the two simulations. In particular, no tax increases were imposed to pay for the increased spending. These simulations are not meant to be realistic (or desirable) policy actions. They are simply meant to illustrate the properties of the model.

Estimated standard errors of the multipliers are also presented in Table 1. These values are computed using a bootstrap procedure discussed in Section 2.7. The exact details for this particular exercise are presented in the appendix to this section.

Table 1 shows that the peak COG multiplier for output is 1.55 after 4 quarters. The multiplier settles down to about 1.2 after about 16 quarters. The peak $TRGHQ$ multiplier for output is 0.79 after 6 quarters. The multiplier settles down to about 0.6 after about 16 quarters. Physical stock effects, interest rate effects, and price effects are the main reasons for the decline in the multipliers after the peak. By 2020:4 the debt/GDP ratio has risen by 8.58 percentage points in the COG case and by 9.75 percentage points in the $TRGHQ$ case.

The estimated standard errors in Table 1 are generally small relative to the size of the multipliers. For example, the four-quarter-ahead COG multiplier for output of 1.55 has an estimated standard error of 0.09. For the 48-quarter-ahead COG multiplier for the debt/GDP ratio of 8.58, the estimated standard error is 0.84. There is somewhat less precision relative to the size of the multiplier for the transfer payment experiment, where the four-quarter-ahead $TRGHQ$ multiplier for output of 0.68 has an estimated standard error of 0.09. The fairly low estimated standard errors are consistent with results Subsection 3.9.2, which show that uncertainty from estimated coefficients is generally small relative to uncertainty from structural error terms. Multiplier uncertainty is from the uncertainty of the coefficient estimates and not also from the uncertainty of the structural error terms because the latter roughly cancel out when computing multipliers.

Table 1
Government Spending Multipliers
Deviations from Baseline in Percentage Points

qtr	Y		U		P		r		debt	
					Spending on Goods (COG)					
2009.1	0.84	(0.06)	-0.18	(0.03)	0.03	(0.01)	0.15	(0.00)	-0.24	(0.04)
2009.2	1.31	(0.08)	-0.39	(0.06)	0.09	(0.02)	0.38	(0.13)	-0.34	(0.05)
2009.3	1.51	(0.09)	-0.56	(0.07)	0.18	(0.03)	0.56	(0.25)	-0.31	(0.07)
2009.4	1.55	(0.09)	-0.66	(0.08)	0.29	(0.05)	0.65	(0.31)	-0.20	(0.08)
2010.1	1.52	(0.12)	-0.71	(0.08)	0.42	(0.06)	0.70	(0.33)	-0.09	(0.10)
2010.2	1.45	(0.12)	-0.72	(0.09)	0.49	(0.08)	0.74	(0.26)	0.07	(0.14)
2010.3	1.36	(0.12)	-0.71	(0.09)	0.58	(0.09)	0.76	(0.22)	0.25	(0.15)
2010.4	1.27	(0.12)	-0.66	(0.09)	0.66	(0.12)	0.74	(0.35)	0.46	(0.17)
2011.1	1.20	(0.13)	-0.62	(0.09)	0.73	(0.14)	0.72	(0.26)	0.71	(0.19)
2011.2	1.16	(0.13)	-0.56	(0.09)	0.78	(0.15)	0.70	(0.19)	0.97	(0.21)
2011.3	1.12	(0.13)	-0.52	(0.08)	0.82	(0.17)	0.68	(0.16)	1.17	(0.23)
2011.4	1.11	(0.12)	-0.50	(0.09)	0.86	(0.18)	0.68	(0.15)	1.38	(0.25)
2012.1	1.10	(0.11)	-0.49	(0.08)	0.90	(0.18)	0.69	(0.14)	1.56	(0.26)
2012.2	1.09	(0.11)	-0.46	(0.08)	0.93	(0.18)	0.67	(0.13)	1.78	(0.27)
2012.3	1.09	(0.10)	-0.44	(0.08)	0.95	(0.19)	0.65	(0.12)	2.00	(0.27)
2012.4	1.10	(0.10)	-0.44	(0.06)	0.97	(0.20)	0.65	(0.12)	2.23	(0.28)
2013.4	1.12	(0.10)	-0.41	(0.06)	1.06	(0.22)	0.63	(0.12)	3.16	(0.31)
2014.4	1.16	(0.10)	-0.42	(0.06)	1.14	(0.23)	0.63	(0.21)	3.93	(0.36)
2015.4	1.19	(0.12)	-0.43	(0.06)	1.21	(0.25)	0.64	(0.20)	4.67	(0.43)
2016.4	1.21	(0.12)	-0.43	(0.06)	1.28	(0.28)	0.64	(0.17)	5.44	(0.45)
2017.4	1.22	(0.12)	-0.43	(0.06)	1.33	(0.30)	0.64	(0.17)	6.22	(0.53)
2018.4	1.22	(0.12)	-0.43	(0.07)	1.38	(0.31)	0.63	(0.17)	7.01	(0.62)
2019.4	1.22	(0.13)	-0.44	(0.06)	1.43	(0.33)	0.63	(0.17)	7.80	(0.72)
2020.4	1.23	(0.13)	-0.44	(0.06)	1.47	(0.33)	0.63	(0.18)	8.58	(0.84)
					Spending on Transfer Payments (TRGHQ)					
2009.1	0.16	(0.03)	-0.03	(0.01)	0.01	(0.00)	0.03	(0.00)	0.16	(0.02)
2009.2	0.36	(0.06)	-0.10	(0.03)	0.03	(0.01)	0.09	(0.04)	0.27	(0.04)
2009.3	0.54	(0.08)	-0.18	(0.04)	0.06	(0.01)	0.18	(0.09)	0.38	(0.06)
2009.4	0.68	(0.09)	-0.25	(0.06)	0.10	(0.02)	0.25	(0.14)	0.50	(0.08)
2010.1	0.76	(0.10)	-0.31	(0.06)	0.16	(0.04)	0.31	(0.17)	0.62	(0.09)
2010.2	0.79	(0.11)	-0.35	(0.06)	0.20	(0.05)	0.36	(0.18)	0.78	(0.10)
2010.3	0.78	(0.11)	-0.38	(0.07)	0.26	(0.06)	0.40	(0.16)	0.95	(0.12)
2010.4	0.75	(0.11)	-0.38	(0.07)	0.31	(0.07)	0.41	(0.22)	1.15	(0.13)
2011.1	0.71	(0.12)	-0.37	(0.07)	0.35	(0.08)	0.42	(0.17)	1.40	(0.14)
2011.2	0.67	(0.11)	-0.35	(0.07)	0.40	(0.08)	0.42	(0.11)	1.63	(0.15)
2011.3	0.63	(0.11)	-0.32	(0.07)	0.43	(0.09)	0.41	(0.10)	1.86	(0.17)
2011.4	0.59	(0.09)	-0.30	(0.06)	0.46	(0.10)	0.40	(0.10)	2.08	(0.17)
2012.1	0.56	(0.10)	-0.28	(0.05)	0.49	(0.11)	0.40	(0.10)	2.30	(0.17)
2012.2	0.54	(0.09)	-0.26	(0.05)	0.52	(0.11)	0.38	(0.08)	2.54	(0.17)
2012.3	0.53	(0.08)	-0.24	(0.05)	0.54	(0.12)	0.36	(0.07)	2.77	(0.18)
2012.4	0.52	(0.08)	-0.23	(0.04)	0.55	(0.12)	0.36	(0.08)	3.02	(0.20)
2013.4	0.52	(0.07)	-0.21	(0.04)	0.62	(0.13)	0.34	(0.09)	3.98	(0.21)
2014.4	0.56	(0.10)	-0.22	(0.03)	0.69	(0.14)	0.35	(0.13)	4.78	(0.24)
2015.4	0.58	(0.09)	-0.23	(0.04)	0.76	(0.16)	0.37	(0.13)	5.55	(0.30)
2016.4	0.60	(0.09)	-0.24	(0.04)	0.82	(0.18)	0.37	(0.11)	6.36	(0.38)
2017.4	0.60	(0.10)	-0.25	(0.04)	0.88	(0.19)	0.38	(0.10)	7.19	(0.46)
2018.4	0.60	(0.12)	-0.25	(0.04)	0.94	(0.19)	0.38	(0.10)	8.04	(0.48)
2019.4	0.60	(0.11)	-0.25	(0.05)	0.99	(0.18)	0.38	(0.11)	8.90	(0.62)
2020.4	0.60	(0.12)	-0.26	(0.05)	1.04	(0.20)	0.39	(0.12)	9.75	(0.75)

• percent deviations for Y and P, absolute deviations for U, r, and $debt$.
$Y = GDPR$ = real GDP, $U = UR$ = unemployment rate, $P = GDPD$ = GDP deflator,
$r = RS$ = three-month Treasury bill rate,
$debt = AGZGDP$ = federal government debt/GDP ratio.
• Estimated standard errors in parentheses.

Table 2
Multiplier Comparisons for Output

qtr	MC	MC^a	RB	CBO	BR	Hall
			Spending on Goods			
1	0.84	0.85	1.05			
2	1.31	1.35	1.24			
3	1.51	1.59	1.35			
4	1.55	1.70	1.44	1.0–2.5	0.44	0.55
5	1.52	1.74	1.51			
6	1.45	1.72	1.53			
7	1.36	1.69	1.54			
8	1.27	1.64	1.57		0.64	
9	1.20	1.59	1.57			
10	1.16	1.56	1.57			
11	1.12	1.52	1.57			
12	1.11	1.48	1.57			
13	1.10	1.46	1.57			
14	1.09	1.43	1.57			
15	1.09	1.40	1.57			
16	1.10	1.38	1.55			

qtr	MC	MC^a	RB	RR	CBO	BR
			Spending on Transfer Payments or Tax Cuts			
1	0.16	0.16	0.00	0.40		
2	0.36	0.37	0.49	0.20		
3	0.54	0.57	0.58	0.70		
4	0.68	0.72	0.66	1.10	0.8–2.1	1.1
5	0.76	0.83	0.75	1.40		
6	0.79	0.89	0.84	1.70		
7	0.78	0.91	0.93	2.50		
8	0.75	0.91	0.99	2.70		
9	0.71	0.90	0.99	3.00		
10	0.69	0.88	0.99	3.08		
11	0.63	0.84	0.99	2.70		
12	0.59	0.81	0.99	2.50		
13	0.56	0.77	0.99			
14	0.54	0.75	0.99			
15	0.53	0.72	0.99			
16	0.52	0.70	0.98			

MC = MC model, Fed rule used, standard errors in Table 1.
MC^a = MC model, Fed rule dropped, standard errors similar to those in Table 1.
RB = Romer and Bernstein (2009), Appendix 1.
CBO = CBO (2010), Table 2.
BR = Barro and Redlick (2010), Table 2, starting date 1939. Standard error 0.06 for 0.44 and 0.06 for 0.20 (= 0.64 - 0.44).
Hall = Hall (2009), Table 1, 1930–2008 sample period. Standard error 0.08.
RR = Romer and Romer (2009), estimated from Figure 4.
Standard errors: 0.5 for 4, 0.7 for 8, 0.87 for 10, 0.9 for 12.

Consider now the multiplier comparisons in Table 2. The MC multipliers, which are taken from Table 1, use the estimated interest rate rule of the Fed, equation 30, which predicts that the Fed will raise the short term interest rate as the economy expands and inflation increases. The Romer-Bernstein and CBO multipliers, on the other hand, are based on the assumption that there is no interest rate response to the government spending increases and tax decreases. For comparison purposes the experiments in Table 1 were repeated with the Fed rule dropped, which means that the short term interest rate is unchanged each quarter from its baseline value. The MC^a multipliers in Table 2 are from these experiments. The CBO multipliers are over "several" quarters, which I have taken to be four in Table 2. In the second half of the table positive values are used, which means an increase in transfer payments or decrease in taxes.

Consider spending on goods first. The main differences are: (1) the multipliers for Barro-Redlick and Hall, based on the reduced form approach, are much smaller than the others,[139] (2) the CBO range is large, and (3) the MC and MC^a multipliers begin to fall after 5 quarters, contrary to those for Romer and Bernstein. This latter result is mainly due to the physical-stock effects in the MC model, which were discussed in the previous section.

The transfer payment and tax results in Table 2 lead to similar conclusions except that the multiplier for Barro-Redlick of 1.1 is similar to the others. Also, Romer and Romer have much higher multipliers after four quarters than the others. After 10 quarters the multiplier is 3.08, which compares to 0.69 for MC, 0.89 for MC^a, and 0.99 for Romer-Bernstein.

5.5.3 The Stimulus Experiment

Stimulus Changes

The results here are based on actual data through 2013:3 (data available as of November 11, 2013). The simulation period is 2009:1–2020:4, 48 quarters. The baseline values for 2009:1–2013:3 are the actual values, and the baseline values for 2013:4–2020:4 are values from a forecast I made on November 11, 2013. This forecast incorporates the stimulus measures (since the stimulus was passed). The estimated Fed rule in the MC model is used for the stimulus experiment. It seems

[139] Hall (2009, Table 2) also reports results from VAR studies. The VAR multipliers after four quarters range from 0.31 to 1.00, also lower than the other multipliers in Table 2. VAR models suffer from the same criticism made in Section 1 about the reduced-form equations, namely that there are many omitted variables.

unrealistic to assume that the Fed would never respond to the stimulus measures, especially over many quarters. This would be contrary to its historical behavior. Using the estimated rule assumes that the Fed is behaving as it has historically.

The simulation that was run for the experiment has the stimulus measures taken away. In order to do this, the stimulus measures have to be chosen. This was done as follows. The stimulus bill has tax cuts, transfer payment increases, and increases in government purchases of goods and services. (Unless otherwise stated, “government” in what follows means federal government.) Some of the transfers are to state and local governments and some are directly to households. In the model it makes no difference whether the federal government makes transfer payments directly to households or makes them to state and local governments if the state and local governments in turn pass on the transfer payments to households. In either case there is an increase in disposable income of the household sector. To keep matters simple in the present experiment, all transfer payment increases are put into federal transfer payments to households. In addition, tax cuts are taken to be increases in transfer payments to households rather than decreases in the personal income tax rate in the model. Most of the tax cuts do not involve cutting tax rates, and so it seems better to put them into transfer payments. Therefore, only two variables are changed for the stimulus experiment, federal transfer payments to households and federal purchases of goods and services.

The timing of expenditures is a major issue in trying to capture the effects of any stimulus package. I have roughly followed the CBO (2009) timing for the present experiment. I have assumed that the nominal value of transfer payments is \$172 billion larger in fiscal 2009, \$370 billion larger in fiscal 2010, \$103 billion larger in fiscal 2011, \$12 billion larger in fiscal 2012, and \$11 billion larger (at an annual rate) in 2012:4. I have roughly spread these increases evenly within the four quarters of the fiscal year. I have assumed that nominal government spending on goods is \$21 billion larger at an annual rate in 2009:2, \$29 billion larger at an annual rate in 2009:3, \$29 billion larger in fiscal 2010, \$31 billion larger is fiscal 2011, \$24 billion larger in fiscal 2012, and \$17 billion larger at an annual rate in 2012:4. No changes in transfer payments and government spending were made for 2009:1. Also, no changes were made after 2012:4. In particular, no tax increases or government spending decreases were imposed. The total nominal government spending increase over the four-year period is \$762 billion, of which \$660 billion is in transfer payments and \$102 billion is in purchases of goods.

The two relevant exogenous policy variables in the model are real federal transfer payments to households, $TRGHQ$, and real federal purchases of goods and services, COG. These are the variables changed for the results in Table 1.

To get the stimulus increases for COG the above nominal increases were divided by predicted values of the government spending deflator from the baseline run. Similarly, to get the stimulus increases for $TRGHQ$ the above nominal increases were divided by predicted values of the GDP deflator from the baseline run. Table 3 presents the stimulus changes for the two variables as a fraction of real GDP from the baseline run. The main increases are between 2009:2 and 2010:3. The increases are slightly larger for 2010 than for 2009.

Results

As noted above, the baseline values are actual values for 2009:1–2013:3 and forecast values for 2013:4–2020:4.[140] If the actual residuals for 2009:1–2013:3 are added to the model (with zero residuals used for 2013:4–2020:4) and a simulation is run for the 2009:1–2020:4 period, the solution values reproduce the baseline values. (Zero residuals are used for 2013:3–2020:4 because these were used for the forecast.) In order to have the experiment with the stimulus measures taken out be consistent with this, the same (actual) residuals for 2009:1–2013:3 were used (with zero residuals used for 2013:4–2020:4). Given these residuals and the new (lower) values of COG and $TRGHQ$, the model was solved for 2009:1–2020:4. This solution is the model's estimate of what the world economy would have been like had there been no stimulus bill. Results are presented in Tables 4 and 5 for selected variables. Table 4 presents results for the United States, and Table 5 presents results for other countries. Note that the only changes made were to COG and $TRGHQ$. No future tax increases or spending cuts were imposed to pay for some of the stimulus. This experiment thus does not necessarily represent a realistic (or desirable) long run policy. It is simply examining the macroeconomic consequences of the stimulus bill with no other changes made. The values in Tables 4 and 5 are baseline values divided by or subtracted from the predicted no-stimulus values.

Values are presented in Table 4 for real GDP, employment, the unemployment rate, the GDP deflator, the three-month Treasury bill rate, the ratio of federal interest payments to GDP, the ratio of the federal government deficit to GDP, and the ratio of the federal government debt to GDP. The cyclical features of the model are immediately evident from Table 4. The stimulus in 2009–2011 has negative effects afterwards. These effects are mostly from the negative physical stock effects

[140]For countries other than the United States not all variable values were available through 2013:3, and when necessary missing values were chosen ahead of time (usually by simple extrapolation).

Table 3
Stimulus Changes for COG and TRGHQ
Percent of Real GDP in Percentage Points

qtr	COG	TRGHQ
2009.1	0.00	0.00
2009.2	0.15	2.20
2009.3	0.21	2.54
2009.4	0.21	2.51
2010.1	0.21	2.50
2010.2	0.20	2.48
2010.3	0.20	2.46
2010.4	0.21	0.68
2011.1	0.21	0.68
2011.2	0.21	0.68
2011.3	0.20	0.67
2011.4	0.16	0.07
2012.1	0.15	0.07
2012.2	0.15	0.07
2012.3	0.15	0.07
2012.4	0.10	0.07

Table 4
Estimated Stimulus Effects
Baseline Values Divided By or Subtracted From Predicted No-Stimulus Values
Percentage Points

qtr	Y	J	U	P	r	int	def	debt	J[a]
2009.1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.
2009.2	0.47	0.14	-0.10	0.02	0.09	0.02	2.23	0.30	175.
2009.3	1.10	0.43	-0.30	0.07	0.16	0.03	2.47	0.58	527.
2009.4	1.66	0.80	-0.54	0.17	0.06	0.04	2.28	0.83	977.
2010.1	2.08	1.18	-0.78	0.35	0.11	0.05	2.16	1.05	1449.
2010.2	2.35	1.54	-0.98	0.46	0.15	0.07	2.07	1.32	1887.
2010.3	2.50	1.83	-1.13	0.64	0.16	0.08	2.03	1.64	2248.
2010.4	2.28	1.98	-1.16	0.83	0.14	0.09	0.33	1.72	2437.
2011.1	1.91	1.98	-1.09	0.97	0.13	0.11	0.47	2.03	2452.
2011.2	1.53	1.87	-0.95	1.10	0.05	0.13	0.59	2.34	2328.
2011.3	1.19	1.70	-0.77	1.18	0.02	0.13	0.69	2.62	2124.
2011.4	0.78	1.46	-0.58	1.25	-0.01	0.13	0.15	2.81	1837.
2012.1	0.44	1.20	-0.40	1.28	-0.08	0.13	0.23	2.99	1513.
2012.2	0.20	0.94	-0.20	1.27	-0.20	0.14	0.30	3.19	1194.
2012.3	0.04	0.71	-0.05	1.26	-0.30	0.11	0.31	3.33	909.
2012.4	-0.08	0.52	0.05	1.18	-0.35	0.11	0.30	3.51	661.
2013.1	-0.22	0.33	0.16	1.12	-0.40	0.09	0.15	3.67	427.
2013.2	-0.29	0.18	0.22	1.06	-0.45	0.10	0.18	3.77	233.
2013.3	-0.29	0.07	0.26	1.00	-0.48	0.07	0.15	3.80	91.
2013.4	-0.26	0.00	0.26	0.94	-0.48	0.06	0.13	3.81	-2.
2014.1	-0.21	-0.04	0.24	0.88	-0.46	0.06	0.11	3.78	-52.
2014.2	-0.15	-0.05	0.21	0.84	-0.43	0.05	0.09	3.74	-70.
2014.3	-0.10	-0.05	0.18	0.80	-0.40	0.04	0.06	3.69	-67.
2014.4	-0.05	-0.04	0.14	0.76	-0.37	0.04	0.05	3.63	-50.
2015.1	-0.01	-0.02	0.10	0.74	-0.33	0.04	0.03	3.57	-26.
2015.2	0.02	0.00	0.07	0.72	-0.30	0.03	0.02	3.51	0.
2015.3	0.04	0.02	0.05	0.70	-0.27	0.03	0.01	3.44	26.
2015.4	0.05	0.04	0.03	0.69	-0.24	0.03	0.00	3.39	48.
2016.4	0.07	0.07	-0.01	0.65	-0.17	0.04	0.00	3.20	98.
2017.4	0.06	0.07	-0.02	0.60	-0.13	0.05	0.00	3.06	101.
2018.4	0.05	0.06	-0.02	0.55	-0.11	0.06	0.01	2.95	88.
2019.4	0.04	0.05	-0.02	0.49	-0.09	0.07	0.02	2.85	77.
2020.4	0.04	0.05	-0.02	0.44	-0.08	0.07	0.03	2.77	71.

[a]thousands of jobs.

- percent deviations for Y, J, and P, absolute deviations for U, r, int, def, and $debt$.
- sum of Y changes = $807 billion (2009 dollars), (0.40 percent).
- average of J changes = 532 thousand (0.40 percent), average U changes = -0.24.
- in 2020:4 federal debt larger by $788 billion ($616 billion in real terms (2009 dollars)).

Y = real GDP, J = employment (jobs), U = unemployment rate, P = GDP deflator,
r = three-month Treasury bill rate, int = federal interest payments/GDP ratio,
def = federal deficit/GDP ratio, $debt$ = federal government debt/GDP ratio.

Table 5
Estimated Stimulus Effects: Other Countries
Baseline Values Divided By Predicted No-Stimulus Values
Percentage Points

qtr	Y_{ca}	Y_{ja}	Y_{uk}	Y_{ge}	Y_{fr}	Y_{me}	Y_{ch}	Y_{id}
2009.1	0.00	0.00	0.00	0.00	0.00			
2009.2	0.08	0.01	0.01	0.01	0.00			
2009.3	0.23	0.03	0.03	0.03	0.01			
2009.4	0.46	0.05	0.06	0.06	0.03	0.29	0.09	0.02
2010.1	0.79	0.09	0.08	0.09	0.04			
2010.2	1.10	0.13	0.10	0.12	0.07			
2010.3	1.37	0.17	0.14	0.15	0.08			
2010.4	1.66	0.19	0.16	0.17	0.09	1.83	0.51	0.14
2011.1	1.95	0.23	0.20	0.22	0.11			
2011.2	1.99	0.22	0.18	0.23	0.11			
2011.3	2.01	0.24	0.18	0.23	0.11			
2011.4	2.06	0.24	0.18	0.23	0.11	2.97	0.76	0.24
2012.1	2.01	0.26	0.19	0.25	0.11			
2012.2	1.81	0.25	0.16	0.22	0.10			
2012.3	1.62	0.22	0.15	0.21	0.08			
2012.4	1.46	0.20	0.12	0.17	0.07	2.66	0.69	0.24
2013.1	1.23	0.19	0.11	0.15	0.06			
2013.2	1.02	0.18	0.10	0.13	0.05			
2013.3	0.79	0.16	0.09	0.10	0.04			
2013.4	0.56	0.15	0.08	0.06	0.02	1.84	0.45	0.13
2014.1	0.35	0.15	0.07	0.04	0.02			
2014.2	0.16	0.14	0.07	0.01	0.01			
2014.3	-0.02	0.14	0.06	-0.02	0.00			
2014.4	-0.18	0.13	0.06	-0.05	-0.01	1.09	0.24	0.05
2015.1	-0.31	0.13	0.06	-0.08	0.00			
2015.2	-0.43	0.13	0.06	-0.10	0.00			
2015.3	-0.53	0.13	0.06	-0.13	0.00			
2015.4	-0.61	0.13	0.06	-0.15	0.00	0.71	0.12	-0.01
2016.4	-0.80	0.14	0.06	-0.21	0.03	0.57	0.07	-0.04
2017.4	-0.80	0.13	0.05	-0.25	0.06	0.53	0.05	-0.06
2018.4	-0.66	0.13	0.05	-0.26	0.09	0.53	0.04	-0.07
2019.4	-0.46	0.12	0.05	-0.25	0.10	0.53	0.04	-0.06
2020.4	-0.23	0.11	0.05	-0.21	0.09	0.53	0.04	-0.04

Y = real GDP
ca = Canada, ja = Japan, uk = United Kingdom, ge = Germany, fr = France, me = Mexico, ch = China, id = India.
Values for Mexico, China, and India are yearly.

(durable stock, housing stock, and capital stock) in the model. There are also slight negative effects from the higher price level.

The peak output effect is in 2010:3, where output is 2.50 percent larger. The peak employment effect is in 2010:4, where employment is 1.98 percent larger (2.437 million jobs). In this quarter the unemployment rate is 1.16 percentage points lower. The GDP deflator effect reaches a peak in 2012:1, where the GDP deflator is 1.28 percent higher.

After the stimulus measures are over in 2012, the negative cyclical features begin to kick in. The peak negative output effect is in 2013:2, where output is 0.29 percent lower. It is interesting to see how much difference the stimulus bill made over the entire 12 year period. As noted at the bottom of Table 4, The sum of the real output changes over the 48 quarters is $807 billion (2009 dollars), which is 0.40 percent of the sum of total real output. The average number of jobs is larger by 532 thousand jobs, which is 0.40 percent of the average number of jobs. The unemployment rate is on average 0.24 percentage points lower.

The results for the three-month Treasury bill rate, the left hand side variable in the estimated interest rate rule, are interesting. There is roughly a zero lower bound constraint in effect until near the end of the stimulus period (in the base data set). The interest rate rule thus calls for very little increase in the interest rate in response to the stimulus measures. The interest rate stays at essentially zero. (Remember that the interest rate is set to zero if the rule calls for a negative rate.) In the base data set there is no longer a zero lower bound constraint beginning about in 2012, and the rule calls for a decrease in the interest rate (from its base values) in response to the cyclical contraction in the economy after the stimulus measures are over. The net effect over the entire period is for the interest rate to be on average lower than it would have been without the stimulus measures.

The federal government deficit and debt increased, as expected. The federal debt as a percent of GDP is always larger than in the base case. At the end of the period it is 2.77 percentage points larger. As noted at the bottom of Table 4, the nominal federal government debt is $788 billion larger in 2020:4. Dividing this figure by the value of the GDP deflator in 2020:4 gives a value of $616 billion in 2009 dollars. This compares to the sum of the real output gain of $807 billion. Comparing $807 billion to $616 billion, which may seem an obvious comparison to make, ignores discounting. The output gains occurs essentially in the first three years, and the debt increase slowly occurs over time.

Table 5 presents output results for other countries. Canada and Mexico have large effects. For China the peak output effect occurs after three years, at 0.76 percent. The cyclical features of the model are also evident in Table 5; they are driven by the cyclical effects on the United States.

The bootstrap procedure used for the results in Table 1 can be used to estimate standard errors for the stimulus experiment. This was done using 100 trials. Again, the estimated standard errors are small relative to the size of the effects. For the sum of the output changes of $807 billion, the estimated standard error is $81 billion; for the average unemployment rate change of -0.24, the estimated standard error is 0.021; and for the average of the employment changes of 532 thousand jobs, the estimated standard error is 58 thousand jobs.

Table 6 compares the present results to those of the CBO (2010) and Romer and Bernstein (2009). For the CBO the ranges are fairly large, and in almost every case the MC estimate is within the CBO range. The estimated uncertainty for the MC estimates is much smaller than is implicit in the CBO ranges. For example, the estimated 2.5 percent increase in output for 2010:3 for the MC model has an estimated standard error of 0.30 percent. This compares to the CBO low and high estimates of 1.3 percent and 4.0 percent, respectively.

Romer and Bernstein's results for 2010:4 are somewhat larger than the MC results. The increase in output is 3.7 percent versus 2.3 percent for the MC model, and the decrease in the unemployment rate is 1.8 percentage points versus 1.2 for the MC model. For employment (jobs) Romer and Berstein estimate an increase in 2010:4 of between 3.3 and 4.1 million, with a point estimate 2.995 million (not shown in Table 6). This compares to 2.437 million for the MC model in Table 4. Although not shown in Table 4, the estimated standard error of this estimate is 0.333 million.

Table 6
Comparison of Stimulus Estimates
Baseline Values Divided By or Subtracted From No-Stimulus Values
Percentage Points

Output

qtr	CBO low	CBO high	RB	MC	standard error
2009.1	0.1	0.1		0.0	0.00
2009.2	0.9	1.5		0.5	0.07
2009.3	1.3	2.7		1.1	0.15
2009.4	1.5	3.5		1.7	0.23
2010.1	1.5	3.9		2.1	0.26
2010.2	1.7	4.5		2.4	0.30
2010.3	1.3	4.0		2.5	0.29
2010.4	1.1	3.4	3.7	2.3	0.32

Unemployment Rate

qtr	CBO low	CBO high	RB	MC	standard error
2009.1	0.0	0.0		0.0	0.00
2009.2	-0.2	-0.3		-0.1	0.03
2009.3	-0.4	-0.7		-0.3	0.06
2009.4	-0.5	-1.1		-0.5	0.11
2010.1	-0.6	-1.5		-0.8	0.14
2010.2	-0.7	-1.8		-1.0	0.17
2010.3	-0.7	-1.9		-1.1	0.18
2010.4	-0.7	-1.9	-1.8	-1.2	0.17

Employment[a]

qtr	CBO low	CBO high	MC	standard error
2009.1	0.0	0.0	0.0	0.00
2009.2	0.3	0.5	0.1	0.03
2009.3	0.7	1.3	0.3	0.08
2009.4	1.0	2.1	0.6	0.12
2010.1	1.2	2.7	0.9	0.15
2010.2	1.4	3.3	1.2	0.19
2010.3	1.3	3.5	1.4	0.20
2010.4	1.2	3.4	1.4	0.22

CBO = CBO (2010), Table 3.
RB = Romer and Bernstein (2009).
[a]employment is the number of people employed, not the number of jobs.

5.5.4 Conclusion

This section provides estimates of the effects on the world economy from the 2009 U.S. stimulus bill. It has the advantage of taking into account many endogenous effects. The results show that the output and employment effects over 12 years are positive, with some redistribution away from 2012–2015 and with an increase in the government debt/GDP ratio. The increase in real output over the 12-year period, 2009–2020, is $807 billion (0.40 percent), and the increase in the average level of employment is 532 thousand jobs (0.40 percent). The estimated standard errors of the stimulus estimates are fairly low.

It is important to remember than the three stimulus experiments discussed in Section 4 do not assume any future tax increases or government spending cuts to pay for the stimulus spending. The MC model has the advantage of being able to estimate the increase in the government debt that would result if no future actions are taken. The increase in the federal debt by 2020:4 is $616 billion in real terms, an increase in the debt/GDP ratio of 2.77 percentage points. The debt rises because of the higher initial spending on goods and transfer payments.

5.5.5 Appendix: Computing Standard Errors

There are 1,689 estimated equations in the MC model, of which 1,379 are trade share equations. The estimation period for the United States is 1954:1–2013:3. The estimation periods for the other countries begin as early as 1962:1 and end as late as 2013:2. The estimation period for most of the trade share equations is 1966:1–2012:4. For each estimated equation there are estimated residuals over the estimation period. Let $\hat{u}_t$ denote the 1689-dimension vector of the estimated residuals for quarter t.[141] Most of the estimation periods have the 1972:1–2007:4 period—144 quarters– in common, and this period is taken to be the "base" period. These 144 observations on $\hat{u}_t$ are used for the draws in the stochastic-simulation

[141] For equations estimated using annual data, the error is put in the first quarter of the year with zeros in the other three quarters (which are never used). If the initial estimate of an equation suggests that the error term is serially correlated, the equation is reestimated under the assumption that the error term follows an autoregressive process (usually first order). The structural coefficients in the equation and the autoregressive coefficient or coefficients are jointly estimated (by 2SLS). The $\hat{u}_t$ error terms are after adjustment for any autoregressive properties, and they are taken to be iid for purposes of the draws. As discussed in the text, the draws are by year—four quarters at a time.

procedure discussed below.[142]

The solution period used to create new data is 1954:1–2020:4—268 quarters. For a given set of coefficient estimates and error terms, the model can be solved dynamically over this period. Equations enter the solution as data become available. For example, for the period 1954:1–1959:4 only the equations for the United States are used. The links from the other countries to the United States are shut off, and the U.S. variables that these links affect are taken to be exogenous. By 1972 almost all the equations are being used. Actual data for the United States end in 2012:3 and somewhat earlier for the other countries. Exogenous variable values from the end of the actual data through 2020:4 are the ones that were chosen for the baseline forecast made on November 11, 2013.

Each trial of the bootstrap procedure is as follows. First, 268 error vectors are drawn with replacement from the 144 vectors in the base period. (Each vector consists of 1,689 errors.) Using these errors and the coefficient estimates base on the actual data, the model is solved dynamically over the 1954:1–2020:4 period. Using the solution values as the new data set, the 1,689 equations are reestimated. Given these new coefficient estimates and the new data, the stimulus experiment is performed for the 2009:1–2020:4 period—as in Tables 4 and 5.[143] The multipliers are recorded. This is one trial. The procedure is then repeated, say, N times. (Note

[142]If an estimation period does not include all of the 1972:1–2007:4 period, zero errors are used for the missing quarters.

[143]Given the new data and new coefficient estimates, residuals can be computed for the 2009:1–2020:4 period—1,689 residuals for each quarter. If these residuals are added to the model and the model is solved for the 2009:1–2020:4 period, the solution values reproduce the values in the new data set. This is taken to be the baseline run. These residuals and the no-stimulus values of COG and $TRGHQ$ are then used for the no-stimulus solution. These no-stimulus solution values can then be compared to the values in the new data set to estimate the stimulus effects.

Another procedure for the stimulus experiment is the following. Compute the new data set and new coefficient estimates as above. Then for trial i draw from the historical error distribution (the 144 observations on $\hat{u}_t$) errors for 2009:1–2020:4. Given these errors, the new data set, and the new coefficient estimates, solve the model twice, once using the stimulus values of G and TR and once using the no-stimulus values. For each variable and quarter record the difference between the two solution values. Do M trials, which gives M values of each difference. Compute the mean of the M values for each difference, and take this as the expected value of the stimulus effect. This procedure is a bootstrap within a bootstrap. For a linear model this procedure is not necessary because the errors cancel out and so each trial gives exactly the same difference for each variable and quarter. For a nonlinear model (which the MC model is) this is not the case, but a common property of models like the MC model—see Subsection 3.9.2—is that predicted values from deterministic simulations are close to mean values from stochastic simulations. This means in the present context that mean values from the second bootstrap procedure would be close to the values computed using the one set of residuals. This second bootstrap procedure was not used here.

that the coefficient estimates used to generate the new data on each trial are the estimates based on the actual data.) This gives N values of each multiplier, from which measures of dispersion can be computed.

The measure of dispersion used in the text is as follows. Rank the N values of a given multiplier by size. Let m_r denote the value below which r percent of the values lie. The measure of dispersion is $(m_{.8413} - m_{.1587})/2$. For a normal distribution this is one standard error.

The experiment done after each new data set and new set of coefficient estimates can be any experiment. For the results in this section three experiments were done using 100 trials each. Two are the ones in Tables 1 and 2, and one is the stimulus experiment. The same random numbers were generated for each experiment, which avoids noise in comparing across experiments. There were no solution failures for any experiment.

5.6 What It Takes To Solve the U.S. Government Deficit Problem[144]

5.6.1 Introduction

This section uses the MC model to estimate what it takes to stabilize the long-run U.S. federal government debt/GDP ratio. The fiscal policy tool is federal transfer payments, $TRGHQ$. This question is complicated in part because of endogeneity issues. A fiscal-policy change designed to decrease the deficit has effects on the macro economy, which in turn affect the deficit. Any analysis of fiscal-policy proposals must take these effects into account: one needs a model of the economy.

The period considered is 2014–2022. The experiments are performed off of a base run. The base run is one in which there are no major changes in U.S. fiscal policy from 2014 on. Aggregate tax rates are taken to be unchanged from their values in 2013:3. Federal government purchases of goods and services and federal transfer payments to households and to state and local governments are assumed to grow at recent historical rates net of the effects of the various stimulus measures.

As will be seen, the base run has an ever increasing debt/GDP ratio. This is, of course, consistent with almost all recent analyzes. Without major fiscal-policy changes, the U.S. government debt/GDP ratio is expected to rise without limit. See, for example, Penner (2011) and CBO (2011). The experiments consist of decreasing transfer payments from the base run beginning in 2014:1. The size of the increase is chosen to stabilize the debt/GDP ratio by 2022.

The results show that decreasing transfer payments by 1 percent of GDP from the base run stabilizes the debt/GDP ratio. The decrease in transfer payments over the nine years is \$2.1 trillion in current dollars and \$1.7 trillion in 2009 dollars. The sum of the real output loss (2009 dollars) over the nine years is \$1.2 trillion, which is 0.7 percent of sum of real output over the nine years from the base run. The average number of jobs per quarter is 929,000 lower, and the average number of people unemployed per quarter is 435,000 higher.

Monetary policy is endogenous in the model; it is determined by the estimated interest rate rule, equation 30. Monetary policy mitigates the fall in output from the fiscal contraction, but it is not powerful enough to eliminate all of the output loss. (See Section 4.4 for results pertaining to the effectiveness of monetary policy.)

[144] This section is an update of the results in Fair (2012c).

5.6.2 Transfer Payments versus Taxes

The use of transfer payments as the government spending variable covers many tax policies as well. Many tax changes are changes in what are sometimes called "tax expenditures"—changing loopholes, deductions, etc.—rather than changes in tax rates. Changes like these are essentially changes in transfer payments. Also, federal grants-in-aid to state and local governments can be considered transfer payments to the extent that state and local governments in turn transfer the money to households. The experiments in this section thus encompass a fairly wide range of policy variables.

It may be the case, however, that changing a tax expenditure like the deductibility of mortgage interest changes behavior enough to have macro implications in addition to implications for the distribution of spending across sectors. Any macro implications would not be captured in the MC model since all tax-expenditure changes are channeled through changes in disposable personal income, variable YD. They are probably small for most tax-expenditure changes, but this is hard to test.

What about tax-rate increases instead of transfer payment decreases or tax expenditure decreases? In the model personal income tax rates ($D1G$ and $D4G$) affect labor supply, and so increasing tax rates does lead to different results than decreasing transfer payments by an equivalent amount. Both affect YD, but there are also labor supply responses. The differences are not, however, large because the labor supply responses are modest. Similar conclusions to those reached in this section would be obtained using tax rates.

5.6.3 Transfer Payment Multipliers

To get an idea of the properties of the MC model regarding changing transfer payments, Table 1 presents transfer payment multipliers for the period 2014:1–2022:4. For the results in the table the level of real transfer payments, $TRGHQ$, was permanently increased by 1.0 percent of real GDP from its baseline values. This is an experiment in which nothing is paid for: no changes to any exogenous variable were made except for transfer payments. The table shows that the peak multiplier for output is 0.68 after 7 quarters. The multiplier settles down to about

Table 1
Transfer Payment Multipliers using the MC Model
Deviations from Baseline in Percentage Points

qtr	GDPR	UR	GDPD	RS	AGZGDP
2014.1	0.06	-0.01	0.01	0.02	0.24
2014.2	0.22	-0.05	0.01	0.06	0.40
2014.3	0.39	-0.12	0.03	0.12	0.53
2014.4	0.53	-0.19	0.07	0.19	0.66
2015.1	0.62	-0.25	0.10	0.24	0.81
2015.2	0.67	-0.29	0.14	0.29	0.99
2015.3	0.68	-0.32	0.18	0.32	1.19
2015.4	0.67	-0.32	0.21	0.33	1.41
2016.1	0.64	-0.32	0.24	0.34	1.64
2016.2	0.61	-0.30	0.27	0.33	1.88
2016.3	0.57	-0.28	0.29	0.32	2.14
2016.4	0.54	-0.26	0.30	0.31	2.40
2017.1	0.51	-0.24	0.31	0.30	2.66
2017.2	0.49	-0.22	0.32	0.28	2.92
2017.3	0.47	-0.20	0.32	0.27	3.18
2017.4	0.45	-0.19	0.32	0.26	3.44
2018.4	0.43	-0.17	0.31	0.23	4.49
2019.4	0.44	-0.17	0.28	0.22	5.51
2020.4	0.47	-0.19	0.22	0.21	6.51
2021.4	0.50	-0.21	0.12	0.20	7.51
2022.4	0.52	-0.23	-0.01	0.19	8.49

$GDPR$ = real GDP
UR = unemployment rate
$GDPD$ = GDP deflator
RS = three-month Treasury bill rate
$AGZGDP$ = nominal federal debt/nominal GDP
- percent deviations for $GDPR$ and $GDPD$, absolute deviations for UR, RS, and $AGZGDP$.
- Experiment is a sustained increase in real transfer payments of 1.0 percent of real GDP

0.5. Physical stock effects and interest rate effects are the main reasons for the decline in the multipliers after the peak. By 2022:4 the debt/GDP ratio has risen by 8.49 percentage points.

5.6.4 The Base Run

The results in this section are based on actual U.S. data through 2013:3 (data available as of November 11, 2013). The base run consists of predicted values for the period 2013:4–2022:4 that I made on November 11, 2013, using the MC model. These values are on my website.[145]

There are two features of the base run's forecast of the macro economy that differ from what was consensus as of the time of the forecast: the economy is more expansive and inflation is higher. Fortunately, results like those in this section are generally not sensitive to changes in a base run. The results of interest are those comparing the predicted values from an alternative run to the predicted values from a base run, and it is generally the case that the differences in these predicted values are not sensitive to the levels in the base run. To the extent that the base run is "off" in levels, so will be the alternative run. Some experimentation (not reported here) suggested that the estimates of the decreases in transfer payments needed to stabilize the debt/GDP ratio are not sensitive to changes in the base run.

The following assumptions were made for the base-run forecast. First, the exogenous federal spending variables that affect federal purchases of goods and services, which are federal purchases of goods, civilian jobs, and military jobs, were chosen to grow at recent past rates abstracting from the effects of the stimulus measures. Second, exogenous federal tax rates were taken to remain unchanged from their 2013:3 values. Third, federal transfer payments to households and to state and local governments were chosen to grow at recent past rates abstracting from the effects of the stimulus measures. Finally, the exogenous state and local government tax and spending variables were chosen to result in a roughly balanced state and local government budget.

The remaining exogenous variables for the United States are either fairly easy to forecast, like population, or are small and not important. Values of each of these variables were chosen to be consistent with recent behavior. The main exogenous variable for each of the other countries is government spending. Remember that exports, export prices, and import prices are all endogenous in the MC model.

[145]For countries other than the United States data were not available as late as 2013:3, and the overall forecast began earlier than 2013:3, with actual values used for the United States until 2013:4.

No assumptions are needed for these. Also, no assumptions are needed about monetary polices because the estimated interest rate rules of the various monetary authorities are used.

Regarding asset prices, one set of asset-price variables consists of the export prices of the oil-exporting countries (roughly the price of oil). These prices have been taken to be exogenous and to grow at historically average rates. An asset-price variable in the US model is the price of housing relative to the GDP deflator. This ratio is taken to be exogenous and to grow at an annual rate of 1.0 percent throughout the forecast period. Exchange rates and the change in U.S. stock prices, which are asset-price variables, are endogenous in the model—there are 23 estimated exchange rate equations and an equation explaining capital gains or losses on the financial assets of the household sector (Flow of Funds data), variable CG. However, very little of the variances of CG and the change in exchange rates are explained by these equations (as would be expected), and the effects on these variables in the model are modest. The base run is thus based on the assumption of no bad asset market reactions even though in this run the debt/GDP ratio continually increases. Since asset-price changes are essentially unpredictable, it would be arbitrary to add large asset-price shocks to the base run. The base run is thus not necessarily realistic in this sense. It is a baseline from which the effects of decreases in transfer payments can be estimated.

Results for the base run are presented in Table 2. This forecast has the unemployment rate falling to 6.5 percent by 2022. The inflation rate is about 2.5 percent over the period. The Fed is predicted to increase the short-term interest rate (three-month Treasury bill rate) to 2.0 percent by 2022.[146] The economy is thus predicted to grow fairly well. It is perhaps not surprising that the model is predicting this given that fiscal policy is expansive and there are no bad asset-price shocks. Interest payments as a percent of GDP rise from 2.37 percent in 2013:3 to 3.61 percent by 2022:4. The deficit as a percent of

[146]Note that by 2015:1 the short-term interest rate is up to 0.5 percent, so there is no longer a zero lower bound. The base run is thus not one in which there is a binding zero lower bound.

Table 2
Base Run: Forecast 2013:4–2022:4
Values are in Percentage Points

qtr	g	u	π	r	int	def	debt
			Actual values				
2006.4	3.16	4.47	1.41	4.90	2.47	1.33	43.1
2007.4	1.47	4.81	1.78	3.39	2.58	2.11	44.4
2008.4	-8.33	6.89	0.60	0.30	2.18	4.70	48.2
2009.4	3.88	9.95	1.17	0.06	2.33	8.79	52.9
2010.4	2.81	9.56	2.06	0.14	2.39	8.57	59.1
2011.4	4.86	8.68	0.52	0.01	2.44	7.60	61.4
2012.4	0.14	7.82	1.34	0.09	2.51	6.57	66.2
2013.1	1.15	7.74	1.76	0.09	2.19	5.16	65.7
2013.2	2.48	7.54	0.58	0.05	2.49	3.96	64.0
2013.3	2.85	7.30	1.90	0.04	2.37	4.71	64.4
			Forecast values				
2013.4	2.40	7.32	2.06	0.16	2.45	4.84	65.1
2014.1	3.41	7.35	2.23	0.22	2.56	4.86	65.4
2014.2	3.61	7.37	2.28	0.23	2.65	4.85	65.7
2014.3	4.19	7.34	2.25	0.27	2.73	4.82	66.0
2014.4	4.46	7.27	2.36	0.39	2.81	4.76	66.1
2015.1	4.47	7.16	2.45	0.54	2.87	4.69	66.2
2015.2	4.33	7.04	2.53	0.71	2.93	4.62	66.3
2015.3	4.12	6.92	2.60	0.86	2.98	4.56	66.4
2015.4	3.88	6.81	2.66	1.01	3.03	4.51	66.5
2016.4	3.16	6.57	2.76	1.45	3.17	4.39	67.0
2017.4	2.90	6.57	2.67	1.66	3.29	4.34	67.8
2018.4	2.86	6.63	2.54	1.75	3.38	4.30	68.6
2019.4	2.91	6.65	2.45	1.80	3.46	4.22	69.3
2020.4	2.96	6.64	2.40	1.85	3.52	4.10	69.9
2021.4	3.01	6.59	2.37	1.92	3.57	3.96	70.3
2022.4	3.08	6.51	2.37	2.02	3.61	3.80	70.5

- g = real GDP, percentage change, annual rate.
- u = unemployment rate (UR).
- π = GDP deflator, percentage change, annual rate.
- r = three-month Treasury bill rate (RS).
- int = federal government interest payments as a percent of GDP ($INTGZGDP$).
- def = federal government deficit (NIPA) as a percent of GDP ($SGPZGDP$).
- debt = federal government debt as a percent of GDP ($AGZGDP$).

GDP falls from 4.71 percent in 2013:3 to 3.80 percent in 2022:4. The debt/GDP ratio, which was 43.1 percent in 2006:4 and 64.4 percent in 2013:3, rises to 70.5 percent by 2022:4. Herein lies the problem.[147]

5.6.5 The Alternative Run: Decreasing Transfer Payments

It turned out that decreasing transfer payments by 1 percent of GDP was enough to stabilize the debt/GDP ratio.[148] The decreases were linearly phased in over three years beginning in 2014:1. No other changes were made for the alternative run.

Before discussing the results, one feature of the model should be stressed, which is that expectations are assumed to be adaptive. If in the present context the government announces that it is going to stabilize the debt/GDP ratio, this has no immediate effect on behavior. There is, for example, no increase in consumer and investor confidence that could increase spending. Spending behavior changes after the decreases in transfer payments take place. Likewise, there are no changes in stock prices and interest rates until the economy begins to respond to the fiscal-policy change. If some of these omitted responses are large, it may be that the debt/GDP ratio could be stabilized with a smaller decrease in transfer payments than 1 percent of GDP. The 1 percent figure is thus an upper bound.

The results from this run are presented in Table 3. This table has two variables not in Table 2: the change in transfer payments from the base run in real terms (2009 dollars) and in nominal terms (current dollars). Comparing Table 3 to Table 2, the decrease in transfer payments is contractionary, as expected. The notes to Table 3 give the sums or averages of the deviations from the base run to the alternative run over the nine years. The sum of the real output loss is \$1.2 trillion, which is 0.7 percent of the sum of real GDP from the base run. The number of unemployed is on average 435,000 larger per quarter. The number of jobs is on average 929,000 smaller per quarter, which is 0.7 percent of the average number of jobs per quarter.

[147]The federal government debt is measured in practice in a variety of ways. The measure used here is the one used in the MC model, which is based on data from the U.S. Flow of Funds Accounts (variable $-AG$ in the model). It is the debt in the hands of the public. To get a sense of how large 78.9 percent is, this number should be compared to earlier values, like 43.1 percent in 2006:4.

[148]For this run the level of real transfer payments was decreased by 1 percent of an estimate of potential real GDP in the model, which is exogenous. Potential real GDP is taken to be $YS + PSI13(JG \cdot HG + JM \cdot HM + JS \cdot HS) + STATP$, which is equation 83 in Table A.3 with YS replacing Y.

Table 3
Alternative Run
Transfer Payments Decreased by One Percent of GDP

qtr	g	u	π	r	int	def	debt	ΔTP^a	ΔTP^b
2014.1	2.84	7.38	2.31	0.20	2.56	4.80	65.5	-15.1	-16.0
2014.2	3.19	7.44	2.19	0.17	2.65	4.73	65.9	-30.5	-33.1
2014.3	3.87	7.46	2.16	0.17	2.73	4.62	66.1	-46.0	-50.7
2014.4	4.19	7.42	2.26	0.24	2.80	4.47	66.2	-61.8	-68.8
2015.1	4.20	7.35	2.32	0.36	2.85	4.32	66.3	-77.8	-87.5
2015.2	4.08	7.26	2.38	0.48	2.90	4.17	66.3	-94.1	-106.8
2015.3	3.88	7.17	2.44	0.60	2.94	4.02	66.3	-110.5	-126.7
2015.4	3.65	7.08	2.49	0.71	2.98	3.88	66.3	-127.2	-147.1
2016.4	2.95	6.94	2.53	1.01	3.05	3.39	66.3	-196.3	-235.2
2017.4	2.96	6.96	2.46	1.14	3.08	3.28	66.3	-202.0	-253.4
2018.4	2.96	6.96	2.39	1.24	3.09	3.14	66.1	-207.8	-271.7
2019.4	2.96	6.94	2.31	1.31	3.10	2.99	65.8	-213.9	-290.3
2020.4	2.98	6.90	2.23	1.36	3.09	2.81	65.5	-220.2	-310.4
2021.4	3.03	6.85	2.16	1.40	3.08	2.61	64.9	-226.7	-333.2
2022.4	3.10	6.77	2.10	1.45	3.06	2.38	64.2	-233.4	-359.2

See notes to Table 2.
- ΔTP^a = change in transfer payments from the base run, 2009 dollars, annual rate.
- ΔTP^b = change in transfer payments from the base run, current dollars, annual rate.
- Sum of real output loss is $1.2 trillion, 0.7 percent of the sum of output from the base run.
- Average number of jobs per quarter is 929,000 lower, 0.7 percent of the average number of jobs per quarter from the base run.
- Average number of people unemployed is 435,000 more.
- Sum of transfer payment decrease is $1.7 trillion in 2009 dollars and $2.1 trillion in current dollars.

The good news is that the debt/GDP ratio is roughly stable. It is 65.5 percent in 2014:1 and 64.2 percent in 2022:4. The deficit as a percent of GDP is down to 2.38 percent by 2022:4. Interest payments as a percent of GDP stabilize at about 3 percent.

The interest rate is lower in the alternative run than in the base run, which is the Fed responding to the higher unemployment and the lower inflation. Although the lower interest rates mitigate the contraction from the transfer payment decrease, they by no means eliminate the contraction. As noted in the Introduction, the effects of interest rate changes in the model are not large enough to eliminate the negative effects on output of a transfer payment decrease of the size considered here.

What is the size of the transfer payment decrease? Table 3 shows that the decrease in transfer payments in 2009 dollars rises to about \$230 billion by 2022 (at an annual rate). In nominal terms the number is about \$360 billion. The sum of the decrease over the ten years is \$1.7 trillion in 2009 dollars and \$2.1 trillion in current dollars.

Figures 1 and 2 give some perspective on the present results. Figure 1 plots the ratio of federal purchases of good and services to GDP for the 1952:1–2022:4 period. Values beyond 2013:3 are predicted values. Values for 2014:1 on are presented for both the base run and the alternative run.[149] The values for the prediction period are low by historical standards. As discussed in Section 3, no major changes in government purchases of goods and services were made for the prediction, and so the ratio is roughly flat.

Figure 2 is the more interesting figure. It plots the ratio of net taxes to GDP, where net taxes is defined to be federal personal income taxes plus federal social security taxes minus federal transfer payments to persons minus federal transfer payments to state and local governments. The value of net taxes was negative in the 2009–2012 period. Revenue of the federal government is also obtained from corporate taxes, indirect business taxes, and a few other items, but this revenue is relatively small. It was roughly the case in the 2009–2012 period that all federal government spending on goods and services was financed by borrowing in that the value of net taxes was negative. The base run has the ratio of net taxes to GDP rising, but to a level that is still low historically. The alternative run, of course, has it rising much more. At the end, the ratio is still low by historical standards, but so is the ratio of purchases of goods and services in Figure 1. Compared to historical averages, less needs to be raised in net taxes if spending on goods and services is low.

Figure 2 is important for getting a sense of how much net taxes has to be raised to stabilize the debt/GDP ratio. The ratio in Figure 2 is conditional on the historically low ratio in Figure 1. Although this is harder to measure, it is probably also conditional, given the aging of the U.S. population, on the future elderly receiving fewer benefits than the past elderly did. The base run assumes that federal transfer

[149] Although COG is exogenous, the values from the two runs differ slightly because of the endogeneity of GDP.

Figure 1
Federal Purchases of Goods and Services as a
Percent of GDP: 1952:1--2022:4

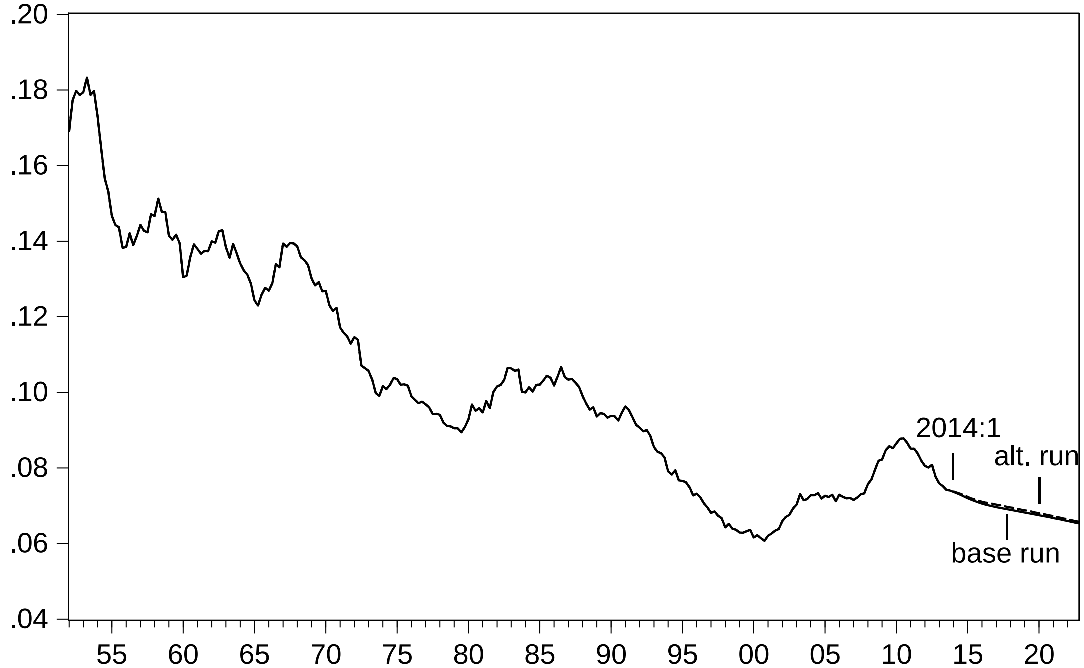
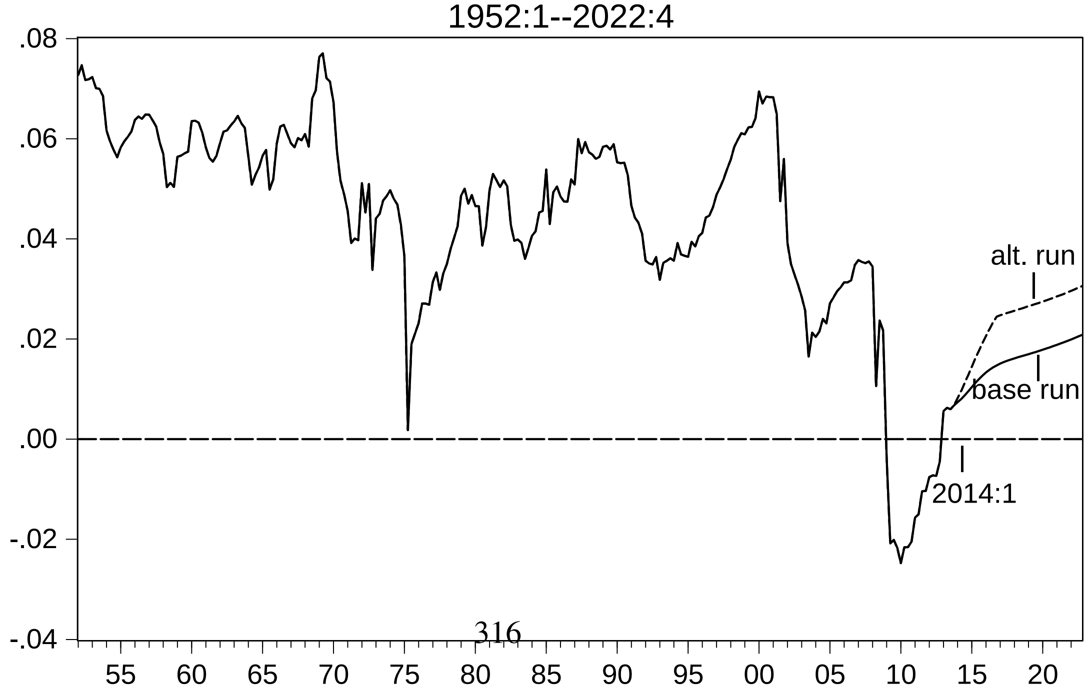


Figure 2
Federal Net Taxes as a Percent of GDP
1952:1--2022:4



payments grow at a constant rate based on recent past growth rates abstracting from the stimulus measures. Implicit in this treatment is the assumption that the future elderly are receiving less than the past elderly because there are more of them. If one wanted to give the future elderly the same, then to keep the same net tax ratio in Figure 2 either other transfer payments would have to be cut or taxes would have to be increased. To repeat, given the spending assumptions in Figure 1, the net tax ratio in Figure 2 must be maintained in some way.

5.6.6 Conclusion

The above results provide estimates of the size of the decrease in transfer payments (or tax expenditures) needed to stabilize the U.S. government debt/GDP ratio. They take into account endogenous effects of changes in fiscal policy on the economy and in turn the effect of changes in the economy on the deficit. The size needed is discussed at the end of Section 5.6.5, particularly around the discussion of Figure 2. Transfer payments need to be decreased by 1 percent of GDP from the base run. The output loss is about 0.7 percent of baseline GDP. Monetary policy helps keep the loss down, but it is not powerful enough in the model to eliminate all of the loss.

Possible caveats to the present results are the following. First, monetary policy might be more powerful than is estimated in the model, which would lessen the output loss. Second, if the process of putting policies in place to stabilize the debt/GDP ratio permanently increases asset prices or animal spirits (like consumer and investor confidence), this would, other things being equal, have a positive effect on output, and this effect is not in the model.

What happens if, say, the government delays doing anything about the debt? An experiment was run in which the decrease in transfer payments began a year later, in 2015:1. In this case decreasing transfer payments by 1 percent of GDP again stabilized the debt/GDP ratio, but at a higher value. The main effect of delaying is to stabilize at a higher ratio with the same percent decrease in transfer payments.

What happens if the government never tackles the debt problem and the debt/GDP ratio never stabilizes? This is where the MC model has little to say. There is nothing in the model that breaks down with rising debt/GDP ratios. What is likely to happen, of course, is that at some point there will be asset-market reactions to the rising ratio, which a model could never predict. The probability of a bad asset-market reaction likely rises as the debt/GDP ratio rises, but the timing cannot be predicted.

5.7 The Financial Crisis and Macroeconomic Activity: 2008-2013[150]

5.7.1 Introduction

Although there is by now a large literature on the financial crisis and the 2008–2009 recession, there are no estimates as far as I am aware of the size of the effects of the crisis on overall macroeconomic activity, on, say, the unemployment rate in 2008–2009. This section provides estimates of the effects of the fall in financial and housing wealth in 2008–2009 on macroeconomic activity. It will be seen that these effects are large and account for a large fraction of the slowdown in activity. The results suggest that much of the 2008–2009 recession was simply due to standard wealth effects on household expenditures.

The extensive literature cited in Brunnermeier and Sannikiov (2014) is theoretical.[151] The various financial frictions that are postulated in this literature are too abstract to be taken directly to macro data. Gilchrist and Zakrajšek (2012) use univariate forecasting equations and VARs to test for the effects of interest rate spreads on various macroeconomic variables. They argue that an increase in their estimate of the excess bond premium reflects shifts in the risk aversion of the financial sector, which leads to a decline in asset prices and a contraction of the supply of credit, which has a negative effect on economic activity. They do not, however, provide estimates of the size of the effects during the 2008–2009 recession. Their excess bond premium variable is examined below. Duygan-Bump, Levkov, and Montoriol-Garriga (2011) test the hypothesis that credit constraints were important in the 2008-2009 recession by examining the financing constraints of small businesses. They also do not provide estimates of the size of the effects during the recession.

The work of Reinhart and Rogoff (2009, 2014) documents the role of financial crises in recessions, arguing, for example, that the subprime crisis in the 2008-2009 recession is not an anomaly in the context of data prior to World War II. This work is descriptive, and no quantitative estimates of the effects of financial crises on economic activity are presented.

Case, Quigley, and Shiller (2012), which is an update of results in Case, Quigley, and Shiller (2005), use data by states to examine housing and financial wealth effects on household spending, where household spending is retail sales.

[150]This section is taken from Fair (2014b).

[151]Important early papers in this literature include Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Bernanke, Gertler, and Gilchrist (1999).

The sample period is 1975:1–2012:2. They find that the effects of housing wealth on spending are larger than the effects of financial wealth on spending. They do not use national income and product accounts (NIPA) data, and there are no estimates of overall effects on the 2008-2009 recession. Some of their estimates are examined below. Zhou and Carroll (2012) also examine wealth effects using state data. They find a strong housing wealth effect, but no financial wealth effect.

Mian, Rao, and Sufi (2013) examine the effects of household wealth on consumption in the 2006-2009 period using consumption and wealth data by zip codes. The data on consumption are constructed using data on auto sales and data from MasterCard Advisors. They also do not use NIPA data, and so obtaining aggregate estimates is limited. Some of their estimates are examined below. Mian and Sufi (2014) examine the effects of changes in housing wealth on employment in the 2007–2009 period using data by counties. Some of their estimates are also examined below.

Carroll, Slacaled, and Sommer (2013) estimate aggregate personal saving equations for the 1966:2–2011:1 period. The find significant coefficient estimates for wealth, for a variable measuring credit constraints (CEA), and for a variable measuring labor income uncertainty ($UnRisk$). CEA is constructed using the question on consumer installment loans from the Federal Reserve's Senior Load Officer Opinion Survey on Bank Lending Practices. CEA is "taken to measure the availability/supply of credit to a typical household through factors other than the level of interest rates." (p. 12) $UnRisk$ is measured "using re-scaled answers to the question about the expected change in unemployment in the Thomson Reuters/University of Michigan Surveys of Consumers." (p. 13) These two variables are examined below.

This section uses the MC model for the estimates. Financial wealth effects versus housing wealth effects on household expenditures in the MC model are examined in Subsection 5.7.2 and then again in Subsection 5.7.4. Subsection 5.7.3 tests various measures of credit conditions, and Subsection 5.7.5 examines large shocks during the 2008:1–2013:3 period. Finally, Subsection 5.7.6 estimates what the 2008–2013 economy would have been like had there been no decrease in financial wealth and housing wealth.

Table 1 presents the variable notation used in this section. The discussion in this section pertains to the U.S. part of the MC model.

Table 1
Variables in the MC Model Referred to in this Section

Variable	Type	Description
AA	endo	Total net wealth, h, B2009$.
$AA1$	endo	Total net financial wealth, h, B2009$.
$AA2$	endo	Total net housing wealth, h, B2009$.
$AG1$	exog	Percent of 16+ population 26-55 minus percent 16-25.
$AG2$	exog	Percent of 16+ population 56-65 minus percent 16-25.
$AG3$	exog	Percent of 16+ population 66+ minus percent 16-25.
AH	endo	Net financial assets, h, B$.
CD	endo	Consumer expenditures for durable goods, B2009$.
CDA	exog	Peak to peak interpolation of CD/POP.
CG	endo	Capital gains(+) or losses(-) on the financial assets of h, B$.
CN	endo	Consumer expenditures for nondurable goods, B2009$.
$cnst$	exog	Constant term.
$cnst2$	exog	0.0 before 1969:1, 0.0125 in 1969:1, 0.0250 in 1969:2, ... , 0.9875 in 1988:3, and 1.0 thereafter.
CS	endo	Consumer expenditures for services, B2009$.
$DELD$	exog	Physical depreciation rate of the stock of durable goods, rate per quarter.
$DELH$	exog	Physical depreciation rate of the stock of housing, rate per quarter.
$GDPD$	endo	GDP price deflator.
$GDPR$	endo	Gross Domestic Product, B2009$.
IHH	endo	Residential investment, h, B2009$.
$IHHA$	exog	Peak to peak interpolation of IHH/POP.
IKF	endo	Nonresidential fixed investment, f, B2009$.
IM	endo	Imports, B2009$.
IVF	endo	Inventory investment, f, B2009$.
JF	endo	Number of jobs, f, millions.
KD	endo	Stock of durable goods, B2009$
KH	endo	Stock of housing, h, B2009$.
MH	endo	Demand deposits and currency, h, B$.
PD	endo	Price deflator for domestic sales.
PH	endo	Price deflator for CS + CN + CD + IHH inclusive of indirect business taxes.
PIV	endo	Price deflator for inventory investment, adjusted.
PKH	endo	Market price of KH.
POP	exog	Noninstitutional population 16+, millions.

Table 1 (continued)

Variable	Type	Description
$PSI14$	exog	Ratio of PKH to PD.
PX	endo	Price deflator for total sales.
RMA	endo	After tax mortgage rate, percentage points.
RS	endo	Three-month Treasury bill rate, percentage points.
RSA	endo	After tax bill rate, percentage points.
UR	endo	Civilian unemployment rate.
YD	endo	Disposable income, h, B$.
YS	endo	Potential output, B2009$.

- h = household sector.
- f = firm sector.
- B$ = Billions of dollars.
- B2009$ = Billions of 2009 dollars.

5.7.2 Financial Wealth versus Housing Wealth in Consumer Expenditure Equations

The aggregate U.S. wealth variable in the MC model is:

$$AA = \frac{AH + MH}{PH} + \frac{PKH \cdot KH}{PH} = AA1 + AA2 \tag{1}$$

where AH is the nominal value of net financial assets of the household sector excluding demand deposits and currency, MH is the nominal value of demand deposits and currency held by the household sector, KH is the real stock of housing, PKH is the market price of KH, and PH is a price deflator relevant to household spending. $(AH + MH)/PH$, denoted $AA1$, is thus real financial wealth, and $(PKH \cdot KH)/PH$, denoted $AA2$, is real housing wealth.

Figures 1 and 2 plot $AA1$ and $AA2$, respectively, for the 1952:1–2013:3 period. Figure 3 plots the ratio of $AA1$ to $AA2$. The ratio fluctuates considerably over time, with a range of 1.3 to 2.4. The peak of $AA2$ is in 2006:1 at $23.9 trillion. The peak of $AA1$ is the last quarter at $44.4 trillion. These values are all in 2009 dollars.

Table 2 presents the MC U.S. estimated equations for consumption of services, CS, and consumption of non durables, CN. The equations are in log per capita terms, and the wealth variable enters as $\log(AA/POP)_{-1}$. Two estimation periods

Figure 1
Financial Wealth, AA1, 1952:1--2013:3
Trillions of 2009 Dollars

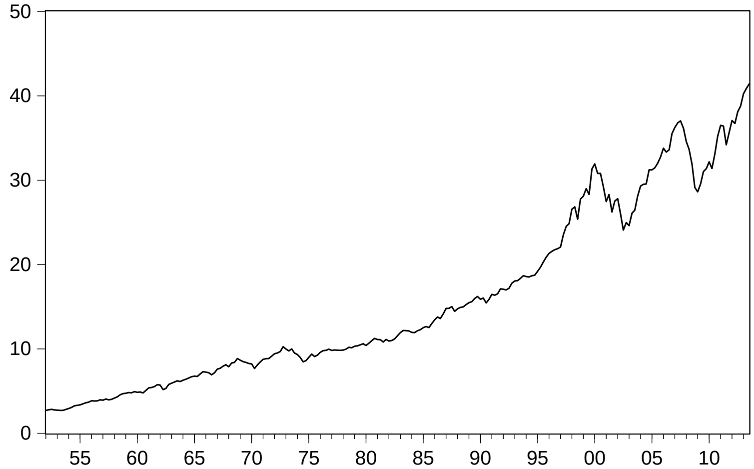


Figure 2
Housing Wealth, AA2, 1952:1--2013:3
Trillions of 2009 Dollars

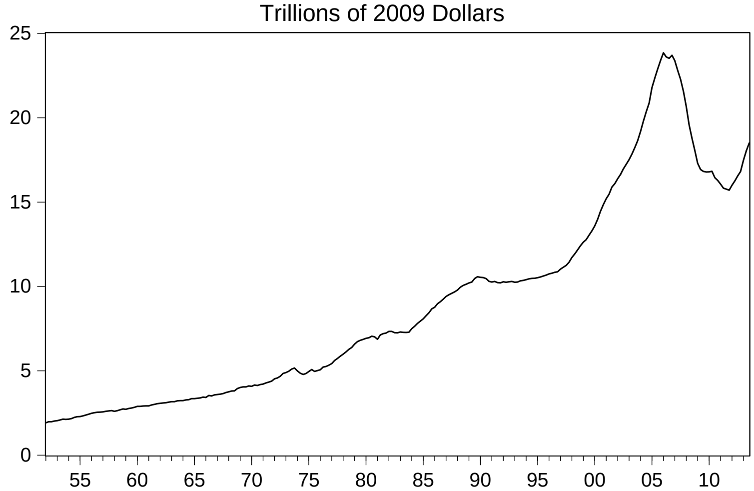


Figure 3
Financial Wealth/Housing Wealth
1952:1--2013:3

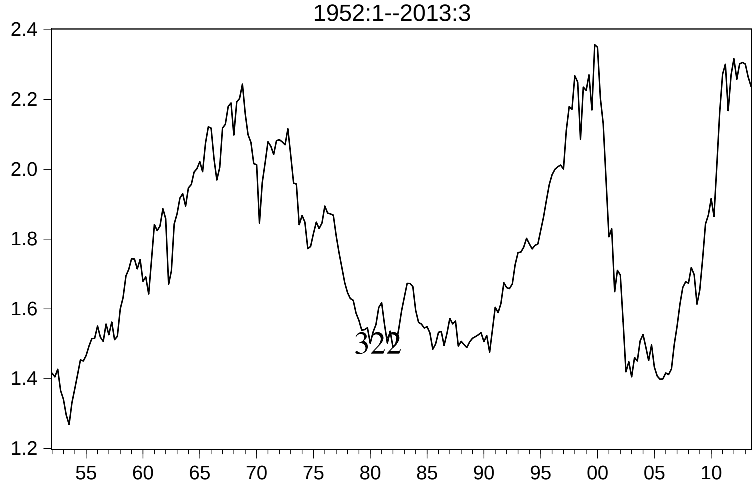


Table 2
Estimates for Consumption of Services (CS) and
Consumption of Non Durables (CN)
Left Hand Side Variables are log(CS/POP)
and log(CN/POP)

	CS		**CN**	
RHS Variable	**1954:1-2013:3**	**1954:1-2007:4**	**1954:1-2013:3**	**1954:1-2007:4**
cnst2	0.022	0.022	-0.015	-0.015
	(6.60)	(5.94)	(-1.87)	(-1.83)
cnst	-0.134	-0.125	-0.341	-0.256
	(-5.86)	(-3.03)	(-5.04)	(-2.95)
$AG1$	-0.052	-0.077	0.124	0.031
	(-2.10)	(-1.51)	(2.45)	(0.35)
$AG2$	-0.288	-0.269	0.124	0.226
	(-8.43)	(-4.90)	(2.09)	(2.18)
$AG3$	0.247	0.287	-0.310	-0.202
	(4.02)	(3.18)	(-2.61)	(-1.34)
$\log(CS/POP)_{-1}$	0.818	0.810		
	(35.22)	(31.74)		
$\log(CN/POP)_{-1}$			0.740	0.754
			(16.95)	(16.81)
$\Delta\log(CN/POP)_{-1}$			0.214	0.193
			(3.59)	(3.06)
$\log[YD/(POP \cdot PH)]$	0.120	0.129	0.119	0.124
	(5.11)	(4.82)	(3.84)	(3.48)
RSA	-0.00115	-0.00110		
	(-5.08)	(-4.80)		
RMA			-0.00092	-0.00095
			(-1.78)	(-1.82)
$\log(AA/POP)_{-1}$	0.0379	0.0377	0.0480	0.0366
	(6.66)	(4.62)	(4.42)	(2.53)
SE	0.00373	0.00378	0.00658	0.00664
R^2	0.999	0.999	0.999	0.999
DW	1.51	1.58	1.95	1.96
End Test (2007:4)—p-value	0.809	–	0.794	–

•t-statistics are in parentheses.
•Estimation method is 2SLS.
•Variables are listed in Table 1.

are used, 1954:1–2013:3 and 1954:1–2007:4, the latter ending before the crisis. The estimates for the first period are the same as those in Tables A1 and A2 in Appendix A.

The lagged wealth variable, $\log(AA/POP)_{-1}$, is significant in both equations for both periods. The interest rate is significant in the CS equation, but has t-statistics of only -1.78 -1.82 in the CN equation. The End test—Andrews (2003)—is a test of the hypothesis that the coefficients are the same both before and after 2007:4. The p-values are large in both cases, and so the hypothesis is not rejected in either case. This result is consistent with the fact that the coefficient estimates for the two periods are fairly similar.

Remember that AA is equal to $AA1 + AA2$, financial wealth plus housing wealth. The wealth variable enters the equations as $\log(AA/POP)_{-1}$, which assumes that financial and housing wealth have the same effect. This can be tested by using as the wealth variable $\log(\lambda AA1+(1-\lambda)AA2)_{-1}$ and estimating λ along with the other structural coefficients. The equations are estimated by 2SLS, and so estimating λ is a non linear 2SLS estimation problem, which is straightforward to solve. If the effects are the same, then λ is 0.5.

For the CS and CN equations the estimates of λ for the two periods are the following. The t-statistics in parentheses are for the hypothesis that $\lambda = 0.5$:

		1954:1-2013:3	**1954:1-2007:4**
CS	$\hat{\lambda}$	0.771	0.883
		(2.46)	(1.44)
CN	$\hat{\lambda}$	0.508	0.679
		(0.08)	(0.58)

Although there is slight evidence that financial wealth has a greater weight in the CS equation (but not in the CN equation), with the hypothesis that $\lambda = 0.5$ rejected for the first period (t-statistic of 2.46), the evidence is only slight, and for the rest of the results in this section the combined AA wealth variable is used in both equations.

Table 3 presents the MC U.S. equation for expenditures on durable goods, CD. The equation is in linear per capita terms. The wealth variable is $(AA/POP)_{-1}$. Again, the estimates for the first period are the same as those in Table A3 in Appendix A. The lagged wealth variable is significant for the first estimation period, but only has a t-statistic of 1.33 for the second. The interest rate is significant for

Table 3
Estimates for Durable Expenditures (CD)
Left Hand Side Variable is CD/POP

RHS Variable	**1954:1-2013:3**	**1954:1-2007:4**
cnst2	0.062	0.039
	(3.97)	(2.72)
cnst	-0.248	-0.169
	(-3.46)	(-2.09)
$AG1$	0.18	0.02
	(1.60)	(0.12)
$AG2$	2.64	2.92
	(6.27)	(5.37)
$AG3$	-2.35	-2.17
	(-5.50)	(-5.27)
^{a}	0.232	0.271
	(5.10)	(5.45)
$(KD/POP)_{-1}$	-0.0277	-0.0245
	(-6.79)	(-5.89)
$YD/(POP \cdot PH)$	0.0639	0.0704
	(6.21)	(5.76)
$RMA \cdot CDA$	-0.0101	-0.0068
	(-3.96)	-2.83)
$(AA/POP)_{-1}$	0.00063	0.00023
	(3.85)	(1.33)
SE	0.01455	0.01250
R^2	0.207	0.224
DW	1.95	2.20
End Test (2007:4)—p-value	0.005	–

^{a}Variable is $DELD(KD/POP)_{-1} - (CD/POP)_{-1}$
•t-statistics are in parentheses.
•Estimation method is 2SLS.
•Variables are listed in Table 1.

both periods. The End test has a p-value of 0.005, so the hypothesis that the coefficients are the same before and after 2007:4 is rejected. It will be seen in Subsection 5.7.4 why this might be so.

Financial versus housing wealth can be tested for durable expenditures by replacing the wealth variable with $(AA1/POP)_{-1}$ and $(AA2/POP)_{-1}$. The results for the two periods are:

	1954:1-2013:3	**1954:1-2007:4**
$(AA1/POP)_{-1}$	0.00043	0.00024
	(2.32)	(1.42)
$(AA2/POP)_{-1}$	0.00106	-0.00022
	(4.06)	(-0.94)
t-statistic for equal coefficients	2.17	0.94

There is slight evidence for the first period that housing has a greater weight, where the hypothesis of equality rejected with a t-statistic of 2.17. For the second period nothing is significant. Because the evidence in favor of housing wealth is slight, as was done for the CS and CN equations, for the rest of the results in this section the combined AA wealth variable is used in the CD equation.

Table 4 presents the MC U.S. equation for housing investment of the household sector, IHH. The equation is similar in form to the CD equation. The wealth variable is $(AA2/POP)_{-1}$, housing wealth, not total wealth. The lagged housing wealth variable is significant for both periods. The hypothesis that the coefficients are the same before and after 2007:4 is not rejected. The coefficient estimates are fairly similar across the two estimation periods.

Regarding housing wealth versus financial wealth in the IHH equation, when $(AA1/POP)_{-1}$ is added to the equation, its t-statistics are 0.67 and 0.04 for the two periods, respectively, and $(AA2/POP)_{-1}$ retains its significance. The t-statistic for the hypothesis that the two coefficients are equal is 2.90 for the first period and 2.71 for the second, so the hypothesis is rejected. The housing wealth variable has thus been used alone in the IHH equation.

The significance of financial wealth in the consumption equations is contrary to results using less aggregate data. As noted in the Introduction, Case, Quigley, and Shiller (2012) (CQS) find stronger effects for housing wealth than for financial wealth on retail sales. In fact, for many of their estimates financial wealth is not

Table 4
Estimates for Housing Investment (IHH)
Left Hand Side Variable is IHH/POP

RHS Variable	**1954:1-2013:3**	**1954:1-2007:4**
cnst2	0.137	0.049
	(1.75)	(0.87)
cnst	0.889	0.804
	(4.16)	(5.31)
^{a}	0.379	0.419
	(7.34)	(7.25)
$(KH/POP)_{-1}$	-0.0377	-0.0506
	(-4.51)	(-5.11)
$YD/(POP \cdot PH)$	0.0790	0.1718
	(2.40)	(4.39)
$RMA_{-1} \cdot IHHA$	-0.0259	-0.0242
	(-5.38)	(-4.83)
$(AA2/POP)_{-1}$	0.00368	0.00376
	(3.77)	(2.88)
RHO1	0.626	0.601
	(8.75)	(7.82)
RHO2	0.326	0.287
	(4.68)	(3.88)
SE	0.01542	0.01610
R^2	0.446	0.395
DW	1.99	1.94
End Test (2007:4)—p-value	0.242	–

^{a}Variable is $DELH(KH/POP)_{-1} - (IHH/POP)_{-1}$
•t-statistics are in parentheses.
•Estimation method is 2SLS.
•RHO1 and RHO2 are first and second order serial correlation coefficient estimates.
•Variables are listed in Table 1.

significant. In the present case financial wealth is significant in the CS, CN, and CD equations with the exception of the shorter estimation period for the CD equation. As discussed above, there is some evidence that financial wealth is more important in the CS equation and that housing wealth is more important in the CD equation, but the evidence is not very strong. Housing wealth does dominate in the IHH equation, but CQS do not examine housing investment. Many assumptions have been used by CQS to create financial wealth data by state, and their negative results for financial wealth could be at least partly due to measurement error. Mian, Rao, and Sufi (2013) also do not find significant financial wealth effects on consumption, but they point out (p. 20) that they do not have the statistical power to estimate financial wealth effects because of lack of good data on financial assets by zip codes. Zhou and Carroll (2012), using data by states like CQS, also find insignificant financial wealth effects but significant housing wealth effects.

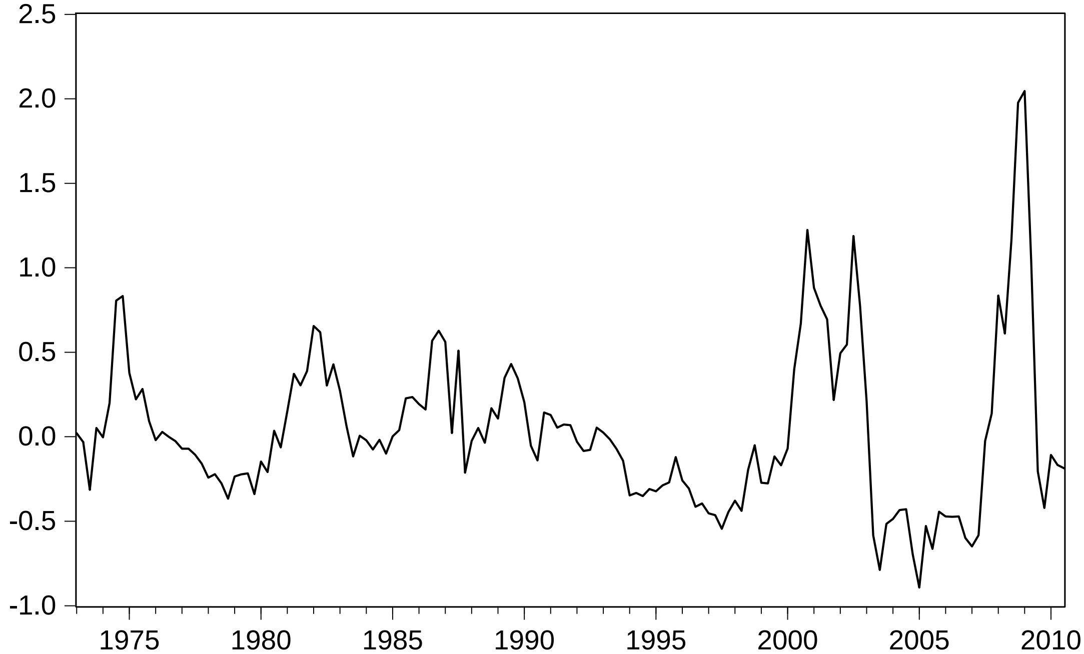
If constructing financial wealth by zip codes or states leads to larger measurement errors than constructing housing wealth by zip codes or states, then this could explain the insignificance of financial wealth versus housing wealth. The present results using aggregate data are quite strong regarding the overall significance of financial wealth. It would be hard, for example, to explain the boom in the U.S. economy in the last half of the 1990s without considering the huge increase in financial wealth in this period from the boom in the stock market.

5.7.3 Testing Measures of Credit Conditions

The consumer expenditure equations in Tables 2–4 do not have explanatory variables measuring credit conditions other than interest rates and wealth. It is of interest to see if other measures might add to the explanatory power. Possible candidates are various interest rate spreads. Stock and Watson (2003) review of the use of interest rate spreads to forecast various macroeconomic variables. Interest rate spreads may incorporate credit conditions not captured in the interest rate and wealth variables. This is straightforward to test by simply adding spread variables to the equations and seeing if they are significant. As noted in the Introduction, Gilchrist and Zakrajšek (2012) create their own interest rate spread variable, an excess bond premium, denoted EBP, and test its predictive power. Figure 4 plots EBP for the period for which data exist, 1973:1–2010:3. The large values during the 2008-2009 recession are evident, and there are also large values in the 2000-2002 period.

When testing the interest rate spread variables, two estimation periods were used, one ending in 2007:4 and one ending in 2013:3 (or 2010:3 for EBP). Since

Figure 4
EBP 1973:1--2010:3



EBP was chosen after the 2008–2009 recession was known, there may be data mining issues for estimation periods including the recession. This is not an issue for the wealth variable used in this section, since it first appeared in the model 30 years ago—Fair (1984). Table 5 presents results for two spread variables, the BAA/AAA bond spread and EBP. For each equation estimates are presented for the interest rate, the wealth variable, and the spread. The BAA/AAA bond spread is not close to being significant in any of the equations. Although not shown in the table, the same was true for the spread between the AAA bond rate and the 10-year government bond rate.

Regarding EBP, it is not significant for the period ending before the recession except for the CN equation, where the t-statistic is -2.05. For the period through the recession it is not significant in the CS equation, but it is in the three others. Adding EBP does not affect the significance of any of the interest rate and wealth variables. They are all significant expect for the wealth variable in the CD equation for the periods ending before the recession. The evidence for EBP is thus mixed, depending on how much weight one puts on possible data mining, since it was created after the recession was known. But it could be that EBP is capturing

Table 5
Testing Interest Rate Spreads

		Interest Rate	Wealth$_{-1}$	Spread$_{-1}$	EBP$_{-1}$
CS	1954:1–2007:4	-0.00110	0.0377	-0.00028	
		(-4.79)	(4.62)	(-0.30)	
	1954:1–2013:3	-0.00116	0.0369	-0.00089	
		(-5.14)	(6.38)	(-1.17)	
	1973:4–2007:4	-0.00151	0.0509		0.00031
		(-5.33)	(4.79)		(0.27)
	1973:4–2010:3	-0.00156	0.0525		-0.00107
		(-5.55)	(5.49)		(-1.41)
CN	1954:1–2007:4	-0.00100	0.0365	0.00026	
		(-1.67)	(2.52)	(0.13)	
	1954:1–2013:3	-0.00079	0.0475	-0.00076	
		(-1.38)	(4.35)	(-0.50)	
	1973:4–2007:4	-0.00121	0.0404		-0.00371
		(-2.31)	(2.56)		(-2.05)
	1973:4–2010:3	-0.00119	0.0445		-0.00494
		(-2.29)	(3.13)		(-3.96)
CD	1954:1–2007:4	-0.00834	0.00031	0.00983	
		(-3.13)	(1.72)	(1.30)	
	1954:1–2013:3	-0.01012	0.00063	0.00024	
		(-3.72)	(3.64)	(0.06)	
	1973:4–2007:4	-0.01013	0.00022		-0.00729
		(-3.17)	(0.99)		(-1.30)
	1973:4–2010:3	-0.01357	0.00058		-0.01682
		(-3.84)	(2.70)		(-4.59)
IHH	1954:1–2007:4	-0.0243	0.00368	0.00216	
		(-4.79)	(2.76)	(0.16)	
	1954:1–2013:3	-0.0269	0.00361	-0.00373	
		(-5.66)	(3.71)	(-0.40)	
	1973:4–2007:4	-0.0259	0.00281		-0.01105
		(-4.96)	(2.02)		(-1.19)
	1973:4–2010:3	-0.0270	0.00395		-0.01531
		(-5.65)	(3.35)		(-2.13)

•Spread is BAA-AAA.
•Estimation method is 2SLS.
•See Tables 2–4 for the interest rate and wealth variables per equation.
•Spread is multiplied by CDA for CD equation. Similarly for EBP.
•Spread is multiplied by $IHHA$ for IHH equation. Similarly for EBP.

some effects on consumer expenditures not captured by the interest rate and wealth variables. This is examined in Section 5.7.7.

Another possible measure of credit conditions is the CEA variable of Carroll, Slacalek, and Sommer (2013), which was mentioned in the Introduction. It was tried (lagged one quarter) in the four consumer expenditure equations for two estimation periods: 1966:2–2007:4 and 1966:2–2011:1. In none of the eight regressions was it significant, and so there is no evidence that it has independent explanatory power. The labor income uncertainty variable, $UnRisk$, was also tried (lagged one quarter), and it was only significant in the CN equation, with t-statistics of -2.45 and -2.28 for the two periods, respectively. There is thus little support for this variable.[152] Whatever information CEA and $UnRisk$ convey, it appears to be captured by variables already in the expenditure equations.

5.7.4 Estimated Effects of Changes in Financial and Housing Wealth

Before considering the 2008–2009 recession, it will be useful to examine the size of the wealth effect in the MC model. How much do household expenditures change when $AA1$ or $AA2$ changes? The size of this wealth effect depends on what is held constant. If the complete MC model is used, then an increase in $AA1$ or $AA2$ increases U.S. household expenditures, which then leads to a multiplier effect on output and at least some increase in inflation. Given the estimated interest rate rule in the model, the Fed responds to the expansion by raising interest rates, which slows down the expansion, and so on. The rest of the world also responds to what the United States is doing, which then feeds back on the United States. The size of the wealth effect with nothing held constant thus depends on many features of the MC model, not just the properties of the U.S. household expenditure equations.

One can focus solely on the properties of the household expenditure equations by taking income and interest rates to be exogenous. The following experiment was performed. The variables $YD/(POP \cdot PH)$, RSA, RMA, $AA1$, and $AA2$ were taken to be exogenous, which isolates the four household expenditure equations from the rest of the model. The estimated residuals were then added to the stochastic equations and taken to be exogenous. This means that when the model is solved using the actual values of all the exogenous variables, a perfect tracking solution is obtained. The actual values are thus the base values. For the first experiment $AA1$, financial wealth, was increased by \$1000 billion in each quarter from the base case, and the model was solved for the 2005:1–2012:4 period. The difference

[152]For the CD equation $UnRisk$ was multiplied by CDA and for the IHH equation it was multiplied by $IHHA$. This was not done for CEA because it has a trend.

for a given quarter between the predicted value of a variable and the actual value is the estimated effect of the $AA1$ change on that variable for that quarter.

The effects on total consumption expenditures ($CS + CN + CD + IHH$) by quarters are presented in Table 6. After four quarters expenditures have risen \$22.0 billion, and after eight quarters they have risen \$33.3 billion. The increases then level off at about \$40 billion. The effect of a sustained increase in wealth on consumption expenditures is thus estimated to be about 4 percent per year ignoring any feedback effects.

The increase in $AA1$ does not affect housing investment, IHH, because it does not appear in the housing investment equation. So in Table 6 IHH is unchanged. If $AA2$ instead of $AA1$ is changed, this changes all four categories of expenditures because $AA2$ appears in all four equations. Results of increasing $AA2$ by \$1000 billion are presented in Table 7. In this case the expenditures peak at about \$60 billion rather than \$40 billion, although the effects wear off faster.[153]

The roughly 4 percent estimate in Table 6 is consistent with results from other approaches. The size of the wealth effect is discussed in Ludvigson and Steindel (1999), where they conclude (p. 30) that "a dollar increase in wealth likely leads to a three-to-four-cent increase in consumption in today's economy," although they argue that there is considerable uncertainty regarding this estimate. Their approach is simpler and less structural than the present one, but the size of their estimate is similar. Starr-McCluer (1998) uses survey data to examine the wealth effect, and she concludes that her results are broadly consistent with a modest wealth effect.

Mian, Rao, and Sufi (2013) (MRS) find 5 to 7 percent effects of housing wealth on consumption (p. 30), although these effects vary considerably across zip codes. These numbers should be compared to the numbers in Table 6 because MRS do not examine housing investment, and so their estimated effects are somewhat higher than the present ones. Zhou and Carroll (2012) find 5 percent effects of housing wealth on consumption (p. 18), slightly higher than the estimates in Table 6.

CQS test for asymmetrical effects and find that the housing wealth elasticity is estimated to be larger in falling markets than in rising markets.[154] Their estimated elasticities are 0.10 and 0.032, respectively. How do these compare with the present results? Take Table 6. Excluding housing investment, $CS + CN + CD$ at the

[153]The main reason the effects wear off faster is that when housing investment is stimulated, the housing stock increases, which over time is a drag on new housing investment. (In Table 4 the coefficient estimate for KH/POP_{-1} is negative.)

[154]No attempt was made in the present study to estimate asymmetrical effects. It is unlikely using aggregate data that any such effects could be estimated even if they exist.

Table 6
Effects on $CS + CN + CD + IHH$ of a Change in $AA1$ of 1000

	Year							
Quarter	2005	2006	2007	2008	2009	2010	2011	2012
1	0.0	26.2	34.4	36.3	39.0	41.4	41.8	41.6
2	8.9	29.2	35.1	36.9	39.9	41.6	41.5	41.4
3	16.3	31.5	35.6	37.3	40.7	41.9	41.4	41.3
4	22.0	33.3	35.9	38.0	41.1	42.1	41.5	40.9

• Units are billions of 2009 dollars

Table 7
Effects on $CS + CN + CD + IHH$ of a Change in $AA2$ of 1000

	Year							
Quarter	2005	2006	2007	2008	2009	2010	2011	2012
1	0.0	56.1	59.9	53.7	50.4	49.0	47.2	45.7
2	23.7	59.2	58.5	52.6	50.2	48.6	46.5	45.4
3	39.7	60.5	56.9	51.5	50.0	48.3	46.0	45.1
4	50.0	60.7	55.2	50.8	49.5	47.9	45.9	44.5

• Units are billions of 2009 dollars

beginning of 2005 was about \$9.4 trillion. Housing wealth, $AA2$, was about \$21.8 trillion. If one takes the change in consumption expenditures to be \$40 billion, then the housing wealth elasticity is (40/9400)/(1000/21800) = 0.09. So this elasticity is close to the CQS elasticity in falling markets of 0.10.

5.7.5 Estimated Shocks: 2008:1–2013:3

If the 2SLS coefficient estimates of the equations in Tables 2–4 are consistent, then consistent estimates of the residuals (actual minus predicted) are available. If credit-condition effects during the 2008–2009 recession have not been captured well by the wealth and interest rate variables in the equations, then one would expect the residuals on average to be negative and large in absolute value during the recession. Table 8 presents residuals that are larger than one standard deviation in absolute value for the 2008:1–2013:3 period, where the main emphasis is on the recessionary period 2008:1–2009:4.

Table 8
Large Absolute-Value Residuals
100(Actual - Predicted)/Standard Error

	Equation						
	1	2	3	4	12	11	27
2008:1		-1.2	-2.4	-1.8		-1.6	
2008:2	-1.2		-1.0				
2008:3	-1.0	-2.1	-1.6	1.9			
2008:4		-2.1	-5.3		-3.2	-1.9	1.1
2009:1	-1.2					-1.4	3.5
2009:2		-1.3	-1.1		2.5		2.0
2009:3			3.1	2.5		-1.6	
2009:4			-1.3				-1.3
2010:1							-1.1
2010:2			1.1				-1.8
2010:3	1.1			-2.6	1.3	1.2	-1.6
2010:4	1.1	1.3	1.1				
2011:1					-1.1	-1.3	
2011:2			-1.7				
2011:3			-1.0		2.7		
2011:4			1.2			1.0	
2012:1				-1.3			
2012:2			-1.4	-1.3			
2012:3							
2012:4	-1.2			-1.2	1.2		
2013:1					-2.1		
2013:2							
2013:3	-1.9		1.9				

Estimation period is 1954:1–2013:3
Equation 1: Service consumption (CS)
Equation 2: Nondurable consumption (CN)
Equation 3: Durable consumption (CD)
Equation 4: Housing investment (IHH)
Equation 12: Nonresidential fixed investment (IKF)
Equation 11: Inventory investment (IVF)
Equation 27: −Imports (IM)

The residuals are not huge except for the CD residual for 2008:4, which is negative and 5.3 times its standard error. CD fell at an annual rate of 25.8 percent in this quarter, much of which was not explained. The error undoubtedly contributes to the rejection of the End test in Table 3. Of the 17 large residuals for the consumer expenditure equations (equations 1–4) for 2008:1–2009:4, 14 are negative. Consumer durable expenditures are hit the hardest. Two of the three large residuals for housing investment are actually positive. For the other three demand equations, there are 10 large residuals for the 2008:1–2009:4 period, of which 6 are negative.

Although 14 of the 17 large residuals for the consumer expenditure equations are negative, only 4 of the 14 are greater than two standard errors. There is thus clearly some of the recession not captured by the equations, but much of it has been.[155] Quantitative estimates of how much has been captured are presented in the next subsection.

5.7.6 What if Financial and Housing Wealth had not Fallen in 2008–2009?

Real financial wealth, $AA1$, and real housing wealth, $AA2$, are plotted in Figures 1 and 2. From 2007:4 to 2009:4 $AA1$ fell by \$4.79 trillion. From 2010 on it recovered well, with a small dip in the middle of 2011. From 2007:4 to 2009:4 $AA2$ fell by \$4.77 trillion, but unlike $AA1$, it had not recovered well by 2013:3. Say these two variables from 2008:1 on had instead behaved normally according to historical experience? What would the macroeconomy have looked like? An answer to this question using the MC model is as follows. The period examined is 2008:1–2013:3.

First, the variable AH, which is in the definition of $AA1$, is the nominal value of net financial assets of the household sector. It is determined by identity 66 in Table A.3 in Appenix A:

$$AH = AH_{-1} + SH - \Delta MH + CG - DISH$$

[155] How does the inclusion of EBP in the four consumer expenditure equations affect the residuals? Consider the 1973:4–2010:3 estimation period. For the four equations without EBP_{-1} added, there are 13 large residuals, of which 10 are negative, for the 2008:1–2009:4 period. (In Table 8 there are 17 large residuals, with 14 negative. The difference is due to the different estimation period. The estimation period when EBP_{-1} is added is considerably shorter.) When EBP_{-1} is added, there are 10 large residuals, of which 6 are negative. The (negative) residual for the CD equation for 2008:4 is 4.9 times its standard error without EBP_{-1} added and 3.8 times when it is added. Quantitative estimates of how much EBP contributes to explaining the recession are presented in Subsection 5.7.7.

where SH is the financial saving of the household sector, MH is its holdings of demand deposits and currency, CG is the value of capital gains (+) or losses (-) on the financial assets held by the household sector (almost all of which is the change in the market value corporate stocks held by the household sector), and $DISH$ is a discrepancy term. CG is constructed from data from the U.S. Flow of Funds accounts. It is highly correlated with the change in the S&P 500 stock price index. Stock prices thus affect AH through CG. There is an equation explaining CG in the model, although, not surprisingly, very little of the variance of CG is explained. The left hand side variable of this equation is $CG/(PX_{-1}YS_{-1})$, where YS is a measure of potential output and PX is a price index. For the experiment here the equation for CG was dropped and $CG/(PX_{-1}YS_{-1})$ was taken in each quarter to be its average over the 1954:1–2007:4 period, which is 0.12623.

Second, the relationship between PKH, the market price of housing, and the deflator for domestic sales in the model, PD, is

$$PKH = PSI14 \cdot PD$$

where PSI14 is taken to be exogenous. This is identity 55 in Table A.3 in Appendix A.[156] An increase in $PSI14$ means that housing prices are rising relative to overall prices. For the experiment $PSI14$ was taken in each quarter to be its value in 2007:4, which is 2.0.

Third, the estimated shocks that occurred during the 2008:1–2013:3 period—the estimated residuals—were assumed to be the same in the new regime. In the estimation these shocks are assumed to be iid.[157]

Fourth, Fed behavior, as reflected in the values of the three-month Treasury bill rate, RS, was assumed to be the same in the new regime. In the model there is an estimated interest rate rule explaining Fed behavior, and this equation has been dropped from the model for the experiment. The rule is a leaning against the wind rule, and so if it were retained, the Fed would be predicted to increase RS from its base values in the more robust economy. For simplicity it seemed best not to compound the effects of wealth changes and interest rate changes, and so RS is taken to be exogenous.

For the experiment the estimated residuals were added to the model for the 2008:1–2013:3 period and taken to be exogenous. This means that when the

[156] As discussed in Appendix A, PKH is constructed from nominal housing stock data from the U.S. Flow of Funds accounts and real housing stock data from the Bureau of Economic Analysis

[157] Remember that serial correlation has been removed from the shocks by the estimation of serial correlation coefficients.

model is solved with no changes in the exogenous variables, there is a perfect tracking solution. Then the two wealth changes were made and the model was solved—the entire MC model, not just the U.S. part. For each endogenous variable and each quarter, the difference between its solution value and its actual value is the estimated effect of the wealth changes on the variable. Because the entire MC model is solved, all the endogenous variables are affected, but the following discussion focuses only on U.S. variables.

Using stochastic simulation and reestimation, standard errors of the estimated effects can be estimated, and this was done. The exact procedure for doing this is discussed in the appendix. Some of the estimated standard errors are reported below. In an experiment like this the main uncertainty comes from changes in the coefficient estimates as new sets of residuals are drawn. The additive error terms wash out because a new set of residuals is the same for both the base simulation and the simulation with the wealth changes.

To summarize, the experiment consists of having U.S. stock prices grow at historical rates, of having housing prices grow at the same rate as overall prices, of using the same shocks, and of having no change in the historical values of the short term interest rate (which are mostly zero). The experiment corresponds to large increases in financial and housing wealth because in reality both U.S. stock prices and housing prices fell dramatically.

Results are presented in Tables 9–11 and Figures 5–14. Table 9 shows the effects on $AA1$ and $AA2$. After 8 quarters financial wealth is \$7.36 trillion higher and housing wealth is \$5.12 trillion higher. These are, of course, huge differences. By the end of the period, 2013:3, financial wealth is about back down to its actual value, but housing wealth is still \$4.33 trillion higher. The estimated standard errors (SE) on the differences are small, but not zero. They are not zero because $AA1$ and $AA2$ depend on more than just $CG/(PX_{-1}YS_{-1}$ and $PSI14$, respectively, which are constant. See equation (1).

Table 10 is the key table. It shows the effects of the changes on the unemployment rate, UR, and private sector jobs, JF. The peak differences are in 2010:3, where the predicted unemployment rate is 6.10 versus 9.50 actual and the predicted number of jobs is 128.79 million versus 122.63 million actual. The standard errors are small relative to the size of the differences. This is a common result—see Subsection 3.9.2. When the uncertainty is only from the coefficient estimates, as here, it tends to be small.

Table 9
Actual and Predicted Values of
Financial Wealth (AA1) and
Housing Wealth (AA2)
(trillions of 2009 Dollars)

	AA1				AA2			
Qtr.	**Act.**	**Pred.**	**Dif.**	**SE**	**Act.**	**Pred.**	**Dif.**	**SE**
2008.1	34.58	36.43	1.84	0.01	20.66	21.65	0.99	0.00
2008.2	33.66	36.67	3.01	0.02	19.58	21.70	2.11	0.00
2008.3	31.88	36.83	4.96	0.04	18.77	21.77	3.00	0.01
2008.4	29.14	37.81	8.68	0.07	18.05	21.85	3.79	0.01
2009.1	28.62	38.55	9.93	0.10	17.30	21.88	4.58	0.02
2009.2	29.55	38.71	9.16	0.13	16.93	21.84	4.91	0.03
2009.3	31.01	38.77	7.76	0.15	16.82	21.87	5.05	0.04
2009.4	31.37	38.73	7.36	0.18	16.78	21.90	5.12	0.05
2010.1	32.17	38.90	6.73	0.20	16.79	21.99	5.20	0.07
2010.2	31.38	39.19	7.81	0.24	16.83	22.08	5.26	0.08
2010.3	33.13	39.35	6.22	0.25	16.44	22.14	5.70	0.09
2010.4	35.23	39.47	4.25	0.28	16.28	22.25	5.97	0.10
2011.1	36.50	39.54	3.04	0.32	16.06	22.30	6.24	0.11
2011.2	36.42	39.58	3.17	0.33	15.83	22.36	6.53	0.11
2011.3	34.19	39.87	5.68	0.36	15.77	22.46	6.69	0.12
2011.4	35.67	40.12	4.45	0.36	15.70	22.51	6.81	0.14
2012.1	37.07	40.15	3.08	0.40	16.00	22.55	6.56	0.14
2012.2	36.73	40.57	3.84	0.39	16.26	22.63	6.36	0.15
2012.3	38.11	40.81	2.70	0.40	16.55	22.70	6.15	0.16
2012.4	38.76	41.13	2.37	0.40	16.81	22.73	5.92	0.17
2013.1	40.25	41.55	1.30	0.41	17.49	22.75	5.26	0.19
2013.2	40.85	42.05	1.20	0.42	18.03	22.80	4.77	0.19
2013.3	41.41	42.29	0.89	0.41	18.50	22.84	4.33	0.19

Table 10
Actual and Predicted Values of
the Unemployment Rate (UR) and
Private Sector Jobs (JF)
(percentage points and millions of jobs)

	UR				JF			
Qtr.	**Act.**	**Pred.**	**Dif.**	**SE**	**Act.**	**Pred.**	**Dif.**	**SE**
2008.1	5.00	5.00	0.00	0.00	131.88	131.88	0.00	0.00
2008.2	5.34	5.27	-0.08	0.02	131.41	131.49	0.08	0.02
2008.3	6.03	5.76	-0.27	0.05	130.52	130.86	0.34	0.06
2008.4	6.89	6.28	-0.61	0.11	128.45	129.27	0.83	0.12
2009.1	8.32	7.19	-1.13	0.19	126.12	127.72	1.59	0.22
2009.2	9.30	7.53	-1.78	0.24	124.17	126.78	2.61	0.37
2009.3	9.63	7.21	-2.42	0.31	123.16	126.90	3.74	0.52
2009.4	9.95	7.04	-2.91	0.33	122.56	127.33	4.78	0.65
2010.1	9.84	6.56	-3.28	0.37	122.42	127.97	5.56	0.75
2010.2	9.67	6.29	-3.38	0.41	122.66	128.67	6.01	0.76
2010.3	9.50	6.10	-3.41	0.39	122.63	128.79	6.16	0.74
2010.4	9.56	6.37	-3.19	0.40	123.08	129.23	6.15	0.78
2011.1	9.04	6.03	-3.01	0.38	123.82	129.83	6.01	0.74
2011.2	9.07	6.38	-2.69	0.38	124.27	129.98	5.72	0.73
2011.3	9.03	6.63	-2.39	0.34	124.78	130.09	5.31	0.69
2011.4	8.68	6.46	-2.22	0.34	125.44	130.34	4.90	0.70
2012.1	8.24	6.12	-2.12	0.33	126.15	130.72	4.57	0.73
2012.2	8.16	6.24	-1.92	0.33	126.79	131.07	4.28	0.67
2012.3	8.03	6.27	-1.76	0.31	127.29	131.29	3.99	0.65
2012.4	7.82	6.14	-1.68	0.29	127.81	131.48	3.67	0.67
2013.1	7.74	6.30	-1.45	0.30	128.17	131.48	3.30	0.67
2013.2	7.54	6.28	-1.26	0.29	128.59	131.46	2.87	0.66
2013.3	7.30	6.30	-1.00	0.28	129.17	131.53	2.36	0.67

Table 11
Actual and Predicted Values of
real GDP (GDPR) and
the GDP Deflator (GDPD)
(billions of 2009 dollars and 2009 = 1.0)

	GDPR				GDPD			
Qtr.	**Act.**	**Pred.**	**Dif.**	**SE**	**Act.**	**Pred.**	**Dif.**	**SE**
2008.1	14895.3	14895.4	0.1	0.1	0.985	0.985	0.000	0.000
2008.2	14969.1	15000.9	31.8	4.6	0.990	0.990	0.000	0.000
2008.3	14895.0	15004.3	109.3	14.5	0.997	0.997	0.001	0.000
2008.4	14574.6	14802.1	227.5	28.4	0.998	1.000	0.002	0.000
2009.1	14372.1	14768.2	396.1	44.0	1.001	1.005	0.004	0.001
2009.2	14356.9	14948.1	591.2	68.0	0.999	1.006	0.007	0.001
2009.3	14402.5	15159.6	757.1	86.7	0.999	1.010	0.011	0.002
2009.4	14540.2	15399.7	859.5	95.5	1.002	1.018	0.016	0.003
2010.1	14597.7	15473.1	875.4	99.7	1.005	1.028	0.023	0.003
2010.2	14738.0	15566.4	828.4	92.8	1.010	1.037	0.027	0.004
2010.3	14839.3	15601.7	762.4	88.9	1.014	1.047	0.032	0.005
2010.4	14942.4	15646.4	704.0	83.7	1.019	1.056	0.037	0.006
2011.1	14894.0	15527.6	633.6	82.4	1.023	1.065	0.042	0.006
2011.2	15011.2	15567.6	556.4	76.2	1.030	1.075	0.045	0.007
2011.3	15062.1	15532.5	470.4	81.0	1.036	1.085	0.048	0.008
2011.4	15242.0	15661.7	419.7	73.8	1.038	1.089	0.051	0.008
2012.1	15381.4	15783.1	401.7	74.9	1.043	1.096	0.054	0.009
2012.2	15427.6	15803.5	375.9	74.1	1.047	1.103	0.056	0.010
2012.3	15534.0	15879.4	345.4	73.8	1.053	1.111	0.058	0.010
2012.4	15539.6	15835.8	296.2	75.0	1.056	1.116	0.059	0.010
2013.1	15583.9	15823.6	239.7	70.6	1.061	1.122	0.061	0.010
2013.2	15679.6	15847.9	168.3	69.9	1.063	1.124	0.062	0.010
2013.3	15790.1	15876.0	85.9	72.1	1.068	1.130	0.062	0.011

Table 11 shows the effects on real GDP, $GDPR$, and the GDP deflator, $GDPD$. In 2010:1 real GDP is higher by $875.4 billion. The GDP deflator is higher by 6.2 percent by the end of the period because of the more robust economy. The standard errors are again relatively small.

To get the big picture, Figures 5–14 plot the six variables in Tables 9–11 plus the three consumption variables and housing investment, CS, CN, CD, and IHH. Figure 6 for housing wealth, $AA2$, shows the small recovery after the initial fall. The results for housing investment, IHH, in Figure 14 are striking. In 2009:4 the actual value is $323.6 billion and the predicted value is $490.5 billion, a 52 percent increase.

The main conclusion from the overall results is that much of the recession and slow recovery from the recession was do to the fall in financial and housing wealth from what wealth would have been had it behaved according to historical norms. It is clear, however, that not all of the recession has been explained. The unemployment rate in the new case still rises, from 5.00 percent in 2008:1 to a peak of 7.53 percent in 2009:2. Some of this increase is likely due to financial effects not captured in the interest rate and wealth variables in the four household expenditure equations in the model. This issue is taken up in the next subsection.

Remember that this experiment takes the wealth changes to be exogenous—actually $CG/(PX_{-1}YS_{-1})$ and $PSI14$ to be exogenous. Households respond to the changes after they have taken place. The wealth changes are not explained. Also, the fall in housing prices before 2008, which likely triggered the future wealth changes, is not explained. Looking at the plots in Figures 1 and 2 from, say, 1995 on, it seems unlikely that the changes in these series could be explained econometrically using macro variables. The changes in $AA1$ and $AA2$ are largely unpredictable. In other words, it is unlikely that estimated equations for $AA1$ and $AA2$ could be obtained that would have picked up the changes that occurred since, say, 1995.[158] The experiment is thus conditional on the wealth changes. Conditional on the changes, conditional on the shocks being the same, and conditional on Fed behavior being the same, it answers the question of what the economy would have been like had the wealth changes been at historical values.

[158]Regressions of $CG/(PX_{-1}YS_{-1})$ and $\log PSI14 - \log PSI14_{-1}$ on lagged values of numerous macroeconomic variables for the 1954:1–2013:3 period yield nothing of interest, as expected.

Figure 5
AA1, Actual and Predicted

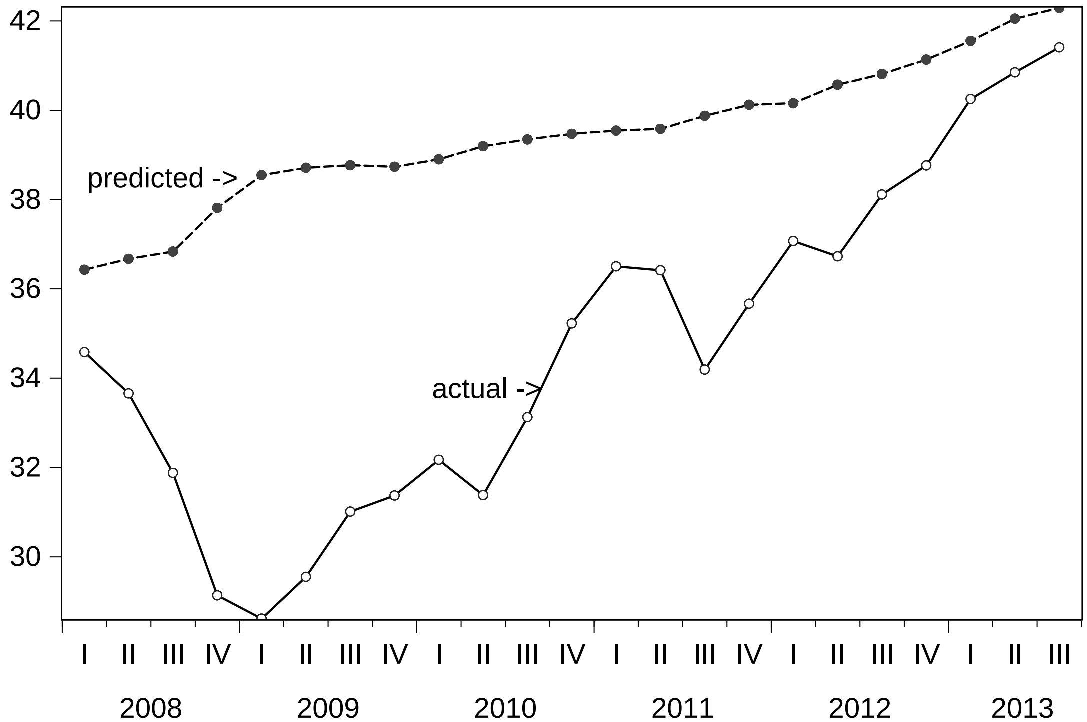


Figure 6
AA2, Actual and Predicted

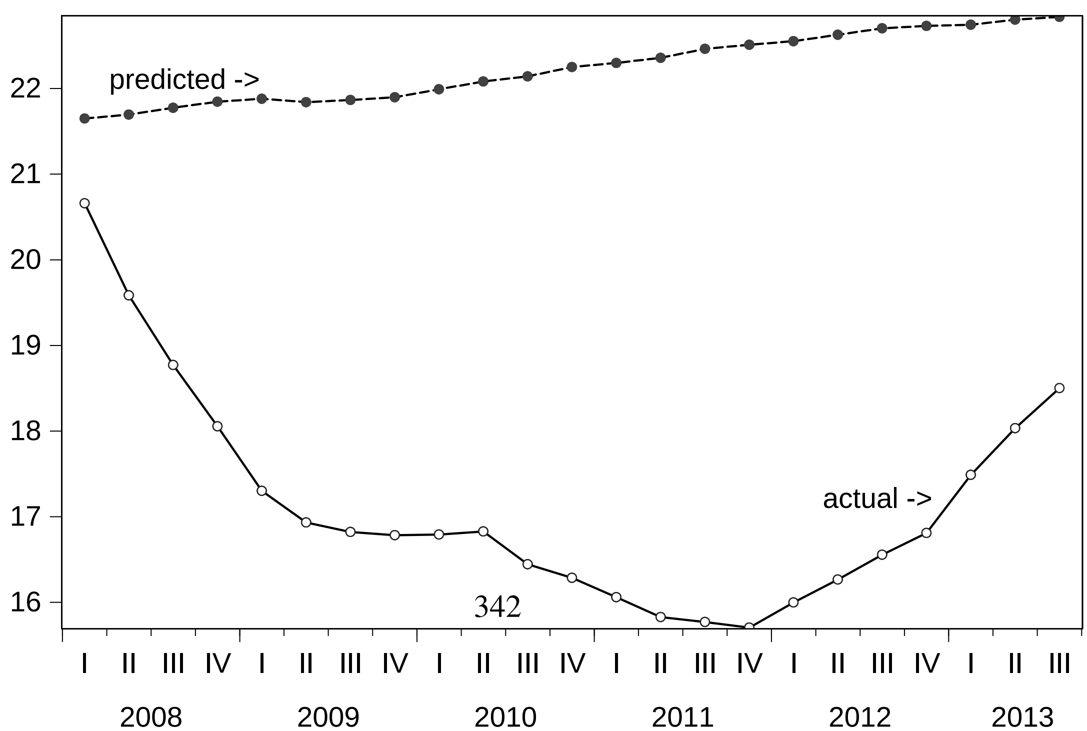


Figure 7
UR Actual and Predicted

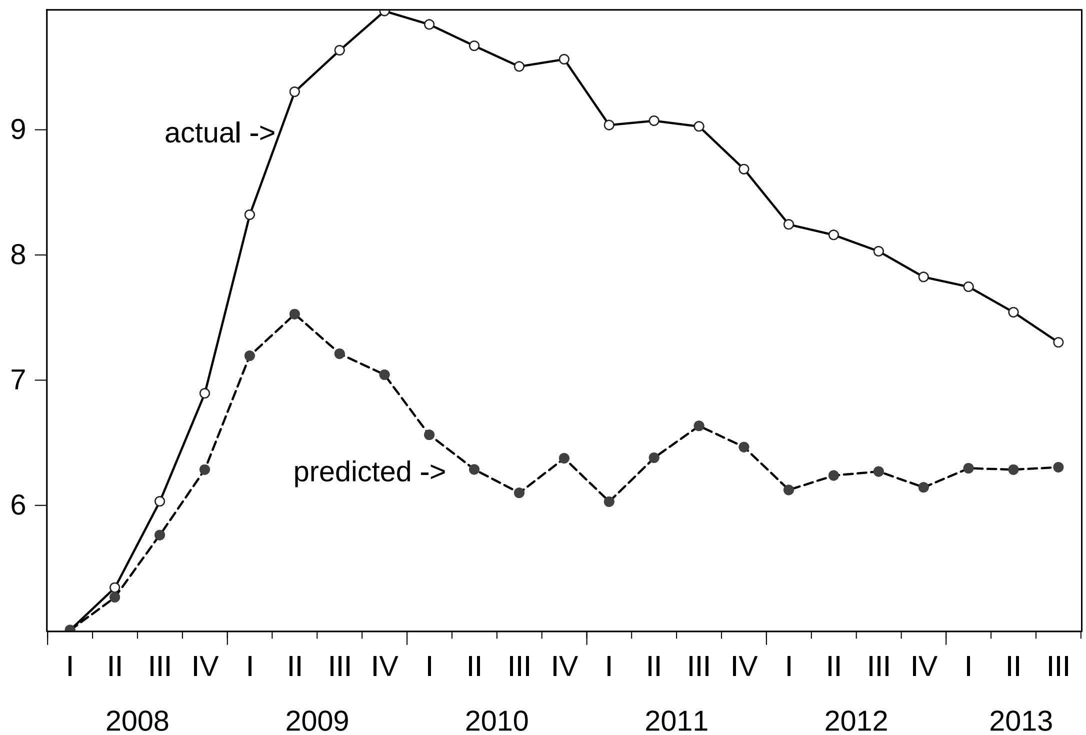


Figure 8
JF Actual and Predicted

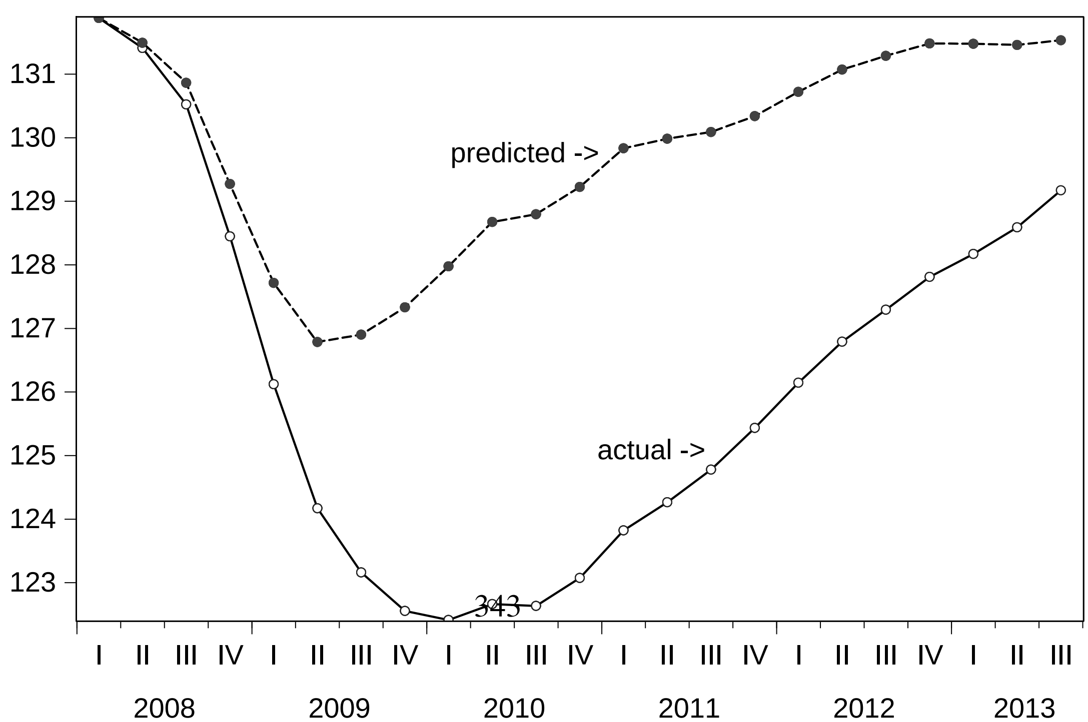


Figure 9
GDPR Actual and Predicted

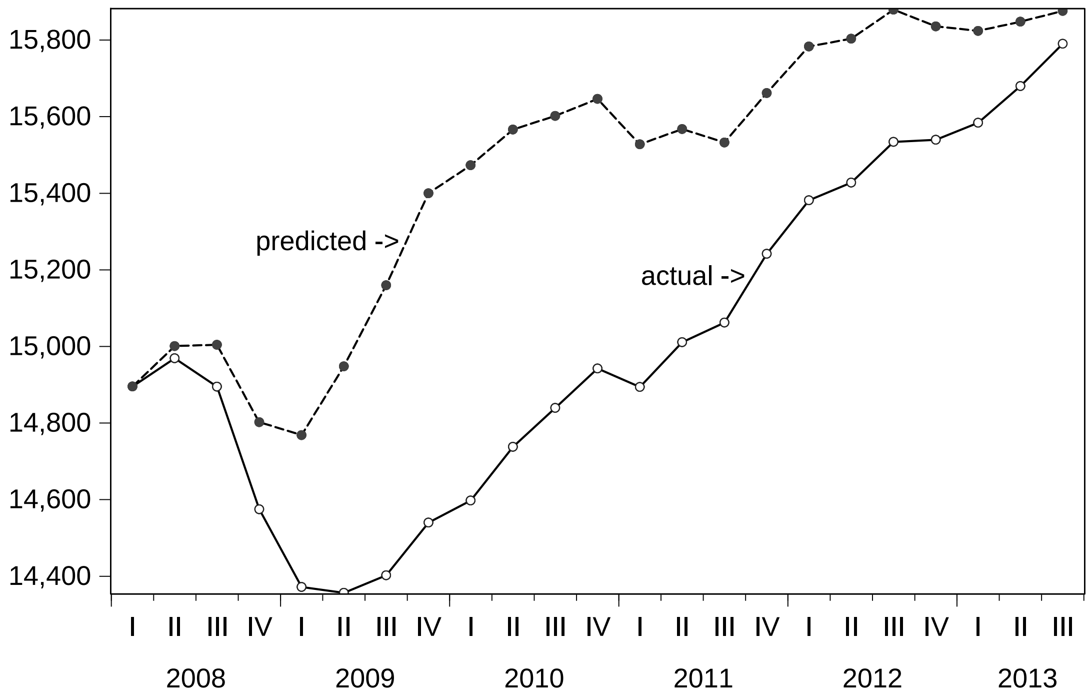


Figure 10
GDPD Actual and Predicted

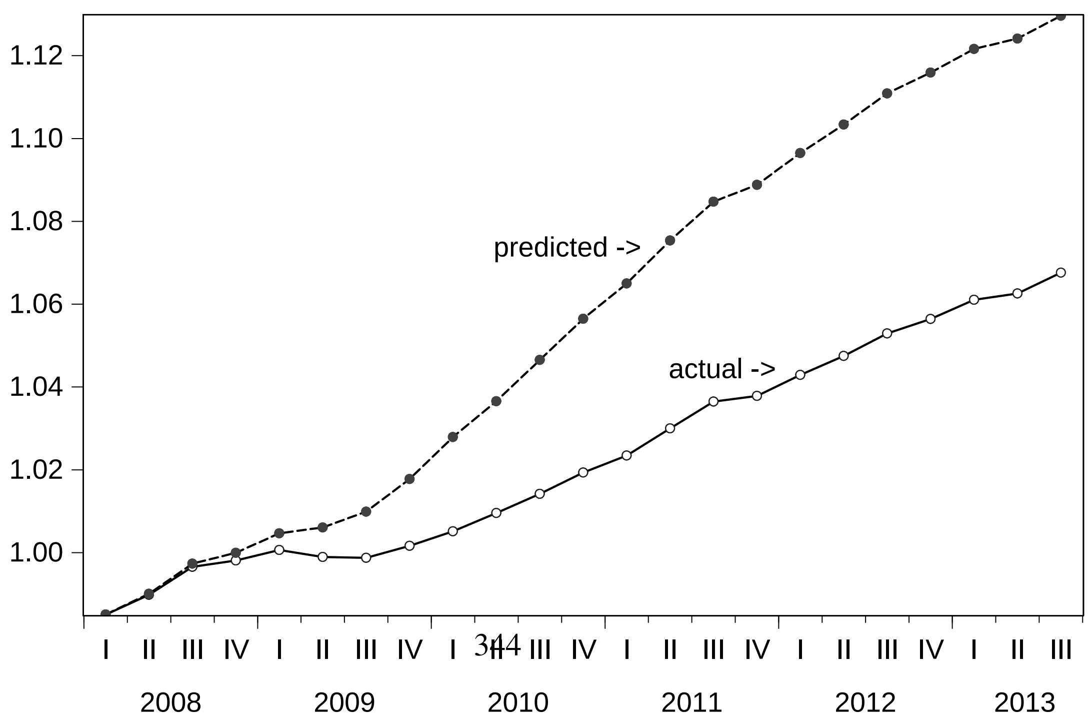


Figure 11
CS Actual and Predicted

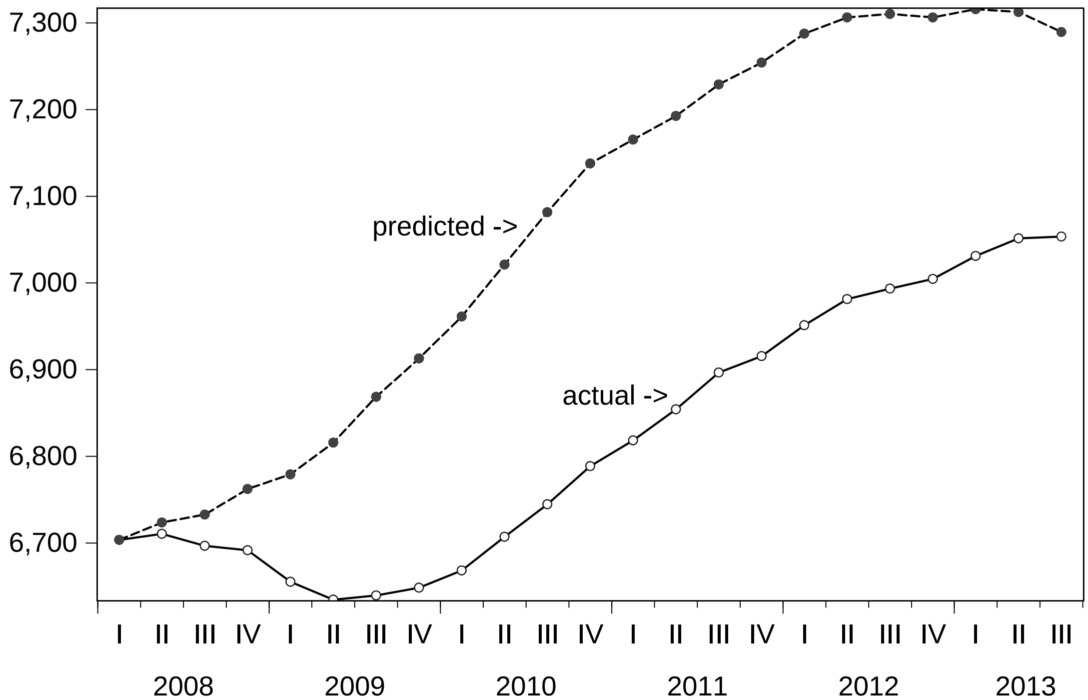


Figure 12
CN Actual and Predicted

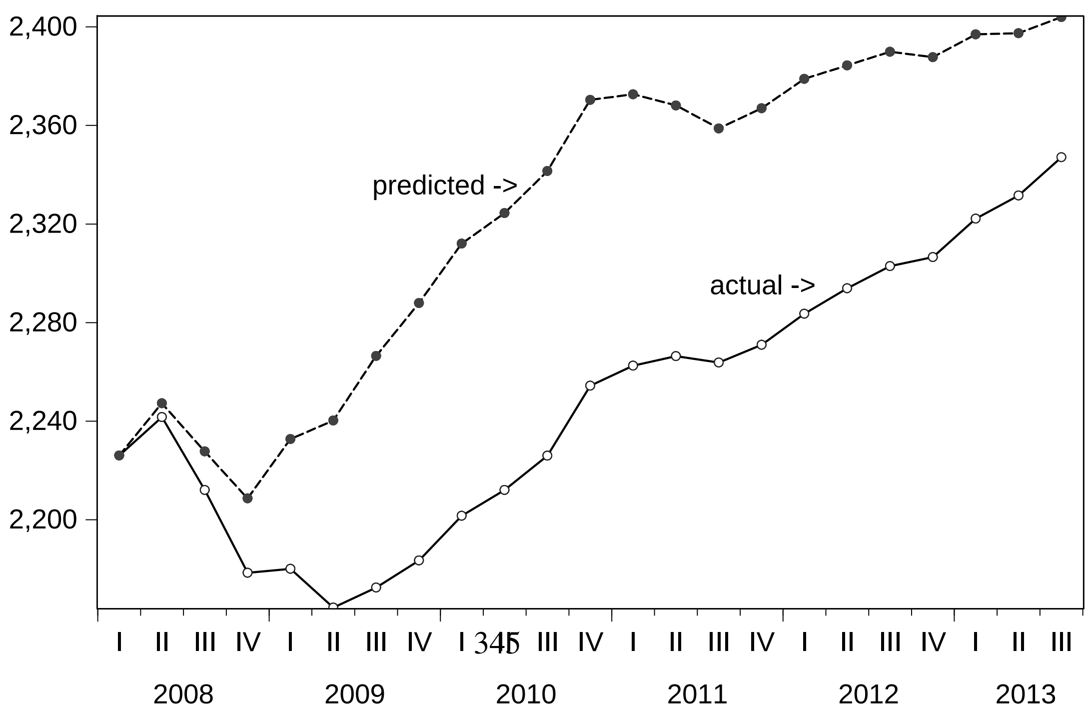


Figure 13
CD Actual and Predicted

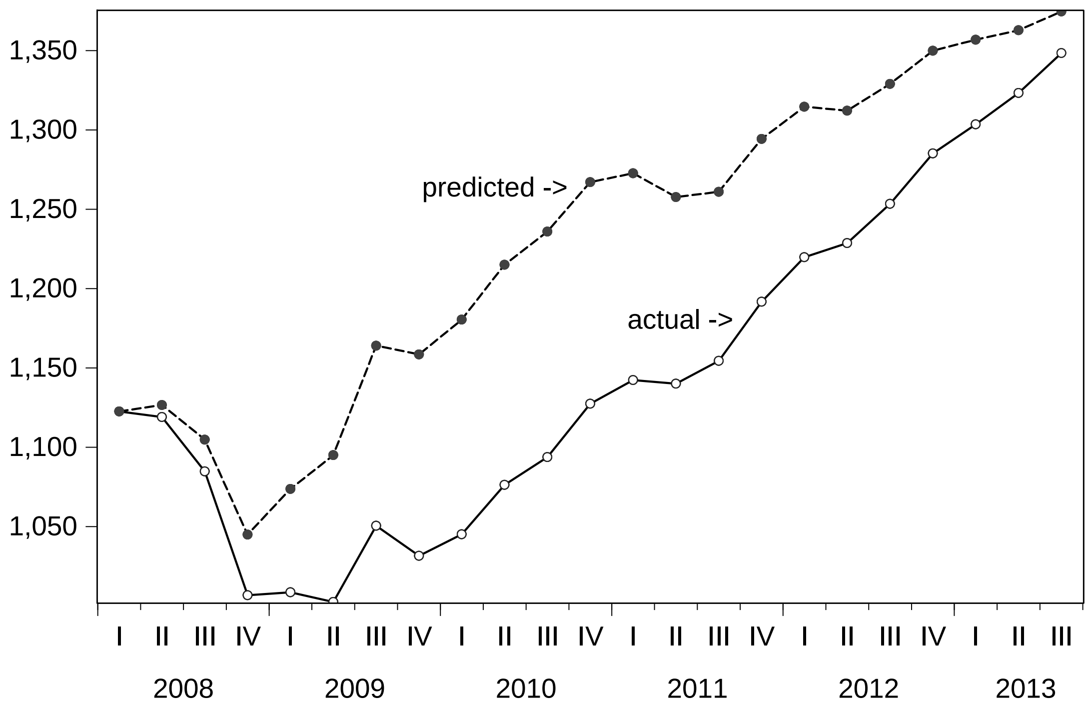
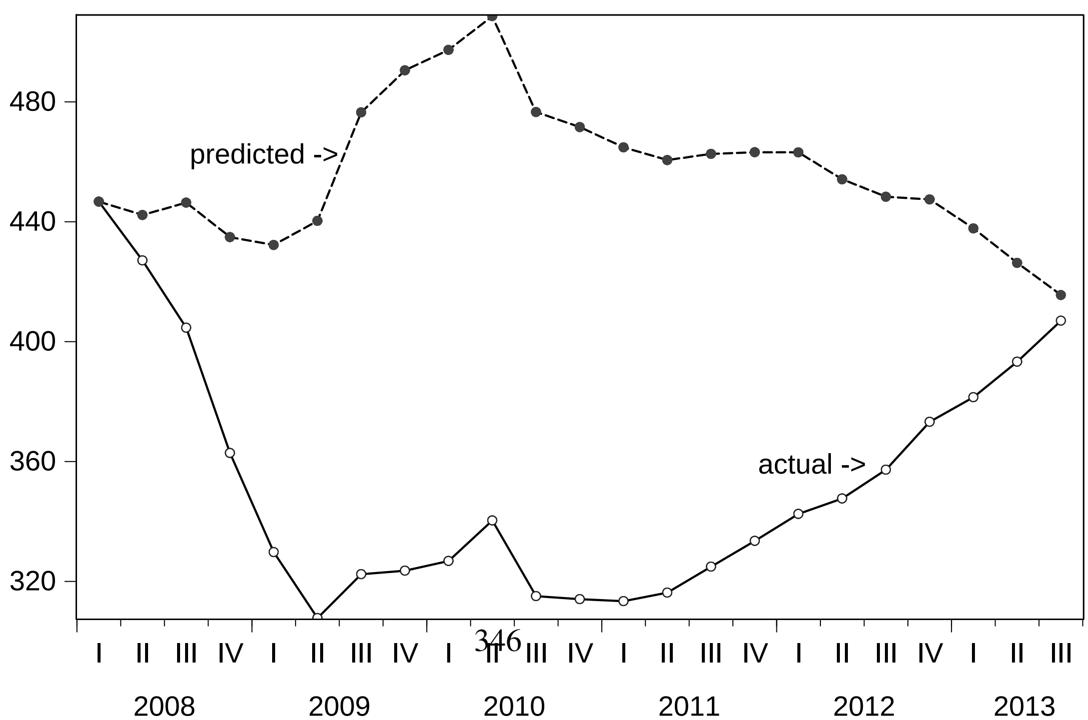


Figure 14
IHH Actual and Predicted



5.7.7 Other Experiments

Can any more of the recession be picked up? In particular, can any other financial effects be picked up? Various measures of credit conditions were tested in Subsection 5.7.3. From the results, the only possible candidate of interest is the EBP variable of Gilchrist and Zakrajšek (2012). Table 5 shows that it is significant in three of the four expenditure equations when they are estimated through the recessionary period. It is possible, however, that EBP is essentially a dummy variable for the 2008–2009 period, chosen after the fact. But if it is picking up actual effects, the following experiment is of interest.

The experiment is as follows. First, the main experiment was rerun with the four household expenditure equations estimated for the 1973:4–2010.3 period, which is the period used when EBP_{-1} is added to the equations. The results for the unemployment rate and jobs are presented in the top half of Table 12. These results differ somewhat from those in Table 10 because of the different estimation period for the four consumer expenditure equations. Second, the fourth of each of the four expenditure equations in Table 5 was used in place of the regular expenditure equations in the model. One can see from Figure 4 that the value of EBP in 2007:2 was quite low. For the experiment this value (-0.5828) was used for 2007:3 on. The rest of the experiment was unchanged. The prediction period was taken to end in 2010:3, since this is the end of the EBP data. So this is an experiment in which wealth doesn't fall and the excess bond premium doesn't rise. The results for the unemployment rate and jobs are presented in the bottom half of Table 12.

Comparing the two sets of results in Table 12, for the version with EBP added UR peaks in 2009:2 at 6.70 versus 7.16 for the regular version. The actual value of UR in this quarter was 9.30, and so roughly the wealth variables lowered it by 2.15 and EBP lowered it by 0.45 more. Figure 4 shows that EBP dropped sharply in 2009:3, which means in the experiment that the stimulative effects from the lower values of EBP are much less. In 2010:3 the predicted unemployment rate for the regular version is 5.70 versus 9.50 actual, and for the version with EBP added the predicted unemployment rate is actually higher at 5.97. In general, EBP is economically important in 2009, but not much otherwise.

Finally, it is of interest to compare the results for jobs in Table 10 with results in Mian and Sufi (2014) (MS). Using estimates for the non-tradeable sector obtained from county data and making some assumptions to aggregate to the entire economy, they estimate that the the fall in housing wealth accounted for 55 percent of the fall in employment between 2007 and 2009. For comparison purposes the above

Table 12
Actual and Predicted Values of
the Unemployment Rate (UR) and
Private Sector Jobs (JF)
(percentage points and millions of jobs)

Regular Version

	UR			JF		
Qtr.	**Act.**	**Pred.**	**Dif.**	**Act.**	**Pred.**	**Dif.**
2008.1	5.00	5.00	0.00	131.88	131.88	0.00
2008.2	5.34	5.25	-0.09	131.41	131.52	0.11
2008.3	6.03	5.70	-0.33	130.52	130.97	0.45
2008.4	6.89	6.15	-0.75	128.45	129.53	1.08
2009.1	8.32	6.95	-1.37	126.12	128.18	2.06
2009.2	9.30	7.16	-2.15	124.17	127.50	3.33
2009.3	9.63	6.74	-2.90	123.16	127.86	4.70
2009.4	9.95	6.52	-3.42	122.56	128.45	5.90
2010.1	9.84	6.04	-3.80	122.42	129.17	6.76
2010.2	9.67	5.82	-3.85	122.66	129.85	7.19
2010.3	9.50	5.70	-3.80	122.63	129.90	7.26

EBP Added

	UR			JF		
Qtr.	**Act.**	**Pred.**	**Dif.**	**Act.**	**Pred.**	**Dif.**
2008.1	5.00	4.97	-0.04	131.88	131.95	0.06
2008.2	5.34	5.13	-0.22	131.41	131.74	0.33
2008.3	6.03	5.50	-0.54	130.52	131.36	0.84
2008.4	6.89	5.86	-1.03	128.45	130.08	1.64
2009.1	8.32	6.58	-1.74	126.12	128.94	2.82
2009.2	9.30	6.70	-2.60	124.17	128.46	4.28
2009.3	9.63	6.28	-3.35	123.16	128.88	5.72
2009.4	9.95	6.21	-3.74	122.56	129.31	6.76
2010.1	9.84	5.93	-3.91	122.42	129.69	7.28
2010.2	9.67	5.93	-3.74	122.66	130.00	7.34
2010.3	9.50	5.97	-3.53	122.63	129.71	7.08

Table 13
Only AA2 Changed
Actual and Predicted Values
of Private Sector Jobs (JF)
(millions of jobs)

Qtr.	**Act.**	**JF Pred.**	**Dif.**
2007.4	132.11		
2008.1	131.88	131.88	0.00
2008.2	131.41	131.46	0.05
2008.3	130.52	130.72	0.20
2008.4	128.45	128.90	0.46
2009.1	126.12	126.94	0.82
2009.2	124.17	125.43	1.25
2009.3	123.16	124.87	1.71
2009.4	122.56	124.67	2.12

experiment was run with only housing wealth ($AA2$) changed. The results for JF are presented in Table 13. Also presented is the actual value of JF for 2007:4. The decline in JF between 2007:4 and 2009:4 was 9.56 million. If housing wealth had not fallen, the estimate is that the decline would have been 2.12 million less, or 7.44 million. 2.12 million is 22 percent of 9.56 million, which is much smaller than the 55 percent estimate of MS. In Table 13, 55 percent would be a fall in JF of 5.36 million due to the fall in housing, which seems high. It may be that some of the assumptions made by MS in moving from the non-tradeable sector results to the aggregate estimates are not realistic. Or it may be that they have overestimated the employment response in the non-tradeable sector.[159]

[159] The employment data used by MS are not the same as the data for JF, and MS use annual changes, not fourth-quarter to fourth-quarter changes. They have a fall in employment between 2007 and 2009 of 5.3 percent, whereas the fall in JF between 2007:4 and 2009:4 in Table 13 is 7.2 percent. The different data might explain part of the 22 versus 55 percent difference.

5.7.8 Conclusion

A standard view of the 2008-2009 financial crisis is that for a variety of reasons, some doing with lack of regulations and some with excessive risk taking, housing prices rose to unsustainable levels between 2002 and 2006. When they started to fall, this set off a chain reaction that led to the financial crisis.[160] The trigger was thus a fall in housing wealth. The results in this section suggest that much of the effect of the financial crisis on macroeconomic activity can be picked up through financial and housing wealth effects on household expenditures. Some of the recession, at least in 2009, not captured by the wealth variables can be captured by the excess bond premium variable of Gilchrist and Zakrajšek (2012), although this variable was created ex post and is only significant ex post. In general much of the 2008-2009 recession is estimated to be simply standard wealth effects at work.

[160]There is some evidence that my wife, Sharon Oster, is the cause of the financial crisis. She was a graduate student with Chip Case at Harvard in the 1970s, and after we were married she introduced Chip to me, which led to our collaborating on an economics text. At some point Chip was interested in finding someone to work with him on housing prices, and I introduced Chip to Bob Shiller. Out of this came the Case-Shiller housing price index. This index for the first time provided financial firms with good data on changes in housing prices. At the time of its release the index had more or less increased every year, and financial firms may have (ex post incorrectly) extrapolated this trend into the future, making mortgage loans under this assumption. Hence the boom in prices and then the collapse. So had Sharon not introduced me to Chip, none of this would have happened.

5.7.9 Appendix: Computing Standard Errors

There are 1,689 estimated equations in the MC model, of which 1,379 are trade share equations. The estimation period for the United States is 1954:1–2013:3. The estimation periods for the other countries begin as early as 1962:1 and end as late as 2013:2. The estimation period for most of the trade share equations is 1966:1–2012:4. For each estimated equation there are estimated residuals over the estimation period. Let $\hat{u}_t$ denote the 1689-dimension vector of the estimated residuals for quarter t.[161] Most of the estimation periods have the 1972:1–2007:4 period—144 quarters– in common, and this period is taken to be the "base" period. These 144 observations on $\hat{u}_t$ are used for the draws in the stochastic-simulation procedure discussed below.[162]

The solution period used to create new data is 1954:1–2013:3—239 quarters. For a given set of coefficient estimates and error terms, the model can be solved dynamically over this period. Equations enter the solution as data become available. For example, for the period 1954:1–1959:4 only the equations for the United States are used. The links from the other countries to the United States are shut off, and the U.S. variables that these links affect are taken to be exogenous. By 1972 almost all the equations are being used.

Each trial of the bootstrap procedure is as follows. First, 239 error vectors are drawn with replacement from the 144 vectors in the base period. (Each vector consists of 1,689 errors.) Using these errors and the coefficient estimates based on the actual data, the model is solved dynamically over the 1954:1–2013:3 period. Using the solution values as the new data set, the 1,689 equations are reestimated. Given these new coefficient estimates and the new data, the experiment in Subsection 5.7.6 is performed for the 2008:1–2013:3 period. The estimated effects are recorded. This is one trial. The procedure is then repeated, say, N times. (Note that the coefficient estimates used to generate the new data on each trial are the estimates based on the actual data.) This gives N values of each estimated effect,

[161] There is a mixture of quarterly and annual equations in the MC model. For equations estimated using annual data, the error is put in the first quarter of the year with zeros in the other three quarters (which are never used). If the initial estimate of an equation suggests that the error term is serially correlated, the equation is reestimated under the assumption that the error term follows an autoregressive process (usually first order). The structural coefficients in the equation and the autoregressive coefficient or coefficients are jointly estimated (by 2SLS). The $\hat{u}_t$ error terms are after adjustment for any autoregressive properties, and they are taken to be iid for purposes of the draws. As discussed in the text, the draws are by year—four quarters at a time.

[162] If an estimation period does not include all of the 1972:1–2007:4 period, zero errors are used for the missing quarters.

from which measures of dispersion can be computed. For the results in Subsection 5.7.6 the number of trials was 100. There were no solution failures for any trial.

The measure of dispersion used in the tables (denoted SE) is as follows. Rank the N values of a given multiplier by size. Let m_r denote the value below which r percent of the values lie. The measure of dispersion is $(m_{.8413} - m_{.1587})/2$. For a normal distribution this is one standard error.

6 Appendix A: The US Model, November 11, 2013

6.1 The US Model in Tables

The tables that pertain to the US model are presented in this appendix. Table A.1 presents the six sectors in the US model: household (h), firm (f), financial (b), foreign (r), federal government (g), and state and local government (s). In order to account for the flow of funds among these sectors and for their balance-sheet constraints, the U.S. Flow of Funds Accounts (FFA) and the U.S. National Income and Product Accounts (NIPA) must be linked. Many of the identities in the US model are concerned with this linkage. Table A.1 shows how the six sectors in the US model are related to the sectors in the FFA. The notation on the right side of this table (H1, FA, etc.) is used in Table A.5 in the description of the FFA data.

Table A.2 lists all the variables in the US model in alphabetical order and the equations in which they appear. Table A.3 lists all the stochastic equations and identities. The coefficient estimates for the stochastic equations are presented in Table A.4, where within this table the coefficient estimates and tests for equation 1 are presented in Table A1, for equation 2 in Table A2, and so on. The results in Table A.4 are discussed in Section 3.6.

The remaining tables provide more detailed information about the model. Tables A.5–A.7 show how the variables were constructed from the raw data, and Table A.9 lists the first stage regressors per equation that were used for the 2SLS estimates. (There is no Table A.8.)

The rest of this appendix discusses the collection of the data and the construction of some of the variables.

6.2 The Raw Data

6.2.1 The NIPA Data

The variables from the NIPA are presented first in Table A.5, in the order in which they appear in the *Survey of Current Business*. The Bureau of Economic Analysis (BEA) uses "chain-type weights" in the construction of real magnitudes, and the data based on these weights have been used here.[163] Because of the use of the chain-type weights, real GDP is not the sum of its real components. To handle this, a discrepancy variable, denoted $STATP$, was created, which is the difference between real GDP and the sum of its real components. ($STATP$ is constructed using equation 83 in Table A.3.) $STATP$ is small in magnitude, and it is taken to be exogenous in the model.

6.2.2 The Other Data

The variables from the FFA are presented next in Table A.5, ordered by their code numbers. Some of these variables are NIPA variables that are not published in the *Survey of Current Business* but that are needed to link the two accounts. Interest rate variables are presented next in the table, followed by employment and population variables. The source for the interest rate data is the website of the Board of Governors of the Federal Reserve System (BOG). The source for the employment and population data is the website of the Bureau of Labor Statistics (BLS). Some of the employment data are unpublished data from the BLS, and these are indicated as such in the table. Data on the armed forces are not published by the BLS, and these data were computed from population data from the U.S. Census Bureau.

Some adjustments that were made to the raw data are presented next in Table A.5. These are explained beginning in the next paragraph. Finally, all the raw data variables are presented at the end of Table A.5 in alphabetical order along with their numbers. This allows one to find a raw data variable quickly. Otherwise, one has to search through the entire table looking for the particular variable. All the raw data variables are numbered with an"R" in front of the number to distinguish them from the variables in the model.

The adjustments that were made to the raw data are as follows. The quarterly social insurance variables R210–R215 were constructed from the annual variables R89-R94 and the quarterly variables R36, R48, and R68. Only annual data are

[163] See Young (1992) and Triplett (1992) for good discussions of the chain-type weights.

available on the breakdown of social insurance contributions between the federal and the state and local governments with respect to the categories "personal," "government employer," and "other employer." It is thus necessary to construct the quarterly variables using the annual data. It is implicitly assumed in this construction that as employers, state and local governments do not contribute to the federal government and vice versa.

The constructed tax variables R216 and R217 pertain to the breakdown of corporate profit taxes of the financial sector between federal and state and local. Data on this breakdown do not exist. It is implicitly assumed in this construction that the breakdown is the same as it is for the total corporate sector.

The tax variables THG (R44) and $TRGHPAY$ (R56) were adjusted to account for the tax surcharge of 1968:3-1970:3 and the tax rebate of 1975:2. The tax surcharge and the tax rebate were taken out of THG and put into $TRGHPAY$. The tax surcharge numbers were taken from Okun (1971), Table 1, p. 171. The tax rebate was $31.2 billion dollars at an annual rate. The two variables were also adjusted in a similar way between 2008:2 and 2011:3 for the effects of the U.S. stimulus bill.

The employment and population data from the BLS are rebenchmarked from time to time, and the past data are not adjusted by the BLS to the new benchmarks. Presented next in Table A.5 are the adjustments that were made to obtain consistent series. These adjustments take the form of various "multiplication factors" for the old data. For the period in question and for a particular variable the old data are multiplied by the relevant multiplication factor to create data for use in the model. The TPOP variables listed in Table A.5 are used to phase out the multiplication factors.

Table A.6 presents the balance-sheet constraints that the data satisfy. The variables in this table are raw data variables. The equations in the table provide the main checks on the collection of the data. If any of the checks are not met, one or more errors have been made in the collection process. Although the checks in the table may look easy, considerable work is involved in having them met.

6.3 Variable Construction

Table A.7 presents the construction of the variables in the model (i.e., the variables in Table A.2) from the raw data variables (i.e., the variables in Table A.5). With a few exceptions, the variables in the model are either constructed in terms of the raw data variables in Table A.5 or are constructed by identities. If the variable is constructed by an identity, the notation"Def., Eq." appears, where the equation number is the identity in Table A.3 that constructs the variable. In a few cases the identity that constructs an endogenous variable is not the equation that determines it in the model. For example, equation 85 constructs LM, whereas stochastic equation 8 determines LM in the model. Equation 85 instead determines E, E being constructed directly from raw data variables. Also, some of the identities construct exogenous variables. For example, the exogenous variables $D2G$ is constructed by equation 49. In the model equation 49 determines TFG, TFG being constructed directly from raw data variables. If a variable in the model is the same as a raw data variable, the same notation is used for both except that variables in the model are in italics and raw data variables are not. For example, consumption expenditures on durable goods is CD as a raw data variable and CD as a variable in the model.

The financial stock variables in the model that are constructed from flow identities need a base quarter and a base quarter starting value. The base quarter values are indicated in Table A.7. The base quarter was taken to be 1971:4, and the stock values for this quarter were taken from the FFA stock values.

There are also a few internal checks on the data in Table A.7 (aside from the balance-sheet checks in Table A.6). The variables for which there are both raw data and an identity available are GDP, MB, $PIEF$, PUG, and PUS. In addition, the saving variables in Table A.6 (SH, SF, and so on) must match the saving variables of the same name in Table A.7. There is also one redundant equation in the model, equation 80, which the variables must satisfy.

There are a few variables in Table A.7 whose construction needs some explanation.

6.3.1 HFS: Peak to Peak Interpolation of HF

HFS is a peak to peak interpolation of HF, hours per job. The peaks are listed in Table A.7. "Flat end" in the table means that the interpolation line was taken to be horizontal from the last peak listed on. The deviation of HF from HFS, which is variable HFF in the model, is used in equation 15, which explains overtime hours. HFS is also used in equations 13 and 14.

6.3.2 HO: Overtime Hours

Data are not available for HO for the first 16 quarters of the sample period (1952:1-1955:4). The equation that explains HO in the model has $\log HO$ on the left hand side and the constant term, HFF, and HFF lagged once on the right hand side. The equation is also estimated under the assumption of a first order autoregressive error term. The missing data for HO were constructed by estimating the $\log HO$ equation for the 1956:1-2013:3 period and using the predicted values from this regression for the (outside sample) 1952:3-1955:4 period as the actual data. The values for 1952:1 and 1952:2 were taken to be the 1952:3 predicted value.

6.3.3 $TAUS$: Progressivity Tax Parameter—s

CREATE THSZ=THS/(PH*POP); CREATE THSZZ=THS/YT;

$TAUS$ is the progressivity tax parameter in the personal income tax equation for state and local governments (equation 48). It was obtained as follows. The sample period 1952:1–2013:3 was divided into the subperiods listed in Table A.7. These were judged from a plot of THS/YT, the ratio of state and local personal income taxes (THS) to taxable income (YT), to be periods of no large tax law changes. Two assumptions were then made about the relationship between THS and YT. The first is that within a subperiod $THS/(POP \cdot PH)$ equals $[D1 + TAUS(YT/(POP \cdot PH))](YT/(POP \cdot PH))$ plus a random error term, where $D1$ and $TAUS$ are constants. The second is that changes in the tax laws affect $D1$ but not $TAUS$. These two assumptions led to the estimation of an equation with $THS/(POP \cdot PH)$ on the left hand side and the constant term, $(YT/(POP \cdot PH))^2$, and the variables $DUMi(YT/(POP \cdot PH))$ on the right hand side, where $DUMi$ is a dummy variable that takes on a value of one in subperiod i and zero otherwise. The estimate of the coefficient of $DUMi(YT/(POP \cdot PH))$ is an estimate of $D1$ for subperiod i. The estimate of the coefficient of $(YT/(POP \cdot PH))^2$ is the estimate of $TAUS$. The estimate of $TAUS$ was .000806, with a t-statistic of 3.14. This procedure is, of course, crude, but at least it provides a rough estimate of the progressivity of the state and local personal income tax system.

Given $TAUS$, $D1S$ is defined to be $THS/YT - (TAUS \cdot YT)/(POP \cdot PH)$ (see Table A.7). In the model $D1S$ is taken to be exogenous, and THS is explained by equation 48 as $[D1S + (TAUS \cdot YT)/(POP \cdot PH)]YT$. This treatment allows a state and local marginal tax rate to be defined in equation 91: $D1SM = D1S + (2 \cdot TAUS \cdot YT)/(POP \cdot PH)$.

6.3.4 $TAUG$: Progressivity Tax Parameter—g

$TAUG$ is the progressivity tax parameter in the personal income tax equation for the federal government (equation 47). A similar estimation procedure was followed for $TAUG$ as was followed above for $TAUS$. The subperiods are listed in Table A.7. The estimate of $TAUG$ was .00724, with a t-statistic of 3.13. Again, this procedure is crude, but it provides a rough estimate of the progressivity of the federal personal income tax system.

Given $TAUG$, $D1G$ is defined to be $THG/YT - (TAUG \cdot YT)/(POP \cdot PH)$ (see Table A.7). In the model $D1G$ is taken to be exogenous, and THG is explained by equation 47 as $[D1G + (TAUG \cdot YT)/(POP \cdot PH)]YT$. This treatment allows a federal marginal tax rate to be defined in equation 90: $D1GM = D1G + (2 \cdot TAUG \cdot YT)/(POP \cdot PH)$.

6.3.5 KD: Stock of Durable Goods

KD is an estimate of the stock of durable goods. It is defined by equation 58:

$$KD = (1 - DELD)KD_{-1} + CD. \tag{58}$$

Given quarterly observations for CD, which are available from the NIPA, quarterly observations for KD can be constructed once a base quarter value and values for the depreciation rate $DELD$ are chosen. End of year estimates of the stock of durable goods are available from the BEA Fixed Assets tables. Given the value of KD at the end of 1952 and given quarterly values of CD for 1953:1–1953:4, a value of $DELD$ can be computed such that the predicted value from equation 58 for 1953:4 matches within a prescribed tolerance level the published BEA value for the end of 1953. This value of $DELD$ can then be used to compute quarterly values of KD for 1953:1, 1953:2, and 1953:3. This process can be repeated for each year, which results in a quarterly series for KD.

6.3.6 KH: Stock of Housing

KH is an estimate of the stock of housing of the household sector. It is defined by equation 59:

$$KH = (1 - DELH)KH_{-1} + IHH. \tag{59}$$

The same procedure was followed for estimating $DELH$ as was followed for estimating $DELD$. The housing stock data are available from the above BEA reference for the durable goods stock data. The BEA residential stock data is for total residential investment, which in the model is $IHH + IHK + IHB$, whereas equation 59 pertains only to the residential investment of the household sector (IHH). The procedure that was used for dealing with this difference is as follows. First, the values for $DELH$ were chosen using total residential investment as the investment series, since this series matched the published stock data. Second, once the values of $DELH$ were chosen, KH was constructed using IHH (not total residential investment). A base quarter value of KH of 2517.7 in 1952:1 was used. This value is .80650 times the computed value for total residential investment for 1952:1. The value .80650 is the average of $IHH/(IHH + IHK + IHB)$ over the sample period.

6.3.7 KK: Stock of Capital

KK is an estimate of the stock of capital of the firm sector. It is determined by equation 92:

$$KK = (1 - DELK)KK_{-1} + IKF. \quad (92)$$

The same procedure was followed for estimating $DELK$ as was followed for estimating $DELD$ and $DELH$. The capital stock data are available from the above BEA reference for the durable goods stock data. The BEA capital stock data is for total fixed nonresidential investment, which in the model is $IKF + IKH + IKB + IKG$, whereas equation 59 pertains only to the fixed non residential investment of the firm sector (IKF). A similar procedure for dealing with this followed here as was followed above for residential investment. First, the values for $DELK$ were chosen using total fixed nonresidential investment as the investment series, since this series matched the published stock data. Second, once the values of $DELK$ were chosen, KK was constructed using IKF (not total fixed nonresidential investment). A base quarter value of KK of 2501.9 in 1952:1 was used. This value is .85831 times the computed value for total fixed nonresidential investment for 1952:1. The value .85831 is the average of $IKF/(IKF + IKH + IKB + IKG)$ over the sample period.

6.3.8 V: Stock of Inventories

V is the stock of inventories of the firm sector (i.e., the nonfarm stock). By definition, inventory investment (IVF) is equal to the change in the stock, which is equation 117:

$$IVF = V - V_{-1}. \tag{117}$$

The stock data on V are in BEA Fixed Assets Table 5.8.6A. For present purposes V was constructed from the formula $V = V_{-1} + IVF$ using the IVF series and base quarter value of 1517.3 in 1996:4. This is the value in NIPA Table 5.8.6A.

6.3.9 Excess Labor and Excess Capital

In the theoretical model the amounts of excess labor and excess capital on hand affect the decisions of firms. In order to test for this in the empirical work, one needs to estimate the amounts of excess labor and capital on hand in each period. This in turn requires an estimate of the technology of the firm sector.

The measurement of the capital stock KK is discussed above. The production function of the firm sector for empirical purposes is postulated to be

$$Y = \min[LAM(JF \cdot HF^a), MU(KK \cdot HK^a)],$$

where Y is production, JF is the number of workers employed, HF^a is the number of hours worked per worker, KK is the capital stock discussed above, HK^a is the number of hours each unit of KK is utilized, and LAM and MU are coefficients that may change over time due to technical progress. The variables Y, JF, and KK are observed; the others are not. For example, data on the number of hours paid for per worker exist, HF in the model, but not on the number of hours actually worked per worker, HF^a.

Equation 92 for KK and the production function A.1 are not consistent with the putty-clay technology of the theoretical model. To be precise with this technology one has to keep track of the purchase date of each machine and its technological coefficients. This kind of detail is not possible with aggregate data, and one must resort to simpler specifications.

Given the production function A.1, excess labor is measured as follows. The log of output per paid for worker hour, $\log[Y/(JF \cdot HF)]$, is first plotted for the 1952:1–2013:3 period. The peaks of this series are then assumed to correspond to cases in which the capital constraint in the production function A.1 is not binding and in which the number of hours worked equals the number of hours paid for. This implies that the values of LAM are observed at the peaks. The values of $\log LAM$ other than those at the peaks are assumed to lie on straight lines between the peaks. This allows LAM to be computed for each quarter.

Since LAM is a measure of potential productivity, an interesting question is how it grows over time. This is discussed in Subsection 5.3.4, where the plot of $\log[Y/(JF \cdot HF)]$ is presented in Figure 6.16a. This plot shows that LAM grew more rapidly in the 1950s and 1960s than it has since. It also shows that the growth rate after 1995 was only slightly larger than before. See also Figure 16.6b.

Coming back to the measurement of excess labor, given an estimate of LAM for a particular quarter and given equation A.1, the estimate of the number of worker hours required to produce the output of the quarter, denoted $JHMIN$ in the model, is simply Y/LAM. This is equation 94 in Table A.3. The actual number of workers hours paid for, $JF \cdot HF$, can be compared to $JHMIN$ to measure the amount of excess labor on hand. The peaks that were used for the interpolations are listed in Table A.7 in the description of LAM.

For the measurement of excess capital there are no data on hours paid for or worked per unit of KK, and thus one must be content with plotting Y/KK. This is, from the production function A.1, a plot of $MU \cdot HK^a$, where HK^a is the average number of hours that each machine is utilized. If it is assumed that at each peak of this series the labor constraint in the production function A.1 is not binding and that HK^a is equal to the same constant, say $\bar{H}$, then one observes at the peaks $MU \cdot \bar{H}$. Interpolation between peaks can then produce a complete series on $MU \cdot \bar{H}$. If, finally, $\bar{H}$ is assumed to be the maximum number of hours per quarter that each unit of KK can be utilized, then $Y/(MU \cdot \bar{H})$ is the minimum amount of capital required to produce Y, denoted $KKMIN$. In the model, $MU \cdot \bar{H}$ is denoted MUH, and the equation determining $KKMIN$ is equation 93 in Table A.4. The actual capital stock (KK) can be compared to $KKMIN$ to measure the amount of excess capital on hand. The peaks that were used for the interpolations are listed in Table A.7 in the description of MUH. "Flat beginning" in the table means that the interpolation line was taken to be horizontal from the beginning of the period to the first peak listed.

6.3.10 YS: Potential Output of the Firm Sector

YS, a measure of the potential output of the firm sector, is defined by equation 98:

$$YS = LAM(JJP \cdot POP - JG \cdot HG - JM \cdot HM - JS \cdot HS). \qquad (98)$$

JJP is the peak or potential ratio of worker hours to population. It is constructed from a peak to peak interpolation of JJ, where JJ is the actual ratio of the total number of worker hours paid for in the economy to the total population 16 and over (equation 95). ("Flat end" in the table means that the interpolation line was taken to be horizontal from the last peak listed on.) $JJP \cdot POP$ is thus the potential number of worker hours. The terms that are subtracted from $JJP \cdot POP$ in equation 98 are, in order, the number of federal civilian worker hours, the number of federal military worker hours, and the number of state and local government worker hours. The entire number in parentheses is thus the potential number of worker hours in the firm sector. LAM is the coefficient LAM in the production function A.1. Since YS in equation 98 is LAM times the potential number of workers in the firm sector, it can be interpreted as the potential output of the firm sector unless the capital input is insufficient to produce YS. This construction of YS is thus based on the assumption that there is always sufficient capital on hand to produce YS.

6.4 The Identities

The identities in Table A.3 are of two types. One type simply defines one variable in terms of others. The other type defines one variable as a rate or ratio times another variable or set of variables, where the rate or ratio has been constructed to have the identity hold. Consider, for example, equation 50:

$$TFS = D2S \cdot PIEF, \tag{50}$$

where TFS is the amount of corporate profit taxes paid from firms (sector f) to the state and local government sector (sector s), $PIEF$ is the level of corporate profits of the firm sector, and $D2S$ is the "tax rate." Data exist for TFS and $PIEF$, and $D2S$ was constructed as $TFS/PIEF$. The variable $D2S$ is then interpreted as a tax rate and is taken to be exogenous. This rate, of course, varies over time as tax laws and other things that affect the relationship between TFS and $PIEF$ change, but no attempt has been made to explain these changes. This general procedure was followed for the other identities involving tax rates.

A similar procedure was followed to handle relative price changes. Consider equation 38:

$$PIH = PSI5 \cdot PD, \tag{38}$$

where PIH is the price deflator for residential investment, PD is the price deflator for total domestic sales, and $PSI5$ is a ratio. Data exist for PIH and PD, and $PSI5$ was constructed as PIH/PD. $PSI5$, which varies over time as the relationship between PIH and PD changes, is taken to be exogenous. This procedure was followed for the other identities involving prices and wages. This treatment means that relative prices and relative wages are exogenous in the model. (Prices relative to wages are not exogenous, however.) It is beyond the scope of the model to explain relative prices and wages, and the foregoing treatment is a simple way of handling these changes.

Many of the identities of the first type are concerned with linking the FFA data to the NIPA data. An identity like equation 66

$$0 = SH - \Delta AH - \Delta MH + CG - DISH \tag{66}$$

is concerned with this linkage. SH is from the NIPA, and the other variables are from the FFA. The discrepancy variable, $DISH$, which is from the FFA, reconciles the two data sets. Equation 66 states that any nonzero value of saving of the household sector must result in a change in AH or MH. There are equations like 66 for each of the other five sectors: equation 70 for the firm sector, 73 for the financial sector, 75 for the foreign sector, 77 for the federal government sector, and 79 for the state and local government sector. Equation 77, for example, is the budget constraint of the federal government sector. Note also from Table A.3 that the saving of each sector (SH, SF, etc.) is determined by an identity. The sum of the saving variables across the six sectors is zero, which is the reason that equation 80 is redundant.

6.5 The Tables for the US Model

Table A.1
The Six Sectors of the US Model

Sector	Corresponding Sector(s) in the Flow of Funds Accounts
1 Household (h)	1 Households and Nonprofit Organizations (H)
2 Firm (f)	2a Nonfinancial Corporate Business (F1) 2b Nonfinancial Noncorporate Business (NN)
3 Financial (b)	3 Financial Business (B) except Government Sponsored Enterprises (CA) and Monetary Authority (MA)
4 Foreign (r)	4 Rest of the World (R)
5 Fed. Gov. (g)	5a Federal Government (US) 5b Government-Sponsored Enterprises (CA) 5c Monetary Authority (MA)
6 S & L Gov. (s)	6 State and Local Governments (S)

- The abbreviations h, f, b, r, g, and s are used throughout this appendix.
- The abbreviations H, F1, NN, B, R, US, CA, MA, and S are used in Table A.5 in the description of the flow of funds data and, when appropriate, in other tables.

Table A.2
The Variables in the US Model in Alphabetical Order

Variable	Eq.	Description	Used in Equations
AA	89	Total net wealth, h, B2009$.	1, 2, 3, 5, 6, 7
AB	73	Net financial assets, b, B$.	none
AF	70	Net financial assets, f, B$.	none
AG	77	Net financial assets, g, B$.	29
$AG1$	exog	Percent of 16+ population 26-55 minus percent 16-25.	1, 2, 3
$AG2$	exog	Percent of 16+ population 56-65 minus percent 16-25.	1, 2, 3
$AG3$	exog	Percent of 16+ population 66+ minus percent 16-25.	1, 2, 3
AH	66	Net financial assets, h, B$.	89
AR	75	Net financial assets, r, B$.	none
AS	79	Net financial assets, s, B$.	none
BO	exog	Bank borrowing from the Fed, B$.	125
BR	exog	Total bank reserves, B$.	125
$CCF1$	67	Capital consumption, F1, B$.	68
CCG	150	Capital consumption, g, B$.	68, 69, 76
$CCGQ$	exog	Capital consumption, g, B2009$.	150
CCH	151	Capital consumption, h, B$.	65, 68, 69
$CCHQ$	exog	Capital consumption, h, B2009$.	151
CCS	152	Capital consumption, s, B$.	68, 69, 78
$CCSQ$	exog	Capital consumption, s, B2009$.	152
CD	3	Consumer expenditures for durable goods, B2009$.	27, 34, 51, 52, 58, 60, 61, 65, 96, 97, 116
CDA	exog	Peak to peak interpolation of CD/POP.	3
CDH	96	Capital expenditures, consumer durable goods, h, B$.	65, 68
CG	25	Capital gains(+) or losses(-) on the financial assets of h, B$.	12, 66
CN	2	Consumer expenditures for nondurable goods, B2009$.	27, 34, 51, 52, 60, 61, 65, 116
COG	exog	Purchases of consumption and investment goods, g, B2009$.	60, 61, 76, 104
COS	exog	Purchases of consumption and investment goods, s, B2009$.	60, 61, 78, 110
CS	1	Consumer expenditures for services, B2009$.	27, 34, 51, 52, 60, 61, 65, 116
CTB	exog	Net capital transfers paid, financial corporations, B$.	72
$CTF1$	exog	Net capital transfers paid, nonfinancial corporations, B$.	69
$CTGB$	exog	Financial stabilization payments, B$.	68, 69
$CTGMB$	exog	Net capital transfers paid, g, less financial stabilization payments, B$.	76
CTH	exog	Net capital transfers paid, h, B$.	65
$CTNN$	exog	Net capital transfers paid, noncorporate business, B$.	69
CTR	exog	Net capital transfers paid, r, B$.	74
CTS	exog	Net capital transfers paid, s, B$.	78
CUR	26	Currency held outside banks, B$.	71, 77
$D1G$	exog	Personal income tax parameter, g.	47, 90
$D1GM$	90	Marginal personal income tax rate, g.	126, 127, 128
$D1S$	exog	Personal income tax parameter, s.	48, 91
$D1SM$	91	Marginal personal income tax rate, s.	126, 127, 128
$D2G$	exog	Profit tax rate, g.	12, 17, 49, 121
$D2S$	exog	Profit tax rate, s.	12, 17, 50, 121
$D3G$	exog	Indirect business tax rate, g.	35, 36, 37, 51
$D3S$	exog	Indirect business tax rate, s.	35, 36, 37, 52
$D4G$	exog	Employee social security tax rate, g.	53, 126
$D5G$	exog	Employer social security tax rate, g.	10, 54
$D6G$	exog	Capital consumption rate for CCF1, g.	67
$D593$	exog	1 in 1959:3; 0 otherwise.	11, 13
$D594$	exog	1 in 1959:4; 0 otherwise.	11
$D601$	exog	1 in 1960:1; 0 otherwise.	11
$D691$	exog	1 in 1969:1; 0 otherwise.	27
$D692$	exog	1 in 1969:2; 0 otherwise.	27
$D714$	exog	1 in 1971:4; 0 otherwise.	27
$D721$	exog	1 in 1972:1; 0 otherwise.	27
$D794823$	exog	1 in 1979:4-1982:3; 0 otherwise.	30

Table A.2 (continued)

Variable	Eq.	Description	Used in Equations
DB	153	Net dividends paid, b, B$.	64, 68, 69, 99, 115
DBQ	exog	Net dividends paid, b, B2009$.	153
$DELD$	exog	Physical depreciation rate of the stock of durable goods, rate per quarter.	3, 58
$DELH$	exog	Physical depreciation rate of the stock of housing, rate per quarter.	4, 59
$DELK$	exog	Physical depreciation rate of the stock of capital, rate per quarter.	92
DF	18	Net dividends paid, f, B$.	64, 69, 99, 115
DG	exog	Net dividends paid, g, B$.	64, 76, 99, 105, 115
$DISB$	exog	Discrepancy for b, B$.	73
$DISF$	exog	Discrepancy for f, B$.	70
$DISG$	exog	Discrepancy for g, B$.	77
$DISH$	exog	Discrepancy for h, B$.	66
$DISR$	exog	Discrepancy for r, B$.	75
$DISS$	exog	Discrepancy for s, B$.	79
DR	154	Net dividends paid, r, B$.	57, 64, 99, 115
DRQ	exog	Net dividends paid, r, B2009$.	154
DS	exog	Net dividends paid, s, B$.	64, 78, 99, 112, 115
E	85	Total employment, civilian and military, millions.	86
EX	exog	Exports, B2009$.	33, 60, 61, 74
$EXPG$	106	Net expenditures, g, B$.	107
$EXPS$	113	Net expenditures, s, B$.	114
FA	exog	Farm gross product, B2009$.	17, 26, 31
GDP	82	Gross Domestic Product, B$.	84, 129
$GDPD$	84	GDP price deflator.	111, 123, 130, 150–169
$GDPR$	83	Gross Domestic Product, B2009$.	84, 122, 130
GNP	129	Gross National Product, B$.	131
$GNPD$	131	GNP price deflator.	none
$GNPR$	130	Gross National Product, B2009$.	131
GSB	155	Gross saving, B, B$.	68, 69, 72
$GSBQ$	exog	Gross saving, B, B2009$.	155
$GSCA$	exog	Gross saving, CA, B$.	68, 69, 76
$GSMA$	exog	Gross saving, MA, B$.	68, 69, 76
$GSNN$	156	Gross saving, NN, B$.	68
$GSNNQ$	exog	Gross saving, NN, B2009$.	156
HF	14	Average number of hours paid per job, f, hours per quarter.	62, 95, 100, 118
HFF	100	Deviation of HF from its peak to peak interpolation.	15
HFS	exog	Peak to peak interpolation of HF.	13, 14, 100
HG	exog	Average number of hours paid per civilian job, g, hours per quarter.	43, 64, 76, 82, 83, 95, 98, 104, 115, 126
HM	exog	Average number of hours paid per military job, g, hours per quarter.	43, 64, 76, 82, 83, 95, 98, 104, 115, 126
HN	62	Average number of non overtime hours paid per job, f, hours per quarter.	43, 53, 54, 64, 67, 68, 115, 121, 126
HO	15	Average number of overtime hours paid per job, f, hours per quarter.	43, 53, 54, 62, 67, 68, 115, 121, 126
HS	exog	Average number of hours paid per job, s, hours per quarter.	43, 64, 78, 82, 83, 95, 98, 110, 115, 126
$IBTG$	51	Indirect business taxes, g, B$.	34, 52, 61, 76, 82, 105
$IBTS$	52	Indirect business taxes, s, B$.	34, 51, 61, 78, 82, 112
IGZ	157	Gross investment, g, B$.	106
$IGZQ$	exog	Gross investment, g, B2009$.	157
IHB	exog	Residential investment, b, B2009$.	27, 60, 61, 72
IHF	exog	Residential investment, f, B2009$.	27, 60, 61, 68
IHH	4	Residential investment, h, B2009$.	27, 34, 59, 60, 61, 65
$IHHA$	exog	Peak to peak interpolation of IHH/POP.	4

Table A.2 (continued)

Variable	Eq.	Description	Used in Equations
IKB	exog	Nonresidential fixed investment, b, B2009$.	27, 60, 61, 72
IKF	92	Nonresidential fixed investment, f, B2009$.	27, 60, 61, 67, 69
IKG	exog	Nonresidential fixed investment, g, B2009$.	60, 61, 76
IKH	exog	Nonresidential fixed investment, h, B2009$.	27, 60, 61, 65
IM	27	Imports, B2009$.	33, 60, 61, 74
INS	exog	Insurance and pension reserves to h from g, B$.	65, 76
$INTF$	exog	Net interest payments, f, B$.	64, 68, 69, 99, 115
$INTG$	29	Net interest payments, g, B$.	56, 64, 76, 99, 106, 115
$INTGR$	56	Net interest payments, g to r, B$.	57, 64, 99, 115
$INTS$	exog	Net interest payments, s, B$.	64, 78, 99, 113, 115
$INTZ$	158	Net interest payments, other, B$.	64, 68, 69, 99, 115
$INTZQ$	exog	Net interest payments, other, B2009$.	158
ISZ	159	Gross investment, s, B$.	113
$ISZQ$	exog	Gross investment, s, B2009$.	159
IVA	exog	Inventory valuation adjustment, B$.	68
IVF	117	Inventory investment, f, B2009$.	68
JF	13	Number of jobs, f, millions.	14, 43, 53, 54, 64, 68, 69, 85, 95, 115, 118, 121
JG	exog	Number of civilian jobs, g, millions.	43, 64, 76, 82, 83, 85, 95, 98, 104, 115, 126
$JHMIN$	94	Number of worker hours required to produce Y, millions.	13, 14
JJ	95	Ratio of the total number of worker hours paid for to the total population 16 and over.	none
JJP	exog	Potential value of JJ.	98
JM	exog	Number of military jobs, g, millions.	43, 64, 76, 82, 83, 85, 87, 95, 98, 104, 115
JS	exog	Number of jobs, s, millions.	43, 64, 78, 82, 83, 85, 95, 98, 110, 115, 126
KD	58	Stock of durable goods, B2009$.	3
KH	59	Stock of housing, h, B2009$.	4, 89
KK	12	Stock of capital, f, B2009$.	92
$KKMIN$	93	Amount of capital required to produce Y, B2009$.	12
$L1$	5	Labor force of men 25-54, millions.	86, 87
$L2$	6	Labor force of women 25-54, millions.	86, 87
$L3$	7	Labor force of all others, 16+, millions.	86, 87
LAM	exog	Amount of output capable of being produced per worker hour.	10, 16, 94, 98
LM	8	Number of"moonlighters": difference between the total number of jobs (establishment data) and the total number of people employed (household survey data), millions.	85
$M1$	81	Money supply, end of quarter, B$.	124
MB	71	Net demand deposits and currency, b, B$.	73
$MDIF$	exog	Net increase in demand deposits and currency of banks in U.S. possessions plus change in demand deposits and currency of private nonbank financial institutions plus change in demand deposits and currency of federally sponsored credit agencies and mortgage pools minus mail float, U.S. government, B$.	81
MF	17	Demand deposits and currency, f, B$.	70, 71, 81
MG	160	Demand deposits and currency, g, B$.	71, 77
MGQ	exog	Demand deposits and currency, g, B2009$.	160
MH	161	Demand deposits and currency, h, B$.	66, 71, 81, 89
MHQ	exog	Demand deposits and currency, h, B2009$.	161
MR	162	Demand deposits and currency, r, B$.	71, 75, 81
MRQ	exog	Demand deposits and currency, r, B2009$.	162
MS	163	Demand deposits and currency, s, B$.	71, 79, 81
MSQ	exog	Demand deposits and currency, s, B2009$.	163
MUH	exog	Amount of output capable of being produced per unit of capital.	93

Table A.2 (continued)

Variable	Eq.	Description	Used in Equations
$NICD$	97	Net investment in consumer durables, h, B$.	65, 68, 69
NNF	exog	Net acquisition of nonproduced nonfinancial assets, f, B$.	69
NNG	exog	Net acquisition of nonproduced nonfinancial assets, g, B$.	76
NNH	exog	Net acquisition of nonproduced nonfinancial assets, h, B$.	65
NNR	exog	Net acquisition of nonproduced nonfinancial assets, r, B$.	74
NNS	exog	Net acquisition of nonproduced nonfinancial assets, s, B$.	78
PCD	37	Price deflator for CD.	34, 51, 52, 61, 65, 96, 97, 116
$PCGDPD$	123	Percentage change in GDPD, annual rate, percentage points.	none
$PCGDPR$	122	Percentage change in GDPR, annual rate, percentage points.	none
$PCM1$	124	Percentage change in M1, annual rate, percentage points.	30
PCN	36	Price deflator for CN.	34, 51, 52, 61, 65, 116
PCS	35	Price deflator for CS.	34, 51, 52, 61, 65, 116
PD	33	Price deflator for X - EX + IM (domestic sales).	12, 30, 35, 36, 37, 38, 39, 40, 41, 42, 55
PEX	32	Price deflator for EX.	33, 61, 74
PF	10	Price deflator for non farm sales.	16, 17, 26, 27, 31, 119
PFA	191	Price deflator for farm sales.	31
PG	40	Price deflator for COG.	61, 76, 104
PH	34	Price deflator for CS + CN + CD + IHH inclusive of indirect business taxes.	1, 2, 3, 4, 6, 7, 8, 89
$PIEF$	67	Before tax profits, f, B$.	18, 25, 49, 50, 121, 132
$PIEFRET$	132	Foreign earnings retained abroad, f, B$.	57, 69
PIH	38	Price deflator for residential investment.	34, 61, 65, 68, 72
PIK	39	Price deflator for nonresidential fixed investment.	21, 61, 65, 68, 72, 76
PIM	exog	Price deflator for IM.	10, 27, 33, 61, 74
PIV	42	Price deflator for inventory investment, adjusted.	67, 82
PKH	55	Market price of KH.	89
POP	120	Noninstitutional population 16+, millions.	1, 2, 3, 4, 5, 6, 7, 8, 26, 27, 47, 48, 90, 91
$POP1$	exog	Noninstitutional population of men 25-54, millions.	5, 120
$POP2$	exog	Noninstitutional population of women 25-54, millions.	6, 120
$POP3$	exog	Noninstitutional population of all others, 16+, millions.	7, 120
$PROD$	118	Output per paid for worker hour ("productivity").	none
PS	41	Price deflator for COS.	61, 78, 110
$PSI1$	exog	Ratio of PEX to PX.	32
$PSI2$	exog	Ratio of PCS to (1 + D3G + D3S)PD.	35
$PSI3$	exog	Ratio of PCN to (1 + D3G + D3S)PD.	36
$PSI4$	exog	Ratio of PCD to (1 + D3G + D3S)PD.	37
$PSI5$	exog	Ratio of PIH to PD.	38
$PSI6$	exog	Ratio of PIK to PD.	39
$PSI7$	exog	Ratio of PG to PD.	40
$PSI8$	exog	Ratio of PS to PD.	41
$PSI9$	exog	Ratio of PIV to PD.	42
$PSI10$	exog	Ratio of WG to WF.	44
$PSI11$	exog	Ratio of WM to WF.	45
$PSI12$	exog	Ratio of WS to WF.	46
$PSI13$	exog	Ratio of gross product of g and s to total employee hours of g and s.	83
$PSI14$	exog	Ratio of PKH to PD.	55
$PSI15$	exog	Ratio of INTGR to INTG.	56
PUG	104	Purchases of goods and services, g, B$.	106
PUS	110	Purchases of goods and services, s, B$.	113
PX	31	Price deflator for total sales.	12, 25, 32, 33, 61, 72, 82, 119
Q	164	Gold and foreign exchange, g, B$.	75,77
QQ	exog	Gold and foreign exchange, g, B2009$.	164

Table A.2 (continued)

Variable	Eq.	Description	Used in Equations
RB	23	Bond rate, percentage points.	12,25,29
$RECG$	105	Net receipts, g, B$.	107
$RECS$	112	Net receipts, s, B$.	114
RM	24	Mortgage rate, percentage points.	128
RMA	128	After tax mortgage rate, percentage points.	2, 3, 4
RNT	165	Rental income, h, B$.	64, 68, 69, 99, 115
$RNTQ$	exog	Rental income, h, B2009$.	165
RS	30	Three-month Treasury bill rate, percentage points.	17, 23, 24, 29, 127
RSA	127	After tax bill rate, percentage points.	1, 26
SB	72	Financial saving, b, B$.	73
SF	69	Financial saving, f, B$.	70
SG	76	Financial saving, g, B$.	77
SGP	107	NIPA surplus (+) or deficit (-), g, B$.	none
SH	65	Saving, h, B$.	66
$SHRPIE$	121	Ratio of after tax profits to the wage bill net of employer social security taxes.	none
$SIFG$	54	Employer social insurance contributions, f to g, B$.	67, 68, 76, 103
$SIFS$	exog	Employer social insurance contributions, f to s, B$.	67, 68, 78, 109
SIG	103	Total employer and employee social insurance contributions to g, B$.	105
$SIGG$	exog	Employer social insurance contributions, g to g, B$.	64, 76, 103, 115, 126
$SIHG$	53	Employee social insurance contributions, h to g, B$.	65, 76, 103, 115
$SIHS$	exog	Employee social insurance contributions, h to s, B$.	65, 78, 109, 115
SIS	109	Total employer and employee social insurance contributions to s, B$.	112
$SISS$	exog	Employer social insurance contributions, s to s, B$.	64, 78, 109, 115, 126
SR	74	Financial saving, r, B$.	75
SRZ	116	Approximate NIPA saving rate, h.	none
SS	78	Financial saving, s, B$.	79
SSP	114	NIPA surplus (+) or deficit (-), s, B$.	none
$STAT$	exog	Statistical discrepancy, B$.	68, 69, 80
$STATP$	exog	Statistical discrepancy relating to the use of chain type price indices, B2009$.	83
$SUBG$	exog	Subsidies less current surplus of government enterprises, g, B$.	68, 69, 76, 106
$SUBS$	exog	Subsidies less current surplus of government enterprises, s, B$.	68, 69, 78, 113
T	exog	1 in 1952:1, 2 in 1952:2, etc.	10, 14, 16
$TAUG$	exog	Progressivity tax parameter in personal income tax equation for g.	47, 90, 99
$TAUS$	exog	Progressivity tax parameter in personal income tax equation for s.	48, 91, 99
TFR	exog	Taxes, f to r, B$.	18, 25, 74, 101
TBG	166	Corporate profit taxes, b to g, B$.	68, 69, 76, 102
$TBGQ$	exog	Corporate profit taxes, b to g, B2009$.	166
TBS	exog	Corporate profit taxes, b to s, B$.	68, 69, 78, 108
TCG	102	Corporate profit tax receipts, g, B$.	105
TCS	108	Corporate profit tax receipts, s, B$.	112
TFA	exog	Corporate profit tax payments, FA, B$.	68, 69, 101
$TF1$	101	Corporate profit tax payments, F1, B$.	69
TFG	49	Corporate profit taxes, f to g, B$.	18, 25, 76, 101, 102
TFS	50	Corporate profit taxes, f to s, B$.	18, 25, 49, 78, 101, 108
$THETA1$	exog	Ratio of PFA to $GDPD$.	111
$THETA2$	exog	Ratio of CDH to $PCD \cdot CD$.	96
$THETA3$	exog	Ratio of $NICD$ to $PCD \cdot CD$.	97
$THETA4$	exog	Ratio of $PIEFRET$ to $PIEF$.	132
THG	47	Personal income taxes, h to g, B$.	65, 76, 101, 115
THS	48	Personal income taxes, h to s, B$.	65, 78, 105, 112, 115

Table A.2 (continued)

Variable	Eq.	Description	Used in Equations
$TRFG$	exog	Transfer payments, f to g, B$.	68, 69, 76, 105
$TRFH$	exog	Transfer payments, f to h, B$.	64, 68, 69, 99, 115
$TRFR$	exog	Transfer payments, f to r, B$.	68, 69, 74
$TRFS$	exog	Transfer payments, f to s, B$.	68, 69, 78, 112
$TRGH$	167	Transfer payments (net), g to h, B$.	65, 76, 99, 106, 115
$TRGHQ$	exog	Transfer payments (net), g to h, B2009$.	167
$TRGR$	exog	Transfer payments (net), g to r, B$.	74, 76, 106
$TRGS$	168	Transfer payments, g to s, B$.	76, 78, 106, 112
$TRGSQ$	exog	Transfer payments, g to s, B2009$.	168
$TRHR$	exog	Transfer payments, h to r, B$.	65, 74, 115
$TRSH$	169	Transfer payments, s to h, excluding unemployment insurance benefits, B$.	65, 78, 99, 111, 115
$TRSHQ$	exog	Transfer payments, s to h, excluding unemployment insurance benefits, B2009$.	169
U	86	Number of people unemployed, millions.	28, 87
UB	28	Unemployment insurance benefits, B$.	65, 78, 99, 111, 115
UBR	128	Unborrowed reserves, B$.	none
UR	87	Civilian unemployment rate.	5, 7, 8, 10, 30
$USAFF$	exog	Contributions for government social insurance, U.S.-affiliated areas, B$.	65, 74, 76, 80, 99
$USOTHER$	exog	Net receipts of factor income from the rest of the world not counting net interest receipts, net dividend receipts, and foreign earnings retained abroad, B$.	57, 68, 69
$USROW$	57	Net receipts of factor income from the rest of the world, B$.	74, 129, 130
V	63	Stock of inventories, f, B2009$.	11, 82, 117
WA	126	After tax wage rate. (Includes supplements to wages and salaries except employer contributions for social insurance.)	6, 7, 8
WF	16	Average hourly earnings excluding overtime of workers in f. (Includes supplements to wages and salaries except employer contributions for social insurance.)	10, 11, 28, 43, 44, 45, 46, 53, 54, 64, 68, 69, 99, 121, 126
WG	44	Average hourly earnings of civilian workers in g. (Includes supplements to wages and salaries including employer contributions for social insurance.)	43, 64, 76, 82, 104, 115, 126
WH	43	Average hourly earnings excluding overtime of all workers. (Includes supplements to wages and salaries except employer contributions for social insurance.)	none
WM	45	Average hourly earnings of military workers. (Includes supplements to wages and salaries including employer contributions for social insurance.)	43, 64, 76, 82, 104, 115, 126
WR	119	Real wage rate of workers in f. (Includes supplements to wages and salaries except employer contributions for social insurance.)	none
WS	46	Average hourly earnings of workers in s. (Includes supplements to wages and salaries including employer contributions for social insurance.)	43, 64, 78, 82, 110, 115, 126
X	60	Total sales, B2009$.	11, 17, 26, 31, 33, 63
XX	61	Total sales, B$.	68, 69, 82
Y	11	Total production, B2009$.	10, 12, 13, 14, 63, 83, 93, 94, 118
YD	115	Disposable income, h, B$.	1, 2, 3, 4, 116
YNL	99	Before tax nonlabor income, h, B$.	none
YS	98	Potential output, B2009$.	12, 25
YT	64	Taxable income, h, B$.	47, 48, 65, 90, 91, 99

- B$ = Billions of dollars.
- B2009$ = Billions of 2009 dollars.

Table A.3
The Equations of the US Model

Eq.	LHS Variable	Explanatory Variables
		STOCHASTIC EQUATIONS
Household Sector		
1	$\log(CS/POP)$	cnst2, cnst, $AG1$, $AG2$, $AG3$, $\log(CS/POP)_{-1}$, $\log[YD/(POP \cdot PH)]$, RSA, $\log(AA/POP)_{-1}$ [Consumer expenditures: services]
2	$\log(CN/POP)$	cnst2, cnst, $AG1$, $AG2$, $AG3$, $\log(CN/POP)_{-1}$, $\Delta\log(CN/POP)_{-1}$, $\log(AA/POP)_{-1}$, $\log[YD/(POP \cdot PH)]$, RMA [Consumer expenditures: nondurables]
3	$\Delta CD/POP$	cnst2, cnst, $AG1$, $AG2$, $AG3$, $DELD(KD/POP)_{-1} - (CD/POP)_{-1}$, $(KD/POP)_{-1}$, $YD/(POP \cdot PH)$, $RMA \cdot CDA$, $(AA/POP)_{-1}$ [Consumer expenditures: durables]
4	$\Delta IHH/POP$	cnst2, cnst, $DELH(KH/POP)_{-1} - (IHH/POP)_{-1}$, $(KH/POP)_{-1}$, $(AA/POP)_{-1}$, $YD/(POP \cdot PH)$, $RMA_{-1}IHHA$, $RHO = 2$ [Residential investment–h]
5	$\log(L1/POP1)$	cnst, $\log(L1/POP1)_{-1}$, $\log(AA/POP)_{-1}$, UR [Labor force–men 25-54]
6	$\log(L2/POP2)$	cnst, $\log(L2/POP2)_{-1}$, $\log(WA/PH)$, $\log(AA/POP)_{-1}$ [Labor force–women 25-54]
7	$\log(L3/POP3)$	cnst, $\log(L3/POP1)_{-1})$, $\log(WA/PH)$, $\log(AA/POP)_{-1}$, UR [Labor force–all others 16+]
8	$\log(LM/POP)$	cnst, $\log(LM/POP)_{-1}$, $\log(WA/PH)$, UR [Number of moonlighters]
Firm Sector		
10	$\log PF$	$\log PF_{-1}$, $\log[WF(1 + D5G)] - \log LAM$, cnst2, TB, cnst, T, $\log PIM$, UR [Price deflator for non farm sales]
11	$\log Y$	cnst, $\log Y_{-1}$, $\log X$, $\log V_{-1}$, $D593$, $D594$, $D601$, $RHO = 3$ [Production–f]
12	$\Delta\log KK$	cnst2, cnst, $\log(KK/KKMIN)_{-1}$, $\Delta\log KK_{-1}$, $\Delta\log Y$, $\Delta\log Y_{-1}$, $\Delta\log Y_{-2}$, $\Delta\log Y_{-3}$, $\Delta\log Y_{-4}$, $\Delta\log Y_{-5}$, $RB_{-2}(1 - D2G_{-2} - D2S_{-2}) - 100(PD_{-2}/PD_{-6}) - 1)$, $(CG_{-2} + CG_{-3} + CG_{-4})/(PX_{-2}YS_{-2} + PX_{-3}YS_{-3} + PX_{-4}YS_{-4})$ [Stock of capital–f]
13	$\Delta\log JF$	cnst, $\log[JF/(JHMIN/HFS)]_{-1}$, $\Delta\log JF_{-1}$, $\Delta\log Y$, $D593$ [Number of jobs–f]
14	$\Delta\log HF$	cnst, $\log(HF/HFS)_{-1}$, $\log[JF/(JHMIN/HFS)]_{-1}$, $\Delta\log Y$, T [Average number of hours paid per job–f]
15	$\log HO$	cnst, HFF, HFF_{-1}, $RHO = 1$ [Average number of overtime hours paid per job–f]
16	$\log WF - \log LAM$	$\log WF_{-1} - \log LAM_{-1}$, $\log PF$, cnst, T, $\log PF_{-1}$ [Average hourly earnings excluding overtime–f]
17	$\log(MF/PF)$	cnst, T, $\log(MF_{-1}/PF)$, $\log(X - FA)$, $RS(1 - D2G - D2S)$ [Demand deposits and currency–f]
18	$\Delta\log DF$	$\log[(PIEF - TFG - TFS - TFR)/DF_{-1}]$ [Dividends paid–f]

Table A.3 (continued)

Eq.	LHS Variable	Explanatory Variables
Financial Sector		
23	$RB - RS_{-2}$	cnst, $RB_{-1} - RS_{-2}$, $RS - RS_{-2}$, $RS_{-1} - RS_{-2}$, $RHO = 1$ [Bond rate]
24	$RM - RS_{-2}$	cnst, $RM_{-1} - RS_{-2}$, $RS - RS_{-2}$, $RS_{-1} - RS_{-2}$ [Mortgage rate]
25	$CG/(PX_{-1} \cdot YS_{-1})$	cnst, ΔRB, $[\Delta(PIEF - TFG - TFS - TFR)]/(PX_{-1} \cdot YS_{-1})$ [Capital gains or losses on the financial assets of h]
26	$\log[CUR/(POP \cdot PF)]$	cnst, $\log[CUR_{-1}/(POP_{-1}PF)]$, $\log[(X - FA)/POP]$, RSA, $RHO = 1$ [Currency held outside banks]
Import Equation		
27	$\log(IM/POP)$	cnst2, cnst, $\log(IM/POP)_{-1}$, $\log[(CS + CN + CD + IHH + IKF + IHB + IHF + IKB + IKH)/POP]$, $\log(PF/PIM)$, $D691$, $D692$, $D714$, $D721$ [Imports]
Government Sectors		
28	$\log UB$	cnst, $\log UB_{-1}$, $\log U$, $\log WF$, $RHO = 1$ [Unemployment insurance benefits]
29	$INTG/(-AG)$	cnst, $[INTG/(-AG)]_{-1}$, $(1/400)[.4RS + .75(.6)(1/8)(RB + RB_{-1} + RB_{-2} + RB_{-3} + RB_{-4} + RB_{-5} + RB_{-6} + RB_{-7})]$, $RHO = 1$
30	RS	cnst, RS_{-1}, $100[(PD/PD_{-1})^4 - 1]$, UR, ΔUR, $PCM1_{-1}$, $D794823 \cdot PCM1_{-1}$, ΔRS_{-1}, ΔRS_{-2} [Three-month Treasury bill rate]

Table A.3 (continued)

Eq.	LHS Variable	Explanatory Variables
		IDENTITIES
31	$PX =$	$[PF(X - FA) + PFA \cdot FA]/X$ [Price deflator for total sales]
32	$PEX =$	$PSI1 \cdot PX$ [Price deflator for EX]
33	$PD =$	$(PX \cdot X - PEX \cdot EX + PIM \cdot IM)/(X - EX + IM)$ [Price deflator for domestic sales]
34	$PH =$	$(PCS \cdot CS + PCN \cdot CN + PCD \cdot CD + PIH \cdot IHH + IBTG + IBTS)/(CS + CN + CD + IHH)$ [Price deflator for (CS + CN + CD + IHH) inclusive of indirect business taxes]
35	$PCS =$	$PSI2(1 + D3G + D3S)PD$ [Price deflator for CS]
36	$PCN =$	$PSI3(1 + D3G + D3S)PD$ [Price deflator for CN]
37	$PCD =$	$PSI4(1 + D3G + D3S)PD$ [Price deflator for CD]
38	$PIH =$	$PSI5 \cdot PD$ [Price deflator for residential investment]
39	$PIK =$	$PSI6 \cdot PD$ [Price deflator for nonresidential fixed investment]
40	$PG =$	$PSI7 \cdot PD$ [Price deflator for COG]
41	$PS =$	$PSI8 \cdot PD$ [Price deflator for COS]
42	$PIV =$	$PSI9 \cdot PD$ [Price deflator for inventory investment]
43	$WH =$	$100[(WF \cdot JF(HN + 1.5HO) + WG \cdot JG \cdot HG + WM \cdot JM \cdot HM + WS \cdot JS \cdot HS)/(JF(HN + 1.5HO) + JG \cdot HG + JM \cdot HM + JS \cdot HS)]$ [Average hourly earnings excluding overtime of all workers]
44	$WG =$	$PSI10 \cdot WF$ [Average hourly earnings of civilian workers–g]
45	$WM =$	$PSI11 \cdot WF$ [Average hourly earnings of military workers]
46	$WS =$	$PSI12 \cdot WF$ [Average hourly earnings of workers–s]
47	$THG =$	$[D1G + ((TAUG \cdot YT)/(POP \cdot PH))]YT$ [Personal income taxes–h to g]
48	$THS =$	$[D1S + ((TAUS \cdot YT)/(POP \cdot PH))]YT$ [Personal income taxes–h to s]
49	$TFG =$	$D2G(PIEF - TFS)$ [Corporate profits taxes–f to g]
50	$TFS =$	$D2S \cdot PIEF$ [Corporate profits taxes–f to s]
51	$IBTG =$	$[D3G/(1 + D3G)](PCS \cdot CS + PCN \cdot CN + PCD \cdot CD - IBTS)$ [Indirect business taxes–g]
52	$IBTS =$	$[D3S/(1 + D3S)](PCS \cdot CS + PCN \cdot CN + PCD \cdot CD - IBTG)$ [Indirect business taxes–s]
53	$SIHG =$	$D4G[WF \cdot JF(HN + 1.5HO)]$ [Employee social insurance contributions–h to g]
54	$SIFG =$	$D5G[WF \cdot JF(HN + 1.5HO)]$ [Employer social insurance contributions–f to g]

Table A.3 (continued)

Eq.	LHS Variable	Explanatory Variables
55	$PKH =$	$PSI14 \cdot PD$ [Market price of KH]
56	$INTGR =$	$PSI15 \cdot INTG$ [Net interest payments, g to r]
57	$USROW =$	$-INTGR + DR + PIEFRET + USOTHER$ [Net receipts of factor income from the rest of the world]
58	$KD =$	$(1 - DELD)KD_{-1} + CD$ [Stock of durable goods]
59	$KH =$	$(1 - DELH)KH_{-1} + IHH$ [Stock of housing–h]
60	$X =$	$CS + CN + CD + IHH + IKF + EX - IM + COG + COS + IKH + IKB + IKG + IHF + IHB$ [Total real sales]
61	$XX =$	$PCS \cdot CS + PCN \cdot CN + PCD \cdot CD + PIH \cdot IHH + PIK \cdot IKF + PEX \cdot EX - PIM \cdot IM + PG \cdot COG + PS \cdot COS + PIK(IKH + IKB + IKG) + PIH(IHF + IHB) - IBTG - IBTS$ [Total nominal sales]
62	$HN =$	$HF - HO$ [Average number of non overtime hours paid per job–f]
63	$V =$	$V_{-1} + Y - X$ [Stock of inventories–f]
64	$YT =$	$WF \cdot JF(HN + 1.5HO) + WG \cdot JG \cdot HG + WM \cdot JM \cdot HM + WS \cdot JS \cdot HS + RNT + INTZ + INTF + INTG - INTGR + INTS + DF + DB + DR + DG + DS + TRFH - TRHR - SIGG - SISS$ [Taxable income–h]
65	$SH =$	$YT - SIHG - SIHS + USAFF - THG - THS - PCS \cdot CS - PCN \cdot CN - PCD \cdot CD + TRGH + TRSH + UB + INS + NICD + CCH - CTH - PIH \cdot IHH - CDH - PIK \cdot IKH - NNH$ [Financial saving–h]
66	$0 =$	$SH - \Delta AH - \Delta MH + CG - DISH$ [Budget constraint–h; (determines AH)]
67	$CCF1 =$	$D6G(PIK \cdot IKF + PIK_{-1} \cdot IKF_{-1} + PIK_{-2} \cdot IKF_{-2} + PIK_{-3} \cdot IKF_{-3})/4$ [Capital consumption, F1]
68	$PIEF =$	$XX + PIV \cdot IVF + SUBS + SUBG + USOTHER - WF \cdot JF(HN + 1.5HO) - RNT - INTZ - INTF - TRFH - NICD - CCH + CDH - TBS - TRFS - CCS - TRFR - DB - GSB - CTGB - GSMA - GSCA - TBG - TRFG - CCG - SIFG - SIFS - GSNN - IVA - CCF1 - TFA - STAT$ [Before tax profits–f]
69	$SF =$	$XX + SUBS + SUBG + PIEFRET + USOTHER - WF \cdot JF(HN + 1.5HO) - RNT - INTZ - INTF - TRFH - NICD - CCH + CDH - TBS - TRFS - CCS - TRFR - DB - GSB - CTGB - GSMA - GSCA - TBG - TRFG - CCG - SIFG - SIFS - STAT - DF - TF1 - TFA - PIK \cdot IKF - PIH \cdot IHF - NNF - CTF1 - CTNN$ [Financial saving–f]
70	$0 =$	$SF - \Delta AF - \Delta MF - DISF$ [Budget constraint–f; (determines AF)]

Table A.3 (continued)

Eq.	LHS Variable	Explanatory Variables
71	$0 =$	$\Delta MB + \Delta MH + \Delta MF + \Delta MR + \Delta MG + \Delta MS - \Delta CUR$ [Demand deposit identity; (determines MB)]
72	$SB =$	$GSB - CTB - PIH \cdot IHB - PIK \cdot IKB$ [Financial saving–b]
73	$0 =$	$SB - \Delta AB - \Delta MB - \Delta(BR - BO) - DISB$ [Budget constraint–b; (determines AB)]
74	$SR =$	$-PEX \cdot EX - USROW + PIM \cdot IM + TFR + TRFR + TRHR + TRGR - USAFF - CTR - NNR$ [Financial saving–r]
75	$0 =$	$SR - \Delta AR - \Delta MR + \Delta Q - DISR$ [Budget constraint–r; (determines AR)]
76	$SG =$	$GSMA + GSCA + THG + IBTG + TBG + TFG + SIHG + SIFG - DG + TRFG - PG \cdot COG - WG \cdot JG \cdot HG - WM \cdot JM \cdot HM - TRGH - TRGR - TRGS - INTG - SUBG + CCG - INS - USAFF - CTGMB - NNG - PIK \cdot IKG + SIGG$ [Financial saving–g]
77	$0 =$	$SG - \Delta AG - \Delta MG + \Delta CUR + \Delta(BR - BO) - \Delta Q - DISG$ [Budget constraint–g; (determines AG unless AG is exogenous)]
78	$SS =$	$THS + IBTS + TBS + TFS + SIHS + SIFS - DS + TRGS + TRFS - PS \cdot COS - WS \cdot JS \cdot HS - TRSH - UB - INTS - SUBS + CCS - CTS - NNS + SISS$ [Financial saving–s]
79	$0 =$	$SS - \Delta AS - \Delta MS - DISS$ [Budget constraint–s; (determines AS)]
80	$0 =$	$SH + SF + SB + SR + SG + SS + STAT + USAFF$ [Redundant equation—for checking]
81	$M1 =$	$M1_{-1} + \Delta MH + \Delta MF + \Delta MR + \Delta MS + MDIF$ [Money supply]
82	$GDP =$	$XX + PIV(V - V_{-1}) + IBTG + IBTS + WG \cdot JG \cdot HG + WM \cdot JM \cdot HM + WS \cdot JS \cdot HS$ [Nominal GDP]
83	$GDPR =$	$Y + PSI13(JG \cdot HG + JM \cdot HM + JS \cdot HS) + STATP$ [Real GDP]
84	$GDPD =$	$GDP/GDPR$ [GDP price deflator]
85	$E =$	$JF + JG + JM + JS - LM$ [Total employment, civilian and military]
86	$U =$	$L1 + L2 + L3 - E$ [Number of people unemployed]
87	$UR =$	$U/(L1 + L2 + L3 - JM)$ [Civilian unemployment rate]
89	$AA =$	$(AH + MH)/PH + (PKH \cdot KH)/PH$ [Total net wealth–h]
90	$D1GM =$	$D1G + (2TAUG \cdot YT)/(POP \cdot PH)$ [Marginal personal income tax rate–g]
91	$D1SM =$	$D1S + (2TAUS \cdot YT)/(POP \cdot PH)$ [Marginal personal income tax rate–s]
92	$IKF =$	$KK + (1 - DELK)KK_{-1}$ [Nonresidential fixed investment–f]
93	$KKMIN =$	Y/MUH [Amount of capital required to produce Y]
94	$JHMIN =$	Y/LAM [Number of worker hours required to produce Y]
95	$JJ =$	$(JF \cdot HF + JG \cdot HG + JM \cdot HM + JS \cdot HS)/POP$ [Ratio of the total number of worker hours paid for to the total population 16 and over]

Table A.3 (continued)

Eq.	LHS Variable	Explanatory Variables
96	$CDH =$	$THETA2 \cdot PCD \cdot CD$ [Capital expenditures, consumer durable goods, h]
97	$NICD =$	$THETA3 \cdot PCD \cdot CD$ [Net investment in consumer durables, h]
98	$YS =$	$LAM(JJP \cdot POP - JG \cdot HG - JM \cdot HM - JS \cdot HS)$ [Potential output]
99	$YNL =$	$RNT + INTZ + INTF + INTG - INTGR + INTS + DF + DB + DR +$ $DG + DS + TRFH + TRGH + TRSH + UB$ [Before-tax nonlabor income–h]
100	$HFF =$	$HF - HFS$ [Deviation of HF from its peak to peak interpolation]
101	$TF1 =$	$TFG + TFS + TFR - TFA$ [Corporate profit tax payments, F1]
102	$TCG =$	$TFG + TBG$ [Corporate profit tax receipts–g]
103	$SIG =$	$SIHG + SIFG + SIGG$ [Total social insurance contributions to g]
104	$PUG =$	$PG \cdot COG + WG \cdot JG \cdot HG + WM \cdot JM \cdot HM$ [Purchases of goods and services–g]
105	$RECG =$	$THG + TCG + IBTG + SIG + TRFG - DG$ [Net receipts–g]
106	$EXPG =$	$PUG + TRGH + TRGR + TRGS + INTG + SUBG - IGZ$ [Net expenditures–g]
107	$SGP =$	$RECG - EXPG$ [NIPA surplus or deficit–g]
108	$TCS =$	$TFS + TBS$ [Corporate profit tax receipts–s]
109	$SIS =$	$SIHS + SIFS + SISS$ [Total social insurance contributions to s]
110	$PUS =$	$PS \cdot COS + WS \cdot JS \cdot HS$ [Purchases of goods and services–s]
111	$PFA =$	$THETA1 \cdot GDPD$ [Price deflator for farm sales]
112	$RECS =$	$THS + TCS + IBTS + SIS + TRGS + TRFS - DS$ [Net receipts–s]
113	$EXPS =$	$PUS + TRSH + UB + INTS + SUBS - ISZ$ [Net expenditures–s]
114	$SSP =$	$RECS - EXPS$ [NIPA surplus or deficit–s]
115	$YD =$	$WF \cdot JF(HN + 1.5HO) + WG \cdot JG \cdot HG + WM \cdot JM \cdot HM + WS \cdot JS \cdot HS +$ $RNT + INTZ + INTF + INTG - INTGR + INTS + DF + DB + DR +$ $DG + DS + TRFH + TRGH + TRSH + UB - SIHG - SIHS + USAFF -$ $THG - THS - TRHR - SIGG - SISS$ [Disposable income–h]
116	$SRZ =$	$(YD - PCS \cdot CS - PCN \cdot CN - PCD \cdot CD)/YD$ [Approximate NIPA saving rate–h]
117	$IVF =$	$V - V_{-1}$ [Inventory investment–f]
118	$PROD =$	$Y/(JF \cdot HF)$ [Output per paid for worker hour:“productivity"]
119	$WR =$	WF/PF [Real wage rate of workers in f]
120	POP	$= POP1 + POP2 + POP3$ [Noninstitutional population 16 and over]

Table A.3 (continued)

Eq.	LHS Variable	Explanatory Variables
121	$SHRPIE =$	$[(1 - D2G - D2S)PIEF]/[WF \cdot JF(HN + 1.5HO)]$ [Ratio of after tax profits to the wage bill net of employer social security taxes]
122	$PCGDPR =$	$100[(GDPR/GDPR_{-1})^4 - 1]$ [Percentage change in GDPR]
123	$PCGDPD =$	$100[(GDPD/GDPD_{-1})^4 - 1]$ [Percentage change in GDPD]
124	$PCM1 =$	$100[(M1/M1_{-1})^4 - 1]$ [Percentage change in M1]
125	$UBR =$	$BR - BO$ [Unborrowed reserves]
126	$WA =$	$100[(1 - D1GM - D1SM - D4G)[WF \cdot JF(HN + 1.5HO)] + (1 - D1GM - D1SM)(WG \cdot JG \cdot HG + WM \cdot JM \cdot HM + WS \cdot JS \cdot HS - SIGG - SISS)]/[JF(HN + 1.5HO) + JG \cdot HG + JM \cdot HM + JS \cdot HS]$ [After tax wage rate]
127	$RSA =$	$RS(1 - D1GM - D1SM)$ [After-tax three-month Treasury bill rate]
128	$RMA =$	$RM(1 - D1GM - D1SM)$ [After-tax mortgage rate]
129	$GNP =$	$GDP + USROW$ [Nominal GNP]
130	$GNPR =$	$GDPR + USROW/GDPD$ [Real GNP]
131	$GNPD =$	$GNP/GNPR$ [GNP price deflator]
132	$PIEFRET =$	$THETA4 \cdot PIEF$ [Foreign earnings retained abroad—f]
Nominal Variables		
150	$CCG =$	$GDPD \cdot CCGQ$
151	$CCH =$	$GDPD \cdot CCHQ$
152	$CCS =$	$GDPD \cdot CCSQ$
153	$DB =$	$GDPD \cdot DBQ$
154	$DR =$	$GDPD \cdot DRQ$
155	$GSB =$	$GDPD \cdot GSBQ$
156	$GSNN =$	$GDPD \cdot GSNNQ$
157	$IGZ =$	$GDPD \cdot IGZQ$
158	$INTZ =$	$GDPD \cdot INTZQ$
159	$ISZ =$	$GDPD \cdot ISZQ$
160	$MG =$	$GDPD \cdot MGQ$
161	$MH =$	$GDPD \cdot MHQ$
162	$MR =$	$GDPD \cdot MRQ$
163	$MS =$	$GDPD \cdot MSQ$
164	$Q =$	$GDPD \cdot QQ$
165	$RNT =$	$GDPD \cdot RNTQ$
166	$TBG =$	$GDPD \cdot TBGQ$
167	$TRGH =$	$GDPD \cdot TRGHQ$
168	$TRGS =$	$GDPD \cdot TRGSQ$
169	$TRSH =$	$GDPD \cdot TRSHQ$
Variables as a percent of GDP		
180	$RECGZGDP =$	$RECG/GDP$
181	$EXPGZGDP =$	$EXPG/GDP$
182	$SGPZGDP =$	$-SGP/GDP$
183	$AGZGDP =$	$-AG/(4 \cdot GDP)$
184	$INTGZGDP =$	$INTG/GDP$
185	$SRZGDP =$	SR/GDP
186	$ASZGDP =$	$-AS/(4 \cdot GDP)$
187	$PCGDPR4 =$	$100 \cdot (GDPR/GDPR_{-4} - 1)$
188	$PCGDPD4 =$	$100 \cdot (GDPD/GDPD_{-4} - 1)$

Table A.4
Coefficient Estimates and Test Results
for the US Equations

See Chapter 1 in Fair (2004) for discussion of the tests.
See Chapter 2 in Fair (2004) for discussion of the equations.

Table A1
Equation 1
LHS Variable is $\log(CS/POP)$

Equation			χ^2 Tests			
RHS Variable	Coef.	t-stat.	Test	χ^2	df	p-value
cnst2	0.02245	6.60	Lags	19.83	4	0.0005
cnst	-0.13399	-5.86	RHO	18.52	4	0.0010
$AG1$	-0.05210	-2.10	Leads +1	8.06	1	0.0045
$AG2$	-0.28777	-8.43	Leads +4	16.04	4	0.0030
$AG3$	0.24734	4.02	Leads +8	14.76	2	0.0006
$\log(CS/POP)_{-1}$	0.81753	35.22				
$\log[YD/(POP \cdot PH)]$	0.12022	5.11				
RSA	-0.00115	-5.08				
$\log(AA/POP)_{-1}$	0.03787	6.66				

SE	0.00373
R^2	1.000
DW	1.51

overid (df = 15, p-value =0.0000)
χ^2 (AGE) = 104.02 (df = 3, p-value = 0.0000)

Stability Test					End Test	
AP	T_1	T_2	λ	Break	p-value	End
14.66	1970.1	1979.4	2.07	1974.2	1.0000	1995.1
11.67	1975.1	1984.4	1.96	1975.4		
12.23	1980.1	1989.4	1.93	1989.1		

Estimation period is 1954.1-2013.3

Table A2
Equation 2
LHS Variable is $\log(CN/POP)$

Equation			χ^2 Tests			
RHS Variable	Coef.	t-stat.	Test	χ^2	df	p-value
cnst2	-0.01467	-1.87	Lags	28.86	4	0.0000
cnst	-0.34113	-5.04	RHO	34.61	4	0.0000
$AG1$	0.12415	2.45	T	0.37	1	0.5415
$AG2$	0.12436	2.09	Leads +1	12.40	1	0.0004
$AG3$	-0.30985	-2.61	Leads +4	12.93	4	0.0116
$\log(CN/POP)_{-1}$	0.74046	16.95	Leads +8	8.07	2	0.0177
$\Delta \log(CN/POP)_{-1}$	0.21435	3.59				
$\log(AA/POP)_{-1}$	0.04795	4.42				
$\log[YD/(POP \cdot PH)]$	0.11918	3.84				
RMA	-0.00092	-1.78				

SE	0.00658
R^2	0.999
DW	1.95

overid (df = 14, p-value =0.0000)
χ^2 (AGE) = 9.04 (df = 3, p-value = 0.0288)

Stability Test					End Test	
AP	T_1	T_2	λ	Break	p-value	End
18.26	1970.1	1979.4	2.07	1976.1	0.9778	1995.1
18.23	1975.1	1984.4	1.96	1976.1		
11.18	1980.1	1989.4	1.93	1983.2		

Estimation period is 1954.1-2013.3

Table A3
Equation 3
LHS Variable is $CD/POP - (CD/POP)_{-1}$

Equation			χ^2 Tests			
RHS Variable	Coef.	t-stat.	Test	χ^2	df	p-value
cnst2	0.06223	3.97	Lags	6.39	4	0.1720
cnst	-0.24766	-3.46	RHO	10.11	4	0.0386
$AG1$	0.18166	1.60	T	11.37	1	0.0007
$AG2$	2.63772	6.27	Leads +1	3.77	1	0.0522
$AG3$	-2.34685	-5.50	Leads +4	16.93	4	0.0020
^{a}	0.23167	5.10	Leads +8	9.21	2	0.0100
$(KD/POP)_{-1}$	-0.02770	-6.79				
$YD/(POP \cdot PH)$	0.06389	6.21				
$RMA \cdot CDA$	-0.01005	-3.96				
$(AA/POP)_{-1}$	0.00063	3.85				

SE	0.01455
R^2	0.207
DW	1.95

overid (df = 10, p-value =0.0000)
χ^2 (AGE) = 94.01 (df = 3, p-value = 0.0000)

Stability Test					End Test	
AP	T_1	T_2	λ	Break	p-value	End
10.68	1970.1	1979.4	2.07	1974.2	0.0111	1995.1
20.92	1975.1	1984.4	1.96	1983.2		
21.69	1980.1	1989.4	1.93	1983.2		

Estimation period is 1954.1-2013.3
^{a}Variable is $DELD(KD/POP)_{-1} - (CD/POP)_{-1}$

Table A4
Equation 4
LHS Variable is $IHH/POP - (IHH/POP)_{-1}$

Equation			χ² Tests			
RHS Variable	Coef.	t-stat.	Test	χ^2	df	p-value
cnst2	0.14633	1.77	Lags	1.61	3	0.6567
cnst	0.58974	2.57	RHO	1.38	2	0.5005
^{a}	0.29966	6.19	T	0.85	1	0.3553
$(KH/POP)_{-1}$	-0.02042	-2.26	Leads +1	0.94	1	0.3324
$YD/(POP \cdot PH)$	0.04392	1.28	Leads +4	3.10	4	0.5418
$RMA_{-1} \cdot IHHA$	-0.02546	-5.34	Leads +8	1.48	2	0.4773
RHO1	0.62310	8.70				
RHO2	0.33274	4.85				

SE	0.01568
R^2	0.424
DW	2.01

overid (df = 19, p-value =0.0055)
χ^2 (AGE) = 5.55 (df = 3, p-value = 0.1356)

Stability Test					End Test	
AP	T_1	T_2	λ	Break	p-value	End
7.61	1970.1	1979.4	2.07	1971.1	0.2444	1995.1
14.32	1975.1	1984.4	1.96	1984.4		
15.02	1980.1	1989.4	1.93	1984.4		

Estimation period is 1954.1-2013.3
^{a}Variable is $DELH(KH/POP)_{-1} - (IHH/POP)_{-1}$

Table A5
Equation 5
LHS Variable is $\log(L1/POP1)$

Equation			χ² Tests			
RHS Variable	Coef.	t-stat.	Test	χ^2	df	p-value
cnst	0.01463	2.62	Lags	6.95	3	0.0734
$\log(L1/POP1)_{-1}$	0.93797	38.33	RHO	60.86	4	0.0000
$\log(AA/POP)_{-1}$	-0.00367	-2.66	T	11.92	1	0.0006
UR	-0.02390	-1.82				

SE	0.00246
R^2	0.992
DW	2.20

overid (df = 10, p-value =0.0000)

Stability Test					End Test	
AP	T_1	T_2	λ	Break	p-value	End
5.10	1970.1	1979.4	2.07	1970.1	0.0000	1995.1
2.73	1975.1	1984.4	1.96	1984.4		
4.09	1980.1	1989.4	1.93	1989.4		

Estimation period is 1954.1-2013.3

Table A6
Equation 6
LHS Variable is $\log(L2/POP2)$

Equation			χ^2 Tests			
RHS Variable	Coef.	t-stat.	Test	χ^2	df	p-value
cnst	0.00844	0.37	Lags	6.96	3	0.0731
$\log(L2/POP2)_{-1}$	0.99123	153.90	RHO	6.71	4	0.1518
$\log(WA/PH)$	0.00585	0.82	T	5.11	1	0.0238
$\log(AA/POP)_{-1}$	-0.00260	-0.56	Leads +1	0.17	1	0.6783
			Leads +4	1.91	4	0.7515
			Leads +8	0.13	2	0.9378
			log PH	3.42	1	0.0644

SE	0.00565
R^2	0.999
DW	1.94

overid (df = 0, p-value =9.9000)

Stability Test					End Test	
AP	T_1	T_2	λ	Break	p-value	End
13.26	1970.1	1979.4	2.07	1973.1	1.0000	1995.1
10.68	1975.1	1984.4	1.96	1976.1		
9.32	1980.1	1989.4	1.93	1980.2		

Estimation period is 1954.1-2013.3

Table A7
Equation 7
LHS Variable is $\log(L3/POP3)$

Equation			χ^2 Tests			
RHS Variable	Coef.	t-stat.	Test	χ^2	df	p-value
cnst	0.04514	2.44	Lags	7.11	4	0.1301
$\log(L3/POP3)_{-1}$	0.96534	64.06	RHO	6.02	4	0.1974
$\log(WA/PH)$	0.02998	2.80	T	0.25	1	0.6155
$\log(AA/POP)_{-1}$	-0.01575	-2.77	Leads +1	0.05	1	0.8263
UR	-0.13446	-4.22	Leads +4	2.99	4	0.5602
			Leads +8	3.53	2	0.1708
			log PH	0.05	1	0.8224

SE	0.00536
R^2	0.986
DW	2.12

overid (df = 9, p-value =0.0641)

Stability Test					End Test	
AP	T_1	T_2	λ	Break	p-value	End
5.19	1970.1	1979.4	2.07	1971.3	0.4111	1995.1
3.88	1975.1	1984.4	1.96	1979.2		
9.27	1980.1	1989.4	1.93	1989.4		

Estimation period is 1954.1-2013.3

Table A8
Equation 8
LHS Variable is $\log(LM/POP)$

Equation			χ^2 Tests			
RHS Variable	Coef.	t-stat.	Test	χ^2	df	p-value
cnst	-0.35537	-5.15	Lags	4.46	3	0.2163
$\log(LM/POP)_{-1}$	0.86570	35.95	RHO	6.25	4	0.1812
$\log(WA/PH)$	0.02602	1.73	T	6.14	1	0.0132
UR	-1.70108	-5.65	Leads +1	0.00	1	0.9940
			Leads +4	3.94	4	0.4146
			Leads +8	0.65	2	0.7212
			$\log PH$	6.83	1	0.0090

SE	0.04634
R^2	0.940
DW	2.04

overid (df = 16, p-value =0.3708)

Stability Test					End Test	
AP	T_1	T_2	λ	Break	p-value	End
7.43	1970.1	1979.4	2.07	1978.1	1.0000	1995.1
7.55	1975.1	1984.4	1.96	1978.1		
8.16	1980.1	1989.4	1.93	1989.4		

Estimation period is 1954.1-2013.3

Table A10
Equation 10
LHS Variable is $\log PF$

Equation			χ^2 Tests			
RHS Variable	Coef.	t-stat.	Test	χ^2	df	p-value
$\log PF_{-1}$	0.90523	76.32	Lags	2.15	4	0.7091
^{a}	0.04809	4.39	RHO	3.11	4	0.5389
cnst2	0.00232	0.38	Leads +1	0.92	1	0.3365
TB	-0.00013	-6.61	Leads +4	2.65	4	0.6188
cnst	0.00606	0.35	Leads +8	2.19	2	0.3351
T	0.00025	6.63	^{b}	0.37	1	0.5423
$\log PIM$	0.03920	13.79	$(YS-Y)/YS$	0.25	1	0.6149
UR	-0.17682	-9.00				

SE	0.00360
R^2	1.000
DW	1.82

overid (df = 8, p-value =0.2801)

Stability Test					End Test	
AP	T_1	T_2	λ	Break	p-value	End
14.09	1970.1	1979.4	2.07	1978.2	1.0000	1995.1
13.96	1975.1	1984.4	1.96	1978.2		
11.12	1980.1	1989.4	1.93	1980.1		

Estimation period is 1954.1-2013.3
^{a}Variable is $\log[WF(1+D5G)] - \log LAM$
^{b}Variable is $\log[(YS-Y)/YS + .04]$

Table A11
Equation 11
LHS Variable is $\log Y$

RHS Variable	Equation Coef.	t-stat.	Test	χ^2 Tests χ^2	df	p-value
cnst	0.32142	3.58	Lags	1.43	2	0.4894
$\log Y_{-1}$	0.35923	8.67	RHO	2.62	1	0.1057
$\log X$	0.80086	18.15	T	0.66	1	0.4161
$\log V_{-1}$	-0.21273	-8.92	Leads +1	1.33	1	0.2494
$D593$	-0.00989	-2.86	Leads +4	2.62	4	0.6225
$D594$	-0.00404	-1.20	Leads +8	1.28	2	0.5263
$D601$	0.00838	2.44				
RHO1	0.40272	5.59				
RHO2	0.39425	5.94				
RHO3	0.15604	2.27				

SE	0.00378
R^2	1.000
DW	2.05

overid (df = 20, p-value =0.0157)

Stability Test AP	T_1	T_2	λ	Break	End Test p-value	End
10.68	1970.1	1979.4	2.07	1970.1	1.0000	1995.1
10.26	1975.1	1984.4	1.96	1982.4		
9.73	1980.1	1989.4	1.93	1982.4		

Estimation period is 1954.1-2013.3

Table A12
Equation 12
LHS Variable is $\Delta \log KK$

RHS Variable	Coef.	t-stat.	Test	χ^2	df	p-value
cnst2	-0.00037	-3.18	Lags	12.86	5	0.0248
cnst	0.00061	2.81	RHO	4.65	4	0.3250
$\log(KK/KKMIN)_{-1}$	-0.00696	-2.77	T	3.04	1	0.0812
$\Delta \log KK_{-1}$	0.90544	57.58	Leads +1	0.01	1	0.9390
$\Delta \log Y$	0.01277	1.43	Leads +4	1.87	4	0.7589
$\Delta \log Y_{-1}$	0.01117	2.35	Leads +8	1.21	2	0.5459
$\Delta \log Y_{-2}$	0.00399	0.94				
$\Delta \log Y_{-3}$	0.00403	1.01				
$\Delta \log Y_{-4}$	0.00656	1.67				
$RBA_{-2} - p^e_{4-2}$	-0.00001	-0.29				
[a]	0.00080	3.98				

SE 0.00044
R^2 0.978
DW 1.76

overid (df = 9, p-value =0.0307)

AP	T_1	T_2	λ	Break	p-value	End
8.34	1970.1	1979.4	2.07	1974.4	0.6222	1995.1
13.13	1975.1	1984.4	1.96	1982.3		
13.34	1980.1	1989.4	1.93	1982.3		

Estimation period is 1954.1-2013.3
[a] Variable is $(CG_{-2} + CG_{-3} + CG_{-4})/(PX_{-2}YS_{-2} + PX_{-3}YS_{-3} + PX_{-4}YS_{-4})$

Table A13
Equation 13
LHS Variable is $\Delta \log JF$

RHS Variable	Coef.	t-stat.	Test	χ^2	df	p-value
cnst	0.00022	0.41	Lags	13.21	3	0.0042
$\log JF/(JHMIN/HFS)_{-1}$	-0.03622	-3.09	RHO	15.92	4	0.0031
$\Delta \log JF_{-1}$	0.60563	15.50	T	0.04	1	0.8513
$\Delta \log Y$	0.26492	6.18	Leads +1	5.38	1	0.0204
$D593$	-0.01810	-5.21	Leads +4	10.60	4	0.0314
			Leads +8	2.37	2	0.3051

SE 0.00336
R^2 0.718
DW 2.17

overid (df = 17, p-value =0.0130)

AP	T_1	T_2	λ	Break	p-value	End
13.12	1970.1	1979.4	2.07	1978.1	1.0000	1995.1
13.06	1975.1	1984.4	1.96	1978.1		
8.83	1980.1	1989.4	1.93	1980.1		

Estimation period is 1954.1-2013.3

Table A14
Equation 14
LHS Variable is $\Delta \log HF$

Equation RHS Variable	Coef.	t-stat.	χ^2 Tests Test	χ^2	df	p-value
cnst	-0.00390	-5.68	Lags	16.12	3	0.0011
$\log(HF/HFS)_{-1}$	-0.17379	-5.85	RHO	13.18	4	0.0104
$\log JF/(JHMIN/HFS)_{-1}$	-0.02673	-2.73	Leads +1	1.47	1	0.2260
$\Delta \log Y$	0.18694	4.45	Leads +4	1.49	4	0.8290
T	0.00001	4.38	Leads +8	4.56	2	0.1021

SE	0.00266
R^2	0.371
DW	2.01

overid (df = 6, p-value =0.0041)

Stability Test AP	T_1	T_2	λ	Break	End Test p-value	End
12.33	1970.1	1979.4	2.07	1978.2	1.0000	1995.1
12.02	1975.1	1984.4	1.96	1978.2		
8.88	1980.1	1989.4	1.93	1980.3		

Estimation period is 1954.1-2013.3

Table A15
Equation 15
LHS Variable is $\log HO$

Equation RHS Variable	Coef.	t-stat.	χ^2 Tests Test	χ^2	df	p-value
cnst	3.93516	37.73	Lags	8.11	2	0.0174
HFF	0.01768	7.97	RHO	8.35	3	0.0394
HFF_{-1}	0.00816	3.68	T	4.59	1	0.0321
RHO1	0.96920	57.39				

SE	0.04715
R^2	0.957
DW	1.65

Stability Test AP	T_1	T_2	λ	Break	End Test p-value	End
2.17	1970.1	1979.4	2.18	1975.2	1.0000	1995.1
5.23	1975.1	1984.4	2.02	1983.1		
6.00	1980.1	1989.4	1.98	1985.3		

Estimation period is 1956.1-2013.3

Table A16
Equation 16
LHS Variable is $\log WF - \log LAM$

Equation			χ² Tests			
RHS Variable	Coef.	t-stat.	Test	χ^2	df	p-value
$\log WF_{-1} - \log LAM_{-1}$	0.93589	52.28	[b]RealWageRes.	0.54	1	0.4644
$\log PF$	0.76647	10.48	Lags	0.31	1	0.5772
cnst	-0.05118	-3.84	RHO	0.97	4	0.9147
T	0.00005	2.02	UR	4.65	1	0.0310
[a] $\log PF_{-1}$	-0.71382	0.00				

SE	0.00769
R^2	0.968
DW	1.91

overid (df = 13, p-value =0.0260)

Stability Test					End Test	
AP	T_1	T_2	λ	Break	p-value	End
2.42	1970.1	1979.4	2.07	1970.1	0.0000	1995.1
2.07	1975.1	1984.4	1.96	1979.2		
2.00	1980.1	1989.4	1.93	1981.1		

Estimation period is 1954.1-2013.3
[a]Coefficient constrained. See the discussion in the text.
[b]Equation estimated with no restrictions on the coefficients.

Table A17
Equation 17
LHS Variable is $\log(MF/PF)$

Equation			χ² Tests			
RHS Variable	Coef.	t-stat.	Test	χ^2	df	p-value
cnst	0.23255	2.72	$\log(MF/PF)_{-1}$	0.75	1	0.3860
$\log(MF_{-1}/PF)$	0.93532	46.62	Lags	7.04	3	0.0708
$\log(X - FA)$	0.02553	3.33	RHO	8.23	4	0.0835
[a]	-0.00573	-3.16	T	2.27	1	0.1320

SE	0.03968
R^2	0.978
DW	1.68

overid (df = 14, p-value =0.0192)

Stability Test					End Test	
AP	T_1	T_2	λ	Break	p-value	End
2.16	1970.1	1979.4	2.07	1975.2	0.0000	1995.1
3.38	1975.1	1984.4	1.96	1984.4		
5.65	1980.1	1989.4	1.93	1986.1		

Estimation period is 1954.1-2013.3
[a]Variable is $[RS(1 - D2G - D2S)]$

Table A18
Equation 18
LHS Variable is $\Delta \log DF$

Equation			χ^2 Tests			
RHS Variable	Coef.	t-stat.	Test	χ^2	df	p-value
^{a}	0.02462	4.37	^{b}Restriction	0.81	1	0.3672
			Lags	0.09	2	0.9546
			RHO	0.92	4	0.9213
			T	0.90	1	0.3423
			cnst	0.04	1	0.8523

SE	0.07077
R^2	0.027
DW	2.56

overid (df = 8, p-value =0.8100)

Stability Test					End Test	
AP	T_1	T_2	λ	Break	p-value	End
1.36	1970.1	1979.4	2.07	1979.4	0.0000	1995.1
1.76	1975.1	1984.4	1.96	1980.2		
1.93	1980.1	1989.4	1.93	1980.2		

Estimation period is 1954.1-2013.3
^{a}Variable is $\log[(PIEF - TFG - TFS - TFR)/DF]_{-1}$
$^b\log DF_{-1}$ added.

Table A23
Equation 23
LHS Variable is $RB - RS_{-2}$

Equation			χ^2 Tests			
RHS Variable	Coef.	t-stat.	Test	χ^2	df	p-value
cnst	0.20558	4.73	^{a}Restriction	0.03	1	0.8600
$RB_{-1} - RS_{-2}$	0.91466	56.88	Lags	0.14	2	0.9331
$RS - RS_{-2}$	0.32720	6.08	RHO	7.51	3	0.0574
$RS_{-1} - RS_{-2}$	-0.27384	-4.33	T	6.79	1	0.0091
RHO1	0.20204	2.98	Leads +1	0.06	1	0.8136
			Leads +4	25.35	4	0.0000
			Leads +8	-119.50	2	9.9000
			p_4^e	0.72	1	0.3944
			p_8^e	1.06	1	0.3035

SE	0.27548
R^2	0.963
DW	2.01

overid (df = 16, p-value =0.1394)

Stability Test					End Test	
AP	T_1	T_2	λ	Break	p-value	End
4.32	1970.1	1979.4	2.07	1979.4	0.4000	1995.1
6.02	1975.1	1984.4	1.96	1982.3		
6.16	1980.1	1989.4	1.93	1982.3		

Estimation period is 1954.1-2013.3
$^a RS_{-2}$ added.

Table A24
Equation 24
LHS Variable is $RM - RS_{-2}$

RHS Variable	Equation Coef.	t-stat.	Test	χ^2 Tests χ^2	df	p-value
cnst	0.38818	5.72	[a]Restriction	0.11	1	0.7451
$RM_{-1} - RS_{-2}$	0.87621	42.81	Lags	0.18	2	0.9119
$RS - RS_{-2}$	0.32617	4.25	RHO	2.43	4	0.6571
$RS_{-1} - RS_{-2}$	-0.13257	-1.32	T	2.56	1	0.1098
			Leads +1	0.30	1	0.5811
			Leads +4	0.61	4	0.9616
			Leads +8	1.94	2	0.3798
			p_4^e	0.66	1	0.4179
			p_8^e	0.74	1	0.3908

SE	0.35141
R^2	0.905
DW	1.84

overid (df = 14, p-value =0.1192)

AP	Stability Test T_1	T_2	λ	Break	End Test p-value	End
4.70	1970.1	1979.4	2.07	1979.4	0.5444	1995.1
13.57	1975.1	1984.4	1.96	1984.4		
13.69	1980.1	1989.4	1.93	1984.4		

Estimation period is 1954.1-2013.3
[a]RS_{-2} added.

Table A25
Equation 25
LHS Variable is $CG/(PX_{-1}YS_{-1})$

RHS Variable	Equation Coef.	t-stat.	Test	χ^2 Tests χ^2	df	p-value
cnst	0.10640	4.77	Lags	2.97	3	0.3956
ΔRB	-0.14541	-1.34	RHO	2.43	4	0.6573
[a]	11.79045	1.13	T	0.02	1	0.8772
			Leads +1	1.51	2	0.4691
			Leads +4	2.20	8	0.9743
			Leads +8	3.01	4	0.5564
			ΔRS	2.74	1	0.0980

SE	0.30756
R^2	0.022
DW	1.84

overid (df = 17, p-value =0.2671)

AP	Stability Test T_1	T_2	λ	Break	End Test p-value	End
1.73	1970.1	1979.4	2.07	1974.4	0.0000	1995.1
2.37	1975.1	1984.4	1.96	1981.2		
3.52	1980.1	1989.4	1.93	1989.4		

Estimation period is 1954.1-2013.3
[a]Variable is $\Delta[(PIEF - TFG - TFS - TFR)]/(PX_{-1}YS_{-1})$

Table A26
Equation 26
LHS Variable is $\log[CUR/(POP \cdot PF)]$

Equation RHS Variable	Coef.	t-stat.	χ^2 Tests Test	χ^2	df	p-value
cnst	-0.05672	-7.26	^{a}	4.89	1	0.0270
$\log[CUR_{-1}/(POP_{-1} \cdot PF)]$	0.95798	137.97	Lags	6.12	3	0.1057
$\log[(X - FA)/POP]$	0.04866	7.53	RHO	14.46	3	0.0023
RSA	-0.00132	-2.67	T	3.51	1	0.0610
RHO1	-0.10513	-1.61				

SE	0.01007
R^2	0.999
DW	1.98

overid (df = 17, p-value =0.1669)

Stability Test AP	T_1	T_2	λ	Break	End Test p-value	End
17.04	1970.1	1979.4	2.07	1977.3	1.0000	1995.1
18.76	1975.1	1984.4	1.96	1982.2		
18.75	1980.1	1989.4	1.93	1982.2		

Estimation period is 1954.1-2013.3
[a]Variable is $\log[CUR/(POP \cdot PF)]_{-1}$

Table A27
Equation 27
LHS Variable is $\log(IM/POP)$

Equation RHS Variable	Coef.	t-stat.	χ^2 Tests Test	χ^2	df	p-value
cnst2	0.03824	2.84	Lags	9.06	3	0.0285
cnst	-0.96097	-5.79	RHO	23.64	4	0.0001
$\log(IM/POP)_{-1}$	0.79089	24.12	T	5.24	1	0.0221
^{a}	0.43291	5.71	Leads +1	2.38	1	0.1227
$\log(PF/PIM)$	0.08963	5.30	Leads +4	11.16	4	0.0248
$D691$	-0.11810	-4.24	Leads +8	3.26	2	0.1958
$D692$	0.13655	4.83	$\log PF$	0.00	1	0.9855
$D714$	-0.08660	-3.12				
$D721$	0.09773	3.49				

SE	0.02756
R^2	0.999
DW	1.62

overid (df = 14, p-value =0.0024)

Stability Test AP	T_1	T_2	λ	Break	End Test p-value	End
5.23	1973.1	1979.4	1.62	1975.3	1.0000	1995.1
5.95	1975.1	1984.4	1.96	1984.2		
6.58	1980.1	1989.4	1.93	1984.2		

Estimation period is 1954.1-2013.3
[a]Variable is $\log[(CS + CN + CD + IHH + IKF + IKH + IKB + IHF + IHB)/POP]$

Table A28
Equation 28
LHS Variable is $\log UB$

RHS Variable	Coef.	t-stat.	Test	χ^2	df	p-value
cnst	0.82003	1.83	Lags	7.76	3	0.0512
$\log UB_{-1}$	0.26153	2.58	RHO	2.72	3	0.4364
$\log U$	1.14738	4.54	T	2.37	1	0.1240
$\log WF$	0.44384	6.79				
RHO1	0.88126	22.32				

SE 0.06593
R^2 0.997
DW 2.28

overid (df = 12, p-value =0.1307)

AP	T_1	T_2	λ	Break	End Test p-value	End
12.99	1970.1	1979.4	2.07	1975.2	0.6889	1995.1
12.74	1975.1	1984.4	1.96	1975.2		
8.30	1980.1	1989.4	1.93	1983.1		

Estimation period is 1954.1-2013.3

Table A29
Equation 29
LHS Variable is $INTG/(-AG)$

RHS Variable	Coef.	t-stat.	Test	χ^2	df	p-value
cnst	0.00147	5.30	[b]Restriction	37.61	2	0.0000
$(INTG/(-AG))_{-1}$	0.79328	23.28	Lags	37.61	2	0.0000
[a]	0.17288	6.11	RHO	89.32	3	0.0000
RHO1	0.21426	2.93	T	35.52	1	0.0000

SE 0.00051
R^2 0.988
DW 2.09

AP	T_1	T_2	λ	Break	End Test p-value	End
26.85	1970.1	1979.4	2.07	1979.2	0.0000	1995.1
29.14	1975.1	1984.4	1.96	1980.3		
32.94	1980.1	1989.4	1.93	1989.4		

Estimation period is 1954.1-2013.3
[a]Variable is $(.4 * (RS/400) + .75 * .6 * (1/8) * (1/400) * (RB + RB_{-1} + RB_{-2} + RB_{-3} + RB_{-4} + RB_{-5} + RB_{-6} + RB_{-7}))$

Table A30
Equation 30
LHS Variable is RS

Equation			χ² Tests			
RHS Variable	Coef.	t-stat.	Test	χ^2	df	p-value
cnst	0.69903	4.68	Lags	7.93	4	0.0940
RS_{-1}	0.91865	51.60	RHO	8.81	4	0.0661
$100 \cdot [(PD/PD_{-1})^4 - 1]$	0.06674	3.95	T	0.89	1	0.3466
UR	-10.84613	-3.59	Leads +1	0.76	2	0.6827
ΔUR	-69.67530	-5.21	Leads +4	3.24	8	0.9184
$PCM1_{-1}$	0.01306	2.54	Leads +8	3.66	4	0.4543
$D794823 \cdot PCM1_{-1}$	0.21724	9.56	p_4^e	0.32	1	0.5690
ΔRS_{-1}	0.26591	4.87	p_8^e	5.52	1	0.0188
ΔRS_{-2}	-0.31506	-6.29				

SE 0.47622
R^2 0.972
DW 1.80

overid (df = 12, p-value =0.1412)

Stability test (1954.1-1979.3 versus 1982.4-2008.3): Wald statistic is 15.22 (8 degrees of freedom, p-value = .0550)

End Test: p-value = 0.9909, End = 1995.1

Estimation period is 1954.1-2008.3

Table A.5
The Raw Data Variables for the US Model

No.	Variable	Table	Line	Description
				NIPA Data
R1	GDPR	1.1.3	1	Real gross domestic product
R2	CD	1.1.3	4	Real personal consumption expenditures, durable goods
R3	CN	1.1.3	5	Real personal consumption expenditures, nondurable goods
R4	CS	1.1.3	6	Real personal consumption expenditures, services
R5	IK	1.1.3	9	Real nonresidential fixed investment
R6	IH	1.1.3	13	Real residential fixed investment
R7	EX	1.1.3	16	Real exports
R8	IM	1.1.3	19	Real imports
R9	PURG	1.1.3	23	Real consumption expenditures and gross investment, federal government
R10	PURS	1.1.3	26	Real consumption expenditures and gross investment, S&L
R11	GDP	1.1.5	1	Gross domestic product
R12	CDZ	1.1.5	4	Personal consumption expenditures, durable goods
R13	CNZ	1.1.5	5	Personal consumption expenditures, nondurable goods
R14	CSZ	1.1.5	6	Personal consumption expenditures, services
R15	IKZ	1.1.5	9	Nonresidential fixed investment
R16	IHZ	1.1.5	13	Residential fixed investment
R17	IVZ	1.1.5	14	Change in private inventories
R18	EXZ	1.1.5	16	Exports
R19	IMZ	1.1.5	19	Imports
R20	PURGZ	1.1.5	23	Consumption expenditures and gross investment, federal government
R21	PURSZ	1.1.5	26	Consumption expenditures and gross investment, S&L
R22	FA	1.3.3	4	Real farm gross domestic product
R23	FAZ	1.3.5	4	Farm gross domestic product
R24	FIUS	1.7.5	2	Income receipts from the rest of the world
R25	FIROW	1.7.5	3	Income payments to the rest of the world
R26	STAT	1.7.5	15	Statistical discrepancy
R28	DC	1.12	16	Net dividends, Total
R29	TRFR	1.12	24	Business current transfer payments to the rest of the world (net)
R30	DCB	1.14	14	Net dividends, corporate business
R31	INTF1	1.14	25	Net interest and miscellaneous payments, nonfinancial corporate business
R32	TCBN	1.14	28	Taxes on corporate income, nonfinancial corporate business
R33	DCBN	1.14	30	Net dividends, nonfinancial corporate business
R34	IVA	1.14	35	Inventory valuation adjustment, corporate business
R35	COMPT	2.1	2	Compensation of employees, received
R36	SIT	2.1	8	Employer contributions for government social insurance
R37	PRI	2.1	9	Proprietors' income with inventory valuation and capital consumption adjustments
R38	RNT	2.1	12	Rental income of persons with capital consumption adjustment
R39	PII	2.1	14	Personal interest income
R40	UB	2.1	21	Government unemployment insurance benefits
R41	TRFH	2.1	24	Other current transfer receipts from business (net)
R42	IPP	2.1	30	Personal interest payments
R43	TRHR	2.1	33	Personal current transfer payments to the rest of the world (net)

Table A.5 (continued)

No.	Variable	Table	Line	Description
R44	THG	3.2	3	Personal current taxes, federal government (see below for adjustments)
R45	RECTXG	3.2	4	Taxes on production and imports, federal government
R46	TCG	3.2	7	Taxes on corporate income, federal government
R47	TRG	3.2	10	Taxes from the rest of the world, federal government
R48	SIG	3.2	11	Contributions for government social insurance, federal government
R49	RECINTG	3.2	13	Interest receipts, federal government
R50	RECDIVG	3.2	14	Dividends, federal government
R51	RECRRG	3.2	15	Rents and royalties, federal government
R52	TRFG	3.2	17	Current transfer receipts from business, federal government
R53	TRHG	3.2	18	Current transfer receipts from persons, federal government
R54	SURPG	3.2	19	Current surplus of government enterprises, federal government
R55	CONGZ	3.2	21	Consumption expenditures, federal government
R56	TRGHPAY	3.2	24	Government social benefits to persons, federal government (see below for adjustments)
R57	TRGR1	3.2	25	Government social benefits to the rest of the world, federal government
R58	TRGS	3.2	27	Grants in aid to atate and local governments, federal government
R59	TRGR2	3.2	28	Other current transfer payments to the rest of the world (net), federal government
R60	PAYINTG	3.2	29	Interest payments, federal government
R61	INTGR	3.2	31	Interest payments, federal government to the rest of the world
R62	SUBSG	3.2	32	Subsidies, federal government
R64	CCG	3.2	44	Consumption of fixed capital, Federal Government
R65	THS	3.3	3	Personal current taxes, S&L
R66	RECTXS	3.3	6	Taxes on production and imports, S&L
R67	TCS	3.3	10	Taxes on corporate income, S&L
R68	SIS	3.3	11	Contributions for government social insurance, S&L
R69	RECINTS	3.3	13	Interest receipts, S&L
R70	RECDIVS	3.3	14	Dividends, S&L
R71	RECRRS	3.3	15	Rents and royalties, S&L
R72	TRFS	3.3	18	Current transfer receipts from business (net), S&L
R73	TRHS	3.3	19	Current transfer receipts from persons, S&L
R74	SURPS	3.3	20	Current surplus of government enterprises, S&L
R75	CONSZ	3.3	22	Consumption expenditures, S&L
R76	TRRSHPAY	3.3	23	Government social benefit payments to persons, S&L
R77	PAYINTS	3.3	24	Interest payments, S&L
R78	SUBSS	3.3	25	Subsidies, S&L
R80	CCS	3.3	37	Consumption of fixed capital, S&L
R81	PROG	3.10.3	15	Real compensation of general government employees, federal
R82	PROS	3.10.3	50	Real compensation of general government employees, S&L
R83	PROGZ	3.10.5	15	Compensation of general government employees, federal
R84	COMPMIL	3.10.5	26	Compensation of general government employees, defense
R85	PROSZ	3/10/5	50	Compensation of general government employees, S&L
R86	TTRFR	4.1	28	Current taxes and transfer payments to the rest of the world from business (net)
R88	IV	5.7.6	1	Real change in private inventories

Table A.5 (continued)

No.	Variable	Table	Line	Description
R89	SIHGA	3.14	3	Employee and self-employed contributions for social insurance to the federal government, annual data only
R90	SIQGA	3.14	5	Government employer contributions for social insurance to the federal government, annual data only
R91	SIFGA	3.14	6	Other employer contributions for social insurance to the federal government, annual data only
R92	SIHSA	3.14	18	Employee and self-employed contributions for social insurance to the S&L governments, annual data only
R93	SIQSA	3.14	20	Government employer contributions for social insurance to the S&L governments, annual data only
R94	SIFSA	3.14	21	Other employer contributions for social insurance to the S&L governments, annual data only

• For Tables 1.1.3, 1.3.3, and 3.10.3, the respective raw data variable was created by multipling the quantity index for a given quarter by the nominal value of the variable in 2009 and then dividing by 100.
• For Table 5.7.6, there is an "A" table and a "B" table. The "A" table is used for data prior to 1998:1, and the "B" table is used for data from 1998:1 on.
• S&L = State and Local Governments.
• R89–R94: Same value for all four quarters of the year. See variables R210–R215 for construction of variables SIHG, SIHS, SIFG, SIGG, SIFS, SISS.

Table A.5 (continued)

Flow of Funds Data

No.	Variable	Code	Description
R95	CDDCF	103020005	Change in checkable deposits and currency, F1, F.102
R96	NFIF1	105000005	Net lending (+) or net borrowing (-), F1, F.102
R97	IHF1	105012005	Residential investment, F1, F.6
R98	NNF	105420005	Net acquisition of nonproduced nonfinancial assets, F1, F.6
R99	CTF1	105440005	Net capital transfers paid, F1, F.9
R100	PIEFRET	106000065	Foreign earnings retained abroad, F1, F.102
R101	PIEF1X	106060005	Profits before tax, F1, F.102
R103	TF1	106231005	Taxes on corporate income, F1, F.102
R104	CCF1	106300015	Capital consumption allowances, F1, F.102
R105	DISF1	107005005	Discrepancy, F1, F.102
R106	CDDCNN	113020005	Change in checkable deposits and currency, NN, F.103
R107	NFINN	115000005	Net lending (+) or net borrowing (-), NN, F.103
R108	IHNN	115012005	Residential Investment, NN, F.6
R109	IKNN	115013005	Nonresidential fixed investment, NN, F.6
R110	IVNN	115020005	Change in inventories, NN, F.103 (only for tesing)
R111	CTNN	115440005	Net capital transfers paid, NN, F.9
R112	GSNN	116300005	Gross saving, NN, F.103
R117	CDDCH1	153020005	Change in checkable deposits and currency, H, F.100
R118	MVCE,	154090005	Total financial assets of Households, H, F.100.
R119	CCE		MVCE is the market value of the assets. CCE is the change in assets excluding capital gains and losses
R120	NFIH1	155000005	Net lending (+) or net borrowing (-), H, F.100
R121	REALEST	155035005	Real estate, H, stock variable, Table B.100, line 3
R122	CDH	155111003	Capital expenditures, consumer durable goods, H, F.100
R123	NICD	155111005	Net investment in consumer durables, H, F.100
R124	NNH	155420003	Net acquisition of nonproduced nonfinancial assets, H, F.6
R125	CTH	155440005	Net capital transfers paid, H, F.9
R126	CCH	156300005	Consumption of fixed capital, H, F.100
R127	USAFF	156600075	Contributions for government social insurance, U.S.-affiliated areas, US, F.105
R128	DISH1	157005005	Discrepancy, H, F.100
R129	IKH1	165013005	Nonresidential fixed investment, H, F.6
R131	NNS	205420003	Net acquisition of nonproduced nonfinancial assets, S, F.6
R132	CTS	205440005	Net capital transfers paid, S, F.9
R133	CDDCS	213020005	Change in checkable deposits and currency, S, F.104
R134	NFIS	215000005	Net lending (+) or net borrowing (-), S, F.104
R135	DISS1	217005005	Discrepancy, S, F.104
R136	CGLDR	263011005	Change in U.S. official reserve assets, R, F.200
R137	CDDCR	263020005	Change in U.S. checkable deposits and currency, R, F.106
R138	CFXUS	263111005	Change in U.S. official reserve assets, R, F.106
R139	NFIR	265000005	Net lending (+) or net borrowing (-), R, F.106
R140	NNR	265420005	Net acquisition of nonproduced nonfinancial assets, R, F.6
R141	CTR	265440005	Net capital transfers paid, R, F.9
R142	DISR1	267005005	Discrepancy, R, F.106
R143	CGLDFXUS	313011005	Change in U.S. official reserve assets, US, F.105
R144	CDDCUS	313020005	Change in checkable deposits and currency, US, F.105
R145	CSDRUS	313111303	Change in SDR allocations, US, F.105
R146	INS	313154015	Insurance and pension reserves, US, F.105
R147	NFIUS	315000005	Net lending (+) or net borrowing (-), US, F.105
R148	CTGB	315410093	Capital transfers paid by US, financial stabilization payments, F.9 (only for testing)
R149	NNG	315420003	Net acquisition of nonproduced nonfinancial assets, US, F.6
R150	CTGMB	315440005	Net capital transfers paid, US, F.105
R151	DISUS	317005005	Discrepancy, US, F.105

Table A.5 (continued)

No.	Variable	Code	Description
R152	CDDCCA	403020005	Change in checkable deposits and currency, CA, F.122
R153	NIACA	404090005	Net acquisition of financial assets, CA, F.122
R154	NILCA	404190005	Net increase in liabilities, CA, F.122
R155	IKCAZ	405013005	Fixed nonresidential investment, CA, F.122
R156	GSCA	406000105	Gross saving, CA, F.122
R157	DISCA	407005005	Discrepancy, CA, F.122
R160	NIDDLZ2	473127003	Net change in liabilities of credit unions of checkable deposits and currency, F.204
R162	IHBZ	645012063	Residential investment, B, F.6
R163	CGLDFXMA	713011005	Change in U.S. official reserve assets, MA, F.108
R164	CFRLMA	713068705	Change in federal reserve loans to domestic banks, MA, F.108
R165	NILBRMA	713113003	Change in depository institution reserves, MA, F.108
R175	CBR	713113003	Change in reserves at Federal Reserve, private depository institutions, F.109
R166	NIDDLRMA	713122605	Net increase in liabilities in the form of checkable deposits and currency of the MA due to the rest of the world, F.108
R167	NIDDLGMA	713123005	Net increase in liabilities in the form of checkable deposits and currency of the MA due to the federal government, F.108
R168	NIDDLCMA	713124003	Net increase in liabilities in the form of checkable deposits and currency of the MA due to government-sponsored enterprises, F.108
R169	NILCMA	713125005	Net increase in liabilities in the form of currency outside banks of the MA, F.108
R170	NIAMA	714090005	Net acquisition of in financial assets, MA, F.108
R171	NILMA	714190005	Net increase in liabilities, MA, F.108
R172	IKMAZ	715013005	Fixed nonresidential investment, MA, F.108
R173	GSMA	716000105	Gross savings, MA, F.108
R174	DISMA	717005005	Discrepancy, MA, F.108
R176	NILVCMA	763025005	Net increase in liabilities in the form of vault cash of commercial banks of the MA, F.108
R178	CVC	763025005	Change in vault cash, private depository institutions, F.109
R179	NIDDLCB3	743127003	Net change in liabilities in the form of checkable deposits and currency, banks in U.S.-affiliated Areas, F.204
R180	CBRB1A	753013003	Change in reserves at federal reserve, foreign banking offices in U.S., F.111
R181	NIDDLCB2	753127005	Net change in liabilities in the form of checkable deposits and currency, foreign banking offices in U.S., F.204
R177	NIDDLCB1	763127005	Net change in liabilities in the form of checkable deposits and currency, U.S.-chartered depository institutions, F.204
R182	CDDCFS	793020005	Net change in assets in the form of checkable deposits and currency of financial sectors, F.204
R183	NFIBB	795000005	Net lending (+) or net borrowing (-), B, F.107
R184	IKBMACA	795013005	Nonresidential fixed investment, B, F.107
R185	CTB	795440005	Net capital transfers paid, B, F.9
R186	GSBBCT	796000105	Gross saving less net capital transfers paid, B, F.107
R187	DISBB	797005005	Discrepancy, B, F.107
R188	MAILFLT1	903023005	Mail Float, US, F.12
R189	MAILFLT3	903028003	Mail Float, S, F.12
R190	MAILFLT2	903029200	Mail Float, private domestic, F.12

Table A.5 (continued)

Interest Rate Data

No.	Variable	Description
R191	RS	Three-month treasury bill rate (secondary market), percentage points. [BOG. Quarterly average.]
R192	RM	Conventional mortgage rate, percentage points. [BOG. Quarterly average.]
R193	RB	Moody's Aaa corporate bond rate, percentage points. [BOG. Quarterly average.]

Labor Force and Population Data

No.	Variable	Description
R194	CE	Civilian employment, SA in millions. [BLS. Quarterly average. See the next page for adjustments.]
R195	U	Unemployment, SA in millions. [BLS. Quarterly average. See the next page for adjustments.]
R196	CL1	Civilian labor force of males 25-54, SA in millions. [BLS. Quarterly average. See the next page for adjustments.]
R197	CL2	Civilian labor force of females 25-54, SA in millions. [BLS. Quarterly average. See the next page for adjustments.]
R198	AF	Total armed forces, millions. [Computed from population data from the U.S. Census Bureau. Quarterly average.]
R199	AF1	Armed forces of males 25-54, millions. [Computed from population data from the U.S. Census Bureau. Quarterly average.]
R200	AF2	Armed forces of females 25-54, millions. [Computed from population data from the U.S. Census Bureau. Quarterly average.]
R201	CPOP	Total civilian noninstitutional population 16 and over, millions. [BLS. Quarterly average. See the next page for adjustments.]
R202	CPOP1	Civilian noninstitutional population of males 25-54, millions. [BLS. Quarterly average. See the next page for adjustments.]
R203	CPOP2	Civilian noninstitutional population of females 25-54, millions. [BLS. Quarterly average. See the next page for adjustments.]
R204	JF	Employment, total private sector, all persons, SA in millions. [BLS, unpublished,"Basic industry data for the economy less general government, all persons."]
R205	HF	Average weekly hours, total private sector, all persons, SA. [BLS, unpublished,"Basic industry data for the economy less general government, all persons."]
R206	HO	Average weekly overtime hours in manufacturing, SA. [BLS. Quarterly average.]
R207	JQ	Total government employment, SA in millions. [BLS. Quarterly average.]
R208	JG	Federal government employment, SA in millions. [BLS. Quarterly average.]
R209	JHQ	Total government employee hours, SA in millions of hours per quarter. [BLS, Table B10. Quarterly average.]

Table A.5 (continued)

No.	Variable	Adjustments to the Raw Data Description
R210	SIHG =	[SIHGA/(SIHGA + SIHSA)](SIG + SIS - SIT) [Employee contributions for social insurance, h to g.]
R211	SIHS =	SIG + SIS - SIT - SIHG [Employee contributions for social insurance, h to s.]
R212	SIFG =	[SIFGA/(SIFGA + SIQGA)](SIG - SIHG) [Employer contributions for social insurance, f to g.]
R213	SIGG =	SIG - SIHG - SIFG [Employer contributions for social insurance, g to g.]
R214	SIFS =	[SIFSA/(SIFSA + SIQSA)](SIS - SIHS) [Employer contributions for social ensurance, f to s.]
R215	SISS =	SIS - SIHS - SIFS [Employer contributions for social insurance, s to s.]
R216	TBG =	[TCG/(TCG + TCS)](TCG + TCS - TCBN) [Corporate profit tax accruals, b to g.]
R217	TBS =	TCG + TCS - TCBN - TBG [Corporate profit tax accruals, b to s.]
	THG =	THG from raw data - TAXADJ
	TRGHPAY=	TRGHPAY from raw data - TAXADJ [TAXADJ (annual rate): 1968:3 = 6.1, 1968:4 = 7.1, 1969:1 = 10.7, 1969:2 = 10.9, 1969:3 = 7.1, 1969:4 = 7.3, 1970:1 = 5.0, 1970:2 = 5.0, 1970:3 = 0.4, 1975:2 = -31.2, 2008.2 = -199.4, 2008.3 = -57.0, 2009.2 = -152.0, 2009.3 = -239.0, 2009.4 = -249.0, 2010.1 = -231.0, 2010.2 = -256.0, 2010.3 = -266.0, 2010.4 = -15.0, 2011.1 = -53.0, 2011.2 = -74.0, 2011.3 = -99.0.]
R218	POP =	CPOP + AF [Total noninstitutional population 16 and over, millions.]
R219	POP1 =	CPOP1 + AF1 [Total noninstitutional population of males 25-54, millions.]
R220	POP2 =	CPOP2 + AF2 [Total noninstitutional population of females 25-54, millions.]

- BLS = Website of the Bureau of Labor Statistics
- BOG = Website of the Board of Governors of the Federal Reserve System
- SA = Seasonally adusted
- For the construction of variables R210, R212, and R214, the annual observation for the year was used for each quarter of the year.

Table A.5 (continued)

Variable	1952:1–1971:4	1952:1–1972:4	1973:1	1952:1–1977:4	1970:1–1989:4
	Adjustments to Labor Force and Population Data				
POP	1.00547	1.00009	1.00006	-	1.0058886-.0000736075TPOP90
POP1	0.99880	1.00084	1.00056	-	1.0054512 -.00006814TPOP90
POP2	1.00251	1.00042	1.00028	-	1.00091654-.000011457TPOP90
(CE+U)	1.00391	1.00069	1.00046	1.00239	1.0107312-.00013414TPOP90
CL1	0.99878	1.00078	1.00052	1.00014	1.00697786-.00008722TPOP90
CL2	1.00297	1.00107	1.00071	1.00123	-
CE	1.00375	1.00069	1.00046	1.00268	1.010617-.00013271TPOP90

- TPOP90 is 79 in 1970:1, 78 in 1970:2, ..., 1 in 1989:3, 0 in 1989:4.

Variable	1990:1–1998:4
POP	1.0014883-.0000413417TPOP99
POP1	.99681716 +.000088412TPOP99
POP2	1.0045032 -.00012509TPOP99
(CE+U)	1.00041798-.000011611TPOP99
CL1	.9967564+.0000901TPOP99
CL2	1.004183-.00011619TPOP99
CE	1.00042068-.000011686TPOP99

- TPOP99 is 35 in 1990:1, 34 in 1990:2, ..., 1 in 1998:3, 0 in 1998:4.

Variable	1990:1–1999:4
POP	1.0165685-.00041421TPOP2000
POP1	1.0188400 -.00047100TPOP2000
POP2	1.0195067 -.00048767TPOP2000
(CE+U)	1.0156403-.00039101TPOP2000
CL1	1.0208284-.00052071TPOP2000
CL2	1.0151172-.00037793TPOP2000
CE	1.0156827-.00039207TPOP2000

- TPOP2000 is 39 in 1990:1, 38 in 1990:2, ..., 1 in 1999:3, 0 in 1999:4.

Variable	1993:1–2002:4
POP	1.0043019-.00010755TPOP2003
POP1	1.0046539 -.00011635TPOP2003
POP2	1.0043621 -.00010905TPOP2003
(CE+U)	1.0042240-.00010560TPOP2003
CL1	1.0046137-.00011534TPOP2003
CL2	1.0042307-.00010577TPOP2003
CE	1.0041995-.00010499TPOP2003

- TPOP2003 is 39 in 1993:1, 38 in 1993:2, ..., 1 in 2002:3, 0 in 2002:4.

Variable	1994:1–2003:4
POP	.9974832+.00006292TPOP2004
POP1	.9982816 +.00004296TPOP2004
POP2	.9966202 +.00008450TPOP2004
(CE+U)	.9970239+.00007440TPOP2004
CL1	.9977729+.00004454TPOP2004
CL2	.9959602+.00010000TPOP2004
CE	.9970481+.00007380TPOP2004

- TPOP2004 is 39 in 1994:1, 38 in 1994:2, ..., 1 in 2003:3, 0 in 2003:4.

Table A.5 (continued)

Variable	1996:1–2005:4
POP	.9997054+.000007365TPOP2006
POP1	.9994935 +.0000126625TPOP2006
POP2	.9994283 +.0000142925TPOP2006
(CE+U)	.9991342 +.000021645TPOP2006
CL1	.9987934+.000030165TPOP2006
CL2	.9986564+.00003359TPOP2006
CE	.9991385 +.0000215375TPOP2006

• TPOP2006 is 39 in 1996:1, 38 in 1996:2, ..., 1 in 2005:3, 0 in 2005:4.

Variable	1997:1–2006:4
POP	1.0013950-.000034875TPOP2007
POP1	1.0009830 -.000024575TPOP2007
POP2	1.0016647 -.0000416175TPOP2007
(CE+U)	1.0010684 -.00002671TPOP2007
CL1	1.0008882-.000022205TPOP2007
CL2	1.0013202-.000033005TPOP2007
CE	1.0010474 -.0000261855TPOP2007

• TPOP2007 is 39 in 1997:1, 38 in 1997:2, ..., 1 in 2006:3, 0 in 2006:4.

Variable	1998:1–2007:4
POP	.9968047+.0000798825TPOP2008
POP1	.9958060+.00010485TPOP2008
POP2	.9976944 +.00005764TPOP2008
(CE+U)	.9958557 +.0001036075TPOP2008
CL1	.9948031+.0001299225TPOP2008
CL2	.9969464+.00007634TPOP2008
CE	.9959135+.0001021625TPOP2008

• TPOP2008 is 39 in 1998:1, 38 in 1998:2, ..., 1 in 2007:3, 0 in 2007:4.

Variable	1999:1–2008:4
POP	.9979450+.000051375TPOP2009
POP1	.9973640+.0000659TPOP2009
POP2	.9984844+.00003789TPOP2009
(CE+U)	.9970910+.000072725TPOP2009
CL1	.9964462+.000088845TPOP2009
CL2	.9977695+.0000557625TPOP2009
CE	.9971608+.00007098TPOP2009

• TPOP2009 is 39 in 1999:1, 38 in 1999:2, ..., 1 in 2008:3, 0 in 2008:4.

Variable	2000:1–2009:4
POP	.9989110+.000027225TPOP2010
POP1	.9978610+.000053475TPOP2010
POP2	.9989019+.0000274525TPOP2010
(CE+U)	.9983693+.0000407675TPOP2010
CL1	.9974105+.0000647375TPOP2010
CL2	.9989507+.0000262325TPOP2010
CE	.9982313+.0000442175TPOP2010

• TPOP2010 is 39 in 2000:1, 38 in 2000:2, ..., 1 in 2009:3, 0 in 2009:4.

Table A.5 (continued)

Variable	2001:1–2010:4
POP	.9985474+.000036315TPOP2011
POP1	.9989740+.000025650TPOP2011
POP2	.9970233+.000074418TPOP2011
(CE+U)	.9967092+.000082270TPOP2011
CL1	.9956715+.000108213TPOP2011
CL2	.9971304+.000071740TPOP2011
CE	.9966082+.000084795TPOP2011

• TPOP2011 is 39 in 2001:1, 38 in 2001:2, ..., 1 in 2010:3, 0 in 2010:4.

Variable	2002:1–2011:4
POP	1.0062764-.000156910TPOP2012
POP1	.9899101+.00002522475TPOP2012
POP2	1.0051234-.000128085TPOP2012
(CE+U)	1.0016822-.000042055TPOP2012
CL1	.9889798+.000275505TPOP2012
CL2	1.0041332-.00010333TPOP2012
CE	1.0015354-.000038385TPOP2012

• TPOP2012 is 39 in 2002:1, 38 in 2002:2, ..., 1 in 2011:3, 0 in 2011:4.

Variable	2003:1–2012:4
POP	1.0005648-.00001412TPOP2013
POP1	1.0003568-.00000892TPOP2013
POP2	1.0007278-.000018195TPOP2013
(CE+U)	1.0008780-.00002195TPOP2013
CL1	1.0006285-.0000157125TPOP2013
CL2	1.0012289-.0000307225TPOP2013
CE	1.0008877-.0000221925TPOP2013

• TPOP2013 is 39 in 2003:1, 38 in 2003:2, ..., 1 in 2012:3, 0 in 2012:4.

Table A.5 (continued)
The Raw Data Variables in Alphabetical Order Matched to R Numbers Above

Var.	No.	Var.	No.	Var.	No.	Var.	No.
AF	R198			MAILFLT3	R189	RECRRG	R51
AF1	R199	DISBB	R187	MVCE	R118	RECRRS	R71
AF2	R200	DISCA	R157	NFIBB	R183	RECTXG	R45
CBRB1A	R180					RECTXS	R66
		DISF1	R105	NFIF1	R96	RM	R192
		DISH1	R128	NFIH1	R120	RNT	R38
CCE	R119	DISMA	R174	NFINN	R107	RS	R191
CCF1	R104	DISR1	R142	NFIR	R139	SIFG	R212
CCG	R64	DISS1	R135	NFIS	R134	SIFGA	R91
CCH	R126	DISUS	R151	NFIUS	R147	SIFS	R214
CCS	R80	EX	R7	NIACA	R153	SIFSA	R94
CD	R2	EXZ	R18	NIAMA	R170	SIG	R48
CDDCCA	R152	FA	R22	NICD	R123	SIGG	R213
CDDCF	R95	FAZ	R23	CVC	R178	SIHGA	R89
		FIROW	R25	NIDDLCB1	R177	SIHSA	R92
CDDCFS	R182	FIUS	R24	NIDDLCB2	R181	SIHG	R210
CDDCH1	R117	GDP	R11	NIDDLCB3	R179	SIHS	R211
CDDCNN	R106	GDPR	R1	NIDDLCMA	R168	SIQGA	R90
CDDCR	R137	GSBBCT	R186	NIDDLGMA	R167	SIQSA	R93
CDDCS	R133	GSCA	R156	NIDDLRMA	R166	SIS	R68
CDDCUS	R144					SISS	R215
CDH	R122	GSMA	R173	NIDDLZ2	R160	SIT	R36
CDZ	R12	GSNN	R112	NILBRMA	R165	STAT	R26
CE	R194	HF	R205	NILCA	R154	SUBSG	R62
CFRLMA	R164	HO	R206	NILCMA	R169	SUBSS	R78
CFXUS	R138	IH	R6	NILMA	R171	SURPG	R54
CGLDFXMA	R163	IHBZ	R162	NILVCMA	R176	SURPS	R74
CGLDFXUS	R143	IHF1	R97	NNF	R98	TBG	R216
CGLDR	R136	IHNN	R108	NNG	R149	TBS	R217
CL1	R196	IHZ	R16	NNH	R124	TCBN	R32
CL2	R197	IK	R5	NNR	R140	TCG	R46
CN	R3	IKBMACA	R184	NNS	R131	TCS	R67
CNZ	R13	IKCAZ	R155	PAYINTG	R60	TF1	R103
COMPMIL	R84			PAYINTS	R77	THG	R44
COMPT	R35	IKH1	R129	PIEF1X	R101	THS	R65
CONGZ	R55	IKMAZ	R172	PIEFRET	R100	TRFG	R52
CONSZ	R75	IKNN	R109	PII	R39	TRFH	R41
CPOP	R201	IKZ	R15	POP	R218	TRFR	R29
CPOP1	R202	IM	R8	POP1	R219	TRFS	R72
CPOP2	R203	IMZ	R19	POP2	R220	TRG	R47
CS	R4	INS	R146	PRI	R37	TRGHPAY	R56
CSDRUS	R145	INTF1	R31	PROG	R81	TRGR1	R57
CSZ	R14	INTGR	R61	PROGZ	R83	TRGR2	R59
CTB	R185	IPP	R42	PROS	R82	TRGS	R58
CTF1	R99	IV	R88	PROSZ	R85	TRHG	R53
CTGB	R148	IVA	R34	PURG	R9	TRHR	R43
CTGMB	R150			PURGZ	R20	TRHS	R73
CTH	R125	IVNN	R110	PURS	R10	TRRSHPAY	R76
CTNN	R111	IVZ	R17	PURSZ	R21	TTRFR	R86
CTR	R141	JF	R204	RB	R193	U	R195
CTS	R132	JG	R208	REALEST	R121	UB	R40
CBR	R175	JHQ	R209	RECDIVG	R50	USAFF	R127
DC	R28	JQ	R207	RECDIVS	R70		
DCB	R30	MAILFLT1	R188	RECINTG	R49		
DCBN	R33	MAILFLT2	R190	RECINTS	R69		

Table A.6
Links Between the National Income and Product Accounts and the Flow of Funds Accounts

Flow of Funds Data (raw data variables)	
SH =	NFIH1 + DISH1
SF =	NFIF1 + DISF1 + NFINN
SB =	NFIBB + DISBB - NIAMA + NILMA - DISMA - NIACA + NILCA - DISCA
SR =	NFIR + DISR1
SG =	NFIUS + DISUS + NIACA - NILCA + DISCA + NIAMA - NILMA + DISMA
SS =	NFIS + DISS1
Variables in the Model on the Right Hand Side	
SHTEST =	YT - SIHG - SIHS + USAFF - THG - THS - PCS*CS - PCN·CN - PCD·CD + TRGH + TRSH + UB + INS + NICD + CCH - CTH - PIH·IHH - CDH - PIK·IKH - NNH
SFTEST =	XX + SUBS + SUBG + USOTHER + PIEFRET - WF·JF(HN + 1.5*HO) - RNT - INTZ - INTF - TRFH - NICD - CCH + CDH - TBS - TRFS - CCS - TRFR - DB - GSB - CTGB - GSMA - GSCA - TBG - TRFG - CCG - SIFG - SIFS - STAT - DF - TF1 - TFA - PIK·IKF - PIH·IHF - NNF - CTF1 - CTNN
SBTEST =	GSB - CTB - PIH·IHB - PIK·IKB
SRTEST =	- PEX·EX - USROW + PIM·IM + TFR + TRFR + TRHR + TRGR - USAFF - CTR - NNR
SGTEST =	GSMA + GSCA + THG + IBTG + TBG + TFG + SIHG + SIFG - DG + TRFG - PG·COG - WG·JG·HG - WM·JM·HM - TRGH - TRGR - TRGS - INTG - SUBG + CCG - INS - USAFF - CTGMB - NNG - PIK·IKG
SSTEST =	THS + IBTS + TBS + TFS + SIHS + SIFS - DS + TRGS + TRFS - PS·COS - WS·JS·HS - TRSH - UB - INTS - SUBS + CCS - CTS - NNS
Tests	
0 =	SH + SF + SB + SR + SG + SS + STAT + USAFF
0 =	SH - SHTEST
0 =	SF - SFTEST
0 =	SB - SBTEST
0 =	SR - SRTEST
0 =	SG - SGTEST
0 =	SS - SSTEST
0 =	-NIDDLCB1 - NIDDLCB2 - NIDDLCB3 - NIDDLZ2 + CDDCFS + CDDCF + MAILFLT1 + MAILFLT2 + CDDCUS - NIDDLRMA - NIDDLGMA + CDDCH1 + CDDCNN + CDDCR + CDDCS - NILCMA + MAILFLT3 - NIDDLCMA
0 =	CBR + CVC - NILBRMA - NILVCMA
0 =	CGLDR - CFXUS + CGLDFXUS + CGLDFXMA - CSDRUS

• See Table A.5 for the definitions of the raw data variables.

Table A.7
Construction of the Variables for the US Model

Variable	Construction (raw data variables on right hand side)
AA	Def., Eq. 89.
AB	Def., Eq. 73. Base Period=1971:4, Value=248.176
AF	Def., Eq. 70. Base Period=1971:4, Value=-388.975
AG	Def., Eq. 77. Base Period=1971:4, Value=-214.587
AH	Def., Eq. 66. Base Period=1971:4, Value=2222.45
AR	Def., Eq. 75. Base Period=1971:4, Value=-18.359
AS	Def., Eq. 79. Base Period=1971:4, Value=-160.5
BO	Sum of CFRLMA. Base Period=1971:4, Value=.039
BR	Sum of CBR+CVC. Base Period=1971:4, Value=35.329
$CCF1$	CCF1
CCG	CCG
$CCGQ$	CCG/$GDPD$
CCH	CCH
$CCHQ$	CCH/$GDPD$
CCS	CCS
$CCSQ$	CCS/$GDPD$
CD	CD
CDA	Peak to peak interpolation of CD/POP. Peak quarters are 1953:1, 1955:3, 1960:2, 1963:2, 1965:4, 1968:3, 1973:2, 1978:4, 1985:1, 1988:4, 1994:1, 1995:4, 2000:3, 2007:2, 2012:1, and 2013:3.
CDH	CDH
CG	$MVCE - MVCE_{-1} - CCE$
CN	CN
COG	PURG-PROG
COS	PURS-PROS
CS	CS
CTB	CTB
$CTF1$	CTF1
$CTGB$	CTBS
$CTGMB$	CTGMB
CTH	CTH
$CTNN$	CTNN
CTR	CTR
CTS	CTS
CUR	Sum of NILCMA. Base Period=1971:4, Value=53.521
$D1G$	Def., Eq. 47
$D1GM$	Def., Eq. 90
$D1S$	Def., Eq. 48
$D1SM$	Def., Eq. 91
$D2G$	Def., Eq. 49
$D2S$	Def., Eq. 50
$D3G$	Def., Eq. 51
$D3S$	Def., Eq. 52
$D4G$	Def., Eq. 53
$D5G$	Def., Eq. 55
$D6G$	Def., Eq. 67
DB	DCB-DCBN
DBQ	DB/$GDPD$
$DELD$	Computed using NIPA asset data
$DELH$	Computed using NIPA asset data
$DELK$	Computed using NIPA asset data
DF	DCBN

Table A.7 (continued)

Variable	Construction (raw data variables on right hand side)
DG	-RECDIVG
$DISB$	DISBB-DISMA-DISCA
$DISF$	DISF1
$DISG$	DISUS+DISCA+DISMA
$DISH$	DISH1
$DISR$	DISR1
$DISS$	DISS1
DR	DC-DCB
DRQ	DR/$GDPD$
DS	-RECDIVS
E	CE+AF
EX	EX
$EXPG$	Def., Eq. 106
$EXPS$	Def., Eq. 113
FA	FA
GDP	Def., Eq. 82, or GDP
$GDPD$	Def., Eq. 84
$GDPR$	GDPR
GNP	Def., Eq. 129
$GNPD$	Def., Eq. 131
GSB	GSB
$GSBQ$	GSB/$GDPD$
$GSCA$	GSCA
$GSMA$	GSMA
$GSNN$	GSNN
$GSNNQ$	GSNN/$GDPD$
$GNPR$	Def., Eq. 130
HF	13·HF
HFF	Def., Eq. 100
HFS	Peak to peak interpolation of HF. The peaks are 1952:4, 1960.3, 1966:1, 1977:2, 1990:1, 2000:1, 2001:4, and 2004:2. Flat end.
HG	JHQ/JQ
HM	520
HN	Def., Eq. 62
HO	13·HO. Constructed values for 1952:1-1955:4.
HS	JHQ/JQ
$IBTG$	RECTXG+RECRRG
$IBTS$	RECTXS+RECRRS
IGZ	PURGZ-CONGZ
$IGZQ$	IGZ/$GDPD$
IHB	IHBZ/(IHZ/IH)
IHF	(IHF1+IHNN)/(IHZ/IH)
IHH	(IHZ-IHF1-IHBZ-IHNN)/(IHZ/IH)
$IHHA$	Peak to peak interpolation of IHH/POP. Peak quarters are 1955:2, 1963:4, 1978:3, 1986:3, 1994:2, 2004:2, 2006.2, and 2007.4. Flat end.
IKB	(IKBMACA-IKMAZ-IKCAZ)/(IKZ/IK)
IKF	(IKZ-IKH1-IKBMACA)/(IKZ/IK)
IKG	((IKCAZ+IKMAZ)/(IKZ/IK)
IKH	IKH1/(IKZ/IK)
IM	IM
INS	INS
$INTF$	INTF1
$INTG$	PAYINTG-RECINTG
$INTGR$	INTGR

Table A.7 (continued)

Variable	Construction (raw data variables on right hand side)
$INTS$	PAYINTS-RECINTS
$INTZ$	PII-IPP-INTF1-(PAYINTG-RECINTG)+INTGR-(PAYINTS-RECINTS)
$INTZQ$	INTZ/$GDPD$
ISZ	PURSZ-CONSZ
$ISZQ$	ISZ/$GDPD$
IVA	IVA
IVF	IV
JF	JF
JG	JG
$JHMIN$	Def., Eq. 94
JJ	Def., Eq. 95
JJP	Peak to peak interpolation of JJ. The peaks are 1952:4, 1955:4, 1959:3, 1969:1, 1973:3, 1979:3, 1985:4, 1990:1, 1995:1, 2000:2, and 2003:2. Flat end.
JM	AF
JS	JQ-JG
KD	Def., Eq. 58. Base Period=1952:1, Value=255.5, Fixed Assets Table 1.2, line 15, 2009 = 100. 2009 dollar value in Fixed Asset Table 1.1, line 15, is 4588.1. Dep. Rate=DELD
KH	Def., Eq. 59. Base Period=1952:1, Value=2517.7, Fixed Assets Table 1.2, line 8, 2009 = 100. 2009 dollar value in Fixed Asset Table 1.1, line 8, is 15708.5. Dep. Rate=DELH
KK	Def., Eq. 92. Base Period=1952:1, Value=2501.9, Fixed Asset Table 1.2, line 4, 2009 = 100. 2009 dollar value in Fixed Assets Table 1.1, line 4, is 18152.8. Dep. Rate=DELK
$KKMIN$	Def., Eq. 93
$L1$	CL1+AF1
$L2$	CL2+AF2
$L3$	Def., Eq. 86
LAM	Computed from peak to peak interpolation of $\log[Y/(JF \cdot HF)]$. Peak quarters are 1955:2, 1966:1, 1973:1, 1992.4, and 2010.4
LM	Def., Eq. 85
$M1$	Def., Eq. 81. Base Period=1971:4, Value=250.218
MB	Def., Eq. 71. Also sum of -NIDDLCB1-NIDDLCB2-NIDDLCB3-NIDDLZ2+CDDCFS-CDDCCA. Base Period=1971:4, Value=-191.73
$MDIF$	CDDCFS-MAILFLT1
MF	Sum of CDDCF+MAILFLT1+MAILFLT2+CDDCNN+MAILFLT3, Base Period= 1971:4, Value=84.075
MG	Sum of CDDCUS+CDDCCA-NIDDLRMA-NIDDLGMA-NIDDLCMA, Base Period=1971:4, Value=10.526
MGQ	MG/$GDPD$
MH	Sum of CDDCH1. Base Period=1971:4, Value=125.813
MHQ	MH/$GDPD$
MR	Sum of CDDCR. Base Period=1971:4, Value=12.723
MRQ	MR/$GDPD$
MS	Sum of CDDCS. Base Period=1971:4, Value=12.114
MSQ	MS/$GDPD$
MUH	Peak to peak interpolation of Y/KK. Peak quarters are 1953:2, 1955:3, 1959:2, 1962:3, 1965:4, 1969:1, 1973:1, 1977:3, 1981:1, 1984:2, 1988:4, 1993:4, 1998:1, 2006:1, 2013.2. Flat beginning.
$NICD$	NICD
NNF	NNF
NNG	NNG
NNH	NNH
NNR	NNR
NNS	NNS
PCD	CDZ/CD
$PCGNPD$	Def., Eq. 122
$PCGNPR$	Def., Eq. 123
$PCM1$	Def., Eq. 124
PCN	CNZ/CN
PCS	CSZ/CS

Table A.7 (continued)

Variable	Construction (raw data variables on right hand side)
PD	Def., Eq. 33
PEX	EXZ/EX
PF	Def., Eq. 31
PFA	FAZ/FA
PG	(PURGZ-PROGZ)/(PURG-PROG)
PH	Def., Eq. 34
$PIEF$	Def., Eq. 67, or PIEF1X
$PIEFRET$	PIEFRET
PIH	IHZ/IH
PIK	IKZ/IK
PIM	IMZ/IM
PIV	IVZ/IV, with the following adjustments: 1954:4 = .2797, 1959:3 = .2449, 1970:1 = .2814, 1971:4 = .2756, 1975:3 = .4265, 1975:4 = .4265, 1983:2 = .7211, 1983:3 = .7211, 1986:4 = .6857, 1987:3 = .7400, 1992:1 = .9053, 1993:3 = .8685, 1996:1 = 1.1245, 2002.1 = .7752, 2003.2 = .8390, 2003.3 = .8390, 2012.4 = 1.2796
PKH	REALEST/KH
POP	POP
$POP1$	POP1
$POP2$	POP2
$POP3$	POP-POP1-POP2
$PROD$	Def., Eq. 118
PS	(PURSZ-PROSZ)/(PURS-PROS)
$PSI1$	Def., Eq. 32
$PSI2$	Def., Eq. 35
$PSI3$	Def., Eq. 36
$PSI4$	Def., Eq. 37
$PSI5$	Def., Eq. 38
$PSI6$	Def., Eq. 39
$PSI7$	Def., Eq. 40
$PSI8$	Def., Eq. 41
$PSI9$	Def., Eq. 42
$PSI10$	Def., Eq. 44
$PSI11$	Def., Eq. 45
$PSI12$	Def., Eq. 46
$PSI13$	(PROG+PROS)/(JHQ + 520AF)
$PSI14$	Def., Eq. 55
$PSI15$	Def., Eq. 56
PUG	Def., Eq. 104 or PURGZ
PUS	Def., Eq. 110 or PURSZ
PX	(CDZ+CNZ+CSZ+IHZ+IKZ+PURGZ-PROGZ+PURSZ-PROSZ+EXZ-IMZ-IBTG-IBTS)/ (CD+CN+CS+IH+IK+PURG-PROG+PURS-PROS+EX-IM)
Q	Sum of CGLDFXUS+CGLDFXMA-CSDRUS. Base Period=1971:4, Value=12.265
QQ	Q/$GDPD$
RB	RB
$RECG$	Def., Eq. 105
$RECS$	Def., Eq. 112
RM	RM
RMA	Def., Eq. 128
RNT	RNT
$RNTQ$	RNT/$GDPD$
RS	RS
RSA	Def., Eq. 127
SB	Def., Eq. 72
SF	Def., Eq. 69
SG	Def., Eq. 76
SGP	Def., Eq. 107
SH	Def., Eq. 65

Table A.7 (continued)

Variable	Construction (raw data variables on right hand side)
$SHRPIE$	Def., Eq. 121
$SIFG$	SIFG
$SIFS$	SIFS
SIG	SIG
$SIGG$	SIGG
$SIHG$	SIHG
$SIHS$	SIHS
SIS	SIS
$SISS$	SISS
SR	Def., Eq. 74
SRZ	Def., Eq. 116
SS	Def., Eq. 78
SSP	Def., Eq. 114
$STAT$	STAT
$STATP$	Def., Eq. 83
$SUBG$	SUBSG - SURPG
$SUBS$	SUBSS - SURPS
T	1 in 1952:1, 2 in 1952:2, etc.
$TAUG$	Determined from a regression. See the discussion in Section 6.3.4. The subperiods are: 1952.1-1953.4, 1954.1-1963.4, 1964.1-1964.1, 1964.2-1964.4, 1965.1-1965.4, 1966.1-1967.4, 1968.1-1970.4, 1971.1-1971.4, 1972.1-1972.4, 1973.1-1973.4, 1974.1-1975.1, 1975.2-1976.4, 1977.1-1978.2, 1978.3-1981.3, 1981.4-1982.2, 1982.3-1983.2, 1983.3-1984.4, 1985.1-1985.1, 1985.2-1985.2, 1985.3-1987.1, 1987.2-1987.2, 1987.3-1987.4, 1988.1-1988.4, 1989.1-1989.4, 1990.1-1990.4, 1991.1-1991.4, 1992.1-1995.1, 1995.2-1996.1, 1996.2-1996.4, 1997.1-1997.4, 1998.1-1998.4, 1999.1-1999.4, 2000.1-2001.2, 2001.3-2001.3, 2001.4-2001.4, 2002.1-2002.4, 2003.1-2003.2, 2003.3-2003.3, 2003.4-2004.4, 2005.1-2005.4, 2006.1-2006.4, 2007.1-2007.4, 2008.1-2008.3, 2008.4-2008.4, 2009.1-2009.1, 2009.2-2009.2, 2009.3-2010.1, 2010.2-2010.3, 2010.4-2010.4, 2011.1-2011.3, 2011.4-2012.4, 2013.1-2013.3.
$TAUS$	Determined from a regression. See the discussion in Section 6.3.3. The subperiods are: 1952.1-1958.4, 1959.1-1966.4, 1967.1-1971.4, 1972.1-2001.2, 2001.3-2004.4, 2005.1-2007.4, 2008.1-2008.1, 2008.2-2008.2, 2008.3-2012.4, 2013.1-2013.3.
TFR	TTRFR - TRFR
TBG	TBG
$TBGQ$	TBG/$GDPD$
TBS	TBS
TCG	TCG
TCS	TCS
TFA	TFA
$TF1$	TF1
TFG	Def., Eq. 102
TFS	Def., Eq. 108
$TF1$	TF1
$THETA1$	PFA/$GDPD$
$THETA2$	CDH/(PCD·CD)
$THETA3$	NICD/(PCD·CD)
$THETA4$	PIEFRET/PIEF
THG	THG
THS	THS
$TRFG$	TRFG
$TRFH$	TRFH
$TRFR$	TRF-TRFH
$TRFS$	TRFS
$TRGH$	TRGHPAY - TRHG
$TRGHQ$	TRGH/$GDPD$
$TRGR$	TRGR1 + TRGR2 - TRG
$TRGS$	TRGS
$TRGSQ$	TRGS/$GDPD$
$TRHR$	TRHR

Table A.7 (continued)

Variable	Construction (raw data variables on right hand side)
$TRSH$	Def., Eq. 111
$TRSHQ$	TRSH/$GDPD$
U	(CE+U)-CE
UB	UB
UBR	Def., Eq. 125
UR	Def., Eq. 87
$USAFF$	USAFF
$USOTHER$	Def., Eq. 57
$USROW$	FIUS-FIROW
V	Def., Eq. 117. Base Period=1996:4, Value=1517.3, Fixed Assets Table 5.8.6A
WA	Def., Eq. 126
WF	WF=[COMPT-PROGZ-PROSZ-(SIT-SIGG-SISS) +PRI]/[JF(HF+.5HO)]
WG	(PROGZ-COMPMIL)/[JG(JHQ/JQ)]
WH	Def., Eq. 43
WM	COMPMIL/(520AF)
WR	Def., Eq. 119
WS	PROSZ/[(JQ-JG)(JHQ/JQ)]
X	Def., Eq. 60
XX	Def., Eq. 61
Y	Def., Eq. 63
YD	Def., Eq. 115
YNL	Def., Eq. 99
YS	Def., Eq. 98
YT	Def., Eq. 64

• The variables in the first column are the variables in the model. They are defined by the identities in Table A.3 or by the raw data variables in Table A.5. A right hand side variable in this table is a raw data variable unless it is in italics, in which case it is a variable in the model. Sometimes the same letters are used for both a variable in the model and a raw data variable.

Table A.9
First Stage Regressors for the US model for 2SLS

Eq.	First Stage Regressors
1	cnst2, cnst, $AG1$, $AG2$, $AG3$, $\log(CS/POP)_{-1}$, $\log[YD/(POP \cdot PH)]_{-1}$, RSA_{-1}, $\log(AA/POP)_{-1}$, T, $\log(1-D1GM-D1SM-D4G)_{-1}$, $\log(IM/POP)_{-1}$, $\log[(JG \cdot HG+JM \cdot HM+JS \cdot HS)/POP]$, $\log(PIM/PF)_{-1}$, $\log[YNL/(POP \cdot PH)]_{-1}$, $100[(PD/PD_{-1})^4-1]_{-1}$, $\log[(COG+COS)/POP]$, $\log[(TRGH+TRSH)/(POP \cdot PH_{-1})]$, RS_{-2}, RB_{-1}, $\log(Y/POP)_{-1}$, $\log(V/POP)_{-1}$, UR_{-1}, $\log[YD/(POP \cdot PH)]$
2	cnst2, cnst, $AG1$, $AG2$, $AG3$, $\log(CN/POP)_{-1}$, $\Delta\log(CN/POP)_{-1}$, $\log(AA/POP)_{-1}$, $\log[YD/(POP \cdot PH)]_{-1}$, RMA_{-1}, $\log(1-D1GM-D1SM-D4G)_{-1}$, $\log(IM/POP)_{-1}$, $\log(EX/POP)_{-1}$, $\log[(JG \cdot HG+JM \cdot HM+JS \cdot HS)/POP]$, $\log(PIM/PF)_{-1}$, $\log[YNL/(POP \cdot PH)]_{-1}$, $100[(PD/PD_{-1})^4-1]_{-1}$, $\log[(COG+COS)/POP]$, $\log[(TRGH+TRSH)/(POP \cdot PH_{-1})]$, RS_{-1}, RS_{-2}, $\log(V/POP)_{-1}$, UR_{-1}, RS_{-1}, RS_{-2}, T
3	cnst2, cnst, $AG1$, $AG2$, $AG3$, $(KD/POP)_{-1}$, $DELD(KD/POP)_{-1}-(CD/POP)_{-1}$, $YD/(POP \cdot PH)$, $(RMA \cdot CDA)_{-1}$, $(AA/POP)_{-1}$, $\log(1-D1GM-D1SM-D4G)_{-1}$, $\log(IM/POP)_{-1}$, $\log(EX/POP)_{-1}$, $\log(PIM/PF)_{-1}$, $\log[YNL/(POP \cdot PH)]_{-1}$, $\log[(COG+COS)/POP]$, $\log[(TRGH+TRSH)/(POP \cdot PH_{-1})]$, $\log(Y/POP)_{-1}$, $\log(V/POP)_{-1}$, T
4	cnst2, cnst, $(KH/POP)_{-1}$, $[YD/(POP \cdot PH)]_{-1}$, $RMA_{-1}IHHA$, $[YD/(POP \cdot PH)]_{-2}$, $RMA_{-2}IHHA_{-1}$, $RMA_{-3}IHHA_{-2}$, $(KH/POP)_{-2}$, $(KH/POP)_{-3}$, $\Delta(IHH/POP)_{-1}$, $\Delta(IHH/POP)_{-2}$, $DELH(KH/POP)_{-1}-(IHH/POP)_{-1}$, $DELH_{-1}(KH/POP)_{-2}-(IHH/POP)_{-2}$, $DELH_{-2}(KH/POP)_{-3}-(IHH/POP)_{-3}$, $\log(1-D1GM-D1SM-D4G)_{-1}$, $\log(IM/POP)_{-1}$, $\log(EX/POP)_{-1}$, $\log[(JG \cdot HG+JM \cdot HM+JS \cdot HS)/POP]$, $\log[YNL/(POP \cdot PH)]_{-1}$, $100[(PD/PD_{-1})^4-1]_{-1}$, $\log[(COG+COS)/POP]$, $\log[(TRGH+TRSH)/(POP \cdot PH_{-1})]$
5	cnst, $\log(L1/POP1)_{-1}$, $\log(AA/POP)_{-1}$, UR_{-1}, $\log(1-D1GM-D1SM-D4G)_{-1}$, $\log(IM/POP)_{-1}$, $\log[(JG \cdot HG+JM \cdot HM+JS \cdot HS)/POP]$, $\log(PIM/PF)_{-1}$, $\log[YNL/(POP \cdot PH)]_{-1}$, $100[(PD/PD_{-1})^4-1]_{-1}$, $\log[(COG+COS)/POP]$, $\log(Y/POP)_{-1}$, $\log(V/POP)_{-1}$, T
6	cnst, $\log(AA/POP)_{-1}$, $\log(WA/PH)_{-1}$, T
7	cnst, $\log(L3/POP1)_{-1})$, $\log(WA/PH)_{-1}$, $\log(AA/POP)_{-1}$, UR_{-1}, $\log(1-D1GM-D1SM-D4G)_{-1}$, $\log(IM/POP)_{-1}$, $\log(EX/POP)_{-1}$, $\log[(JG \cdot HG+JM \cdot HM+JS \cdot HS)/POP]$, $\log(PIM/PF)_{-1}$, $100[(PD/PD_{-1})^4-1]_{-1}$, $\log[(TRGH+TRSH)/(POP \cdot PH_{-1})]$, $\log(Y/POP)_{-1}$, T
8	cnst, $\log(LM/POP)_{-1}$, $\log(WA/PH)_{-1}$, UR_{-1}, $\log(1-D1GM-D1SM-D4G)_{-1}$, $\log(IM/POP)_{-1}$, $\log(EX/POP)_{-1}$, $\log[(JG \cdot HG+JM \cdot HM+JS \cdot HS)/POP]$, $\log(PIM/PF)_{-1}$, $\log[YNL/(POP \cdot PH)]_{-1}$, $100[(PD/PD_{-1})^4-1]_{-1}$, $\log[(COG+COS)/POP]$, $\log[(TRGH+TRSH)/(POP \cdot PH_{-1})]$, RS_{-1}, RS_{-2}, RB_{-1}, $\log(Y/POP)_{-1}$, $\log(V/POP)_{-1}$, $\log(AA/POP)_{-1}$, T

Table A.9 (continued)

Eq.	First Stage Regressors
10	$\log PF_{-1}$, $\log[[WF(1+D5G)] - \log LAM]_{-1}$, cnst2, cnst, TB, T, $\log(PIM/PF)_{-1}$, UR_{-1}, $\log(1 - D1GM - D1SM - D4G)_{-1}$, $\log(IM/POP)_{-1}$, $\log(EX/POP)_{-1}$, $\log[YNL/(POP \cdot PH)]_{-1}$, $\log[(COG + COS)/POP]$, $\log[(TRGH + TRSH)/(POP \cdot PH_{-1})]$, $\log(Y/POP)_{-1}$, $\log(AA/POP)_{-1}$
11	cnst, $\log Y_{-1}$, $\log V_{-1}$, $D593$, $D594$, $D601$, $\log Y_{-2}$, $\log Y_{-3}$, $\log Y_{-4}$, $\log V_{-2}$, $\log V_{-3}$, $\log V_{-4}$, $D601_{-1}$, $D601_{-2}$, $D601_{-3}$, T, $\log(1 - D1GM - D1SM - D4G)_{-1}$, $\log(IM/POP)_{-1}$, $\log(EX/POP)_{-1}$, $\log(PIM/PF)_{-1}$, $\log[YNL/(POP \cdot PH)]_{-1}$, $100[(PD/PD_{-1})^4 - 1]_{-1}$, $\log[(COG+COS)/POP]$, $\log[(TRGH+TRSH)/(POP \cdot PH_{-1})]$, RS_{-1}, RB_{-1}, UR_{-1}
12	cnst2, cnst, $\log KK_{-1}$, $\log KK_{-2}$, $\log Y_{-1}$, $\log Y_{-2}$, $\log Y_{-3}$, $\log Y_{-4}$, $\log Y_{-5}$, $\log(KK/KKMIN)_{-1}$, $RB_{-2}(1-D2G_{-2}-D2S_{-2}) - 100(PD_{-2}/PD_{-6}) - 1)$, $(CG_{-2}+CG_{-3}+CG_{-4})/(PX_{-2}YS_{-2}+PX_{-3}YS_{-3}+PX_{-4}YS_{-4})$, $\log(1-D1GM-D1SM-D4G)_{-1}$, $\log(EX/POP)_{-1}$, $\log[(JG \cdot HG + JM \cdot HM + JS \cdot HS)/POP]$, $\log[YNL/(POP \cdot PH)]_{-1}$, $\log[(TRGH+TRSH)/(POP \cdot PH_{-1})]$, UR_{-1}, $\log(AA/POP)_{-1}$, T
13	cnst, $\log[JF/(JHMIN/HFS)]_{-1}$, $\Delta \log JF_{-1}$, $\Delta \log Y_{-1}$, $D593$, $\log(1 - D1GM - D1SM - D4G)_{-1}$, $\log(IM/POP)_{-1}$, $\log(EX/POP)_{-1}$, $\log[(JG \cdot HG + JM \cdot HM + JS \cdot HS)/POP]$, $\log(PIM/PF)_{-1}$, $\log[YNL/(POP \cdot PH)]_{-1}$, $100[(PD/PD_{-1})^4 - 1]_{-1}$, $\log[(COG+COS)/POP]$, $\log[(TRGH + TRSH)/(POP \cdot PH_{-1})]$, RS_{-1}, RS_{-2}, RB_{-1}, $\log(Y/POP)_{-1}$, $\log(V/POP)_{-1}$, UR_{-1}, $\log(AA/POP)_{-1}$, T
14	cnst, $\log(HF/HFS)_{-1}$, $\log[JF/(JHMIN/HFS)]_{-1}$, $\Delta \log Y_{-1}$, T, $\log[(JG \cdot HG + JM \cdot HM + JS \cdot HS)/POP]$, $\log(PIM/PF)_{-1}$, $100[(PD/PD_{-1})^4 - 1]_{-1}$, RS_{-1}, RS_{-2}, UR_{-1}
17	cnst, T, $\log(MF/PF)_{-1}$, $\log(X - FA)_{-1}$, $RS(1 - D2G - D2S)_{-1}$, T, $\log(1 - D1GM - D1SM - D4G)_{-1}$, $\log(IM/POP)_{-1}$, $\log(EX/POP)_{-1}$, $\log[(JG \cdot HG + JM \cdot HM + JS \cdot HS)/POP]$, $\log(PIM/PF)_{-1}$, $\log[YNL/(POP \cdot PH)]_{-1}$, $100[(PD/PD_{-1})^4 - 1]_{-1}$, $\log[(COG+COS)/POP]$, RS_{-2}, RB_{-1}, $\log(Y/POP)_{-1}$, $\log(V/POP)_{-1}$, UR_{-1}
18	cnst, $\log[(PIEF - TFG - TFS)/DF_{-1}]_{-1}$, $\log[(JG \cdot HG + JM \cdot HM + JS \cdot HS)/POP]$, $\log(PIM/PF)_{-1}$, $100[(PD/PD_{-1})^4 - 1]_{-1}$, RS_{-1}, RS_{-2}, UR_{-1}

Table A.9 (continued)

Eq.	First Stage Regressors
23	cnst, RB_{-1}, RB_{-2}, RS_{-1}, RS_{-2}, RS_{-3}, $\log(1 - D1GM - D1SM - D4G)_{-1}$, $\log(IM/POP)_{-1}$, $\log(EX/POP)_{-1}$, $\log[(JG \cdot HG + JM \cdot HM + JS \cdot HS)/POP]$, $\log(PIM/PF)_{-1}$, $\log[YNL/(POP \cdot PH)]_{-1}$, $100[(PD/PD_{-1})^4 - 1]_{-1}$, $\log[(COG + COS)/POP]$, $\log[(TRGH + TRSH)/(POP \cdot PH_{-1})]$, $\log(Y/POP)_{-1}$, $\log(V/POP)_{-1}$, $\log(AA/POP)_{-1}$, UR_{-1}, T
24	cnst, RM_{-1}, RS_{-1}, RS_{-2}, $\log(1 - D1GM - D1SM - D4G)_{-1}$, $\log(IM/POP)_{-1}$, $\log(EX/POP)_{-1}$, $\log[(JG \cdot HG + JM \cdot HM + JS \cdot HS)/POP]$, $\log(PIM/PF)_{-1}$, $\log[YNL/(POP \cdot PH)]_{-1}$, $100[(PD/PD_{-1})^4 - 1]_{-1}$, $\log[(COG + COS)/POP]$, $\log[(TRGH + TRSH)/(POP \cdot PH_{-1})]$, $\log(Y/POP)_{-1}$, $\log(V/POP)_{-1}$, $\log(AA/POP)_{-1}$, UR_{-1}, T
25	cnst, ΔRB_{-1}, $[[\Delta(PIEF - TFG - TFS + PX \cdot PIEB - TBG - TBS)]/(PX_{-1} \cdot YS_{-1})]_{-1}$, T, $\log(1 - D1GM - D1SM - D4G)_{-1}$, $\log(IM/POP)_{-1}$, $\log(EX/POP)_{-1}$, $\log[(JG \cdot HG + JM \cdot HM + JS \cdot HS)/POP]$, $\log(PIM/PF)_{-1}$, $\log[YNL/(POP \cdot PH)]_{-1}$, $100[(PD/PD_{-1})^4 - 1]_{-1}$, $\log[(COG + COS)/POP]$, $\log[(TRGH + TRSH)/(POP \cdot PH_{-1})]$, RS_{-1}, RS_{-2}, RB_{-1}, $\log(Y/POP)_{-1}$, $\log(V/POP)_{-1}$, UR_{-1}, $\log(AA/POP)_{-1}$
26	cnst, $\log[CUR_{-1}/(POP_{-1}PF)]_{-1}$, $\log[(X - FA)/POP]_{-1}$, RSA_{-1}, $\log[CUR_{-1}/(POP_{-1}PF_{-1})]$, T, $\log(1 - D1GM - D1SM - D4G)_{-1}$, $\log(IM/POP)_{-1}$, $\log(EX/POP)_{-1}$, $\log[(JG \cdot HG + JM \cdot HM + JS \cdot HS)/POP]$, $\log(PIM/PF)_{-1}$, $\log[YNL/(POP \cdot PH)]_{-1}$, $100[(PD/PD_{-1})^4 - 1]_{-1}$, $\log[(COG + COS)/POP]$, $\log[(TRGH + TRSH)/(POP \cdot PH_{-1})]$, RS_{-2}, RB_{-1}, $\log(Y/POP)_{-1}$, $\log(V/POP)_{-1}$, UR_{-1}, $\log(AA/POP)_{-1}$
27	cnst2, cnst, $\log(IM/POP)_{-1}$, $\log[(CS + CN + CD + IHH + IKF + IHB + IHF + IKB + IKH)/POP]_{-1}$, $\log(PF/PIM)_{-1}$, $D691$, $D692$, $D714$, $D721$, $\log(1 - D1GM - D1SM - D4G)_{-1}$, $\log(EX/POP)_{-1}$, $\log[(JG \cdot HG + JM \cdot HM + JS \cdot HS)/POP]$, $\log[YNL/(POP \cdot PH)]_{-1}$, $100[(PD/PD_{-1})^4 - 1]_{-1}$, $\log[(COG + COS)/POP]$, $\log[(TRGH + TRSH)/(POP \cdot PH_{-1})]$, RS_{-1}, RB_{-1}, $\log(Y/POP)_{-1}$, $\log(V/POP)_{-1}$, UR_{-1}, $\log(AA/POP)_{-1}$, T
28	cnst, $\log UB_{-1}$, $\log U_{-1}$, $\log WF_{-1}$, $\log UB_{-2}$, $\log(1 - D1GM - D1SM - D4G)_{-1}$, $\log(IM/POP)_{-1}$, $\log[(JG \cdot HG + JM \cdot HM + JS \cdot HS)/POP]$, $\log(PIM/PF)_{-1}$, $\log[YNL/(POP \cdot PH)]_{-1}$, $100[(PD/PD_{-1})^4 - 1]_{-1}$, $\log[(COG + COS)/POP]$, $\log[(TRGH + TRSH)/(POP \cdot PH_{-1})]$, RS_{-1}, RS_{-2}, T
30	cnst, RS_{-1}, $100[(PD/PD_{-1})^4 - 1]_{-1}$, UR_{-1}, ΔUR_{-1}, $PCM1_{-1}$, $D794823 \cdot PCM1_{-1}$, ΔRS_{-1}, ΔRS_{-2}, T, $\log(1 - D1GM - D1SM - D4G)_{-1}$, $\log(IM/POP)_{-1}$, $\log(EX/POP)_{-1}$, $\log[(JG \cdot HG + JM \cdot HM + JS \cdot HS)/POP]$, $\log(PIM/PF)_{-1}$, $\log[YNL/(POP \cdot PH)]_{-1}$, $\log[(COG + COS)/POP]$, $\log[(TRGH + TRSH)/(POP \cdot PH_{-1})]$, $\log(Y/POP)_{-1}$, $\log(V/POP)_{-1}$, $\log(AA/POP)_{-1}$

7 Appendix B: The ROW Model, November 11, 2013

7.1 The ROW Model in Tables

The tables that pertain to the ROW model are presented in this appendix. Table B.1 lists the countries in the model. The 38 countries for which structural equations are estimated are Canada (CA) through Peru (PE). Countries 40 through 59 are countries for which only trade share equations are estimated. The countries that make up the EMU are listed at the bottom of Table B.1. EMU is denoted EU in the model.

A detailed description of the variables per country is presented in Table B.2, where the variables are listed in alphabetical order. Data permitting, each of the countries has the same set of variables. Quarterly data were collected for countries 2 through 14, and annual data were collected for the others. Countries 2 through 14 will be referred to as "quarterly" countries, and the others will be referred to as "annual" countries. The way in which each variable was constructed is explained in brackets in Table B.2. All of the data with potential seasonal fluctuations have been seasonally adjusted.

Table B.3 lists the stochastic equations and the identities. The functional forms of the stochastic equations are given, but not the coefficient estimates. The coefficient estimates for all the countries are presented in Table B.4, where within this table the coefficient estimates and tests for equation 1 are presented in Table B1, for equation 2 in Table B2, and so on. The results in Table B.4 are discussed in Section 2.4. Table B.3 also lists the equations that pertain to the trade and price links among the countries, and it explains how the quarterly and annual data are linked for the trade share calculations. Table B.5 lists the links between the US and ROW models, and Table B.6 explains the construction of the balance of payments data—data for variables S and TT.

The rest of this appendix discusses the collection of the data and the construction of some of the variables.

7.2 The Raw Data

The data sets for the countries other than the United States (i.e., the countries in the ROW model) begin in 1960. The sources of the data are the IMF and OECD. Data from the IMF are international financial statistics (IFS) data and direction of trade (DOT) data. Data from the OECD are quarterly national accounts data, annual national accounts data, quarterly labor force data, and annual labor force data. These are the "raw" data. As noted above, the way in which each variable was constructed is explained in brackets in Table B.2. When "IFS" precedes a number or letter in the table, this refers to the IFS variable number or letter. Some variables were constructed directly from IFS and OECD data (i.e., directly from the raw data), and some were constructed from other (already constructed) variables. The construction of the EU variables is listed near the end of Table B.2.

7.3 Variable Construction

7.3.1 S, TT, and A: Balance of Payments Variables

One important feature of the data collection is the linking of the balance of payments data to the other export and import data. The two key variables involved in this process are S, the balance of payments on current account, and TT, the value of net transfers. The construction of these variables and the linking of the two types of data are explained in Table B.6. Quarterly balance of payments data do not generally begin as early as the other data, and the procedure in Table B.6 allows quarterly data on S to be constructed as far back as the beginning of the quarterly data for merchandise imports and exports ($M\$$ and $X\$$).

The variable A is the net stock of foreign security and reserve holdings. It is constructed by summing past values of S from a base period value of zero. The summation begins in the first quarter for which data on S exist. This means that the A series is off by a constant amount each period (the difference between the true value of A in the base period and zero). In the estimation work the functional forms were chosen in such a way that this error was always absorbed in the estimate of the constant term. It is important to note that A measures only the net asset position of the country vis-à-vis the rest of the world. Domestic wealth, such as the domestically owned housing stock and plant and equipment stock, is not included.

7.3.2 V: Stock of Inventories

Data on inventory investment, denoted $V1$ in the ROW model, are available for each country, but not data on the stock of inventories, denoted V. By definition $V = V_{-1} + V1$. Given this equation and data for $V1$, V can be constructed once a base period and base period value are chosen. The base period was chosen for each country to be the quarter or year prior to the beginning of the data on $V1$. The base period value was taken to be the value of Y in the base period for the quarterly countries and the value of $.25Y$ for the annual countries.

7.3.3 Excess Labor

Good capital stock data are not available for countries other than the US. If the short run production function for a country is one of fixed proportions and if capital is never the constraint, then the production function can be written:

$$Y = LAM(J \cdot H^a), \tag{1}$$

where Y is production, J is the number of workers employed, and H^a is the number of hours worked per worker. LAM is a coefficient that may change over time due to technical progress. The notation in equation (1)is changed slightly from that in equation A.1 for the US. J is used in place of JF because there is no disaggregation in the ROW model between the firm sector and other sectors. Similarly, H^a is used in place of HF^a. Note also that Y refers here to the total output of the country (real GDP), not just the output of the firm sector. Data on Y and J are available. Contrary to the case for the US, data on the number of hours paid for per worker (denoted HF in the US model) are not available.

Given the production function (1), excess labor is measured as follows for each country. $\log(Y/J)$ is first plotted for the sample period. This is from equation (1) a plot of $\log(LAM \cdot H^a)$. If it is assumed that at each peak of this plot H^a is equal to the same constant, say $\bar{H}$, then one observes at the peaks $\log(LAM \cdot \bar{H}$. Straight lines are drawn between the peaks (peak to peak interpolation), and $\log(LAM \cdot \bar{H}$ is assumed to lie on the lines. If, finally, $\bar{H}$ is assumed to be the maximum number of hours that each worker can work, then $Y/(LAM \cdot \bar{H})$ is the minimum number of workers required to produce Y, which is denoted $JMIN$ in the ROW model. $LAM \cdot \bar{H}$ is simply denoted LAM, and the equation determining $JMIN$ is equation I-11 in Table B.3. The actual number of workers on hand, J, can be compared to $JMIN$ to measure the amount of excess labor on hand.

7.3.4 YS: Potential Output

A measure of potential output, YS, is constructed for each country from peak-to-peak interpolations of $\log Y$. Given YS, a gap variable is constructed as $(YS - Y)/YS$, which is denoted ZZ in the ROW model. ZZ is determined by equation I-12 in Table B.3.

7.4 The Identities

The identities for each country are listed in Table B.3. There are up to 16 identities per country. Equation I-1 links the non NIPA data on imports (i.e., data on M and MS) to the NIPA data (i.e., data on IM). The variable $IMDS$ in the equation picks up the discrepancy between the two data sets. It is exogenous in the model. Equation I-2 is a similar equation for exports. Equation I-3 is the income identity; equation I-4 defines inventory investment as the difference between production and sales; and equation I-5 defines the stock of inventories as the previous stock plus inventory investment.

Equation I-6 defines S, the current account balance. Equation I-7 defines A, the net stock of foreign security and reserve holdings, as equal to last period's value plus S. (Remember that A is constructed by summing past values of S.)

Equation I-8 links M, total merchandise imports in 2005 lc, to $M95\$A$, merchandise imports from the countries in the trade share matrix in 2005$. The variable $M95\$B$ is the difference between total merchandise imports (in 2005$) and merchandise imports (in 2005$) from the countries in the trade share matrix. It is exogenous in the model.

Equation I-9 links E, the average exchange rate for the period, to EE, the end of period exchange rate. If the exchange rate changes fairly smoothly within the period, then E is approximately equal to $(EE + EE_{-1})/2$. A variable $PSI1$ was defined to make the equation $E = PSI1[(EE + EE_{-1})/2]$ exact, which is equation I-9. One would expect $PSI1$ to be approximately one and not to fluctuate much over time, which is generally the case in the data.

Equation I-10 defines the civilian unemployment rate, UR. $L1$ is the labor force. J is total employment. UR is equal to the number of people unemployed divided by the civilian labor force.

Equations I-11 pertains to the measurement of excess labor, and equation I-12 defines the demand pressure variable. These were discussed above.

Equation I-13 links PM, the import price deflator obtained from the IFS data, to PMP, the import price deflator computed from the trade share calculations. The variable that links the two, $PSI2$, is taken to be exogenous.

Equation I-14 links the exchange rate relative to the U.S. dollar, E, to the exchange rate relative to the German DM, H. This equation is used to determine H when equation 9 determines E, and it is used to determine E when equation 9 determines H.

Equation I-15 determines NW, an estimate of the net worth of the country. Net worth is equal to last period's net worth plus investment plus net exports.

Finally, equation I-16 defines the country's export price index in terms of U.S. dollars.

7.5 The Linking Equations

The equations that pertain to the trade and price links among countries are presented next in in Table B.3. All imports and exports in this part of the table are merchandise imports and exports only. The equations L-1 determine the trade share coefficients, a_{ij}. The estimation of the trade share equations is discussed in Section 2.4. a_{ij} is the share of i's merchandise exports to j out of total merchandise imports of j. Given a_{ij} and $M00\$A_j$, the total merchandise imports of j, the equations L-2 determine the level of exports from i to j, $XX00\$_{ij}$. The equations L-3 then determine the total exports of country i by summing $XX00\$_{ij}$ over j.

The equations L-4 link export prices to import prices. The price of imports of country i, PMP_i, is a weighted average of the export prices of other countries (except for country 59, the "all other" category, where no data on export prices were collected). The weight for country j in calculating the price index for country i is the share of country j's exports imported by i.

The equations L-5 define a world price index for each country, which is a weighted average of the 58 countries' export prices except the prices of the oil exporting countries. The world price index differs slightly by country because the own country's price is not included in the calculations. The weight for each country is its share of total exports of the relevant countries.

7.6 Solution of the MC Model

The way in which the US and ROW models are linked is explained in Table B.5. The two key variables that are exogenous in the US model but become endogenous in the overall MC model are exports, EX, and the price of imports, PIM. EX depends on $X00\$_{US}$, which is determined in Table B.3. PIM depends on PM_{US}, which depends on PMP_{US}, which is also determined in Table B.3.

Feeding into Table B.3 from the US model are PX_{US} and $M95\$A_{US}$. PX_{US} is determined is the same way that PX is determined for the other countries, namely by equation 11. In the US case $\log PX_{US} - \log PW\$_{US}$ is regressed on $\log GDPD - \log PW\$_{US}$. The equation is:

$$\log PX_{US} - \log PW\$_{US} = \lambda(\log GDPD - \log PW\$_{US})$$

This equation is estimated under the assumption of a second order autoregressive error for the 1962:1–2012:4 period. The estimate of λ is .854 with a t-statistic of 28.50. The estimates (t-statistics) of the two autoregressive coefficients are 1.40 (21.77) and $-.41$ (-6.31), respectively. The standard error is .0113. Given the predicted value of PX_{US} from this equation, PEX is determined by the identity listed in Table B.5: $PEX = DEL3 \cdot PX_{US}$. This identity replaces identity 32 in Table A.3 in the US model.

$M00\$A_{US}$, which, as just noted, feeds into Table B.3, depends on M_{US}, which depends on IM. This is shown in Table B.5. IM is determined by equation 27 in the US model. Equation 27 is thus the key equation that determines the U.S. import value that feeds into Table B.3.

Because some of the countries are annual, the overall MC model is solved a year at a time. A solution period must begin in the first quarter of the year. In the following discussion, assume that year 1 is the first year to be solved. The overall MC model is solved as follows:

1. Given values of $X00\$$, PMP, and $PW\$$ for all four quarters of year 1 for each quarterly country and for year 1 for each annual country, all the stochastic equations and identities are solved. For the annual countries "solved" means that the equations are passed through k_1 times for year 1, where k_1 is determined by experimentation (as discussed below). For the quarterly countries "solved" means that quarter 1 of year 1 is passed through k_1 times, then quarter 2 k_1 times, then quarter 3 k_1 times, and then quarter 4 k_1 times. The solution for the quarterly countries for the four quarters of year 1 is a dynamic simulation in the sense that the predicted values of the endogenous

variables from previous quarters are used, when relevant, in the solution for the current quarter.

2. Given from the solution in step 1 values of E, PX, and $M00\$A$ for each country, the calculations in Table B.3 can be performed. Since all the calculations in Table B.3 are quarterly, the annual values of E, PX, and $M00\$A$ from the annual countries have to be converted to quarterly values first. This is done in the manner discussed at the bottom of Table B.3. The procedure in effect takes the distribution of the annual values into the quarterly values to be exogenous. The second task is to compute $PX\$$ using equation L-1. Given the values of $PX\$$, the third task is to compute the values of α_{ij} from the trade share equations—see equation (9) in Subsection 3.7.2. This solution is also dynamic in the sense that the predicted value of α_{ij} for the previous quarter feeds into the solution for the current quarter. (Remember that the lagged value of α_{ij} is an explanatory variable in the trade share equations.) The fourth task is to compute $X95\$$, PMP, and $PW\$$ for each country using equations L-2, L-3, and L-4. Finally, for the annual countries the quarterly values of these three variables are then converted to annual values by summing in the case of $X95\$$ and averaging in the case of PMP and $PW\$$.

3. Given the new values of $X00\$$, PMP, and $PW\$$ from step 2, repeat step 1 and then step 2. Keep repeating steps 1 and 2 until they have been done k_2 times. At the end of this, declare that the solution for year 1 has been obtained.

4. Repeat steps 1, 2, and 3 for year 2. If the solution is meant to be dynamic, use the predicted values for year 1 for the annual countries and the predicted values for the four quarters of year 1 for the quarterly countries, when relevant, in the solution for year 2. Continue then to year 3, and so on.

I have found that going beyond $k_1 = 10$ and $k_2 = 10$ leads to very little change in the final solution values.

7.7 The Tables for the ROW Model

Table B.1
The Countries and Variables in the MCI Model

Quarterly Countries			**Local Currency**	**Trade Share Equations Only**		
1	US	United States	U.S. Dollar (mil.)	40	TU	Turkey
2	CA	Canada	Can. Dollar (mil.)	41	PD	Poland
3	JA	Japan	Yen (bil.)	42	RU	Russia
4	AU	Austria	Euro (mil.)	43	UE	Ukraine
5	FR	France	Euro (mil.)	44	EG	Egypt
6	GE	Germany	Euro (mil.)	45	IS	Israel
7	IT	Italy	Euro (mil.)	46	KE	Kenya
8	NE	Netherlands	Euro (mil.)	47	BA	Bangladesh
9	ST	Switzerland	Swiss Franc (bil.)	48	HK	Hong Kong
10	UK	United Kingdom	Pound Sterling (mil.)	49	SI	Singapore
11	FI	Finland	Euro (mil.)	50	VI	Vietnam
12	AS	Australia	Aust. Dollar (mil.)	51	NI	Nigeria
13	SO	South Africa	Rand (mil.)	52	AL	Algeria
14	KO	Rep. of Korea	Won (bil.)	53	IA	Indonesia
Annual Countries				54	IN	Iran
15	BE	Belgium	Euro (mil.)	55	IQ	Iraq
16	DE	Denmark	Den. Kroner (bil.)	56	KU	Kuwait
17	NO	Norway	Nor. Kroner (bil.)	57	LI	Libya
18	SW	Sweden	Swe. Kroner (bil.)	58	UA	United Arab Emirates
19	GR	Greece	Euro (mil.)	59	AO	All Other
20	IR	Ireland	Euro (mil.)			
21	PO	Portugal	Euro (mil.)			
22	SP	Spain	Euro (mil.)			
23	NZ	New Zealand	N.Z. Dollar (mil.)			
24	SA	Saudi Arabia	Riyals (bil.)			
25	VE	Venezuela	Bolivares (bil.)			
26	CO	Colombia	Col. Pesos (bil.)			
27	JO	Jordan	Jor. Dinars (mil.)			
28	SY	Syria	Syr. Pound (mil.)			
29	ID	India	Ind. Rupee (bil.)			
30	MA	Malaysia	Ringgit (mil.)			
31	PA	Pakistan	Pak. Rupee (bil.)			
32	PH	Philippines	Phil. Peso (bil.)			
33	TH	Thailand	Baht (bil.)			
34	CH	China	Yuan (bil.)			
35	AR	Argentina	Arg. Peso (mil.)			
36	BR	Brazil	Reais (mil.)			
37	CE	Chile	Chi. Peso (bil.)			
38	ME	Mexico	New Peso (mil.)			
39	PE	Peru	Nuevos Soles (mil.)			

• The countries that make up the EMU, denoted EU in the model, are AU, FR, GE, IT, NE, FI, BE, IR, PO, SP, GR. (GR begins in 2001.) (Luxembourg, which is also part of the EMU, is not in the model.)
• Prior to 1999:1 the currency is Schillings for AU, Fr. Francs for FR, DM for GE, Lira for IT, Guilders for NE, Markkaa for FI, Bel. Francs for BE, Irish Pounds for IR, Escudes for PO, Pesetas for SP, and Drachmas for GR (prior to 2001:1). The units are in euro equivalents. For example, in 1999:1 the Lira was converted to the euro at 1936.27 Liras per euro, and 1936.27 was used to convert the Lira to its euro equivalent for 1998:4 back.
• The NIPA base year is 2005 for all countries except BE (2010), NO (1995), IR (2010), PO (2006), SP (2000), NZ (1995), ME (2003).

Table B.2
The Variables for a Given Country in Alphabetical Order

Variable	Eq. No.	Description
a_{ij}	L-1	Share of i's merchandise exports to j out of total merchandise imports of j. [See below]
A	I-7	Net stock of foreign security and reserve holdings, end of quarter, in lc. [$A_{-1} + S$. Base value of zero used for the quarter prior to the beginning of the data.]
C	2	Personal consumption in constant lc. [OECD data or IFS96F/PY]
E	9 or I-14	Exchange rate, average for the period, lc per $. [IFSRF]
EE	I-9	Exchange rate, end of period, lc per $. [IFSAE]
EX	I-2	Total exports (NIPA) in constant lc. [OECD data or (IFS90C or IFS90N)/ PY]
$EXDS$	exog	Discrepancy between NIPA export data and other export data in constant lc. [$EX - PX00(E00 \cdot X00\$ + XS)$.]
$E00$	exog	E in 2005, 2005 lc per 2005 $. [IFSRF in 2005]
F	10	Three-month forward exchange rate, lc per $. [IFSB]
G	exog	Government purchases of goods and services in constant lc. [OECD data or (IFS91F or IFS91FF)/PY] (Denoted GZ for countries CO and TH.)
H	9	Exchange rate, average for the period, lc per DM euro. [E/E_{GE}]
I	3	Gross fixed investment in constant lc. [OECD data or IFS93/PY]
IM	I-1	Total imports (NIPA) in constant lc. [OECD data or IFS98C/PY]
$IMDS$	exog	Discrepancy between NIPA import data and other import data in constant lc. [$IM - PM00(M + MS)$]
J	13	Total employment in millions. [OECD data or IFS67 or IFS67E or IFS67EY or IFS67EYC]
$JMIN$	I-13	Minimum amount of employment needed to produce Y in millions. [Y/LAM]
LAM	exog	Computed from peak-to-peak interpolation of $\log(Y/J)$.
$L1$	14	Labor force in millions. [OECD data]
M	1	Total merchandise imports (fob) in 2005 lc. [IFS71V/PM]
MS	exog	Other goods, services, and income (debit) in 2005 lc, BOP data. [((IFS78AED+IFS78AHD)E)/PM]
$M00\$A$	I-8	Merchandise imports (fob) from the trade share matrix in 2005 $. [See below]
$M00\$B$	exog	Difference between total merchandise imports and merchandise imports from the trade share matrix in 2005 $ (i.e., imports from countries other than the 44 in the trade share matrix). [$M/E00 - M00\$A$]
$M1$	6	Money supply in lc. [IFS34 or IFS34A.N+IFS34B.N or IFS35L.B or IFS39MAC or IFS59MA or IFS59MC]
NW	I-15	National Wealth in constant lc. [$NW_{-1} + I + V1 + EX - IM$. Base value of zero used for the quarter prior to the beginning of the data.]
PM	I-13	Import price deflator, 2005 = 1.0. [IFS75/100]
PMP	L-4	Import price index from DOT data, 2005 = 1.0. [See below]
$PM00$	exog	PM in the NIPA base year divided by PM in 2005.
POP	exog	Population in millions. [IFS99Z]
$POP1$	exog	Population of labor-force-age in millions. [OECD data]
$PSI1$	exog	[$(EE + EE_{-1})/2]/E$]
$PSI2$	exog	[PM/PMP]
$PW\$$	L-5	World price index, $/2005$. [See below]
PX	11	Export price index, 2005 = 1.0. [IFS74/100. If no IFS74 data for t, then $PX_t = PX\$_t(E_t/E00_t$, where $PX\$_t$ is defined next.]

Table B.2 (continued)

Variable	Eq. No.	Description
$PX\$$	I-16	Export price index, \$/2005\$, 2005 = 1.0. [$(E00 \cdot PX)/E$. If no IFS74 data at all, then $PX\$_t = PX_{USt}$ for all t. If IFS74 data only from t through $t+h$, then for $i > 0$, $PX\$_{t-i} = PX\$_t(PX_{USt-i}/PX_{USt}$ and $PX\$_{t+h+i} = PX\$_{t+h}(PX_{USt+k+i}/PX_{USt}$.
$PX00$	exog	PX in the NIPA base year divided by PX in 2005.
PY	5	GDP or GNP deflator, equals 1.0 in the NIPA base year. [OECD data or (IFS99B/IFS99B.P]
RB	8	Long term interest rate, percentage points. [IFS61]
RS	7	Three-month interest rate, percentage points. [IFS60 or IFS60B or IFS60C or IFS60L or IFS60P]
S	I-6	Total net goods, services, and transfers in lc. Current account balance. [See Table B.6] (Denoted SZ for countries CO and TH.)
$STAT$	exog	Statistical discrepancy in constant lc. [$Y - C - I - G - EX + IM - V1$]
T	exog	Time trend. [For quarterly data, 1 in 1952.1, 2 in 1952.2, etc.; for annual data, 1 in 1952, 2 in 1953, etc.]
TT	exog	Total net transfers in lc. [See Table B.7]
UR	I-10	Unemployment rate. [$(L1 - J)/L1$]
V	I-5	Stock of inventories, end of period, in constant lc. [$V_{-1} + V1$. Base value of zero was used for the period (quarter or year) prior to the beginning of the data.]
$V1$	I-4	Inventory investment in constant lc. [OECD data or IFS93I/PY]
W	not used	Nominal wage rate. [IFS65..C or IFS65A or IFS65EY or IFS65UMC]
X	I-3	Final sales in constant lc. [$Y - V1$] (Denoted XZ for country PE.)
XS	exog	Other goods, services, and income (credit) in 2005 lc. BOP data. [(E(IFS78ADD+IFS78AGD))/PX]
$X00\$$	L-3	Merchandise exports from the trade share matrix in 2005 \$. [See below]
$XX00\$_{ij}$	L-2	Merchandise exports from i to j in 2005\$. [See below]
Y	4	Real GDP or GNP in constant lc. [OECD data or IFS99B.P or IFS99B.R]
YS	exog	Potential value of Y. [From a peak-to-peak interpolation of log Y.]
ZZ	I-12	Demand pressure variable. [$\log Y - \log YS$]

Construction of variables related to the trade share matrix:

The raw data are:

$XX\$_{ij}$	Merchandise exports from i to j in \$, $i, j = 1, ..., 58$ [DOT data. 0 value used if no data]
$X\$_i$	Total merchandise exports (fob) in \$. $i = 1, ..., 39$ [IFS70/E or IFS70D]

The constructed variables are:

$XX\$_{i59} =$	$X\$_i - \sum_{j=1}^{58} XX\$_{ij}, i = 1, ..., 39$
$XX00\$_{ij} =$	$XX\$_{ij}/PX\$_i, i = 1, ..., 39, j = 1, ..., 59$ and $i = 40, ..., 58, j = 1, ..., 58$
$M00\$A_i =$	$\sum_{j=1}^{58} XX00\$_{ji}, i = 1, ..., 58; M00\$A_{59} = \sum_{j=1}^{39} XX00\$_{j59}$
$a_{ij} =$	$XX00\$_{ij}/M00\$A_j, i = 1, ..., 39, j = 1, ..., 59$ and $i = 40, ..., 58, j = 1, ..., 58$
$X00\$_i =$	$\sum_{j=1}^{59} XX00\$_{ij}, i = 1, ..., 39; X00\$_i = \sum_{j=1}^{58} XX00\$_{ij}, i = 40, ..., 58$
$PMP_i =$	$(E_i/E00_i) \sum_{j=1}^{58} a_{ji} PX\$_j, i = 1, ..., 39$
$PW\$_i =$	$(\sum_{j=1}^{58} PX\$_j X00\$_j)/(\sum_{j=1}^{58} X00\$_j), i = 1, ..., 39$ An element in this summation is skipped if $j = i$. This summation also excludes the oil exporting countries, which are SA, VE, NI, AL, IA, IN, IQ, KU, LI, UA.

- Variables available for trade share only countries are $M00\$A$, $PX\$$, $X00\$$.
- lc = local currency
- IFSxxxxx = variable number xxxxx from the IFS data

Table B.2 (continued)
The EU Variables

Variable	Eq. No.	Description
E	9	Exchange rate, average for the period, euro per \$. [IFSRF]
PY	[ ]	GDP deflator. $[(\sum_{i=1}^{6} PY_i Y_i)/Y_{EU}$, where the summation is for $i =$ GE, AU, FR, IT, NE, FI.]
RB	8	Long term interest rate, percentage points. [IFS61]
RS	7	Three-month interest rate, percentage points. [IFS60]
Y	[ ]	Real GDP in constant euros. $[Y_{GE} + \sum_{i=1}^{5}[Y_i/(E00_i/E00_{GE})]$, where the summation is for $i =$ AU, FR, IT, NE, FI.]
YS	[ ]	Potential value of Y_{EU}. $[YS_{GE} + \sum_{i=1}^{5}[YS_i/(E00_i/E00_{GE})]$, where the summation is for $i =$ AU, FR, IT, NE, FI.]
ZZ	I-18	Demand pressure variable. $[\log Y_{EU} - \log YS_{EU}]$

Table B.3
The Equations for a Given Country

STOCHASTIC EQUATIONS

Eq.	LHS Variable	Explanatory Variables
1	$\log(IM/POP)$	cnst, $\log(IM/POP)_{-1}$, $\log(PY/PM)$, $\log[(C+I+G)/POP]$ [Total Imports (NIPA), constant lc]
2	$\log(C/POP)$	cnst, $\log(C/POP)_{-1}$, RS or RB, $\log(Y/POP)$ [Consumption, constant lc]
3	$\log I$	cnst, $\log I_{-1}$, $\log Y$, RS or RB [Fixed Investment, constant lc]
4	$\log Y$	$\log Y_{-1}$, $\log X$, $\log V_{-1}$ [Real GDP, constant lc]
5	$\log PY$	cnst, $\log PY_{-1}$, $\log PM$, ZZ, T [GDP Price Deflator, base year = 1.0]
6	$\log[M1/(POP \cdot PY)]$	cnst, $\log[M1/(POP \cdot PY)]_{-1}$ or $\log[M1_{-1}/(POP_{-1}PY)]$, RS, $\log(Y/POP)$ [Money Supply, lc]
7	RS	cnst, RS_{-1}, $100[(PY/PY_{-1})^4 - 1]$, ZZ, RS_{GE}, RS_{US} [Three-Month Interest Rate, percentage points]
8	$RB - RS_{-2}$	cnst, $RB_{-1} - RS_{-2}$, $RS - RS_{-2}$, $RS_{-1} - RS_{-2}$ [Long Term Interest Rate, percentage points]
9	$\Delta \log E$	cnst, $\log(PY/PY_{US} - \log E_{-1}$, $.25\log[(1 + RS/100)/(1 + RS_{US}/100)]$ [Exchange Rate, lc per \$] [For all countries but AU, FR, IT, NE, ST, UK, FI, BE, DE, NO, SW, GR, IR, PO, and SP]
9	$\Delta \log H$	cnst, $\log(PY/PY_{GE} - \log H_{-1}$, $.25\log[(1 + RS/100)/(1 + RS_{GE}/100)]$ [Exchange Rate, lc per DM] [For countries AU, FR, IT, NE, ST, UK, FI, BE, DE, NO, SW, GR, IR, PO, and SP]
10	$\log F$	$\log EE$, $.25\log[(1 + RS/100)/(1 + RS_{US}/100)]$ [Three-Month Forward Rate, lc per \$]
11	$\log PX - \log[PW\$(E/E00)]$	$\log PY - \log[PW\$(E/E00)]$ [Export Price Index, 2005 = 1.0]
13	$\Delta \log J$	cnst, T, $\log(J/JMIN)_{-1}$, $\Delta \log Y$, $\Delta \log Y_{-1}$ [Employment, millions]
14	$\log(L1/POP1)$	cnst, T, $\log(L1/POP1)_{-1}$, UR [Labor Force, millions]

Table B.3 (continued)

IDENTITIES

Eq.	LHS Variable	Explanatory Variables
I-1	$M =$	$(IM - IMDS)/PM00 - MS$ [Merchandise Imports, 2005 lc]
I-2	$EX =$	$PX00(E00 \cdot X00\$ + XS) + EXDS$ [Total Exports (NIPA), constant lc]
I-3	$X =$	$C + I + G + EX - IM + STAT$ [Final Sales, constant lc]
I-4	$V1 =$	$Y - X$ [Inventory Investment, constant lc]
I-5	$V =$	$V_{-1} + V1$ [Inventory Stock, constant lc]
I-6	$S =$	$PX(E00 \cdot X00\$ + XS) - PM(M + MS) + TT$ [Current Account Balance, lc]
I-7	$A =$	$A_{-1} + S$ [Net Stock of Foreign Security and Reserve Holdings, lc]
I-8	$M00\$A =$	$M/E00 - M00\$B$ [Merchandise Imports from the Trade Share Calculations, 2005 $]
I-9	$EE =$	$2PSI1 \cdot E - EE_{-1}$ [Exchange Rate, end of period, lc per $]
I-10	$UR =$	$(L1 - J)/L1$ [Unemployment Rate]
I-11	$JMIN =$	Y/LAM [Minimum Required Employment, millions]
I-12	$ZZ =$	$\log Y - \log YS$ [Demand Pressure Variable]
I-13	$PM =$	$PSI2 \cdot PMP$ [Import Price Deflator, 2005 = 1.0]
I-14	E	$E = H \cdot E_{GE}$ [Exchange Rate: lc per $] [Equation relevant for countries AU, FR, IT, NE, ST, UK, FI, BE, DE, NO, SW, GR, IR, PO, and SP only]
I-15	$NW =$	$NW_{-1} + I + V1 + EX - IM$ [National Wealth, constant lc]
I-16	$PX\$ =$	$(E00/E)PX$ [Export Price Index, $/2005$]

• From 1999:1 on for GE: $E_{GE} = E_{EU}$, $RS_{GE} = RS_{EU}$, and $RB_{GE} = RB_{EU}$. From 1999:1 on for an EU country i (except GE): $H_i = 1.0$, $RS_i = RS_{EU}$, and $RB_i = RB_{EU}$.
• $PX\$$ and $M00\$A$ are exogenous for trade share only countries.

Table B.3 (continued)

Equations that Pertain to the Trade and Price Links Among Countries

L-1	$a_{ij} =$	fraction of country i's exports imported by j. Computed from trade share equations [Trade Share Coefficients]
L-2	$XX00\$_{ij} =$	$a_{ij}M00\$A_j, i = 1,...,39, j = 1,...,59$ and $i = 40,...,58, j = 1,...,58$ [Merchandise Exports from i to j, 2005$]
L-3	$X00\$_i =$	$\sum_{j=1}^{59} XX00\$_{ij}, i = 1,...,39$
	$X00\$_i =$	$\sum_{j=1}^{58} XX00\$_{ij}, i = 40,...,58$ [Total Merchandise Exports, 2005$]
L-4	$PMP_i =$	$(E_i/E00_i)\sum_{j=1}^{58} a_{ji}PX\$_j, i = 1,...,39$ [Import Price Deflator, 2005 = 1.0]
L-5	$PW\$_i =$	$(\sum_{j=1}^{58} PX\$_j X00\$_j)/\sum_{j=1}^{58} X00\$_j), i = 1,...,39$ An element in this summation is skipped if $j = i$. This summation also excludes the oil exporting countries, which are SA, VE, NI, AL, IA, IN, IQ, KU, LI, UA. [World Price Index, $/2005$]

Trade Share Equations

• For each i, j equation, the left hand side variable is $\log(a_{ijt} + .00001)$. The three right hand side variables are the constant, $\log(a_{ijt-1} + .00001)$, and $PX\$_{it}/(\sum_{k=1}^{58} a_{kjt-1}PX\$_{kt})$, where the summation excludes the oil exporting countries, which are SA, VE, NI, AL, IA, IN, IQ, KU, LI, UA. Also, an element in the summation is skipped if $k = j$.

Linking of the Annual and Quarterly Data

• Quarterly data exist for all the trade share calculations, and all these calculations are quarterly. Feeding into these calculations from the annual models are predicted annual values of $PX\$_i$, $M00\$A_i$, and E_i. For each of these three variables the predicted value for a given quarter was taken to be the predicted annual value multiplied by the ratio of the actual quarterly value to the actual annual value. This means in effect that the distribution of an annual value into its quarterly values is taken to be exogenous.

• Once the quarterly values have been computed from the trade share calculations, the annual values of $X00\$_i$ that are needed for the annual models are taken to be the sums of the quarterly values. Similarly, the annual values of PMP_i and $PW\$_i$ are taken to be the averages of the quarterly values.

Table B.4
Coefficient Estimates and Test Results
for the ROW Equations

ρ = first order autoregressive coefficient of the error term.
† = variable is lagged one period.
t-statistics are in parentheses.

Table B1: Coefficient Estimates for Equation 1

$$\log(IM/POP) = a_1 + a_2 \log(IM/POP)_{-1} + a_3 \log(PY/PM)$$
$$+ a_4 \log[(C + I + G)/POP)]$$

	a_1	a_2	a_3	a_4	ρ	SE	DW
Quarterly							
CA	-0.160	0.943	0.086	0.069	0.320	0.0279	2.05
	(-0.55)	(37.55)	(2.47)	(1.35)	(4.40)		1961.2–2013.2
JA	-0.014	0.963	0.033	0.028	0.134	0.0319	2.03
	(-0.09)	(50.16)	(2.90)	(0.79)	(1.77)		1966.1–2013.2
AU	-0.674	0.925	0.024	0.145		0.0283	1.86
	(-1.47)	(37.89)	(0.69)	(2.04)			1970.1–2013.1
FR	-1.321	0.874	0.065	0.258		0.0275	1.92
	(-3.62)	(28.98)	(4.13)	(3.87)			1961.1–2012.4
GE	-0.113	0.988	0.022	0.025		0.0232	1.95
	(-0.50)	(87.47)	(1.43)	(0.72)			1971.1–2012.4
IT	-0.672	0.914	0.051	0.150		0.0351	1.81
	(-2.71)	(40.99)	(3.44)	(3.22)			1961.1–2013.1
NE	-0.031	0.947	0.069	0.055		0.0216	1.86
	(-0.13)	(48.75)	(3.77)	(1.29)			1961.1–2013.1
ST	-0.399	0.907	0.042	0.213	0.338	0.0227	2.08
	(-1.35)	(22.09)	(1.28)	(1.60)	(3.91)		1971.1–2013.2
UK	-2.193	0.754	0.022	0.467		0.0277	1.90
	(-5.14)	(16.41)	(1.34)	(5.27)			1961.1–2013.2
FI	-0.269	0.952	0.034	0.073		0.0538	2.50
	(-0.95)	(35.29)	(1.15)	(1.37)			1965.1–2013.2
AS	-1.812	0.841	0.076	0.327		0.0373	1.46
	(-4.69)	(24.47)	(3.50)	(4.72)			1966.1–2013.2
SO	-0.350	0.914	0.017	0.113		0.0703	1.83
	(-0.82)	(26.17)	(0.51)	(1.64)			1962.1–2012.4
KO	-0.652	0.897		0.170		0.0497	1.87
	(-3.41)	(33.71)		(3.73)			1974.1–2012.4
Annual							
BE	0.082	0.877	0.276	0.116		0.0504	2.03
	(0.06)	(8.68)	(3.00)	(0.53)			1962–2012
DE	-1.265	0.826	0.047	0.380		0.0514	2.05
	(-1.86)	(12.22)	(0.43)	(2.19)			1962–2012
NO	0.340	0.667	0.197	0.219		0.0518	1.65
	(0.77)	(4.97)	(2.93)	(1.35)			1962–2012
SW	-0.400	0.938	0.030	0.128		0.0592	1.97
	(-0.41)	(9.27)	(0.34)	(0.50)			1965–2012
GR	-0.372	0.900	0.144	0.129		0.0762	1.68
	(-0.37)	(10.87)	(1.40)	(0.74)			1962–2009
IR	-1.711	0.813	0.118	0.356		0.0692	1.01
	(-1.64)	(10.25)	(1.22)	(2.04)			1962–2012
PO	-1.385	0.295	0.482	0.773		0.0706	1.53
	(-2.48)	(2.68)	(5.71)	(5.63)			1962–2012
SP	-1.699	0.681	0.312	0.453		0.0712	1.29
	(-1.18)	(7.97)	(4.35)	(2.09)			1962–2012
NZ	-3.149	0.660	0.275	0.609		0.0716	1.83
	(-1.45)	(4.72)	(2.73)	(1.86)			1962–2011
SA	-0.649	0.564		0.499		0.1426	0.96
	(-1.71)	(4.93)		(3.14)			1970–2012
VE	-2.488	0.112		1.401		0.1281	1.18
	(-6.14)	(0.99)		(6.89)			1962–2012
CO	-2.163	0.140	0.116	0.941		0.0751	1.49
	(-1.83)	(0.98)	(2.49)	(5.20)			1970–2012

Table B1: Coefficient Estimates for Equation 1

	a_1	a_2	a_3	a_4	ρ	SE	DW
JO	-1.038	0.320		0.771		0.1040	1.01
	(-1.35)	(2.42)		(4.62)			1978–2009
SY	-4.594	0.304	0.097	1.042		0.1253	1.37
	(-3.63)	(2.29)	(2.68)	(4.74)			1965–2009
ID	-1.008	0.846		0.398		0.1032	1.82
	(-1.91)	(8.79)		(2.00)			1962–2012
MA	-1.266	0.793		0.339		0.0944	1.58
	(-1.75)	(10.70)		(2.36)			1972–2012
PA	-1.030	0.420		0.600		0.0895	1.39
	(-3.18)	(3.56)		(4.19)			1974–2012
PH	-1.276	0.781		0.475		0.1592	2.13
	(-1.52)	(8.88)		(1.82)			1962–2012
TH	-0.968	0.737		0.454		0.1034	1.64
	(-3.01)	(9.30)		(3.30)			1962–2012
CH	-0.850	0.574		0.581		0.1111	1.40
	(-2.83)	(4.98)		(3.34)			1984–2011
BR	-2.514	0.764		0.453		0.1008	2.23
	(-0.96)	(4.89)		(1.31)			1995–2011
CE	-2.322	0.343		0.851		0.1022	1.18
	(-3.57)	(2.27)		(4.26)			1978–2012
ME	-2.775	0.825	0.294	0.404		0.1542	1.36
	(-1.71)	(11.72)	(2.15)	(2.10)			1962–2011
PE	-8.972			1.803		0.0746	1.14
	(*****)			(21.44)			1992–2012

Table B1: Test Results for Equation 1

	Lags	log PY	RHO	T	Stability			End Test		overid	
	p-val	p-val	p-val	p-val	AP	df	λ	p-val	End	p-val	df
Quarterly											
CA	0.000	0.319	0.000	0.122	23.23	5	5.086	1.000	1998.4		
JA	0.046	0.000	0.059	0.001	21.68	5	6.257	0.352	1998.3		
AU	0.758	0.000	0.000	0.003	28.82	4	4.872	1.000	1998.3		
FR	0.170	0.800	0.037	0.713	7.76	4	3.226	0.602	1998.3	0.004	5
GE	0.067	0.466	0.429	0.033	8.75	4	4.573	0.345	1998.4		
IT	0.118	0.938	0.023	0.012	11.48	4	3.204	1.000	1998.3	0.000	5
NE	0.130	0.661	0.362	0.009	4.65	4	1.859	0.979	1998.4	0.032	5
ST	0.000	0.947	0.000	0.000	38.85	5	1.428	0.412	1998.3		
UK	0.197	0.977	0.311	0.065	7.25	4	3.453	0.890	1998.3	0.006	5
FI	0.000	0.893	0.001	0.373	35.37	4	2.223	1.000	1998.3	0.000	4
AS	0.000	0.564	0.000	0.012	8.07	4	4.945	1.000	1998.2	0.000	6
SO	0.253	0.650	0.000	0.767	7.26	4	9.078	0.910	1998.3		
KO	0.763		0.000	0.001	18.91	3	3.643	0.512	1998.4		
Annual											
BE	0.328	0.642	0.947	0.001	34.83	4	7.384	0.056	1996	0.006	5
DE	0.327	0.134	0.872	0.000	43.52	4	7.384	0.636	1998	0.000	5
NO	0.278	0.385	0.010	0.913	32.81	4	7.384	0.955	1998	0.194	5
SW	0.024	0.719	0.786	0.023	39.17	4	9.411	0.105	1998	0.000	5
GR	0.281	0.002	0.137	0.004	10.36	4	8.251	0.240	1998	0.045	5
IR	0.011	0.284	0.000	0.122	21.52	4	4.003	0.045	1998		
PO	0.012	0.863	0.244	0.677	5.29	4	7.384	0.938	1995		
SP	0.031	0.579	0.000	0.039	13.97	4	7.384	0.273	1998		
NZ	0.289	0.000	0.000	0.000	22.73	4	7.638	1.000	1998	0.000	5
SA	0.057		0.000	0.255	15.57	3	4.510	0.857	1998		
VE	0.315		0.000	0.832	2.12	3	3.363	0.682	1998		
CO	0.043	0.277	0.000	0.004	5.92	4	4.510	1.000	1998		
JO	0.023		0.000	0.314							
SY	0.326	0.206	0.002	0.116	9.66	4	6.911	1.000	1998		
ID	0.602		0.478	0.231	5.40	3	7.384				
MA	0.484		0.140	0.767	10.79	3	3.901	0.917	1998		
PA	0.079		0.000	0.000	3.93	3	3.335	0.000	1998		
PH	0.318		0.327	0.048	21.66	3	7.384	1.000	1999		
TH	0.723		0.000	0.102	2.84	3	7.384	0.045	1998		
CH	0.031		0.062	0.591							
CE	0.558		0.000	0.004	1.13	3	2.043				
ME	0.384	0.000	0.000	0.000	13.84	4	7.638	1.000	1998		

Table B2: Coefficient Estimates for Equation 2

$$\log(C/POP) = a_1 + a_2 \log(C/POP)_{-1} + a_3 RS + a_4 RB + a_5 \log(Y/POP)$$

	a_1	a_2	a_3	a_4	a_5	ρ	SE	DW
Quarterly								
CA	-0.006	0.901		-0.0012†	0.094		0.0073	2.09
	(-0.30)	(48.94)		(-5.78)	(5.08)		1961.2–2013.2	
JA	0.130	0.906		-0.0011	0.069	-0.232	0.0099	2.05
	(6.25)	(27.77)		(-3.07)	(2.24)	(-3.23)	1966.1–2013.2	
AU	0.134	0.944	-0.0004		0.037		0.0123	2.74
	(3.30)	(26.32)	(-0.73)		(1.08)		1970.1–2013.1	
FR	-0.014	0.810	-0.0002		0.180		0.0069	1.96
	(-0.67)	(28.73)	(-1.58)		(6.35)		1961.1–2012.4	
GE	0.025	0.931			0.063		0.0095	2.31
	(1.05)	(45.32)			(2.97)		1963.1–2013.2	
IT	0.033	0.937			0.056		0.0071	1.36
	(0.76)	(29.50)	(-0.05)		(1.61)		1961.1–2013.1	
NE	0.163	0.942		-0.0004	0.036		0.0097	2.14
	(4.25)	(43.94)		(-0.76)	(1.75)		1967.1–2013.1	
ST	0.044	0.926		-0.0014	0.047		0.0045	1.95
	(3.26)	(28.80)		(-3.96)	(1.67)		1977.1–2013.2	
UK	-0.342	0.805		-0.0013	0.226		0.0104	1.63
	(-5.35)	(25.70)		(-4.35)	(6.11)		1961.1–2013.2	
FI	0.026	0.845	-0.0004		0.142		0.0127	2.43
	(0.92)	(25.00)	(-1.51)		(4.27)		1961.1–2013.2	
AS	-0.078	0.899			0.104		0.0070	1.99
	(-2.89)	(42.39)			(4.68)		1966.1–2013.2	
SO	0.084	0.955	-0.0006†		0.034		0.0195	2.23
	(0.67)	(35.27)	(-2.00)		(1.20)		1962.1–2012.4	
KO	0.201	0.861		-0.0009	0.106		0.0186	1.89
	(3.88)	(19.69)		(-1.57)	(2.71)		1974.1–2012.4	
Annual								
BE	0.228	0.805			0.162		0.0134	1.62
	(3.11)	(9.38)			(1.90)		1962–2012	
DE	0.235	0.623			0.287		0.0216	1.82
	(3.79)	(6.36)			(3.51)		1962–2012	
NO	0.041	0.934			0.053		0.0217	1.55
	(0.97)	(19.75)			(1.28)		1962–2012	
SW	0.245	0.641			0.272		0.0165	1.14
	(3.42)	(7.76)			(4.26)		1965–2012	
GR	0.009	0.881	-0.0014		0.117		0.0213	1.39
	(0.06)	(22.05)	(-2.47)		(2.36)		1962–2009	
IR	1.136	0.567		-0.0034	0.296		0.0257	1.25
	(6.14)	(7.26)		(-2.91)	(5.17)		1962–2012	
PO	0.247	0.618		-0.0015	0.341		0.0345	1.50
	(2.30)	(8.03)		(-1.65)	(4.47)		1962–2012	
SP	0.182	0.478			0.476		0.0132	0.99
	(3.00)	(6.80)			(6.76)		1962–2012	

Table B2: Coefficient Estimates for Equation 2

	a_1	a_2	a_3	a_4	a_5	ρ	SE	DW
NZ	0.033	0.562		-0.0027	0.415		0.0169	1.47
	(0.24)	(5.66)		(-3.26)	(4.32)			1962–2011
SA	-0.019	0.846			0.119		0.1411	2.16
	(-0.04)	(10.00)			(1.14)			1970–2012
VE	-0.494	0.727			0.412		0.0756	1.87
	(-1.68)	(9.28)			(3.49)			1962–2012
CO	0.712	0.494			0.403		0.0226	1.71
	(3.67)	(4.92)			(4.89)			1970–2012
SY	1.512				0.827		0.0659	1.12
	(4.03)				(24.21)			1965–2009
ID	0.192	0.170	-0.0029		0.665		0.0250	1.88
	(4.53)	(1.57)	(-3.04)		(8.04)			1962–2012
MA	0.367	0.516			0.411		0.0433	1.28
	(2.07)	(4.00)			(3.76)			1972–2012
PA	-0.036	0.859			0.147		0.0346	1.60
	(-0.46)	(8.18)			(1.44)			1974–2012
PH	-0.034	0.875	-0.0025		0.135		0.0194	1.77
	(-0.49)	(16.30)	(-4.09)		(2.61)			1962–2012
TH	0.067	0.436			0.479		0.0249	1.58
	(3.24)	(6.06)			(7.73)			1962–2012
CH	-0.117	0.594	-0.0029		0.330		0.0249	1.28
	(-2.17)	(6.41)	(-0.95)		(4.38)			1984–2011
AR	1.129	0.385			0.468		0.0499	1.38
	(1.22)	(2.65)			(4.90)			1994–2012
BR	0.750	0.452			0.441		0.0160	0.79
	(1.46)	(3.65)			(4.90)			1995–2011
CE	0.237	0.490			0.453		0.0383	1.52
	(1.52)	(5.90)			(6.44)			1978–2012
ME	0.364	0.492			0.457		0.0302	0.80
	(1.77)	(5.25)			(5.06)			1962–2011
PE	1.091	0.455			0.404		0.0200	1.39
	(2.57)	(3.75)			(5.29)			1992–2012

Table B2: Test Results for Equation 2

	Lags	RHO	T	Leads	Stability			End Test		overid	
	p-val	p-val	p-val	p-val	AP	df	λ	p-val	End	p-val	df
Quarterly											
CA	0.128	0.225	0.099	0.005	8.63	4	5.086	1.000	1998.4		
JA	0.297	0.091	0.509	0.009	6.05	5	6.257	1.000	1998.3	0.040	4
AU	0.000	0.000	0.810	0.334	47.02	4	4.872	1.000	1998.3	0.008	4
FR	0.083	0.000	0.845	0.305	15.27	4	3.226	1.000	1998.3		
GE	0.001	0.016	0.000	0.752	13.66	3	3.332	1.000	1998.4		
IT	0.002	0.000	0.000	0.025	13.81	4	3.204	0.935	1998.3	0.004	4
NE	0.231	0.090	0.000	0.146	6.69	4	1.977	1.000	1998.4		
ST	0.004	0.015	0.026	0.001	19.32	4	1.428	1.000	1998.3	0.013	4
UK	0.331	0.000	0.007	0.004	18.30	4	3.453	0.824	1998.3		
FI	0.001	0.000	0.355	0.225	11.38	4	2.115	0.462	1998.3	0.025	3
AS	0.952	0.744	0.690	0.785	1.97	3	4.945	1.000	1998.2	0.881	4
SO	0.068	0.120	0.000	0.004	10.12	4	9.078	1.000	1998.3	0.000	4
KO	0.520	0.315	0.033	0.059	11.08	4	3.643	0.977	1998.4	0.006	3
Annual											
BE	0.199	0.185	0.002	0.353	11.50	3	7.384	1.000	1996	0.060	4
DE	0.621	0.030	0.220	0.411	3.25	3	7.384	0.773	1998	0.053	5
NO	0.119	0.101	0.130	0.252	19.09	3	7.384	1.000	1998	0.023	4
SW	0.002	0.000	0.004	0.730	9.98	3	6.224	1.000	1998	0.001	4
GR	0.257	0.003	0.000	0.177	14.14	4	8.251	0.960	1998		
IR	0.000	0.004	0.044	0.511	16.84	4	4.003	1.000	1998	0.001	3
PO	0.059	0.020	0.000	0.051	15.45	4	7.384	0.938	1995	0.006	3
SP	0.002	0.000	0.000	0.398	15.53	3	7.384	0.364	1998	0.014	4
NZ	0.086	0.030	0.968	0.198	8.71	4	7.638	1.000	1998	0.188	3
SA	0.730	0.519	0.915	0.142	0.41	3	4.510	0.571	1998		
VE	0.900	0.664	0.709	0.328	3.24	3	7.384	0.955	1998		
CO	0.950	0.003	0.549	0.992	1.53	3	1.130	0.000	1998		
SY	0.747	0.002	0.753	0.170	4.96	2	6.911	0.591	1998		
ID	0.372	0.318	0.030	0.905	6.61	4	7.384				
MA	0.011	0.001	0.354	0.514	4.91	3	3.901	0.000	1998		
PA	0.490	0.002	0.542	0.880	23.77	3	3.335	0.200	1998		
PH	0.563	0.517	0.017	0.289	9.50	4	7.384	1.000	1999		
TH	0.503	0.000	0.002	0.731	7.89	3	7.384	0.000	1998		
CH	0.069	0.007	0.400	0.175							
CE	0.405	0.001	0.000	0.001	0.25	3	2.043				
ME	0.009	0.000	0.290	0.586	46.91	3	7.638	0.217	1998		

Table B3: Coefficient Estimates for Equation 3

$$\log I = a_1 + a_2 \log I_{-1} + a_3 \log Y + a_4 RS + a_5 RB$$

	a_1	a_2	a_3	a_4	a_5	SE	DW
Quarterly							
CA	-0.357	0.913	0.106		-0.0013†	0.0212	1.40
	(-2.51)	(35.27)	(3.15)		(-2.56)		1961.2–2013.2
AU	0.470	0.933	0.017		-0.0047	0.0246	2.38
	(3.37)	(28.38)	(0.53)		(-2.64)		1970.1–2013.1
FR	0.227	0.957	0.021		-0.0020†	0.0171	1.98
	(4.90)	(56.32)	(1.28)		(-4.54)		1961.1–2012.4
GE	0.437	0.804	0.139		-0.0016	0.0337	2.06
	(3.26)	(20.20)	(4.38)		(-0.96)		1963.1–2013.2
IT	0.282	0.939	0.032		-0.0008†	0.0197	1.28
	(3.95)	(38.57)	(1.83)		(-2.13)		1961.1–2013.1
NE	0.261	0.821	0.133		-0.0045†	0.0527	2.57
	(2.37)	(19.20)	(3.64)		(-2.27)		1961.1–2013.1
ST	0.023	0.967	0.021		-0.0035	0.0212	1.52
	(0.18)	(26.89)	(0.42)		(-1.79)		1971.1–2013.2
UK	0.017	0.904	0.082		-0.0012†	0.0325	2.15
	(0.17)	(28.03)	(2.54)		(-1.65)		1961.1–2013.2
FI	0.253	0.927	0.038	-0.0012		0.0418	2.28
	(2.88)	(31.72)	(1.67)	(-1.38)			1961.1–2013.2
AS	-0.227	0.953	0.062		-0.0013	0.0266	1.78
	(-1.79)	(42.22)	(2.08)		(-2.03)		1966.1–2013.2
SO	0.074	0.955	0.037		-0.0034†	0.0380	2.12
	(0.82)	(67.52)	(2.72)		(-4.43)		1962.1–2012.4
KO	0.103	0.960	0.028			0.0487	1.83
	(1.23)	(40.12)	(1.05)				1974.1–2012.4
Annual							
BE	0.462	0.599	0.321		-0.0151	0.0465	1.81
	(1.87)	(6.50)	(3.73)		(-4.56)		1962–2012
DE	-0.907	0.574	0.459		-0.0084	0.0612	1.61
	(-2.45)	(6.06)	(3.89)		(-3.17)		1962–2012
NO	0.174	0.896	0.066	-0.0063		0.0662	1.45
	(1.26)	(13.32)	(1.21)	(-2.36)			1962–2012
SW	-0.005	0.696	0.244	-0.0072		0.0566	1.31
	(-0.02)	(7.15)	(2.75)	(-2.59)			1965–2012
GR	0.314	0.507	0.412	-0.0132		0.0885	1.69
	(0.85)	(4.57)	(3.76)	(-4.32)			1962–2009
IR	0.649	0.923	0.012		-0.0040	0.1035	0.98
	(1.84)	(9.34)	(0.12)		(-0.74)		1962–2012
PO	0.434	0.850	0.095		-0.0012	0.0784	0.96
	(1.39)	(6.69)	(0.74)		(-0.48)		1962–2012
SP	0.664	0.869	0.071	-0.0048		0.0650	0.80
	(1.47)	(9.10)	(0.62)	(-2.09)			1962–2012

Table B3: Coefficient Estimates for Equation 3

$\log I = a_1 + a_2 \log I_{-1} + a_3 \log Y + a_4 RS + a_5 RB$

	a_1	a_2	a_3	a_4	a_5	SE	DW
NZ	-1.589	0.619	0.474		-0.0062	0.0755	1.19
	(-2.21)	(5.00)	(2.88)		(-1.89)		1962–2011
ID	-1.841	0.616	0.518			0.0512	1.46
	(-3.63)	(5.60)	(3.61)				1962–2012
PA	0.226	0.799	0.134			0.0882	1.00
	(0.73)	(7.67)	(1.20)				1974–2012
CH	-1.287	0.462	0.630	-0.0097		0.0694	0.95
	(-1.91)	(2.27)	(2.57)	(-1.19)			1984–2011

Table B3: Test Results for Equation 3

	Lags	RHO	T	Leads	Stability			End Test		overid	
	p-val	p-val	p-val	p-val	AP	df	λ	p-val	End	p-val	df
Quarterly											
CA	0.000	0.000	0.169	0.074	5.15	4	5.086	1.000	1998.4	0.010	4
AU	0.007	0.027	0.773	0.180	15.53	4	4.872	1.000	1998.3	0.488	4
FR	0.888	0.072	0.057	0.427	6.50	4	3.226	0.645	1998.3	0.487	4
GE	0.029	0.162	0.006	0.605	7.73	4	3.332	1.000	1998.4		
IT	0.000	0.000	0.000	0.000	8.85	4	3.204	0.326	1998.3	0.000	4
NE	0.000	0.000	0.004	0.027	2.94	4	1.859	1.000	1998.4	0.000	4
ST	0.000	0.000	0.054	0.001	10.75	4	5.162	0.961	1998.3		
UK	0.213	0.353	0.000	0.927	3.86	4	3.453	0.000	1998.3	0.000	4
FI	0.020	0.012	0.000	0.024	13.08	4	2.115	1.000	1998.3	0.000	4
AS	0.127	0.030	0.152	0.081	4.79	4	4.945	0.739	1998.2	0.085	4
SO	0.465	0.127	0.000	0.650	7.63	4	9.078	1.000	1998.3	0.004	4
KO	0.154	0.325	0.000	0.032	7.92	3	3.643	1.000	1998.4	0.037	5
Annual											
BE	0.329	0.394	0.008	0.362	5.32	4	7.384	1.000	1996	0.215	4
DE	0.082	0.075	0.000	0.683	12.76	4	7.384	0.864	1998	0.003	4
NO	0.030	0.033	0.131	0.828	5.04	4	7.384	0.591	1998	0.051	5
SW	0.000	0.007	0.253	0.636	15.97	4	6.224	0.474	1998	0.055	4
GR	0.356	0.456	0.022	0.307	17.61	4	8.251	0.560	1998	0.059	4
IR	0.000	0.000	0.001	0.002	6.27	4	4.003	0.000	1998		
PO	0.000	0.000	0.000	0.072	12.06	4	7.384	0.125	1995	0.041	4
SP	0.000	0.000	0.286	0.000	2.98	4	7.384	0.000	1998	0.000	4
NZ	0.000	0.001	0.968	0.117	7.44	4	7.638	1.000	1998	0.206	4
ID	0.113	0.003	0.178	0.671	4.25	3	7.384				
PA	0.000	0.000	0.132	0.809	0.64	3	3.335	0.000	1998		
CH	0.000	0.002	0.092	0.000							

Table B4: Coefficient Estimates for Equation 4

$\log Y = a_1 + a_2 \log Y_{-1} + a_3 \log X + a_4 \log V_{-1}$

	a_1	a_2	a_3	a_4	ρ	Implied Values See eq. (17) in Section 3.6.4 λ	α	β	SE	DW
Quarterly										
FR	0.196	0.140	0.880	-0.0353	0.776	0.860	0.041	0.585	0.0042	1.88
	(4.27)	(7.78)	(47.17)	(-1.94)	(15.95)				1961.1–2012.4	
IT	-0.001	0.441	0.562	-0.0035	0.479	0.559	0.006	0.954	0.0068	1.94
	(-0.02)	(9.88)	(12.26)	(-2.16)	(6.96)				1961.1–2013.1	
AS	0.317	0.471	0.552	-0.0504	0.115	0.529	0.095	0.460	0.0077	2.01
	(4.50)	(8.93)	(10.41)	(-3.89)	(1.35)				1966.1–2013.2	
Annual										
PA	-0.099	0.075	0.956	-0.0207		0.925	0.022	1.508	0.0035	1.19
	(-1.77)	(2.16)	(29.90)	(-1.93)					1974–2012	

Table B4: Test Results for Equation 4

	Lags p-val	RHO p-val	T p-val	Leads p-val	Stability AP	df	λ	End Test p-val	End
Quarterly									
FR	0.000	0.338	0.127	0.489	27.48	5	3.226	0.989	1998.3
IT	0.850	0.971	0.349	0.001	13.56	5	3.204	1.000	1998.3
AS	0.129	0.110	0.757	0.033	2.36	5	2.306	1.000	1998.2
Annual									
PA	0.008	0.011	0.175	0.246	11.34	4	3.335	0.800	1998

Table B5: Coefficient Estimates for Equation 5

$$\log PY = a_1 + a_2 \log PY_{-1} + a_3 \log PM + a_4 ZZ + a_5 T$$

	a_1	a_2	a_3	a_4	a_5	ρ	SE	DW
Quarterly								
CA	0.028	0.997	0.009	0.08937†	-0.00009	0.586†	0.0068	2.09
	(1.19)	(50.59)	(0.61)	(2.19)	(-0.87)	(10.03)	1961.2–2013.2	
JA	0.018	0.986	0.005		-0.00009	0.479†	0.0071	2.07
	(3.55)	(184.07)	(1.33)		(-3.41)	(7.78)	1966.1–2013.2	
AU	-0.007	0.974	0.014	0.06307†	0.00005	-0.237†	0.0066	2.00
	(-1.15)	(167.00)	(2.21)	(2.35)	(1.82)	(-3.17)	1970.1–2013.1	
FR	-0.096	0.895	0.075	0.18887†	0.00046	0.402†	0.0071	2.09
	(-3.06)	(40.05)	(5.47)	(4.15)	(3.15)	(5.99)	1961.1–2012.4	
GE	0.014	0.997	0.004	0.10216†	-0.00004		0.0066	2.09
	(2.33)	(175.07)	(0.97)	(4.05)	(-1.47)		1963.1–2013.2	
IT	0.026	0.953	0.051	0.14664†	-0.00007		0.0099	1.48
	(2.95)	(208.40)	(13.31)	(4.89)	(-1.70)		1961.1–2013.1	
NE	0.016	0.992	0.004	0.15859†	-0.00004		0.0066	1.58
	(1.32)	(105.05)	(0.72)	(6.65)	(-0.78)		1967.1–2013.1	
ST	0.005	0.981	0.010	0.09725†		0.497†	0.0036	2.01
	(0.51)	(120.04)	(1.12)	(5.31)	(-0.05)	(7.28)	1971.1–2013.2	
UK	-0.057	0.896	0.090†	0.15854†	0.00029	0.431†	0.0099	2.27
	(-2.66)	(53.37)	(6.93)	(3.97)	(2.88)	(6.59)	1961.1–2013.2	
FI	0.015	0.961	0.040	0.04139	-0.00004		0.0145	2.31
	(1.22)	(102.34)	(5.07)	(1.71)	(-0.66)		1961.1–2013.2	
AS	0.021	0.994		0.15584†	-0.00006	0.469†	0.0089	2.18
	(1.26)	(164.91)		(2.74)	(-0.73)	(6.79)	1969.1–2013.2	
SO	0.024	0.954	0.048		-0.00003		0.0171	1.87
	(0.67)	(103.05)	(5.85)		(-0.18)		1962.1–2012.4	
KO	-0.007	0.973		0.11682†	0.00007		0.0144	1.76
	(-0.42)	(196.69)		(3.70)	(0.80)		1974.1–2012.4	
Annual								
BE	0.093	0.976	0.053	0.31668†	-0.00160		0.0197	0.39
	(2.37)	(30.43)	(1.96)	(1.61)	(-1.94)		1962–2012	
DE	0.044	0.921	0.076		-0.00056		0.0218	0.42
	(1.07)	(21.37)	(2.02)		(-0.60)		1962–2012	
NO	-0.247	0.827	0.093	0.39819†	0.00612		0.0360	1.60
	(-1.04)	(6.12)	(1.11)	(1.73)	(1.24)		1962–2012	
SW	0.202	0.898	0.153	0.24029†	-0.00392		0.0181	0.89
	(5.41)	(32.18)	(6.67)	(2.43)	(-4.70)		1965–2012	
IR	0.015	0.846	0.159†	0.31197†	-0.00022		0.0367	0.73
	(0.15)	(11.45)	(2.70)	(3.43)	(-0.10)		1962–2012	
PO	-0.297	0.731	0.278	0.31567†	0.00673		0.0233	1.19
	(-6.07)	(40.71)	(18.71)	(2.83)	(6.28)		1962–2012	
SP	0.320	0.985	0.104†	0.53546†	-0.00596		0.0344	0.28
	(3.37)	(25.99)	(3.00)	(2.46)	(-2.80)		1962–2012	
NZ	-0.037	0.724	0.276	0.14016†	0.00167		0.0343	1.41
	(-0.40)	(16.11)	(8.09)	(0.95)	(0.75)		1962–2011	
JO	0.162	0.872	0.159		-0.00217		0.0374	1.89
	(0.95)	(12.10)	(3.72)		(-0.56)		1978–2009	
SY	-0.135	0.895	0.113		0.00427		0.0655	1.34
	(-0.60)	(18.71)	(4.28)		(0.85)		1965–2009	

Table B5: Coefficient Estimates for Equation 5

	a_1	a_2	a_3	a_4	a_5	ρ	SE	DW
MA	-0.926	0.352	0.259		0.01837		0.0356	1.78
	(-5.25)	(3.13)	(4.59)		(5.28)			1972–2012
PA	0.287	0.724	0.292		-0.00482		0.0329	1.75
	(0.96)	(10.70)	(6.30)		(-0.74)			1974–2012
PH	0.014	0.800	0.175†	0.22256†	0.00146		0.0642	1.65
	(0.05)	(8.35)	(2.92)	(1.05)	(0.26)			1962–2012
TH	0.092	0.763	0.178	0.31402†	-0.00049		0.0349	1.02
	(0.96)	(10.18)	(4.00)	(3.02)	(-0.24)			1962–2012
CH	-0.561	0.544	0.189†	0.51177	0.01371		0.0380	1.14
	(-2.75)	(4.50)	(3.55)	(2.07)	(3.02)			1984–2011
CE	-0.313	0.593	0.341	0.54102†	0.00721		0.0489	1.32
	(-2.42)	(7.91)	(4.41)	(2.78)	(2.56)			1978–2012

Table B5: Test Results for Equation 5

	Lags-1	Lags-2	RHO	Stability			End Test		overid	
	p-val	p-val	p-val	AP	df	λ	p-val	End	p-val	df
Quarterly										
CA	0.449	0.103	0.300	24.93	6	5.086	0.011	1998.4		
JA	0.000	0.000	0.000	77.28	5	6.257	0.986	1998.3		
AU	0.181	0.727	0.640	3.96	6	4.872	1.000	1998.3	0.000	6
FR	0.328	0.001	0.069	25.55	6	3.226	1.000	1998.3	0.000	7
GE	0.688	0.002	0.001	8.34	5	3.332	1.000	1998.4	0.000	6
IT	0.000	0.000	0.000	41.52	5	3.204	1.000	1998.3	0.000	6
NE	0.005	0.000	0.014	10.29	5	1.977	1.000	1998.4		
ST	0.682	0.810	0.885	4.76	6	1.428	1.000	1998.3	0.034	6
UK	0.000	0.000	0.000	26.53	6	3.453	1.000	1998.3	0.000	7
FI	0.013	0.000	0.075	10.79	5	2.115	1.000	1998.3	0.000	4
AS	0.037	0.047	0.011	15.24	5	5.911	0.561	1998.2		
SO	0.009	0.013	0.006	24.03	4	9.078	0.876	1998.3	0.000	5
KO	0.515	0.478	0.802	3.63	4	3.643	1.000	1998.4	0.001	7
Annual										
BE	0.000	0.000	0.000	98.21	5	7.384	1.000	1996		
DE	0.000	0.000	0.000	98.77	4	7.384	1.000	1998		
NO	0.002	0.000	0.001	8.61	5	7.384	0.182	1998	0.000	4
SW	0.000	0.000	0.000	26.08	5	6.224	1.000	1998	0.001	4
IR	0.000	0.000	0.000	43.83	5	4.003	1.000	1998	0.000	4
PO	0.050	0.005	0.008	38.19	5	7.384	1.000	1995	0.002	4
SP	0.000	0.000	0.000	98.77	5	7.384	1.000	1998	0.000	4
NZ	0.033	0.151	0.043	9.54	5	7.638	1.000	1998	0.034	4
JO	0.987	0.987	0.786							
SY	0.004	0.011	0.003	20.82	4	6.911	1.000	1998		
MA	0.014	0.000	0.005	5.86	4	3.901	0.583	1998		
PA	0.377	0.159	0.559	6.15	4	3.335	0.300	1998		
PH	0.568	0.211	0.231	34.68	5	7.384	1.000	1999		
TH	0.000	0.026	0.000	63.20	5	7.384	0.727	1998		
CH	0.010	0.001	0.010					44	0.004	4
CE	0.071	0.005	0.063	19.41	5	2.043				

Table B6: Coefficient Estimates for Equation 6

$$\log[M1/(POP \cdot PY)] = a_1 + a_2 \log[M1/(POP \cdot PY)]_{-1}$$
$$+ a_3 \log[M1_{-1}/(POP_{-1} \cdot PY)] + a_4 RS + a_5 \log(Y/POP)$$

	a_1	a_2	a_3	a_4	a_5	SE	DW
Quarterly		real	nominal				
CA	-0.287		0.931	-0.0029	0.103	0.0272	2.29
	(-2.55)		(53.97)	(-2.61)	(4.08)		1968.1–2008.4
GE	-0.374	0.969		-0.0033	0.078	0.0340	2.28
	(-2.25)	(63.21)		(-2.34)	(2.31)		1966.1–2012.4
NE	-0.073		0.924	-0.0046	0.091	0.0203	2.16
	(-1.33)		(63.04)	(-5.96)	(4.53)		1961.1–2012.4
ST	0.070	0.971		-0.0077	0.019	0.0355	1.82
	(0.80)	(59.52)		(-3.87)	(0.48)		1976.1–2013.1
UK	0.144	0.976		-0.0030	0.004	0.0137	2.13
	(1.74)	(121.26)		(-6.07)	(0.69)		1971.2–2006.1
FI	-1.115		0.742	-0.0063	0.416	0.0680	1.84
	(-3.28)		(15.05)	(-3.08)	(4.78)		1970.1–2012.4
AS	-0.528		0.946	-0.0032	0.115	0.0239	2.03
	(-3.98)		(72.65)	(-3.54)	(4.30)		1966.1–2012.4
KO	0.099		0.926		0.054	0.0597	2.46
	(1.49)		(24.23)		(1.42)		1974.1–2012.4
Annual							
BE	0.506	0.924		-0.0092	0.024	0.0341	1.84
	(2.12)	(21.68)		(-5.32)	(0.98)		1962–2012
DE	-0.576		0.777	-0.0078	0.299	0.0497	1.96
	(-1.98)		(12.79)	(-2.81)	(3.05)		1962–2008
SW	0.021	0.966		-0.0071	0.039	0.0404	1.88
	(0.13)	(13.68)		(-3.94)	(0.71)		1965–2008
IR	-0.211		0.749	-0.0214	0.260	0.1656	2.20
	(-0.10)		(8.39)	(-1.09)	(1.12)		1983–2012
PO	-0.663	0.843		-0.0009	0.215	0.1254	1.48
	(-0.98)	(10.76)		(-0.35)	(1.59)		1962–2012
SP	0.748		0.861	-0.0016	0.053	0.0500	1.12
	(3.22)		(8.16)	(-0.80)	(0.52)		1962–2004
VE	-1.652	0.842		-0.0019	0.732	0.1911	1.77
	(-1.80)	(9.77)		(-1.17)	(1.91)		1962–2012
ID	-0.731		0.648	-0.0005	0.411	0.0483	1.72
	(-2.59)		(4.97)	(-0.24)	(2.79)		1962–2012
PA	-0.188		0.844	-0.0099	0.206	0.0682	1.69
	(-0.58)		(6.95)	(-1.80)	(1.33)		1974–2007
PH	-0.416		0.714	-0.0094	0.272	0.0765	2.17
	(-1.27)		(8.43)	(-2.57)	(2.86)		1962–2007

Table B6: Test Results for Equation 6

	[a]N vs R	Lags	RHO	T	Stability			End Test		overid	
	p-val	p-val	p-val	p-val	AP	df	λ	p-val	End	p-val	df
Quarterly											
CA	0.041	0.284	0.009	0.558	15.96	4	6.476	1.000	1998.4	0.158	5
GE	0.882	0.078	0.000	0.000	48.44	4	3.676	1.000	1998.4	0.000	4
NE	0.351	0.015	0.250	0.437	6.52	4	1.868	0.000	1998.4		
ST	0.009	0.000	0.050	0.025	16.31	4	1.000	0.031	1998.3	0.131	5
UK	0.000	0.036	0.037	0.061	3.59	4	17.141	0.433	1998.3	0.274	4
FI	0.007	0.083	0.000	0.000	35.00	4	2.457	1.000	1998.3	0.009	4
AS	0.701	0.313	0.237	0.952	3.25	4	5.018	0.521	1998.2	0.114	4
KO	0.732	0.003	0.003	0.054	3.12	3	3.643	1.000	1998.4	0.009	5
Annual											
BE	0.721	0.869	0.554	0.464	3.78	4	7.384	0.056	1996		
DE	0.932	0.381	0.883	0.019	2.37	4	4.902	0.615	1998		
SW	0.155	0.920	0.675	0.692	2.92	4	7.208	0.000	1998		
IR	0.710	0.480	0.580	0.683	1.40	4	1.000	0.000	1998		
PO	0.002	0.030	0.048	0.101	41.15	4	7.384	1.000	1995		
SP	0.048	0.421	0.048	0.138	2.75	4	7.384	0.000	1998		
VE	0.341	0.229	0.000	0.000	16.83	4	7.384	0.091	1998		
ID	0.340	0.872	0.329	0.950	6.73	4	7.384				
PA	0.508	0.106	0.610	0.981	5.67	4	3.850	0.000	1998		
PH	0.367	0.171	0.471	0.351	3.89	4	9.061	0.414	1999		

Table B7: Coefficient Estimates for Equation 7

$RS = a_1 + a_2 RS_{-1} + a_3 PCPY + a_4 ZZ + a_5 RS_{GE} + a_6 RS_{US}$

	a_1	a_2	a_3	a_4	a_5	a_6	ρ	SE	DW
Quarterly									
EU	0.46	0.849	0.040†	23.0		0.08		0.616	1.79
	(3.72)	(32.20)	(2.17)	(6.79)		(3.98)		1972.2–2012.4	
CA	0.20	0.816		6.6		0.23		0.780	1.63
	(1.48)	(22.72)		(3.05)		(4.99)		1972.2–2013.2	
JA	-0.12	0.810	0.112			0.11	0.394	0.540	2.07
	(-0.92)	(21.08)	(4.91)			(3.28)	(4.59)	1972.2–2013.2	
AU	1.39	0.752		43.0		0.08		0.709	1.89
	(4.66)	(18.16)		(7.06)		(2.51)		1972.2–1998.4	
FR	-0.22	0.742	0.032	2.5	0.20	0.17		0.884	1.62
	(-0.59)	(15.60)	(0.98)	(0.32)	(3.41)	(3.52)		1972.2–1998.4	
GE	0.67	0.850	0.040†	32.9		0.08		0.716	1.95
	(2.72)	(24.79)	(1.35)	(6.98)		(2.53)		1972.2–1998.4	
IT	1.17	0.876	0.064	20.8			0.324	1.074	1.93
	(2.05)	(16.61)	(2.09)	(2.04)			(2.94)	1972.2–1998.4	
NE	0.03	0.516		8.1	0.36	0.16		1.440	1.68
	(0.05)	(7.19)		(1.16)	(4.03)	(2.39)		1972.2–1998.4	
ST	0.17	0.840	0.138				0.452	0.565	1.89
	(1.20)	(14.00)	(2.21)				(5.17)	1972.2–2013.2	
UK	0.41	0.812	0.011	7.1		0.23		0.869	1.52
	(2.51)	(23.34)	(0.63)	(3.58)		(5.70)		1972.2–2013.2	
FI	1.02	0.939		6.9				0.940	1.77
	(2.75)	(32.90)		(3.44)				1972.2–1998.4	
AS	0.17	0.888	0.022	11.3		0.14		0.974	1.72
	(0.91)	(35.72)	(0.97)	(2.62)		(3.71)		1972.2–2013.2	
SO	0.37	0.900				0.13	0.482	0.978	2.01
	(0.79)	(21.32)				(2.59)	(5.25)	1972.2–2012.4	
Annual									
BE	0.71	0.600	0.025	34.2	0.42			1.491	2.33
	(0.65)	(3.54)	(0.25)	(1.26)	(2.22)			1972–1998	
DE	-0.52	0.631	0.222	14.2	0.50			1.999	2.35
	(-0.75)	(6.48)	(2.12)	(1.04)	(3.24)			1972–2012	
NO	0.01	0.802		25.3	0.33			1.433	2.10
	(0.01)	(11.26)		(2.92)	(3.41)			1972–2012	
SW	-0.29	0.761	0.046	9.7		0.34		1.638	2.52
	(-0.53)	(7.77)	(0.45)	(0.99)		(2.91)		1972–2012	
IR	2.69		0.157		0.25	0.73		2.051	1.85
	(2.12)		(2.25)		(1.33)	(3.88)		1972–1998	
PO	-0.73	0.761	0.328	38.5				2.583	1.89
	(-0.56)	(8.52)	(4.21)	(2.81)				1972–1998	
SP	1.83	0.555	0.195			0.21		3.009	2.40
	(0.88)	(3.07)	(1.72)			(0.72)		1972–1998	
NZ	0.52	0.746	0.311	2.5				1.722	1.52
	(0.68)	(7.92)	(5.01)	(0.46)				1979–2011	
ID	0.28	0.828	0.262					1.563	1.89
	(0.24)	(11.35)	(4.23)					1972–2012	
PA	1.95	0.668	0.147	15.9				1.395	2.24
	(2.18)	(6.81)	(3.37)	(2.09)				1974–2012	
PH	1.29	0.675	0.164			0.29		2.375	1.52
	(1.13)	(8.19)	(3.45)			(2.11)		1972–2012	

Table B7: Test Results for Equation 7

	Lags	RHO	T	Stability			End Test		overid	
	p-val	p-val	p-val	AP	df	λ	p-val	End	p-val	df
Quarterly										
EU	0.539	0.096	0.658					44	0.154	3
CA	0.000	0.011	0.205	11.01	4	4.121	1.000	1998.4	0.000	6
JA	0.105	0.227	0.079	7.80	5	4.121	1.000	1998.3	0.034	7
AU	0.773	0.165	0.067	2.04	4	2.696			0.311	6
FR	0.544	0.115	0.052	5.17	6	2.696			0.012	4
GE	0.826	0.644	0.372	2.53	5	2.696			0.290	5
IT	0.364	0.207	0.940	1.21	5	2.696	0.510	1998.3	0.022	7
NE	0.179	0.010	0.582	2.44	5	1.125			0.050	5
ST	0.550	0.186	0.083	4.39	4	1.308	1.000	1998.3	0.017	7
UK	0.002	0.001	0.043	7.67	5	4.121	1.000	1998.3	0.004	5
FI	0.448	0.270	0.688	0.82	3	1.468			0.370	4
AS	0.379	0.042	0.091	8.29	5	4.121	1.000	1998.2	0.001	5
SO	0.462	0.904	0.018	9.34	4	4.173	0.062	1998.3	0.016	6
Annual										
BE	0.017	0.186	0.868	5.58	5	2.469				
DE	0.057	0.136	0.301	6.69	5	3.901	1.000	1998		
NO	0.216	0.714	0.815	8.19	4	3.901	0.833	1998		
SW	0.325	0.039	0.215	5.06	5	3.901	1.000	1998		
IR	0.979	0.950	0.098	4.74	4	2.469				
PO	0.980	0.788	0.043	4.00	4	2.469				
SP	0.575	0.123	0.449	1.98	4	2.469				
NZ	0.390	0.095	0.775	7.09	4	4.077	1.000	1998		
ID	0.655	0.730	0.906	3.43	3	3.901				
PA	0.421	0.376	0.297	8.34	4	3.335	0.100	1998		
PH	0.117	0.122	0.290	12.01	4	3.901	1.000	1999		

Table B8: Coefficient Estimates for Equation 8

$$RB - RS_{-2} = a_1 + a_2(RB_{-1} - RS_{-2}) + a_3(RS - RS_{-2}) + a_4(RS_{-1} - RS_{-2})$$

For annual, RS_{-1} replaces RS_{-2}

	a_1	a_2	a_3	a_4	ρ	SE	DW
Quarterly							
EU	0.071	0.940	0.322	-0.303		0.3637	1.65
	(1.60)	(43.26)	(4.14)	(-3.08)		1970.3–2012.4	
CA	0.097	0.922	0.343	-0.301		0.3755	1.96
	(2.28)	(37.74)	(2.54)	(-1.84)		1961.2–2013.2	
JA	0.036	0.894	0.433	-0.462		0.4076	2.09
	(0.90)	(24.71)	(2.21)	(-1.62)		1966.1–2013.2	
AU	0.064	0.948	0.170	-0.070	0.392	0.2639	1.92
	(0.97)	(30.56)	(1.60)	(-0.87)	(4.17)	1970.1–1998.4	
FR	0.042	0.947	0.278	-0.174	0.296	0.3857	2.00
	(0.67)	(25.77)	(1.94)	(-1.25)	(3.02)	1961.1–1998.4	
GE	0.117	0.926	0.489	-0.500		0.3983	1.95
	(2.43)	(41.26)	(5.35)	(-4.27)		1963.1–2013.2	
IT	0.097	0.705	0.495	-0.292	0.584	0.4434	1.95
	(1.04)	(7.27)	(3.89)	(-3.13)	(5.02)	1961.1–1998.4	
NE	0.179	0.872	0.366	-0.297		0.4907	1.95
	(2.47)	(20.14)	(3.56)	(-3.19)		1961.1–1998.4	
ST	0.021	0.945	0.519	-0.518		0.2887	1.95
	(0.82)	(49.10)	(5.16)	(-3.70)		1971.1–2013.2	
UK	0.018	0.973	0.300	-0.310		0.4496	1.63
	(0.41)	(38.16)	(1.75)	(-1.49)		1961.1–2013.2	
AS	0.021	0.953	0.345	-0.373		0.4696	1.66
	(0.50)	(25.03)	(2.30)	(-2.27)		1966.1–2013.2	
SO	0.149	0.920	0.837	-1.151		0.6480	1.99
	(1.79)	(23.19)	(2.57)	(-2.57)		1962.1–2012.4	
KO	0.120	0.913	0.387	-0.160		0.9987	2.05
	(0.99)	(21.54)	(2.47)	(-0.87)		1974.1–2012.4	
Annual							
BE	0.509	0.753	0.378			0.6957	1.43
	(2.05)	(7.89)	(5.64)			1962–1998	
DE	0.348	0.727	0.419			1.0940	1.71
	(1.65)	(7.36)	(5.42)			1962–2012	
NO	-0.042	0.851	0.451			0.6330	1.80
	(-0.47)	(9.35)	(6.93)			1962–2012	
IR	0.475	0.545	0.468			1.1547	1.47
	(2.04)	(4.55)	(6.21)			1962–1998	
PO	0.065	0.785	0.386			1.2882	1.69
	(0.31)	(9.01)	(5.17)			1962–1998	
NZ	-0.145	0.865	0.491			0.4776	2.33
	(-1.35)	(7.69)	(7.02)			1989–2011	
TH	0.012	0.831	0.339			0.9981	2.23
	(0.07)	(11.36)	(5.61)			1978–2012	

Table B8: Test Results for Equation 8

	[a]Restr.	Lags	RHO	T	Leads	Stability			End Test		overid	
	p-val	p-val	p-val	p-val	p-val	AP	df	λ	p-val	End	p-val	df
Quarterly												
EU	0.254	0.002	0.001	0.625	0.221					44	0.000	6
CA	0.057	0.131	0.737	0.556	0.066	4.25	4	5.086	1.000	1998.4	0.069	5
JA	0.006	0.028	0.562	0.077	0.009	2.81	4	6.257	1.000	1998.3	0.026	5
AU	0.380	0.080	0.867	0.034	0.232	2.47	5	3.475			0.016	6
FR	0.223	0.214	0.879	0.041	0.189	3.84	5	2.445			0.162	6
GE	0.226	0.004	0.006	0.326	0.293	3.76	4	3.332	1.000	1998.4	0.001	5
IT	0.194	0.447	0.717	0.244	0.206	6.90	5	2.445			0.882	6
NE	0.489	0.369	0.021	0.126	0.465	3.51	4	1.104			0.471	5
ST	0.013	0.014	0.815	0.054	0.027	4.99	4	1.428	0.373	1998.3	0.008	5
UK	0.759	0.282	0.006	0.046	0.682	5.66	4	3.453	1.000	1998.3	0.003	5
AS	0.189	0.226	0.003	0.097	0.091	8.73	4	4.945	1.000	1998.2	0.005	5
SO	0.326	0.011	0.033	0.035	0.300	4.61	4	9.078	0.573	1998.3	0.069	5
KO	0.513	0.676	0.542	0.063		4.69	4	3.643	1.000	1998.4	0.010	5
Annual												
BE	0.372	0.158	0.030	0.002	0.499	12.00	3	24.156				
DE	0.775	0.793	0.243	0.027	0.535	8.52	3	7.384	1.000	1998		
NO	0.318	0.416	0.440	0.029	0.759	6.00	3	7.384	0.955	1998		
IR	0.708	0.538	0.013	0.004	0.786	9.70	3	3.136				
PO	0.002	0.002	0.160	0.004	0.115	7.86	3	6.370				
NZ	0.005	0.127	0.345	0.086	0.468	1.00	0	0.000				
TH	0.081	0.391	0.469	0.691	0.617	6.47	3	2.333	1.000	1998		

Table B9: Coefficient Estimates for Equation 9

$$\Delta \log E = a_1 + \lambda[\log(PY/PY_{US}) - \log E_{-1}]$$
$$+.25\lambda\beta \log[(1 + RS/100)/(1 + RS_{US}/100)]$$
or
$$\Delta \log H = a_1 + \lambda[\log(PY/PY_{GE}) - \log H_{-1}]$$
$$+.25\lambda\beta \log[(1 + RS/100)/(1 + RS_{GE}/100)]$$

	a_1	λ	$\lambda\beta$	ρ	SE	DW
Quarterly						
EU	-0.028	0.087	-1.479	0.305	0.0463	1.95
	(-2.63)	(2.47)	(-1.37)	(3.51)	1972.2–2012.4	
CA	0.009	0.050	-0.583	0.384	0.0242	1.91
	(2.22)		(-0.63)	(5.29)	1972.2–2013.2	
JA	-0.127	0.050	-1.336	0.293	0.0478	1.92
	(-19.79)		(-1.45)	(3.79)	1972.2–2013.2	
AU	0.003	0.050		0.476	0.0044	2.13
	(3.52)			(5.68)	1972.2–1998.4	
FR	0.011	0.172		0.219	0.0199	2.04
	(3.98)	(3.33)		(1.96)	1972.2–1998.4	
GE	-0.031	0.088	-1.960	0.302	0.0490	1.98
	(-2.32)	(2.03)	(-1.55)	(2.78)	1972.2–1998.4	
IT	0.024	0.050		0.335	0.0333	1.95
	(4.99)			(3.64)	1972.2–1998.4	
NE	0.007	0.050	-1.546		0.0092	2.02
	(7.86)		(-5.53)		1972.2–1998.4	
ST	-0.326	0.050			0.0227	1.58
	(******)				1977.1–2013.2	
UK	-0.002	0.050	-0.403		0.0399	1.42
	(-0.43)		(-0.72)		1972.2–2013.2	
FI	0.009	0.077	-0.446	0.354	0.0295	2.00
	(1.03)	(1.37)	(-0.47)	(3.05)	1972.2–1998.4	
AS	0.012	0.049		0.300	0.0461	1.95
	(1.47)	(1.68)		(3.56)	1972.2–2013.2	
KO	0.010	0.109		0.366	0.0472	1.93
	(1.64)	(2.40)		(3.92)	1974.1–2012.4	
Annual						
BE	0.036	0.171			0.0288	1.39
	(3.09)	(2.11)			1972–1998	
DE	-0.228	0.050			0.0255	0.88
	(-57.16)				1972–2012	
NO	-0.220	0.050			0.0521	1.44
	(-26.97)				1972–2012	
SW	-1.559	0.331			0.0599	1.85
	(-3.86)	(3.93)			1972–2012	
GR	0.150	0.299			0.0667	0.96
	(5.35)	(1.84)			1972–2000	
IR	0.069	0.141			0.0618	0.96
	(2.92)	(1.15)			1972–1998	

Table B9: Coefficient Estimates for Equation 9

	a_1	λ	$\lambda\beta$	ρ	SE	DW
PO	0.190	0.353			0.0951	0.56
	(2.89)	(1.52)				1972–1998
SP	0.054	0.168			0.0722	1.27
	(3.89)	(1.16)				1972–1998
NZ	0.064	0.050	-5.586		0.1006	1.23
	(2.55)		(-2.49)			1978–2011
PH	-0.790	0.265			0.0971	1.02
	(-2.13)	(2.26)				1972–2012

Table B9: Test Results for Equation 9

	[a]Restr.	Lags	RHO	T	Stability			End Test		overid	
	p-val	p-val	p-val	p-val	AP	df	λ	p-val	End	p-val	df
Quarterly											
EU	0.804	0.426	0.432	0.957					44	0.478	4
CA	0.906	0.337	0.138	0.078	0.91	3	4.121	0.000	1998.4	0.084	7
JA	0.609	0.538	0.118	0.516	2.39	3	4.121	1.000	1998.3	0.019	7
AU	0.003	0.039	0.183	0.004	3.51	2	2.696			0.009	7
FR	0.176	0.518	0.490	0.504	1.96	3	2.696			0.403	6
GE	0.903	0.757	0.947	0.842	4.36	4	2.696			0.437	6
IT	0.001	0.915	0.519	0.004	4.39	2	2.696			0.114	7
NE	0.736	0.839	0.007	0.408	0.32	2	1.125			0.104	7
ST	0.088	0.012	0.023	0.017	1.88	1	1.513	0.259	1998.3	0.003	7
UK	0.000	0.000	0.001	0.000	7.50	2	4.121	1.000	1998.3	0.000	7
FI	0.255	0.900	0.634	0.237	0.98	4	1.468			0.095	6
AS	0.526	0.471	0.272	0.177	1.68	3	4.121	0.000	1998.2	0.047	6
KO	0.036	0.402	0.181	0.076	10.00	3	3.643	0.070	1998.4	0.555	6
Annual											
BE	0.940	0.134	0.127	0.969	24.80	2	2.469				
DE	0.000	0.000	0.000	0.000	24.57	1	3.901	1.000	1998		
NO	0.033	0.102	0.101	0.008	2.78	1	3.901	0.583	1998		
SW	0.403	0.414	0.638	0.490	2.90	2	3.901	1.000	1998		
GR	0.002	0.003	0.001	0.000	10.98	2	7.528	0.125	1998		
IR	0.000	0.001	0.000	0.000	5.68	2	2.469				
PO	0.026	0.000	0.000	0.005	8.75	2	2.469				
SP	0.003	0.051	0.002	0.008	4.34	2	2.469	0.500	1998		
NZ	0.546	0.016	0.034	0.045	2.77	2	6.953	0.000	1998		
PH	0.271	0.001	0.000	0.108	1.35	2	3.901	1.000	1999		

Table B10: Coefficient Estimates for Equation 10

$\log F = a_1 \log EE + a_2(.25) \log[(1 + RS/100)/(1 + RS_{US}/100)]$

	a_1	a_2	ρ	SE	DW
Quarterly					
CA	0.9824	1.761	0.793	0.0096	2.28
	(49.23)	(3.68)	(11.64)		1972.2–1997.3
JA	1.0010	1.182	0.359	0.0087	1.84
	(1301.92)	(6.85)	(4.47)		1972.2–2006.3
AU	0.9930	1.049	0.250	0.0058	2.10
	(299.71)	(8.25)	(2.60)		1972.2–1998.4
FR	1.0076	0.644		0.0071	1.54
	(333.90)	(4.78)			1972.2–1989.3
GE	0.9960	1.198	0.720	0.0032	2.21
	(250.42)	(10.89)	(10.67)		1972.2–1998.4
IT	0.9967	1.057		0.0105	1.74
	(257.91)	(8.62)			1976.3–1998.4
NE	0.9921	1.154		0.0086	1.91
	(184.37)	(6.31)			1972.2–1990.4
ST	1.0003	1.115		0.0059	1.48
	(11391.90)	(14.92)			1972.2–2013.2
UK	1.0028	1.246	0.199	0.0049	2.01
	(742.91)	(12.21)	(2.32)		1972.2–2006.3
FI	0.9897	1.177	0.555	0.0088	2.42
	(128.83)	(4.65)	(5.52)		1972.2–1989.3
AS	1.0038	1.142		0.0065	1.95
	(458.71)	(15.96)			1976.1–2006.4

Table B11: Coefficient Estimates for Equation 11

$\log PX - \log[PW\$(E/E00)] = \lambda[\log PY - \log[PW\$(E/E00)]$

	λ	ρ_1	ρ_2	SE	DW
Quarterly					
CA	0.652	1.279	-0.298	0.0167	1.96
	(14.67)	(19.06)	(-4.48)		1961.2–2012.4
JA	0.394	1.267	-0.277	0.0134	1.96
	(17.14)	(17.95)	(-3.96)		1966.1–2012.4
AU	0.868	0.822	0.162	0.0091	2.00
	(37.21)	(10.86)	(2.16)		1970.1–2012.4
FR	0.766	0.960	0.031	0.0116	1.99
	(27.00)	(13.49)	(0.45)		1961.1–2012.4
GE	0.800	1.055	-0.069	0.0081	1.99
	(41.52)	(14.86)	(-0.98)		1963.1–2012.4
IT	0.622	0.920	0.041	0.0179	1.98
	(16.24)	(13.14)	(0.60)		1961.1–2012.4
NE	0.557	1.158	-0.168	0.0166	2.01
	(12.53)	(16.83)	(-2.47)		1961.1–2012.4
ST	0.860	0.814	0.168	0.0120	2.06
	(34.37)	(10.56)	(2.19)		1971.1–2012.4
UK	0.692	1.028	-0.040	0.0190	2.00
	(17.23)	(14.69)	(-0.58)		1961.1–2012.4
FI	0.452	0.965	0.028	0.0228	1.97
	(9.27)	(13.96)	(0.40)		1961.1–2012.4
AS	0.492	1.345	-0.375	0.0321	1.90
	(8.15)	(19.81)	(-5.53)		1966.1–2012.4
SO	0.636	0.860	0.115	0.0403	2.01
	(11.58)	(12.23)	(1.63)		1962.1–2012.4
KO	0.865	1.152	-0.166	0.0379	1.90
	(12.44)	(13.71)	(-2.00)		1974.1–2012.4
Annual					
BE	0.497	1.065	-0.150	0.0252	1.99
	(8.38)	(7.44)	(-1.04)		1962–2012
DE	0.607	1.059	-0.115	0.0184	1.96
	(12.78)	(7.40)	(-0.85)		1962–2012
SW	0.499	1.169	-0.263	0.0311	1.77
	(6.53)	(7.97)	(-1.87)		1965–2012
IR	0.494	1.170	-0.191	0.0277	1.97
	(7.62)	(8.11)	(-1.35)		1962–2012
SP	0.534	1.107	-0.145	0.0343	1.70
	(7.16)	(7.79)	(-1.06)		1962–2012

Table B11: Coefficient Estimates for Equation 11

	λ	ρ_1	ρ_2	SE	DW
NZ	0.496	0.890	-0.056	0.0709	1.94
	(3.62)	(6.05)	(-0.40)		1962–2012
CO	0.789	1.012	-0.055	0.1199	1.96
	(3.43)	(6.10)	(-0.34)		1972–2012
ID	0.535	0.876	-0.169	0.0591	1.94
	(6.30)	(6.14)	(-1.02)		1962–2012
MA	0.764	0.846	0.010	0.1176	1.94
	(2.87)	(5.11)	(0.06)		1972–2012
PA	0.196	0.948	-0.019	0.0672	2.17
	(1.19)	(6.74)	(-0.13)		1974–2012
TH	0.542	0.934	-0.408	0.0593	1.82
	(6.99)	(7.10)	(-3.18)		1962–2012
CH	0.500	1.151	-0.251	0.0426	1.97
		(6.22)	(-1.32)		1984–2012
ME	0.500	1.171	-0.228	0.0547	1.92
		(8.41)	(-1.67)		1962–2012

Table B11: Test Results for Equation 11

	[a]Restr.	Stability			End Test	
	p-val	AP	df	λ	p-val	End
Quarterly						
CA	0.005	3.05	3	3.147	0.000	1998.4
JA	0.000	2.73	3	6.354	1.000	1998.3
AU	0.000	7.72	3	4.906	1.000	1998.3
FR	0.013	10.52	3	3.226	0.688	1998.3
GE	0.000	7.45	3	3.376	0.966	1998.4
IT	0.187	1.40	3	3.226	0.000	1998.3
NE	0.000	8.03	3	1.868	0.579	1998.4
ST	0.024	5.62	3	1.435	0.000	1998.3
UK	0.010	2.66	3	3.501	0.796	1998.3
FI	0.000	18.27	3	2.137	0.570	1998.3
AS	0.002	5.13	3	5.018	0.000	1998.2
SO	0.023	2.15	3	9.078	1.000	1998.3
KO	0.000	15.13	3	3.643	0.116	1998.4
Annual						
BE	0.000	2.34	3	7.384	0.000	1996
DE	0.632	1.18	3	7.384	0.591	1998
SW	0.000	17.72	3	6.224	1.000	1998
IR	0.281	-8.71	3	4.003	1.000	1998
SP	0.004	3.03	3	7.384	1.000	1998
NZ	0.000	5.96	3	7.384	0.591	1998
CO	0.053	4.06	3	4.510	1.000	1998
ID	0.237	1.84	3	7.384		
MA	0.614	3.34	3	3.901	1.000	1998
PA	0.105	1.70	3	3.335	1.000	1998
TH	0.303	3.22	3	7.384	1.000	1998

Table B13: Coefficient Estimates for Equation 13

$\Delta \log J = a_1 + a_2 T + a_3 \log(J/JMIN)_{-1} + a_4 \Delta \log Y + a_5 \Delta \log Y_{-1}$

	a_1	a_2	a_3	a_4	a_5	ρ	SE	DW
Quarterly								
CA	0.004	-0.00001	-0.148	0.311	0.153		0.0040	1.64
	(3.17)	(-1.79)	(-5.29)	(3.58)	(3.27)		1961.2–2013.2	
JA	0.005	-0.00002	-0.045	0.075			0.0033	2.11
	(4.97)	(-4.40)	(-3.33)	(3.40)			1966.1–2013.2	
AU	-0.002	0.00003	-0.108	0.045			0.0062	2.20
	(-0.78)	(2.26)	(-3.65)	(0.32)			1970.1–2013.1	
FR		0.00001	-0.100	0.128		-0.359	0.0043	1.93
	(0.05)	(1.52)	(-5.35)	(4.37)		(-5.31)	1961.1–2012.4	
GE	-0.008	0.00004	-0.120	0.575			0.0057	1.86
	(-4.55)	(4.74)	(-3.69)	(4.83)			1963.1–2013.2	
IT	-0.001	0.00001	-0.111	0.153			0.0054	2.05
	(-0.48)	(1.78)	(-5.34)	(3.76)			1961.1–2013.1	
NE	0.003		-0.154	0.251			0.0038	1.52
	(3.76)	(0.07)	(-6.00)	(4.76)			1961.1–2013.1	
ST	0.004		-0.190	0.040			0.0068	2.37
	(2.21)	(0.03)	(-6.07)	(0.39)			1971.1–2013.2	
UK	-0.003	0.00003	-0.120	0.120		0.449	0.0030	2.15
	(-3.08)	(4.59)	(-6.50)	(5.79)		(7.05)	1961.1–2013.2	
FI	-0.001	0.00001	-0.085	0.209			0.0068	1.52
	(-0.33)	(1.09)	(-5.06)	(2.83)			1961.1–2013.2	
AS	0.008	-0.00001	-0.209	0.095		0.400	0.0041	2.06
	(5.08)	(-1.39)	(-6.39)	(3.23)		(5.48)	1966.1–2013.2	
Annual								
BE	-0.022	0.00053	-0.197	0.439			0.0092	1.70
	(-4.60)	(4.98)	(-1.84)	(5.36)			1962–2012	
DE	-0.001	0.00002	-0.410	0.309			0.0113	1.55
	(-0.12)	(0.19)	(-5.51)	(4.14)			1962–2012	
NO	-0.010	0.00032	-0.166	0.405			0.0129	0.85
	(-1.48)	(2.21)	(-1.98)	(3.24)			1962–2012	
SW		0.00007	-0.135	0.380			0.0132	0.99
	(-0.01)	(0.49)	(-2.92)	(4.12)			1965–2012	
IR	-0.030	0.00048	-0.365	0.584			0.0163	1.49
	(-4.99)	(3.07)	(-3.82)	(7.91)			1962–2012	

Table B13: Test Results for Equation 13

	Lags	RHO	Leads	Stability			End Test		overid	
	p-val	p-val	p-val	AP	df	λ	p-val	End	p-val	df
Quarterly										
CA	0.000	0.005	0.198	8.20	5	5.086	0.933	1998.4	0.002	5
JA	0.518	0.016	0.028	15.28	4	6.257	1.000	1998.3		
AU	0.000	0.115	0.163	5.86	4	4.300	0.672	1998.4	0.006	6
FR	0.061	0.118	0.003	13.73	5	1.729	1.000	1998.3		
GE	0.001	0.577	0.600	11.79	4	3.332	0.788	1998.4	0.073	6
IT	0.002	0.155	0.214	5.61	4	3.204	1.000	1998.3		
NE	0.000	0.000	0.060	50.02	4	3.204	0.957	1998.4	0.000	6
ST	0.011	0.006	0.183	12.43	4	1.428	1.000	1998.3	0.000	6
UK	0.069	0.024	0.015	6.15	5	3.453	0.187	1998.3		
FI	0.000	0.000	0.021	43.31	4	2.115	1.000	1998.3	0.000	7
AS	0.358	0.210	0.000	6.31	5	4.945	1.000	1998.2		
Annual										
BE	0.271	0.147	0.101	8.28	4	7.384	0.556	1996		
DE	0.290	0.050	0.108	5.91	4	7.384	0.955	1998		
NO	0.000	0.000	0.131	9.68	4	7.384	0.636	1998		
SW	0.000	0.000	0.015	17.37	4	6.224	0.842	1998		
IR	0.114	0.016	0.546	6.01	4	4.003	0.045	1998		

Table B14: Coefficient Estimates for Equation 14

$\log(L1/POP1) = a_1 + a_2 T + a_3 \log(L1/POP1)_{-1} + a_4 UR$

	a_1	a_2	a_3	a_4	SE	DW
Quarterly						
JA	-0.012	0.00001	0.972	-0.120	0.0031	2.18
	(-1.69)	(0.90)	(56.65)	(-2.73)		1966.1–2013.2
AU	-0.069	0.00010	0.901	-0.073	0.0059	2.41
	(-3.33)	(2.75)	(29.99)	(-0.80)		1970.1–2013.1
ST	-0.014	0.00006	0.980	-0.160	0.0066	2.78
	(-1.05)	(1.98)	(37.58)	(-1.91)		1977.1–2013.2
FI	-0.023		0.949	-0.026	0.0056	2.19
	(-3.14)	(-0.25)	(55.08)	(-1.92)		1961.1–2013.2
AS	-0.053	0.00006	0.899	-0.040	0.0039	1.77
	(-3.44)	(3.55)	(30.88)	(-2.59)		1966.1–2013.2
Annual						
BE	-0.115	0.00014	0.827	-0.026	0.0072	2.06
	(-2.16)	(1.46)	(10.14)	(-0.69)		1962–2012
NO	-0.023	0.00026	0.952	-0.184	0.0124	1.03
	(-0.60)	(0.85)	(15.76)	(-1.13)		1962–2012
SW	-0.074	0.00047	0.841	-0.296	0.0074	1.55
	(-3.18)	(3.51)	(17.11)	(-4.33)		1965–2012
IR	-0.107	0.00079	0.816	-0.228	0.0123	2.28
	(-4.15)	(4.68)	(18.58)	(-4.13)		1962–2012

Table B14: Test Results for Equation 14

	Lags	RHO	Stability			End Test		overid	
	p-val	p-val	AP	df	λ	p-val	End	p-val	df
Quarterly									
JA	0.104	0.126	11.81	4	6.257	1.000	1998.3	0.003	5
AU	0.012	0.010	1.91	4	4.872	0.161	1998.3	0.263	5
ST	0.000	0.000	6.80	4	1.513	1.000	1998.3	0.001	5
FI	0.068	0.017	11.66	4	2.115	0.692	1998.3	0.000	5
AS	0.104	0.344	6.77	4	4.945	1.000	1998.2	0.381	5
Annual									
BE	0.391	0.737	7.95	4	3.092	0.111	1996		
NO	0.000	0.000	20.53	4	7.384	0.636	1998		
SW	0.003	0.102	5.65	4	6.224	0.263	1998		
IR	0.717	0.329	11.31	4	4.003	0.727	1998		

Table B.5
Links Between the US and ROW Models

The data on the variables for the United States that are needed when the US model is imbedded in the MCF model were collected as described in Table B.2. These variables are (with the US subscript dropped): $EXDS$, $IMDS$, M, MS, $M00\$A$, $M00\$B$, PM, PMP, $PSI2$, $PW\$$, PX (= $PX\$$), S, TT, XS, and $X00\$$. The PX_{US} variable here is not the same as the PX variable for the United States in Appendix A. The variable here is denoted $USPX$ in the MCF model. The PX variable for the United States is the price deflator of total sales of the firm sector.

Variable	Determination
$X00\$_{US}$	Determined in Table B.3
PMP_{US}	Determined in Table B.3
$PW\$_{US}$	Determined in Table B.3
PX_{US}	Determined by an equation that is equivalent to equation 11 for the other countries. See the discussion in Section B.6.
$PEX =$	$DEL3 \cdot PX_{US}$. In the US model by itself, PEX is determined as $PSI1 \cdot PX$, which is equation 32 in Table A.2. This equation is dropped when the US model is linked to the ROW model. $DEL3$ is constructed from the data as PEX/PX_{US} and is taken to be exogenous.
$PM_{US} =$	$PSI2_{US} PMP_{US}$. This is the same as equation I-19 for the other countries.
$PIM =$	$DEL4 \cdot PM_{US}$. PIM is an exogenous variable in the US model by itself. $DEL4$ is constructed from the data as PIM/PM_{US} and is taken to be exogenous.
$EX =$	$(X00\$_{US} + XS_{US} + EXDS_{US})/1000$. This is the same as equation I-2 for the other countries. EX is an exogenous variable in the US model by itself. $EXDS_{US}$ is constructed from the data as $1000EX - X00\$_{US} - XS_{US}$ and is taken to be exogenous.
$M_{US} =$	$1000IM - MS_{US} - IMDS_{US}$. This is the same as equation I-1 for the other countries. $IMDS_{US}$ is constructed from the data as $1000IM - M_{US} - MS_{US}$ and is taken to be exogenous.
$M00\$A_{US} =$	$M_{US} - M00\$B_{US}$. This is the same as equation I-8 for the other countries.
$S_{US} =$	$PX_{US}(X00\$_{US} + XS_{US}) - PM_{US}(M_{US} + MS_{US}) + TT_{US}$. This is the same as equation I-6 for the other countries.

• The new exogenous variables for the US model when it is linked to the ROW model are $DEL3$, $DEL4$, $EXDS_{US}$, $IMDS_{US}$, $M00\$B_{US}$, MS_{US}, $PSI2_{US}$, TT_{US}, and XS_{US}. EX and PIM are exogenous in the US model by itself, but endogenous when the US model is linked to the ROW model.

Table B.6
Construction of the Balance of Payments Data: Data for S and TT

The relevant raw data variables are:

$M\$'$	Goods imports (fob) in \$, BOP data. [IFS78ABD or IFS1A9DX]
$M\$$	Goods imports (fob) in \$. [IFS71V/E]
$X\$'$	Goods exports (fob) in \$, BOP data. [IFS78AAD or IFS1A9CX]
$X\$$	Goods exports (fob) in \$. [IFS70/E]
$MS\$$	Services and income (debit) in \$, BOP data. [IFS78AED + IFS78AHD or IFS1B9DX + IFS1C9DX]
$XS\$$	Services and income (credit) in \$, BOP data. [IFS78ADD + IFS78AGD or IFS1B9CX + IFS1C9CX]
$XT\$$	Current transfers, n.i.e., (credit) in \$, BOP data. [IFS78AJD or IFS1D9CA]
$MT\$$	Current transfers, n.i.e., (debit) in \$, BOP data. [IFS78AKD or IFS1D9DA]

When quarterly data on all the above variables were available, then $S\$$ and $TT\$$ were constructed as:

$$S\$ = X\$' + XS\$ - M\$' - MS\$ + XT\$ - MT\$$$

$$TT\$ = S\$ - X\$ - XS\$ + M\$ + MS\$$$

where $S\$$ is total net goods, services, and transfers in \$ (balance of payments on current account) and $TT\$$ is total net transfers in \$.

When only annual data on $M\$'$ were available and quarterly data were needed, interpolated quarterly data were constructed using $M\$$. Similarly for $MS\$$.

When only annual data on $X\$'$ were available and quarterly data were needed, interpolated quarterly data were constructed using $X\$$. Similarly for $XS\$$, $XT\$$, and $MT\$$.

When no data on $M\$'$ were available, then $M\$'$ was taken to be $\lambda M\$$, where λ is the last observed value of $M\$'/M\$$. Similarly for $MS\$$ (where λ is the last observed annual value of $MS\$/M\$$.)

When no data on $X\$'$ were available, then $X\$'$ was taken to be $\lambda X\$$, where λ is the last observed value of $X\$'/X\$$. Similarly for $XS\$$ (where λ is the last observed annual value of $XS\$/X\$$), for $XT\$$ (where λ is the last observed annual value of $XT\$/X\$$), and for $MT\$$ (where λ is the last observed annual value of $MT\$/X\$$).

The above equations for $S\$$ and $TT\$$ were then used to construct quarterly data for $S\$$ and $TT\$$.

After data on $S\$$ and $TT\$$ were constructed, data on S and TT were constructed as:

$$S = E \cdot S\$$$

$$TT = E \cdot TT\$$$

Note from MS and XS in Table B.2 and from $MS\$$ and $XS\$$ above that

$$MS\$ = (PM \cdot MS)/E$$

$$XS\$ = (PX \cdot XS)/E$$

Note also from Table B.2 that

$$M\$ = (PM \cdot M)/E$$

$$X\$ = (E00 \cdot PX \cdot X00\$)/E$$

Therefore, from the above equations, the equation for S can be written

$$S = PX(E00 \cdot X00\$ + XS) - PM(M + MS) + TT$$

which is equation I-6 in Table B.3.

8 References in Fair (1984)

References in Fair (1984).

9 References

References

[1] Aastveit, Knut Are, Karsten R. Gerdrup, and Anne Sofie Jore, 2011, "Short-Term Forecasting of GDP and Inflation in Real-Time: Norges Bank's System for Averaging Models," Norges Bank, Staff Memo, No. 09.

[2] Adelman, I., and F.L. Adelman, 1959, "The Dynamic Properties of the Klein-Goldberger Model," *Econometrica*, 27, 596–625.

[3] Akerlof, George, A., William T. Dickens, and George L. Perry, 1996, "The Macroeconomics of Low Inflation," *Brookings Papers on Economic Activity*, 1–76.

[4] Allen, Polly Reynolds, 1973, "A Portfolio Approach to International Capital Flows," *Journal of International Economics*, 3, 135-160.

[5] Almon, Shirley, 1965, "The Distributed Lag Between Capital Appropriations and Expenditures," *Econometrica*, 33, 178-196.

[6] Alogoskoufis, George, Charles Bean, Giuseppe Bertola, Daniel Cohen, Juan Dolado, and Gilles Saint-Paul, 1995, *Unemployment: Choices for Europe*, London: Centre for Economic Policy Research.

[7] Amman, Hans M., and David A. Kendrick, 1999, "Linear Quadratic Optimization for Models with Rational Expectations and Learning," *Macroeconomic Dynamics*.

[8] Andrews, Donald W.K., 2003, "End-of-Sample Instability Tests," *Econometrica*, 71, 1661–1694.

[9] Andrews, Donald W.K., and Ray C. Fair, 1988, "Inference in Nonlinear Econometric Models with Structural Change," *Review of Economic Studies*, 615–640.

[10] Andrews, Donald W. K., and Werner Ploberger, 1994, "Optimal Tests When a Nuisance Parameter is Present Only Under the Alternative," *Econometrica*, 62, 1383–1414.

[11] Auerbach, Alan J., and Yuriy Gorodnichenko, 2012, "Measuring the Output Responses to Fiscal Policy," *The American Economic Review: Economic Policy*, 4, 1–27.

[12] Barro, Robert J., 1977, "Unanticipated Money Growth and Unemployment in the United States," *The American Economic Review*, 67, 101-115.

[13] Barro, Robert J., and Charles J. Redlick, 2011, "Macroeconomic Effects from Government Purchases and Taxes," *Quarterly Journal of Economics*, 126, 51–102.

[14] Berkowitz, J. and L. Kilian, 2000, "Recent Developments in Bootstrapping Time Series," *Econometric Reviews*, 19, 1–48.

[15] Bernanke, Ben S., and Mark Gertler, 1989, "Agency Costs, Net Worth, and Business Fluctuations," *American Economic Review*, 79, 14–331.

[16] Bernanke, Ben S., Mark Gertler, and Simon Gilchrist, 1999, "The Financial Accelerator in a Quantitative Business Cycle Framework," in *Handbook of Macroeconomics*, Vol. 1, edited by John B. Taylor and Michael Woodford, 1341–1393, Amsterdam: Elsevier Science, North-Holland.

[17] Binder, Michael, M. Hashem Pesaran, and S. Hossein Samiei, 2000, "Solution of Nonlinear Rational Expectations Models with Applications to Finite-Horizon Life-Cycle Models of Consumption," *Computational Economics*, 15, 25–57.

[18] Black, Stanley W., 1973, *International Money Markets and Flexible Exchange Rates*, Studies in International Finance, No. 32, Princeton: Princeton University Press.

[19] Blinder, Alan S., 1981, "Retail Inventory Behavior and Business Fluctuations," *Brookings Papers on Economic Activity*, 443–505.

[20] Blinder, Alan S., and Janet L. Yellen, 2001, *The Fabulous Decade: Macroeconomic Lessons from the 1990s*, New York: The Century Foundation Press.

[21] Bloom, Nicholas, 2009, "The Impact of Uncertainty Shocks," *Econometrica*, 77, 623–685.

[22] Branson, William H., 1974, "Stocks and Flows in International Monetary Analysis," in A. Ando, R. Herring, and R. Martson, eds., *International Aspects of Stabilization Policies*, Boston: Federal Reserve Bank of Boston, 27-50.

[23] Brown, B.W., and R.S. Mariano, 1984, "Residual-Based Procedures for Prediction and Estimation in a Nonlinear Simultaneous System," *Econometrica*, 52, 321–343.

[24] Brunnermeier, Markus K., and Yuliy Sannikov, 2014, "A Macroeconomic Model with a Financial Sector," *American Economic Review*, 104, 371–421.

[25] Carroll, Christopher, Jiri Slacalek, and Martin Sommer, 2013, "Dissecting Saving Dynamics: Measuring Wealth, Precautionary, and Credit Effects," February 17,

[26] Case, Karl E., John M. Quigley, and Robert J. Shiller, 2005, "Comparing Wealth Effects: The Stock Market vs. the Housing Mrket," *Advances in Macroeconomics*, 5, 1–34.

[27] Case, Karl E., John M. Quigley, and Robert J. Shiller, 2012, "Wealth Effects Revisited 1975–2012," Cowles Foundation Discussion Paper No. 1884, December.

[28] Chauvet, Marcelle, and Simon Potter, 2005, "Forecasting Recessions Using the Yield Curve," *Journal of Forecasting*, 24, 77–103.

[29] Chong, Yock Y., and David F. Hendry, 1986, "Econometric Evaluation of Linear Macro-Econometric Models," *Review of Economic Studies*, 53, 671-690.

[30] Chow, Gregory C., 1968, "The Acceleration Principle and the Nature of Business Cycles," *The Quarterly Journal of Economics*, 82, 403-418.

[31] Chow, Gregory C., 1975, *Analysis and Control of Dynamic Economic Systems*. New York: John Wiley & Sons.

[32] Chow, Gregory C., 1981, *Econometric Analysis by Control Methods*, New York: John Wiley & Sons.

[33] Chow, Gregory C., 1989, "Rational Versus Adaptive Expectations in Present Value Models," *The Review of Economics and Statistics*, 71, 376–384.

[34] Christian, James W., 1968, "A Further Analysis of the Objectives of American Monetary Policy," *Journal of Finance*, 23, 465–477.

[35] Clarida, Richard, Jordi Galí, and Mark Gertler, 1999, "The Science of Monetary Policy: A New Keynesian Perspective," *The Journal of Economic Literature*, 37, 1661–1707.

[36] Clarida, Richard, Jordi Galí, and Mark Gertler, 2000, "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *The Quarterly Journal of Economics*, 115, 147–180.

[37] Coenen, Günter, Christopher J. Erceg, Charles Freedman, Davide Furceri, Michael Kumhof, René Lalonde, Douglas Laxton, Jesper Lindé, Annabelle Mourougane, Dirk Muir, Susana Mursula, Carlos de Resende, John Roberts, Werner Roeger, Stephen Snudden, Mathias Trabandt, and Jan in't Veld, 2012, "Effects of Fiscal Stimulus in Structural Models," *American Economic Journal: Macroeconomics*, 4, 22–68.

[38] Congressional Budget Office, 2009, Letter to the Honorable Charles E. Grassley, March 2.

[39] Congressional Budget Office, 2010, "Estimated Impact of the American Recovery and Reinvestment Act on Employment and Economic Output From October 2009 through December 2009," February.

[40] Congressional Budget Office, 2011, "An Analysis of the President's Budgetary Proposals for Fiscal Year 2012," April.

[41] Cooper, J. Phillip, and Charles R. Nelson, 1975, "The Ex-Ante Prediction Performance of the St. Louis and FRB-MIT-Penn Econometric Models and Some Results on Composite Predictions," *Journal of Money, Credit, and Banking*, 7, 1-32.

[42] Council of Economic Advisers, 2005, *Economic Report of the President*, United States Government Printing Office, February.

[43] Cumby, Robert E., John Huizinga, and Maurice Obstfeld, 1983, "Two-Step Two-Stage Least Squares Estimation in Models with Rational Expectations," *Journal of Econometrics*, 21, 333–355.

[44] Davidson, Russell, and James G. MacKinnon, 1981, "Several Tests of Model Specification in the Presence of Alternative Hypotheses," *Econometrica*, 40, 781-793.

[45] DeLong, J. Bradford and Lawrence H. Summers, 1986, "Is Increased Price Flexibility Stabilizing?", *The American Economic Review*, 78, 1031-1043.

[46] DeLong, J. Bradford, and Lawrence H. Summers, 2012, "Fiscal Policy in a Depressed Economy," March 20.

[47] Dewald, William G., and Harry G. Johnson, 1963, "An Objective Analysis of the Objectives of American Monetary Policy, 1952–61," in *Banking and Monetary Studies*, Deane Carson, ed., Homewood, Illinois: Richard D. Irwin, 171–189.

[48] Diebold, Francis X., and Glenn D. Rudebusch, 1989, "Scoring the Leading Indicators," *Journal of Business*, 62, 369–391.

[49] Dornbusch, Rudiger, 1976, "Capital Mobility, Flexible Exchange Rates and Macroeconomic Equilibrium," in E. Claassen and P. Salin, eds., *Recent Developments in International Monetary Economics*, Amsterdam: North-Holland, 261-278.

[50] Duygan-Bump, Burcu, Alexey Levkov, and Judit Montoriol-Garriga, 2011, "Financing Constraints and Unemployment: Evidence from the Great Recession," Federal Reserve Bank of Boston, Working Paper No. QAU10-6, December 14.

[51] Efron, B., 1979, "Bootstrap Methods: Another Look at the Jackknife," *The Annals of Statistics*, 7, 1–26.

[52] Eisner, Robert, 1997, "A New View of the NAIRU," in P. Davidson and J. Kregel, (eds.), *Improving the Global Economy: Keynesianism and the Growth in Output and Employment*, Cheltinham, U.K., and Brookfield: Edward Algar.

[53] Fair, Ray C., 1969, *The Short-Run Demand for Workers and Hours*. North-Holland Publishing Co.

[54] Fair, Ray C., 1970, "The Estimation of Simultaneous Equation Models with Lagged Endogenous Variables and First Order Serially Correlated Errors," *Econometrica*, 38, 507–516.

[55] Fair, Ray C., 1974a, *A Model of Macroeconomic Activity. Volume I: The Theoretical Model*, Cambridge, MA: Ballinger Publishing Co.

[56] Fair, Ray C., 1974b, "On the Solution of Optimal Control Problems as Maximization Problems," *Annals of Economic and Social Measurement*, 3, 135–154.

[57] Fair, Ray C., 1974c, "On the Robust Estimation of Econometric Models," *Annals of Economic and Social Measurement*, 667–677.

[58] Fair, Ray C., 1976, *A Model of Macroeconomic Activity. Volume II: The Empirical Model*, Cambridge, MA: Ballinger Publishing Co.

[59] Fair, Ray C., 1978a, "The Use of Optimal Control Techniques to Measure Economic Performance," *International Economic Review*, 289–309.

[60] Fair, Ray C., 1978b, "The Sensitivity of Fiscal Policy Effects to Assumptions about the Behavior of the Federal Reserve," *Econometrica*, 46, 1165–1179.

[61] Fair, Ray C., 1979a, "A Model of the Balance of Payments," *Journal of International Economics*, 9, 25-46.

[62] Fair, Ray C., 1979b, "An Analysis of a Macro-Econometric Model with Rational Expectations in the Bond and Stock Markets," *The American Economic Review*, 539–552.

[63] Fair, Ray C., 1980a, "Estimating the Expected Predictive Accuracy of Econometric Models," *International Economic Review*, 21, 355–378.

[64] Fair, Ray C., 1980b, "Estimating the Uncertainty of Policy Effects in Nonlinear Econometric Models," *Econometrica*, 48, 1381–1391.

[65] Fair, Ray C., 1984, *Specification, Estimation, and Analysis of Macroeconometric Models*, Cambridge, MA: Harvard University Press.

[66] Fair, Ray C., 1985, "Excess Labor and the Business Cycle," *The American Economic Review*, 239–245.

[67] Fair, Ray C., 1987, "International Evidence on the Demand for Money," *The Review of Economics and Statistics*, 473–480.

[68] Fair, Ray C., 1988, "Sources of Economic Fluctuations in the United States," *The Quarterly Journal of Economics*, 313-332.

[69] Fair, Ray C., 1989, "The Production Smoothing Model is Alive and Well," *Journal of Monetary Economics*, 23, 353-370.

[70] Fair, Ray C., 1992, "The Cowles Commission Approach, Real Business Cycle Theories, and New Keynesian Economics," in Michael T. Belongia and Michelle R. Garfinkel, eds., *The Business Cycle: Theories and Evidence*, Kluwer Academic Publishers, 133–147.

[71] Fair, Ray C., 1993a, "Testing the Rational Expectations Hypothesis in Macroeconometric Models," *Oxford Economic Papers*, 45, 169–190.

[72] Fair, Ray C., 1993b, "Estimating Event Probabilities in Macroeconometric Models," in J.H. Stock and M.W. Watson, eds., *Business Cycles, Indicators, and Forecasting*, Chicago: The University of Chicago Press, 157–176.

[73] Fair, Ray C., 1994, *Testing Macroeconometric Models*, Cambridge, MA: Harvard University Press.

[74] Fair, Ray C., 1998, "Estimated Stabilization Costs of the EMU," *National Institute Economic Review*, 164, 90–99.

[75] Fair, Ray C., 1999, "Estimated Inflation Costs Had European Unemployment Been Reduced in the 1980s by Macro Policies," *Journal of Macroeconomics*, 1–28.

[76] Fair, Ray C., 2000, "Testing the NAIRU Model for the United States," *The Review of Economics and Statistics*, 82, 64–71.

[77] Fair, Ray C., 2002, "On Modeling the Effects of Inflation Shocks," *Contributions to Macroeconomics*, Vol. 2, No. 1, Article 3. http://www.bepress.com/bejm/contributions/vol2/iss1/art3.

[78] Fair, Ray C., 2003a, "Bootstrapping Macroeconomeric Models," *Studies in Nonlinear Dynamics & Econometrics*, Vol. 7, No. 4, Article 1. http://www.bepress.com/snde/vol7/iss4/art1.

[79] Fair, Ray C., 2003b, "Optimal Control and Stochastic Simulation of Large Nonlinear Models with Rational Expectations," *Computational Economics*, 245–256.

[80] Fair, Ray C., 2004a, *Estimating How the Macroeconomy Works*. Cambridge, MA: Harvard University Press.

[81] Fair, Ray C., 2004b, "Testing for a New Economy in the 1990s," *Business Economics*, 43–53.

[82] Fair, Ray C., 2005a, "Estimates of the Effectiveness of Monetary Policy," *Journal of Money, Credit, and Banking*, 645–660.

[83] Fair, Ray C., 2005b, "Policy Effects in the Post Boom U.S. Economy," *Topics in Macroeconomics*, Vol. 5: Iss. 1, Article 19.

[84] Fair, Ray C., 2007, "A Comparison of Five Federal Reserve Chairmen: Was Greenspan the Best?" *The B.E. Journal of Macroeconomics*, Vol. 7: Iss. 1 (Contributions), Article 12.

[85] Fair, Ray C., 2010a, "Estimated Macroeconomic Effects of a Chinese Yuan Appreciation," *Business Economics*, 233–243.

[86] Fair, Ray C., 2010b, "Estimated Macroeconomic Effects of the U.S. Stimulus Bill," *Contemporary Economic Policy*, 28, 439–452.

[87] Fair, Ray C., 2012a, "Has Macro Progressed?" *Journal of Macroeconomics*, 34, 2–10.

[88] Fair, Ray C., 2012b, "Analyzing Macroeconomic Forecastability," *Journal of Forecasting*, 31, 99–108.

[89] Fair, Ray C., 2012c, "What It Takes To Solve the U.S. Government Deficit Problem," *Contemporary Economic Policy*, 30, 618–628. v

[90] Fair, Ray C., 2014a, "Is Fiscal Stimulus a Good Idea," March.

[91] Fair, Ray C., 2014b, "The Financial Crisis and Macroeconomic Activity: 2008–2013," December.

[92] Fair, Ray C., and Kathryn M. Dominguez, 1991, "Effects of the Changing U.S. Age Distribution on Macroeconomic Equations," *The American Economic Review*, 81, 1276–1294.

[93] Fair, Ray C., and E. Philip Howrey, 1996, "Evaluating Alternative Monetary Policy Rules," *Journal of Monetary Economics*, 38, 173–193.

[94] Fair, Ray C., and William R. Parke, 1980, "Full-Information Estimates of a Nonlinear Macroeconometric Model," *Journal of Econometrics*, 269–291.

[95] Fair, Ray C., and Robert J. Shiller, 1989, "The Informational Content of Ex Ante Forecasts," *The Review of Economics and Statistics*, 71, 325–331.

[96] Fair, Ray C., and Robert J. Shiller, 1990, "Comparing information in Forecasts from Econometric Models," *The American Economic Review*, 80, 375–389.

[97] Fair, Ray C., and John B. Taylor, 1983, "Solution and Maximum Likelihood Estimation of Dynamic Rational Expectations Models," *Econometrica*, 51, 1169–1185.

[98] Fair, Ray C., and John B. Taylor, 1990, "Full information Estimation and Stochastic Simulation of Models with Rational Expectations," *Journal of Applied Econometrics*, 5, 381–392.

[99] Fay, Jon A., and James L. Medoff, 1985, "Labor and Output Over the Business Cycle: Some Direct Evidence," *The American Economic Review*, 75, 638–655.

[100] Federal Reserve Board, 2000, "FRB/US Equation Documentation for the VAR-Based Expectations Version of the Model."

[101] Feldstein, Martin, and James H. Stock, 1993, "The Use of a Monetary Aggregate to Target Nominal GDP," NBER Working Paper No. 4304.

[102] Finan, Frederico S., and Robert Tetlow, 1999, "Optimal Control of Large, Forward-Looking Models," Board of Governors of the Federal Reserve System.

[103] Freedman, D., 1981, "Bootstrapping Regression Models," *The Annals of Statistics*, 9, 1218–1228.

[104] Freedman, D., 1984, "On Bootstrapping Two-Stage Least-Squares Estimates in Stationary Linear Models," *The Annals of Statistics*, 12, 827–842.

[105] Frenkel, Jacob A., and Harry G. Johnson, eds., 1976, *The Monetary Approach to the Balance of Payments*, Toronto: University of Toronto Press.

[106] Frenkel, Jacob A., and Carlos A. Rodrigues, 1975, "Portfolio Equilibrium and the Balance of Payments: A Monetary Approach," *The American Economic Review*, 65, 674-688.

[107] Fuhrer, Jeffrey C., 1995, "The Phillips Curve Is Alive and Well," *New England Economic Review*, 41–56.

[108] Gerdrup, Karsten R., and Jon Nicolaisen, 2011, "On the Purpose of Models—The Norges Bank Experience," Norges Bank, Staff Memo, No. 06.

[109] Gilchrist Simon, and Egon Zakrajšek, 2012, "Credit Spreads and Business Cycle Fluctuations," *American Economic Review*, 102, 1692–1720.

[110] Girton, Lance, and Dale W. Henderson, 1976, "Financial Capital Movements and Central Bank Behavior in a Two-Country Portfolio Balance Model," *Journal of Monetary Economics*, 2, 33-61.

[111] Gordon, Robert J., 1980, "Comments on George L. Perry, 'Inflation in Theory and Practice," *Brookings Papers on Economic Activity*, 249–257.

[112] Gordon, Robert J., 1997, "The Time-Varying NAIRU and its Implications for Economic Policy," *Journal of Economic Perspectives*, 11, 11–32.

[113] Gordon, Robert J., 2000a, Comment on Dale W. Jorgenson and Kevin J. Stiroh, "Raising the Speed Limit: U.S. Economic Growth in the Information Age," *Brookings Papers on Economic Activity*, 1, 212–222.

[114] Gordon, Robert J., 2000b, "Does the 'New Economy' Measure up to the Great Inventions of the Past?" *The Journal of Economic Perspectives*, 14, 49–74.

[115] Gordon, Robert J., and Stephen R. King, 1982, "The Output Cost of Disinflation in Traditional and Vector Autoregressive Models," *Brookings Papers on Economic Activity*, 205–242.

[116] Granger, Clive W. J., and Paul Newbold, 1986, *Forecasting Economic Time Series*, 2nd ed., New York: Academic Press.

[117] Hall, P., 1988, "Theoretical Comparison of Bootstrap Confidence Intervals," *The Annals of Statistics*, 16, 927–953.

[118] Hall, P., 1992, *The Bootstrap and Edgeworth Expansion*, New York: Springer-Verlag.

[119] Hall, P., and J.L. Horowitz, 1996, "Bootstrap Critical Values for Tests Based on Generalized-Method-of-Moments Estimators," *Econometrica*, 64, 891–916.

[120] Hall, Robert E., 1986, "The Role of Consumption in Economic Fluctuations," in Robert J. Gordon, ed., *The American Business Cycle: Continuity and Change*, Chicago: University of Chicago Press.

[121] Hall, Robert E., 2009, "By How Much Does GDP Rise if the Government Buys More Output?" *Brookings Papers on Economic Activity*, Fall, 183–231.

[122] Hallett, Andrew Hughes, Patrick Minford, and Anupam Rastogi, 1993, "The European Monetary System: Achievements and Survival," in Bryant, Ralph C., Peter Hooper, and Catherine L. Mann, eds., *Evaluating Policy Regimes: New Research in Empirical Macroeconomics*, Washington, D.C.: The Brookings Institution, 617–668.

[123] Hamilton, James D., 1983, "Oil and the Macroeconomy since World War II," *Journal of Political Economy*, 91, 228-248.

[124] Hansen, Lars Peter, 1982, "Large Sample Properties of Generalized Method of Moments Estimators," *Econometrica*, 50, 1029–1054.

[125] Härdle, W., J. Horowitz, and J. Kreiss, 2001, "Bootstrap Methods for Time Series," memo.

[126] Hendry, David F., and Jean-Francois Richard, 1982, "On the Formulation of Empirical Models in Dynamic Economics," *Journal of Econometrics*, 20, 3-33.

[127] Hendry, David F., Adrian R. Pagan, and J. Denis Sargan, 1984, "Dynamic Specifications," in Z. Griliches and M. D. Intriligator, eds., *Handbook of Econometrics*, Amsterdam: North-Holland, 1023–1100.

[128] Horowitz, J.L., 1997, "Bootstrap Methods in Econometrics: Theory and Numerical Performance," Chapter 7 in D.M. Kreps and K.F. Wallis, *Advances in Economics and Econometrics: Theory and Applications*, Volume III, Cambridge: Cambridge University Press, 188–222.

[129] Hurwicz, L., 1950, "Least-Squares Bias in Time Series," in T.C. Koopmans, ed., *Statistical Inference in Dynamic Economic Models*," Cowles Commission Monograph No. 10, New York: John Wiley & Sons, Inc., 365–383.

[130] Judd, John P., and Glen D. Rudebusch, 1998, "Taylor's Rule and the Fed: 1970–1997," *FRBSF Economic Review*, 3, 3–16.

[131] Kenen, Peter B., 1969, "The Theory of Optimum Currency Areas," in Mundell, Robert A., and Alexander A. Swoboda, eds., *Monetary Problems of the International Economy*, Chicago: University of Chicago Press, 41–60.

[132] Khoury, Salwa S., 1990, "The Federal Reserve Reaction Function: A Specification Search," in *The Political Economy of American Monetary Policy*, Thomas Mayer, ed., Cambridge: Cambridge University Press, 27–49.

[133] Kilian, L., 1998, "Small-Sample Confidence Intervals for Impulse Response Functions," *The Review of Economics and Statistics*, 80, 218–230.

[134] King, Stephen R., "Is Increased Price Flexibility Stabilizing?: Comment," 1988, *The American Economic Review*, 78, 267-272.

[135] Kiyotaki, Nobuhiro, and John Moore, 1997, "Credit Cycles," *Journal of Political Economy*, 105, 211–248.

[136] Kouri, Penti J. K., 1976, "The Exchange Rate and the Balance of Payments in the Short Run and in the Long Run," *Scandinavian Journal of Economics*, 2, 280-304.

[137] Krugman, Paul. R., 1996, "Stay on Their Backs," *The New York Times Magazine*, 36–37, February 4.

[138] Krugman, Paul, 2010, "Chinese New Year," *The New York Times*, January 1.

[139] Kydland, F.E., and E.C. Prescott, 1982, "Time To Build and Aggregate Fluctuations," *Econometrica*, 50, 1342-1370.

[140] Layard, Richard, Stephen Nickell, and Richard Jackman, 1991, *Unemployment*, Oxford: Oxford University Press.

[141] Levin, Andrew, Volker Wieland, and John C. Williams, 1999, "Robustness of Simple Monetary Policy Rules under Model Uncertainty," in Taylor (1999a), 263–316.

[142] Li, H., and G.S. Maddala, 1996, "Bootstrapping Time Series Models," *Econometric Reviews*, 15, 115–158.

[143] Lilien, David M., 1982, "Sectoral Shifts and Cyclical Unemployment," *Journal of Political Economy*, 90, 777-793.

[144] Lucas, Robert E., Jr., 1976, "Econometric Policy Evaluation: A Critique," in K. Brunner and A.H. Meltzer, eds., *The Phillips Curve and Labor Markets*, Amsterdam: North-Holland.

[145] Ludvigson, Sydney, and Charles Steindel, 1999, "How Important Is the Stock Market Effect on Consumption," *Economic Policy Review*, Federal Reserve Bank of New York, 5, 29–51.

[146] MacKinnon, J.G., 2002, "Bootstrap Inference in Econometrics," presidential address at the 2002 Annual Meeting of the Canadian Economics Association.

[147] Mankiw, N. Gregory, 1994, *Macroeconomics*, New York: Worth Publishers, second edition.

[148] Masson, Paul R., and Stephen Symansky, 1992, "Evaluating the EMS and EMU Using Stochastic Simulations: Some Issues," in Barrell, Ray, and John Whitley, eds., *Macroeconomic Policy Coordination in Europe: The ERM and Monetary Union*, London: Sage Publications, 12–34.

[149] Maason, Paul R., and Bart G. Turtelboom, 1997, "Characteristics of the Euro, the Demand for Reserves, and Policy Coordination Under the EMU," in Masson, Paul R., Thomas H. Krueger, and Bart G. Turtelboom, *EMU and the International Monetary System*, Washington, D.C.: International Monetary Fund, 194–224.

[150] McCallum, B. T., 1976, "Rational Expectations and the Natural Rate Hypothesis: Some Consistent Results," *Econometrica*, 44, 43-52.

[151] McCallum, Bennett T., and Edward Nelson, 1999, "Performance of Operational Policy Rules in an Estimated Semiclassical Structural Model," in Taylor (1999a), 15–45.

[152] McKinnon, Ronald, 1963, "Optimum Currency Areas," *The American Economic Review*, 53, 717–725.

[153] McNees, Stephen K., 1981, "The Methodology of Macroeconometric Model Comparisons," in J. Kmenta and J. B. Ramsey, eds., *Large Scale Macroeconometric Models*, Amsterdam: North-Holland, 397–442.

[154] McNees, Stephen K., 1986, "Modeling the Fed: A Forward-Looking Monetary Policy Reaction Function," *New England Economic Review*, November/December, 3–8.

[155] McNees, Stephen K., 1992, "A Forward-Looking Monetary Policy Reaction Function: Continuity and Change," *New England Economic Review*, November/December, 3–13.

[156] Mian, Atif, Kamalesh Rao, and Amir Sufi, 2013, "Household Balance Sheets, Consumption, and the Economic Slump," June.

[157] Mian, Atif and Amir Sufi, 2014, "What Explains the 2007-2008 Drop in Employment?" February.

[158] Mizon, Grayham and Richard, Jean-Francois, 1986, "The Encompassing Principle and Its Application to Testing Non-Nested Hypotheses," *Econometrica*, 54, 657-678.

[159] Mundell, Robert A., 1961, "A Theory of Optimum Currency Areas," *The American Economic Review*, 50, 657–665.

[160] Murphy, Kevin M., and Robert H. Topel, 1985, ""Estimation and Inference in Two-Step Econometric Models," *Journal of Business and Economic Statistics*, 3, 370-379.

[161] Myhrman, Johan, 1976, "Balance-of-Payments Adjustment and Portfolio Theory: A Survey," in E. Claassen and P. Salin, eds., *Recent Developments in International Monetary Economics*, Amsterdam: North-Holland, 203-237.

[162] Nelson, Charles R., 1972, "The Prediction Performance of the FRB-MIT-Penn Model of the U.S. Economy," *The American Economic Review*, 62, 902-917.

[163] Nordhaus, William D., 2000, "Productivity Growth and the New Economy," Cowles Foundation Discussion Paper No. 1284.

[164] *OECD Economic Outlook*, 2005, May.

[165] Okun, Arthur M., 1971, "The Personal Tax Surcharge and Consumer Demand, 1968–70," *Brookings Papers on Economic Activity*, 167–204.

[166] Oliner, Stephen D., and Daniel E. Sichel, 2000, "The Resurgence of Growth in the Late 1990s: Is Information Technology the Story?" *The Journal of Economic Perspectives*, 14, 3–22.

[167] Orcutt, G.H., 1948, "A Study of the Autoregressive Nature of the Time Series Used for Tinbergen's Model of the Economic System of the United States, 1919–1932," *Journal of the Royal Statistical Society*, Series B, 1–45.

[168] Parke, William R., 1982, "An Algorithm for FIML and 3SLS Estimation of Large Nonlinear Models," *Econometrica*, 50, 81-96.

[169] Penner, Rudolph G., 2011, "Will It Take a Crisis to Fix Fiscal Policy?" *Business Economics*, 62–70.

[170] Pigou, Arthur C., 1929, *Industrial Fluctuations*, 2nd ed., London: Macmillian.

[171] Poole, William, 1976, "Rational Expectations in the Macro Model," *Brookings Papers*, 2, 463–505.

[172] Ramey, Valerie A., 2011, "Can Government Purchases Stimulate the Economy?" *Journal of Economic Literature*, 49, 673–685.

[173] Reifschneider, David, Robert Tetlow, and John Williams, 1999, "Aggregate Disturbances, Monetary Policy, and the Macroeconomy: The FRB/US Perspective," *Federal Reserve Bulletin*, January, 1–19.

[174] Reinhart, Carmen M., and Kenneth S. Rogoff, 2009, *This Time is Different: Eight Centuries of Financial Folly*, Princeton, New Jersey: Princeton University Press.

[175] Reinhart, Carmen M., and Kenneth S. Rogoff, 2014, "Recovery from Financial Crises: Evidence from 100 Episodes," NBER Working Paper 19823, January.

[176] Romer, David, 2000, "Keynesian Macroeconomics without the LM Curve," *The Journal of Economic Perspectives*, 14, 149–169.

[177] Romer, Christina D., and Jared Bernstein, 2009, "The Job Impact of the American Recovery and Reinvestment Plan," Council of Economic Advisers, January 10.

[178] Romer, Christina D., and David H. Romer, 2010, "The Macroeconomic Effects of Tax Changes: Estimates Based on a New Measure of Fiscal Shocks," *The American Economic Review*, 100, 763–801.

[179] Rotemberg, Julio J., and Michael Woodford, 1999, "Interest Rate Rules in an Estimated Sticky Price Model," in Taylor (1999a), 57–119.

[180] Rudebusch, Glenn D., 1999, "Is the Fed Too Timid? Monetary Policy in an Uncertain World."

[181] Rudebusch, Glenn D., and John C. Williams, 2008, "Forecasting Recessions: The Puzzle of the Enduring Power of the Yield Curve," Working Paper 2007-16, Federal Reserve Bank of San Francisco, July.

[182] Rudebusch, Glenn D., and Lars E. O. Svensson, 1999, "Policy Rules for Inflation Targeting," in Taylor (1999a), 203–246.

[183] Runkle, D.E., 1987, "Vector Autoregressions and Reality," *Journal of Business & Economic Statistics*, 5, 437–442.

[184] Shiller, Robert J., 1987, "Ultimate Sources of Aggregate Variability," *The American Economic Review*, 77, 87-92.

[185] Staiger, Douglas, James Stock, and Mark Watson, 1997, "The NAIRU, Unemployment and Monetary Policy," *Journal of Economic Perspectives*, 11, 33–49.

[186] Starr-McCluer, Martha, 1998, "Stock Market Wealth and Consumer Spending," Board of Governors of the Federal Reserve System.

[187] Stock, James, H., and Mark W. Watson, 1998, "Median Unbiased Estimation of Coefficient Variance in a Time-Varying Parameter Model," *Journal of the American Statistical Association*, 93, 349-358.

[188] Stock, James, H., and Mark W. Watson, 2003, "Forecasting Output and Inflation: The Role of Asset Prices," *Journal of Economic Literature*, 61, 788–829.

[189] Stoker, Thomas M., 1986, "Simple Tests of Distributional Effects on Macroeconomic Equations," *Journal of Political Economy*, 94, 763-795.

[190] Svensson, Lars E. O., 2003, "What is Wrong with Taylor Rules? Using Judgement in Monetary Policy through Targeting Rules," *Journal of Economic Literature*, 41, 426–477.

[191] Taylor, John B., 1993, "Discretion Versus Policy Rules in Practice," *Carnegie-Rochester Series on Public Policy*, 39, 195–214.

[192] Taylor, John B., 1999a, *Monetary Policy Rules*, Chicago: The University of Chicago Press.

[193] Taylor, John B., 1999b, "Introduction," in Taylor (1999a), 1–14.

[194] Taylor, John B., 1999c, "A Historical Analysis of Monetary Policy Rules," in Taylor (1999a), 319–341.

[195] Taylor, John B., 2000, "Teaching Modern Macroeconomics at the Principles Level," *The American Economic Review*, 90, 90–94.

[196] Taylor, John B., and Harald Uhlig, 1990, "Solving Nonlinear Stochastic Growth Models: A Comparison of Alternative Solution Methods," *Journal of Business and Economic Statistics*, 8, 1-21.

[197] Tinbergen, Jan, 1939, *Statistical Testing of Business-Cycle Theories, Volume 2, Business Cycles in the United States of America, 1919–1932*. Geneva: League of Nations.

[198] Triplett, Jack E., 1992, "Economic Theory and BEA's Alternative Quantity and Price Indexes," *Survey of Current Business*, 72, 49–52.

[199] von Hagen, Jürgen, and Manfred J. M. Neumann, 1994, "Real Exchange Rates within and between Currency Areas: How Far Away is the EMU?," *Review of Economics and Statistics*, 76, 236–244.

[200] Weller, Christian, Josh Bivens, and Max Sawicky, 2004, "Macro Policy Lessons from the Recent Recession," *Challenge*, 47, 42-72, May-June.

[201] White, Halbert, 1980, "A Heteroskedasticity Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity," *Econometrica*, 48, 817–838.

[202] White, Halbert, and Ian Domowitz, 1984, "Nonlinear Regression with Dependent Observations," *Econometrica*, 52, 143–161.

[203] Wooldridge, Jeffrey M., 2000, *Introductory Econometrics: A Modern Approach*, South-Western College Publishing.

[204] Wyplosz, Charles, 1997, "EMU: Why and How It Might Happen," *Journal of Economic Perspectives*, 11, 3–22.

[205] Young, Allan H., 1992, "Alternative Measures of Change in Real Output and Prices," *Survey of Current Business*, 72, 32–48.

[206] Zandi, Mark M., 2004, "Assessing President Bush's Fiscal Policies," *The Dismal Scientist*, www.dismal.com, July.

[207] Zhou, Xia, and Christopher D. Carroll, 2012, "Dynamics of Wealth and Consumption: New and Improved Measures for U.S. States," *The B.E. Journal of Macroeconomics*: Vol. 12: Iss. 2 (Advances), Article 4.