O'ZBEKISTON RESPUBLIKASI OLIY VA O'RTA MAXSUS TA'LIM VAZIRLIGI

TOSHKENT MOLIYA INSTITUTI

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IQTISODCHILAR UCHUN MATEMATIKA

mustaqil ta'lim bo'yicha praktikum

O'zbekiston Respublikasi Oliy va o'rta maxsus ta'lim vazirligining Muvofiqlashtiruvchi kengashi tomonidan o'quv qo'llanma sifatida tavsiya etilgan

TOSHKENT
"IQTISOD-MOLIYA"
2019

UO'K: 372.851.001.76(063)
KBK: 66.4 (0)
X-22

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Iqtisodchilar uchun matematika. Mustaqil ta'lim bo'yicha praktikum: *O'quv qo'llanma /* A.R.Xashimov, G.S.Xujaniyozova; T., "Iqtisod-Moliya", 2019. – 400 b.

Ushbu qo'llanma iqtisodiyot ta'lim yo'nalishida tahsil olayotgan bakalavrlar uchun mo'ljallangan bo'lib, "Iqtisodchilar uchun matematika" fan dasturiga moslashtirib yozilgan. Qo'llanmada chiziqli algebra, analitik geometriya, matematik tahlil, oddiy differensial tenglamalar va qatorlar nazariyasiga doir materiallar keltirilgan. Bu qo'llanmaga kirilgan materiallar ma'lum bir doiradagi iqtisodiy muammolarning matematik modellarini tuzish hamda ularning optimal yechimini topishda yordam beradi. Qo'llanma talabalarning fan bo'yicha nazariy bilimlarini chuqurlashtirishga, ma'ruza darslarida berilgan tushunchalarni kengroq bilib olishiga yordam beradi. Qo'llanmaga mavzular bo'yicha nazorat ish variantlari kiritilgan.

Qo'llanma iqtisod ta'lim yo'nalishidagi bakalavriat uchun mo'ljallangan ta'lim standartlari va o'quv rejasi talablariga javob beradi.

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ISBN 978-9943-13-776-9

KIRISH

O'zbekiston Respublikasi Prezidentining "Oliy ta'lim tizimini yanada rivojlantirish chora-tadbirlari to'g'risida"gi (2017-yil 20-aprel) PQ-2909-sonli qarorida mamlakatimizni ijtimoiy-iqtisodiy rivojlantirishning ustuvor vazifalaridan kelib chiqqan holda, kadrlar tayyorlash mazmunini tubdan qayta ko'rish, xalqaro standartlar darajasiga mos oliy ma'lumotli mutaxassislar tayyorlash, oliy ta'limda ilm-fanni yanada rivojlantirish, uning akademik ilm-fan bilan integratsiyalashuvini kuchaytirish lozimligi ta'kidlangan.

Ilm-fan jadal taraqqiy etayotgan, zamonaviy axborot-kommunikatsiya tizimlari vositalari keng joriy etilgan jamiyatda turli fan sohalarida bilimlarning tez yangilanib borishi, ta'lim oluvchilar oldiga ularni jadal egallash bilan bir qatorda, muntazam va mustaqil ravishda bilim olish vazifasini qo'ymoqda.

Iqtisodchilar uchun matematika fanini o'qitishdan maqsad talabalarni iqtisodiyot sohasida duch keladigan nazariy va amaliy masalalarni yechishda qo'llaniladigan matematik apparatning asoslari bilan tanishtirish, ularning mantiqiy fikrlash qobiliyatini oshirish, ilmiy adabiyotlarni mustaqil o'rganish ko'nikmalarini shakllantirishdan iborat.

Iqtisodchilar uchun matematika fani ishlab chiqarish jarayoni bilan bevosita bog'lanmagan. Lekin "Iqtisodchilar uchun matematika" fani matematik modellashtirish metodi yordamida ishlab chiqarishni takomillashtirish boyicha muqobil qarorlar qabul qilishda qo'llaniladigan sonli usullarni o'rgatadi. Bu esa eng ilmiy asoslangan maqbul yechimlar qabul qilishga qodir bo'lgan iqtisodchi kadrlarni tayyorlashga yordam beradi.

"Iqtisodchilar uchun matematika" fanining har bir bo'limi keng qamrovli bo'lganligi sababli, ularni faqat auditoriya mashg'ulotlarida chuqur o'rganib bo'lmaydi. Bu esa fan bo'yicha mustaqil ta'limga ajratilgan soatlardan samarali foydalanishni taqozo etadi.

Mustaqil ta'lim – bilish, tafakkur etish jarayonlarini o'zida mujassamlashtirib, texnika, texnologiyalarning yangilanib borayotgan hozirgi sharoitda, shaxsni hayotga va mehnat qilishga tayyorlashning samarali yo'llaridan biri hisoblanadi. Mustaqil ta'limni bajarish talabaning shaxs sifatida rivojlanishiga olib keladi.

Mazkur o'quv qo'llanma iqtisodiyot sohasidagi barcha bakalavriat ta'lim yo'nalishlarida ta'lim olayotgan talabalarga mo'ljallangan bo'lib, ta'lim standartlari va o'quv rejasi talablariga javob beradi. Qo'llanmani yozishga mualliflarning Toshkent moliya institutida o'tkazgan bir necha yillik tajribasi asos bo'ldi.

Qo'llanmada har bir mavzuga doir mustaqil bajarish uchun 25 ta variantda topshiriqlar keltirilgan.

Talabalar guruh jurnalidagi o'zining tartib nomeriga mos bo'lgan variantlarni alohida daftarga bajarishlari lozim. Ular u yoki bu matematik tushunchaga doir mustaqil ishlarni bajarib bo'lishgach olgan javoblarini *Mathcad* amaliy dasturlar paketi yordamida tekshirib ko'radilar. Qo'llanmada talabaning bu faoliyati ham mustaqil ishlar tarkibiga kiritilgan.

I BOB. MATRITSA VA DETERMINANTLAR

1.1.Matritsalar ustida amallar. Texnologik matritsa

Matematikada matritsa va determinant tushunchalaridan ko‘pgina iqtisodiy masalalarning matematik modelini qurishda keng foydalanamiz. Matritsa tushunchasi ko‘p tarmoqli axborotlarni tartiblashga va ular ustidagi masalalarni yechishga yordam beradi. Matematikaning iqtisoddagi tatbiqlarida chiziqli tenglamalar sistemasini yechishga to‘g‘ri keladi. Bunday sistemalarni yechishda, ular bilan bog‘liq bo‘lgan kvadrat matritsalarni xarakterlash uchun determinant deb nomlanuvchi son mos qo‘yiladi.

1-ta’rif. O‘lchamlari $m \times n$ bo‘lgan matritsa deb, satrlar soni m ga, ustunlar soni n ga teng bo‘lgan, $m \cdot n$ ta sondan tashkil topgan to‘g‘ri to‘rtburchak shakldagi jadvalga aytiladi.

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}.$$

Matritsadagi a_{ij} - son (bu yerda birinchi indeks satr nomerini, ikkinchisi esa ustun nomerini ko‘rsatadi va ularning kesishgan joyida a_{ij} element turadi $i = 1,2,...,m$ va $j = 1,2,...,n$) matritsaning elementi deb ataladi.

2-ta’rif. Satrlar soni ham, ustunlar soni ham n ga teng bo‘lgan, ya’ni $n \times n$ o‘lchamli matritsa $n-$tartibli kvadrat matritsa deb ataladi.

$A = (a_{ij})$ kvadrat matritsa uchun $i \neq j \Rightarrow a_{ij} = 0$ munosabat o‘rinli bo‘lsa, u holda A matritsa diagonal matritsa deyiladi.

$$A = \begin{pmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & a_{mn} \end{pmatrix}.$$

$A = (a_{ij})$ kvadrat matritsada $i \neq j \Rightarrow a_{ij} = 0$ $i = j \Rightarrow a_{ii} = 1$ bo‘lsa, u holda bu matritsa birlik matritsa deyiladi va bu matritsa odatda E harfi bilan belgilanadi, ya’ni

$$E = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{pmatrix}.$$

Agar $A=(a_{ij})$ kvadrat matritsa uchun $a_{ij}=0$, $i>j$ ($i<j$) munosabat bajarilsa, u holda A matritsaga yuqori (quyi) uchburchak matritsa deyiladi.

$1\times n$ o'lchamli matritsaga satr matritsa, $m\times 1$ o'lchamli matritsaga esa ustun matritsa deyiladi,

Vektorlar algebrasida ustun-matritsa va satr-matritsalarni mos ravishda ustun-vektor va satr-vektor deb ataladi. Bu matritsalarning elementlari esa ularning koordinatalari deyiladi.

3-ta'rif. Agar A va B matritsalar bir xil o'lchamga ega bo'lib, ularning barcha mos elementlari o'zaro teng bo'lsa, bunday matritsalar teng deyiladi va $A=B$ ko'rinishda yoziladi.

Matritsalar ustida qo'shish, songa ko'paytirish va ko'paytirish amallari bajariladi.

O'lchamlari aynan teng bo'lgan matritsalar ustidagina qo'shish amali bajariladi. A va B matritsalarni qo'shish uchun, ularning mos elementlari qo'shiladi:

$$A+B=\left(a_{ij}+b_{ij}\right).$$

Xuddi shuningdek, ikkita matritsa ayirmasi, ya'ni $A-B=C$ ham matritsalarni qo'shish kabi amalga oshiriladi: $A-B=\left(a_{ij}-b_{ij}\right)$.

Biror haqiqiy λ sonni matritsaga ko'paytirish uchun bu son matritsaning har bir elementiga ko'paytiriladi: $\lambda\left(a_{ij}\right)=\left(\lambda a_{ij}\right)$.

Har bir elementi nolga teng bo'lgan matritsaga nol matritsa deyiladi.

Matritsalarni qo'shish va songa ko'paytirish amallari quyidagi xossalarga bo'ysinadi:

1) $A+B=B+A$,
2) $(A+B)+C=A+(B+C)$,
3) $\lambda(A+B)=\lambda A+\lambda B$,
4) $(\lambda_1+\lambda_2)A=\lambda_1 A+\lambda_2 A$,
5) $\lambda_1(\lambda_2 A)=(\lambda_1\lambda_2)A$,
6) $A+O=O+A=A$; (O – nol matritsa)
7) $A+(-A)=O$;
8) $1\cdot A=A$;

9) Agar $\lambda=0$ bo'lsa, u holda $\lambda A=O$ nol matritsa bo'ladi.

Bu yerda $A,B,C-$bir xil o'lchamli ixtiyoriy matritsalar, $O-A,B,C$ matritsalar bilan bir xil o'lchamli nol matritsa, λ_1,λ_2 –ixtiyoriy sonlar.

4-ta'rif. A matritsaning ustunlar soni B matritsaning satrlari soniga teng bo'lsa, A va B matritsalar o'zaro zanjirlangan matritsalar deyiladi.

Ko'paytirish amali o'zaro zanjirlangan matritsalar ustida bajariladi. $m\times k$ o'lchamli A matritsaning $k\times n$ o'lchamli B matritsaga ko'paytmasi deb, elementlari

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{ik}b_{kj} = \sum_{s=1}^{k} a_{is}b_{sj}$$

ko'rinishida aniqlanadigan $m \times n$ o'lchamli $C = (c_{ij})$ matritsaga aytiladi, bunda $i = 1,2,\dots,m;\ j = 1,2,\dots n$.

Ko'rish mumkinki $C = AB$ matritsaning c_{ij} elementi, A matritsaning i – satr vektori bilan B matritsaning j – ustun vektorini skalyar ko'paytmasidan iborat.

Matritsalarni ko'paytirish amali quyidagi xossalarga bo'ysinadi:

1) $(A + B)C = AC + BC$;

2) $C(A + B) = CA + CB$;

3) $\lambda(AB) = (\lambda A)B = A(\lambda B)$;

4) $(AB)C = A(BC)$;

5) $AE = EA = A$.

Agar A va B matritsalar uchun $AB = BA$ $(AB = -BA)$ munosabat o'rinli bo'lsa, u holda A va B matritsalar kommutativ (antikommutativ) matritsalar deyiladi.

A kvadrat matritsani $m\,(m > 1)$ butun musbat darajaga ko'tarish quyidagicha amalga oshiriladi: $A^m = \underbrace{A \cdot A \cdot \ldots \cdot A}_{m \text{ marta}}$.

Agar A matritsada barcha satrlar mos ustunlar bilan almashtirilsa, u holda A matritsaga transponirlangan matritsa hosil bo'ladi va u A^T ko'rinishda belgilanadi.

Matritsalar ustida bajarilgan transponirlash amali quyidagi xossalarga bo'ysinadi:

1. $(A^T)^T = A$;
2. $(\lambda A)^T = \lambda A^T$;
3. $(A + B)^T = A^T + B^T$;
4. $(AB)^T = B^T A^T$.

Agar A kvadrat matritsada $A = A^T$ $(A = -A^T)$ munosabat o'rinli bo'lsa, u holda bunday matritsaga simmetrik (kososimmetrik) matritsa deyiladi.

5-ta'rif. Nolmas satrlarga ega A matritsada har qanday k – nolmas satrning birinchi noldan farqli elementi $(k-1)$ – nolmas satrning birinchi noldan farqli elementidan o'ngda tursa, u holda A pog'onasimon matritsa deyiladi.

Masalan $A=\begin{pmatrix} 1 & 0 & 2 & 3 & -5 \\ 0 & 0 & 4 & 0 & 1 \\ 0 & 0 & 0 & 7 & 0 \end{pmatrix}$ matritsa pog'onasimon matritsadir.

Texnologik matritsa

Iqtisodiy masalalarni matematik modellashtirishda, ya'ni, iqtisodiy muammoni matematik ifodalar yordamidagi ifodasida, matritsalardan keng foydalaniladi. Bunda muhim tushunchalardan biri texnologik matritsa tushunchasidir. Bu matritsa, masalan, bir nechta turdagi resurslardan bir nechta tovar turlarini ishlab chiqarishni rejalashtirish (programmalashtirish), tarmoqlararo balansni modellashtirish kabi muhim iqtisodiy masalalarda asosiy rolni o'ynaydi.

Faraz qilaylik o'rganilayotgan iqtisodiy jarayonda n xil mahsulot ishlab chiqarish uchun m xil ishlab chiqarish faktorlari (resurslar) zarur bo'lsin. i–mahsulotning bir birligini ishlab chiqarish uchun $j-$turdagi resursdan a_{ij} miqdori sarflansin. a_{ij} elementlardan tuzilgan $m\times n$ o'lchamli A matritsa texnologik matritsa deb ataladi.

1-turdagi mahsulotdan x_1 miqdorda, 2-turdagi mahsulotdan x_2 miqdorda, ..., $n-$turdagi mahsulotdan x_n birlik miqdorda ishlab chiqarilishi talab qilinsin. Bu rejani $X=\begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix}$ ustun vektor ($n\times 1$ o'lchamli matritsa) shaklida ifodalaymiz. U holda 1-turdagi resurs sarfi $a_{11}x_1+\dots+a_{1n}x_n$ ga, ikkinchi turdagi resurs sarfi $a_{21}x_1+\dots+a_{2n}x_n$ ga teng. Umumlashtiradigan bo'lsak, ishlab chiqarish rejasini bajarish uchun zarur bo'lgan $j-$turdagi resurs sarfi $a_{j1}x_1+\dots+a_{jn}x_n$ birlikka teng. Bu miqdorlarni ustun vektor sifatida yozsak aynan AX ko'paytmani hosil qilamiz.

$j-$mahsulotning bir birligining narxi c_j bo'lsin. Narxlar vektorini $C=(c_1,\dots,c_n)$ ko'rinishda ifodalaymiz. U holda CX ko'paytma, matritsalarni ko'paytirish qoidasiga ko'ra, skalyar miqdor, ya'ni sondan iborat. Bu son ishlab chiqarishdan olingan daromadni ifodalaydi.

$i-$ turdagi resurs zahirasi miqdori b_i birlikka teng bo'lsin. Resurs zahiralari vektorini ustun vektor shaklida ifodalaymiz: $B=\begin{pmatrix} b_1 \\ b_2 \\ \dots \\ b_m \end{pmatrix}$. U holda $AX \le B$ tengsizlik ishlab chiqarishda resurs zahiralari hisobga olinishi zarurligini bildiradi. Bu vektor tengsizlik AX vektorning har bir elementi B vektorning mos elementidan katta emasligini bildiradi. $AX \le B$ shartni qanoatlantiruvchi X rejani joiz reja, deb ataymiz. Ma'nosidan kelib chiqadigan bo'lsak, har qanday X rejaning elementlari musbat sonlardan iborat bo'lishi zarur.

Misol. Korxona ikki turdagi transformatorlar ishlab chiqaradi. 1-turdagi transformator ishlab chiqarish uchun 5 kg temir va 3 kg sim, 2-turdagi transformator ishlab chiqarish uchun 3 kg temir va 2 kg sim sarflanadi. Bir birlik transformatorlarni sotishdan mos ravishda 6 va 5 sh.p.b. miqdorida daromad olinadi. Korxonaning omborida 4,5 tonna temir va 3 tonna sim mavjud. Texnologik matritsa, narxlar vektori va resurs zahirasini ifodalovchi vektorni tuzing. $\begin{pmatrix} 500 \\ 600 \end{pmatrix}$, $\begin{pmatrix} 600 \\ 600 \end{pmatrix}$ rejalar joiz reja bo'la oladimi?

Yechish. Korxona ikki turdagi resursdan foydalanib 2 turdagi mahsulot ishlab chiqaradi. Narxlar vektori $C=(6,5)$. Resurs zahiralari vektori $B=\begin{pmatrix} 4500 \\ 3000 \end{pmatrix}$. Texnologik (resurs sarfi normasi) matritsa $A=\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$.

$X=\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ rejani qaraymiz. Bu rejani bajarishdagi resurs sarfi

$$AX=\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} 5x_1+3x_2 \\ 3x_1+2x_2 \end{pmatrix}$$

ga teng. Bu sarf zahiradan oshib ketmasligi kerak, ya'ni $AX \le B$ yoki

$$5x_1+3x_2 \le 4500,$$
$$3x_1+2x_2 \le 3000.$$

Joiz reja yuqoridagi tengsizliklarni qanoatlantirishi zarur.

1) $X=\begin{pmatrix} 500 \\ 600 \end{pmatrix}$ rejani qaraymiz. U holda

$$AX=\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 500 \\ 600 \end{pmatrix}=\begin{pmatrix} 4300 \\ 2700 \end{pmatrix}<\begin{pmatrix} 4500 \\ 3000 \end{pmatrix},$$

ya'ni bu reja joiz reja. Bu reja asosida olinadigan daromad miqdori $CX=(6\quad 5)\begin{pmatrix}500\\600\end{pmatrix}=(6000)$ sh.p.b. ga teng.

2) $X=\begin{pmatrix}600\\600\end{pmatrix}$ rejani qaraymiz. U holda

$$AX=\begin{pmatrix}5&3\\3&2\end{pmatrix}\begin{pmatrix}600\\600\end{pmatrix}=\begin{pmatrix}4800\\3000\end{pmatrix}.$$

Bundan ko'rish mimkinki, 1-turdagi resurs sarfi 4800 ga teng bo'lib, resurs zahirasi 4500 dan katta. Shu sababli, qaralayotgan reja joiz reja emas.

Misollar

1. $2A+3B^T$ chiziqli kombinatsiyani toping. Bu yerda $A=\begin{pmatrix}1&2&3\\0&1&-1\end{pmatrix}$, $B=\begin{pmatrix}-2&2\\3&1\\0&-1\end{pmatrix}$.

Yechish. $B=\begin{pmatrix}-2&2\\3&1\\0&-1\end{pmatrix}$ matritsani transponirlab $B^T=\begin{pmatrix}-2&3&0\\2&1&-1\end{pmatrix}$ matritsani topamiz.

$$2A+3B=2\cdot\begin{pmatrix}1&2&3\\0&1&-1\end{pmatrix}+3\cdot\begin{pmatrix}-2&3&0\\2&1&-1\end{pmatrix}=\begin{pmatrix}2&4&6\\0&2&-2\end{pmatrix}+\begin{pmatrix}-6&9&0\\6&3&-3\end{pmatrix}=$$

$$=\begin{pmatrix}2-6&4+9&6+0\\0+6&2+3&-2+3\end{pmatrix}=\begin{pmatrix}-4&13&6\\6&5&1\end{pmatrix}.$$

2. $A=\begin{pmatrix}1&2&3\\1&0&-1\end{pmatrix}$, $B=\begin{pmatrix}3&4&5\\6&0&-2\\7&1&8\end{pmatrix}$ matritsalar berilgan. AB va BA (agar ular mavjud bo'lsa) ko'paytmani toping.

Yechish. $AB=\begin{pmatrix}1&2&3\\1&0&-1\end{pmatrix}\begin{pmatrix}3&4&5\\6&0&-2\\7&1&8\end{pmatrix}=\begin{pmatrix}1\cdot3+2\cdot6+3\cdot7&1\cdot4+2\cdot0+3\cdot1&1\cdot5+2\cdot(-2)+3\cdot8\\1\cdot3+0\cdot6+(-1)\cdot7&1\cdot4+0\cdot0+(-1)\cdot1&1\cdot5+0\cdot(-2)+(-1)\cdot8\end{pmatrix}=$

$=\begin{pmatrix}36&7&25\\-4&3&-3\end{pmatrix}.$

BA ko'paytma mavjud emas. B matritsaning ustunlari soni A matritsaning satrlari soniga mos emas $(3\neq2)$.

3. Agar $f(x)=-2x^2+5x+9$, $A=\begin{pmatrix}1&2\\3&0\end{pmatrix}$ bo'lsa, $f(A)$ matritsali ko'phadning qiymatini toping.

Yechish. $A^2=A\cdot A=\begin{pmatrix}1&2\\3&0\end{pmatrix}\cdot\begin{pmatrix}1&2\\3&0\end{pmatrix}=\begin{pmatrix}1\cdot1+2\cdot3&1\cdot2+2\cdot0\\3\cdot1+0\cdot3&3\cdot2+0\cdot0\end{pmatrix}=\begin{pmatrix}7&2\\3&6\end{pmatrix},$

$$f(A)=-2A^2+5A+9E=-2\begin{pmatrix}7&2\\3&6\end{pmatrix}+5\begin{pmatrix}1&2\\3&0\end{pmatrix}+9\begin{pmatrix}1&0\\0&1\end{pmatrix}=$$

$$=\begin{pmatrix}-14&-4\\-6&-12\end{pmatrix}+\begin{pmatrix}5&10\\15&0\end{pmatrix}+\begin{pmatrix}9&0\\0&9\end{pmatrix}=\begin{pmatrix}0&6\\9&-3\end{pmatrix}.$$

4. Korxona mahsulotning n turini ishlab chiqaradi, ishlab chiqarish mahsulot hajmlari $A_{1\times n}$ matritsa bilan berilgan. j – mintaqada mahsulotning i–turi birligini sotilish narxi $B_{n\times k}$ matritsa bilan berilgan, bu yerda k – mahsulot sotilayotgan mintaqalar soni.

Mintaqalar bo'yicha daromad matritsasi C ni toping.

$A_{1\times3}=(100\;2000\;100)$ bo'lsin.

$$B_{3\times4}=\begin{pmatrix}2&3&1&5\\1&3&2&2\\2&4&2&4\end{pmatrix}.$$

Yechish. Daromad $C_{1\times k}=A_{1\times n}\cdot B_{n\times n}$ matritsa bilan aniqlanadi, ya'ni $c_{ij}=\sum_{i=1}^{n}a_{1i}\cdot b_{ij}$ – bu j – mintaqadagi korxonaning daromadi:

$$C=(100,2000,100)\begin{pmatrix}2&3&1&5\\1&3&2&2\\2&4&2&4\end{pmatrix}=(600\quad 1300\quad 700\quad 1300).$$

1.2. Determinantlar

A kvadrat matritsaning skalyar (sonli) miqdorni aniqlovchi determinant tushunchasining kiritilishi chiziqli tenglamalar sistemasini yechish bilan chambarchas bog'liq.

Bizga n–tartibli

$$A=\begin{pmatrix}a_{11}&a_{12}&\dots&a_{1n}\\a_{21}&a_{22}&\dots&a_{2n}\\\dots&\dots&\dots&\dots\\a_{n1}&a_{n2}&\dots&a_{nn}\end{pmatrix}$$

kvadrat matritsa berilgan bo'lsin.

n–tartibli determinant $\det(A)$, $|A|$ yoki

$$\begin{vmatrix}a_{11}&a_{12}&\dots&a_{1n}\\a_{21}&a_{22}&\dots&a_{2n}\\\dots&\dots&\dots&\dots\\a_{n1}&a_{n2}&\dots&a_{nn}\end{vmatrix}$$

kabi belgilanadi.

n-tartibli determinant $n!$ ta hadning yig'indisidan iborat va bu yig'indining har bir hadi matritsaning turli satrlari va turli ustunlarida joylashgan n ta elementi ko'paytmasidan ibora. Yuqorida aytilgan ko'paytmalarning yarmi ($n!/2$ tasi) o'z ishorasi bilan, qolgan yarmi qarama-qarshi ishora bilan olingan.

2-tartibli kvadrat matritsaning determinanti quyidagicha aniqlanadi:

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}.$$

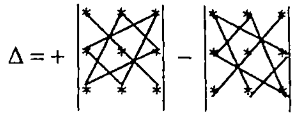
Uchinchi tartibli determinant uchun quyidagi ifodani olamiz:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}.$$

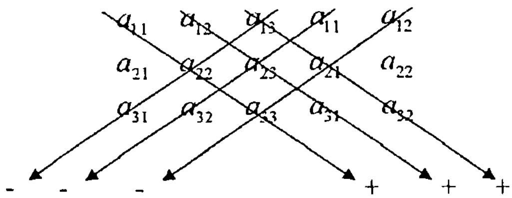
Uchinchi tartibli determinantda o'z ishorasi va qarama-qarshi ishora bilan olinadigan hadlarni eslab qolish uchun odatda ikki xil usuldan foydalaniladi. Bular uchburchak va Sarryus usullari deb nomlanadi.

Determinantlarni hisoblash metodlari

Uchburchak usuli. Uchinchi tartibli determinantni hisoblashning uchburchak usuli quyidagicha sxematik ko'rinishda amalga oshiriladi:



Uchinchi tartibli determinantni hisoblashning Sarryus qoidasi quyidagicha amalga oshiriladi. Determinant ustunlarining o'ng yoniga chapdagi birinchi va ikkinchi ustunlar ko'chirib yoziladi. Hosil bo'lgan kengaytirilgan jadvalda bosh diagonal yo'nalishida joylashgan elementlar ko'paytirilib musbat ishora bilan, ikkilamchi diagonal yo'nalishidagi elementlar ko'paytirilib manfiy ishora bilan olinib yig'indi tuziladi. Bu yig'indi uchinchi tartibli determinantning qiymatidan iborat. Buni sxema ko'rinishida quyidagicha tasvirlash mumkin:



Bizga n-tartibli kvadrat matritsa berilgan bo'lsin.

6-ta'rif. n-tartibli A kvadrat matritsaning $1 \le k \le n-1$ shartni qanoatlantiruvchi ixtiyoriy k ta satrlari va k ta ustunlari kesishgan joyda turgan elementlardan tashkil topgan k-tartibli matritsaning determinanti d determinantning k-tartibli minori deb ataladi.

k-tartibli minor sifatida A kvadrat matritsaning $n-k$ ta satr va $n-k$ ta ustunini o'chirishdan hosil bo'lgan determinant, deb ham qarash mumkin.

7-ta'rif. Matritsaning diagonal elementlari yordamida hosil bo'lgan minorlar bosh minorlar deb ataladi.

8-ta'rif. n-tartibli A kvadrat matritsada k-tartibli M minor turgan satrlar va ustunlar o'chirib tashlangandan so'ng qolgan $(n-k)$-tartibli M' minorga M minorning to'ldiruvchisi deyiladi va aksincha.

M minor va uning M' to'ldiruvchi minorini sxematik ravishda quyidagicha tasvirlash mumkin:

$$d = \begin{vmatrix} a_{11} & \dots & a_{1k} & a_{1k+1} & \dots & a_{1n} \\ \dots & \boxed{M} & \dots & \dots & \dots & \dots \\ a_{k1} & \dots & a_{kk} & a_{kk+1} & \dots & a_{kn} \\ a_{k+11} & \dots & a_{k+1k} & a_{k+1k+1} & \dots & a_{k+1n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & \dots & a_{nk} & a_{nk+1} & \dots & a_{nn} \end{vmatrix}.$$

Shunday qilib, determinantning o'zaro to'ldiruvchi minorlar jufti haqida gapirish mumkin. Xususiy holda, a_{ij} element va determinantning i-satri va j-ustunini o'chirishdan hosil bo'lgan $(n-1)$-tartibli minor o'zaro to'ldiruvchi minorlar juftini hosil qiladi.

9-ta'rif. a_{ij} minorning (elementning) algebraik to'ldiruvchisi deb $A_{ij} = (-1)^{i+j} M_{ij}$ songa aytiladi.

Laplas teoremasi. Determinantning qiymati uning ixtiyoriy satr (ustun) elementlari bilan, shu elementlarga mos algebraik to'ldiruvchilar ko'paytmalari yig'indisiga teng, ya'ni:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \dots & \dots & \dots & \dots \\ a_{i1} & a_{i2} & \dots & a_{in} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix} = a_{i1}A_{i1} + a_{i2}A_{i2} + \dots + a_{in}A_{in} = \sum_{j=1}^{n} (-1)^{i+j} a_{ij} M_{ij}$$

Bu formulaga Δ determinantni i satr elementlari bo'yicha yoyish formulasi deyiladi.

Determinantning biror satr (ustun) elementlari bilan uning boshqa satri (ustuni) elementlari algebraik to'ldiruvchilari ko'paytmalarining yig'indisi nolga teng.

1-xossa. Transponirlash natijasida determinantning qiymati o'zgarmaydi.

2-xossa. Determinantda ikkita satr (ustun) o'rinlari almashtirilsa, determinant ishorasi o'zgaradi.

3-xossa. Agar determinant ikkita bir xil satr (ustun)ga ega bo'lsa, u holda uning qiymati nolga teng.

4-xossa. Determinantning biror satri (ustuni) elementlarini $k \neq 0$ songa ko'paytirish determinantni shu songa ko'paytirishga teng kuchlidir yoki biror satr (ustun) elementlarining umumiy ko'paytuvchisini determinant belgisidan chiqarish mumkin.

5-xossa. Agar determinant ikkita satr (ustun)ning mos elementlari proporsional bo'lsa, u holda uning qiymati nolga teng.

6-xossa. Agar determinant biror satr (ustun)ning barcha elementlari nolga teng bo'lsa, u holda uning qiymati nolga teng.

7-xossa. Agar determinant biror satr (ustun)ning har bir elementi ikkita qo'shiluvchidan iborat bo'lsa, u holda berilgan determinant ikkita determinant yig'indisiga teng bo'ladi, ulardan birining tegishli satri (ustuni) birinchi qo'shiluvchilaridan, ikkinchisining tegishli satri (ustuni) esa ikkinchi qo'shiluvchilaridan qolgan elementlari berilgan determinant elementlaridan iborat.

8-xossa. Agar determinantning biror satri (ustuni) elementlariga boshqa satr (ustun)ning mos elementlarini biror songa ko'paytirib qo'shilsa, determinantning qiymati o'zgarmaydi.

9-xossa. Agar determinant satr (ustun) laridan biri uning qolgan satr (ustun) larining chiziqli kombinatsiyasidan iborat bo'lsa, determinant nolga teng.

Misollar

1. Ikkinchi tartibli determinantni hisoblang: $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$.

Yechish. $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1 \cdot 4 - 2 \cdot 3 = -2.$

2. Uchinchi tartibli determinantni ixtiyoriy satr yoki ustun elementlari bo'yicha yoyib hisoblang: $\begin{vmatrix} 3 & 2 & 1 \\ 2 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix}$.

Yechish. Determinantni birinchi satr elementlari bo'yicha yoyib hisoblaymiz.

$$3\cdot\begin{vmatrix} 5 & 3 \\ 4 & 2 \end{vmatrix}-2\cdot\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix}+1\cdot\begin{vmatrix} 2 & 5 \\ 3 & 4 \end{vmatrix}=$$

$$=3\cdot(5\cdot2-3\cdot4)-2\cdot(2\cdot2-3\cdot3)+1\cdot(2\cdot4-5\cdot3)=$$

$$=3\cdot(-2)-2\cdot(-5)+1\cdot(-7)=-3.$$

3. Uchburchak qoidasidan foydalanib determinantni hisoblang: $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix}$.

Yechish. Oltita qo'shiluvchidan faqat bittasi noldan farqli:

$+1\cdot2\cdot3=6.$

4. To'rtinchi tartibli determinantni hisoblang.

$$\Delta=\begin{vmatrix} a & 0 & 3 & 5 \\ 0 & 0 & b & 2 \\ 1 & c & 2 & 3 \\ 0 & 0 & 0 & d \end{vmatrix}.$$

Yechish. To'rtinchi satr elementlari bo'yicha yoyib hisoblaymiz

$$\Delta=(+d)\cdot\begin{vmatrix} a & 0 & 3 \\ 0 & 0 & b \\ 1 & c & 2 \end{vmatrix}=\begin{bmatrix} 2-satr\ bo'yicha \\ yoyamiz \end{bmatrix}=$$

$$=d\cdot(-b)\cdot\begin{vmatrix} a & 0 \\ 1 & c \end{vmatrix}=-d\cdot b\cdot a\cdot c.$$

1.3. Matritsa rangi. Teskari matritsa

Ixtiyoriy o'lchamli matritsaning bir necha satr yoki ustunlarini o'chirishdan hosil bo'lgan kvadrat matritsa determinantiga matritsa osti minori deyiladi. Bu kvadrat matritsa tartibi matritsa osti minorining tartibi deyiladi. Agar berilgan matritsa kvadrat shaklda bo'lsa, uning eng katta tartibli minori o'ziga teng.

Masalan, $A=\begin{pmatrix} 4 & 5 & 7 \\ 2 & 1 & 4 \\ 3 & 7 & 0 \end{pmatrix}$ matritsaning 1-satr va 1-ustunini o'chirishdan

2-tartibli minor $M_{11}=\begin{vmatrix}1 & 4\\ 7 & 0\end{vmatrix}$, 2-satr va 3-ustunini o‘chirishdan 2-tartibli minor $M_{23}=\begin{vmatrix}4 & 5\\ 3 & 7\end{vmatrix}$ va hokazo minorlarni hosil qilish mumkin.

10-ta'rif. A matritsaning rangi deb, noldan farqli matritsa osti minorlarining eng katta tartibiga aytiladi va $rang(A)=r(A)$ ko‘rinishida ifodalanadi.

Matritsa rangining xossalari:

1) agar A matritsa $m\times n$ o‘lchovli bo‘lsa, u holda $rangA\le \min(m;n)$;

2) A matritsaning barcha elementlari nolga teng bo‘lsa, u holda $rangA=0$;

3) agar A matritsa $n-$tartibli kvadrat matritsa va $|A|\neq 0$ bo‘lsa, u holda $rangA=n$.

Misol. $A=\begin{pmatrix}1 & -2\\ -2 & 4\\ 3 & -7\end{pmatrix}$ matritsa rangini aniqlang.

Yechish. Berilgan matritsa (3×2) o‘lchamli bo‘lgani uchun satrlar va ustunlar sonini taqqoslaymiz va kichigini, ya`ni 2 ni tanlaymiz. Matritsadan ikkinchi tartibli minorlar ajratamiz va ularning qiymatini hisoblaymiz. Bu jarayonni noldan farqli ikkinchi tartibli minor topilguncha davom ettiramiz:

$$M_1=\begin{vmatrix}1 & -2\\ -2 & 4\end{vmatrix}=0,\quad M_2=\begin{vmatrix}1 & -2\\ 3 & -7\end{vmatrix}=-1\neq 0.$$

Berilgan matritsadan noldan farqli eng yuqori ikkinchi tartibli minor ajraldi. Demak, ta'rifga binoan, A matritsa rangi 2 ga teng, ya'ni $rang(A)=2$.

Matritsa rangini aniqlashning yuqoridagi usuli «minorlar ajratib hisoblash» usuli deb ataladi.

Matritsa rangi uning ustida quyidagi almashtirishlar bajarganda o‘zgarmaydi:

1. Matritsa biror satri (ustuni) har bir elementini biror noldan farqli songa ko‘paytirganda;
2. Matritsa satrlari (ustunlari) o‘rinlari almashtirilganda;
3. Matritsa biror satri (ustuni) elementlariga uning boshqa parallel satri (ustuni) mos elementlarini biror noldan farqli songa ko‘paytirib, so‘ngra qo‘shganda;

4. Matritsa transponirlanganda.

Teorema. Elementar almashtirishlar matritsa rangini o‘zgartirmaydi.

Matritsa rangini aniqlashni aniq misollarda ko‘rib chiqaylik.

Misol. $A=\begin{pmatrix} 3 & 1 & -2 & -1 \\ 2 & -1 & 1 & -2 \\ -5 & -2 & 3 & 1 \end{pmatrix}$

matritsada birinchi satrni 2 ga va ikkinchi satrni -3 ga ko‘paytirib, birinchini ikkinchiga qo‘shsak, so‘ngra yana birinchi satrni 5 ga, uchunchi satrni 3 ga ko‘paytirib, natijalarni qo‘shsak,

$$\begin{pmatrix} 3 & 1 & -2 & -1 \\ 0 & 5 & -7 & 4 \\ 0 & -1 & -1 & -2 \end{pmatrix}$$

matritsa hosil bo‘ladi.

Bu matritsada ikkinchi satrni 1 ga, uchunchi satrni 5 ga ko‘paytirib, ikkinchi satrni uchinchi satrga qo‘shsak,

$$\begin{pmatrix} 3 & 1 & -2 & -1 \\ 0 & 5 & -7 & 4 \\ 0 & 0 & -12 & -6 \end{pmatrix}$$

matritsa hosil bo‘ladi. Yana

$$B=\begin{pmatrix} 2 & -3 & 3 & 0 \\ -4 & 2 & -4 & 5 \\ -2 & -1 & -1 & 5 \end{pmatrix}$$

matritsani olib, yuqoridagi singari almashtirishlarni bajarsak,

$$B=\begin{pmatrix} 2 & -3 & 3 & 0 \\ 0 & -4 & 2 & 5 \\ 0 & -4 & 2 & 5 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & -3 & 3 & 0 \\ 0 & -4 & 2 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

hosil bo‘ladi.

A va B matritsaga qo‘llanilgan almashtirishlarning mohiyati quyidagidan iborat: m satrli matritsa berilgan holda birinchi va ikkinchi satrlarni, undan keyin birinchi va uchinchi satrlarni, ..., nihoyat, birinchi va $m-$ satrlarni shunday sonlarga ko‘paytiramizki, tegishli songa ko‘paytirilgan birinchi satrni navbat bilan boshqa hamma satrlarga qo‘shganimizda ikkinchi satrdan boshlab birinchi ustun elementlari nollarga aylanadi. So‘ngra ikkinchi satr yordamida keyingi hamma satrlar bilan yana shunday almashtirishlarni bajaramizki, uchinchi satrdan boshlab, ikkinchi ustun elementlari nollarga aylanadi. Undan keyin to'rtinchi satrdan boshlab uchinchi ustun elementlari nollarga aylanadi va hokazo. Shu tariqa bu jarayon oxirigacha davom ettiriladi.

Agar matritsaning qandaydir satrlari boshqa satrlari orqali chiziqli ifodalangan bo'lsa, u holda shu almashtirishlar natijasida, bunday satrlarning hamma elementlari nollarga (ya'ni bunday satrlar nol satrlarga) aylanadi.

Birorta elementi noldan farqli satrni nolmas satr deb atasak, yuqoridagi almashtirishlardan keyin hosil bo'lgan matritsaning rangi nolmas satrlar soniga teng bo'ladi, chunki bunday satrlar chiziqli erkli satrlarni bildiradi.

Yuqorida qo'llaniladigan almashtirishlar matritsani elementar almashtirishlardan iborat bo'lgani uchun, ular matritsaning rangini o'zgartirmaydi. Shu sababli, birinchi misolda $r(A)=3$ bo'ladi, chunki A da uchta nolmas satr bor. Ikkinchi misolda esa $r(B)=2$ bo'ladi.

Yuqoridagi muhokamalardan quyidagi xulosaga kelamiz: pog'onasimon matritsaning rangi uning nolmas satrlari soniga teng.

Elementar almashtirishlar yordamida matritsani pog'onasimon matritsaga keltirish mumkin

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1r} & \dots & a_{1k} \\ 0 & a_{22} & \dots & a_{2r} & \dots & a_{2k} \\ . & . & . & . & . & . \\ 0 & 0 & \dots & a_{rr} & \dots & a_{rk} \end{pmatrix},$$

bu yerda $a_{ii}\neq 0,\ i=1,\dots,r,\ r\leq k.$

Pog'onasimon matritsaning rangi r ga teng.

Masalan, yuqoridagi misollarda $r(A)=3, r(B)=1, r(C)=1$ bo'ladi.

11-ta'rif. Agar A kvadrat matritsa uchun $AA^{-1}=A^{-1}A=E$ tenglik bajarilsa, A^{-1} matritsa A matritsaga teskari matritsa deyiladi.

Bu yerda E birlik matritsa bo'lib, uning o'lchami A matritsaning o'lchami bilan bir xil.

Agar A xosmas matritsa bo'lsa $(\det A\neq 0)$, u holda uning uchun yagona A^{-1} matritsa mavjud bo'ladi va u quyidagi tenglik bilan aniqlanadi:

$$A^{-1}=\frac{1}{\det A}\tilde{A}.$$

Bu yerda $\tilde{A}$ matritsa A kvadrat matritsaning har bir elementini unga mos algebraik to'ldiruvchisi bilan almashtirish natijasida olingan matritsa ustida transponirlash amalini bajarishdan hosil bo'lgan matritsa.

$$\tilde{A} = \begin{pmatrix} A_{11} & A_{21} & \dots & A_{n1} \\ A_{12} & A_{22} & \dots & A_{n2} \\ \dots & \dots & \dots & \dots \\ A_{1n} & A_{2n} & \dots & A_{nn} \end{pmatrix}.$$

Xos matritsa $(\det A = 0)$ uchun teskari matritsa mavjud emas. Boshqacha qilib aytganda, biror $n-$tartibli matritsa uchun $r<n$ bo'lsa, uning uchun teskari matritsa mavjud emas.

Endi A kvadrat matritsaga A^{-1} teskari matritsani elementar almashtirishlar yordamida topamiz. Bu usul quyidagicha amalga oshiriladi:

1. A matritsaning o'ng tarafiga tartibi uning tartibiga teng bo'lgan E birlik matritsa yoziladi va kengaytirilgan $A|E$ matritsa tuziladi.

2. Parallel ravishda $A|E$ kengaytirilgan matritsaning chap va o'ng qismlari satr (ustun)lari ustida elementar almashtirishlar bajarilib, chap qismi birlik matritsa ko'rinishiga keltiriladi. U holda uning o'ng qismida A^{-1} teskari matritsa hosil bo'ladi.

$$A|E \sim E|A^{-1}.$$

Teskari matritsa quyidagi xossalariga ega:

1) $(A^{-1})^{-1} = A$;

2) $(A^T)^{-1} = (A^{-1})^T$;

3) $(AB)^{-1} = B^{-1}A^{-1}$.

X noma'lum matritsaning oddiy ko'rinishdagi matritsali tenglamasi quyidagi ko'rinishlarda bo'ladi

$$A \cdot X = B, \quad (1)$$

$$X \cdot A = B, \quad (2)$$

$$A \cdot X \cdot C = B \quad (3)$$

Ushbu tenglamada A, B, C, X – shunday o'lchamli matritsalarki, barcha foydalaniladigan amallarda ko'paytirish amali bajariladi va tenglikning ikkala tomonida bir xil o'lchamli matritsalar joylashgan.

Agar 1-va 2- tenglamalarda A xosmas matritsa bo'lsa, u holda yechim quyidagicha ifodalanadi.

$$X = A^{-1} \cdot B$$

$$X = B \cdot A^{-1}.$$

Agar 3- tenglamada A xosmas matritsa bo'lsa, u holda yechim quyidagicha ifodalanadi.

$$X = A^{-1} \cdot B \cdot C^{-1}.$$

Misollar

1. Elementar almashtirishlar metodida quyidagi matritsaning rangini toping:

$$\begin{pmatrix} 2 & -1 & 5 & 6 \\ 1 & 1 & 3 & 5 \\ 1 & -5 & 1 & -3 \end{pmatrix}.$$

Yechish. Elementar almashtirishlar yordamida matritsani pog'onasimon ko'rinishga keltiramiz

$$\begin{pmatrix} 2 & -1 & 5 & 6 \\ 1 & 1 & 3 & 5 \\ 1 & -5 & 1 & -3 \end{pmatrix} \begin{matrix} \\ 2\cdot II - I \\ 2\cdot III - I \end{matrix} \sim \begin{pmatrix} 2 & -1 & 5 & 6 \\ 0 & 3 & 1 & 4 \\ 0 & -9 & -3 & -12 \end{pmatrix} \begin{matrix} \\ \\ III + 3\cdot II \end{matrix} \sim$$

$$\sim \begin{pmatrix} 2 & -1 & 5 & 6 \\ 0 & 3 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Hosil bo'lgan pog'onasimon matritsa ikkita noldan farqli satrga ega, demak uning rangi ikkiga teng. Shuning uchun berilgan matritsaning rangi ikkiga teng.

2. Berilgan matritsaga teskari matritsani toping: $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{pmatrix}.$

Yechish. 1) A matritsaning determinantini topamiz:

$$\det A = 1\cdot\begin{vmatrix} 5 & 6 \\ 8 & 0 \end{vmatrix} - 2\cdot\begin{vmatrix} 4 & 6 \\ 7 & 0 \end{vmatrix} + 3\cdot\begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix} =$$

$$= -48 - 2\cdot(-42) + 3\cdot(32-35) = -48 + 84 - 9 = 27 \neq 0.$$

$\det A \neq 0$ demak A^{-1} mavjud.

2) A matritsa barcha elementlarining algebraik to'ldiruvchilarini topamiz:

$$A_{11} = (-1)^{1+1}\cdot\begin{vmatrix} 5 & 6 \\ 8 & 0 \end{vmatrix} = 5\cdot 0 - 6\cdot 8 = -48;$$

$$A_{12} = (-1)^{1+2}\cdot\begin{vmatrix} 4 & 6 \\ 7 & 0 \end{vmatrix} = -(4\cdot 0 - 6\cdot 7) = 42;$$

$$A_{13} = (-1)^{1+3}\cdot\begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix} = 4\cdot 8 - 5\cdot 7 = -3;$$

$$A_{21} = -\begin{vmatrix} 2 & 3 \\ 8 & 0 \end{vmatrix} = 24; \quad A_{22} = \begin{vmatrix} 1 & 3 \\ 7 & 0 \end{vmatrix} = -21;$$

$$A_{23}=-\begin{vmatrix}1 & 2\\ 7 & 8\end{vmatrix}=6;\quad A_{31}=\begin{vmatrix}2 & 3\\ 5 & 6\end{vmatrix}=-3;$$

$$A_{32}=-\begin{vmatrix}1 & 3\\ 4 & 6\end{vmatrix}=6;\quad A_{33}=\begin{vmatrix}1 & 2\\ 4 & 5\end{vmatrix}=-3;$$

3) $\tilde{A}=\left(A_{ij}\right)^T=\begin{pmatrix}-48 & 24 & -3\\ 42 & -21 & 6\\ -3 & 6 & -3\end{pmatrix}$ matritsani yozamiz.

4) A^{-1} matritsani topamiz:

$$A^{-1}=\frac{1}{\det A}\cdot\tilde{A}=\frac{1}{27}\cdot\begin{pmatrix}-48 & 24 & -3\\ 42 & -21 & 6\\ -3 & 6 & -3\end{pmatrix}=\begin{pmatrix}-\frac{16}{9} & \frac{8}{9} & -\frac{1}{9}\\ \frac{14}{9} & -\frac{7}{9} & \frac{2}{9}\\ -\frac{1}{9} & \frac{2}{9} & -\frac{1}{9}\end{pmatrix}.$$

Tekshiramiz: $A^{-1}\cdot A=\begin{pmatrix}-\frac{16}{9} & \frac{8}{9} & -\frac{1}{9}\\ \frac{14}{9} & -\frac{7}{9} & \frac{2}{9}\\ -\frac{1}{9} & \frac{2}{9} & -\frac{1}{9}\end{pmatrix}\cdot\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 0\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix};$

$$A\cdot A^{-1}=\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 0\end{pmatrix}\cdot\begin{pmatrix}-\frac{16}{9} & \frac{8}{9} & -\frac{1}{9}\\ \frac{14}{9} & -\frac{7}{9} & \frac{2}{9}\\ -\frac{1}{9} & \frac{2}{9} & -\frac{1}{9}\end{pmatrix}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}.$$

3. Elementar almashtirishlar yordamida berilgan matritsaga teskari matritsani toping. $A=\begin{pmatrix}1 & 1 & 1\\ 1 & 2 & -1\\ 2 & 2 & 4\end{pmatrix}.$

Yechish. (3×6) o‘lchamli $\Gamma=\left(A/E\right)$ kengaytirilgan matritsani yozamiz. Avval matritsaning satrlari ustida elementar almashtirishlar bajarib uni pog‘onasimon ko‘rinishga keltiramiz $\Gamma_1=\left(A_1/B\right)$, keyin $\Gamma_2=\left(E/A^{-1}\right)$ ko‘rinishga keltiramiz.

$$\Gamma=\left(\begin{array}{ccc|ccc}1&1&1&1&0&0\\1&2&-1&0&1&0\\2&2&4&0&0&1\end{array}\right)\begin{array}{l}\\II-I\\III-2\cdot I\end{array}\sim$$

$$\sim\Gamma_1=\left(\begin{array}{ccc|ccc}1&1&1&1&0&0\\0&1&-2&-1&1&0\\0&0&2&-2&0&1\end{array}\right)II+III\quad\sim$$

$$\sim\left(\begin{array}{ccc|ccc}1&1&1&1&0&0\\0&1&0&-3&1&1\\0&0&2&-2&0&1\end{array}\right)\begin{array}{l}\\ \\III\div 2\end{array}\sim$$

$$\sim\left(\begin{array}{ccc|ccc}1&1&1&1&0&0\\0&1&0&-3&1&1\\0&0&1&-1&0&\frac{1}{2}\end{array}\right)\begin{array}{l}I-II-III\\ \\ \\ \end{array}\sim$$

$$\sim\left(\begin{array}{ccc|ccc}1&0&0&5&-1&-\frac{3}{2}\\0&1&0&-3&1&1\\0&0&1&-1&0&\frac{1}{2}\end{array}\right)=\Gamma_2$$

Demak, $A^{-1}=\begin{pmatrix}5&-1&-\frac{3}{2}\\-3&1&1\\-1&0&\frac{1}{2}\end{pmatrix}$.

Tekshiramiz: $AA^{-1}=\begin{pmatrix}1&1&1\\1&2&-1\\2&2&4\end{pmatrix}\begin{pmatrix}5&-1&-\frac{3}{2}\\-3&1&1\\-1&0&\frac{1}{2}\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$.

$$A^{-1}A=\begin{pmatrix}5&-1&-\frac{3}{2}\\-3&1&1\\-1&0&\frac{1}{2}\end{pmatrix}\begin{pmatrix}1&1&1\\1&2&-1\\2&2&4\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}.$$

4.Matritsali tenglamani yeching

$$\begin{pmatrix} -1 & 2 \\ 2 & -3 \end{pmatrix} \cdot X = \begin{pmatrix} -2 & 3 \\ 1 & -4 \end{pmatrix}.$$

Yechish. Berilgan matritsali tenglamani $A \cdot X = B$ ko'rinishda yozamiz. Uning yechimi $X = A^{-1} \cdot B$ bo'ladi. (Agar A^{-1} matritsa mavjud bo'lsa)

1) A matritsaning determinantini topamiz: $\det A = \begin{vmatrix} -1 & 2 \\ 2 & -3 \end{vmatrix} = -1 \neq 0.$

Demak, A^{-1} teskari matritsa mavjud, tenglama yechimga (yagona) ega.

2) Teskari matritsani topamiz

$$A^{-1} = \frac{1}{\det A} \cdot \tilde{A} = (-1) \cdot \begin{pmatrix} -3 & -2 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}.$$

3) X matritsani topamiz

$$X = A^{-1}B = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} -2 & 3 \\ 1 & -4 \end{pmatrix} = \begin{pmatrix} -4 & 1 \\ -3 & 2 \end{pmatrix}.$$

1.4. Talabaning mustaqil ishi

1-topshiriq

1-misolda berilgan matritsalarning chiziqli kombinatsiyasini toping.

2-misolda matritsalar ko'paytmasi AB va BA ni toping (agar ular mavjud bo'lsa).

3-misolda $f(A)$ matritsali ko'phadning qiymatini toping.

1-variant

1. $2A^T + 3B$,

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 3 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & 3 & 0 \\ 2 & 1 & 1 \end{pmatrix}.$$

2. $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 4 & 5 \\ 6 & 0 & -2 \\ 7 & 1 & 8 \end{pmatrix}.$

3. $f(x) = -2x^2 + 5x + 9, \quad A = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}.$

2-variant

1. $A^T - 3E, \quad A = \begin{pmatrix} 2 & 5 & -1 \\ -1 & -3 & 0 \\ 2 & 3 & -2 \end{pmatrix}.$

2. $A=\begin{pmatrix}3 & -2\\5 & -4\end{pmatrix}$, $B=\begin{pmatrix}3 & 4\\2 & 5\end{pmatrix}$.

3. $f(x)=3x^3+x^2+2$, $A=\begin{pmatrix}1 & 5\\0 & -3\end{pmatrix}$.

3-variant

1. $4A-5B^T$, $A=\begin{pmatrix}2 & -1 & 0\\3 & 4 & -2\\-3 & 1 & 5\end{pmatrix}$, $B=\begin{pmatrix}3 & -2 & 0\\1 & 1 & 2\\2 & 3 & -4\end{pmatrix}$.

2. $A=\begin{pmatrix}4 & 0 & -2 & 3 & 1\end{pmatrix}$, $B=\begin{pmatrix}3\\1\\-1\\5\\2\end{pmatrix}$.

3. $f(x)=2x^3-3x^2+5$, $A=\begin{pmatrix}1 & 2\\-2 & 3\end{pmatrix}$.

4-variant

1. $3A^T+4B$, $A=\begin{pmatrix}7 & 0 & -5\\-2 & 2 & 3\\3 & 1 & 2\\-4 & -1 & 0\end{pmatrix}$, $B=\begin{pmatrix}2 & -1 & -3 & 1\\7 & -1 & 0 & 4\\8 & -2 & 1 & 5\end{pmatrix}$.

2. $A=\begin{pmatrix}1 & 2\\3 & 6\end{pmatrix}$, $B=\begin{pmatrix}2 & 6\\-1 & -3\end{pmatrix}$.

3. $f(x)=3x^2-5x+2$, $A=\begin{pmatrix}1 & 2 & 0\\0 & 2 & -1\\-2 & 1 & 4\end{pmatrix}$.

5-variant

1. $3A-2B^T$, $A=\begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}$, $B=\begin{pmatrix}0 & 1\\1 & -2\end{pmatrix}$.

2. $A=\begin{pmatrix}2 & 1\\5 & 3\end{pmatrix}$, $B=\begin{pmatrix}3 & -1\\-5 & 2\end{pmatrix}$.

3. $f(x)=x^3-6x^2+9x+4$, $A=\begin{pmatrix}1 & 0 & 0\\0 & 2 & -1\\0 & 1 & 4\end{pmatrix}$.

6-variant

1. $2B-5A^T,\ A=\begin{pmatrix} 0 & -6 \\ 2 & 4 \\ 4 & 0 \end{pmatrix},\ B=\begin{pmatrix} 0 & 5 & 10 \\ -15 & 10 & 0 \end{pmatrix}.$

2. $A=\begin{pmatrix} 2 & 4 \\ -5 & 7 \end{pmatrix},\quad B=\begin{pmatrix} 2 & -2 \\ -2 & 3 \end{pmatrix}.$

3. $f(x)=2x^2-3x+1,\ A=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$

7-variant

1. $A^T-2E,\quad A=\begin{pmatrix} 2 & 3 \\ 3 & -2 \end{pmatrix}.$

2. $A=\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix},\quad B=\begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}.$

3. $f(x)=3x^2+2x+5,\ A=\begin{pmatrix} 2 & -3 \\ 0 & 4 \end{pmatrix}.$

8-variant

1. $4A^T-7B,\ A=\begin{pmatrix} 1 & 2 & 5 \\ -2 & 0 & -1 \\ 5 & -3 & 0 \\ 3 & 1 & 4 \end{pmatrix},\ B=\begin{pmatrix} 0 & 2 & 7 & -5 \\ -8 & 1 & 3 & 0 \\ 4 & 2 & -2 & 5 \end{pmatrix}.$

2. $A=\begin{pmatrix} 1 & -2 & 3 & 0 \end{pmatrix},\quad B=\begin{pmatrix} 5 \\ -3 \\ -4 \\ 1 \end{pmatrix}.$

3. $f(x)=2x^3-x^2+3,\ A=\begin{pmatrix} -1 & 2 \\ -3 & 1 \end{pmatrix}.$

9-variant

1. $5A^T-3B,\quad A=\begin{pmatrix} 1 & 3 & -1 \\ -2 & 5 & 2 \\ 0 & 1 & 4 \end{pmatrix},\ B=\begin{pmatrix} 5 & 1 & -2 \\ -3 & 2 & 7 \\ 4 & 0 & -1 \end{pmatrix}.$

2. $A=\begin{pmatrix} 2 & 0 & 3 \\ -1 & 2 & 1 \end{pmatrix},\quad B=\begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix}.$

3. $f(x)=x^2-3x+2,\ A=\begin{pmatrix} 1 & -3 & 0 \\ 0 & 2 & 1 \\ 3 & -3 & 2 \end{pmatrix}.$

10-variant

1. $3A+2B^T$, $A=\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}$, $B=\begin{pmatrix}-1 & 5\\ 3 & -2\end{pmatrix}$.

2. $A=\begin{pmatrix}3 & 5 & -1\\ 2 & -2 & 0\end{pmatrix}$, $B=\begin{pmatrix}2 & 4\\ -3 & 0\\ 5 & 1\end{pmatrix}$.

3. $f(x)=4x^3-2x^2+3x-2$, $A=\begin{pmatrix}-2 & 3\\ 1 & 0\end{pmatrix}$.

11-variant

1. A^T-3B, $A=\begin{pmatrix}3 & 0\\ -2 & 1\\ 2 & -4\end{pmatrix}$, $B=\begin{pmatrix}-2 & 5 & 1\\ 1 & 0 & 2\end{pmatrix}$.

2. $A=\begin{pmatrix}-2 & 3 & 1\\ 5 & 4 & 4\\ 2 & -1 & -5\end{pmatrix}$, $B=\begin{pmatrix}1 & -2 & -3\\ 0 & -3 & 1\\ 4 & -4 & 5\end{pmatrix}$.

3. $f(x)=3x^2+5x-2$, $A=\begin{pmatrix}2 & 3 & -3\\ 0 & 1 & 4\\ 5 & -2 & 1\end{pmatrix}$.

12-variant

1. $7A^T-4B$, $A=\begin{pmatrix}1 & -1 & 1\\ 0 & 2 & -1\\ 1 & -1 & 1\end{pmatrix}$, $B=\begin{pmatrix}3 & 2 & 0\\ 3 & 2 & 0\\ 3 & 2 & 1\end{pmatrix}$.

2. $A=\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$, $B=\begin{pmatrix}2 & 0\\ -3 & 1\end{pmatrix}$.

3. $f(x)=x^3-x^2+5$, $A=\begin{pmatrix}1 & 0 & 1\\ 3 & -1 & 0\\ 0 & 0 & 2\end{pmatrix}$.

13-variant

1. $3A-2C^T$, $A=\begin{pmatrix}4 & 1\\ 1 & 4\end{pmatrix}$ $C=\begin{pmatrix}1 & 2\\ 3 & 5\end{pmatrix}$.

2. $A=\begin{pmatrix}1 & 3\\ 2 & 5\end{pmatrix}$, $B=\begin{pmatrix}-5 & 3\\ 2 & -1\end{pmatrix}$.

3. $f(x)=2x^3-x^2+3x-2,\quad A=\begin{pmatrix}2 & -3 & 4\\ 0 & 5 & -1\\ -2 & -1 & 3\end{pmatrix}.$

14-variant

1. $5B^T-2C,\quad B=\begin{pmatrix}3 & 1\\ 0 & -7\end{pmatrix}\quad C=\begin{pmatrix}1 & 3\\ 2 & 5\end{pmatrix}.$

2. $A=(1\quad -3),\quad B=\begin{pmatrix}-3 & 2 & 0\\ -2 & 5 & -1\end{pmatrix}.$

3. $f(x)=2x^2-5x+3\quad A=\begin{pmatrix}2 & 4\\ 1 & 0\end{pmatrix}.$

15-variant

1. $3A-2C^T,\quad A=\begin{pmatrix}1 & 0 & 2\\ 4 & 3 & 7\\ -4 & 0 & 6\end{pmatrix},\quad C=\begin{pmatrix}4 & 2 & 2\\ -5 & 1 & 5\\ 7 & 3 & 8\end{pmatrix}.$

2. $A=\begin{pmatrix}-5 & 0 & 3\\ 4 & 1 & -1\\ 2 & -3 & 2\\ 1 & 5 & 3\end{pmatrix},\quad B=\begin{pmatrix}3 & 0\\ -2 & 1\\ 4 & 3\end{pmatrix}.$

3. $f(x)=3x^2-2x+5,\quad A=\begin{pmatrix}1 & -2 & 3\\ 2 & -4 & 1\\ 3 & -5 & 2\end{pmatrix}.$

16-variant

1. $3B^T-2C,\quad B=\begin{pmatrix}3 & 1\\ 1 & -2\end{pmatrix}\quad C=\begin{pmatrix}1 & 3\\ 2 & 5\end{pmatrix}.$

2. $A=\begin{pmatrix}1 & 0 & 0\\ -2 & 1 & 3\end{pmatrix},\quad B=\begin{pmatrix}4 & 5\\ 0 & 1\\ -3 & 0\end{pmatrix}.$

3. $f(x)=x^3-7x^2+13x-5,\quad A=\begin{pmatrix}5 & 2 & -3\\ 1 & 3 & -1\\ 2 & 2 & 1\end{pmatrix}.$

17-variant

1. $2A^T+3B,\quad A=\begin{pmatrix}3 & -3\\ -1 & 0\\ 2 & -4\end{pmatrix},\quad B=\begin{pmatrix}0 & 4 & 1\\ 1 & 0 & -2\end{pmatrix}.$

2. $A=\begin{pmatrix}-1 & 2\\ 8 & 9\end{pmatrix}$, $B=\begin{pmatrix}4 & 0\\ -1 & 3\end{pmatrix}$.

3. $f(x)=x^2-2x$, $A=\begin{pmatrix}1 & 3 & 1\\ 2 & 0 & 4\\ 1 & 2 & 3\end{pmatrix}$.

18-variant

1. A^T-3B, $A=\begin{pmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{pmatrix}$, $B=\begin{pmatrix}-2 & 5 & 1\\ 1 & 0 & 2\end{pmatrix}$.

2. $A=\begin{pmatrix}1 & 1 & 2\\ 2 & 1 & 3\end{pmatrix}$, $B=\begin{pmatrix}4 & 5\\ 3 & 1\\ 3 & 0\end{pmatrix}$.

3. $f(x)=x^2+4x$, $A=\begin{pmatrix}1 & 3 & -1\\ 2 & 1 & 2\\ 0 & 1 & 0\end{pmatrix}$.

19-variant

1. A^T+B, $A=\begin{pmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{pmatrix}$, $B=\begin{pmatrix}-6 & 4 & 1\\ 1 & 4 & 2\end{pmatrix}$.

2. $A=\begin{pmatrix}4 & 0 & 1\\ 6 & 11 & -8\end{pmatrix}$ $B=\begin{pmatrix}1 & 1\\ 1 & 1\\ 0 & 1\end{pmatrix}$.

3. $f(x)=x^2-3x$, $A=\begin{pmatrix}2 & 3 & -1\\ 2 & 4 & -1\\ 1 & 1 & 0\end{pmatrix}$.

20-variant

1. $4A-B^T$, $A=\begin{pmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{pmatrix}$, $B=\begin{pmatrix}-2 & 5 & 1\\ 1 & 0 & 2\end{pmatrix}$.

2. $A=\begin{pmatrix}2 & 5\\ 4 & 3\end{pmatrix}$, $B=\begin{pmatrix}2 & 5\\ -2 & 1\end{pmatrix}$.

3. $f(x)=x^2+4x-1$, $A=\begin{pmatrix}1 & 2 & 3\\ -1 & 5 & 3\\ 7 & 5 & 4\end{pmatrix}$.

21-variant

1. A^T+2C, $A=\begin{pmatrix}1&2\\3&4\\5&6\end{pmatrix}$, $C=\begin{pmatrix}5&5&1\\0&-1&9\end{pmatrix}$.

2. $A=\begin{pmatrix}2&4\\-5&7\end{pmatrix}$, $B=\begin{pmatrix}2&-2\\-2&3\end{pmatrix}$.

3. $f(x)=x^2+3x-4$, $A=\begin{pmatrix}1&-1&3\\3&0&3\\7&8&4\end{pmatrix}$.

22-variant

1. A^T-B, $A=\begin{pmatrix}2&-1&0\\3&4&-2\\-3&1&5\end{pmatrix}$, $B=\begin{pmatrix}3&1&2\\-2&1&3\\0&2&-4\end{pmatrix}$.

2. $A=\begin{pmatrix}1&0&0\\-2&1&3\end{pmatrix}$, $B=\begin{pmatrix}4&5\\0&1\\-3&0\end{pmatrix}$.

3. $f(x)=x^2-4x+2$, $A=\begin{pmatrix}1&0&0\\-2&1&0\\0&-2&1\end{pmatrix}$.

23-variant

1. $A+B^T$, $A=\begin{pmatrix}7&-1&0\\-1&7&-2\\0&-2&7\end{pmatrix}$, $B=\begin{pmatrix}3&1&2\\1&-5&1\\2&1&-4\end{pmatrix}$.

2. $A=\begin{pmatrix}2&5\\4&3\end{pmatrix}$, $B=\begin{pmatrix}2&5\\-2&1\end{pmatrix}$.

3. $f(x)=x^2+2x-3$, $A=\begin{pmatrix}2&-1&0\\0&2&-1\\0&0&2\end{pmatrix}$.

24-variant

1. A^T+2B, $A=\begin{pmatrix}2&-1&0\\4&4&4\\-3&0&5\end{pmatrix}$, $B=\begin{pmatrix}3&1&2\\3&1&2\\4&4&4\end{pmatrix}$.

2. $A=\begin{pmatrix} -2 & 3 \\ 5 & 2 \end{pmatrix}$, $B=\begin{pmatrix} 2 & -5 \\ 0 & -1 \end{pmatrix}$.

3. $f(x)=x^2-2x$, $A=\begin{pmatrix} 1 & -1 & 2 \\ 3 & 2 & 2 \\ 5 & 4 & 2 \end{pmatrix}$.

25-variant

1. $A^T-5\cdot C$, $A=\begin{pmatrix} 1 & 2 \\ 0 & 4 \\ -9 & 3 \end{pmatrix}$, $C=\begin{pmatrix} 8 & 3 & 1 \\ 0 & -1 & 9 \end{pmatrix}$.

2. $A=\begin{pmatrix} -2 & 1 & -1 \\ 1 & -3 & -3 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 0 \\ 6 & 4 \\ 0 & -1 \end{pmatrix}$.

3. $f(x)=x^3-3x+1$, $A=\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$.

2-topshiriq

1- misolda berilgan matritsa rangini toping.

2-misolda berilgan matritsaga teskari matritsani ikki usulda toping.

3-misolda matritsali tenglamani yeching. Natijani Mathcad dasturida tekshiring.

1-variant

1. $A=\begin{pmatrix} 1 & -3 & -2 & -2 \\ -3 & 10 & 2 & 1 \\ 7 & -24 & -2 & 1 \end{pmatrix}$.

2. $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$.

3. $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}X=\begin{pmatrix} 4 & -6 \\ 2 & 1 \end{pmatrix}$.

2-variant

1. $A=\begin{pmatrix} 1 & 2 & -1 & 2 \\ 9 & -12 & 15 & 0 \\ -2 & 6 & -6 & 2 \end{pmatrix}$.

2. $\begin{pmatrix} 2 & 1 & 1 \\ 5 & 1 & 3 \\ 2 & 1 & 2 \end{pmatrix}$.

3. $\begin{pmatrix} 2 & 3 \\ 5 & 8 \end{pmatrix} X = \begin{pmatrix} -3 & 2 \\ 1 & 4 \end{pmatrix}$.

3-variant

1. $A = \begin{pmatrix} 1 & 4 & -3 & 61 \\ 4 & 10 & 2 & -46 \\ 34 & -20 & 40 & 0 \end{pmatrix}$.

2. $\begin{pmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 7 & 9 & 1 \end{pmatrix}$.

3. $\begin{pmatrix} 3 & 4 & 1 \\ 2 & 3 & 1 \\ 5 & 2 & 2 \end{pmatrix} X = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 0 & 0 & 2 \end{pmatrix}$.

4-variant

1. $A = \begin{pmatrix} 2 & 1 & -2 & 3 \\ -3 & 0 & 1 & 1 \\ 5 & 1 & -3 & 2 \end{pmatrix}$.

2. $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$.

3. $\begin{pmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} X = \begin{pmatrix} 3 & 4 & 1 \\ 2 & 3 & 1 \\ 5 & 2 & 2 \end{pmatrix}$.

5-variant

1. $A = \begin{pmatrix} 3 & 2 & 1 & 3 \\ 6 & 4 & 3 & 5 \\ 9 & 6 & 5 & 7 \end{pmatrix}$.

2. $\begin{pmatrix} 1 & 3 & 4 \\ 2 & 0 & 3 \\ -2 & 1 & -3 \end{pmatrix}$.

3. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} X = \begin{pmatrix} 3 & 5 \\ 5 & 9 \end{pmatrix}$.

6-variant

1. $A=\begin{pmatrix}1&2&3&4\\2&3&4&5\\3&4&5&6\\4&5&6&7\end{pmatrix}.$

2. $\begin{pmatrix}1&2&3\\2&0&-1\\3&-1&4\end{pmatrix}.$

3. $\begin{pmatrix}1&2&-3\\3&2&-4\\2&-1&0\end{pmatrix}X=\begin{pmatrix}1&-3&0\\10&2&7\\10&7&8\end{pmatrix}.$

7-variant

1. $A=\begin{pmatrix}1&8&-1&2\\2&-1&8&5\\1&10&-6&8\end{pmatrix}.$

2. $\begin{pmatrix}2&7&3\\3&9&4\\1&5&3\end{pmatrix}.$

3. $\begin{pmatrix}2&1\\1&1\end{pmatrix}X=\begin{pmatrix}1&0&7\\8&1&2\end{pmatrix}.$

8-variant

1. $A=\begin{pmatrix}7&1&4&6\\-5&0&3&4\\3&-1&-9&0\end{pmatrix}.$

2. $\begin{pmatrix}1&-1&-1\\-1&2&1\\-1&1&2\end{pmatrix}.$

3. $\begin{pmatrix}3&4\\2&3\end{pmatrix}X=\begin{pmatrix}2&-3\\7&4\end{pmatrix}.$

9-variant

1. $A=\begin{pmatrix}1&2&-1&2\\-2&-4&8&5\\-1&8&-6&10\end{pmatrix}.$

2. $\begin{pmatrix}5&8&-1\\2&-3&2\\1&2&3\end{pmatrix}.$

3. $\begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ 0 & -2 & 1 \end{pmatrix} \cdot X \cdot = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{pmatrix}$.

10-variant

1. $A = \begin{pmatrix} 1 & 2 & -1 & 2 \\ -2 & -4 & 8 & -16 \\ -1 & -2 & 1 & -2 \end{pmatrix}$.

2. $\begin{pmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 0 \end{pmatrix}$.

3. $\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \cdot X = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$.

11-variant

1. $A = \begin{pmatrix} 1 & 2 & 1 & 4 \\ 0 & 5 & -1 & 4 \\ -1 & 3 & 4 & 6 \end{pmatrix}$.

2. $\begin{pmatrix} 3 & 4 & 2 \\ 2 & -4 & -3 \\ 1 & 5 & 1 \end{pmatrix}$.

3. $\begin{pmatrix} -5 & 6 \\ -4 & 5 \end{pmatrix} X = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$.

12-variant

1. $A = \begin{pmatrix} 1 & 3 & -4 & -2 \\ -2 & -6 & 8 & 4 \\ -1 & -3 & 4 & 2 \end{pmatrix}$.

2. $\begin{pmatrix} 1 & 1 & 2 \\ 2 & -1 & 2 \\ 4 & 1 & 4 \end{pmatrix}$.

3. $\begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix} \cdot X \cdot \begin{pmatrix} -1 & \frac{1}{2} \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -2 & 4 \end{pmatrix}$.

13-variant

1. $A = \begin{pmatrix} -3 & -1 & 8 & -2 \\ 2 & -2 & -3 & -7 \\ 1 & 11 & -12 & 34 \end{pmatrix}$.

2. $\begin{pmatrix} 1 & 1 & -1 \\ 8 & 3 & -6 \\ -4 & -1 & 3 \end{pmatrix}$.

3. $\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \cdot X = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$.

14-variant

1. $A = \begin{pmatrix} -1 & -4 & 3 & -61 \\ 2 & 5 & 1 & -23 \\ 17 & -10 & 20 & 0 \end{pmatrix}$.

2. $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 6 & 4 \\ -4 & -14 & -6 \end{pmatrix}$.

3. $X \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

15-variant

1. $A = \begin{pmatrix} -2 & -1 & 2 & -3 \\ -12 & 0 & 4 & 4 \\ 5 & 1 & -3 & 2 \end{pmatrix}$.

2. $\begin{pmatrix} 1 & 1 & 1 \\ -2 & -1 & -2 \\ 2 & 3 & 3 \end{pmatrix}$.

3. $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cdot X = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

16-variant

1. $A = \begin{pmatrix} -3 & -2 & -1 & -3 \\ 6 & 4 & 3 & 5 \\ 9 & 6 & 5 & 7 \end{pmatrix}$.

2. $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 6 & 4 \\ 3 & 10 & 8 \end{pmatrix}$.

3. $\begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ 0 & -2 & 1 \end{pmatrix} \cdot X = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$.

17-variant

1. $A=\begin{pmatrix}-1 & -2 & 1 & -2\\ 3 & -4 & 5 & 0\\ -1 & 3 & -3 & 1\end{pmatrix}$.

2. $\begin{pmatrix}1 & 1 & 1\\ 1 & 2 & 1\\ 1 & 1 & 2\end{pmatrix}$.

3. $X\cdot\begin{pmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{pmatrix}=\begin{pmatrix}0 & 0 & 1\\ 0 & 2 & 0\\ 3 & 0 & 0\end{pmatrix}$.

18-variant

1. $A=\begin{pmatrix}7 & 1 & 4 & 6\\ -5 & 0 & 3 & 4\\ -3 & 1 & 9 & 0\end{pmatrix}$.

2. $\begin{pmatrix}1 & 1 & 1\\ 1 & 2 & -1\\ 2 & 2 & 4\end{pmatrix}$.

3. $X\cdot\begin{pmatrix}4 & 3\\ -5 & -4\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$.

19-variant

1. $A=\begin{pmatrix}-3 & -24 & 3 & -6\\ 2 & -1 & 8 & 5\\ 1 & 10 & -6 & 8\end{pmatrix}$.

2. $\begin{pmatrix}5 & 3 & 1\\ 1 & -3 & -2\\ -5 & 2 & 1\end{pmatrix}$.

3. $\begin{pmatrix}1 & -2 & -1\\ -3 & 2 & 2\\ 3 & -1 & -2\end{pmatrix}\cdot X=\begin{pmatrix}1 & 0\\ 2 & -2\\ -3 & 1\end{pmatrix}$.

20-variant

1. $A=\begin{pmatrix}-1 & -2 & -3 & -4\\ 2 & 3 & 4 & 5\\ 3 & 4 & 5 & 6\\ 4 & 5 & 6 & 7\end{pmatrix}$.

2. $\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{pmatrix}$.

3. $X\cdot\begin{pmatrix}1 & 2 & -3\\ 3 & 2 & -4\\ 2 & -1 & 0\end{pmatrix}=\begin{pmatrix}1 & -3 & 0\\ 2 & 2 & -1\\ -1 & -2 & 4\end{pmatrix}$.

21-variant

1. $A=\begin{pmatrix}1 & 2 & -1 & 2\\ -2 & -4 & 8 & 5\\ 4 & -32 & 24 & -40\end{pmatrix}$.

2. $\begin{pmatrix}3 & 2 & 1\\ 2 & 3 & 1\\ 2 & 1 & 3\end{pmatrix}$.

3. $\begin{pmatrix}1 & 2 & 3\\ 2 & 4 & 6\\ 3 & 6 & 9\end{pmatrix}\cdot X=\begin{pmatrix}1\\ 2\\ 10\end{pmatrix}$.

22-variant

1. $A=\begin{pmatrix}1 & 2 & -1 & 3\\ 2 & 4 & -8 & 16\\ -5 & -10 & 5 & -10\end{pmatrix}$.

2. $\begin{pmatrix}2 & -3 & 1\\ 4 & -5 & 2\\ 5 & -7 & 3\end{pmatrix}$.

3. $\begin{pmatrix}1 & -2 & 3\\ 2 & 3 & -1\\ 0 & -2 & 1\end{pmatrix}\cdot X=\begin{pmatrix}7\\ 0\\ 7\end{pmatrix}$.

23-variant

1. $A=\begin{pmatrix}1 & 3 & -4 & -2\\ -2 & -6 & 8 & 4\\ 7 & 21 & -28 & -14\end{pmatrix}$.

2. $\begin{pmatrix}1 & 2 & -3\\ 3 & 2 & -4\\ 2 & -1 & 0\end{pmatrix}$.

3. $X\cdot\begin{pmatrix}-1 & 1\\ 0 & 1\end{pmatrix}=\begin{pmatrix}2 & 0\\ -1 & 3\end{pmatrix}$.

24-variant

1. $A=\begin{pmatrix}1 & 2 & 1 & 4\\ 0 & -5 & 1 & -4\\ -2 & 6 & 8 & 12\end{pmatrix}$.

2. $\begin{pmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{pmatrix}$

3. $\begin{pmatrix} 1 & -1 \\ -2 & 2 \end{pmatrix} \cdot X = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$.

25-variant

1. $\begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 4 \\ 3 & -1 & 2 \end{pmatrix}$

2. $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$.

3. $\begin{pmatrix} -1 & 2 \\ 2 & -3 \end{pmatrix} \cdot X = \begin{pmatrix} -2 & 3 \\ 1 & -4 \end{pmatrix}$.

3-topshiriq

1-misolda ikkinchi tartibli determinantlarni hisoblang.

2-misolda uchinchi tartibli determinantlarni qulay usulda hisoblang.

3-misolda tenglama yoki tengsizlikni yeching.

4-misolda to‘rtinchi tartibli determinantlarni determinant xossalaridan foydalanib, nollar yig'ib hisoblang, biror satr yoki ustun elementlari bo‘yicha yoyib hisoblang.

1-variant

1. $\begin{vmatrix} \sin^2\alpha & \cos^2\alpha \\ \sin^2\beta & \cos^2\beta \end{vmatrix}$.

2. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$.

3. $\begin{vmatrix} x^2 & 4 & 9 \\ x & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 0$.

4. $\begin{vmatrix} 1 & 3 & -5 & 7 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{vmatrix}$.

2-variant

1. $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$.

2. $\begin{vmatrix} a & 1 & a \\ -1 & a & 1 \\ a & -1 & a \end{vmatrix}$.

3. $\begin{vmatrix} x^2 & 3 & 2 \\ x & -1 & 1 \\ 0 & 1 & 4 \end{vmatrix} = 0.$

4. $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{vmatrix}$.

3-variant

1. $\begin{vmatrix} -1 & 4 \\ -5 & 2 \end{vmatrix}$.

2. $\begin{vmatrix} 1 & b & 1 \\ 0 & b & 0 \\ b & 0 & -b \end{vmatrix}$.

3. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1-x & 1 \\ 1 & 1 & 2-x \end{vmatrix} = 0.$

4. $\begin{vmatrix} 7 & -3 & 0 & 4 \\ 2 & 1 & 1 & 5 \\ 3 & 6 & -1 & -3 \\ 8 & 1 & 1 & 1 \end{vmatrix}$.

4-variant

1. $\begin{vmatrix} \sqrt{a} & -1 \\ a & \sqrt{a} \end{vmatrix}$.

2. $\begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix}$.

3. $\begin{vmatrix} 4 & 2 & 2 \\ -5 & 1 & -6 \\ 3 & 1 & x+1 \end{vmatrix} = 0$.

4. $\begin{vmatrix} 2 & -3 & 4 & 1 \\ 4 & -2 & 3 & 2 \\ a & b & c & d \\ 3 & -1 & 4 & 3 \end{vmatrix}$.

5-variant

1. $\begin{vmatrix} a+b & a-b \\ a-b & a+b \end{vmatrix}$.

2. $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$.

3. $\begin{vmatrix} \sin 2x & -\sin 3x \\ \cos 2x & \cos 3x \end{vmatrix} = 0$.

4. $\begin{vmatrix} a & 1 & 2 & 0 \\ b & 3 & 1 & 4 \\ c & 0 & 1 & 2 \\ d & 1 & 1 & 0 \end{vmatrix}$.

6-variant

1. $\begin{vmatrix} 1 & 2 \\ -3 & -4 \end{vmatrix}$.

2. $\begin{vmatrix} \sin^2\alpha & \cos^2\alpha & \cos 2\alpha \\ \sin^2\beta & \cos^2\beta & \cos 2\beta \\ \sin^2\gamma & \cos^2\gamma & \cos 2\gamma \end{vmatrix}$.

3. $\begin{vmatrix} 2 & 3 & 4 \\ 2 & 4-x & 4 \\ 2 & 3 & 7+x \end{vmatrix} = 0$.

4. $\begin{vmatrix} -1 & 3 & 1 & 2 \\ -5 & 8 & 2 & 7 \\ 4 & -5 & 3 & -2 \\ -7 & 8 & 4 & 5 \end{vmatrix}$.

7-variant

1. $\begin{vmatrix} 2 & 6 \\ 3 & 4 \end{vmatrix}$.

2. $\begin{vmatrix} \sin^2\alpha & \cos^2\alpha & 1 \\ \sin^2\beta & \cos^2\beta & 1 \\ \sin^2\gamma & \cos^2\gamma & 1 \end{vmatrix}$.

3. $\begin{vmatrix} x-2 & y+3 \\ 7-y & x+4 \end{vmatrix} = -34$.

4. $\begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$.

8-variant

1. $\begin{vmatrix} 1 & 3 \\ 4 & 7 \end{vmatrix}$.

2. $\begin{vmatrix} a & a^2+1 & (a+1)^2 \\ b & b^2+1 & (b+1)^2 \\ c & c^2+1 & (c+1)^2 \end{vmatrix}$.

3. $\begin{vmatrix} 3 & 2 & -1 \\ x+2 & 0 & 1 \\ -2 & 3-x & 1 \end{vmatrix} < 0$.

4. $\begin{vmatrix} 3 & 5 & 7 & 2 \\ 7 & 6 & 3 & 7 \\ 5 & 4 & 3 & 5 \\ -5 & -6 & -5 & -4 \end{vmatrix}$.

9-variant

1. $\begin{vmatrix} -3 & 2 \\ 2 & -3 \end{vmatrix}$.

2. $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix}.$

3. $\begin{vmatrix} 2x-3 & 4 \\ -x & -3 \end{vmatrix} = 0.$

4. $\begin{vmatrix} 2 & -3 & 4 & 1 \\ 4 & -2 & 3 & 2 \\ a & b & c & d \\ 3 & -1 & 4 & 3 \end{vmatrix}.$

10-variant

1. $\begin{vmatrix} (a+b)^2 & (a-b)^2 \\ (a-b) & (a+b) \end{vmatrix}.$

2. $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 4 & 6 & 7 \end{vmatrix}.$

3. $\begin{vmatrix} x+3 & x+1 \\ x-1 & x-2 \end{vmatrix} = 0.$

4. $\begin{vmatrix} 0 & -a & -b & -d \\ a & 0 & -c & -e \\ b & c & 0 & 0 \\ d & e & 0 & 0 \end{vmatrix}.$

11-variant

1. $\begin{vmatrix} (a+b)^2 & (a-b)^2 \\ (a-b)^2 & (a+b)^2 \end{vmatrix}.$

2. $\begin{vmatrix} 1 & 1 & 0 \\ 2 & 3 & 1 \\ 0 & 2 & 3 \end{vmatrix}.$

3. $\begin{vmatrix} 3-x & x+2 \\ x+1 & x-1 \end{vmatrix} = 6$

4. $\begin{vmatrix} 3 & -1 & 4 & 2 \\ 5 & 2 & 0 & 1 \\ 0 & 2 & 1 & -3 \\ 6 & -2 & 9 & 8 \end{vmatrix}.$

12-variant

1. $\begin{vmatrix} 2 & -3 \\ 5 & -4 \end{vmatrix}$.

2. $\begin{vmatrix} \sin\alpha & \cos\alpha & 1 \\ \sin\beta & \cos\beta & 1 \\ \sin\gamma & \cos\gamma & 1 \end{vmatrix}$.

3. $\begin{vmatrix} x-2 & y+3 \\ 1-y & x-2 \end{vmatrix} = -4$.

4. $\begin{vmatrix} 2 & 3 & -3 & 4 \\ 2 & 1 & -1 & 2 \\ 6 & 2 & 1 & 0 \\ 2 & 3 & 0 & -5 \end{vmatrix}$.

13-variant

1. $\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix}$.

2. $\begin{vmatrix} 2 & 1 & -3 \\ 0 & 1 & -1 \\ 3 & -2 & 1 \end{vmatrix}$.

3. $\begin{vmatrix} -3 & x-1 & 1 \\ x+2 & 2 & 3 \\ 0 & 1 & x \end{vmatrix} = 6$.

4. $\begin{vmatrix} 2 & -1 & 1 & 0 \\ 0 & 1 & 2 & -1 \\ 3 & -1 & 2 & 3 \\ 3 & 1 & 6 & 1 \end{vmatrix}$.

14-variant

1. $\begin{vmatrix} x & x-1 \\ x^2+x+1 & x^2 \end{vmatrix}$.

2. $\begin{vmatrix} 2 & 0 & 3 \\ 7 & 1 & 6 \\ 6 & 0 & 5 \end{vmatrix}$.

3. $\begin{vmatrix} 2 & 0 & -1 \\ 1 & x+5 & 2-x \\ 3 & -1 & 2 \end{vmatrix} \le 4$.

4. $\begin{vmatrix} 2 & 1 & 3 & 4 \\ a & b & c & d \\ 4 & -2 & 5 & -1 \\ 3 & 1 & 7 & 1 \end{vmatrix}$.

15-variant

1. $\begin{vmatrix} \cos\varphi & \sin\varphi \\ \sin\varphi & \cos\varphi \end{vmatrix}$.

2. $\begin{vmatrix} 3 & 0 & 2 \\ -5 & 3 & -1 \\ 6 & 0 & 3 \end{vmatrix}$.

3. $\begin{vmatrix} x+2 & 4 & -1 \\ -2 & 2 & x-1 \\ 1 & 3 & 0 \end{vmatrix} = 0.$

4. $\begin{vmatrix} 5 & a & 2 & -1 \\ 4 & b & 4 & -3 \\ 2 & c & 3 & -2 \\ 4 & d & 5 & -4 \end{vmatrix}$.

16-variant

1. $\begin{vmatrix} \alpha & 3\alpha \\ \beta & 3\beta \end{vmatrix}$.

2. $\begin{vmatrix} 5 & 6 & 3 \\ 0 & 2 & 0 \\ 7 & -4 & 5 \end{vmatrix}$.

3. $\begin{vmatrix} -3 & 2 & 1 \\ x-1 & 0 & 7 \\ 2 & -1 & 3 \end{vmatrix} = 0.$

4. $\begin{vmatrix} 2 & -3 & 4 & 1 \\ 4 & -2 & 3 & 2 \\ a & b & c & d \\ 3 & -1 & 4 & 3 \end{vmatrix}$.

17-variant

1. $\begin{vmatrix} x^2 & x \\ xy^2 & y^2 \end{vmatrix}$.

2. $\begin{vmatrix} 0 & 1 & 0 \\ 2 & 3 & 4 \\ 0 & 5 & 0 \end{vmatrix}$.

3. $\begin{vmatrix} -1 & 3 & -2 \\ 2-3x & 0 & 5 \\ 3 & 2 & 1 \end{vmatrix} \geq 0$.

4. $\begin{vmatrix} 7 & 3 & 2 & 6 \\ 8 & -9 & 4 & 9 \\ 7 & -2 & 7 & 3 \\ 5 & -3 & 3 & 4 \end{vmatrix}$.

18-variant

1. $\begin{vmatrix} \alpha & \beta \\ 0 & 0 \end{vmatrix}$.

2. $\begin{vmatrix} 0 & x & 0 \\ y & 0 & 0 \\ 0 & 0 & z \end{vmatrix}$.

3. $\begin{vmatrix} \sin 2x & \sin x \\ \cos x & \cos 2x \end{vmatrix} = 0$.

4. $\begin{vmatrix} 3 & 2 & 2 & 2 \\ 9 & -8 & 5 & 10 \\ 5 & -8 & 5 & 8 \\ 6 & -5 & 4 & 7 \end{vmatrix}$.

19-variant

1. $\begin{vmatrix} tg\varphi & 1 \\ -1 & tg\varphi \end{vmatrix}$.

2. $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix}$.

3. $\begin{vmatrix} x-2 & y+3 \\ -y-3 & x-2 \end{vmatrix} = 0$.

4. $\begin{vmatrix} 5 & a & 2 & -1 \\ 4 & b & 4 & -3 \\ 2 & c & 3 & -2 \\ 4 & d & 5 & -4 \end{vmatrix}$.

20-variant

1. $\begin{vmatrix} \sin\alpha & \cos\alpha \\ \sin\beta & \cos\beta \end{vmatrix}$.

2. $\begin{vmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{vmatrix}$.

3. $\begin{vmatrix} 6 & 3 & x-1 \\ 2x & 1 & 0 \\ 4 & x+2 & 2 \end{vmatrix} = 0.$

4. $\begin{vmatrix} 0 & 5 & 2 & 0 \\ 8 & 3 & 5 & 4 \\ 7 & 2 & 4 & 1 \\ 0 & 4 & 1 & 0 \end{vmatrix}$.

21-variant

1. $\begin{vmatrix} a^2-b^2 & a^4-b^4 \\ 1 & a^2+b^2 \end{vmatrix}$.

2. $\begin{vmatrix} 1 & 1 & 1 \\ 5 & 7 & 8 \\ 25 & 49 & 64 \end{vmatrix}$.

3. $\begin{vmatrix} 2 & 0 & 3 \\ -1 & 7 & x-3 \\ 5 & -3 & 6 \end{vmatrix} = 0.$

4. $\begin{vmatrix} 3 & -5 & 2 & -4 \\ -3 & 4 & -5 & 3 \\ -5 & 7 & -7 & 5 \\ 8 & -8 & 5 & -6 \end{vmatrix}$.

22-variant

1. $\begin{vmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{vmatrix}$.

2. $\begin{vmatrix} 3 & 2 & -1 \\ -2 & 2 & 3 \\ 4 & 2 & -3 \end{vmatrix}$.

3. $\begin{vmatrix} -1 & 0 & 2x+3 \\ 3-x & 1 & 1 \\ 2x+1 & -1 & 2 \end{vmatrix} = 0.$

4. $\begin{vmatrix} 2 & -3 & 4 & 1 \\ 4 & -2 & 3 & 2 \\ 1 & 1 & 1 & 1 \\ 3 & -1 & 4 & 3 \end{vmatrix}$.

23-variant

1. $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$.

2. $\begin{vmatrix} 2 & 1 & 3 \\ 5 & 3 & 2 \\ 1 & 4 & 3 \end{vmatrix}$.

3. $\begin{vmatrix} x+3 & x-1 \\ 7-x & x-1 \end{vmatrix} = 0.$

4. $\begin{vmatrix} 1 & 1 & 3 & 4 \\ 2 & 0 & 0 & 8 \\ 3 & 0 & 0 & 2 \\ 4 & 4 & 7 & 5 \end{vmatrix}$.

24-variant

1. $\begin{vmatrix} x & xy \\ 1 & y \end{vmatrix}$.

2. $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$.

3. $\begin{vmatrix} 2x-1 & x+1 \\ x+2 & x-1 \end{vmatrix} = -6.$

4. $\begin{vmatrix} 0 & -a & -b & -d \\ a & 0 & -c & -e \\ b & c & 0 & 0 \\ d & e & 0 & 0 \end{vmatrix}$.

25-variant

1. $\begin{vmatrix} -3 & 5 \\ 0 & 0 \end{vmatrix}$.

2. $\begin{vmatrix} 3 & 2 & 1 \\ 2 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix}$.

3. $\begin{vmatrix} 2x+1 & 3 \\ x+5 & 2 \end{vmatrix} = 0.$

4. $\begin{vmatrix} -2 & -3 & 0 & 2 \\ 1 & -1 & 2 & 2 \\ 3 & -1 & 5 & -2 \\ 0 & -2 & 4 & 1 \end{vmatrix}$.

4-topshiriq

Iqtisodiy mazmundagi masalalarning matematik modelini tuzing. Mathcad dasturida yeching.

1. Biror sohada m ta zavod mahsulotning n ta turini ishlab chiqaradi. $A_{m\times n}$ matritsa birinchi kvartalda har bir zavoddagi mahsulot hajmlarini belgilaydi. $B_{m\times n}$ matritsa esa mos ravishda ikkinchisida. $(a_{ij};b_{ij})$ mos ravishda birinchi va ikkinchi kvartalda i–zavodda j – turdagi mahsulot hajmlari

$$A=\begin{pmatrix}2&3&7\\1&2&2\\4&1&5\\2&1&3\end{pmatrix};\qquad B=\begin{pmatrix}3&0&2\\2&4&1\\4&3&2\\5&2&4\end{pmatrix}.$$

a) mahsulot hajmlari;

b) ishlab chiqarish hajmlarining mahsulot turlari va zavodlar bo‘yicha, birinchi kvartalga qaraganda ikkinchi kvartaldagi orttirmasini toping.

2. Korxona mahsulotning n turini ishlab chiqaradi, ishlab chiqarish mahsulot hajmlari $A_{1\times n}$ matritsa bilan berilgan. j – mintaqada mahsulotning i–turi birligining sotilish narxi $B_{n\times k}$ matritsa bilan berilgan, bu yerda k – mahsulot sotilayotgan mintaqalar soni.

Mintaqalar bo‘yicha daromad matritsasi C ni toping.

$A_{1\times 3}=(100,2000,100)$ bo‘lsin.

$$B_{3\times 4}=\begin{pmatrix}2&3&1&5\\1&3&2&2\\2&4&2&4\end{pmatrix}.$$

3. Korxona m turdagi resurslarni qo‘llab, n turdagi mahsulot ishlab chiqaradi. j – turdagi mahsulot birligini ishlab chiqarishga ketgan i–tovar resurslari xarajatlarining normalari $A_{m\times n}$ matritsa bilan berilgan. Vaqtning ma’lum oralig‘ida korxona $X_{n\times 1}$ matritsa bilan yozilgan har bir x_{ij} turdagi mahsulot miqdorini ishlab chiqargan bo‘lsin.

Vaqtning berilgan davrida barcha mahsulotning har bir turini ishlab chiqarishga ketgan resurslarning to‘la xarajatlar matritsasi S ni aniqlang. Berilgan

$$A_{4\times3}=\begin{pmatrix}2&5&3\\0&1&8\\1&3&1\\2&2&3\end{pmatrix},\quad X_{3\times1}=\begin{pmatrix}100\\80\\110\end{pmatrix}.$$

4. 3-masalaning shartida resurslarning har bir turining narxi ko'rsatilgan bo'lsin. Narx $P_{1\times m}$ matritsa bilan beriladi. Agar $P=(10,\ 20,\ 10,\ 10)$ bo'lsa, vaqtning berilgan oralig'ida barcha sarf qilingan resurslarning to'la narxini aniqang.

5. Zavod birdan qo'shimcha sozlashni talab etadigan, (40% holda) yoki birdan ishlatadigan (60 % holda) dvigatellarni ishlab chiqaradi. Statistik ma'lumotlar shuni ko'rsatadiki 65 % holda qo'shimcha sozlashni talab etadigan dvigatellar bir oydan so'ng 65 % qo'shimcha sozlashni talab etadi, 35% holda esa bir oydan keyin yaxshi ishlaydi. Dastlab sozlashni talab etmagan dvigatellar esa 20 % holda bir oydan so'ng sozlashni talab etadi va 80% holda yaxshi ishlaydi.

Ishlab chiqarilganidan ikki oy o'tgach, yaxshi ishlaydigan yoki sozlashni talab etadigan dvigatellar ulushi qanday? 3 oydan keyinchi?

6. Uchta zavod to'rt turdagi mahsulot ishlab chiqaradi. Agar oyma - oy ishlab chiqarilgan

$$A_1=\begin{pmatrix}2&3&1&2\\4&2&2&1\\5&4&4&2\end{pmatrix};\ A_2=\begin{pmatrix}1&4&12&2\\3&3&3&2\\4&5&4&3\end{pmatrix};\ A_3=\begin{pmatrix}2&5&3&1\\3&4&3&1\\4&4&4&4\end{pmatrix}.$$

mahsulot hajmlari matritsalari berilgan bo'lsa: a) kvartalda ishlab chiqarilgan mahsulot matritsasini toping; b) har bir oyda ishlab chiqarilgan mahsulotlarining B_1 va B_2 orttirma matritsasini toping va natijalarni tahlil qiling.

7. Korxona mahsulotning n turini ishlab chiqaradi, ishlab chiqarish mahsulot hajmlari $A_{1\times n}$ matritsa bilan berilgan. j – mintaqada mahsulotning i–turi birligini sotilish narxi $B_{n\times k}$ matritsa bilan berilgan, bu yerda k – mahsulot sotilayotgan mintaqalar soni.

Mintaqalar bo'yicha daromad matritsasi C ni aniqlang.

$$A=(10;40;10;20);\qquad B=\begin{pmatrix}2&1&2\\4&2&1\\3&3&1\\2&4&4\end{pmatrix}.$$

Mahsulotni sotishda uchta mintaqadan qaysi biri ko'proq foyda berishini aniqlang.

8. Korxona uch turdagi mebel ishlab chiqaradi va uni to‘rt mintaqada sotadi.

$B=(b_{ij})=\begin{pmatrix} 2 & 5 & 1 & 2 \\ 1 & 8 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$ matritsa j - mintaqada i–turdagi mebel birligini sotish narxini belgilaydi. Agar bir oyda (turlar bo‘yicha) mebel sotilishi

$A=\begin{pmatrix} 200 \\ 80 \\ 100 \end{pmatrix}$ matritsasi bilan berilgan bo‘lsa, har bir mintaqada korxonaning daromadini aniqlang.

9. Korxona m turdagi resurslarni qo‘llab, n turdagi mahsulot ishlab chiqaradi. j - turdagi mahsulot birligini ishlab chiqarishga ketgan i–tovar resurslari xarajatlarining normalari $A_{m\times n}$ matritsa bilan berilgan. Vaqtning ma’lum oralig‘ida korxona $X_{n\times 1}$ matritsa bilan yozilgan har bir x_{ij} turdagi mahsulot miqdorini ishlab chiqargan bo‘lsin.

1) agar xarajatlar normalari $A=\begin{pmatrix} 2 & 1 \\ 2 & 2 \\ 3 & 1 \end{pmatrix}$ matritsa bilan va mahsulot 2 turining har birini ishlab chiqarish hajmi $X=\begin{pmatrix} 200 \\ 300 \end{pmatrix}$ bilan berilgan bo‘lsa, bir oylik mahsulotni ishlab chiqarishga ketgan uch turdagi resurslarning to‘la xarajatlarini;

2) agar har bir resurs birliklarining narxi $P=(50,\ 10,\ 20)$ bilan berilgan bo‘lsa, barcha sarf qilingan resurslarning narxini toping.

10. Sotuvchi spektaklga narxi 100 so‘m bo‘lgan 1 tadan 5 tagacha bilet sotib olib, spektakldan oldin har birini 200 so‘mdan sotishi mumkin. Sotib olingan biletlar soniga (matritsa satri) va sotish natijalari (matritsa ustuni)ga bog‘liq holda sotuvchining daromad matritsasini tuzing.

11. Sifat bo‘yicha ikki xil o‘simlik yog‘lari uchta do‘konda sotiladi. A matritsa – bu mahsulotlarning do‘konlarda 1-kvartalda sotilish hajmlari, B matritsa – bu 2-kvartalda (ming so‘mlarda). 1) 2 ta kvartalda sotish hajmlarini, 2) 2-kvartalda birinchisiga nisbatan sotilishning o‘sishini aniqlang.

$$A=\begin{pmatrix} 2 & 5 \\ 7 & 3 \\ 2 & 4 \end{pmatrix};\quad B=\begin{pmatrix} 3 & 4 \\ 6 & 4 \\ 2 & 5 \end{pmatrix}.$$

12. Sifat bo'yicha ikki xil o'simlik yog'lari uchta do'konda sotiladi. A matritsa - bu mahsulotlarning do'konlarda 1-kvartalda sotilish hajmlari, B matritsa- bu 2-kvartalda (ming so'mlarda). 1) 2 ta kvartalda sotish hajmlarini, 2) 2-kvartalda birinchisiga nisbatan sotilishning o'sishini aniqlang.

$$A=\begin{pmatrix}3 & 4\\ 6 & 4\\ 2 & 3\end{pmatrix};\quad B=\begin{pmatrix}1 & 2\\ 5 & 3\\ 3 & 3\end{pmatrix}.$$

13. Sifat bo'yicha ikki xil o'simlik yog'lari uchta do'konda sotiladi. A matritsa-bu mahsulotlarning do'konlarda 1-kvartalda sotilish hajmlari, B matritsa- bu 2-kvartalda (ming so'mlarda). 1) 2 ta kvartalda sotish hajmlarini, 2) 2-kvartalda birinchisiga nisbatan sotilishning o'sishini aniqlang.

$$A=\begin{pmatrix}2 & 2\\ 3 & 4\\ 4 & 1\end{pmatrix};\quad B=\begin{pmatrix}1 & 2\\ 3 & 4\\ 4 & 2\end{pmatrix}.$$

14. Korxona ikki turdagi resurslardan foydalanib, uch turdagi mahsulotni ishlab chiqaradi. j – turdagi mahsulot birligini ishlab chiqarishga ketgan i–turdagi resurslar xarajatlarining normasi A xarajatlar matritsasi bilan berilgan, kvartal bo'yicha mahsulot ishlab chiqarilishi esa X matritsa bilan, har bir turdagi resurs birligining narxi P matritsa bilan berilgan.

$$A=\begin{pmatrix}2 & 1 & 1\\ 1 & 2 & 3\end{pmatrix};\quad X=\begin{pmatrix}10\\ 20\\ 10\end{pmatrix};\quad P=(5\quad 2).$$

1. Har bir turdagi resurslarning to'la xarajatlar S matritsasini aniqlang.
2. Barcha sarf etilgan resurslarning to'la narxini aniqlang.

15. Korxona ikki turdagi resurslardan foydalanib, uch turdagi mahsulotni ishlab chiqaradi. j – turdagi mahsulot birligini ishlab chiqarishga ketgan i–turdagi resurslar xarajatlarining normasi A xarajatlar matritsasi bilan berilgan, kvartal bo'yicha mahsulot ishlab chiqarilishi esa X matritsa bilan, har bir turdagi resurs birligining narxi P matritsa bilan berilgan.

$$A=\begin{pmatrix}1 & 2 & 1\\ 3 & 2 & 1\end{pmatrix};\quad X=\begin{pmatrix}10\\ 10\\ 10\end{pmatrix};\quad P=(2\quad 4).$$

1. Har bir turdagi resurslarning to'la xarajatlar S matritsasini aniqlang.
2. Barcha sarf etilgan resurslarning to'la narxini aniqlang.

16. Korxona ikki turdagi resurslardan foydalanib, uch turdagi mahsulotni ishlab chiqaradi. j – turdagi mahsulot birligini ishlab chiqarishga ketgan i–turdagi resurslar xarajatlarining normasi A xarajatlar matritsasi bilan berilgan, kvartal bo'yicha mahsulot ishlab chiqarilishi esa X matritsa bilan, har bir turdagi resurs birligining narxi P matritsa bilan berilgan.

$$A=\begin{pmatrix} 3 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix}; \quad X=\begin{pmatrix} 20 \\ 10 \\ 10 \end{pmatrix}; \quad P=(1 \quad 3).$$

1. Har bir turdagi resurslarning to'la xarajatlar S matritsasini aniqlang.
2. Barcha sarf etilgan resurslarning to'la narxini aniqlang.

17. Korxona ikki turdagi resurslardan foydalanib, uch turdagi mahsulotni ishlab chiqaradi. Bir birlik mahsulot uchun xomashyo xarajatlari normasi $A=\begin{pmatrix} 2 & 3 & 1 \\ 1 & 4 & 2 \end{pmatrix}$ matritsasi bilan berilgan. Har bir turdagi xomashyo birligining narxi $P=(2; \ 3)$ matritsa bilan berilgan. Mahsulot ishlab chiqarilishi esa $X=\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ matritsa bilan berilgan bo'lsa, korxonaning kunlik xarajatini aniqlang.

18. Korxona har kuni 4 turdagi mahsulot ishlab chiqaradi, ularning asosiy ishlab chiqarish – iqtisodiy ko'rsatkichlari 1-jadvalda keltirilgan.

1-jadval

Mahsulot turi, korxona tartib raqami №	**Mahsulot miqdori, bir.**	**Xomashyo xarajati, kg**	**Vaqt normasi s/mah.**	**Mahsulot bahosi, pul.bir./mah.**
1	20	5	10	30
2	50	2	5	15
3	30	7	15	45
4	40	4	8	40

Quyidagi kunlik ko'rsatkichlarni aniqlash talab etiladi: S – xomashyo xarajatlari, T – ish vaqtining sarfi va P – korxona chiqarayotgan mahsulotning narxi.

19. Korxona 4 xil xomashyo turini qo'llab 4 xil mahsulot ishlab chiqaradi. Xomashyo xarajatlarining normalari A matritsaning elementlari sifatida berilgan:

$$A=\begin{pmatrix} 2 & 3 & 4 & 5 \\ 1 & 2 & 5 & 6 \\ 7 & 2 & 3 & 2 \\ 4 & 5 & 6 & 8 \end{pmatrix}$$

mahsulot chiqarishning berilgan rejasida mos ravishda 60, 50, 35, 40 birlik. Mahsulotning har bir turiga talab etiladigan xomashyo xarajatlarini toping.

20. 2-jadvalda 3 xil xomashyo turini qo'llagan holda 4 xil mahsulotni ishlab chiqaruvchi 5 ta korxonaning kunlik ishlab chiqarishi haqida ma'lumot berilgan, hamda bir yilda har bir korxonaning ish muddati va har bir xom ashyoning narxi keltirilgan.

Har bir korxonaning har bir turdagi mahsulot bo'yicha yillik ishlab chiqarish unumdorligini toping.

2-jadval

Mahsulot turi, korxona tartib raqami №	**Korxonaning ishlab chiqarish unumdorligi, mah./kun.**					**Har bir turdagi mahsulot birligini ishlab chiqarish uchun sarflanadigan xomashyolar miqdori (kg)**		
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**
1	4	5	3	6	7	2	3	4
2	0	2	4	3	0	3	5	6
3	8	15	0	4	6	4	4	5
4	3	10	7	5	4	5	8	6
	Bir yildagi ish kunlari miqdori					Xomashyo turlari narhi (sh.p.b)		
	1	2	3	4	5	1	2	3
	200	150	170	120	140	40	50	60

21. 2-jadval asosida har bir korxonaning mahsulotning har bir turi bo'yicha yillik talabini toping.

22. 2-jadval asosida ko'rsatilgan ko'rinishda va sonda mahsulotlarni ishlab chiqarish uchun zarur bo'lgan xomashyo sotib olish uchun har bir korxonaning yillik kreditini toping.

23. Firma 5 turdagi mahsulotni ikkita sexda ishlab chiqaradi. Firmaning bir oyda ishlab chiqargan mahsulotlari taqsimoti quyidagi jadvalda berilgan:

3-jadval

Mahsulot turlari	1	2	3	4	5
1-sexda bir oyda ishlab chiqarilgan mahsulotlar miqdori	139	160	205	340	430
2-sexda bir oyda ishlab chiqarilgan mahsulotlar miqdori	122	130	145	162	152

Firma ishlab chiqarish uskunalarini yangilash natijasida ishlab chiqarishni 17 %ga oshirdi. Firma ishlab chiqarish uskunalarini yangilagandan keyin, firmaning bir oyda ishlab chiqargan mahsulotlari taqsimoti qanday bo'ladi?

24. Korxona 3 xil xomashyo turini qo'llab 4 xil mahsulot ishlab chiqaradi. Xomashyo xarajatlarining normalari A matritsaning elementlari sifatida berilgan:

$$A=\begin{pmatrix} 2 & 5 & 3 & 1 \\ 4 & 1 & 5 & 3 \\ 1 & 2 & 4 & 4 \end{pmatrix}$$

mahsulot chiqarishning berilgan rejasi mos ravishda 40, 100, 50, 120 birlik. Mahsulotning har bir turiga talab etiladigan xomashyo xarajatlarini toping.

25. Korxona m turdagi resurslarni qo'llab, n turdagi mahsulot ishlab chiqaradi. j – turdagi mahsulot birligini ishlab chiqarishga ketgan i–tovar resurslari xarajatlarining normalari $A_{m \times n}$ matritsa bilan berilgan. Vaqtning ma'lum oralig'ida korxona $X_{n \times 1}$ matritsa bilan yozilgan har bir x_{ij} turdagi mahsulot miqdorini ishlab chiqargan bo'lsin.

a) agar xarajatlar normalari $A=\begin{pmatrix} 5 & 4 & 6 \\ 10 & 8 & 12 \\ 3 & 5 & 4 \\ 9 & 6 & 3 \\ 2 & 8 & 10 \end{pmatrix}$ matritsa bilan va mahsulot 3 turining har birini ishlab chiqarish hajmi $X=\begin{pmatrix} 1 \\ 7 \\ 4 \end{pmatrix}$ bilan berilgan bo'lsa, bir oylik mahsulotni ishlab chiqarishga ketgan 5 turdagi resurslarning to'la xarajatlarini;

b) agar har bir resurs birliklarining narxi $P=\begin{pmatrix}7 & 4 & 5 & 1 & 2\end{pmatrix}$ bilan berilgan bo‘lsa, barcha sarf qilingan resurslarning narxini toping.

1.5. Mathcad dasturida hisoblash

Chiziqli algebra masalalarini Mathcad da yechishni shartli ravishda ikki guruhga ajratamiz. Birinchi – bu oddiy matritsaviy amallar, matritsa elementlari ustida aniqlangan arifmetik amallarni bajarish uchun. Ikkinchi guruh –murakkabroq amallarni bajarish uchun, ya’ni determinantlarni hisoblash, teskari matritsani topish, xos vetor va xos qiymatlarni hisoblash, chiziqli algebraik tenglamalar sistemasini yechish.

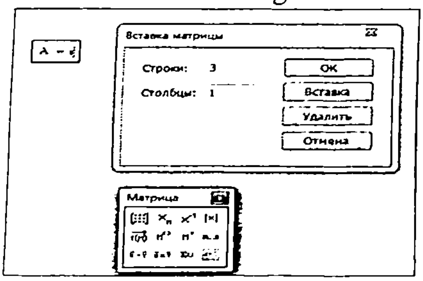
Matritsali operatorlar

Matritsali operatorlar vektorlar yoki matritsalar ustida turli amallarni bajarish uchun mo‘ljallangan.

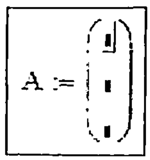
Vektor va matritsalarni oddiy va ko‘rgazmali usulda yaratish quyidagicha bajariladi:

1. **Matritsa** (Matrix) panelidan **Матрица** (Matrix) yoki **Vektor** (Vektor) tugmasini, yoki <Ctrl>+<M>, yoki menyular punktidan **Вставка/ Матрица** (Insert/ Matrix) tugmasini bosing.

2. **Вставка / Матрица** (Insert/ Matrix) dialog oynasining matritsa satr va ustunlariga butun sonlarni kiriting. Masalan 3ga 1 o‘lchamli matritsani yaratish 1-rasmda ko‘rsatilgan



1-rasm. Matritsa yaratish



2-rasm. Yaratilgan matritsa natijasi

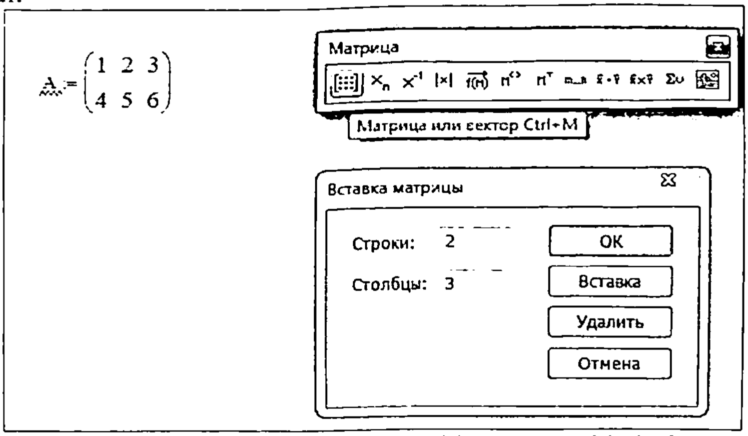
3. **Ok** yoki **Вставка** (Insert) tugmasini bosing natijada hujjatda aniqlangan o‘lchamdagi matritsa hosil bo‘ladi.

4. O‘rinto‘ldirgichlarga matritsa elementlarining qiymatlarini kiriting. Matritsaning bir elementidan boshqa elementiga sichqoncha ko‘rsatgichidan yoki strelka klavishi yordamida o‘tish mumkin.

Matritsalar ustida amallar

Transponirlash

Transponirlash (transpose) simvolini kiritish **Матрица** (Matrix) instrumentlar paneli yoki <Ctrl>+<M> tugmasini bosish yordamida amalga oshiriladi.



3-rasm. Matritsani kiritish va ular ustida amallarni bajarish **Матрица** paneli yordamida amalga oshiriladi.

1-misol. Matritsa va vektorlarni transponirlang.

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}$$

$$(1.01 \quad 3.03 \quad 4.04 \quad 5.05)^T = \begin{pmatrix} 1.01 \\ 3.03 \\ 4.04 \\ 5.05 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}^T \rightarrow (x \quad y \quad z)$$

Qo‘shish va ayirish

Mathcad da qo‘shish va ayirish amallarini bajarish uchun mos ravishda “+” va “-”simvollari qo‘llaniladi.

O‘lchamlari aynan teng bo‘lgan matritsalar ustidagina qo‘shish amali bajariladi, aks holda xato haqida axborot beriladi. *A* va *B* matritsalarni qo‘shish uchun, ularning mos elementlari qo‘shiladi.

2-misol. Matritsalarni qo‘shish, ayirish va ishorasini almashtirish.

$$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} + \begin{pmatrix} u & v & w \\ x & y & z \end{pmatrix} \rightarrow \begin{pmatrix} a+u & b+v & c+w \\ d+x & y+e & f+z \end{pmatrix}$$

$$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} - \begin{pmatrix} u & v & w \\ x & y & z \end{pmatrix} \rightarrow \begin{pmatrix} a-u & b-v & c-w \\ d-x & e-y & f-z \end{pmatrix}$$

$$-\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -1 & -2 & -3 \\ -4 & -5 & -6 \end{pmatrix}$$

Ba'zida matritsa yoki vektorning barcha elementlari yig'indisini hisoblash kerak bo'ladi. Buning uchun yordamchi operator mavjud (2-misoldagi birinchi va ikkinchi satrlar) **Матрица** (Matrix) instrumentlar panelidan **Сумма вектора** (Vector Sum) tugmasi yoki <Ctrl>+<4> klavishini bosish yordamida.

Shuningdek 3-misolda kvadrat matritsaning diagonal elementlari yig'indisini topish amali ko'rsatilgan. Bu amal tr: funksiyasida amalga oshiriladi:

tr (A)- A kvadrat matritsaning diagonal elementlari yig'indisi.

3-misol.

$$tr\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) \rightarrow a + d$$

$$\sum \begin{pmatrix} a \\ b \\ c \\ k \end{pmatrix} \rightarrow a + b + c + k$$

Ko'paytirish

Ko'paytirish amali faqat va faqat o'zaro zanjirlangan matritsalar ustida bajariladi. Ya'ni $m \times k$ o'lchamli matritsani $k \times n$ o'lchamli matritsaga ko'paytirish mumkin. Ko'paytma simvolini kiritish uchun yulduzcha <*> klavishasini bosish kerak.

4-misol. Matritsalarni ko'paytiring.

$$\begin{pmatrix} a & b & c \\ d & f & s \end{pmatrix} \cdot \begin{pmatrix} u & x \\ v & y \\ w & z \end{pmatrix} \rightarrow \begin{pmatrix} a \cdot u + b \cdot v + c \cdot w & a \cdot x + b \cdot y + c \cdot z \\ d \cdot u + f \cdot v + s \cdot w & d \cdot x + f \cdot y + s \cdot z \end{pmatrix}$$

$$\begin{pmatrix} a & b & c \\ d & f & s \end{pmatrix} \cdot \begin{pmatrix} u & v & w \\ x & y & z \end{pmatrix} \rightarrow$$

4-misolning ikkinchi satrida matritsalar zanjirlanmagan, shuning uchun natija yo'q. Tenglik kiritish belgisidan so'ng bo'sh joy turadi. Mathcad redaktoridagi ifodaning o'zi qizil rangda belgilanadi. Bu ifodaga kursor begisini qo'ysak birinchi matritsaning ustunlari soni ikkinchi matritsaning satrlari soniga teng emasligi haqida xabar keladi.

Matritsani skalyar kattalikka ko'paytirish va bo'lish amali ham aniqlangan. Ko'paytirish simvoli ikkita matritsani ko'paytirishdagi kabi kiritiladi. Skalyar kattalikni ixtiyoriy o'lchamdagi matritsaga ko'paytirish mumkin.

5-misol. Matritsani skalyar kattalikka ko'paytiring.

$$\begin{pmatrix} a & b & c \\ d & f & s \end{pmatrix} \cdot n \rightarrow \begin{pmatrix} a\cdot n & b\cdot n & c\cdot n \\ d\cdot n & f\cdot n & n\cdot s \end{pmatrix}$$

Kvadrat matritsaning determinanti

Matritsa determinantini hisoblash operatorini kiritishda **Матрица** (Matrix) instrumentlar panelidan **Определител** (Determinant) tugmasi bosiladi yoki klaviaturadan <\> (<Shift>+<\>) teriladi.

6-misol. Kvadrat matritsaning determinantini hisoblang.

$$\left|\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right| \rightarrow a\cdot d - b\cdot c$$

$$\left|\begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ 5 & 4 & 3 \end{pmatrix}\right| = 16$$

Matritsa rangi

Mathcadda matritsa rangini hisoblash uchun **rank** funksiyasi kiritiladi.

rank (A) - matritsa rangi; A- matritsa.

7-misol. Matritsa rangini hisoblash.

$$A := \begin{pmatrix} 1 & 2 & 3 \\ 4 & 8 & 12 \\ 1 & 7 & 1 \end{pmatrix} \qquad \left|\begin{pmatrix} 1 & 2 & 3 \\ 4 & 8 & 12 \\ 1 & 7 & 1 \end{pmatrix}\right| = 0$$

$$\text{rank}\left(\begin{pmatrix} 1 & 2 & 3 \\ 4 & 8 & 12 \\ 1 & 7 & 1 \end{pmatrix}\right) = 2 \qquad \text{rank}(A) = 2$$

Teskari matritsa

Teskari matritsani topish amalini kiritish uchun **Матрица** (Matrix) instrumentlar panelidan **Обращение** (Inverse) tugmasini bosing.

8 - misol. Teskari matritsani toping va natijani tekshiring.

$$\begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 & 1 \\ -0.5 & 0.5 & -0.5 \\ 0 & 0 & 1 \end{pmatrix}$$

Tekshirish:

$$\begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 1 \\ -0.5 & 0.5 & -0.5 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Kvadrat matritsani darajaga oshirish

Kvadrat matritsalar uchun $n-$ darajaga oshirish amalini formal qo‘llash mumkin. Buning uchun n butun son bo‘lishi kerak. Berilgan amallarning natijasi 1-jadvalda keltirilgan. A matritsani $n-$darajaga oshirish uchun **Калкулятор** (Calculator) panelidan **Возведения в степен** (Raise in Power) tugmasini yoki <^> klavishini bosing. Keyin uning o‘rinto‘ldirgichga $n-$darajaning qiymatini kiriting.

1-jadval. Matritsani darajaga oshirish qoidasi

n	A^n
0	A matritsa bilan bir xil o‘lchovdagi birlik matritsa
1	A matritsaning o‘zi
-1	A^{-1} – A matritsaga teskari matritsa
2,3,...	$A\cdot A,\ (A\cdot A)\cdot A,...$
$-2,-3,...$	$A^{-1}\cdot A^{-1},\ (A^{-1}\cdot A^{-1})\cdot A^{-1},...$

9-misol. Kvadrat matritsani butun sonli darajaga oshirish.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}^{-2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0.25 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}^{0} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}^{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}^{2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

Matritsa normasi

Mathcadda kvadrat matritsaning turli normalarini hisoblash uchun to'rtta funksiya bor.

norm1 (A) – L1 fazoda norma;

norm2 (A) – L2 fazoda norma;

norme (A) – L1 evklid norma (euclidean norm);

normi (A) – L1 max - norma yoki ∞ - norma (infinity norm);

A-kvadrat matritsa.

10-misol. Matritsaning turli normalarini hisoblash.

$$A := \begin{pmatrix} 1 & 2 & 3 \\ -4 & 5 & -6 \\ -7 & 8 & 9 \end{pmatrix} \qquad B := \begin{pmatrix} 100 & 200 & 300 \\ -400 & 500 & -600 \\ -700 & 800 & 900 \end{pmatrix}$$

$$\text{norm1}(A) = 18 \qquad \text{norm1}(B) = 1.8 \times 10^3$$

$$\text{norm2}(A) = 14.213 \qquad \text{norm2}(B) = 1.421 \times 10^3$$

$$\text{normi}(A) = 24 \qquad \text{normi}(B) = 2.4 \times 10^3$$

$$\text{norme}(A) = 16.882 \qquad \text{norme}(B) = 1.688 \times 10^3$$

$$\max(A) = 9 \qquad \max(B) = 900$$

Diagonal matritsa yaratish

Mathcad da matritsalarni quyidagi funksiyalar yordamida quriladi:

identity (n) - $n \times n$ o'lchovli birlik matritsa

diag (v) – diagonalida v vektorning elementlari joylashgan diagonal matritsa

n – butun son

v – vektor

11-misol. Berilgan o'lchovdagi birlik va diagonal matritsani yaratish.

$$\text{identity}(3) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad v := \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\text{diag}(v) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Matritsaning o'lchovi

Matritsa yoki vektorlarning xarakteristikalari haqida ma'lumot olish uchun quyidagi funksiyalarni ko'rib chiqamiz:

- rows(A) – satrlar soni;
- cols (A) – ustunlar soni;
- length (v) –vektor elementlari soni;

- last (v) – vektorning oxirgi elementi indeksi;
- A - matritsa yoki vektor;
- v- vektor.

12-misol. Matritsa o‘lchovi

$$A := \begin{pmatrix} 1 & 2 \\ 4 & 3 \\ 5 & 6 \\ 0 & 7 \end{pmatrix}$$

rows(A) = 4 cols(A) = 2

13-misol. Vektor o‘lchovi

$$v := \begin{pmatrix} 1 \\ 3 \\ 4 \\ 5 \end{pmatrix}$$

last(v) = 3
length(v) = 4
rows(v) = 4
cols(v) = 1

II BOB. CHIZIQLI ALGEBRAIK TENGLAMALAR SISTEMASI

2.1. Chiziqli algebraik tenglamalar sistemasi nazariyasi va asosiy tushunchalar

Asosiy tushunchalar. Kroneker-Kapelli teoremasi

Ushbu

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2, \\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \end{cases} \qquad (1)$$

tenglamalar sistemasiga n noma'lumli m ta chiziqli tenglamalar sistemasi deyiladi. Bu yerda a_{11}, a_{12}, ..., a_{mn} sonlar sistemaning koeffitsiyentlari, $x_1, x_2, \ldots, x_n$ lar noma'lumlar, b_1, $b_2, \ldots, b_m$ sonlar esa ozod hadlar deyiladi. a_{ij} koeffitsiyentda birinchi indeks i tenglamaning nomerini, ikkinchi indeks j esa nomalumning nomerini bildiradi.

(1) sistema va uning yechimlarini to'liqroq tahlil qilish uchun ba'zi tushunchalarni kirtib olishimiz kerak.

Oldingi paragraflarda aytilganidek matritsaning ustunlarini ustun vektor, satrlarini esa satr vektor sifatida qarash mumkin.

Bizga

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}, \quad Y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}, \quad \Theta = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

ustun vektorlar va λ_1 λ_2, ..., λ_m haqiqiy sonlar berilgan bo'lsin.

1-ta'rif. Agar Y vektor uchun

$$Y = \lambda_1 X_1 + \lambda_2 X_2 + \ldots + \lambda_m X_m$$

tenglik o'rinli bo'lsa, u holda Y vektor X_1, X_2, ..., X_m vektorlarning chiziqli kombinatsiyasidan iborat deyiladi.

Bu yerda λ_1, λ_2, ..., λ_m sonlar chiziqli kombinatsiya koeffitsiyentlari deb ataladi.

Agar X_1, X_2, ..., X_m vektorlar uchun

$$\Theta = \lambda_1 X_1 + \lambda_2 X_2 + \ldots + \lambda_m X_m$$

tenglik faqat va faqat $\lambda_1 = \lambda_2 = ... = \lambda_m = 0$ holdagina bajarilsa, u holda $X_1, X_2, ..., X_m$ vektorlar chiziqli erkli vektorlar deb ataladi.

Agar $X_1, X_2, ..., X_m$ vektorlar uchun

$$\Theta = \lambda_1 X_1 + \lambda_2 X_2 + ... + \lambda_m X_m$$

tenglik hech bo'lmaganda bitta λ_i uchun $\lambda_i \neq 0$ bo'lgan holda bajarilsa, u holda $X_1, X_2, ..., X_m$ vektorlar chiziqli bog'liq vektorlar deb ataladi

Teorema (Bazis minorlar haqida teorema). Matritsaning har qanday ustuni (satri) bazis ustunlarning (satrlarning) chiziqli kombinatsiyasidan iborat bo'lib, uning bazis ustun (satr) vektorlari chiziqli erkli bo'ladi.

2-ta'rif. (1) sistemaning **yechimi** deb shunday $\alpha_1, \alpha_2, \ldots, \alpha_n$ sonlarga aytiladiki, agar bu sonlar $x_1, x_2, \ldots, x_n$ noma'lumlarning o'rniga qo'yilganda (1) sistemadagi tenglamalar ayniyatga aylanadi.

Chiziqli tenglamalar sistemasi kamida bitta yechimga ega bo'lsa, u holda bunday sistema birgalikda deyiladi.

Bitta ham yechimga ega bo'lmagan chiziqli tenglamalar sistemasi birgalikda bo'lmagan sistema deyiladi.

Birgalikda bo'lgan sistema yagona yechimga ega bo'lsa aniq sistema va cheksiz ko'p yechimga ega bo'lsa aniqmas sistema deb ataladi.

Masalan,

$\begin{cases} x - y = 2, \\ 2x + y = 7 \end{cases}$ sistema birgalikda va aniq, chunki u yagona $x = 3, y = 1$ yechimga ega.

$\begin{cases} x - y = 1, \\ 2x - 2y = 2, \\ 3x - 3y = 3 \end{cases}$ sistema esa birgalikda, ammo aniqmas, chunki bu sistema $x = \alpha$, $y = -1 + \alpha$ ko'rinishdagi cheksiz ko'p yechimga ega, bunda α - ixtiyoriy haqiqiy son.

$\begin{cases} x + y + z = 1, \\ 3x + 3y + 3z = 5 \end{cases}$ sistema yechimga ega bo'lmaganligi sababli birgalikda emas.

Keltirilgan misollardan ko'rinadiki (1) sistema yagona yechimga ega, yechimga ega emas yoki cheksiz ko'p yechimga ega bo'lishi mumkin ekanligi kelib chiqadi.

Agar ikki sistemaning yechimlari bir xil sonlar to'plamidan iborat bo'lsa, bunday sistemalar **teng kuchli** yoki **ekvivalent** deyiladi.

Odatda sistemani yechish uchun avvalo unda elementar almashtirish bajariladi. Bunda almashtirilgan sistema dastlabki sistemaga ekvivalent bo'lishi kerak. Sistemani ekvivalent sistemaga aylantiradigan almashtirishlar elementar almashtirishlar deb ataladi. Sistema uchun bajariladigan **elementar almashtirishl**arni keltiramiz:

1) tenglamalarning o'rinlarini almashtirish;

2) tenglamalardan ixtiyoriy birini noldan farqli songa ko'paytirish;

3) sistemadagi tenglamalardan ixtiyoriy birining ikkala tarafini biror songa ko'paytirib tenglamadagi boshqa tenglamaga mos ravishda qo'shish;

4) sistemadagi $0\cdot x_1+0\cdot x_2+...+x_n=0$ ko'rinishdagi tenglamalarni tashlab yuborish.

(1) chiziqli tenglamalar sistema noma'lumlari oldidagi koeffitsiyentlardan

$$A=\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

matritsani tuzib olamiz. Bu matritsa sistemaning **asosiy matritsasi** deb ataladi. A matritsaning o'ng tarafiga ozod hadlarni qo'shib yozish orqali hosil qilingan

$$A|B=\begin{pmatrix} a_{11} & a_{12} & \dots a_{1n} & b_1 \\ a_{21} & a_{22} & \dots a_{2n} & b_2 \\ \dots & \dots & \dots\dots & \dots \\ a_{m1} & a_{m2} & a_{mn} & b_m \end{pmatrix}$$

matritsaga esa (1) chiziqli tenglamalar sistemasining **kengaytirilgan matritsasi** deyiladi.

Endi yuqorida qo'yilgan savollarga javob beradigan quyidagi teoremani keltiramiz.

Teorema (Kroneker-Kapelli teoremasi). Chiziqli tenglamalar sistemasi birgalikda bo'lishi uchun uning asosiy va kengaytirilgan matritsalarining ranglari teng bo'lishi zarur va yetarli.

Misol. Tenglamalar sistemasining birgalikda bo'lish-bo'lmasligini tekshiring:

$$\begin{cases} x_1 + x_2 + 3x_3 = 2, \\ x_1 - x_2 + x_3 = 0, \\ 3x_1 - 2x_2 - 2x_3 = -5. \end{cases}$$

Yechish. Sistemaning matritsasi va kengaytirilgan matritsasi ranglarini topamiz :

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & -1 & 1 \\ 1 & -2 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 3 \\ 0 & -2 & -2 \\ 0 & -3 & -5 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & -3 & -5 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{pmatrix},$$

$rangA = 3;$

$$A|B = \begin{pmatrix} 1 & 1 & 3 & 2 \\ 1 & -1 & 1 & 0 \\ 1 & -2 & -2 & -5 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 3 & 2 \\ 0 & -2 & -2 & -2 \\ 0 & -3 & -5 & -7 \end{pmatrix} \sim$$

$$\begin{pmatrix} 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & -3 & -5 & -7 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 3 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -2 & -4 \end{pmatrix},$$

$rangB = 3$. $rangA = rangB$. Demak, Kroneker-Kapelli teoremasiga ko'ra tenglamalar sistemasi birgalikda.

Izoh. Ba'zan chiziqli tenglamalar sistemasini yechimini ta'minlaydigan shartni aniqlashdan ko'ra uni yechimga ega emaslik sharti topiladi.

2.2. Chiziqli algebraik tenglamalar sistemasini yechishning Gauss va Gauss-Jordan usullari

Chiziqli tenglamalar sistemasining yechishda davom etamiz. n ta noma'lumli m ta chiziqli tenglamalar sistemasi berilgan bo'lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2, \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = b_m. \end{cases} \qquad (1)$$

Gauss metodi

(1) ko'rinishdagi sistemani bu usulda yechish qulaylik tug'diradi. Uning mohiyati noma'lumlarni ketma-ket yo'qotishdan iborat bo'lib, yechish ikki bosqichda (dastlab chapdan o'ngga, so'ngra o'ngdan chapga qarab) amalga oshiriladi.

1 – bosqich. Chapdan o'ngga, ya'ni (1) sistemani uchburchak ko'rinishga keltirishdan iborat. Buning uchun, $a_{11} \neq 0$ deb (agar $a_{11} = 0$ bo'lsa, 1- tenglama $a_{i1} \neq 0$ bo'lgan i-tenglama bilan o'rin almashtiriladi) birinchi tenglamaning chap va o'ng tomoni a_{11} ga bo'linadi. So'ngra, 1-tenglama $-\frac{a_{i1}}{a_{11}}$ ga ko'paytirilib, i – tenglamaga mos ravishda qo'shiladi. Bunda, sistemaning 2-tenglamasidan boshlab x_1 noma'lum yo'qotiladi. Bu jarayonni $(n-1)$ marotaba takrorlab quyidagi uchburchaksimon sistema hosil qilinadi:

$$\begin{cases} x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1, \\ \quad a_{22}^1 x_2 + a_{23}^{(1)}x_3 + \ldots + a_{2n}^{(1)}x_n = b_2^{(1)}, \\ \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \\ \qquad\qquad a_{nn}^{(n-1)}x_n = b_n^{(n-1)}. \end{cases} \qquad (2)$$

2–bosqich. O'ngdan chapga, oxirgi sistemani yechimini topishdan iborat. Buning uchun (2) sistemadagi tenglamadan $x_n = \frac{b_n^{(n-1)}}{a_{nn}^{(n-1)}}$ yechim topiladi, so'ngra, topilgan x_n undan oldingi tenglamaga (faqat x_n va x_{n-1} noma'lumlarni o'zida saqlovchi) qo'yiladi va undan x_{n-1} topiladi. Shu jarayon davom ettirilib, nihoyat 1-tenglamadan x_1 topiladi.

Gauss usuli bilan yechilganda sistema uchburchaksimon shaklga kelsa, u holda sistema yagona yechimga ega bo'ladi. Agar sistema trapetsiyasimon shaklga kelsa cheksiz ko'p yechimga ega bo'ladi.

Misol. Faraz qilamiz, korxona A, B, C turdagi mahsulotlarni ishlab chiqarish uchun S_1, S_2, S_3 xom ashyolardan foydalanadi. Bitta mahsulotni tayyorlash uchun xom ashyolarning bir kunlik sarf normasi quyida berilgan jadvaldagidek bo'lsin:

Xom ashyo turi	Bitta mahsulotni tayyorlash uchun xom ashyoning sarf normasi			Xom ashyoning bir kunlik sarf miqdori
	A	B	C	
S_1	5	3	4	2700
S_2	2	1	1	800
S_3	3	2	2	1600

Har bir tur mahsulotning bir kunlik ishlab chiqarish hajmi topilsin.

Yechish. Agar korxona bir kunda A mahsulotdan x_1 dona, V mahsulotdan x_2 dona va C mahsulotdan x_3 dona ishlab chiqarsa, u holda yuqoridagi jadvalga asosan:

$$\begin{cases} 5x_1 + 3x_2 + 4x_3 = 2700, \\ 2x_1 + x_2 + x_3 = 800, \\ 3x_1 + 2x_2 + 2x_3 = 1600 \end{cases}$$

tenglamalar sistemasiga ega bo'lamiz. Bu sistemani yuqorida keltirilgan usullardan biri bilan yechsak: (200; 300; 200). Bu esa korxona bir kunda A mahsulotdan 200 dona, B mahsulotdan 300 dona va C mahsulotdan 200 dona ishlab chiqarishini bildiradi.

Chiziqli tenglamalar sistemasini yechishning Gauss-Jordan usuli

Gauss – Jordan usulinig (Gauss usulining Jordan modifikatsiyasi) mazmun-mohiyati quyidagidan iborat: dastlabki normal ko'rinishda berilgan sistemaning kengaytirilgan $A|B$ matritsasi quriladi. Yuqorida keltilgan sistemani teng kuchligini saqlovchi elementar almashtirishlar yordamida, kengaytirilgan matritsaning chap qismida birlik matritsa hosil qilinadi. Bunda birlik matritsadan o'ngda yechimlar ustuni hosil bo'ladi. Gauss - Jordan usulini quyidagicha sxematik ifodalash mumkin:

$$A|B - E|X.$$

Chiziqli tenglamalar sistemasini yechish Gauss-Jordan usuli noma'lumlarni ketma-ket yo'qotishning Gauss strategiyasi va teskari matritsa qurishning Jordan taktikasiga asoslanadi. Teskari matritsa oshkor shaklda qurilmaydi, balki o'ng ustunda bir yo'la teskari matritsaning ozod hadlar ustuniga ko'paytmasi–yechimlar ustuni quriladi.

Misol. Quyidagi chiziqli tenglamalar sistemasini Jordan – Gauss metodi yordamida yeching:

$$\begin{cases} x_1 - x_2 + 2x_3 - x_4 = 1, \\ x_1 + x_2 + x_3 + x_4 = 4, \\ 2x_1 + 3x_2 - x_3 = -6, \\ 5x_1 + 2x_2 + 5x_3 - 6x_4 = 0. \end{cases}$$

Yechish. Chiziqli tenglamalar sistemasi koyeffitsentlaridan kengaytirilgan matritsa tuzamiz. Tenglamalar ustida bajariladigan almashtirishlar yordamida asosiy matritsani quyidagicha birlik matritsaga keltirib javobni topamiz:

$$\begin{pmatrix}1&1&2&3&1\\3&-1&-1&-2&-4\\2&3&-1&-1&-6\\1&2&3&-1&-4\end{pmatrix}\Rightarrow\begin{pmatrix}1&1&2&3&1\\0&4&7&11&7\\0&-1&5&7&8\\0&1&1&-4&-5\end{pmatrix}\Rightarrow\begin{pmatrix}1&1&2&3&1\\0&1&1&-4&-5\\0&-1&5&7&8\\0&4&7&11&7\end{pmatrix}\Rightarrow$$

$$\Rightarrow\begin{pmatrix}1&0&-1&7&6\\0&1&1&-4&-5\\0&0&6&3&3\\0&0&3&27&27\end{pmatrix}\Rightarrow\begin{pmatrix}1&0&-1&7&6\\0&1&1&-4&-5\\0&0&1&9&9\\0&0&2&1&1\end{pmatrix}\Rightarrow\begin{pmatrix}1&0&0&16&15\\0&1&0&-13&-14\\0&0&1&9&9\\0&0&0&-17&-17\end{pmatrix}\Rightarrow$$

$$\Rightarrow\begin{pmatrix}1&0&0&16&15\\0&1&0&-13&-14\\0&0&1&9&9\\0&0&0&1&1\end{pmatrix}\Rightarrow\begin{pmatrix}1&0&0&0&-1\\0&1&0&0&-1\\0&0&1&0&0\\0&0&0&1&1\end{pmatrix}\Rightarrow\begin{cases}x_1=-1,\\x_2=-1,\\x_3=0,\\x_4=1.\end{cases}$$

Misol. Tenglamalar sistemasini Gauss – Jordan usulida yeching

$$\begin{cases}5x_1+2x_2+3x_3+3x_4=1,\\2x_1-2x_2+5x_3+2x_4=4,\\3x_1+4x_2+2x_3+2x_4=-2.\end{cases}$$

Yechimi. Berilgan sistemada kengaytirilgan matritsani ajratib olamiz

$$A|B=\begin{pmatrix}5&2&3&3&\vdots&1\\2&-2&5&2&\vdots&4\\3&4&2&2&\vdots&-2\end{pmatrix}$$

va unga Gauss – Jordan yoki noma'lumlarni to'liq yo'qotish usulini tatbiq yetamiz:

$$A|B=\begin{pmatrix}1&\frac{2}{5}&\frac{3}{5}&\frac{3}{5}&\vdots&\frac{1}{5}\\0&-\frac{14}{5}&\frac{19}{5}&\frac{4}{5}&\vdots&\frac{18}{5}\\0&\frac{14}{5}&\frac{1}{5}&\frac{1}{5}&\vdots&-\frac{13}{5}\end{pmatrix}\sim\begin{pmatrix}1&0&\frac{8}{7}&\frac{5}{7}&\vdots&\frac{5}{7}\\0&1&-\frac{19}{14}&-\frac{2}{7}&\vdots&-\frac{9}{7}\\0&0&4&1&\vdots&1\end{pmatrix}\sim$$

$$\sim\begin{pmatrix}1&0&0&\frac{3}{7}&\vdots&\frac{3}{7}\\0&1&0&\frac{3}{56}&\vdots&-\frac{53}{56}\\0&0&1&\frac{1}{4}&\vdots&\frac{1}{4}\end{pmatrix}.$$

Sistema zinapoya ko'rinishiga keldi:

$$\begin{cases} x_1 \qquad\qquad + \frac{3}{7}x_4 = \frac{3}{7}, \\ \qquad x_2 \qquad + \frac{3}{56}x_4 = -\frac{53}{56}. \\ \qquad\qquad x_3 + \frac{1}{4}x_4 = \frac{1}{4}. \end{cases}$$

Bu yerda x_1, x_2 va x_3 o'zgaruvchilarni bazis sifatida qabul qilamiz, chunki sistema asosiy matritsasining rangi 3 ga teng va x_1, x_2 va x_3 o'zgaruvchilar oldidagi koeffitsiyentlardan tuzilgan determinant

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \neq 0.$$

Bu birlik determinant sistema asosiy matritsasining bazis minorlaridan bo'ladi. Erkli o'zgaruvchi bo'lib esa x_4 xizmat qiladi.

Oxirgi sistemadan

$$\begin{cases} x_1 = \frac{3}{7} - \frac{3}{7}x_4, \\ x_2 = -\frac{53}{56} - \frac{3}{56}x_4, \\ x_3 = \frac{1}{4} - \frac{1}{4}x_4. \end{cases}$$

ega bo'lamiz. Shunday qilib, berilgan sistemaning umumiy yechimini

$$X = \begin{pmatrix} \frac{3}{7} - \frac{3}{7}x_4 \\ -\frac{53}{56} - \frac{3}{56}x_4 \\ \frac{1}{4} - \frac{1}{4}x_4 \end{pmatrix}.$$

ko'rinishda tasvirlash mumkin.

Agar $x_4 = 2$ deb olsak, u holda berilgan sistemaning $X_1 = \begin{pmatrix} -\frac{3}{7} \\ -\frac{59}{56} \\ -\frac{1}{4} \end{pmatrix}$

ko'rinishdagi xususiy yechimini topamiz.

Agar $x_4 = 0$ ni olsak berilgan sistemaning quyidagi bazis yechimiga ega bo'lamiz:

$$X_b = \begin{pmatrix} \frac{3}{7} \\ -\frac{53}{56} \\ \frac{1}{4} \end{pmatrix}.$$

2.3. Chiziqli tenglamalar sistemasini yechishning Kramer qoidasi va matritsalar usuli

Ushbu n noma'lumli n ta chiziqli tenglamalar sistemasi berilgan bo'lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2, \\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n. \end{cases} \qquad (1)$$

(1) sistemada quyidagi belgilashlarni kiritamiz:

$$A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}, \quad B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}.$$

Bunda, A – noma'lumlar oldida turgan koeffitsiyentlardan tuzilgan matritsa; X – noma'lumlardan tuzilgan matritsa; B – ozod hadlardan tuzilgan matritsa.

U holda (1)sistemani

$$AX = B \qquad (2)$$

ko'rinishda ifodalash mumkin.

Faraz qilamiz $|A| \neq 0$ bo'lsin. U holda A matritsa uchun A^{-1} teskari matritsa mavjud. $AX = B$ tenglikning har ikkala tomonini A^{-1} ga chapdan ko'paytiramiz:

$$A^{-1}AX = A^{-1}B, \quad EX = A^{-1}B, \quad X = A^{-1}B .$$

Hosil bo'lgan $X = A^{-1}B$ ifoda chiziqli tenglamalar sistemasining matritsalar usuli bilan yechish formulasidan iborat.

Misol. Chiziqli tenglamalar sistemasini matritsalar usuli bilan yeching:

$$\begin{cases} 2x_1 + 2x_2 - 3x_3 = 5, \\ x_1 - x_2 + x_3 = 0, \\ 3x_1 + x_2 + x_3 = 2. \end{cases}$$

Yechish. A, X, B matritsalarni tuzib olamiz:

$$A = \begin{pmatrix} 2 & 2 & -3 \\ 1 & -1 & 1 \\ 3 & 1 & 1 \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad B = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}.$$

Bundan, $|A| = -12 \neq 0$.

Teskari matritsani topamiz:

$$A_{11} = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -2, \quad A_{12} = -\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = 2, \quad A_{13} = \begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix} = 4,$$

$$A_{21} = -\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = -5, \quad A_{22} = \begin{vmatrix} 2 & -3 \\ 3 & 1 \end{vmatrix} = 11, \quad A_{23} = -\begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} = 4,$$

$$A_{31} = \begin{vmatrix} 2 & -3 \\ -1 & 1 \end{vmatrix} = -1, \quad A_{32} = -\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = -5, \quad A_{33} = \begin{vmatrix} 2 & 2 \\ 1 & -1 \end{vmatrix} = -4,$$

$$A^{-1} = -\frac{1}{12}\begin{pmatrix} -2 & -5 & -1 \\ 2 & 11 & -5 \\ 4 & 4 & -4 \end{pmatrix}.$$

Bundan,

$$X = A^{-1}B = -\frac{1}{12}\begin{pmatrix} -2 & -5 & -1 \\ 2 & 11 & -5 \\ 4 & 4 & -4 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} = -\frac{1}{12}\begin{pmatrix} -10+0-2 \\ 10+0-10 \\ 20+0-8 \end{pmatrix} = -\frac{1}{12}\begin{pmatrix} -12 \\ 0 \\ 12 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}.$$

Demak, $x_1 = 1, \ x_2 = 0, \ x_3 = -1$.

Misol. Quyidagi tenglamani yeching.

$$\begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix} X \begin{pmatrix} 1 & 3 \\ 3 & 6 \end{pmatrix} = \begin{pmatrix} 4 & 0 \\ 5 & 8 \end{pmatrix}$$

Yechish. Tenglamaga quyidagi belgilashlarni kiritamiz:

$$A = \begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 3 \\ 3 & 6 \end{pmatrix}, \quad C_1 = \begin{pmatrix} 4 & 0 \\ 5 & 8 \end{pmatrix}.$$

U holda berilgan tenglama

$$A \cdot X \cdot B = C$$

ko‘rinishni oladi.

Agar AXB ifodaning chap tomondan A^{-1} va o‘ng tomondan B^{-1} ga ko‘paytirsak, hamda $A^{-1}A = E$, $EX = X$, $BB^{-1} = E$ va $XE = X$ ekanligini hisobga olsak quyidagi yechimga ega bo‘lamiz:

$$X = A^{-1}CB^{-1} = -\frac{1}{2}\begin{pmatrix} 1 & -1 \\ -2 & 0 \end{pmatrix} \cdot \begin{pmatrix} 4 & 0 \\ 5 & 8 \end{pmatrix} \cdot \begin{pmatrix} -2 & 1 \\ 1 & -\frac{1}{3} \end{pmatrix} =$$

$$= -\frac{1}{2}\begin{pmatrix} -1 & -8 \\ -8 & 0 \end{pmatrix} \cdot \begin{pmatrix} -2 & 1 \\ 1 & -\frac{1}{3} \end{pmatrix} = \begin{pmatrix} 3 & -\frac{5}{6} \\ -8 & 4 \end{pmatrix}.$$

Teorema (Kramer teoremasi). Agar (1) sistemadagi A matritsa Δ determinanti noldan farqli bo‘lsa, u holda (1) sistema yagona yechimga ega bo‘ladi va bu yechim quyidagi formulalar bilan topiladi:

$$x_1 = \frac{\Delta x_1}{\Delta}, \quad x_2 = \frac{\Delta x_2}{\Delta}, \quad \cdots \quad x_n = \frac{\Delta x_n}{\Delta}. \qquad (3)$$

bu yerda Δx_i determinant - A matritsa i – ustunini B ozod had ustuni bilan almashtirish orqali hosil qilingan matritsa determinanti ($i = 1,\ 2,\ ...,\ n$).

(3) formulalarga **Kramer formulalari** va sistemani bu formulalar bo‘yicha yechish qoidasiga **Kramer qoidasi** deyiladi.

Agar $\Delta = 0$ va $\Delta x_1, \Delta x_2, \ldots, \Delta x_n$ lardan birortasi noldan farqli bo‘lsa, u holda (1) sistema yechimga ega bo‘lmaydi.

Agar $\Delta = 0$ va $\Delta x_1 = \Delta x_2 = ... = \Delta x_n = 0$ bo‘lsa, u holda (1) sistema cheksiz ko‘p yechimga ega bo‘ladi.

Misol. Quyidagi chiziqli tenglamalar sistemasining

$$\begin{cases} -2x_1 + x_2 - x_3 = 7, \\ 4x_1 + 2x_2 + 3x_3 = -5, \\ x_1 + 3x_2 - 2x_3 = 1 \end{cases}$$

yechimini Kramer formulalari yordamida toping.

Yechish. Sistemaning asosiy Δ determinantini hisoblaymiz. Bunda

$$\Delta=\begin{vmatrix}-2 & 1 & -1\\ 4 & 2 & 3\\ 1 & 3 & -2\end{vmatrix}=27.$$

$\Delta\neq 0$ bo'lganligi sababli berilgan sistema aniq sistemani tashkil qiladi va u yagona yechimga ega bo'ladi. Bu yechim Kramer formulalari yordamida quyidagicha topiladi:

$$x_1=\frac{\Delta x_1}{\Delta}=\frac{\begin{vmatrix}7 & 1 & -1\\ -5 & 2 & 3\\ 1 & 3 & -2\end{vmatrix}}{27}=-\frac{81}{27}=-3,$$

$$x_2=\frac{\Delta x_2}{\Delta}=\frac{\begin{vmatrix}-2 & 7 & -1\\ 4 & -5 & 3\\ 1 & 1 & -2\end{vmatrix}}{27}=\frac{54}{27}=2,$$

$$x_3=\frac{\Delta x_3}{\Delta}=\frac{\begin{vmatrix}-2 & 1 & 7\\ 4 & 2 & -5\\ 1 & 3 & 1\end{vmatrix}}{27}=\frac{27}{27}=1.$$

Demak, sistemaning yechimi: (-3; 2; 1).

2.4. Talabaning mustaqil ishi

Topshiriq

1-misolda chiziqli tenglamalar sistemasini Kramer, teskari matritsa va Gauss - Jordan metodida yeching.

2-misolda chiziqli tenglamalar sistemasini Gauss metodida yeching.

3-misolda berilgan chiziqli tenglamalar sistemasining birgalikda yoki birgalikda emasligini tekshiring, birgalikda bo'lgan sistema uchun umumiy va bitta xususiy yechimini toping.

1-variant

1. $\begin{cases}x_1+3x_2-5x_3=-1\\ 2x_1-x_2+3x_3=4\\ 3x_1+2x_2-5x_3=0\end{cases}$

2. $\begin{cases} x_1 - 3x_2 - 5x_3 + x_4 = 3 \\ -5x_1 + 7x_2 + x_3 + 11x_4 = 65 \\ 2x_1 - 6x_2 + 3x_3 + 12x_4 = 4 \\ 3x_1 - 5x_2 - 4x_3 - 3x_4 = -17 \end{cases}$

3. $\begin{cases} x_1 + x_2 + x_3 = 3 \\ 2x_1 + 2x_2 + 2x_3 = 6 \end{cases}$

2-variant

1. $\begin{cases} x_1 + 2x_2 + x_3 = 8 \\ -2x_1 + 3x_2 - 3x_3 = -5 \\ 3x_1 - 4x_2 + 5x_3 = 10 \end{cases}$

2. $\begin{cases} 5x_1 + 4x_3 + 2x_4 = 3 \\ x_1 - x_2 + 2x_3 + x_4 = 1 \\ 4x_1 + x_2 + 2x_3 = 1 \\ x_1 + x_2 + x_3 + x_4 = 0 \end{cases}$

3. $\begin{cases} -5x_1 - 6x_2 + 2x_3 - 6x_4 = 4 \\ -x_1 + 3x_2 + 3x_3 - 8x_4 = -5 \\ 3x_1 + 7x_2 + x_3 - 2x_4 = -7 \\ -x_1 - 2x_2 = 2. \end{cases}$

3-variant

1. $\begin{cases} 3x_1 + x_2 = -9 \\ x_1 - 2x_2 - x_3 = 5 \\ 3x_1 + 44x_2 - 2x_3 = 13 \end{cases}$

2. $\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 = 5 \\ x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ x_1 + 3x_3 + 4x_4 = 2 \\ x_1 + x_2 + 5x_3 + 6x_4 = 1 \end{cases}$

3. $\begin{cases} -x_1 - 2x_2 - 6x_3 + 3x_4 = -1 \\ 2x_1 + 5x_2 + 14x_3 - 7x_4 = 3 \\ 3x_1 + 7x_2 + 20x_3 - 10x_4 = 4 \\ -x_2 - 2x_3 + x_4 = -1. \end{cases}$

4-variant

1. $\begin{cases} 2x_1 + 3x_2 + 2x_3 = 7, \\ 3x_1 - 5x_2 + 2x_3 = 0, \\ x_1 - 5x_2 + 3x_3 = -1. \end{cases}$

2. $\begin{cases} 2x_2 - x_3 + 2x_4 = -3, \\ x_1 + x_2 + 3x_3 = 10 \\ -2x_1 + x_2 - 3x_3 + 2x_4 = -12 \\ 3x_1 + 2x_2 - x_4 = 3 \end{cases}$

3. $\begin{cases} x_1 - 2x_2 + x_3 - x_4 + 3x_5 = 2 \\ 2x_1 - 4x_2 + 3x_3 - 2x_4 + 6x_5 = 5 \\ 3x_1 - 6x_2 + 4x_3 - 3x_4 + 9x_5 = 7 \end{cases}$

5-variant

1. $\begin{cases} 2x_1 + x_2 = 5, \\ x_1 + 3x_3 = 16, \\ 5x_2 - x_3 = 10. \end{cases}$

2. $\begin{cases} x_1 + 2x_2 - x_3 + x_4 = 1 \\ 3x_1 - x_2 + 2x_3 - x_4 = -1 \\ 2x_1 - 2x_2 + 3x_3 = 5 \\ 2x_1 + 3x_2 - 2x_3 + x_4 = -3. \end{cases}$

3. $\begin{cases} 3x_1 + 4x_2 + x_3 + 2x_4 = 3 \\ 6x_1 + 8x_2 + 2x_3 + 5x_4 = 7 \\ 9x_1 + 12x_2 + 3x_3 + 10x_4 = 13 \end{cases}$

6-variant

1. $\begin{cases} x_1 + x_2 - 2x_3 = 6 \\ 2x_1 + 3x_2 - 7x_3 = 16 \\ 5x_1 + 2x_2 + x_3 = 16 \end{cases}$

2. $\begin{cases} x_1 + x_2 + x_3 + x_4 = 10 \\ x_1 + x_2 - x_3 - x_4 = -4 \\ x_1 - x_2 + x_3 - x_4 = -2 \end{cases}$

3. $\begin{cases} 3x_1 + 2x_2 + x_3 = 5 \\ 2x_1 + 3x_2 + x_3 = 1 \\ 2x_1 + x_2 + 3x_3 = 11 \\ 3x_1 + 4x_2 - x_3 = -5 \end{cases}$

7-variant

1. $\begin{cases} 5x_1 + 8x_2 + x_3 = 2 \\ 3x_1 - 2x_2 + 6x_3 = -7 \\ 2x_1 + x_2 - x_3 = -5 \end{cases}$

2. $\begin{cases} x_1 + 3x_2 + 4x_3 - 2x_4 = 2 \\ -3x_1 - 7x_2 - 8x_3 + 2x_4 = -4 \\ 2x_1 - x_2 + 3x_3 = 4 \\ 2x_1 + 4x_2 + 4x_3 = 3 \end{cases}$

3. $\begin{cases} 3x_1 + x_2 - 5x_3 = 0 \\ x_1 - 2x_2 - x_3 = 0 \\ 2x_1 + 3x_2 - 4x_3 = 0 \\ x_1 + 5x_2 - 3x_3 = 0 \end{cases}$

8-variant

1. $\begin{cases} x_1 + 2x_2 + 3x_3 = 5 \\ 4x_1 + 5x_2 + 6x_3 = 8 \\ 7x_1 + 8x_2 = 2. \end{cases}$

2. $\begin{cases} x_2 + 3x_3 - x_4 = 10 \\ x_1 + 3x_2 + 8x_3 - x_4 = 22 \\ 4x_1 + 2x_2 - 3x_4 = 11 \end{cases}$

3. $\begin{cases} 3x_1 - x_2 = 5 \\ 2x_1 + 3x_2 = 4 \\ x_1 + \frac{1}{3}x_2 = \frac{5}{3} \\ x_1 + 1{,}5x_2 = 2 \end{cases}$

9-variant

1. $\begin{cases} 2x_1 - 3x_2 + x_3 = -7 \\ x_1 + 2x_2 - 3x_3 = 14 \\ -x_1 - x_2 + 5x_3 = -18 \end{cases}$

2. $\begin{cases} 6x_1 - 5x_2 + 4x_3 + 7x_4 = 28 \\ 5x_1 - 8x_2 + 5x_3 + 8x_4 = 36 \\ 9x_1 - 8x_2 + 5x_3 + 10x_4 = 42 \\ 3x_1 + 2x_2 + 2x_3 + 2x_4 = 2. \end{cases}$

3. $\begin{cases} -x_1 + x_2 - 3x_3 = 5 \\ 3x_1 - x_2 - x_3 = 2 \\ 2x_1 + x_2 - 9x_3 = 0 \end{cases}$

10-variant

1. $\begin{cases} 2x_1 + 3x_2 + 2x_3 = 7, \\ 3x_1 - 5x_2 + 2x_3 = 0, \\ x_1 - 5x_2 + 3x_3 = -1. \end{cases}$

2. $\begin{cases} x + 2y - 3z - t = 10, \\ -2x - 3y + 7z = -23, \\ 2x + 6y - 5z - 5t = 18 \\ -x + 3z - 4t = -11 \end{cases}$

3. $$\begin{cases} 2\sqrt{5}x_1 - x_2 + \sqrt{5}x_3 = 1 \\ 10x_1 - \sqrt{5}x_2 + 5x_3 = \sqrt{5} \\ -2x_1 + \dfrac{\sqrt{5}}{5}x_2 - x_3 = -\dfrac{1}{\sqrt{5}}. \end{cases}$$

11-variant

1. $$\begin{cases} x_1 + 2x_2 + 3x_3 = 3 \\ 2x_1 + 6x_2 + 4x_3 = 6 \\ 3x_1 + 10x_2 + 8x_3 = 21 \end{cases}$$

2. $$\begin{cases} 2x_1 + 6x_2 + x_3 = 0 \\ x_1 + 2x_2 - 2x_3 + 4x_4 = 0 \\ -x_1 + 4x_2 + 5x_3 - 4x_4 = 0 \\ 3x_1 + x_3 + 2x_4 = 0. \end{cases}$$

3. $$\begin{cases} x_1 + 2x_2 + 3x_3 + x_4 = 1, \\ 3x_1 + 13x_2 + 13x_3 + 5x_4 = 3, \\ 3x_1 + 7x_2 + 7x_3 + 2x_4 = 12, \\ x_1 + 5x_2 + 3x_3 + x_4 = 7, \\ 4x_1 + 5x_2 + 6x_3 + x_4 = 19. \end{cases}$$

12-variant

1. $$\begin{cases} x_1 + 2x_2 + 3x_3 = 8 \\ 4x_1 + 5x_2 + 6x_3 = 19 \\ 7x_1 + 8x_2 = 1 \end{cases}$$

2. $$\begin{cases} 3x_1 - 5x_2 + 2x_3 - 4x_4 = 0 \\ -3x_1 + 4x_2 - 5x_3 + 3x_4 = -2 \\ -5x_1 + 7x_2 - 7x_3 + 5x_4 = -2 \\ 8x_1 - 8x_2 + 5x_3 - 6x_4 = -5. \end{cases}$$

3. $$\begin{cases} x_1 + 2x_2 = 3 \\ -2x_1 + 3x_2 = 0 \\ -2x_1 - 4x_2 = 1 \end{cases}$$

13-variant

1. $$\begin{cases} x_1 + 2x_2 + 3x_3 = 4 \\ 2x_1 + 6x_2 + 4x_3 = -6 \\ 3x_1 + 10x_2 + 8x_3 = -8 \end{cases}$$

2. $$\begin{cases} x_1 + 2x_2 - 4x_3 = 1, \\ 2x_1 + x_2 - 5x_3 = -1, \\ x_1 - x_2 - x_3 = -2. \end{cases}$$

3. $\begin{cases} x_1 - \sqrt{3}x_2 = 1 \\ \sqrt{3}x_1 - 3x_2 = \sqrt{3} \\ -\dfrac{\sqrt{3}}{3}x_1 + x_2 = -\dfrac{\sqrt{3}}{3}. \end{cases}$

14-variant

1. $\begin{cases} 3x_1 - 2x_2 + x_3 = -10, \\ 2x_1 + 3x_2 - 4x_3 = 16, \\ x_1 - 4x_2 + 3x_3 = -18. \end{cases}$

2. $\begin{cases} 2x_2 - x_3 + 2x_4 = -3, \\ x_1 + x_2 + 3x_3 = 10, \\ -2x_1 + x_2 - 3x_3 + 2x_4 = -12, \\ 3x_1 + 2x_2 - x_4 = 3. \end{cases}$

3. $\begin{cases} 3x_1 + 4x_2 + 2x_3 = 8 \\ 2x_1 - 4x_2 - 3x_3 = -1 \\ x_1 + 5x_2 + x_3 = 0 \end{cases}$

15-variant

1. $\begin{cases} 3x_1 + 2x_2 + x_3 = -8 \\ 2x_1 + 3x_2 + x_3 = -3 \\ 2x_1 + x_2 + 3x_3 = -1 \end{cases}$

2. $\begin{cases} 3x_1 + 2x_2 + 2x_3 + 2x_4 = 9, \\ 9x_1 - 8x_2 + 5x_3 + 10x_4 = 16, \\ 5x_1 - 8x_2 + 5x_3 + 8x_4 = 10, \\ 6x_1 - 5x_2 + 4x_3 + 7x_4 = 12. \end{cases}$

3. $\begin{cases} 2x_1 - x_2 - x_3 = 0 \\ 3x_1 + 4x_2 - 2x_3 = 0 \\ 3x_1 - 2x_2 + 4x_3 = 0 \end{cases}$

16-variant

1. $\begin{cases} 2x_1 - 3x_2 - x_3 = -6 \\ 3x_1 + 4x_2 + 3x_3 = -5 \\ x_1 + x_2 + x_3 = -2 \end{cases}$

2. $\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 = 0, \\ 7x_1 + 14x_2 + 20x_3 + 27x_4 = 0, \\ 5x_1 + 10x_2 + 16x_3 + 19x_4 = -2, \\ 3x_1 + 5x_2 + 6x_3 + 13x_4 = 5. \end{cases}$

3. $\begin{cases} 8x_1 + 6x_2 + 5x_3 + 2x_4 = 21 \\ 3x_1 + 3x_2 + 2x_3 + x_4 = 10 \\ 4x_1 + 2x_2 + 3x_3 + x_4 = 8 \\ 3x_1 + 5x_2 + x_3 + x_4 = 15 \\ 7x_1 + 4x_2 + 5x_3 + 2x_4 = 18. \end{cases}$

17-variant

1. $\begin{cases} 2x_1 + 2x_2 - x_3 = 4 \\ 3x_2 + 4x_3 = -5 \\ x_1 + x_3 = -2 \end{cases}$

2. $\begin{cases} 2x_1 + 3x_2 - x_3 + x_4 = -3, \\ 3x_1 - x_2 + 2x_3 + 4x_4 = 8, \\ x_1 + x_2 + 3x_3 - 2x_4 = 6, \\ -x_1 + 2x_2 + 3x_3 + 5x_4 = 3. \end{cases}$

3. $\begin{cases} x_1 + 2x_2 + 3x_3 - x_4 = 8 \\ 2x_1 - x_2 - 4x_3 + 3x_4 = 1 \\ 4x_1 - 7x_2 - 18x_3 + 11x_4 = -13 \\ 3x_1 + x_2 - x_3 + 2x_4 = 9 \end{cases}$

18-variant

1. $\begin{cases} x_1 + 2x_2 + 3x_3 = 6, \\ 2x_1 + 3x_2 - x_3 = 4, \\ 3x_1 + x_2 - 4x_3 = 0. \end{cases}$

2. $\begin{cases} 2x_1 + 2x_2 + 11x_3 + 5x_4 = 2, \\ x_1 - x_2 + 5x_3 + 2x_4 = 1, \\ 2x_1 + 2x_2 + 3x_3 + 2x_4 = -3, \\ x_1 + 3x_2 + 3x_3 + 4x_4 = -3. \end{cases}$

3. $\begin{cases} 2x_1 - x_2 + 3x_3 - 5x_4 = 1 \\ x_1 - x_2 - 5x_3 = 2 \\ 3x_1 - 2x_2 - 2x_3 - 5x_4 = 3 \\ 7x_1 - 5x_2 - 9x_3 + 10x_4 = 8 \end{cases}$

19-variant

1. $\begin{cases} 2x_1 + 2x_2 - x_3 = 5, \\ 4x_1 + 3x_2 - x_3 = 8, \\ 8x_1 + 5x_2 - 3x_3 = 16. \end{cases}$

2. $\begin{cases} 2x_1 + 3x_2 - 3x_3 + x_4 = 0, \\ 3x_1 - 2x_2 + 4x_3 - 2x_4 = 3, \\ 2x_1 + x_2 + 3x_4 = 4 \\ 3x_1 + 3x_2 - 4x_3 + 2x_4 = 2. \end{cases}$

3. $\begin{cases} 3x_1 - x_2 + 2x_3 = 2 \\ 4x_1 - x_2 + 3x_3 = 3 \\ x_1 + 3x_2 = 0 \\ 5x_1 + 3x_3 = 3 \end{cases}$

20-variant

1. $\begin{cases} 2x_1 - x_2 + 3x_3 = 3, \\ 3x_1 + 3x_2 - x_3 = 8, \\ 8x_1 + 5x_2 + x_3 = 16. \end{cases}$

2. $\begin{cases} 2x_1 + 4x_2 + 4x_3 + 6x_4 = 18, \\ 4x_1 + 2x_2 + 5x_3 + 7x_4 = 24, \\ 3x_1 + 2x_2 + 8x_3 + 5x_4 = 13, \\ 2x_1 + 8x_2 + 7x_3 + 3x_4 = 6. \end{cases}$

3. $\begin{cases} 2x_1 - 3x_2 = -2 \\ x_1 + 2x_2 = 2{,}5 \\ -2x_1 - 4x_2 = -5 \\ 2\sqrt{3}x_1 - 3\sqrt{3}x_2 = -2\sqrt{3} \end{cases}$

21-variant

1. $\begin{cases} 4x_1 - 2x_2 - 5x_3 + x_4 = 2, \\ 3x_1 - 3x_2 + x_3 + 5x_4 = -4, \\ 2x_1 + 2x_2 - 4x_4 = -4, \\ 2x_1 - x_2 - 4x_3 + 9x_4 = 21. \end{cases}$

2. $\begin{cases} 2x_1 + x_2 + x_3 = 4, \\ x_1 - x_2 + x_3 = 0, \\ 3x_1 + x_2 + 2x_3 = 5. \end{cases}$

3. $\begin{cases} 4x_1 - 3x_2 + 2x_3 = 21 \\ 2x_1 + 5x_2 - 3x_3 = 4 \\ 5x_1 + 6x_2 - 2x_3 = 18 \end{cases}$

22-variant

1. $\begin{cases} x_1 + x_2 - 6x_3 - 4x_4 = 6, \\ 2x_1 - x_2 - 6x_3 - 4x_4 = 2, \\ 4x_1 + 3x_2 + 9x_3 + 2x_4 = 6, \\ 5x_1 + 2x_2 + 3x_3 + 8x_4 = -7. \end{cases}$

2. $\begin{cases} x_1 + 2x_2 + 3x_3 = 0, \\ 2x_1 + 4x_2 + 5x_3 = -1, \\ 3x_1 + 5x_2 + 6x_3 = 1. \end{cases}$

3. $\begin{cases} 3x_1 - x_2 + 2x_3 = 0 \\ 4x_1 - 3x_2 + 3x_3 = 0 \\ x_1 + 3x_2 = 0 \end{cases}$

23-variant

1. $\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 = 0, \\ 7x_1 + 14x_2 + 20x_3 + 27x_4 = 0, \\ 5x_1 + 10x_2 + 16x_3 + 19x_4 = -2, \\ 3x_1 + 5x_2 + 6x_3 + 13x_4 = 5. \end{cases}$

2. $\begin{cases} x_1 + 2x_2 + 3x_3 = 2, \\ 2x_1 + x_2 + 2x_3 = 2, \\ 3x_1 + 2x_2 + 4x_3 = 3, \\ x_1 + 3x_2 + 4x_3 = -3. \end{cases}$

3. $\begin{cases} x_1 + x_2 - x_3 = 0 \\ 8x_1 + 3x_2 - 6x_3 = 0 \\ 4x_1 - x_2 + 3x_3 = 0 \end{cases}$

24-variant

1. $\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 = 0, \\ 2x_1 + 3x_2 + 4x_3 - 5x_4 = -12, \\ 3x_1 + 4x_2 - 5x_3 - 6x_4 = 4, \\ 4x_1 - 5x_2 - 6x_3 - 7x_4 = -26. \end{cases}$

2. $\begin{cases} x_1 + 2x_2 - x_3 + x_4 = 3, \\ x_1 + x_2 + x_4 = 3, \\ 2x_1 - x_2 + x_3 = 2, \\ 3x_1 + x_3 + 2x_4 = 6. \end{cases}$

3. $\begin{cases} x_1 + x_2 + x_3 = 3, \\ 2x_1 - x_2 + x_3 = 2, \\ x_1 + 4x_2 + 2x_3 = 5. \end{cases}$

25-variant

1. $\begin{cases} x_1 - 2x_2 + 3x_3 - 4x_4 = 29, \\ 2x_1 - 3x_2 + 4x_3 - 5x_4 = 39, \\ 3x_1 + x_2 - 5x_3 - x_4 = -6, \\ 4x_1 - 3x_2 + 6x_3 - x_4 = 33. \end{cases}$

2. $\begin{cases} x_1 + 2x_2 + x_3 = 8, \\ x_2 + 3x_3 + x_4 = 15, \\ 4x_1 + x_3 + x_4 = 11, \\ x_1 + x_2 + 5x_4 = 23. \end{cases}$

3. $\begin{cases} x_1 + x_2 - x_3 = -4, \\ x_1 + 2x_2 - 3x_3 = 0, \\ -2x_1 - 2x_3 = 3. \end{cases}$

2.5. Mathcad dasturida hisoblash

Mathcad dasturida chiziqli tenglamalar sistemasini yechishni ikki variantda amalga oshirish mumkin.

1. Given/Find hisoblash bloki (taqribiy iteratsion algoritm);
2. lsolve o‘rnatish funksiyasi (Gauss algoritmi).

lsolve o‘rnatish funksiyasini qo‘llab chiziqli tenglamalar sistemasini yechish uchun sistema $Ax = b$ matritsali ko‘rinishda yoziladi.

$$A := \begin{pmatrix} 6 & 4 & 5 \\ 4 & 3 & 1 \\ 5 & 2 & 3 \end{pmatrix} \quad b := \begin{pmatrix} 2400 \\ 1450 \\ 1550 \end{pmatrix}$$

$$\text{lsolve}(A, b) = \begin{pmatrix} 150 \\ 250 \\ 100 \end{pmatrix}$$

Tekshirish

$$A \cdot \text{lsolve}(A, b) - b = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

1- misol. Korxona xom ashyoning uch turini qo‘llab, mahsulotning uch turini ishlab chiqaradi. Ishlab chiqarishning zaruriy xarakteristikalari jadvalda ko‘rsatilgan. Xom ashyoning berilgan zaxiralarida mahsulotning har bir turini ishlab chiqarish hajmini aniqlash talab etiladi.

Xomashyo turi	**Mahsulot turlari uchun xomashyo sarfi**			**Xom ashyo hajmi**
	1	**2**	**3**	
1	6	4	5	2400
2	4	3	1	1450
3	5	2	3	1550

Yechish. Mahsulotni ishlab chiqarish hajmlarini x_1, x_2, x_3 orqali belgilaymiz U holda hom ashyoning har bir turi uchun zahiralarining to'liq ishlatilishi shartida balans munosabatlarini yozish mumkin. Ular uch noma'lumli uchta tenglamalar sistemasini tashkil qiladi.

$$\begin{cases} 6x_1 + 4x_2 + 5x_3 = 2400 \\ 4x_1 + 3x_2 + x_3 = 1450 \\ 5x_1 + 2x_2 + 3x_3 = 1550 \end{cases}$$

Mathcad da chiziqli tenglamalar sistemasi Gauss ululida quyidagi tartibda yechiladi:

1. Berilgan sistemadagi noma'lumlar oldidagi koeffitsiyentlar va ozod hadlar matritsali ko'rinishda ifodalanadi.
2. augment (A,b) funksiyasi yordamida sistemaning kengaytirilgan matritsasi ifodalanadi.
3. rref(A) funksiyasidan foydalanib, kengaytirilgan matritsa pog'onasimon ko'rinishga keltiriladi.
4. Matritsaning oxirgi ustuni sistemaning yechimi sifatida olinadi.
5. $Ax-b$ hisoblashni bajarish. Natijada nol matritsa hosil bo'lsa, masala to'g'ri yechilgan bo'ladi.

ORIGIN := 1

$$A := \begin{pmatrix} 6 & 4 & 5 \\ 4 & 3 & 1 \\ 5 & 2 & 3 \end{pmatrix}$$ - noma'lumlar oldidagi koeffitsientlardan tuzilgan matritsa

$$b := \begin{pmatrix} 2400 \\ 1450 \\ 1550 \end{pmatrix}$$ - ozod hadlardan tuzilgan matritsa

P := augment(A ,b) $$P = \begin{pmatrix} 6 & 4 & 5 & 2.4 \times 10^3 \\ 4 & 3 & 1 & 1.45 \times 10^3 \\ 5 & 2 & 3 & 1.55 \times 10^3 \end{pmatrix}$$ - kengaytirilgan matritsa

$R := rref(P)$ $\quad R = \begin{pmatrix} 1 & 0 & 0 & 150 \\ 0 & 1 & 0 & 250 \\ 0 & 0 & 1 & 100 \end{pmatrix}$ - pog'onasimon matritsa

$n := cols(R)$

$x := R^{\langle n \rangle}$ $\quad x = \begin{pmatrix} 150 \\ 250 \\ 100 \end{pmatrix}$ - sistemaning yechimi

Tekshirish

$$A \cdot x - b = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

III BOB. CHIZIQLI FAZO

3.1. Arifmetik vektor fazo

1-ta'rif. n o'lchovli haqiqiy arifmetik fazo deb, mumkin bo'lgan barcha n ta haqiqiy sonlarning tartiblangan tizimlari to'plamiga aytiladi. R^n yozuv bilan belgilanadi.

Har bir alohida olingan $(a_1, a_2, ..., a_n)$ tizim R^n fazo arifmetik vektori deyiladi. $a_1, a_2, ..., a_n$ haqiqiy sonlarga $\bar{a}$ vektorning mos koordinatalari yoki komponentlari deyiladi. Tizim koordinatalari soni n esa $\bar{a}$ arifmetik vektor o'lchovi deyiladi.

2-ta'rif. Ikki n o'lchovli $\bar{a} = (a_1, a_2, ..., a_n)$ va $\bar{b} = (b_1, b_2, ..., b_n)$ arifmetik vektorlar berilgan bo'lsin. $\overline{a_i} = \overline{b_i}\,(i = 1, 2, ..., n)$ munosabatlar o'rinli, ya'ni vektorlarning har bir mos koordinatalari o'zaro teng bo'lsa, $\bar{a}$ va $\bar{b}$ vektorlarga o'zaro teng vektorlar deyiladi. $\bar{a}$ va $\bar{b}$ vektorlarning tengligi $\bar{a} = \bar{b}$ ko'rinishda yoziladi.

3-ta'rif. n ta nollardan iborat $(0, 0, ..., 0)$ tizimga n o'lchovli nol vektor deyiladi va $\bar{0}$ kabi belgilanadi.

$\bar{a}$ vektorning qarama-qarshi vektori deb, $-\bar{a} = (-1)\bar{a}$ vektorga aytiladi.

n o'lchovli arifmetik vektorlar ustida chiziqli amallar quyidagicha bajariladi:

1. Berilgan $\bar{a}$ va $\bar{b}$ vektorlarni qo'shganda ularning mos koordinatalari qo'shiladi:

$$\bar{a} + \bar{b} = (a_1 + b_1, a_2 + b_2, ..., a_n + b_n).$$

2. Berilgan $\bar{a}$ vektorni λ haqiqiy songa ko'paytirganda uning har bir koordinatasi λ marta ortadi:

$$\lambda\bar{a} = (\lambda a_1, \lambda a_2, ..., \lambda a_n).$$

Vektorlar ustida chiziqli amallar quyidagi xossalarga bo'ysunadi:

1) $\bar{a} + \bar{b} = \bar{b} + \bar{a}$,

2) $(\bar{a} + \bar{b}) + \bar{c} = \bar{a} + (\bar{b} + \bar{c})$,

3) $\bar{a} + \bar{0} = \bar{a}$

4) $\bar{a} + (-\bar{a}) = \bar{0}$,

5) $\lambda(\bar{a} + \bar{b}) = \lambda\bar{a} + \lambda\bar{b}$,

6) $(\lambda + \mu)\bar{a} = \lambda\bar{a} + \mu\bar{a}$,

7) $\lambda(\mu\vec{a})=(\lambda\mu)\vec{a}$,

8) $1\cdot\vec{a}=\vec{a}$

bu yerda $\vec{a},\vec{b}$ va $\vec{c}$ lar n o'lchovli vektorlar, λ va μ haqiqiy sonlar, $\vec{0}$ esa n o'lchovli nol vektor.

4-ta'rif. $\vec{a}=(a_1,a_2,...,a_n)$ va $\vec{b}=(b_1,b_2,...,b_n)$ arifmetik vektorlarning skalyar ko'paytmasi deb, vektorlar mos koordinatalari ko'paytmalarining yig'indisiga teng songa aytiladi.

$$(\vec{a},\vec{b})=(a_1+b_1,a_2+b_2,...,a_n+b_n).$$

Vektorlarning skalyar ko'paytmasi quyidagi xossalarga ega:

1) $(\vec{a},\vec{b})=(\vec{b},\vec{a})$.

2) $(\lambda\vec{a},\vec{b})=\lambda(\vec{b},\vec{a})$.

3) $(\vec{a}+\vec{b},\vec{c})=(\vec{a},\vec{c})+(\vec{b},\vec{c})$.

4) Agar $\vec{a}\neq\vec{0}$ bo'lsa $(\vec{a},\vec{a})\geq 0$, va agar $\vec{a}=\vec{0}$ bo'lsa $(\vec{a},\vec{a})=0$.

5-ta'rif. Berilgan $\vec{a}=(a_1,a_2,...,a_n)$ vektorning moduli yoki uzunligi (normasi) deb, quyidagi formula bo'yicha aniqlanadigan nomanfiy songa aytiladi:

$$|\vec{a}|=\sqrt{(\vec{a},\vec{a})}.$$

Ikkita n o'lchovli $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak deb

$$\cos\varphi=\frac{(\vec{a},\vec{b})}{|\vec{a}||\vec{b}|},\quad \varphi\in[0,\pi].$$

shartlarni qanoatlantiruvchi φ burchakka aytiladi, bunda $\vec{a}\neq\vec{0}$ va $\vec{b}\neq\vec{0}$.

Agar $(\vec{a},\vec{b})=0$ bo'lsa, $\vec{a}$ va $\vec{b}$ vektorlar ortogonal deyiladi.

Misollar

1-misol. $\vec{a}=(4;5;3;1)$ va $\vec{b}=(-3;2;0;3)$ vektorlar berilgan. $3\vec{a}+2(\vec{b}-\vec{a})+\vec{a}-(\vec{a}-2\vec{b})$ vektorni toping.

Yechish. Vektorlar ustida bajariladigan amallarning xossalaridan foydalanib ifodani soddalashtiramiz:

$$3\vec{a}+2\left(\vec{b}-\vec{a}\right)+\vec{a}-\left(\vec{a}-2\vec{b}\right)=3\vec{a}+2\vec{b}-2\vec{a}+\vec{a}-\vec{a}+2\vec{b}=\vec{a}+4\vec{b}.$$

$$\vec{a}+4\vec{b}=\left(4+4(-3);5+4\cdot 2;3+4\cdot 0;1+4\cdot 3\right)=\left(-8;13;3;13\right).$$

2-misol. $\vec{a}=(1;-1;-1;-1)$ va $\vec{b}=(-1;1;1;-1)$ vektorlar uzunliklari va ular orasidagi burchakni toping.

Yechish. Vektorlar uzunliklarini va ularning skalyar ko'paytmasini topamiz:

$$\left|\vec{a}\right|=\sqrt{\left(\vec{a},\vec{a}\right)}=\sqrt{1^2+(-1)^2+(-1)^2+(-1)^2}=2.$$

$$\left|\vec{b}\right|=\sqrt{\left(\vec{b},\vec{b}\right)}=\sqrt{(-1)^2+1^2+1^2+(-1)^2}=2.$$

$$\left(\vec{a},\vec{b}\right)=1\cdot(-1)+(-1)\cdot 1+(-1)\cdot 1+(-1)\cdot(-1)=-2.$$

$$\cos\varphi=\frac{\left(\vec{a},\vec{b}\right)}{\left|\vec{a}\right|\left|\vec{b}\right|}=\frac{-2}{2\cdot 2}=-\frac{1}{2}$$

va $\frac{2\pi}{3}$ ga teng.

3.2. Bir jinsli chiziqli tenglamalar sistemasining fundamental yechimlari tizimi

Ushbu

$$\begin{cases} a_{11}x_1+a_{12}x_2+....+a_{1n}x_n=0, \\ a_{21}x_1+a_{22}x_2+....+a_{2n}x_n=0, \\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots \\ a_{m1}x_1+a_{m2}x_2+...+a_{mn}x_n=0. \end{cases} \qquad (1)$$

ko'rinishdagi sistemaga n ta noma'lumli m ta bir jinsli chiziqli tenglamalar sistemasi deyiladi. Bu yerda a_{11}, a_{12}, ..., a_{mn} sonlar sistemaning koeffitsiyentlari, $x_1, x_2, \ldots, x_n$ lar noma'lumlar deyiladi. a_{ij} koeffitsiyentda birinchi indeks i tenglamaning nomerini, ikkinchi indeks j esa nomalumning nomerini bildiradi.

O'z-o'zidan ko'rinib turibdiki bir jinsli chiziqli tenglamalar sistemasi har doim birgalikda $(r(A)=r(B))$, ya'ni $x_1=x_2=...=x_n=0$ (trivial) yechimga ega.

Teorema. Bir jinsli chiziqli tenglamalar sistemasi nolmas yechimga ega bo'lishi uchun, uning asosiy matritsasining rangi r noma'lumlar soni n dan kichik bo'lishi zarur va yetarli, ya'ni $r < n$.

Faraz qilamiz, n noma'lumli n ta bir jinsli chiziqli tenglamalar sistemasi berilganbo'lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = 0, \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = 0, \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n = 0. \end{cases} \qquad (2)$$

Teorema. n noma'lumli n ta bir jinsli chiziqli tenglamalar sistemasi nolmas yechimga ega bo'lishi uchun, uning Δ determinanti nolga teng bo'lishi zarur va yetarli, ya'ni $\Delta = 0$.

Misol. Quyidagi bir jinsli chiziqli tenglamalar sistemasini yeching:

$$\begin{cases} x_1 - 2x_2 + 4x_3 = 0, \\ 2x_1 - 3x_2 + 5x_3 = 0. \end{cases}$$

Yechish. Asosiy matritsani tuzib olamiz va uning rangini topamiz:

$$A = \begin{pmatrix} 1 & -2 & 4 \\ 2 & -3 & 5 \end{pmatrix}, \qquad r(A) = 2 \quad \left(\Delta = \begin{vmatrix} 1 & -2 \\ 2 & -3 \end{vmatrix} = 1 \neq 0 \right), \qquad n = 3.$$

Sistema cheksiz ko'p yechimlar to'plamiga ega, chunki $r < n$. Shuning uchun uni quyidagicha yozib olamiz:

$$\begin{cases} x_1 - 2x_2 = -4x_3, \\ 2x_1 - 3x_2 = -5x_3. \end{cases}$$

Oxirgi sistemani Kramer usulida yechamiz:

$$\Delta_{x_1} = \begin{vmatrix} -4x_3 & -2 \\ -5x_3 & -3 \end{vmatrix} = 2x_3, \; \Delta_{x_2} = \begin{vmatrix} 1 & -4x_3 \\ 2 & -5x_3 \end{vmatrix} = 3x_3$$

bo'lganligi sababli, umumiy yechim

$$\begin{cases} x_1 = \dfrac{\Delta_{x_1}}{\Delta} \\ x_2 = \dfrac{\Delta_{x_2}}{\Delta} \end{cases} \Rightarrow \begin{cases} x_1 = 2x_3 \\ x_2 = 3x_3 \end{cases}$$

Umumiy yechimga $x_3 = 0$ qiymatni quyib $\begin{cases} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{cases}$ xususiy yechimga ega bo'lamiz. Shuningdek, $x_3 = 1$ deb olsak $\begin{cases} x_1 = 2 \\ x_2 = 3 \\ x_3 = 1 \end{cases}$ xususiy yechim hosil bo'ladi va hakozo.

(2) sistemaning $x_1 = k_1,\ x_2 = k_2, ..., x_n = k_n$ yechimlarini $e_1 = (k_1,\ k_2,\ ...,\ k_n)$ satr ko'rinishida yozib olamiz.

Bir jinsli chiziqli tenglamalar sistemasining yechimlari quyidagi xossalarga ega:

1) Agar $e_1 = (k_1,\ k_2, ..., k_n)$ satr (2) sistemaning yechimi bo'lsa, u holda $\lambda e_1 = (\lambda k_1,\ \lambda k_2,\ ...,\ \lambda k_n)$ satr ham shu sistemaning yechimi bo'ladi. Bu yerda λ ixtiyoriy son.

2) Agar $e_1 = (k_1, k_2,\ ...,\ k_n)$ va $e_2 = (l_1,\ l_2,\ ...,\ l_n)$ satrlar (1) sistemaning yechimlari bo'lsa, u holda ixtiyoriy c_1 va c_2 sonlar uchun ularning

$$c_1e_1 + c_2e_2 = (c_1k_1 + c_2l_1,\ c_1k_2 + c_2l_2, ...,\ c_1 k_n + c_2l_n)$$

chiziqli kombinatsiyasi ham shu sistemaning yechimi bo'ladi.

Bir jinsli bo'lmagan chiziqli tenglamalar sistemasi yechimlari uchun yuqorida keltirilgan da'volar o'rinli emas.

Yechimlarning keltirilgan xossalarini to'g'riligiga ishonch hosil qilish uchun ularni sistemaga qo'yish kifoya.

Keltirilgan xossalardan bir jinsli chiziqli tenglamalar sistemasi yechimlarining har qanday chiziqli kombinatsiyasi ham uning yechimi bo'la olishligi kelib chiqadi. Shuning uchun (2) sistemaning shunday chiziqli erkli yechimlar sistemasini topish masalasi qo'yiladiki, qolgan barcha yechimlar uning chiziqli kombinatsiyasidan iborat bo'lsin.

1-ta'rif. Agar (2) sistemaning barcha yechimlari $F_1,\ F_2,\ ...,\ F_k$ chiziqli erkli yechimlar sistemasining chiziqli kombinatsiyasidan iborat bo'lsa, u holda bunday $F_1,\ F_2, ..., F_k$ yechimlarga sistemaning **fundamental yechimlari** deyiladi.

Teorema. Agar (2) bir jinsli chiziqli tenglamalar sistemasining rangi r ga teng bo'lib, noma'lumlar soni n dan kichik bo'lsa, u holda (2) sistemaning har qanday fundamental yechimlar sistemasi $n-r$ ta yechimdan iborat bo'ladi.

(2) bir jinsli chiziqli tenglamalar sistemasining umumiy yechimi quyidagi ko'rinishda bo'ladi:

$$\lambda_1 F_1 + \lambda_2 F_2 + ... + F_k \lambda_k .$$

Bunda $F_1, F_2, ..., F_k$ –biror bir fundamental yechimlar sistemasi, $\lambda_1, \lambda_2, ..., \lambda_k$ - ixtiyoriy sonlar va $k = n-r$.

Bir jinsli chiziqli tenglamalar sistemasining fundamental yechimlar sistemasi quyidagicha quriladi:

1. Bir jinsli sistemaning umumiy yechimi topiladi;

2. $n-r$ o'lchovli $n-r$ ta chiziqli erkli satr matritsa tanlaniladi. Masalan, har biri $n-r$ o'lchovli

$$\begin{cases} e_1 \ (1,0,...,0) \\ e_2 \ (0,1,...,0) \\ .... \\ e_{n-r} (0,0,...,1) \end{cases}$$

satr matritsalar sistemani tanlash mumkin;

3. Umumiy yechimni topish uchun erkli noma'lumlari o'rniga, masalan, e_1 satr matritsaning mos elementlari qo'yilib, bazis noma'lumlar aniqlanadi va F_1 fundamental yechim quriladi. Xuddi shunday usulda $e_2, e_3, ..., e_{n-r}$ satr matritsalardan foydalanib, mos ravishda $F_2, F_3, ..., F_{n-r}$ fundamental yechimlar quriladi.

1-misol. Quyidagi

$$\begin{cases} 3x_1 + x_2 - 8x_3 + 2x_4 + x_5 = 0 \\ 2x_1 - 2x_2 - 3x_3 - 7x_4 + 2x_5 = 0 \\ x_1 - 5x_2 + 2x_3 - 16x_4 + 3x_5 = 0 \\ x_1 + 11x_2 - 12x_3 + 34x_4 - 5x_5 = 0 \end{cases}$$

chiziqli tenglamalar sistemasining fundamental yechimlar sistemasini toping.

Yechish. Bu sistemada $r = 2$, $n = 5$. Demak, sistemaning har qanday fundamental yechimlar sistemasi $n-r = 3$ ta yechimdan iborat bo'ladi. Bu yerda x_3, x_4, x_5 noma'lumlarni erkli noma'lumlar deb hisoblab sistemani yechamiz va quyidagi umumiy yechimni hosil qilamiz:

$$\begin{cases} x_1 = \frac{19}{8}x_3 + \frac{3}{8}x_4 - \frac{1}{2}x_5 \\ x_2 = \frac{7}{8}x_3 + \frac{25}{8}x_4 + \frac{1}{5}x_5 \end{cases}$$

So'ngra uchta chiziqli erkli uch o'lchovli satr matritsa tanlaymiz: $e_1(1,0,0)$, $e_2(0,1,0)$, $e_3(0,0,1)$. Bularni har birining koordinatalarini umumiy yechimdagi erkli noma'lumlar o'rniga mos ravishda qo'yilib x_1, x_2 larning qiymatlari topiladi. Bu esa berilgan tenglamalar sistemasining izlangan fundamental yechimlar sistemasini biridan iborat,ya'ni

$$F_1 = \begin{pmatrix} \frac{19}{8} \\ \frac{7}{8} \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad F_2 = \begin{pmatrix} \frac{3}{5} \\ -\frac{25}{8} \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad F_3 = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

Berilgan bir jinsli chiziqli tenglamalar sistemaning umumiy yechimini quyidagi ko'rinishda yozish mumkin:

$$X = \lambda_1 F_1 + \lambda_2 F_2 + \lambda_3 F_3 = \lambda_1 \begin{pmatrix} \frac{19}{8} \\ \frac{7}{8} \\ 1 \\ 0 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} \frac{3}{5} \\ -\frac{25}{8} \\ 0 \\ 1 \\ 0 \end{pmatrix} + \lambda_3 \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

Bu yerda, F_2, F_3, F_3 – berilgan sistemaning fundamental yechimlari sistemasi, λ_1, λ_2, λ_3 – ixtiyoriy haqiqiy sonlar.

Bir jinsli bo'lmagan chiziqli tenglamalar sistemasi berilgan bo'lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2, \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_n. \end{cases} \qquad (3)$$

(3) sistemadagi ozod hadlarini nol bilan almashtirishdan hosil bo'lgan (1) bir jinsli chiziqli tenglamalar sistemasi (3) sistema uchun **keltirilgan**

sistema deyiladi. (1) va (3) sistemalarning yechimlari orasida muhim bog'lanish mavjud bo'lib, uni quyidagi teorema ko'rsatadi:

Teorema. Bir jinsli bo'lmagan chiziqli tenglamalar sistemasining umumiy yechimi, unga mos keltirilgan sistema fundamental yechimi bilan berilgan sistemaning xususiy yechimi yig'indisiga teng, ya'ni

$$X_{у.мў.ч.} = X_{хуcус} + \lambda_1 F_1 + \lambda_2 F_2 + \ldots + F_k \lambda_k .$$

Bu teoremani quyidagi masalada qaraymiz.

2-misol. Quyidagi bir jinsli bo'lmagan

$$\begin{cases} 3x_1 + x_2 - 8x_3 + 2x_4 + x_5 = 7 \\ 2x_1 - 2x_2 - 3x_3 - 7x_4 + 2x_5 = 2 \\ x_1 - 5x_2 + 2x_3 - 16x_4 + 3x_5 = -3 \\ x_1 + 11x_2 - 12x_3 + 34x_4 - 5x_5 = 13 \end{cases}$$

chiziqli tenglamalar sistemasining umumiy yechimlari topilsin.

Yechish. Berilgan bir jinsli bo'lmagan chiziqli tenglamalar sistemasiga mos keltirilgan sistemaning fundamental yechimi (1-misol) quyidagicha:

$$X = \lambda_1 F_1 + \lambda_2 F_2 + \lambda_3 F_3 = \lambda_1 \begin{pmatrix} \frac{19}{8} \\ \frac{7}{8} \\ 1 \\ 0 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} \frac{3}{5} \\ -\frac{25}{8} \\ 0 \\ 1 \\ 0 \end{pmatrix} + \lambda_3 \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

Endi berilgan bir jinsli bo'lmagan chiziqli tenglamalar sistemasining ixtiyoriy xususiy yechimini topamiz. Buning uchun $x_3 = x_4 = x_5 = 0$ deb olsak, $x_1 = 2$ va $x_2 = 1$ yechimga ega bo'lamiz. Shunday qilib, bir jinsli bo'lmagan chiziqli tenglamalar sistemasining umumiy yechimi

$$X_{у.мў.ч.} = X_{хуcус.} + \lambda_1 F_1 + \lambda_2 F_2 + \lambda_3 F_3 = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \lambda_1 \begin{pmatrix} \frac{19}{8} \\ \frac{7}{8} \\ 1 \\ 0 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} \frac{3}{5} \\ -\frac{25}{8} \\ 0 \\ 1 \\ 0 \end{pmatrix} + \lambda_3 \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

3.3. Chiziqli fazo

1-ta'rif. Agar elementlari ixtiyoriy tabiatli bo'lgan L to'plam berilgan va bu toplam elementlari orasida qo'shish va songa ko'paytirish amallari kiritilgan, ya'ni

1) ixtiyoriy $x \in L$ va $y \in L$ elementlar juftiga x va y elementlarning yig'indisi, deb ataluvchi yagona $z = x + y \in L$ element mos qo'yilgan;

2) $x \in L$ element va $\lambda \in K$ (K-haqiqiy yoki kompleks sonlar to'plami) songa x vektorning λ songa ko'paytmasi deb ataluvchi yagona $z = \lambda x \in L$ element mos qo'yilgan bo'lib, aniqlangan bu qo'shish va songa ko'paytirish amallari quyidagi 8 ta aksiomani bajarsa, u holda L to'plam chiziqli (yoki vektor) fazo deyiladi:

1. Qo'shish kommutativ, $x + y = y + x$;

2. Qo'shish assotsiativ, $(x + y) + z = x + (y + x)$;

3. L to'plamda barcha x elementlar uchun $x + \theta = x$ shartni qanoatlantiradigan nol element θ mavjud;

4. L to'plamda har qanday x element uchun $x + (-x) = \theta$ shartni qanoatlantiradigan $-x$ qarama-qarshi element mavjud;

5. $\alpha(x + y) = \alpha x + \alpha y$;

6. $(\alpha + \beta)x = \alpha x + \beta x$;

7. $\alpha(\beta x) = (\alpha\beta)x$;

8. $1 \cdot x = x$.

Bundan keyin biz chiziqli fazo elementlarini vektorlar deb aytamiz.

Chiziqli fazoni aniqlovchi aksiomalardan, quyidagi xossalarni ajratish mumkin:

1) Har qanday chiziqli fazo uchun yagona θ-nol vektor mavjud.

2) Har qanday chiziqli fazoda har bir x vektor uchun unga qarama-qarshi bo'lgan yagona $(-x)$ vektor mavjud.

3) Har qanday chiziqli fazoda har bir vektor uchun $0 \cdot x = 0$ tenglik o'rinli.

Izoh. $y - x$ vektorlar ayirmasi deb, y va $-x$ vektorlar yig'indisi tushuniladi.

2-ta'rif. L chiziqli fazodan olingan $x_1, x_2, \ldots, x_n$ elementlar va $\lambda_i \in R, (i = 1 \ldots n)$ sonlar yordamida qurilgan $\lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + \ldots + \lambda_n x_n$ ifodaga $x_1, x_2, \ldots, x_n$ - elementlarning chiziqli kombinatsiyasi deyiladi.

3-ta'rif. Agar $y=\lambda_1 x_1+\lambda_2 x_2+\cdots+\lambda_n x_n$ tenglik o'rinli bo'lsa, u holda y element $x_1,x_2,...,x_n$ elementlarning chiziqli kombinatsiyasidan iborat deyiladi.

4-ta'rif. Agar $\lambda_1,\lambda_2,...,\lambda_n$ koeffitsiyentlardan hech bo'lmaganda bittasi noldan farqli bo'lganda

$$\lambda_1 x_1+\lambda_2 x_2+\cdots+\lambda_n x_n=\theta$$

tenglik o'rinli bo'lsa, u holda $x_1,x_2,...,x_n$ elementlar chiziqli bog'liq deyiladi.

Agar

$$\lambda_1 x_1+\lambda_2 x_2+\cdots+\lambda_n x_n=\theta$$

tenglik $\lambda_1,\lambda_2,...,\lambda_n$ koeffitsiyentlardan barchasi nolga teng bo'lgandagina o'rinli bo'lsa, u holda $x_1,x_2,...,x_n$ - elementlar chiziqli erkli deyiladi. Bu yerda, θ -chiziqli fazoning nol elementi.

5-ta'rif. Agar L chiziqli fazoda n ta chiziqli erkli elementlar mavjud bo'lib, har qanday $n+1$ ta element chiziqli bog'liq bo'lsa, u holda L chiziqli fazoning o'lchovi n ga teng deyiladi.

6-ta'rif. n o'lchovli L chiziqli fazoda har qanday n ta chiziqli erkli vektorlar sistemasi bu fazoning bazisi deyiladi.

Odatda bazis vektorlar sistemasi $e_1, e_2,...,e_n$ kabi belgilanadi.

Teorema. n o'lchovli L chiziqli fazoning har bir elementi bazis vektorlarining chiziqli kombinatsiyasi ko'rinishida bir qiymatli yoziladi.

$x=\mu_1 e_1+\mu_2 e_2+...+\mu_n e_n$ tenglik $x\in L$ elementning $\{\ e_1,e_2,...,e_n\ \}$ bazis vektorlari bo'yicha yoyilmasi deyiladi, $\lambda_1,\lambda_2,...,\lambda_n$ sonlarga esa x elementning bu bazis vektorlar bo'yicha koordinatalari deyiladi.

7-ta'rif. Agar chiziqli fazo cheksiz sondagi chiziqli erkli vektorlar sistemasiga ega bo'lsa, u holda bunday chiziqli fazoga cheksiz o'lchovli chiziqli fazo deyiladi.

8-ta'rif. L chiziqli fazoning V qism to'plamining o'zi ham L da aniqlangan elementlarni qo'shish va elementlarni songa ko'paytirish amallariga nisbatan chiziqli fazo bo'lsa, u holda V fazo L fazoning chiziqli qism fazosi deyiladi.

Teorema. L fazoning bo'sh bo'lmagan V qism to'plami uning chiziqli qism fazosi bo'lishi uchun quyidagi shartlarning bajarilishi yetarli:

1. Agar x va y vektorlar V ga tegishli bo'lsa, u holda $x+y$ vektor ham V ga tegishli bo'lishi;

2. Agar x vektor V ga tegishli bo'lsa, u holda αx vektor ham α sonning istalgan qiymatida V ga tegishli bo'lishi.

9-ta'rif. n o'lchovli haqiqiy L chiziqli fazoning har bir x va y vektorlar juftligi uchun mos ravishda skalyar ko'paytma, deb ataluvchi (x, y) haqiqiy son mos qo'yilgan bo'lib, quyidagi shartlar bajarilsa, L chiziqli fazoda skalyar ko'paytma aniqlangan, deyiladi:

1) $(x, x) \geq 0$, ixtiyoriy $x \in L$ uchun $(x, x) = 0 \Leftrightarrow x = \theta$;

2) $(x, y) = (y, x)$;

3) $(x + y, z) = (x, z) + (y, z)$;

4) $(\alpha x, y) = \alpha(x, y)$.

10-ta'rif. Agar n o'lchovli haqiqiy chiziqli fazoda skalyar ko'paytma aniqlangan bo'lsa, bu fazo n o'lchovli Yevklid fazosi deyiladi va E^n ko'rinishda belgilanadi.

Misol. Korxona jadvalda ko'rsatilgan miqdorda 4 turdagi mahsulot ishlab chiqaradi.

Mahsulot turlari	B_1	B_2	B_3	B_4
Mahsulot miqdori (birlik)	50	80	20	120
Bir birlik mahsulot uchun xomashyo sarfi	7	3,5	10	4
Mahsulot hajmining o'zgarishi	+5	-4	-2	+10

Mahsulot ishlab chiqarish uchun sarflanadigan umumiy xomashyo miqdori va mahsulot hajmining o'zgarishidagi uning o'zgarishini toping.

Yechish. Umumiy xomashyo miqdori S $x = (50; 80; 20; 120)$ va $y = (7; 3{,}5; 10; 4)$ vektorlarning skalyar ko'paytmasi bo'ladi:

$$S = (x, y) = 50 \cdot 7 + 80 \cdot 3{,}5 + 20 \cdot 10 + 120 \cdot 4 = 1310(kg)$$

Skalyar ko'paytmaning xossasidan, umumiy xomashyo miqdorining o'zgarishini topamiz.

$$\Delta S = (x + \Delta x, y) - (x, y) = (\Delta x, y) = +5 \cdot 7 - 4 \cdot 3{,}5 - 2 \cdot 10 + 10 \cdot 4 = 41(kg).$$

Har qanday n o'lchovli haqiqiy arifmetik fazoda skalyar ko'paytmani aniqlash orqali uni Yevklid fazosiga aylantirish mumkin.

Yevklid fazosida x vektorning uzunligi (normasi) deb uning skalyar kvadratidan olingan kvadrat ildizga aytiladi:

$$|x| = \sqrt{(x, x)} = \sqrt{x_1^2 + x_2^2 + \ldots + x_n^2}$$

Vektorning uzunligi uchun quyidagi xossalar o'rinlidir:

1. $|x| = 0$ bo'ladi faqat va faqat $x = 0$ bo'lsagina;

2. $|\lambda x| = |\lambda| \cdot |x|$, bunda $\lambda \in R$;

3. $|(x,y)| \le |x| \cdot |y|$ (Koshi-Bunyakovskiy tengsizligi);

4. $|x+y| \le |x| + |y|$ (uchburchak tengsizligi).

Noldan farqli vektorlardan tashkil topgan vektorlar sistemasidagi vektorlarning har qanday ikki jufti o'zaro ortogonal bo'lsa, u holda sistema ortogonal vektorlar sistemasi deb ataladi.

Teng o'lchovli $a_1, a_2, ..., a_k$ chiziqli erkli vektorlar sistemasi ustida ortogonal vektorlar sistemasini qurish, ya'ni uni $b_1, b_2, ..., b_k$ ortogonal vektorlar sistemasi bilan almashtirish mumkin. Buning uchun Shmidt formulalaridan foydalanamiz:

1) $b_1 = a_1$, deb olib keyingi qadamda

$$2) b_t = a_t - \sum_{i=1}^{t-1} \frac{(b_i \cdot a_t)}{(b_i \cdot b_i)} b_i, \quad t = 2, 3, ..., k$$

Masalan, $\vec{a}_1(1,1,1)$, $\vec{a}_2(0,1,1)$, $\vec{a}_3(0,0,1)$ vektorlar sistemasi ustida ortogonal vektorlar sistemasini quramiz.

Birinchi navbatda $\vec{a}_1(1,1,1)$, $\vec{a}_2(0,1,1)$, $\vec{a}_3(0,0,1)$ vektorlar sistemasining rangini aniqlab olamiz

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

$rang(\vec{a}_1, \vec{a}_2, \vec{a}_3) = 3$ bo'lganligi sababli bu sistemadagi vektorlar chiziqli erkli. Sistemani ortogonal sistemaga aylantirish uchun Shmidt formulasidan foydalanamiz:

1) $\vec{b}_1 = \vec{a}_1(1,1,1)$;

$$2) \vec{b}_2 = \vec{a}_2 - \frac{(\vec{b}_1, \vec{a}_2)}{(\vec{b}_1, \vec{b}_1)} \vec{b}_1 = (0,1,1) - \frac{2}{3}(1,1,1) = \left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right);$$

$$3) \vec{b}_3 = \vec{a}_3 - \frac{(\vec{b}_1, \vec{a}_3)}{(\vec{b}_1, \vec{b}_1)} \vec{b}_1 - \frac{(\vec{b}_2 \vec{a}_3)}{(\vec{b}_2 \vec{b}_2)} \vec{b}_2 = \left(0; -\frac{1}{2}; \frac{1}{2}\right).$$

Berilgan vektorlar sistemasi ustida qurilgan ortogonal sistema vektorlarini butun koordinatali vektorlarga aylantirish uchun $\vec{c}_1 = \vec{b}_1(1,1,1)$; $\vec{b}_2 = \left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right)$ ni unga kollinear bo'lgan $\vec{c}_2(-2,1,1) = \frac{1}{3}\vec{b}_2$ bilan;

$\bar{b}_3=\left(0;-\frac{1}{2};\frac{1}{2}\right)$ ni esa unga kollinear bo'lgan $\bar{c}_3(0,-1,1)=\frac{1}{2}\bar{b}_3$ bilan almashtirib va $\bar{c}_1=\bar{b}_1(1,1,1)$ belgilash kiritib: $\bar{c}_1(1,1,1)$, $\bar{c}_2(-2,1,1)$, $\bar{c}_3(0,-1,1)$ ortogonal vektorlar sistemasini hosil qilamiz.

Nol bo'lmagan b vektorning birlik vektori, deb $\frac{b}{|b|}$ vektorga aytiladi.

Har bir vektori birlik vektorga keltirilgan ortoganal sistemaga ortonormal vektorlar sistemasi deyiladi.

Yuqoridagi misolda topilgan ortogonal $\bar{c}_1(1,1,1)$, $\bar{c}_2(-2,1,1)$, $\bar{c}_3(0,-1,1)$ vektorlar sistemasini ortonormal vektorlar sistemasiga keltiramiz.

$$\frac{\bar{b}_1}{|\bar{b}_1|}=\frac{1}{\sqrt{1^2+1^2+1^2}}(1,1,1)=\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$$

$$\frac{\bar{b}_2}{|\bar{b}_2|}=\frac{1}{\sqrt{(-2)^2+1^2+1^2}}(-2,1,1)=\left(-\frac{2}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}}\right)$$

$$\frac{\bar{b}_3}{|\bar{b}_3|}=\frac{1}{\sqrt{0^2+(-1)^2+1^2}}(0,-1,1)=\left(0,-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$$

n o'lchovli Yevklid fazosida $e_1,e_2,...,e_n$ vektorlar $i\neq j$ da $(e_i,e_j)=0$ bo'lsa ortogonal bazis, $i=1,2,...,n$ da $|e_i|=1$ bo'lsa ortonormallangan bazis tashkil qiladi.

3.4. Chiziqli operatorlar va ularning xossalari

Matritsalar algebrasining asosiy tushunchalaridan biri – chiziqli operatorlar tushunchasidir. Faraz qilaylik bizga L, L_1 chiziqli fazolar berilgan bo'lsin.

1-ta'rif. Agar biror $\tilde{A}$ qoida yoki qonun bo'yicha har bir $x\in L$ elementga $y\in L_1$ element mos qo'yilgan bo'lsa, u holda L fazoni L_1 fazoga o'tkazuvchi $\tilde{A}$ operator (almashtirish, akslantirish) aniqlangan deyiladi va $y=\tilde{A}(x)$ ko'rinishda belgilanadi.

2-ta'rif. Agar ixtiyoriy $x,y\in L$, $\lambda\in R$ uchun:

1) $\tilde{A}(x+y)=\tilde{A}(x)+\tilde{A}(y)$ (operatorning additivligi);

2) $\tilde{A}(\lambda x)=\lambda\tilde{A}(x)$ (operatorning bir jinsliligi) munosabatlar o'rinli bo'lsa, u holda bu operator chiziqli operator deyiladi.

Endi operatorning bir jinsli ekanligini tekshiramiz. Ma'lumki, $ka_1 = (kx_1, ky_1)$. U holda

$$\tilde{A}(ka_1) = \tilde{A}(kx_1, ky_1) = (kx_1, ky_1, kx_1 + ky_1) = k(x_1, y_1, x_1 + y_1) = k\tilde{A}(a_1).$$

Demak, biz o'rganayotgan operator chiziqli operatordir.

$y = \mathring{A}(x) \in L_1$ element $x \in L$ elementning aksi, $x \in L$ elementning o'zi esa $y \in L_1$ elementning proobrazi deyiladi. Agar $L = L_1$ bo'lsa, u holda $\tilde{A}$ operator L fazoni o'zini o'ziga akslantiruvchi operator bo'ladi. Biz ko'proq fazoni o'zini o'ziga akslantiruvchi operatorlarni o'rganamiz.

Teorema. Har bir $\tilde{A}: L^n \to L^n$ chiziqli operatorga berilgan bazisda n-tartibli matritsa mos keladi va aksincha har bir n-tartibli matritsaga n o'lchovli chiziqli fazoni, n o'lchovli chiziqli fazoga akslantiruvchi $\tilde{A}$ chiziqli operator mos keladi.

Isbot. Faraz qilaylik $\tilde{A}: L^n \to L^n$ chiziqli operator bo'lsin. Agar $\{e_1, e_2, ..., e_n\} \subseteq L^n$ vektorlar sistemasi L^n fazoning bazisi bo'lsa, u holda ixtiyoriy $x \in L^n$ elementni bu bazis elementlari orqali yozish mumkin:

$$x = x_1 e_1 + ... + x_n e_n. \qquad (1)$$

Bu yerda biz $\tilde{A}$ operatorning chiziqliligidan foydalanib, $\tilde{A}(x)$ni quyidagicha yoza olamiz:

$$\mathring{A}(x) = \mathring{A}(x_1 e_1 + ... + x_n e_n) = x_1 \mathring{A}(e_1) + ... + x_n \mathring{A}(e_n). \qquad (2)$$

Bu yerda har bir $\mathring{A}(e_i)$ $(i = \overline{1,n})$ elementlar o'z navbatida L^n fazoning elementlari bo'lganligi sababli, bu elementlarni ham $\{e_1, e_2, ..., e_n\}$ bazis orqali yozish mumkin:

$$\mathring{A}(e_i) = a_{1i} e_1 + ... + a_{ni} e_n. \qquad (3)$$

U holda (3) dan foydalanib (2) ifodani quyidagicha yozish mumkin:

$$\begin{aligned} \mathring{A}(x) &= x_1(a_{11}e_1 + a_{21}e_2 + ... + a_{n1}e_n) + x_2(a_{12}e_1 + a_{22}e_2 + ... + a_{n2}e_n) + ... \\ &+ x_n(a_{1n}e_1 + a_{2n}e_2 + ... + a_{nn}e_n) = (a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n)e_1 + \\ &+ (a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n)e_2 + ... + (a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n)e_n \end{aligned} \qquad (4)$$

Ikkinchi tomondan $y = \mathring{A}(x)$ element ham $\{e_1, e_2, ..., e_n\}$ bazis elementlari bo'yicha quyidagi yoyilmaga ega:

$$y = \mathring{A}(x) = y_1 e_1 + y_2 e_2 + ... + y_n e_n. \qquad (5)$$

Vektorning bitta bazis bo'yicha yoyilmasi yagonaligidan (4) va (5) tengliklarning o'ng tomonlarini tenglashtirib, quyidagini olamiz.

$$\begin{cases} y_1 = a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \\ y_2 = a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ y_n = a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n \end{cases}$$

yoki matritsa ko‘rinishida $Y = AX$, bu yerda

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}, \quad Y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

3-ta’rif. $A = (a_{ij})$ $(i, j = 1,2,\ldots,n)$ matritsa $\mathring{A}$ operatorning $\{e_1, e_2, \ldots, e_n\}$ bazisdagi matritsasi, $A = (a_{ij})$ $(i, j = 1,2,\ldots,n)$ matritsaning rangi esa $\mathring{A}$ operatorning rangi deyiladi.

L fazoning barcha vektorlarini θ nol vektorga akslantiruvchi $O(x) = \theta$ operator nol operator, $\mathring{E}(x) = x$ tenglikni qanoatlantiruvchi operator birlik operatori deb ataladi.

Chiziqli operatorlar ustida bajariladigan amallar bilan tanishib chiqamiz. R^n chiziqli fazoda $\mathring{A}$, $\mathring{B}$ chiziqli operatorlar berilgan bo‘lsin.

4-ta’rif. $\left(\mathring{A} + \mathring{B}\right)(x) = \mathring{A}(x) + \mathring{B}(x)$ tenglik bilan aniqlanadigan operatorni $\mathring{A}$, $\mathring{B}$ operatorlarning yig‘indisi deb ataladi.

$\mathring{A} + \mathring{B}$ operator chiziqlidir.

Haqiqatan ham, ixtiyoriy $x, y \in R^n$ vektorlar va $\alpha \in R$ son uchun:

1) $\left(\tilde{A} + \tilde{B}\right)(x + y) = \tilde{A}(x + y) + \tilde{B}(x + y) =$

$= \tilde{A}(x) + \tilde{A}(y) + \tilde{B}(x) + \tilde{B}(y) = \left(\tilde{A} + \tilde{B}\right)(x) + \left(\tilde{A} + \tilde{B}\right)(y);$

2) $\left(\tilde{A} + \tilde{B}\right)(\alpha x) = \tilde{A}(\alpha x) + \tilde{B}(\alpha x) = \alpha(\tilde{A}(x)) + \alpha(\tilde{B}(x)) =$

$= \alpha(\tilde{A}(x) + \tilde{B}(x)) = \alpha\left[(\tilde{A} + \tilde{B})(x)\right]$

munosabatlar o‘rinli. Bu esa $\mathring{A} + \mathring{B}$ operator chiziqli ekanligini ko‘rsatadi.

5-ta’rif. $\left(\tilde{A}\tilde{B}\right)(x) = \tilde{B}\left(\tilde{A}(x)\right)$ tenglik bilan aniqlanadigan, ya’ni $\mathring{A}$, $\mathring{B}$ operatorlarni ketma-ket bajarishdan hosil bo‘lgan $\mathring{A}\mathring{B}$ operator $\mathring{A}$, $\mathring{B}$ operatorlarning ko‘paytmasi deyiladi.

$\mathring{A}\mathring{B}$ operator chiziqlidir.

Haqiqatan ham, ixtiyoriy $x,y\in R^n$ vektorlar va $\alpha\in R$ son uchun:

1) $\left(\tilde{A}\tilde{B}\right)(x+y)=\tilde{B}[\tilde{A}(x+y)]=\tilde{B}(\tilde{A}(x)+\tilde{A}(y))=\left(\tilde{A}\tilde{B}\right)(x)+\left(\tilde{A}\tilde{B}\right)(y);$

2) $\left(\tilde{A}\tilde{B}\right)(\alpha x)=\tilde{B}[\tilde{A}(\alpha x)]=\tilde{B}[\alpha(\tilde{A}(x))]=\alpha[\tilde{B}(\tilde{A}(x))]=\alpha[(\tilde{A}\tilde{B})(x)]$

munosabat o'rinli. Bu esa $\mathring{A}\mathring{B}$ operator chiziqli ekanligini ko'rsatadi.

6-ta'rif. $\left(\alpha\tilde{A}\right)(x)=\alpha\left(\tilde{A}(x)\right)$ tenglik bilan aniqlanadigan $\alpha\mathring{A}$ operator $\mathring{A}$ operatorlarning α songa ko'paytmasi deyiladi.

$\alpha\mathring{A}$ operator chiziqlidir.

Haqiqatan ham, ixtiyoriy $x,y\in R^n$ vektorlar va $\alpha,\beta\in R$ sonlar uchun:

1) $\left(\alpha\tilde{A}\right)(x+y)=\alpha[\tilde{A}(x+y)]=\alpha(\tilde{A}(x)+\tilde{A}(y))=$

$=\alpha(\tilde{A}(x))+\alpha(\tilde{A}(y))=\left(\alpha\tilde{A}\right)(x)+\left(\alpha\tilde{A}\right)(y);$

2) $\left(\alpha\tilde{A}\right)(\beta x)=\alpha[\tilde{A}(\beta x)]=\alpha[\beta(\tilde{A}(x))]=\beta[\alpha(\tilde{A}(x))]=\beta[(\alpha\tilde{A})(x)]$

munosabat o'rinli. Bu esa $\alpha\mathring{A}$ operator chiziqli ekanligini ko'rsatadi.

Yuqoridagilardan quyidagi xulosalarni chiqarish mumkin.

I. Ixtiyoriy bazisda chiziqli operatorlar yig'indisining matritsasi bu operatorlarning o'sha bazisdagi matritsalari yig'indisiga teng.

II. Ixtiyoriy bazisda chiziqli operatorlar ko'paytmasining matritsasi bu operatorlarning o'sha bazisdagi matritsalari ko'paytmasiga teng.

III. Biror bir bazisda $\mathring{A}$ chiziqli operatorning α songa ko'paytmasini beruvchi matritsa bu operatorning shu bazisdagi matritsasini α songa ko'paytirilganiga teng.

7-ta'rif. $\tilde{A}(x)$ operator uchun $\tilde{A}\tilde{A}^{-1}=\tilde{A}^{-1}\tilde{A}=\tilde{E}$ munosabat o'rinli bo'lsa, u holda $\mathring{A}^{-1}$ operator $\mathring{A}$ operatorga teskari operator deb ataladi.

Shuni ta'kidlab o'tish kerakki, $\tilde{A}(x)$ operatorga teskari operator mavjud bo'lishi uchun (bu holda $\tilde{A}(x)$ operator aynimagan operator, deb ataladi) uning har qanday bazisdagi A matritsasi aynigan bo'lmasligi zarur va etarlidir.

Bitta chiziqli operatorning turli bazislardagi matritsalari orasidagi bog'lanish haqidagi teoremani keltiramiz.

Teorema. Agar $\mathring{A}$ chiziqli operatorning $\{e_1,e_2,...,e_n\}$ va $\{e_1^*,e_2^*,...,e_n^*\}$ bazislardagi matrisalari mos ravishda A va A^* matrisalardan iborat bo'lsa, u holda $A^*=C^{-1}AC$ munosabat o'rinli bo'ladi.

Bu yerda C o'tish matritsasi deb ataladi.

Shuni ta'kidlab o'tish kerakki, $\tilde{A}(x)$ operatorga teskari operator mavjud bo'lishi uchun (bu holda $\tilde{A}(x)$ operator aynimagan operator, deb ataladi) uning har qanday bazisdagi A matritsasi aynigan bo'lmasligi zarur va etarlidir.

Agar $\overset{o}{A}$ chiziqli operator va λ son uchun

$$\tilde{A}(x) = \lambda x$$

tenglik o'rinli bo'lsa, u holda λ son $\tilde{A}(x)$ operatorning xos soni, unga mos x vektorga esa operatorning xos vektori deb ataladi.

Yuqoridagi tenglikni operatorning matritsasidan foydalanib yozsak, u holda quyidagi tenglamalar sistemasini hosil qilamiz:

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = \lambda \cdot x_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = \lambda \cdot x_2 \\ ------------- \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = \lambda \cdot x_n \end{array}\right\} \Rightarrow \left.\begin{array}{l} (a_{11} - \lambda)x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = 0 \\ a_{21}x_1 + (a_{22} - \lambda)x_2 + \cdots + a_{2n}x_n = 0 \\ ------------- \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + (a_{nn} - \lambda)x_n = 0 \end{array}\right\}$$

Bundan

$$[A - \lambda E] \cdot X = 0.$$

Ma'lumki bir jinsli sistema har doim nol yechimga ega. Sistema nolmas yechimga ega bo'lishi uchun esa uning koeffitsiyentlaridan tuzilgan determinantning qiymati nolga teng bo'lishi zarur va etarli, ya'ni

$$|A - \lambda E| = \begin{vmatrix} a_{11} - \lambda & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} - \lambda & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} - \lambda \end{vmatrix} = 0 \qquad (6)$$

$|A - \lambda E|$ determinant λ ga nisbatan n darajali ko'phaddir. Bu ko'phad $\tilde{A}(x)$ operatorning xarakteristik ko'phadi deb ataladi. (6) tenglama $\tilde{A}(x)$ operatorning xarakteristik tenglamasi deyiladi. Chiziqli operatorning xarakteristik ko'phadi bazisni tanlashga bog'liq emas.

Xalqaro savdo modeli. Ko'pgina iqtisodiy masalalarning matematik modeli chiziqli modellarga keltirilishi sababli, chiziqli fazo elementlari iqtisodiyotda o'zining muhim o'rnini egallagan.

Matritsaning xos vektori va xos qiymatini topishga olib keladigan iqtisodiy jarayonning matematik modeli sifatida xalqaro savdo modelini keltirish mumkin.

$S_1, S_2, \cdots, S_n$ n-ta mamlakat bo'lib, ularning milliy daromadlari mos ravishda $x_1, x_2, \ldots, x_n$ larga teng bo'lsin. a_{ij} - S_j-mamlakatning S_i-mamlakatdan sotib olgan tovarlarga sarf qilgan milliy daromadning ulushi bo'lsin. Milliy daromad to'laligicha mamlakat ichida va boshqa mamlakatlardan tovar xarid uchun sarf bo'ladi deb hisoblaymiz, ya'ni

$$\sum_{i=1}^{n} a_{ij} = 1, \quad j = 1, 2, \ldots, n$$

tenglik o'rinli bo'lishi kerak. Quyidagi matritsani qaraylik

$$A = \begin{pmatrix} a_{11} & a_{12} \ldots a_{1n} \\ a_{21} & a_{22} \cdots a_{2n} \\ ------- \\ a_{n1} & a_{n2} \cdots a_{nn} \end{pmatrix},$$

bu matritsa savdo-sotiqning strukturaviy matritsasi deb nomlanadi. Istalgan S_i $(i = \overline{1,n})$ mamlakat uchun ichki va tashqi savdodan hosil bo'lgan tushimi $P_i = a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n$ tenglik orqali aniqlanadi. Mamlakat olib borayotgan savdo-sotiqning muvozanatda bo'lishi uchun, har bir mamlakat savdosi kamomadsiz bo'lishi kerak, ya'ni har bir mamlakat savdosidan hosil bo'lgan tushum uning milliy daromadidan kam bo'lmasligi kerak. Ya'ni

$$P_i \geq x_i, \quad i = \overline{1,n}$$

Agar $P_i > x_i$ deb faraz qilsak, u holda quyidagini hosil qilamiz,

$$Pi = \sum_{k=1}^{n} a_{ik}x_k > x_i, \quad i = \overline{1,n}$$

bu yerdan

$$\sum_{i=1}^{n} P_i > \sum_{i=1}^{n} x_i,$$

ya'ni,

$$\sum_{i=1}^{n} P_i = \sum_{i=1}^{n}\sum_{k=1}^{n} a_{ik}x_k = \sum_{k=1}^{n}\left(\sum_{i=1}^{n} a_{ik}\right)x_k = \sum_{k=1}^{n} x_k > \sum_{k=1}^{n} x_i$$

ekanligi kelib chiqadi, bu esa qarama-qarshilikdir. Demak $P_i \geq x_i$ tengsizlik o'rniga $P_i = x_i$ tenglik o'rinli bo'lishligi kelib chiqadi. Iqtisodiy nuqtai nazardan bu tushunarli holatdir, chunki mamlakatlarning barchasi bir paytda foyda ko'rolmaydi. Mamlakatlar milliy daromadi uchun $X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$ vektorni kiritsak u holda $P_i = x_i$, ya'ni $\sum_{k=1}^{n} a_{ik} x_k = x_i$, $i=\overline{1,n}$ tengliklardan quyidagi tenglamani hosil qilamiz: $AX = X$, ya'ni, qaralayotgan masala A-matritsaning $\lambda = 1$ xos qiymatiga mos keladigan xos vektorini topish masalasiga kelar ekan.

Misollar

1-misol. R^3 fazoda $\{e_1, e_2, ..., e_n\}$ bazisda chiziqli operator matritsasi

$$A = \begin{pmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{pmatrix}$$

berilgan bo'lsin. $x = 4e_1 - 3e_2 + e_3$ vektorning $y = \mathring{A}(x)$ aksini toping.

Yechish. Yuqorida qayd qilingan formulaga ko'ra

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 4 \\ -1 & 5 & 6 \\ 1 & 8 & 2 \end{pmatrix} \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ -13 \\ -18 \end{pmatrix}$$

Demak, $y = 10e_1 - 13e_2 - 18e_3$.

2-misol. $\{e_1, e_2\}$ bazisda chiziqli operator matritsasi $A = \begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix}$ berilgan bo'lsin. Yangi $\begin{cases} e_1^* = e_1 - 2e_2 \\ e_1^* = 2e_1 + e_2 \end{cases}$ bazisdagi chiziqli operator matritsasini toping.

Yechish. O'tish matritsasi $C = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$, unga teskari matritsa $C^{-1} = \frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$. Demak, yangi bazisda operatorning matritsasi quyidagi ko'rinishda bo'ladi:

$$A^* = C^{-1}AC = \frac{1}{5}\begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 17 & 6 \\ 6 & 8 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 20 \end{pmatrix}.$$

3-misol. $\tilde{A}(x) = (2x_1 - x_2 + 2x_3, 5x_1 - 3x_2 + 3x_3, -x_1 - 2x_3)$ operatorning xos soni va xos vektorlarini toping.

Yechish. Avval $\tilde{A}$ operatorning matritsasini tuzib olamiz:

$$A=\begin{pmatrix} 2 & -1 & 2 \\ 5 & -3 & 3 \\ -1 & 0 & -2 \end{pmatrix}.$$

Berilgan operatorga mos keluvchi bir jinsli tenglamalar sistemasi quyidagi ko'rinishni oladi:

$$\begin{cases} (2-\lambda)x_1 - x_2 + 2x_3 = 0 \\ 5x_1 - (3+\lambda)x_2 + 3x_3 = 0 \\ -x_1 - (2+\lambda)x_3 = 0. \end{cases}$$

Bundan xarakteristik ko'phadni topamiz:

$$p(\lambda) \equiv \begin{vmatrix} 2-\lambda & -1 & 2 \\ 5 & -3-\lambda & 3 \\ -1 & 0 & -2-\lambda \end{vmatrix} = -(\lambda+1)^3.$$

Demak, xos son $\lambda=-1$ ekan. Bu sonni sistemaga qo'ysak,

$$\begin{cases} 3x_1 - x_2 + 2x_3 = 0, \\ 5x_1 - 2x_2 + 3x_3 = 0, \\ -x_1 - x_3 = 0. \end{cases}$$

Bundan $x_1 = x_2$, $x_1 = -x_3$. Demak, $X=\alpha\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$.

4-misol. Uch mamlakatdagi savdoning strukturali matrisasi quyidagi ko'rinishga ega:

$$A=\begin{pmatrix} 0,2 & 0,3 & 0,2 \\ 0,6 & 0,4 & 0,6 \\ 0,2 & 0,3 & 0,2 \end{pmatrix}.$$

Balanslangan savdo uchun bu mamlakatlarning milliy daromadlari nisbatini aniqlang.

Yechish. $(A-E)x=0$ tenglamani yechib yoki

$$\begin{pmatrix} -0,8 & 0,3 & 0,2 \\ 0,6 & -0,6 & 0,6 \\ 0,2 & 0,3 & -0,8 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

Sistemani Gauss metodida yechib, $\lambda=1$ xos qiymatga mos x xos vektorni topamiz. Demak $x=(c;\ 2c;\ c)$. Olingan natijadan balanslangan savdo uchun bu uch mamlakatlarning milliy daromadlari nisbati $1:2:1$ bo'ladi.

3.5. Kvadratik formalar

$$\Phi(x_1, x_2, ..., x_n) = \sum_{i,k=1}^{n} a_{ik} x_i x_k \quad (1)$$

ko'rinishdagi funksiya kvadratik forma deyiladi. Agar $a_{ik} = a_{ki}$ bo'lsa u holda (1) kvadratik forma simmetrik deyiladi.

$$A = \|a_{ik}\| = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}, \quad a_{ik} = a_{ki} \quad (2)$$

simmetrik kvadratik matritsani (1) kvadratik formaning matritsasi deymiz. (1) kvadratik forma

$$\Phi = XAX' \quad (3)$$

matritsalar ko'paytmasi shaklida ifodalanishi mumkin. Bu yerda $X = (x_1, x_2, ..., x_n)$ va X' o'zgaruvchilarning vektor - satri va vektor – ustuni.

1-ta'rif. Kvadratik formaning koeffitsiyentlari $i \neq j$ da $a_{ij} = 0$ bo'lsa, kvadratik forma kanonik ko'rinishga ega bo'ladi, ya'ni:

$$\Phi = a_{11}x_1^2 + a_{22}x_2^2 + ... + a_{nn}x_n^2. \quad (8)$$

2-ta'rif. Haqiqiy koeffitsiyentlarga ega kvadratik forma kanonik ko'rinishidagi barcha koeffitsiyentlari 1 yoki -1 bo'lsa, kvadratik forma normal ko'rinishga ega deyiladi.

Kvadratik formani kanonik ko'rinishga keltirishning turli usullari mavjud. Ulardan birini keltiramiz.

Lagranj usuli. Lagranj usuli to'la kvadratlarni chiqarishdan iborat. Avval x_1 ga ega bo'lgan qo'shiluvchilardan to'la kvadrat ajratiladi. So'ngra x_2 ga ega bo'lgan qo'shiluvchilardan va hokazo.

Misol. Kvadratik formani normal va kanonik ko'rinishga keltiring.

$$\Phi(x_1, x_2, x_3) = x_1^2 + 4x_1x_2 + 4x_1x_3 + x_2^2 + 4x_2x_3 + x_3^2.$$

Yechish. Lagranj metodini qo'llab

$$\Phi(x_1,x_2,x_3)=\left[x_1^2+2x_1(2x_2+2x_3)+(2x_2+2x_3)^2\right]-$$
$$-(2x_2+2x_3)^2+x_2^2+4x_2x_3+x_3^2=(x_1+2x_2+2x_3)^2-$$
$$-4x_2^2-8x_2x_3-4x_3^2+x_2^2+4x_2x_3+x_3^2=y_1^2-3x_2^2-4x_2x_3-3x_3^2=$$
$$=y_1^2-3\left[x_2^2+2x_2(2x_3/3)+(2x_3/3)^2\right]+3(2x_3/3)^2-3x_3^2=$$
$$=y_1^2-3(x_2+2x_3/3)^2-5x_3^2/3=y_1^2-3y_2^2-5y_3^2/3,$$

bu yerda $y_1=x_1+2x_2+2x_3$, $y_2=x_2+2x_3/3$, $y_3=x_3$. Ma'lumki o'zgaruvchilarni bu almashtirilishi maxsus emas. Normal ko'rinishga keltirish uchun bu kvadratik formada $y_1=z_1, y_2=\frac{z_2}{\sqrt{3}}$, $y_3=\sqrt{\frac{3}{5}}z_3$ o'zgaruvchilarni almashtirishni qo'llash zarur u holda

$$\Phi(z_1,z_2,z_3)=z_1^2-z_2^2-z_3^2$$

ni hosil qilamiz.

Kvadratik formalar nazariyasining fundamental holatini teorema ko'rinishida ifodalaymiz.

Teorema (kvadratik forma inersiya qonuni). Haqiqiy koeffitsiyentlarga ega bo'lgan (5) kvadratik formani normal ko'rinishga keltirishning ixtiyoriy usulida koeffitsiyentlar 1 bo'lgan kvadratlar soni, koeffitsiyentlari -1 bo'lgan kvadratlar soni ham bir xil bo'ladi.

$$\Phi(x_1,x_2,...,x_n)=\sum_{i,k=1}^{n}a_{ik}x_ix_k$$

Simmetrik kvadratik formani ko'rib chiqamiz. Uning matritsasi

$$A=\|a_{ik}\|=\begin{pmatrix}a_{11} & a_{12} & ... & a_{1n}\\ a_{21} & a_{22} & ... & a_{2n}\\ ... & ... & ... & ...\\ a_{n1} & a_{n2} & ... & a_{nn}\end{pmatrix},\quad a_{ik}=a_{ki}\quad (2)$$

3-ta'rif. Agar bir vaqtda nolga teng bo'lmagan $x_1,x_2,...,x_m$ o'zgaruvchilarning ixtiyoriy qiymatlari uchun ko'rsatilgan forma musbat (manfiy) qiymatlarga ega bo'lsa kvadratik forma musbat aniqlangan (manfiy aniqlangan) deyiladi.

Ushbu ikkala hol **ishora aniqlovchi** formalar nomi bilan birlashgan. Agar (2) kvadratik forma ham musbat ham manfiy qiymatlarga ega bo'lsa, u holda u ishora o'zgaruvchi deyiladi.

Quyidagi tasdiqni isbotlash qiyin emas. n o'zgaruvchili $\Phi(x_1,x_2,...,x_n)$kvadratik forma musbat aniqlangan deyiladi, agar uning normal ko'rinishi n kvadratlarga ega bo'lsa, ya'ni $y_1^2+y_2^2+...+y_n^2$ ko'rinishga ega bo'lsa. Kvadratik formalar nazariyasiga muvofiq musbat aniqlangan forma uchun uning matritsasining barcha n ta xos qiymatlari musbat bo'lishi kerak.

Shunga o'xshab manfiy aniqlangan kvadratik formaning normal ko'rinishiga barcha n ta kvadratlar "minus" ishora bilan kirishi kerak. Kvadratik formadagi matritsaning barcha xos qiymatlari minus bo'lishi kerak.

Ammo kvadratik formani normal yoki kanonik ko'rinishga keltirish usuli ko'rganimizdek murakkab hisoblanadi. Bu orada kvadratik formaning ishorasini uning dastlabki ko'rinishi bo'yicha aniqlash zarurligi tug'iladi. Kvadratik formaning ishora aniqlilik mezonini keltiramiz.

$$\Delta_1=a_{11},\quad \Delta_2=\begin{vmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{vmatrix},\quad \Delta_3=\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix},...$$

$$\Delta_n=\begin{vmatrix}a_{11} & a_{12} & ... & a_{1n}\\ a_{21} & a_{22} & ... & a_{2n}\\ ... & ... & ... & ...\\ a_{n1} & a_{n2} & ... & a_{nn}\end{vmatrix}$$

Minorlar (2) kvadratik formadagi A matritsaning bosh minorlari deyiladi.

Teorema (Silvestr mezoni). (1) kvadratik forma musbat aniqlangan bo'lishi uchun

$$\Delta_1>0,\ \Delta_2>0,...,\ \Delta_n>0\ (3)$$

shartlar bajarilishi zarur va yetarli. (1) kvadratik forma manfiy aniqlangan bo'lishi uchun Δ_1, Δ_2,..., Δ_m bosh minorlarning ishora almashinishi zarur va yetarli, bunda $\Delta_1<0$.

Misollar

1-misol. $\Phi(x_1,x_2,x_3)=5x_1^2-6x_1x_2-8x_1x_3+2x_2^2+4x_2x_3+x_3^2$ kvadratik forma berilgan uni matritsali ko'rinishda yozing.

Yechish. Bu kvadratik forma matritsasi quyidagi ko'rinishga ega asosiy diagonalda o'zgaruvchilarning kvadratlaridagi koeffitsiyentlar joylashgan.

$$\Phi=(x_1,x_2,x_3)\begin{pmatrix}5 & -3 & -4\\ -3 & 2 & 2\\ -4 & 2 & 1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}.$$

2-misol. $\Phi(x_1,x_2,x_3)=-6x_1^2+4x_1x_2+4x_1x_3-x_2^2+4x_2x_3+x_3^2$ kvadratik forma ishora aniqlanganligini silvestor mezoni bo'yicha toping.

Yechish. Bu kvadratik formaning matritsasi

$$A=\begin{pmatrix}-6 & 2 & 2\\ 2 & -1 & 2\\ 2 & 2 & 1\end{pmatrix}.$$

ko'rinishga ega. Uning minorlarini hisoblaymiz

$$\Delta_1=a_{11}=-6\quad \Delta_2=\begin{vmatrix}-6 & 2\\ 2 & -1\end{vmatrix}=2,\ \Delta_3=\begin{vmatrix}-6 & 2 & 2\\ 2 & -1 & 2\\ 2 & 2 & 1\end{vmatrix}=-14.$$

Asosiy minorlarning ishoralari minusdan boshlab almashinishi tufayli berilgan kvadratik forma manfiy aniqlangan hisoblanadi.

3.6. Iqtisodiy masalalarni yechishning ba`zi metodlari.

Leontev modeli

Matritsalar algebrasi elementlarining qo'llanilishi ko'plab iqtisodiy masalalarni hal etishning asosiy usullaridan biri hisoblanadi. Aksariyat iqtisodiy ob'yekt va jarayonlarning matematik modellari matritsalar yordamida sodda va kompakt ko'rinishda tasvirlanadi. Hozirgi kunda matritsalar tabiiy va amaliy jarayonlarning matematik modellarini tuzishda muhim apparat sifatida qo'llanilmoqda.Ushbu masala ma'lumotlar bazasini ishlab chiqishda va qo'llashda dolzarb masalaga aylangan. Ular bilan ishlashda deyarli barcha ma'lumotlar matritsali shaklda saqlanadi va qayta ishlanadi. Shuning uchun ma'lumotlar bazasi bilan ishlash dolzarb masalalardan hisoblanadi.

Matritsali hisoblar. Quyida korxonalar ish faoliyatini o'rganish va tahlil qilishda matritsaviy hisoblardan foydalanishga misollar ko'rib chiqamiz.

Misol. Jadvalda 3 xil xomashyo turidan foydalangan holda 4 xil mahsulotni ishlab chiqaruvchi 5 ta korxonaning kunlik ishlab chiqarishi haqida ma'lumot berilgan, hamda bir yilda har bir korxonaning ish muddati va har bir xomashyoning narxi keltirilgan.

Mahsulot turi	**Korxonalarning mehnat unumdorligi (bir kunda ishlab chiqarilgan mahsulot miqdori)**					**Xomashyo sarfi (bir birlik mahsulot uchun)**		
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**
1	4	5	3	6	7	2	3	4
2	0	2	4	3	0	3	5	6
3	8	15	0	4	6	4	4	5
4	3	10	7	5	4	5	8	6
Bir yildagi ish kunlari soni						**Xomashyo bahosi**		
	1	2	3	4	5	1	2	3
	200	150	170	120	140	40	50	60

Topshiriqlar:

1. Har bir korxonaning har bir turdagi mahsulot bo'yicha yillik ishlab chiqarish unumdorligini toping.
2. Har bir korxonaning xomashyoning har bir turi bo'yicha yillik talabini toping.
3. Jadval asosida ko'rsatilgan turlarda va miqdorda mahsulotlarni ishlab chiqarish uchun zarur bo'lgan xomashyolarni sotib olish uchun har bir korxonaning yillik kreditini toping.

Yechish. Ushbu misolda bizni qiziqtirayotgan ishlab chiqarishning butun iqtisodiy spektrni xarakterlovchi matritsalarni tuzishimiz kerak, so'ngra esa ular ustida bajariladigan amallar yordamida berilgan masalaning yechimini olishimiz mumkin. Eng avvalo korxonalarning mahsulotning barcha turlari bo'yicha ishlab chiqarish unumdorligi matritsasini keltiramiz.

Ishlab chiqarish unumdorligi

$$\begin{matrix} & 1\ 2\ 3\ 4\ 5 & & \\ A= & \begin{pmatrix} 4 & 5 & 3 & 6 & 7 \\ 0 & 2 & 4 & 3 & 0 \\ 8 & 15 & 0 & 4 & 6 \\ 3 & 10 & 7 & 5 & 4 \end{pmatrix} & \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{matrix} \text{max } sulot \\ turi \\ \downarrow \end{matrix} \end{matrix}$$

Bu matritsaning har bir ustuni alohida korxonaning mahsulotning har bir turi bo'yicha kunlik ishlab chiqarish unumdorligiga mos keladi. Bundan kelib chiqadiki j- korxonada mahsulotning har bir turi bo'yicha yillik ishlab chiqarish unumdorligi A matritsada j- ustunning har bir korxona uchun yildagi ish kunlari soniga $(j=1,2,3,4,5)$ ko'paytirishdan kelib chiqadi. Shunday qilib har bir korxonada mahsulotning har bir turi bo'yicha yillik ishlab chiqarish unumdorligi quyidagi matritsa bilan tavsiflanadi.

$$A_{yil}=\begin{pmatrix} 800 & 750 & 510 & 720 & 980 \\ 0 & 300 & 680 & 360 & 0 \\ 1600 & 2250 & 0 & 480 & 840 \\ 600 & 1500 & 1190 & 600 & 560 \end{pmatrix}$$

Mahsulot birligiga ketadigan xomashyo xarajatlari matritsasi (bu ko'rsatgichlar shartga ko'ra har bir korxona uchun bir xil) quyidagi ko'rinishga ega.

Mahsulot turi

1 2 3 4

$$B=\begin{pmatrix} 2 & 3 & 4 & 5 \\ 3 & 5 & 4 & 8 \\ 4 & 6 & 5 & 6 \end{pmatrix}\begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \quad \begin{matrix} xom\,ashyo \\ turi \\ \downarrow \end{matrix}$$

Korxonalardagi xomashyoning har bir turi bo'yicha kunlik xarajat B matritsani A matritsaga ko'paytirish bilan aniqlanadi:

$$B\cdot A=\begin{pmatrix} 55 & 126 & 53 & 62 & 58 \\ 68 & 165 & 85 & 89 & 77 \\ 74 & 167 & 78 & 92 & 82 \end{pmatrix}.$$

Bu yerda $i-$ satr xomashyo turidan nomeriga mos keladigan $j-$ ustun esa korxonaning nomeriga mos keladi $(i=1,2,3\ j=1,2,3,4,5)$. Masalaning 2-savoliga javobni A_{yil} matritsa kabi, ya'ni BA matritsa ustunlarini korxonaning yillik ish kunlari soniga ko'paytirish orqali olamiz bu har bir korxonaning xomashyoning har bir turiga yillik talabini beradi.

$$B\cdot A_{yil}=\begin{pmatrix} 11000 & 18900 & 9010 & 7440 & 8120 \\ 13600 & 24750 & 14450 & 10680 & 10780 \\ 14800 & 25050 & 13260 & 11040 & 11480 \end{pmatrix}.$$

Xomashyo narxi matritsasini kiritamiz $Q=(40\quad 50\quad 60)$. U holda har bir korxona uchun xomashyoning umumiy yillik zahirasi Q narx matritsasini BA matritsaga ko'paytirish orqali hosil qilinadi.

$$P = QBA_{yil} = (2008000 \quad 3496500 \quad 1878500 \quad 1494000 \quad 1552600).$$

Demak xomashyoni sotib olish uchun korxonalarning kreditlashtirish summasi P matritsaning elementlari bilan aniqlanadi.

Chiziqli tenglamalar sistemasidan foydalanish. Chiziqli tenglamalar sistemasini tuzishga va yechishga olib keluvchi masalalarni ko'rib chiqamiz.

Misol. Korxona xomashyoning uch turini qo'llab, mahsulotning uch turini ishlab chiqaradi. Ishlab chiqarishning zaruriy xarakteristikalari jadvalda ko'rsatilgan. Xomashyoning berilgan zaxiralarida mahsulotning har bir turini ishlab chiqarish hajmini aniqlash talab etiladi.

Xomashyo turi	Mahsulot turlari uchun xomashyo sarfi			Xomashyo hajmi
	1	2	3	
1	6	4	5	2400
2	4	3	1	1450
3	5	2	3	1550

Yechish. Mahsulot ishlab chiqarish hajmlarini x_1, x_2, x_3 orqali belgilaymiz. U holda homashyoning har bir turi uchun zahiralarining to'liq ishlatilishi shartida balans munosabatlarini yozish mumkin. Ular uch noma'lumli uchta tenglamalar sistemasini tashkil qiladi.

$$\begin{cases} 6x_1 + 4x_2 + 5x_3 = 2400 \\ 4x_1 + 3x_2 + x_3 = 1450 \\ 5x_1 + 2x_2 + 3x_3 = 1550 \end{cases}$$

Ushbu tenglamalar sistemasini ixtiyoriy usul bilan yechib, homashyoning berilgan zahiralarida mahsulot ishlab chiqarish hajmlarini topamiz. Har bir tur bo'yicha mos ravishda $x_1 = 150,\ x_2 = 250,\ x_3 = 100$ ni tashkil qiladi.

Tarmoqlararo balansning matematik modeli. Chiziqli algebra usullari masalan, chiziqli tenglamalar sistemasi nazariyasi keng ko'lamda iqtisodiyotni rejalashtirish va tashkil etish bilan bog'liq masalalarni yechishda qo'llaniladi. Biz quyida asosan tarmoqlararo balansning matematik modeli bilan tanishamiz.

Iqtisodiyotni sonli tahlil qilish xususan, ijtimoiy mahsulot ishlab chiqarish jarayonini tahlil qilish masalasi o'zaro ishlab chiqarish mahsulotlari va xizmatlar oqimlarini o'rganishga keltiriladi.

Shu nuqtai-nazardan iqtisodiy sistema har biri biror-bir turdagi mahsulot ishlab chiqarishga moslashgan tarmoqlardan iborat, deb qaralishi mumkin. Ishlab chiqarilgan mahsulotlar o'zaro ayirboshlanadi va natijada tarmoqlar orasida mahsulot oqimlari vujudga keladi. O'zaro mahsulot oqimlarining vujudga kelishi muqarrardir, chunki har bir tarmoq o'z mahsulotini ishlab chiqarish jarayonida o'zga tarmoq mahsulotidan foydalanadi yoki uni sarflaydi.

Iqtisodiyotni normal rivojlanishining asosiy shartlaridan biri barcha tarmoqlar bo'yicha ishlab chiqarish sarflari va umumiy yig'indi mahsulot orasida balansning mavjudligidir. Bunda ishlab chiqarilgan mahsulotning bir qismi ishlab chiqarish tarmoqlari sohasiga qaytmasligini va shaxsiy ehtiyojni qondirishga, jamg'arishga sarflanishini yoki eksportga chiqarilishini e'tiborga olish talab etiladi.

Iqtisodiy sistemaning yalpi mahsuloti uning n ta o'zaro bog'liq tarmoqlarida ishlab chiqariladi deylik. Ishlab chiqarish sikli yakunlanadigan vaqtni o'z ichiga olgan davrni qaraymiz.

$x_1, x_2, ..., x_n$ – mos ravishda, birinchi, ikkinchi, ..., n – tarmoqlarning natural birliklarda ishlab chiqaradigan yalpi mahsulot hajmlari bo'lsin. Aytaylik, qaralayotgan davrda x_1 – metallurgiya tarmog'ining tonna hisobida ishlab chiqaradigan metall miqdori, x_2 – kimyo tarmog'ining ishlab chiqaradigan mahsuloti miqdori, x_3 – avtomobilsozlik tarmog'ining ishlab chiqaradigan yengil avtomobillari soni bo'lsin va hokazo.

$x(x_1, x_2, ..., x_n)$ – sistemaning yalpi mahsulot vektori deyiladi.

k – tarmoqning x_k birlik mahsulotini ishlab chiqarish uchun i – tarmoq mahsuloti sarfini x_{ik} orqali belgilaymiz. Masalan, misolimizda x_3 dona avtomobil ishlab chiqarish uchun 1-tarmoq mahsuloti, ya'ni metallning sarfi miqdorini x_{13} bilan belgilaymiz. i-tarmoqning ishlab chiqarish sohasiga qaytmaydigan yakuniy mahsulot miqdori y_i bo'lsin. U holda $y(y_1, y_2, ..., y_n)$ – sistemaning yakuniy mahsulot vektori deyiladi.

Sistemaning i-tarmog'i mahsuloti x_i uchun moddiy balans sxemasini «mahsulot ishlab chiqarish va uni taqsimlash» prinsipi bo'yicha quyidagicha tasvirlash mumkin.

Ishlab chiqarish iste'moli	Yakuniy mahsulot	Yalpi mahsulot
x_{11} x_{12} L x_{1n}	y_1	x_1
x_{21} x_{22} L x_{2n}	y_2	x_2
L L L L	K	L
x_{n1} x_{n2} L x_{nn}	y_n	x_n

Moddiy balansning oqimlar tenglamalarini

$$x_i = \sum_{k=1}^{n} x_{ki} + y_i, \quad i = 1,2,\ldots,n$$

ko'rinishda yozish mumkin.

Yuqoridagilarni quyidagi jadvalda tasvirlash mumkin

$i \setminus k$	1	2	...	n	$\sum x$
1	x_{11}	x_{12}	...	x_{1n}	$\sum_{k=1}^{n} x_{1k}$
2	x_{21}	x_{22}	...	x_{2n}	$\sum_{k=1}^{n} x_{2k}$
...	...	...	...	...	...
n	x_{n1}	x_{n2}	...	x_{nn}	$\sum_{k=1}^{n} x_{nk}$
yalpi mahsulot	x_1	x_2	...	x_n	
yakuniy mahsulot	y_1	y_2	...	y_n	

k – mahsulotning bir (shartli) birligini ishlab chiqarish uchun i-mahsulotning bevosita sarfi miqdori a_{ik} bo'lsin. a_{ik} kattaliklarga bevosita xarajat koeffitsiyentlari yoki texnologik koeffitsiyentlar, deyiladi.

Masalan, misolimizga qaytsak, a_{13} – 1 dona avtomobil ishlab chiqarish uchun bevosita sarflanadigan metall miqdoridir.

O'z-o'zidan ko'rinadiki, i – mahsulotning k–tarmoqqa jami sarfi x_{ik} k–tarmoqning bir birlik mahsulotini ishlab chiqarish uchun i–mahsulotning bevosita sarfi a_{ik} ning ushbu tarmoq ishlab chiqaradigan mahsulot miqdori x_k ga ko'paytirilganiga teng.

$x_{ik} = a_{ik} x_k$ ya'ni, ishlab chiqarish sarflarida chiziqlilik prinsipi o'rinli bo'lsin. U holda

$$x_i = \sum_{k=1}^{n} a_{ik} x_k + y_i, \quad i = 1,2,\ldots,n$$

Oxirgi sistemani, o'z navbatida, vektor-matritsa ko'rinishida quyidagicha yozish mumkin:

$$X - AX = Y \text{ yoki } (E - A)X = Y \qquad \text{(L)}$$

Bu yerda, E – n-tartibli birlik matritsa, $A = (a_{ik})$ – bevosita xarajat koeffitsiyentlari matritsasi yoki texnologik matritsa deb ataladi. a_{ik} kattaliklarni o'zgarmas deb qaraymiz.

(L) tenglamaga Leontevning chiziqli modeli deyiladi. Agar $Y = \theta$ bo'lsa, Leontev modeli yopiq, $Y \neq \theta$ bo'lganda esa model ochiq deyiladi.

Masala quyidagi hollarning biri ko'rinishida qo'yilishi mumkin:

1. Yakuniy mahsulot hajmlari vektori Y ga qarab sistema yalpi mahsulot hajmi vektori X ni hisoblash;

2. X ga qarab Y ni hisoblash.

Rejalashtirishning asosiy masalalaridan biri bu birinchi masaladir, ya'ni Y vektorning berilishiga qarab, X vektorni hisoblashdir. Leontevning ochiq modeliga tegishli asosiy masala – tegishli model ixtiyoriy yakuniy ehtiyoj Y ni qondira oladimi, degan savolga javob berishdan iborat. Ma'nosiga ko'ra X nomanfiy bo'lgani uchun iqtisodiy sistema A matritsa qanday bo'lganda nomanfiy yechimga ega bo'lishini tekshirishdan iborat.

$X_0 - AX_0$ vektorning nomanfiyligini ta'minlaydigan manfiymas X_0 vektor mavjud bo'lsa, A matritsaga (shu jumladan, modelga) samarali matritsa (model), deyiladi.

Ochiq model uchun A matritsaning samaralilik zaruriy va yetarli shartlari isbotlangan. Ularning biriga ko'ra, ochiq (L) model samarali bo'lishi uchun manfiymas A matritsaning barcha xos qiymatlari moduli bo'yicha 1 dan kichik bo'lishi yetarli.

Agar (2) modelda nomanfiy A matritsa samarali bo'lsa, u holda ixtiyoriy berilgan nomanfiy Y vektor uchun (L) tenglamalar sistemasi yagona manfiymas X yechimga ega bo'ladi. Boshqacha aytganda, har bir yakuniy mahsulot nomanfiy Y vektoriga, yagona manfiymas ishlab chiqarish hajmi X vektori mos keladi.

A matritsa samarali bo'lsa, nomanfiy $(E - A)^{-1}$ matritsa mavjud bo'lib, asosiy masala yechimi

$$X = (E - A)^{-1} Y$$

formula bo'yicha topiladi.

Misol. Quyidagi

Ishlab chiqarish sohasi	Iste'mol qilish		Yakuniy mahsulot	Yalpi ishlab chiqarish
	energetika	mashinasozlik		
energetika	7	21	72	100
mashinasozlik	12	15	123	150

jadvaldan foydalanib, agar energetika tarmog'ni ikki marta oshirib mashinasozlikni o'zgartirmasak, har bir tarmoqdagi zaruriy yalpi ishlab chiqarish hajmini toping.

Yechish. Bu yerda

$x_1 = 100, \quad x_2 = 150, \quad x_{11} = 7, \quad x_{12} = 21, \quad x_{21} = 12, \quad x_{22} = 15, \quad y_1 = 72, \quad y_2 = 123.$

U holda

$$a_{11} = 0{,}07, \quad a_{12} = 0{,}14, \quad a_{21} = 0{,}12, \quad a_{22} = 0{,}1.$$

Yani

$$A = \begin{pmatrix} 0{,}07 & 0{,}14 \\ 0{,}12 & 0{,}1 \end{pmatrix}.$$

Bundan foydalanib

$$(E - A)^{-1} = \frac{1}{0{,}8202} \begin{pmatrix} 0{,}9 & 0{,}14 \\ 0{,}12 & 0{,}93 \end{pmatrix}$$

matritsani topamiz.

Shart bo'yicha $Y = \begin{pmatrix} 144 \\ 123 \end{pmatrix}$. $X = (E - A)^{-1} Y$ formuladan foydalansak

$$X = \frac{1}{0{,}8202} \begin{pmatrix} 0{,}9 & 0{,}14 \\ 0{,}12 & 0{,}93 \end{pmatrix} \begin{pmatrix} 144 \\ 123 \end{pmatrix} = \begin{pmatrix} 179{,}0 \\ 160{,}5 \end{pmatrix}.$$

Demak, energetika tarmog'idagi yalpi ishlab chiqarishni 179,0 sh. b. gacha, mashinasozlikda esa 160,5 sh. b. gacha orttirish kerak.

3.7. Talabaning mustaqil ishi

1-topshiriq

1-misol sharti variantda berilgan.

2-misolda bir jinsli chiziqli tenglamalar sistemasining fundamental yechimlari tizimini toping.

1-variant

1. $\vec{a} = 2\vec{m} + 4\vec{n}$, va $\vec{b} = \vec{m} - \vec{n}$, bu yerda $\vec{m}$ va $\vec{n}$–birlik vektorlar ular orasidagi burchak 120^0ga teng. $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchakni toping.

$$2.\begin{cases}5x_1+6x_2-2x_3+7x_4+4x_5=0,\\2x_1+3x_2-x_3+4x_4+2x_5=0,\\7x_1+9x_2-3x_3+5x_4+6x_5=0,\\5x_1+9x_2-3x_3+x_4+6x_5=0.\end{cases}$$

2-variant

1. $\vec{a}=(3;2;-4;1)$, $\vec{b}=(1;-7;2;0)$ vektorlar berilgan. $2\left(3\vec{a}+2\vec{b}\right)-3\vec{a}+\vec{b}+7\left(\vec{a}-\vec{b}\right)$ vektorni toping.

$$2.\begin{cases}x_1+2x_2+4x_3-3x_4=0,\\3x_1+5x_2+6x_3-4x_4=0,\\4x_1+5x_2-2x_3+3x_4=0,\\3x_1+8x_2+24x_3-19x_4=0.\end{cases}$$

3-variant

1. $\vec{a}=-2\vec{j}+\vec{k}$, $\vec{b}=2\vec{i}+\vec{j}$ vektorlarga qurilgan parallelogramm diagonallari orasidagi burchakni toping.

$$2.\begin{cases}x_1-\sqrt{3}x_2=0,\\\sqrt{3}x_1-3x_2=0,\\-\sqrt{2}x_1+\sqrt{6}x_2=0,\\2x_1-\sqrt{12}x_2=0.\end{cases}$$

4-variant

1. Vektorlar uzunliklari berilgan $\left|\vec{a}\right|=11$; $\left|\vec{b}\right|=23$; $\left|\vec{a}-\vec{b}\right|=30$. $\left|\vec{a}+\vec{b}\right|$ni aniqlang.

$$2.\begin{cases}x_1+2x_2+3x_3=0,\\4x_1+5x_2+6x_3=0,\\7x_1+8x_2+9x_3=0.\end{cases}$$

5-variant

1. α va β ning qanday qiymatlarida $\vec{a}=-2\vec{i}+3\vec{j}+\beta\vec{k}$ va $\vec{b}=\alpha\vec{i}-6\vec{j}+2\vec{k}$ vektorlar a) kolleniar b) ortogonal bo'ladi.

$$2.\begin{cases}x_1-2x_2+3x_3=0,\\-x_1+2x_2-3x_3=0,\\2x_1-4x_2+6x_3=0,\\-3x_1+6x_2-9x_3=0.\end{cases}$$

6-variant

1. Oxy tekisligida $\vec{OA}=\vec{a}=2\vec{i}$, $\vec{OB}=\vec{b}=3\vec{i}+3\vec{j}$ va $\vec{OC}=\vec{c}=2\vec{i}+6\vec{j}$ vektorlarni yasang. $\vec{c}$ ni $\vec{a}$ va $\vec{b}$ vektorlar orqali analitik va geometrik ifodalang.

2. $\begin{cases} x_1 - x_3 = 0, \\ x_2 - x_4 = 0, \\ -x_1 + x_3 - x_5 = 0, \\ -x_2 + x_4 - x_6 = 0, \\ -x_3 + x_5 = 0, \\ -x_4 + x_6 = 0. \end{cases}$

7-variant

1. $\vec{a}=(2;1;0)$, $\vec{b}=(1;-1;2)$, $\vec{c}=(2;2;-1)$ va $\vec{d}=(3;7;-7)$ vektorlar berilgan. $\vec{a}$ ni $\vec{b}$ $\vec{c}$, $\vec{d}$ vektorlar orqali ifodalang.

2. $\begin{cases} 2x_1 - x_2 + x_3 = 0, \\ 4x_1 - 2x_2 + 2x_3 = 0, \\ 6x_1 - 3x_2 + 3x_3 = 0. \end{cases}$

8-variant

1. $\vec{a}=2\vec{i}+3\vec{j}-6\vec{k}$ vektor uzunligi va uning yo'naltiruvchi kosinuslarini toping.

2. $\begin{cases} x_1 + 2x_2 + 3x_3 = 0, \\ 4x_1 + 5x_2 + 6x_3 = 0, \\ 7x_1 + 8x_2 + 10x_3 = 0. \end{cases}$

9-variant

1. Vektor Oy va Oz o'qlari bilan mos ravishda 60^0 va 120^0 burchak tashkil qiladi. Ox o'qi bilan qanday burchak tashkil qiladi.

2. $\begin{cases} x_1 + 2x_2 + 4x_3 - 3x_4 = 0, \\ 3x_1 + 5x_2 + 6x_3 - 4x_4 = 0, \\ 4x_1 + 5x_2 - 2x_3 + 3x_4 = 0, \\ 3x_1 + 8x_2 + 24x_3 - 19x_4 = 0. \end{cases}$

10-variant

1. $\vec{a}=6\vec{i}-8\vec{j}+5\sqrt{2}\vec{k}$ va $\vec{b}=2\vec{i}-4\vec{j}+\sqrt{2}\vec{k}$ vektorlar berilgan. $\vec{a}-\vec{b}$ vektorning Ox o'qi bilan hosil qilgan burchakni toping.

2. $\begin{cases} 6x_1+9x_2+2x_3=0, \\ -4x_1+x_2+x_3=0, \\ 5x_1+7x_2+4x_3=0, \\ 2x_1+5x_2+3x_3=0. \end{cases}$

11-variant

1. m ning qanday qiymatlarida $\vec{a}=m\vec{i}-3\vec{j}+2\vec{k}$ va $\vec{b}=\vec{i}+2\vec{j}-m\vec{k}$ vektorlar perpendikulyar.

2. $\begin{cases} 3x_1+5x_2+3x_3+2x_4+x_5=0, \\ 5x_1+7x_2+6x_3+4x_4+3x_5=0, \\ 7x_1+9x_2+9x_3+6x_4+5x_5=0, \\ 4x_1+8x_2+3x_3+2x_4=0. \end{cases}$

12-variant

1. $\vec{a}=\vec{i}+\vec{j}+2\vec{k}$ vektorning $\vec{b}=\vec{i}-\vec{j}+4\vec{k}$ vektordagi proeksiyasini toping.

2. $\begin{cases} 3x_1+5x_2-4x_3+2x_4=0, \\ 2x_1+4x_2-6x_3+3x_4=0, \\ 11x_1+17x_2-8x_3+4x_4=0. \end{cases}$

13-variant

1. $\vec{a}=3\vec{i}-6\vec{j}-\vec{k}$, $\vec{b}=\vec{i}+4\vec{j}-5\vec{k}$, $\vec{c}=3\vec{i}+4\vec{j}+2\vec{k}$ vektorlar berilgan. $\vec{a}+\vec{c}$ vektorning $\vec{b}+\vec{c}$ vektordagi proeksiyasini toping.

2. $\begin{cases} 5x_1+7x_2+6x_3-2x_4+2x_5=0, \\ 8x_1+9x_2+9x_3-3x_4+4x_5=0, \\ 7x_1+x_2+6x_3-2x_4+6x_5=0, \\ 4x_1-x_2+3x_3-x_4+4x_5=0. \end{cases}$

14-variant

1. $\vec{c}=\vec{i}+2\vec{j}+2\vec{k}$ vektordagi proeksiyasi 1 ga teng bo'lgan, $\vec{a}=\vec{i}+\vec{k}$, va $\vec{b}=2\vec{j}-\vec{k}$, vektorlarga perpendikulyar $\vec{d}$ vektorni toping.

$$2.\begin{cases} x_1+x_2-2x_3+2x_4=0,\\ 3x_1+5x_2+6x_3-4x_4=0,\\ 4x_1+5x_2-2x_3+3x_4=0,\\ 3x_1+8x_2+24x_3-19x_4=0. \end{cases}$$

15-variant

1. $\vec{a}=(1;-1;2)$ va $\vec{b}=(1;0;1)$ vektorlar uzunliklarini va ular orasidagi burchakni topíng.

$$2.\begin{cases} -x_1+2x_2-3x_3+4x_4=0,\\ 3x_1-5x_2+2x_3-10x_4=0. \end{cases}$$

16-variant

1. $M_1=(1;2;3)$ va $M_2=(3;-4;6)$ nuqtalar berilgan. $\overline{M_1M_2}$ vektorning uzunligi va uning yo'naltiruvchi kosinuslarini toping.

$$2.\begin{cases} x_1-x_2-2x_3+3x_4=0,\\ -2x_1+x_2-x_3-5x_4=0. \end{cases}$$

17-variant

1. M nuqtaning radius vektori Oy o'qi bilan 60^0, Oz o'qi bilan 45^0 li burchak tashkil qiladi, uning uzunligi $r=8$. M nuqtaning absissasi manfiy bo'lsa uni toping.

$$2.\begin{cases} x_1+2x_2+3x_3+4x_4=0,\\ 2x_1+2x_2+5x_3+8x_4=0. \end{cases}$$

18-variant

1. $\vec{a}=\vec{i}+2\vec{j}+3\vec{k}$ va $\vec{b}=6\vec{i}+4\vec{j}-2\vec{k}$ vektorlar orasidagi burchakni toping.

$$2.\begin{cases} x_1-x_2-2x_3+3x_4=0,\\ x_1+2x_2-4x_4=0,\\ 2x_1+x_2+2x_3-x_4=0,\\ x_1-4x_2+x_3+10x_4=0. \end{cases}$$

19-variant

1. m ning qanday qiymatlarida $\vec{a}=m\vec{i}+3\vec{j}+4\vec{k}$ va $\vec{b}=4\vec{i}+m\vec{j}-7\vec{k}$ vektorlar perpendikulyar.

$$2.\begin{cases} x_1-2x_2+3x_3=0,\\ -x_1+2x_2-3x_3=0,\\ 2x_1-4x_2+6x_3=0. \end{cases}$$

20-variant

1. $\vec{a}=(1;6;1)$, $\vec{b}=(0;1;-2)$, $\vec{c}=(1;-1;0)$ va $\vec{d}=(2;-1;3)$ vektorlar berilgan. $\vec{a}$ ni $\vec{b}\ \vec{c}, \vec{d}$ vektorlar orqali ifodalang.

$$2.\begin{cases} 3x_1+2x_2+x_3=0, \\ 2x_1+5x_2+3x_3=0, \\ 3x_1+4x_2+2x_3=0. \end{cases}$$

21-variant

1. α va β ning qanday qiymatlarida $\vec{a}=-2\vec{i}+3\vec{j}+\alpha\vec{k}$ va $\vec{b}=\beta\vec{i}-6\vec{j}+2\vec{k}$ vektorlar kollinear.

$$2.\begin{cases} 2x_1+3x_2-4x_3+x_4=0, \\ 3x_1-x_2+2x_3-x_4=0, \\ -2x_1+2x_2-3x_3+5x_4=0. \end{cases}$$

22-variant

1. $\vec{c}=(9;4)$, $\vec{a}=(1;2)$, $\vec{b}=(2;-3)$ vektorlar berilgan. $\vec{c}$ ni $\vec{a}, \vec{b}$ vektorlar orqali ifodalang.

$$2.\begin{cases} 2x_1-4x_2+5x_3+3x_4=0, \\ 3x_1-6x_2+4x_3+2x_4=0, \\ 4x_1-8x_2+17x_3+11x_4=0. \end{cases}$$

23-variant

1. $\vec{a}=(2;3)$, $\vec{b}=(1;-3)$, $\vec{c}=(-1;3)$ vektorlar berilgan. α ning qanday qiymatlarida $\vec{p}=\vec{a}+\alpha\vec{b}$ va $\vec{q}=\vec{a}+2\vec{c}$ vektorlar kollinear.

$$2.\begin{cases} x_1-2x_2-3x_3=0, \\ 2x_1+3x_2+x_3=0, \\ 5x_1-3x_2-8x_3=0. \end{cases}$$

24-variant

1. $\vec{a}=(1;1;1)$ va $\vec{b}=(0;1;1)$ vektorlar uzunliklarini va ular orasidagi burchakni toping.

$$2.\begin{cases} x_1+2x_2-x_3+x_4=0, \\ 2x_1-3x_2+x_3-2x_4=0. \end{cases}$$

25-variant

1. Tekislikda uch vektor joylashgan $\vec{a}$, $\vec{b}$, $\vec{c}$. va $|\vec{a}|=2$, $|\vec{b}|=3$, $|\vec{c}|=5$, $(\vec{a}\,\hat{}\,\vec{b})=60^0$, $(\vec{b}\,\hat{}\,\vec{c})=60^0$. $\vec{d}=-\vec{a}+\vec{b}-\vec{c}$ vektorning uzunligini toping.

2. $\begin{cases} x_1+2x_2-x_3=0, \\ 2x_1-3x_2+x_3=0. \end{cases}$

2-topshiriq Misollar sharti variantda berilgan.

1-variant

1. $A=\begin{pmatrix} 1 & 2 & 3 \\ -1 & 0 & 4 \\ 3 & 1 & -5 \end{pmatrix}$ (e_1,e_2,e_3) bazisdan (e_1',e_2',e_3') bazisga o`tish matritsasi berilgan. e_3' vektorning (e_1,e_2,e_3) bazisdagi koordinatalarini toping.

2. Agar (e_1,e_2,e_3) bazisda $\bar{A}$ chiziqli operator $\begin{pmatrix} -1 & 0 & 2 \\ 2 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$ matritsa bilan berilgan va $x=2e_1+4e_2-e_3$ bo`lsa, $y=\bar{A}(x)$ vektorning koordinatalarini toping.

3. $L=2x_1^2-3x_3^2-4x_1x_2+4x_1x_3-8x_2x_3$ kvadratik formani kanonik ko`rinishga keltiring.

2-variant

1. (e_1,e_2,e_3) bazisda $x=(4;0;-12)$ vektor berilgan. Bu vektorning $(e_1'=e_1+2e_2+e_3;e_2'=2e_1+3e_2+4e_3;e_3'=3e_1+4e_2+3e_3)$ bazisdagi koordinatalarini toping.

2. (e_1,e_2,e_3) bazisda $\bar{A}$ operator $A=\begin{pmatrix} 0 & -2 & 1 \\ 3 & 1 & 0 \\ 2 & -1 & 1 \end{pmatrix}$ matritsaga ega. Agar $e_1'=3e_1+e_2+2e_3, e_2'=2e_1+e_2+2e_3, e_3'=-e_1+2e_2+5e_3$ bo'lsa, (e_1',e_2',e_3') bazisda $\bar{A}$ operatorning matritsasini toping.

3. Kvadratik formani kanonik ko`rinishga keltiring: $L(x_1,x_2,x_3)=x_1^2+2x_1x_2+2x_2x_3$.

3-variant

1. $\vec{b}=(1;m;3)$ vektor $\vec{a_1}=(2;3;7)$ $\vec{a_2}=(3;-2;4)$ va $\vec{a_3}=(-1;1;-1)$ vektorlar orqali chiziqli ifodalanadigan m ning barcha qiymatlarini toping.

2. (e_1,e_2) bazisda $\tilde{A}$ chiziqli operator $A=\begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$ matritsa bilan berilgan, $x=e_1$ bo`lsa, $y=\tilde{A}(x)$ vektorning koordinatalarini toping.

3. Kvadratik formani kanonik ko`rinishga keltiring: $L(x_1,x_2,x_3)=x_1^2+x_3^2-4x_2x_3$.

4-variant

1. (e_1,e_2,e_3) bazisda $a_1=(1;1;1)$ $a_2=(0;2;3)$ va $a_3=(0;1;5)$ vektorlar berilgan. (e_1,e_2,e_3) bazisda $d=2e_1-e_2+e_3$ vektorning koordinatalarini toping.

2. Biror bazisda $A=\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$ matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

3. Kvadratik formani kanonik ko`rinishga keltiring.
$L(x_1,x_2,x_3)=x_1^2+3x_2^2+4x_3^2+2x_1x_2+2x_1x_3+6x_2x_3$.

5-variant

1. $A=\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 2 \\ -1 & 3 & 1 \end{pmatrix}$ (e_1,e_2,e_3) bazisdan (e_1',e_2',e_3') bazisga o`tish matritsasi berilgan. e_1,e_2,e_3 vektorlarning (e_1',e_2',e_3') bazisdagi koordinatalarini toping.

2. Biror bazisda $A=\begin{pmatrix} 3 & 1 & 3 \\ 0 & 3 & 1 \\ 0 & 0 & 4 \end{pmatrix}$ matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

3. Kvadratik forma qanday aniqlanganligini toping:
$L(x_1,x_2,x_3)=x_1^2+4x_2^2+3x_3^2+2x_1x_2$.

6-variant

1. Ortonormallangan bazis tashkil qiluvchi e_1,e_2,e_3 vektorlar berilgan. $x=5e_1+e_3$ va $y=e_1+e_2+e_3$ vektorlar orasidagi burchakni toping.

2. Chiziqli operatorning (e_1,e_2,e_3) bazisdagi matritsasi $A=\begin{pmatrix} 2 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ ko‘rinishga ega. Agar $e_1'=2e_1+e_2-e_3$; $e_2'=2e_1-e_2+2e_3$; $e_3'=3e_1+e_3$ bo‘lsa, chiziqli operatorning (e_1',e_2',e_3') bazisdagi matritsasini toping.

3. Kvadratik forma qanday aniqlanganligini toping:

$L(x_1,x_2,x_3)=2x_2^2-x_1^2-x_1x_3+2x_2x_3-2x_3^2$.

7-variant

1. Biror bazisda $\vec{a_1}=(-2;0;1)$ $\vec{a_2}=(1;-1;0)$ va $\vec{a_3}=(0,1,2)$ vektorlar berilgan. $\vec{a_4}=(2;3;4)$ vektor $\vec{a_1},\vec{a_2},\vec{a_3}$ vektorlarning chiziqli kombinatsiyasi bo'ladimi.

2. (e_1,e_2) bazisda chiziqli $\tilde{A}$ operator $\begin{pmatrix} -1 & 1 \\ 2 & 1 \end{pmatrix}$ matritsa bilan berilgan $x=(2;-1)$ bo'lsa, $y=\tilde{A}(x)$ vektorning koordinatalarini toping.

3. Kvadratik forma qanday aniqlanganligini toping:

$L(x_1,x_2,x_3)=x_1^2+x_2^2+x_3^2+4x_1x_2+6x_1x_3+4x_2x_3$.

8-variant

1. (e_1,e_2,e_3) bazisdan $(e_1'=e_2+e_3; e_2'=-e_1+2e_3; e_3'=e_1+e_2)$ bazisga o'tish matritsasini toping.

2. Biror bazisda $A=\begin{pmatrix} 1 & 2 & 0 \\ 3 & 6 & 0 \\ 2 & 3 & 5 \end{pmatrix}$ matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

3. Kvadratik forma qanday aniqlanganligini toping:

$L(x_1,x_2,x_3)=3x_1^2+3x_3^2+4x_1x_2+4x_1x_3-2x_2x_3$.

9-variant

1. $A=\begin{pmatrix} 1 & 2 & -1 \\ 3 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}$ (e_1,e_2,e_3) bazisdan (e_1',e_2',e_3') bazisga o'tish matritsasi berilgan. e_2' vektorning (e_1,e_2,e_3) bazisdagi koordinatalarini toping.

2. Chiziqli operatorning (e_1,e_2) bazisdagi matritsasi $A=\begin{pmatrix} -3 & 1 \\ 2 & -1 \end{pmatrix}$ ko'rinishga ega. Agar $e_1'=e_2$, $e_2'=e_1+e_2$ bo'lsa, chiziqli operatorning (e_1',e_2') bazisdagi matritsasini toping.

3. Kvadratik forma rangini toping:

$L(x_1,x_2,x_3)=x_1^2+x_2^2+x_3^2+2x_1x_2$.

10-variant

1. Biror bazisda $\vec{a_1}=(2;1)$ va $\vec{a_2}=(-1;3)$ vektorlar berilgan. $\vec{b}=(1;m)$ vektor $\vec{a_1},\vec{a_2}$ vektorlar orqali chiziqli ifodalanadigan m ning barcha qiymatlarini toping.

2. Biror bazisda $A=\begin{pmatrix} 0 & 2 & 0 \\ 1 & 5 & 4 \\ 0 & 3 & 0 \end{pmatrix}$. matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

3. Kvadratik forma rangini toping: $L(x_1,x_2,x_3)=2x_1^2-x_2^2+3x_3^2+2x_1x_2+6x_1x_3$.

11-variant

1. (e_1,e_2) bazisda $a_1=2e_1+e_2$, $a_2=e_1-2e_2$ vektorlar berilgan a_1, a_2 vektorlar bazis tashkil qilishini isbotlang. $a_3=3e_1+2e_2$ vektorning (a_1,a_2) bazisdagi koordinatalarini toping.

2. Biror bazisda $A=\begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

3. Kvadratik forma rangini toping:
$L(x_1,x_2,x_3)=2x_1^2+4x_2^2+9x_3^2+4x_1x_2+6x_1x_3+12x_2x_3$.

12-variant

1. Biror bazisda $\vec{a}_1=(1;2;1)$ va $\vec{a}_2=(2;1;1)$ $\vec{a}_3=(-1;-2;-1)$ vektorlar berilgan. $\vec{b}=(2;3;m)$ vektor $\vec{a}_1,\vec{a}_2,\vec{a}_3$ vektorlar orqali chiziqli ifodalanadigan m ning barcha qiymatlarini toping.

2. Biror bazisda $A=\begin{pmatrix} 7 & -2 & -2 \\ 2 & 2 & -1 \\ 0 & 0 & 3 \end{pmatrix}$ matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

3. m parametrning qanday qiymatlarida kvadratik forma ishorasi aniqlangan bo'ladi: $L=mx_1^2+x_2^2+4x_1x_2$.

13-variant

1. $\vec{b}=(5;9;m)$ vektor $\vec{a}_1=(4;4;3)$, $\vec{a}_2=(7;2;1)$ va $\vec{a}_3=(4;1;6)$ vektorlar orqali chiziqli ifodalanadigan m ning barcha qiymatlarini toping.

2. Biror bazisda $A=\begin{pmatrix} 1 & 2 & -2 \\ -1 & 0 & 3 \\ 1 & 3 & 0 \end{pmatrix}$ matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

3. m parametrning qanday qiymatlarida kvadratik forma ishorasi aniqlangan bo'ladi: $L=-x_1^2+mx_2^2-4x_1x_2+6x_1x_3+10x_2x_3$.

14-variant

1. Quyidagi vektorlar sistemalari chiziqli bog'liq yoki chiziqli erkliligini ko'rsating: $\vec{a}_1=(-7;5;19)$, $\vec{a}_2=(-5;7;-7)$, $\vec{a}_3=(-8;7;14)$

2. Biror bazisda $A=\begin{pmatrix}1&0&3\\0&3&0\\0&0&0\end{pmatrix}$ matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

3. Kvadratik forma musbat aniqlangan m parametrning barcha qiymatlarini toping: $L=2x_1^2+x_2^2+mx_3^2+2x_1x_2-2x_1x_3-2x_2x_3$.

15-variant

1. $A=\begin{pmatrix}2&-1\\3&5\end{pmatrix}$ (e_1,e_2) bazisdan (e_1',e_2') bazisga o'tish matritsasi berilgan. e_1,e_2 vektorlarning (e_1',e_2') bazisdagi koordinatalarini toping.

2. Biror bazisda $A=\begin{pmatrix}1&1&8\\0&2&0\\1&0&-1\end{pmatrix}$. matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

3. Kvadratik forma musbat aniqlangan m parametrning barcha qiymatlarini toping: $L=mx_1^2+2x_2^2+3x_3^2+2x_1x_2+2x_1x_3+4x_2x_3$.

16-variant

1. (e_1,e_2,e_3) vektorlar ortonormallangan bazisni tashkil qiladi. $x=3\overline{e}_2-\overline{e}_3$ va $y=4\overline{e}_1+\overline{e}_2-2\overline{e}_3$ vektorlar orasidagi burchakni toping.

2. Biror bazisda $A=\begin{pmatrix}2&0&-6\\1&3&2\\-1&0&1\end{pmatrix}$. matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

3. Kvadratik forma musbat aniqlangan m parametrning barcha qiymatlarini toping: $L=2mx_1^2+x_2^2+2x_3^2+2x_1x_2+2x_2x_3$.

17-variant

1. Quyidagi vektorlar sistemalari chiziqli bog'liq yoki chiziqli erkliligini ko'rsating: $\vec{a}_1=(1;8;-1)$, $\vec{a}_2=(-2;3;3)$, $\vec{a}_3=(4;-11;9)$.

2. Biror bazisda $A=\begin{pmatrix}2&-1&0\\-1&2&0\\1&-1&1\end{pmatrix}$. matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

3. Kvadratik forma musbat aniqlangan m parametrning barcha qiymatlarini toping: $L=2x_1^2+mx_2^2+2x_3^2+2x_1x_2+6x_1x_3+4x_2x_3$.

18-variant

1. R^3 uch o'lchovli fazoda $\vec{a_1}=(1;1;1)$, $\vec{a_2}=(1;0;1)$, $\vec{a_3}=(2;1;2)$ vektorlar bazis tashkil qiladimi.

2. Biror bazisda $A=\begin{pmatrix} 3 & -1 & 1 \\ 0 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$. matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

3. Kvadratik forma manfiy aniqlangan m parametrning barcha qiymatlarini toping: $L=-x_1^2+mx_2^2+3x_3^2+4x_1x_2+2x_1x_3+2x_2x_3$.

19-variant

1. $\vec{b}=(1;3;5)$ vektor $\vec{a_1}=(3;2;5)$, $\vec{a_2}=(2;4;7)$ va $\vec{a_3}=(5;6;m)$ vektorlar orqali chiziqli ifodalanadigan m ning barcha qiymatlarini toping.

2. Biror bazisda $A=\begin{pmatrix} 5 & -1 & -1 \\ 0 & 4 & -1 \\ 0 & -1 & 4 \end{pmatrix}$. matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

3. Kvadratik forma manfiy aniqlangan m parametrning barcha qiymatlarini toping: $L=-2x_1^2-2x_2^2+mx_3^2+2x_1x_2+4x_1x_3-2x_2x_3$.

20-variant

1. R^4 to'rt o'lchovli fazoda $\vec{a_1}=(1;1;1;1)$, $\vec{a_2}=(1;0;1;0)$, $\vec{a_3}=(0;-1;0;1)$, $\vec{a_4}=(1;0;0;1)$ vektorlar bazis tashkil qiladimi.

2. Biror bazisda $A=\begin{pmatrix} 1 & 0 & 0 \\ 3 & 6 & 4 \\ 2 & 3 & 5 \end{pmatrix}$. matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

3. Kvadratik forma manfiy aniqlangan m parametrning barcha qiymatlarini toping: $L=2mx_1^2+x_2^2+x_3^2-4x_1x_2+2x_1x_3+2x_2x_3$.

21-variant

1. Biror bazisda $a_1=(4;5;2)$, $a_2=(3;0;1)$, $a_3=(-1;4;2)$ va $b=(5;7;8)$. vektorlar berilgan. a_1,a_2,a_3 vektorlar bazis tashkil qilishini ko'rsating. b vektorning bu bazisdagi koordinatalarini toping.

2. Biror bazisda $A=\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 1 & 0 & 0\end{pmatrix}$. matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

3. Kvadratik forma manfiy aniqlangan m parametrning barcha qiymatlarini toping: $L=-x_1^2-2x_2^2+2mx_3^2+2x_1x_2+2x_1x_3-6x_2x_3$.

22-variant

1. Biror bazisda $a_1=(3;-5;2)$, $a_2=(4;5;1)$, $a_3=(-3;0;-4)$ va $b=(-4,5;-16)$ vektorlar berilgan. a_1,a_2,a_3 vektorlar bazis tashkil qilishini ko'rsating. b vektorning bu bazisdagi koordinatalarini toping.

2. e_1,e_2,e_3 bazisda chiziqli $\tilde{A}$ operator $\begin{pmatrix}0 & 0 & 3\\ 1 & 1 & 4\\ 3 & 1 & 5\end{pmatrix}$ matritsa bilan berilgan $x=-\bar{e}_1+2\bar{e}_2+\bar{e}_3$ bo`lsa, $y=A(x)$ vektorning koordinatalarini toping.

3. Kvadratik forma qanday aniqlanganligini toping:
$L(x_1,x_2,x_3)=2x_1^2+x_2^2+4x_3^2+2x_1x_2-4x_1x_3-2x_2x_3$.

23-variant

1. Biror bazisda $a_1=(-2;3;5)$, $a_2=(1;-3;4)$, $a_3=(7;8;-1)$ va $b=(1;20;1)$. vektorlar berilgan. a_1,a_2,a_3 vektorlar bazis tashkil qilishini ko'rsating. b vektorning bu bazisdagi koordinatalarini toping.

2. Chiziqli operatorning (e_1,e_2) bazisdagi matritsasi $A=\begin{pmatrix}2 & 1\\ 0 & 3\end{pmatrix}$ ko'rinishga ega. Agar $e_1=3e_1'-e_2'$, $e_2=e_1'+e_2'$ bo'lsa, chiziqli operatorning (e_1',e_2') bazisdagi matritsasini toping.

3. Kvadratik formani kanonik ko`rinishga keltiring.
$L(x_1,x_2,x_3)=x_1^2+2x_2^2+7x_3^2+2x_1x_2+2x_1x_3+4x_2x_3$.

24-variant

1. Quyidagi vektorlar sistemalari chiziqli bog'liq yoki chiziqli erkliligini ko'rsating: $\vec{a}_1=(1;4;6)$, $\vec{a}_2=(1;-1;1)$, $\vec{a}_3=(1;1;3)$.

2. Biror bazisda $A=\begin{pmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{pmatrix}$. matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

3. Kvadratik forma qanday aniqlanganligini toping:
$L=9x_1^2+4x_2^2+x_3^2+6x_1x_3-42x_2x_3$.

25-variant

1. (e_1, e_2, e_3) vektorlar ortogonal bazisni tashkil qiladi. Agar $|e_1|=1$, $|e_2|=2$, $|e_3|=2$ bo`lsa, $x=2\bar{e}_1-3\bar{e}_2+4\bar{e}_3$ va $y=\bar{e}_1+\bar{e}_2-5\bar{e}_3$ vektorlar uzunliklarini va ularning skalyar ko`paytmasini toping.

2. Biror bazisda $A=\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 2\end{pmatrix}$. matritsasi bilan berilgan chiziqli operatorning xos qiymatlari va xos vektorlarini toping.

3. Kvadratik forma qanday aniqlanganligini toping: $L=x_1^2+2x_2^2+2x_1x_2+4x_2x_3+5x_3^2$.

3-topshiriq

Iqtisodiy mazmundagi masalalarning matematik modelini quring va yeching. Hisoblash ishlarini Mathcad dasturida bajaring.

1. Uch mamlakatdagi savdoning strukturali matrisasi quyidagi ko'rinishga ega:

$$A=\begin{pmatrix}0 & 0,25 & \frac{1}{3}\\ 0,5 & 0,5 & \frac{1}{3}\\ 0,5 & 0,25 & \frac{1}{3}\end{pmatrix}.$$

Balanslangan savdo uchun bu mamlakatlarning milliy daromadlari nisbatini aniqlang.

2. To'rt mamlakatdagi savdoning strukturali matrisasi quyidagi ko'rinishga ega:

$$A=\begin{pmatrix}0,2 & 0,3 & 0,2 & 0,2\\ 0,4 & 0,3 & 0,1 & 0,2\\ 0,3 & 0,3 & 0,5 & 0,2\\ 0,1 & 0,1 & 0,2 & 0,4\end{pmatrix}.$$

byudjetlar yig'indisi $x_1+x_2+x_3+x_4=6270$ (pul birligi) bo'lgan shartda balanslangan kamomadsiz savdoni qanoatlantiruvchi bu mamlakatlarning byudjetlari topilsin.

3. Tarmoqlarning yakuniy mahsulotlari va bevosita sarf xarajat koeffitsiyentlari quyidagi jadvalda keltirilgan:

Ishlab chiqarish sohasi		Istemol qilish		Yakuniy mahsulot
		Sanoat	Qishloq xo'jaligi	
Ishlab chiqarish	Sanoat	0,3	0,25	300
	Qishloq xo'jaligi	0,15	0,12	100

a) yalpi mahsulotning rejadagi hajmi, tarmoqlararo yetkazib berish ko'rsatkichlarini;

b) agar qishloq xo'jaligi mahsulotlariga jami talab 20% ga, sanoat mahsulotlariga talab esa 10% ga oshsa, har bir sohadagi yakuniy mahsulot hajmini hisoblang.

4. $A=\begin{pmatrix}0,1 & 0,5\\ 0,3 & 0,2\end{pmatrix}$ bevosita xarajatlar matritsasi berilgan.

a) $Y=\begin{pmatrix}400\\ 500\end{pmatrix}$ yakuniy mahsulot ishlab chiqarishni ta'minlaydigan X yalpi mahsulot vektorini;

b) yakuniy mahsulot ishlab chiqarish $\Delta Y=\begin{pmatrix}100\\ 50\end{pmatrix}$ ga ko'payganida vektor orttirmasi ΔX ni toping.

5. Savdoning strukturali matritsasi A uchun xalqaro savdo modelida milliy daromadlarning muvozanat vektorini toping. Bu mamlakatlarning daromadlari yig'indisi 402 shartli pul birligiga teng.

$$A=\begin{pmatrix}0,3 & 0,4 & 0,2\\ 0,4 & 0,5 & 0,7\\ 0,3 & 0,1 & 0,1\end{pmatrix}.$$

6. Savdoning strukturali matritsasi A uchun xalqaro savdo modelida milliy daromadlarning muvozanat vektorini toping. Bu mamlakatlarning daromadlari yig'indisi 402 shartli pul birligiga teng.

$$A=\begin{pmatrix}0,6 & 0,3 & 0,5\\ 0,3 & 0,4 & 0,1\\ 0,1 & 0,3 & 0,4\end{pmatrix}.$$

7. Ikki tarmoqdan iborat sistema ishi ma'lum muddat davomida quyidagi jadvaldagi ma'lumotlar bilan xarakterlanadi:

Ishlab chiqarish	Iste'mol qilish		Sof mahsulot
	I	II	
I	100	160	240
II	275	40	85

bevosita xarajatlar matritsasini hisoblang.

8. O'tgan davrda bir necha tarmoqlar sistemasi ishi haqidagi ma'lumotlar va kelgusi davrda Y_1 yakuniy mahsulot ishlab chiqarish rejasi quyidagi jadvalda keltirilgan:

Ishlab chiqarish	Iste'mol qilish		Sof mahsulot	Reja Y_1
	I	II		
I	80	120	300	350
II	70	30	200	300

Bevosita va to'la xarajat matritsasi, shuningdek Y_1 yakuniy mahsulot ishlab chiqarishni ta'minlovchi reja asosida yalpi mahsulot ishlab chiqarish rejasini toping.

9. Savdoning strukturali matritsasi A uchun xalqaro savdo modelida milliy daromadlarning muvozanat vektorini toping. Bu mamlakatlarning daromadlari yig'indisi 402 shartli pul birligiga teng.

$$A=\begin{pmatrix} \frac{2}{5} & \frac{3}{10} & \frac{1}{10} \\ \frac{1}{10} & \frac{2}{5} & \frac{3}{10} \\ \frac{1}{2} & \frac{3}{10} & \frac{3}{5} \end{pmatrix}.$$

10. $A=\begin{pmatrix} 0,3 & 0,2 \\ 0,4 & 0,1 \end{pmatrix}$ bevosita xarajatlar matritsasi berilgan. $\Delta X=\begin{pmatrix} 200 \\ 100 \end{pmatrix}$ yalpi mahsulot vektori o'zgarishida ΔY yakuniy vektor o'zgarishini toping.

11. Savdoning strukturali matritsasi A uchun xalqaro savdo modelida milliy daromadlarning muvozanat vektorini toping. Bu mamlakatlarning daromadlari yig'indisi 402 shartli pul birligiga teng.

$$A=\begin{pmatrix} \frac{3}{10} & \frac{1}{5} & \frac{2}{5} \\ \frac{3}{10} & \frac{1}{10} & \frac{1}{10} \\ \frac{2}{5} & \frac{7}{10} & \frac{1}{2} \end{pmatrix}.$$

12. Biror tarmoqlararo balansning S to'la xarajatlar matritsasi berilgan.
a) ΔY_1 yakuniy mahsulot orttirmasini ta'minlaydigan, ΔX_1 yalpi ishlab chiqarish orttirmasini
b) ΔX_2 yalpi ishlab chiqarish orttirmasiga mos ΔY_2 yakuniy mahsulot orttirmasini toping.

$$S=\begin{pmatrix}1,5 & 0,2 & 0,1\\ 0,5 & 1,5 & 0,3\\ 0,2 & 0,1 & 1,1\end{pmatrix};\quad a)\ \Delta Y_1=\begin{pmatrix}10\\30\\20\end{pmatrix};\quad b)\ \Delta X_2=\begin{pmatrix}5\\-10\\20\end{pmatrix}.$$

13. $A=\begin{pmatrix}0,5 & 0,3\\ 0,2 & 0,4\end{pmatrix}$ bevosita xarajatlar matritsasi berilgan.

a) S to'la xarajatlar matritsasini;

b) $Y=\begin{pmatrix}1200\\840\end{pmatrix}$ yakuniy mahsulot ishlab chiqarishni ta'minlaydigan X yalpi mahsulot vektorini;

c) $\Delta X=\begin{pmatrix}1500\\1000\end{pmatrix}$ yalpi ishlab chiqarish orttirmasiga mos, yakuniy mahsulot vektori orttirmasi ΔY ni toping.

14. Uch mamlakatdagi savdoning strukturali matrisasi quyidagi ko'rinishga ega:

$$A=\begin{pmatrix}0,3 & 0,9 & 0,5\\ 0 & 0,1 & 0,3\\ 0,7 & 0 & 0,2\end{pmatrix}.$$

Balanslangan savdo uchun bu mamlakatlarning milliy daromadlari nisbatini aniqlang.

15. $A=\begin{pmatrix}0,2 & 0,3\\ 0,6 & 0,2\end{pmatrix}$ bevosita xarajatlar matritsasi berilgan. $\Delta X=\begin{pmatrix}100\\120\end{pmatrix}$ yalpi mahsulot vektori o'zgarishida ΔY yakuniy vektor o'zgarishini toping.

16. Uch mamlakatdagi savdoning strukturali matrisasi quyidagi ko'rinishga ega:

$$A=\begin{pmatrix}0,3 & 0,3 & 0,8\\ 0,6 & 0,1 & 0,1\\ 0,1 & 0,6 & 0,1\end{pmatrix}.$$

Balanslangan savdo uchun bu mamlakatlarning milliy daromadlari nisbatini aniqlang.

17. Bevosita xarajatlar matritsasi $A=\begin{pmatrix} 0,1 & 0,5 \\ 0,2 & 0,3 \end{pmatrix}$ va yalpi ishlab chiqarish vektori $X=\begin{pmatrix} 800 \\ 900 \end{pmatrix}$ berilgan. $Y=\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ yakuniy mahsulot vektorining y_1, y_2 komponentlarini toping.

18. To'la xarajatlar matritsasi $S=\begin{pmatrix} 1,125 & 0,125 \\ 0,125 & 1,125 \end{pmatrix}$ va $Y=\begin{pmatrix} 80 \\ 80 \end{pmatrix}$ yakuniy mahsulot vektorin berilgan. $X=\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ yalpi ishlab chiqarish vektorining x_1, x_2 komponentlarini toping.

19. Uch mamlakatdagi savdoning strukturali matrisasi quyidagi ko'rinishga ega:

$$A=\begin{pmatrix} 0,5 & 0,3 & 0,6 \\ 0,4 & 0,3 & 0,1 \\ 0,1 & 0,4 & 0,3 \end{pmatrix}.$$

Balanslangan savdo uchun bu mamlakatlarning milliy daromadlari nisbatini aniqlang.

20. $A=\begin{pmatrix} 0,1 & 0,4 \\ 0,5 & 0,2 \end{pmatrix}$ bevosita xarajatlar matritsasi berilgan. $\Delta X=\begin{pmatrix} 140 \\ 100 \end{pmatrix}$ yalpi mahsulot vektori o'zgarishida ΔY yakuniy vektor o'zgarishini toping.

21. Uch mamlakatdagi savdoning strukturali matrisasi quyidagi ko'rinishga ega:

$$A=\begin{pmatrix} 0,7 & 0,3 & 0,4 \\ 0,2 & 0,5 & 0,1 \\ 0,1 & 0,2 & 0,5 \end{pmatrix}.$$

Balanslangan savdo uchun bu mamlakatlarning milliy daromadlari nisbatini aniqlang.

22.. $A=\begin{pmatrix} 0,3 & 0,2 \\ 0,4 & 0,1 \end{pmatrix}$ bevosita xarajatlar matritsasi berilgan.

$\Delta Y=\begin{pmatrix} 55 \\ 110 \end{pmatrix}$ yakuniy mahsulot vektori o'zgarishida ΔX yalpi mahsulot vektori o'zgarishini toping.

23. $A=\begin{pmatrix}0,1 & 0,4\\ 0,5 & 0,2\end{pmatrix}$ bevosita xarajatlar matritsasi berilgan. $\Delta Y=\begin{pmatrix}52\\ 104\end{pmatrix}$ yakuniy mahsulot vektori o'zgarishida ΔX yalpi mahsulot vektori o'zgarishini toping.

24. Uch mamlakat savdoning strukturali matrisasi quyidagi ko'rinishga ega

$$A=\begin{pmatrix}0,2 & 0,3 & 0,4\\ 0,5 & 0,4 & 0,2\\ 0,3 & 0,3 & 0,4\end{pmatrix}.$$

uchinchi mamlakatning byudjeti 1100 shartli pul birligiga teng bo'lgan shartida balanslangan kamomadsiz savdoni qanoatlantiruvchi birinchi va ikkinchi mamlakatlarning byudjetlari topilsin.

25. $A=\begin{pmatrix}0,2 & 0,3\\ 0,6 & 0,2\end{pmatrix}$ bevosita xarajatlar matritsasi berilgan.

$\Delta Y=\begin{pmatrix}92\\ 138\end{pmatrix}$ yakuniy mahsulot vektori o'zgarishida ΔX yalpi mahsulot vektori o'zgarishini toping.

3.8. Mathcad dasturida hisoblash

1- misol. Vektor modulini toping.

$$\left|\begin{pmatrix}x\\ y\\ z\end{pmatrix}\right| \rightarrow \sqrt{(|x|)^2+(|y|)^2+(|z|)^2}$$

$$\left|\begin{pmatrix}1\\ 3\\ 4\end{pmatrix}\right| \rightarrow \sqrt{26}$$

Skalyar ko'paytma

2-misol. Vektorlarning skalyar ko'paytmasi.

$$\begin{pmatrix}a\\ b\\ c\end{pmatrix}\cdot\begin{pmatrix}x\\ y\\ z\end{pmatrix} \rightarrow a\cdot\bar{x}+b\cdot\bar{y}+c\cdot\bar{z}$$

Bir nechta (ikkitadan ortiq) vektorlarni ko'paytirishda ehtiyot bo'lish kerak. Qavslarni qo'yishga qarab ko'paytmaning natijasi butunlay o'zgaradi. Buni quyidagi misolda ko'ramiz.

3-misol. Vektorlarning skalyar ko‘paytmasini uchinchi vektorga ko‘paytirish.

$$\left[\begin{pmatrix}1\\2\\3\end{pmatrix}\cdot\begin{pmatrix}4\\5\\6\end{pmatrix}\right]\cdot\begin{pmatrix}7\\8\\9\end{pmatrix}=\begin{pmatrix}224\\256\\288\end{pmatrix}$$

$$\begin{pmatrix}1\\2\\3\end{pmatrix}\cdot\left[\begin{pmatrix}4\\5\\6\end{pmatrix}\cdot\begin{pmatrix}7\\8\\9\end{pmatrix}\right]=\begin{pmatrix}122\\244\\366\end{pmatrix}$$

Vektor ko‘paytma

× simvol vektor ko‘paytmani bildiradi. **Матрица** (Matrix) panelidan **Векторное произведение** (Cross Product) tugmasini bosish yoki <Ctrl>+<8> klavishini birgalikda bosish bilan kiritish mumkin.

4-misol. Ikki vektorning vektor ko‘paytmasi

$$\begin{pmatrix}a\\b\\c\end{pmatrix}\times\begin{pmatrix}x\\y\\z\end{pmatrix}\rightarrow\begin{pmatrix}b\cdot z-c\cdot y\\c\cdot x-a\cdot z\\a\cdot y-b\cdot x\end{pmatrix}$$

Matritsaning xos vektorlari va xos qiymatlari

$$A:=\begin{pmatrix}1&2\\3&4\end{pmatrix}$$

$$\mathrm{eigenvals}(A)=\begin{pmatrix}-0.372\\5.372\end{pmatrix}$$

$$\mathrm{eigenvecs}(A)=\begin{pmatrix}-0.825&-0.416\\0.566&-0.909\end{pmatrix}$$

$$\mathrm{eigenvec}(A,-0.372)=\begin{pmatrix}0.825\\-0.566\end{pmatrix}$$

$$-0.372\cdot\begin{pmatrix}0.825\\-0.566\end{pmatrix}-\begin{pmatrix}1&2\\3&4\end{pmatrix}\cdot\begin{pmatrix}0.825\\-0.566\end{pmatrix}=\begin{pmatrix}10\times10^{-5}\\-4.48\times10^{-4}\end{pmatrix}$$

IV BOB. ANALITIK GEOMETRIYA ELEMENTLARI

4.1. Tekislikda to'g'ri chiziq

$$Ax + By + C = 0, \ (A^2 + B^2 \neq 0) \qquad (1)$$

(1) tenglamaga to'g'ri chiziqning umumiy tenglamasi deyiladi.

To'g'ri chiziq umumiy tenglamasining xususiy hollarini qaraymiz:

1) Agar (1) tenglamada $C=0$ bo'lsa, u holda (0,0) nuqta $Ax+By=0$ tenglamani qanoatlantiradi, ya'ni to'g'ri chiziq koordinatalar boshidan o'tadi.

2) Agar (1) tenglamada $B=0$ bo'lsa, u holda tenglama $Ax+C=0$ yoki $x=-\frac{C}{A}=a$ ko'rinishni oladi va to'g'ri chiziq Oy o'qiga parallel bo'ladi.

3) Agar (1) tenglamada $A=0$ bo'lsa, tenglama $By+C=0$ yoki $y=-\frac{C}{A}=b$ ko'rinishni oladi, to'g'ri chiziq Ox o'qiga parallel bo'ladi.

4) Agar (1) tenglamada $B=C=0$ bo'lsa, tenglama $Ax=0$ yoki $x=0$ bo'lib, bu Oy o'qining tenglamasi bo'ladi.

5) Agar (1) tenglamada $A=C=0$ bo'lsa, tenglama $By=0$ yoki $y=0$ bo'lib, bu Ox o'qining tenglamasi bo'ladi.

$B\neq 0$ bo'lsin. U holda to'g'ri chiziqning $Ax+By+C=0$ umumiy tenglamasini y ga nisbatan yechmiz va $-\frac{A}{B}=k;\ -\frac{C}{B}=b$ deb belgilab:

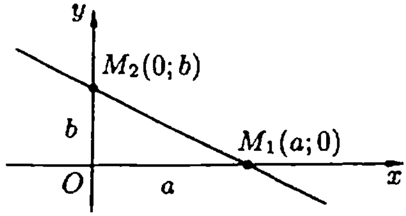
$$y = kx + b \qquad (2)$$

tenglama hosil bo'ladi. Bu to'g'ri chiziqning burchak koeffitsiyentli tenglamasi deyiladi. $k=tg\varphi$ to'g'ri chiziqning burchak koeffitsiyenti deyiladi, bu yerda φ to'g'ri chiziqning Ox o'qining musbat yo'nalishi bilan hosil qilgan burchagi, b to'g'ri chiziqning Oy o'q bilan (2) tenglamaning kesishish nuqtasi.

To'g'ri chiziqning $Ax+By+C=0$ umumiy tenglamasida $A\cdot B\cdot C\neq 0$ bo'lsin. U holda tenglamada almashtirlar bajarib tegishli belgilashlar kiritsak:

$$Ax+By=-C;\ \frac{x}{-\frac{C}{A}}+\frac{y}{-\frac{C}{B}}=1,\ \ -\frac{C}{A}=a;\ \ -\frac{C}{B}=b$$

$$\frac{x}{a}+\frac{y}{b}=1 \qquad (3)$$



(1) tenglama (3) ko‘rinishiga keladi. (3) ko‘rinishdagi tenglama to‘g’ri chiziqning kesmalar boyicha tenglamasi deyiladi. a va b to‘g’ri chiziqning mos ravishda Ox va Oy oqlarni kesish nuqtalari bo‘ladi.

Berilgan ikki: $M_1(x_1,y_1)$, $M_2(x_2,y_2)$ nuqtalardan o‘tuvchi to‘g’ri chiziq tenglamasini topamiz. M_1 nuqtadan o‘tuvchi to‘g’ri chiziq tenglamasi

$$A(x-x_1)+B(y-y_1)=0. \quad (4)$$

Agar bu to‘g’ri chiziq M_2 nuqtadan ham o‘tsa

$$A(x_2-x_1)=-B(y_2-y_1) \quad (5)$$

shart bajariladi. (4) va (5) dan

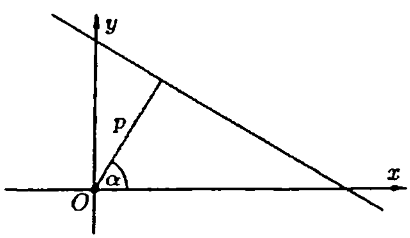
$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1} \quad (6)$$

hosil bo‘ladi. (6) tenglamaga berilgan ikkita nuqtadan o‘tuvchi to‘g’ri chiziq tenglamasi deyiladi.

Agar to‘g‘ri chiziq umumiy tenglamasining ikki tomonini $\lambda=1/\pm\sqrt{A^2+B^2}$ songa ko‘paytirsak (λ – normallashtiruvchi ko‘paytuvchi, ildiz oldidagi ishorani shunday tanlaymizki $\lambda C<0$ bo‘lsin),

$$x\cos\alpha+y\sin\alpha-p=0 \quad (7)$$

ga ega bo‘lamiz. (7) tenglikka to‘g‘ri chiziqning normal tenglamasi deyiladi. Bu yerda p koordinatalar boshidan to‘g‘ri chiziqqa tushirilgan perpendikulyarning uzunligi, α – perpendikulyar bilan Ox o‘qining musbat yo‘nalishi orasidagi burchak.



Endi $y=k_1x+b_1$, (I) va $y=k_2x+b_2$ (II) to'g'ri chiziqlar orasidagi burchakni topamiz. $\theta=\varphi_2-\varphi_1$. Bundan

$tg\theta=tg(\varphi_2-\varphi_1)=\frac{tg\varphi_2-tg\varphi_1}{1+tg\varphi_1\, tg\varphi_2}$, $tg\varphi_1=k_1$, $tg\varphi_2=k_2$ ekanligini va to'g'ri chiziqlar orasidagi burchak ta'rifini hisobga olsak:

$$tg\theta=\left|\frac{k_2-k_1}{1+k_1k_2}\right| \qquad (8)$$

ga bo'lamiz. (8) tenglamaga ikki to'g'ri chiziq orasidagi burchakni topish formulasi deyiladi.

Agar to'g'ri chiziqlar parallel bo'lsa, $\theta=0^0$ va $tg\theta=0$ bo'ladi. Bundan $k_2-k_1=0$ yoki $k_2=k_1$ kelib chiqadi va bu to'g'ri chiziqlarning parallelik shartini ifodalaydi. Agar to'g'ri chiziqlar perpendikulyar bo'lsa $\varphi_2-\varphi_1=90^0$ bo'ladi. Bundan $\varphi_2=90^0+\varphi_1$, $tg\varphi_2=tg(90^0+\varphi_1)=-ctg\varphi_1$ yoki $tg\varphi_1\cdot tg\varphi_2=-1$ yoki $k_1k_2=-1$ hosil bo'ladi. Bu ikki to'g'ri chiziqni perpendikulyarlik sharti deyiladi.

Umumiy tenglamalari $A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ bo'lgan to'g'ri chiziqlar orasidagi burchak, quyidagi formula bo'yicha hisoblanadi:

$$\cos\theta=\frac{|\vec{n}_1\cdot\vec{n}_2|}{|\vec{n}_1|\cdot|\vec{n}_2|}=\frac{|A_1A_2+B_1B_2|}{\sqrt{A_1^2+B_1^2}\cdot\sqrt{A_2^2+B_2^2}} \qquad (9)$$

Bundan ikkita to'g'ri chiziqning

$$\frac{A_1}{A_2}=\frac{B_1}{B_2} \qquad \text{parallellik;}$$

$$A_1A_2+B_1B_2=0 \qquad \text{perpendikulyarlik}$$

shartlari kelib chiqadi.

Berilgan nuqtadan berilgan to'g'ri chiziqqacha masofa. Faraz qilamiz $M(x_0,y_0)$ nuqta va $l: Ax+By+C=0$ to'g'ri chiziq berilgan bo'lsin. $M(x_0,y_0)$ nuqtadan l to'g'ri chiziqqacha bo'lgan masofa deyilganda $M(x_0,y_0)$ nuqtadan l to'g'ri chiziqqa tushurilgan perpendikulyar uzunligi tushuniladi. $d=MN$ masofani aniqlash uchun:

a) $M(x_0,y_0)$ nuqtadan o'tuvchi va berilgan to'g'ri chiziqqa perpendikulyar bo'lgan MN to'g'ri chiziq tenglamasi tuziladi; b) bu tenglamalar sistemasi yechilib ularning $N(x_1,y_1)$ kesishish nuqtasi topiladi; c) ikki nuqta orasidagi masofa formulaga asosan $d=MN$ masofa topiladi. Natijada,

$$d=\left|\frac{Ax_0+By_0+C}{\sqrt{A^2+B^2}}\right| \qquad (10)$$

formulaga ega bo'lamiz. (10) ga berilgan nuqtadan berilgan to'g'ri chiziqqacha masofani topish formulasi deyiladi.

Agar $A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ to'g'ri chiziqlar kesishsa, u holda $A_1x+B_1y+C_1+\lambda(A_2x+B_2y+C_2)=0$ (11) tenglama (λ – sonli ko'paytuvchi) to'g'ri chiziqlarning kesishgan nuqtasidan o'tadigan to'g'ri chiziqni anglatadi. (11) da λ ga turli qiymatlar berib, markaz deb ataluvchi kesishgan nuqtadan o'tuvchi to'g'ri chiziqlar dastasiga tegishli har xil to'g'ri chiziqlarni hosil qilamiz.

Misollar

1. $(2;6)$ nuqtadan o'tuvchi va Ox o'q bilan $arctg5$ burchak tashkil etuvchi to'g'ri chiziqning tenglamasini tuzing.

Yechish. To'g'ri chiziqning burchak koeffitsiyentli tenglamasini tuzish uchun k va b ni hisoblash kerak. $k=tg(arctg5)=5$, b ni hisoblash uchun $y=kx+b$ tenglamaga k ning topilgan qiymatini hamda x va y o'zgaruvchilarning o'rniga berilgan nuqtaning koordinatalarini qo'yamiz. $6=5\cdot 2+b$ bu yerdan $b=-4$. Izlanayotgan tenglama $y=5x-4$.

2. $3x+2y+6=0$ to'g'ri chiziqning Ox o'qqa og'ish burchagini hisoblang.

Yechish. $3x+2y+6=0$ tenglamani y ga nisbatan yechib, $y=-\frac{3}{2}x-3$ ni hosil qilamiz, bu yerdan $k=-\frac{3}{2}$, biroq $k=tg\alpha$; demak, $tg\alpha=-\frac{3}{2}$. Jadvaldan $\alpha=180^0-56^019'=123^041'$ ni topamiz.

3. To'g'ri chiziq Ox o'qdan 3 ga, Oy o'qdan 5 ga teng kesma ajratadi. Bu to'g'ri chiziqning tenglamasini tuzing

Yechish. Masala shartida $a=3$ va $b=5$. Bu qiymatlarni $\frac{x}{a}+\frac{y}{b}=1$ – to‘g‘ri chiziqning koordinata o‘qlaridan ajratgan kesmalari bo‘yicha tenglamasiga qo‘yamiz: $\frac{x}{3}+\frac{y}{5}=1$ ga ega bo‘lamiz.

4. $M(-2;4)$ nuqtadan $2x-3y+6=0$ to‘g‘ri chiziqqa parallel bo‘lib o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzing.

Yechish. $2x-3y+6=0$ to‘g‘ri chiziqning burchak koeffitsiyentini topish uchun uning tenglamasini $y$ga nisbatan yechamiz: $y=\frac{2}{3}x+2$, bu yerdan burchak koeffitsiyenti $k_1=\frac{2}{3}$. Izlanayotgan to‘g‘ri chiziqning burchak koeffitsiyenti berilgan to‘g‘ri chiziqning burchak koeffitsiyentiga teng bo‘ladi, chunki to‘g‘ri chiziqlar parallel, ya’ni $k_1=k_2=\frac{2}{3}$. Izlanayotgan to‘g‘ri chiziq $M(-2;4)$ nuqtadan o‘tadi va $k_2=\frac{2}{3}$ burchak koeffitsiyentga ega bo‘ladi. Bu qiymatlarni berilgan nuqtadan berilgan yo‘nalishda o‘tuvchi to‘g‘ri chiziqning tenglamasiga qo‘yib, $y-4=\frac{2}{3}(x+2)$ yoki $2x-3y+16=0$ ni hosil qilamiz.

5. $M(2;3)$ nuqtadan $5x-4y-20=0$ to‘g‘ri chiziqqa perpendikulyar bo‘lib o‘tuvchi to‘g‘ri chiziqning tenglamasini tuzing.

Yechish. $5x-4y-20=0$ to‘g‘ri chiziqning burchak koeffitsiyenti $k_1=\frac{5}{4}$. Izlanayotgan to‘g‘ri chiziqning burchak koeffitsiyentini $k_2=-\frac{1}{k_1}$ formula bo‘yicha topamiz: $k_2=-\frac{1}{k_1}=-\frac{4}{5}$. $k_2=-\frac{4}{5}$ ni va $M(2;3)$ nuqtaning koordinatalarini to‘g‘ri chiziqlar dastasining tenglamasiga qo‘yib, $y-y_0=k(x-x_0)$ $y-3=-\frac{4}{5}(x-2)$ yoki $4x+5y-23=0$ ni hosil qilamiz.

6. $M(-4,\ 1)$ nuqtadan $3x+4y-8=0$ to‘g’ri chiziqqacha masofani toping?

Yechish. Nuqtadan to‘g’ri chiziqqacha masofani hisoblash formulasini qo‘llaymiz:

$$d=\frac{|3\cdot(-4)+4\cdot1-8|}{\sqrt{3^2+4^2}}=\frac{16}{5}.$$

4.2. Tekislikda ikkinchi tartibli egri chiziqlar

1-ta'rif. x va y o'zgaruvchilarga nisbatan ikkinchi darajali

$$Ax^2+2Bxy+Cy^2+2Dx+2Ey+F=0\ \left(A^2+B^2+C^2\neq0\right)\quad(1)$$

ko'rinishidagi algebraik tenglama bilan aniqlangan chiziq, ikkinchi tartibli egri chiziq deyiladi.

(1) da agar $A=C\neq0,\ B=0$ bo'lsa aylana tenglamasi hosil bo'ladi.

2-ta'rif. Fiksirlangan $M_0(a,b)$ nuqtadan bir xil R- masofada yotgan nuqtalarning geometrik o'rniga aylana deyiladi.

Bu yerda $M_0(a,b)$ nuqta markaz deb, R masofa esa radius deb ataladi.

Aylana tenglamasini keltirib chiqaramiz. $M(x,y)$ aylana chizig'ida yotgan ixtiyoriy nuqta bo'lsin. Ta'rifga ko'ra $|M_0M|=R$

$$\sqrt{(x-a)^2+(y-b)^2}=R$$

Bundan quyidagi tenglamani topamiz

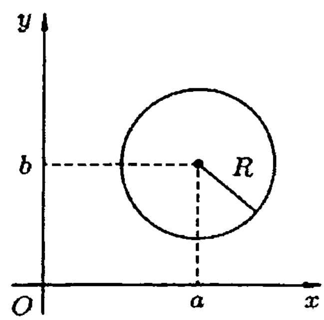
$$(x-a)^2+(y-a)^2=R^2\qquad(2)$$

(2) aylananing kanonik tenglamasi deb ataladi. Agar aylana markazi koordinatalar boshi O(0,0) nuqtada bo'lsa, uning tenglamasi $x^2+y^2=R^2$ ko'rinishni oladi.

Misol. $x^2+y^2-6x-7=0$ aylana markazining koordinatasini va radiusini toping.

Yechish. Tenglamada x va y ga nisbatan to'la kvadrat ajratamiz: $(x-3)^2+y^2=7+9=16=4^2$. Bundan $R=4$ va aylana markazi $M_0(3,0)$ ni topamiz.

3-ta'rif. Fiksirlangan F_1 va F_2 nuqtalargacha bo'lgan masofalar yig'indisi o'zgarmas $2a$ kattalikka teng bo'lgan nuqtalarning geometrik o'rniga ellips deyiladi.



Bu yerda F_1 va F_2 nuqtalar ellipsning fokuslari deb ataladi. $F_1(-c,0)$ va $F_2(c,0)$ nuqtalar Ox o'qida joylashgan bo'lsin. U holda ta'rifga ko'ra ellips tenglamasi $|F_1M|+|F_2M|=2a$ bo'ladi. Uchburchak tengsizligiga asosan $2a>2c$ ekanligi ko'rinadi. Ellips tenglamasida koordinatalarga o'tsak, $\sqrt{(x-c)^2+y^2}+\sqrt{(x+c)^2+y^2}=2a$ ni hosil qilamiz. Bu tenglamani soddalashtirib $\frac{x^2}{a^2}+\frac{y^2}{a^2-c^2}=1$ ko'rinishga keltiramiz. $a^2-c^2=b^2$ belgilash kiritib ellipsning kanonik tenglamasi deb ataluvchi

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad (3)$$

tenglamani hosil qilamiz.

(3) ko'rinishdagi tenglama bilan berilgan ellips uchun quyidagi xossalarni keltirish mumkin:

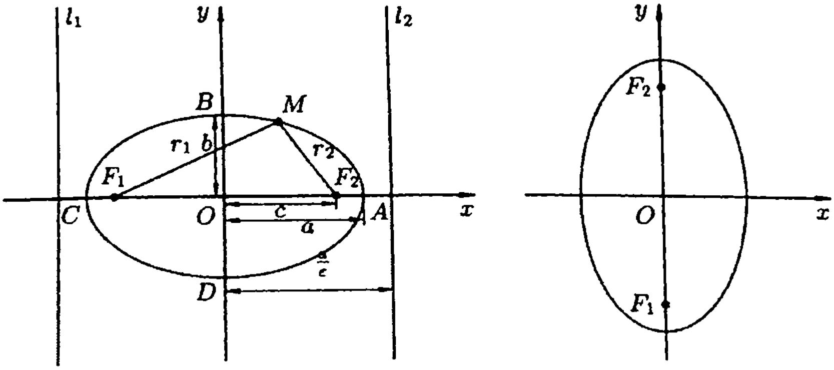
1) Ellips grafigi Ox va Oy o'qlariga nisbatan simmetrik joylashgan;

2) Koordinatalar boshi uning simmetriya markazi;

3) Koordinata o'qlari simmetriya o'qlari bo'ladi.

Fokuslar joylashgan o'q ellipsning fokus (fokal) o'qi deyiladi.

Ellipsning koordinata o'qlari bilan kesishgan nuqtalari uning uchlari deyiladi. (3) tenglamadan $A_1(a,0), A_2(-a,0)$ uchlarni, $B_1(0,b), B_2(0,-b)$ uchlarni topamiz. $|A_2A_1|=2a, |B_2B_1|=2b$ kesmalar $(a>b)$ mos ravishda ellipsning katta (fokal) o'qi va kichik (fokal) o'qi deyiladi. a va b kesmalar esa mos ravishda katta yarim o'q va kichik yarim o'q deyiladi. Ellips rasmini chizish uchun uni $x\geq 0$ *va* $y\geq 0$ bo'lgan holda tekshirish yetarli. Agar x 0 dan a gacha o'sib borsa, y b dan 0 gacha kamayadi. (3) tenglamada $b=a$ bo'lsa, tenglama $x^2+y^2=a^2$ ko'rinishni oladi, ya'ni aylana tenglamasiga ega bo'lamiz. Demak, aylana ellipsning xususiy holidir (ellips fokuslari birlashsa, c=0 holat).



O‘qlari koordinata o‘qlariga parallel bo‘lgan ellipsning kanonik tenglamasi $\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1$ ko‘rinishda bo‘ladi. (x_0,y_0) ellips markazining koordinatalari.

Ellips fokuslari orasidagi $2c$ masofani katta o‘q $2a$ ga nisbati uning ekssentrisiteti deyiladi va ε bilan belgilanadi: $\varepsilon=\frac{2c}{2a}$ *yoki* $\varepsilon=\frac{c}{a}$

Har qanday ellips uchun $0<\varepsilon<1$ bo‘lib, ε ellipsning cho‘zinchoqligini yoki siqilganligini bildiradi.

Tenglamalari $x=\pm\frac{a}{\varepsilon}=\pm\frac{a^2}{c}$ bo‘lgan l_1 va l_2 to‘g’ri chiziqlar ellipsning direktrisalari deyiladi. Ellipsning ixtiyoriy nuqtasidan fokusgacha bo‘lgan masofa r_i $(i=1,2)$ ni mos direktrisasigacha bo‘lgan masofa d_i $(i=1,2)$ ga nisbati o‘zgarmas son bo‘lib ε ekssentrisitetga teng, yani $\frac{r_1}{d_1}=\frac{r_2}{d_2}=\varepsilon$

4-ta’rif. Fiksirlangan F_1 va F_2 nuqtalargacha bo‘lgan masofalar ayirmasining absolyut qiymati o‘zgarmas $2a$ kattalikka teng bo‘lgan nuqtalarning geometrik o‘rniga giperbola deyiladi.

Bu yerda F_1 va F_2 nuqtalar giperbolaning fokuslari deb ataladi.

Giperbola tenglamasini topish uchun giperbolaga tegishli $M(x,y)$ nuqtani hamda $F_1(-c,0)$, $F_2(c,0)$ nuqtalarni olib, ta’rifga asosan $|F_1M-F_2M|=2a$ tenglikni hosil qilamiz. Bundan koordinatalardagi tenglamalarga o‘tamiz va ma’lum bir soddalashtirishlardan so‘ng

$$\frac{x^2}{a^2}-\frac{y^2}{c^2-a^2}=1$$

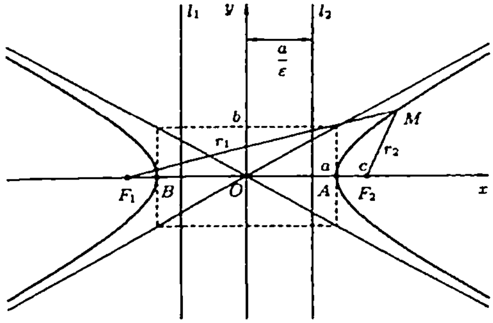
tenglamaga ega bo‘lamiz. $c>a$ ekanligini hisobga olib, $b^2=c^2-a^2$ belgilash kiritamiz va giperbolaning kanonik tenglamasini topamiz:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad (4)$$

(4) tenglamani (x,y) nuqta qanoatlantiradi, demak Ox va Oy o‘qlari giperbolaning simmetriya orqali bo‘ladi. Fokuslar yotgan o‘q giperbolaning fokal o‘qi deyiladi. Simmetriya o‘qlarining kesishish nuqtasi simmetriya markazi, ya’ni giperbola markazi deyiladi.

Agar (4) tenglamada $y=0$ deb, $x=\pm a$ ni topamiz. $A_1(a,0)$, $A_2(-a,0)$ nuqtalar giperbolaning haqiqiy uchlari deyiladi va ular orasidagi masofa $2a$ ga teng bo‘ladi. (4) tenglamada $x=0$ deb, $y^2=-b^2$, $y=\pm bi$ ni topamiz. Bu esa

(4) formula bilan aniqlanqdigan giperbola Oy o‘q bilan kesishmasligini bildiradi. $B_1(0,b)$, $B_2(0,-b)$ nuqtalar giperbolaning mavhum uchlari deyiladi va ular orasidagi masofa $2b$ ga teng bo‘ladi.



$A_1A_2 = 2a$ kesma giperbolaning haqiqiy (fokal) o‘qi, $B_1B_2 = 2b$ kesma esa mavhum o‘qi deyiladi. a va b kesmalar giperbolaning haqiqiy va mavhum yarim o‘qlari deyiladi.

$y = \pm\frac{b}{a}x$ to‘g’ri chiziqlar (4) giperbolaning asimptotalari deyiladi.

Bu to‘g’ri chiziqlar markazi koordinatalar boshida bo‘lib, tomonlari $2a$ va $2b$ ga teng bo‘lgan to‘g’ri to‘rtburchak (giperbolaning asosiy to‘rtburchagi) dioganallaridan o‘tadi. Giperbolaning grafigini chizishda oldin asimptotalarini chizish maqsadga muvofiq.

Giperbolani tekshirish uchun $x \geq 0$, $y \geq 0$ holatni tekshirish yetarli bo‘ladi.

Ellipsdagi kabi giperbola uchun ham $\varepsilon = \frac{c}{a}$ tenglik bilan aniqlanuvchi kattalik giperbolaning ekssentrisiteti deyiladi. Giperbola uchun $\varepsilon > 1$.

Ekssentrisitet giperbolaning asosiy to‘g’ri to‘rtburchagini aniqlaydi.

Fokal o‘qqa perpendikulyar bo‘lgan tenglamalari $x = \pm\frac{a}{\varepsilon} = \pm\frac{a^2}{c}$ bo‘lgan l_1 va l_2 to‘g’ri chiziqlar giperbolaning direktrisalari deyiladi.

Giperbolaning ixtiyoriy nuqtasidan fokusgacha masofaning shu nuqtadan mos direktrisagacha bo‘lgan masofaga nisbati o‘zgarmas miqdordir: $\frac{r_1}{d_1} = \frac{r_2}{d_2} = \varepsilon$ (4-rasm).

Agar giperbolada $b=a$ bo‘lsa, giperbola teng yonli giperbola deyiladi, uning tenglamasi $x^2 - y^2 = a^2$ ko‘rinishda bo‘ladi. Uning asimptolalari $y = \pm x$

bo‘lib o‘zaro perpendikulyar hamda asosiy to‘g’ri to‘rtburchagi kvadratdan iborat bo‘ladi.

Simmetriya markazi $M_0(x_0,y_0)$ nuqtada va simmetriya o‘qlari koordinata o‘qlariga parallel bo‘lgan giperbolaning tenglamasi

$$\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{b^2}=1$$

ko‘rinishda bo‘ladi.

Agar giperbolaning haqiqiy o‘qi Oy o‘qida yotsa yoki unga parallel bo‘lsa, tenglamasi mos ravishda

$$\frac{y^2}{b^2}-\frac{x^2}{a^2}=1; \qquad \frac{(y-y_0)^2}{b^2}-\frac{(x-x_0)^2}{a^2}=1$$

ko‘rinishda bo‘ladi.

5-ta’rif. Berilgan F nuqtadan berilgan va berilgan to‘g’ri chizig’idan bir xil uzoqlikda yotuvchi nuqtalarning geometrik o‘rniga parabola deyiladi.

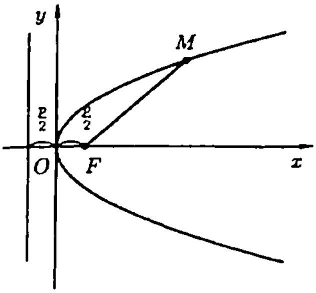
Bu yerda F nuqta fokus, to‘g’ri chiziq esa direktrisa deb ataladi. $F\left(\frac{p}{2},0\right)$ – fokus nuqtadan o‘tib direktrisaga perpendikulyar bo‘lgan chiziqni Ox o‘q sifatida qabul qilamiz va yo‘nalishini direktrisadan fokusga qarab olamiz. Oy o‘qini direktrisa bilan fokus orasidagi kesmaning o‘rtasidan perpendikulyar qilib olamiz. U holda ta’rifga ko‘ra

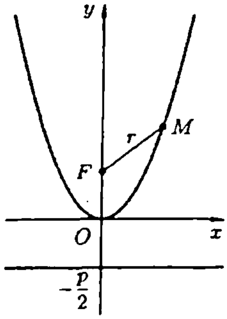
$$\sqrt{(x-\frac{p}{2})^2+y^2}=\sqrt{(x+\frac{p}{2})^2}.$$

Bu tenglamani soddalashtirib parabolaning

$$y^2=2px \qquad (5)$$

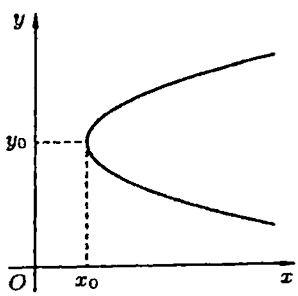
kanonik tenglamasini hosil qilamiz, bu erda p fokusdan direktrisagacha masofa bo‘lib, parabolaning parametri deyiladi. (5) tenglamadan x manfiy qiymatlarni qabul qilmasligi ko‘rinadi, ya’ni parabolaning barcha nuqtalari Oy o‘qdan o‘ngda joylashgan. x ning har bir qiymatiga y ning ikkita qiymati mos keladi. Bu qiymatlar bir-biridan faqat ishorasi bilan farq qiladi, yani Ox o‘qining musbat qismi parabolaning simmetriya o‘qi bo‘ladi va (0,0) nuqtadan o‘tadi. x 0 dan boshlab qiymatlar qabul qilib o‘sib borsa, $|y|$ ham o‘sadi.

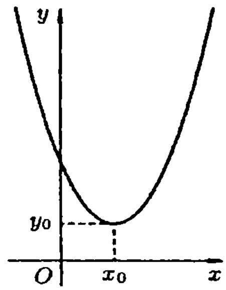




Ta'rifga ko'ra parabola uchun ekssentrisitet $\varepsilon=1$, direktrisa tenglamasi $x=-\frac{p}{2}$ bo'ladi. Fokal radius $r=x+\frac{p}{2}$ formula bilan hisoblanadi.

Agar parabolaning simmetriya o'qi $0y$ o'qida bo'lsa, uholda uning tenglamasi $x^2=2py$ bo'lib, fokusi $F\left(0,\frac{p}{2}\right)$, direktrisasi $y=-\frac{p}{2}$ bo'ladi. Fokal radius $r=y+\frac{p}{2}$ formula bilan hisoblanadi.





Agar parabolaning uchi (x_0,y_0) nuqtada bo'lib, simmetriya o'qi koordinata o'qlaridan birortasiga parallel bo'lsa, uning tenglamasi $(y-y_0)^2=2P(x-x_0)$ yoki $(x-x_0)^2=2P(y-y_0)$ bo'ladi.

Misollar

1. $(-1;3)$, $(0;2)$, $(1;-1)$ nuqtalar orqali o'tuvchi aylana tenglamasini yozing.

Yechish. Aylana tenglamasini $(x-a)^2+(y-b)^2=R^2$ ko'rinishida izlaymiz. Berilgan nuqtalarning koordinatalarini tenglama qo'yib quyidagi tenglamalar sistemasiga ega bo'lamiz:

$$\begin{cases}(-1-a)^2+(3-b)^2=R^2,\\ a^2+(2-b)^2=R^2,\\ (1-a)^2+(-1-b)^2=R^2.\end{cases}$$

Sistemadan a, b va R ning qiymatlarini aniqlaymiz.

Sistemaning birinchi ikkita tenglamasidan quyidagilarni olamiz: $(-1-a)^2+(3-b)^2=a^2+(2-b)^2$, $1+2a+a^2+9-6b+b^2=a^2+4-4b+b^2$, $a-b=-3$; sistemaning ikkinchi va uchinchi tenglamasidan quyidagilarni olamiz: $a^2+(2-b)^2=(1-a)^2+(-1-b)^2$, bu yerdan $a-3b=-1$.

$$\begin{cases}a-b=-3,\\ a-3b=-1.\end{cases}$$

sistemani yechib $a=-4, b=-1$ ni topamiz. a va b ning qiymatlarini boshlang'ich sistemaning ikkinchi tenglamasiga qo'yib R^2 ni topamiz: $16+9=R^2$, $R^2=25$. Izlanayotgan tenglama $(x+4)^2+(y+1)^2=25$.

Aylana tenglamasini $x^2+y^2+2Dx+2Ey+F=0$ ko'rinishida ham izlash mumkin. Berilgan uchta nuqtaning koordinatalarini aylana tenglamasiga qo'yib quyidagi sistemaga ega bo'lamiz:

$$\begin{cases}10-2D+6E+F=0,\\ 4+4E+F=0,\\ 2+2D-2E+F=0.\end{cases}$$

Sistemani yechib $D=4$, $E=1$, $F=-8$ ni topamiz, izlanayotgan aylana tenglamasi $x^2+y^2+8x+2y-8=0$.

2. $24x^2+49y^2=1176$ ellips tenglamasi berilgan

1)ellips yarim o'qlari uzunliklari;

2)fokuslari koordinatalarini;

3)ellips ekssentrisiteti;

4)direktrisalari tenglamalari va ular orasidagi masofa

5)ellipsning F_1 chap fokusidan 12 ga teng masofada yotuvchi nuqtasining koordinatalarini toping.

Yechish. $24x^2+49y^2=1176$ tenglamaning ikkala tomonini 1176 ga bo'lib kanonik tenglamasiga keltiramiz: $\frac{x^2}{49}+\frac{y^2}{24}=1$.

1)bu yerdan $a^2=49,\ b^2=24$ demak, $a=7,\ b=2\sqrt{6}$.

2)$c^2=a^2-b^2$ munosabatdan foydalanib, $c^2=7^2-\left(2\sqrt{6}\right)^2=25,\ c=5$ ni topamiz. Demak, $F_1=(-5;0)$ va $F_2=(5;0)$.

3)$\varepsilon=\frac{c}{a}$ formuladan foydalanib $\varepsilon=\frac{5}{7}$ ni topamiz.

4)Direktrisa tenglamasi $x=\pm\frac{7}{\frac{5}{7}},\quad x=\frac{49}{5}$ va $x=-\frac{49}{5}$. Direktrisalar orasidagi masofa $d=\frac{49}{5}-\left(-\frac{49}{5}\right)=\frac{98}{5}=19{,}6$.

5)$r_1=a+\varepsilon x$ formula bo'yicha F_1 nuqtadan 12 ga teng masofada yotuvchi nuqtasining absissasini topamiz: $12=7+\frac{5}{7}x,\quad x=7$. x ning qiymatini ellips tenglamasiga qo'yib, bu nuqtaning ordinatasini topamiz: $24\cdot 49+49y^2=1176,\quad 49y^2=0,\quad y=0$. $A(7;0)$ nuqta masala shartini qanoatlantiradi.

3. $M_1=\left(2;-4\sqrt{3}\right)$ va $M_2=\left(-1;2\sqrt{15}\right)$ nuqtalar orqali o'tuvchi ellips tenglamasini tuzing.

Yechish. Ellips $M_1=\left(2;-4\sqrt{3}\right)$ va $M_2=\left(-1;2\sqrt{15}\right)$ nuqtalar orqali o'tadi, M_1 va M_2 nuqtalarning koordinatalari ellips tenglamasini qanoatlantiradi. Quyidagi tenglamalar sistemasiga ega bo'lamiz:

$$\begin{cases}\frac{4}{a^2}+\frac{48}{b^2}=1,\\ \frac{1}{a^2}+\frac{60}{b^2}=1.\end{cases}$$

Bu sistemani yechib $a^2=16,\ b^2=64$ ga ega bo'lamiz. Shuning uchun izlanayotgan ellips tenglamasi $\frac{x^2}{16}+\frac{y^2}{64}=1$.

4. $5x^2-4y^2=20$ giperbola tenglamasi berilgan. Quyidagilarni toping:

1)giperbola yarim o'qlari uzunliklari;

2)fokuslari koordinatalarini;

3)giperbola ekssentrisiteti;

4)asimptotalari va direktrisalari tenglamalari;

5) $M(3;2,5)$ nuqtadagi fokal radiuslarini.

Yechish. $5x^2-4y^2=20$ tenglamaning ikkala tomonini 20 ga bo'lib giperbolaning kanonik tenglamasiga keltiramiz: $\frac{x^2}{4}-\frac{y^2}{5}=1$.

Bu yerdan:

1) $a^2=4,\ b^2=5$, bundan $a=2,\ b=\sqrt{5}$.

2) $c^2=a^2+b^2$ munosabatdan foydalanib, $c^2=4+5,\ c=3$ ni topamiz. Demak, $F_1=(-3;0)$ va $F_2=(3;0)$.

3) $\varepsilon=\frac{a}{c}$ formuladan foydalanib $\varepsilon=\frac{3}{2}$ ni topamiz.

4)asimptotalari va direktrisalari tenglamalari $y=\pm\frac{\sqrt{5}}{2}x$ va $x=\pm\frac{4}{3}$;

5) $M(3;2,5)$ nuqta giperbolaning o'ng pallasida yotadi. $r_1=a+\varepsilon x$, $r_2=-a+\varepsilon x$ formulalardan foydalanib fokal radiuslarini topamiz: $r_1=2+\frac{3}{2}\cdot 3=6,5,\ r_2=-2+\frac{3}{2}\cdot 3=2,5$.

5. Fokuslari Oy o'qida yotgan va fokuslari orasidagi masofa 10 ga teng, haqiqiy o'qi uzunligi 8 ga teng giperbola tenglamasini tuzing.

Yechish. Izlanayotgan giperbola tenglamasi $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$ ko'rinishda. Masala shartidan $2c=10,\ c=5;\ 2b=8,\ b=4.\ c^2=a^2+b^2$ munosabatdan foydalanib, kichik yarim o'q a ni topamiz: $25=a^2+16,\ a^2=9,\ a=3$. Izlanayotgan giperbola tenglamasi $\frac{y^2}{16}-\frac{x^2}{9}=1$.

6. $x^2=4y$ parabola berilgan. Parabolaning fokusi koordinatalarini, direktrisasi tenglamasini, $M(4;4)$ nuqtadagi fokal radiusi uzunligini toping.

Yechish. Parabola $x^2=2py$ ko'rinishidagi kanonik tanglamasi bilan berilgan. Shuning uchun, $2p=4,\ p=2$. $F\left(0;\frac{p}{2}\right)$ formuladan fokusi $(0;1)$

koordinataga ega, $y=-\frac{p}{2}$ direktrisa tenglamasidan $y=-1$; $M(4;4)$ nuqtadagi fokal radiusi $r=y+\frac{p}{2}=4+1=5$ ga teng.

4.3. Fazoda tekislik va to`g`ri chiziq tenglamalari

$M_0(x_0,y_0,z_0)$ nuqtadan o'tuvchi $\vec{n}=(A;B;C)$ vektorga perpendikulyar tekislik tenglamasi:

$$A(x-x_0)+B(y-y_0)+C(z-z_0)=0. \qquad (1)$$

$Ax+By+Cz+D=0$ (2) tenglama tekislikning umumiy tenglamasi deyiladi. (2) umumiy tenglamani uning koeffitsiyentlariga nisbatan tirli holatlarda ko'rib chiqamiz:

1) $D=0$ bo'lsa, tenglama $Ax+By+Cz=0$ ko'rinishida bo'lib, uni $O(0,0,0)$ nuqtaning koordinatalari qanoatlantiradi, ya'ni bu tekislik koordinatalar boshidan o'tadi.

2) $C=0$ bo'lsa, tenglama $Ax+By+D=0$ ko'rinishni oladi va bu XOY tekislikdagi proeksiyasi $Ax+By+D=0$ to'g'ri chiziqdan iborat bo'lgan tekislik tenglamasi bo'ladi. Bu tekislik Oz o'qiga parallel.

Xuddi shunga o'xshash, $B=0$ (Oy o'qiga parallel bo'lgan) va $A=0$ (Ox o'qiga parallel bo'lgan) bo'lgandagi $Ax+Cz+D=0$ va $By+Cz+D=0$ tekisliklarni hosil qilamiz.

3) $C=D=0$ bo'lsa, tenglama $Ax+By=0$ ko'rinishda bo'lib, u koordinatalar boshidan va Oz o'qidan o'tuvchi tekislik tenglamasi bo'ladi.

Xuddi shunga o'xshash, $Ax+Cz=0$ $(B=D=0)$ va $By+Cz=0$ $(A=D=0)$ tenglamalar Oy va Ox o'qlaridan o'tuvchi tekisliklarni beradi.

4) $B=C=0$ bo'lsa, $Ax+D=0$, $x=-\frac{D}{A}$ hosil bo'ladi. Bu esa YOZ tekisligiga parallel bo'lgan tekislik tenglamasidir.

Xuddi shunga o'xshash, $y=-\frac{D}{B}$ $(A=C=0)$ va $z=-\frac{D}{C}$ $(A=B=0)$ tenglamalar XOZ va XOY tekisliklariga parallel bo'lgan tekisliklarning tenglamalaridir.

5) $B=C=D=0$ bo'lsa, $Ax=0$ yoki $x=0$ bo'lib, bu YOZ tekislikning tenglamasidir.

Xuddi shunga o'xshash, $y=0$ $(A=C=D=0)$ va $z=0$ $(A=B=D=0)$ tenglamalar mos ravishda XOZ va XOY tekisliklarning tenglamalaridan iboratdir.

Tekislikning (2) umumiy tenglamasini quyidagicha o'zgartirib yozamiz $(A\cdot B\cdot C\cdot D\neq 0)$

$$Ax+By+Cz=-D \Rightarrow \frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1. \qquad (3)$$

Bu yerda, $a=-\frac{D}{A}$, $b=-\frac{D}{B}$, $c=-\frac{D}{C}$.

(3) tekislikni kesmalar boyicha tenglamasi deyiladi.

Bunda a,b,c tekislikning koordinat o'qlarida kesib o'tgan nuqtalari.

Misol. $2x-3y+z-6=0$ tenglamani kesmalardagi tenglamaga keltiramiz. Buning uchun tenglamaning har bir hadini 6 ga bo'lamiz va

$$\frac{x}{3}+\frac{y}{-2}+\frac{z}{6}=1$$

tenglamani hosil qilamiz.

Bitta to'g'ri chiziqda yotmagan $M_0(x_0,y_0,z_0)$, $M_1(x_1,y_1,z_1)$, $M_2(x_2,y_2,z_2)$ nuqtalardan o'tuvchi tekislik tenglamasini tuzish talab qilinsin. Ma'lumki, bitta M_0 nuqtadan o'tuvchi tekislik tenglamasi

$$A(x-x_0)+B(y-y_0)+C(z-z_0)=0 \quad (M_0)$$

ko'rinishda bo'lsadi, M_1 va M_2 nuqtalarning ham bu tekislikda yotishini talab qilamiz, u holda:

$$A(x_1-x_0)+B(y_1-y_0)+C(z_1-z_0)=0 \quad (M_1)$$

$$A(x_2-x_0)+B(y_2-y_0)+C(z_2-z_0)=0 \quad (M_2)$$

tenglamalarni hosil qilamiz. Bu tenglamalarni birgalikda yechamiz. Ma'lumki, $(M_0),(M_1),(M_2)$ tenglamalar sistemasi noldan farqli yechimga ega bo'lishi uchun uning determinanti nolga teng bo'lishi kerak, yani

$$\begin{vmatrix} x-x_0 & y-y_0 & z-z_0 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix}=0 \text{ yoki } \begin{vmatrix} 1 & x & y & z \\ 1 & x_0 & y_0 & z_0 \\ 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \end{vmatrix}=0 \quad (4)$$

Bu tenglama (yuqorida bitta tenglamani ikki xil yozilishi tasvirlangan) berilgan uchta nuqtadan o'tuvchi yagona tekislik tenglamasidir.

Ikki tekislik orasidagi ikki yoqli burchakning chiziqli burchagi uchun bu tekisliklarga normal vektorlar yo'naltiruvchi bo'lgan to'g'ri chiziqlar orasidagi burchak qabul qilinadi. Demak,

$$A_1x + B_1y + C_1z + D_1 = 0$$

va

$$A_2x + B_2y + C_2z + D_2 = 0$$

umumiy tenglamala bilan berilgan tekisliklar orasidagi burchak φ bo'lsa, φ uchun $\vec{n}_1(A_1, B_1, C_1)$, $\vec{n}_2(A_2, B_2, C_2)$ vektorlar yotgan to'g'ri chiziqlar orasidagi burchak qabul qilinadi. Bu burchak kosinusini topamiz:

$$\cos\varphi = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1| \cdot |\vec{n}_2|}$$

yoki

$$\cos\varphi = \frac{|A_1A_2 + B_1B_2 + C_1C_2|}{\sqrt{A_1^2 + B_1^2 + C_1^2} \cdot \sqrt{A_2^2 + B_2^2 + C_2^2}}$$

Bu ikki tekislik orasidagi burchakni topish formulasi bo'lib, bu burchak $0 \le \varphi \le \pi$ oraliqda topiladi.

Agar tekisliklar parallel bo'lsa $\vec{n}_1 = \lambda\vec{n}_2$ yoki $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$ bo'ladi, agar tekisliklar perpendikulyar bo'lsa, $(\vec{n}_1, \vec{n}_2) = 0$ yoki $A_1A_2 + B_1B_2 + C_1C_2 = 0$ bo'ladi.

Agar tekislik umumiy $Ax + By + Cz + D = 0$ tenglamasi bilan berilgan bo'lsa, M_1 nuqtadan bu tekislikkacha masofani topish uchun bu tenglamani normallashtirish kerak. Buning uchun tenglamaning ikkala tamonini ham normallashtiruvchi kupayuvchi $M = \pm\frac{1}{|\vec{n}|} = \pm\frac{1}{\sqrt{A^2 + B^2 + C^2}}$ ga ko'paytirish kerak. U holda d masofa quyidagi formula bilan hisoblanadi:

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{|\vec{n}|}$$

Misol. $M_1(-2, 1, 0)$ nuqtadan $2x - 6y + 3x - 4 = 0$ tekislikgacha masofa topilsin.

Yechish. $d = \frac{|2 \cdot (-2) - 6 \cdot 1 + 3 \cdot 0 - 4|}{\sqrt{4 + 36 + 9}} = \frac{|-4 - 6 - 4|}{7} = 2.$

Agar fazoda to‘g’ri chiziqda yotuvchi $M_0(x_0, y_0, z_0)$ nuqta va unga parallel $\bar{s}(m, n, p)$ $(|\bar{s}| \neq 0)$ vektor berilgan bo‘lsa, u holda bu to‘g’ri chiziqning tenglamasini tuzish mumkin.

Faraz qilamiz $M(x, y, z)$ nuqta to‘g’ri chiziqda yotgan ixtiyoriy nuqta bo‘lsin. U holda $\overline{M_0M} = t\bar{s}$ bo‘ladi. Bu yerda t parametr M nuqtaning joylashishiga qarab ixtiyoriy haqiqiy sonni qabul qilishi mumkin. U holda ta’rifga asosan $\overline{M_0M} = \bar{r} - \bar{r}_0$ ekanligini hisobga olsak, to‘g’ri chiziqning

$$\bar{r} - \bar{r}_0 = \bar{s}t \quad (5)$$

vektor tenglamasini hosil qilamiz.

(5) tenglamani koordinatalar ko‘rinishida ifodalasak

$$x - x_0 = mt, \quad y - y_0 = nt, \quad z - z_0 = pt$$

$$\left.\begin{aligned} x &= x_0 + mt \\ y &= y_0 + nt \\ z &= z_0 + pt \end{aligned}\right\} \quad (6)$$

tenglamalarni hosil qilamiz.

(6) tenglamaga to‘g’ri chiziqning parametrik tenglamalari deyiladi.

(6) sistemada t parametrni yo‘qotamiz

$$\frac{x - x_0}{m} = \frac{y - y_0}{n} = \frac{z - z_0}{p}. \quad (7)$$

(7) to‘g’ri chiziqning kanonik tenglamasi deyiladi.

Bunda $\bar{s}$ vektor to‘g’ri chiziqning yo‘naltiruvchi vektori deyiladi. Uning proeksiyalari (koordinatalari) m, n, p to‘g’ri chiziqning yo‘naltiruvchi koeffitsiyentlari deyiladi. Agar $\bar{s} = \overline{s^0}$ bo‘lsa m, n, p o‘rniga $\cos\alpha, \cos\beta, \cos\gamma$ hosil bo‘lib, ular to‘g’ri chiziqning yo‘naltiruvchi kosinuslari deyiladi. α, β, γ burchaklar $\overline{s^0}$ yoki $\bar{s}$ vektorning koordinata o‘qlari bilan hosil qilgan burchaklaridir.

Yo‘naltiruvchi kosinuslar $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ shartni qanoatlantiradi.

(7) ko‘rinishdagi tenglamalarni ($m \neq 0$ deb olinganda)

$$\left.\begin{aligned} \frac{x - x_0}{m} &= \frac{y - y_0}{n} \\ \frac{x - x_0}{m} &= \frac{z - z_0}{p} \end{aligned}\right\} \quad (8)$$

shaklda yozish mumkin.

(8) dagi har bir tenglama alohida-alohida tekislik tenglamasi bo'lib, birgalikda bu tekisliklarning kesishidan hosil bo'lgan to'g'ri chiziqni beradi.

Demak, to'g'ri chiziqni ikkita tekislikning kesishish chizig'i deb qarash mumkin. Boshqacha aytganda, har qanday ikkita parallel bo'lmagan tekisliklarning tenglamalari birgalikda

$$\left.\begin{aligned} A_1x + B_1y + C_1z + D_1 = 0 \\ A_2x + B_2y + C_2z + D_2 = 0 \end{aligned}\right\} \qquad (9)$$

to'g'ri chiziqning umumiy tenglamasi deyiladi.

To'g'ri chiziqning (9) umumiy tenglamasidan (7) kanonik tenglamasini hosil qilish mumkin. Buni quyidagi masalada ko'ramiz.

Misol. To'g'ri chiziqning ushbu $\begin{cases} 2x + y - z + 1 = 0 \\ 3x - y + 2z - 3 = 0 \end{cases}$ umumiy tenglamasini kanonik shaklga keltiring.

Yechish. Buning uchun berilgan sistemani x va y larga nisbatan yechib

$$x = -\frac{1}{5}z + \frac{5}{2},$$
$$y = \frac{7}{5}z - \frac{9}{5}$$

tenglamalarni topamiz. Sistemani z ga niabatan yechib

$$z = \frac{x - 2/5}{-1/5},$$
$$z = \frac{y + 9/5}{7/5}$$

tengliklarni hosil qilamiz. Endi ularni tenglashtirib

$$\frac{x - 2/5}{-1/5} = \frac{y + 9/5}{7/5} = \frac{z}{1}$$

to'g'ri chiziqning kanonik tenglamasini topamiz.

Noma'lumlardan biriga ixtiyoriy qiymat berib (5) sistemani qolgan ikkita noma'lumlarga nisbatan yechib to'g'ri chiziqdagi nuqtalarning koordinatalarini topamiz.

To'g'ri chiziqning $\vec{s}$ yo'naltiruvchi vektori (5) sistemadagi tekisliklarning $\vec{n}_1(A_1, B_1, C_1)$ va $\vec{n}_2(A_2, B_2, C_2)$ normal vektorlarining har biriga perpendikulyar, demak $\vec{s} = [\vec{n}_1 \cdot \vec{n}_2]$.

Ikkita to'g'ri chiziq kanonik tenglamalari bilan berilgan bo'lsin:

$$\frac{x-x_1}{m_1}=\frac{y-y_1}{n_1}=\frac{z-z_1}{p_1},$$
$$\frac{x-x_2}{m_2}=\frac{y-y_2}{n_2}=\frac{z-z_2}{p_2} \qquad (6)$$

$M_1(x_1, y_1, z_1)$ va $M_2(x_2, y_2, z_2)$ nuqtalar ham berilgan bo'lib, ulardan o'tuvchi to'g'ri chiziq tenglamalarini tuzish talab qilinsin. M_1 ni to'g'ri chiziqda yotuvchi nuqta uchun qabul qilib, to'g'ri chiziqni $\overline{M_1M_2}(x_2-x_1, y_2-y_1, z_2-z_1)$ vektorga parallel desak, izlanayotgan to'g'ri chiziqlarning tenglamalari

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1} \qquad (7)$$

bo'ladi.

(7) tenglamaga fazoda ikkita nuqtadan o'tuvchi to'g'ri chiziq tenglamasi deyiladi.

Fazoda ikki to'g'ri chiziq orasidagi burchak shu to'g'ri chiziqlarning yo'naltiruvchi $\vec{s}_1(m_1,n_1,p_1)$ va $\vec{s}_2(m_2,n_2,p_2)$ vektorlar orasidagi φ burchak bilan aniqlanadi, ya'ni

$$\cos\varphi=\frac{|\vec{s}_1\cdot\vec{s}_2|}{|\vec{s}_1|\cdot|\vec{s}_2|}=\frac{|m_1m_2+n_1n_2+p_1p_2|}{\sqrt{m_1^2+n_1^2+p_1^2}\cdot\sqrt{m_2^2+n_2^2+p_2^2}} \qquad (8)$$

Agar to'g'ri chiziqlar perpendikulyar bo'lsa, $\varphi=90^0$ bo'lib $\cos 90^0=0$ bo'ladi, shuning uchun $m_1m_2+n_1n_2+p_1p_2=0$ (perpendikulyar sharti).

Agar to'g'ri chiziqlar parallel bo'lsa $\vec{s}_1(m_1,n_1,p_1)$ va $\vec{s}_2(m_2,n_2,p_2)$ vektorlarning proektsiyalari (koordinatalari) proportsional bo'ladi $\frac{m_1}{m_2}=\frac{n_1}{n_2}=\frac{p_1}{p_2}$ (paralellik sharti).

$\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}$ to'g'ri chiziq va $Ax+By+Cz+D=0$ tekislik orasidagi burchak φ uchun to'g'ri chiziq va uni tekislikdagi proektsiyasi orasidagi burchak qabul qilinadi. $\vec{s}(m,n,p)$ yo'naltiruvchi va $\vec{n}(A,B,C)$ normal vektorlar orasidagi burchak $\frac{\pi}{2}-\varphi$ ga teng. U holda $\cos\left(\frac{\pi}{2}-\varphi\right)=\sin n\varphi$.

$$\sin\varphi=\frac{|\bar{n}\cdot\bar{s}|}{|\bar{n}|\cdot|\bar{s}|}=\frac{|Am+Bn+Cp|}{\sqrt{A^2+B^2+C^2}\cdot\sqrt{m^2+n^2+p^2}}$$

$\varphi\le\frac{\pi}{2}$ bo'lgani uchun kasr surati modul bilan olingan.

To'g'ri chiziq va tekislik parallel bo'lsa, $Am+Bn+Cp=0$; perpendikulyar bo'lsa, $\frac{A}{m}=\frac{B}{n}=\frac{C}{p}$ shart bajariladi.

Ikkita (6) to'g'ri chiziqni bir tekislikda yotish sharti

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ m_1 & n_1 & p_1 \\ m_2 & n_2 & p_2 \end{vmatrix}=0 \text{ dan iborat.}$$

$\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}$ to'g'ri chiziq va $Ax+By+Cz+D=0$ tekislikning kesishish nuqtasi topish uchun ularning tenglamalarini birgalikda yechish kerak $(Am+Bn+Cp\neq 0)$.

Misol. Berilgan $\frac{x+2}{3}=\frac{y-2}{-1}=\frac{z+1}{2}$ to'g'ri chiziq va $2x+3y+3z-8=0$ tekislikning kesishish nuqtasi topilsin.

Yechish. $\frac{x+2}{3}=\frac{y-2}{-1}=\frac{z+1}{2}=t$ deb belgilab $x=3t-2,\ y=-t-2,\ z=2t-1$ larni topamiz. Bu qiymatlarni tekislik tenglamasiga qoysak $t=1$ ga ega bo'lamiz. Bundan, yana o'rniga qoyib $x=y=z=1$ ni topamiz.

Misollar

1. $M_0(3;5;-8)$ nuqtadan $6x-3y+2z-28=0$ tekislikkacha bo'lgan masofani toping.

Yechish. Nuqtadan tekislikkacha bo'lgan masofa formulasidan foydalanib, $d=\frac{|6\cdot 3-3\cdot 5+2\cdot(-8)-28|}{\sqrt{6^2+3^2+2^2}}=\frac{41}{7}$ ni topamiz.

2. $M(2;3;5)$ nuqtadan o'tib, $\overline{N}=4\bar{i}+3\bar{j}+2\bar{k}$ vektorga perpendikulyar tekislik tenglamasini tuzing.

Yechish. $A(x-x_0)+B(y-y_0)+C(z-z_0)=0$ formuladan foydalanamiz. $4(x-2)+3(y-3)+2(z-5)=0$, ya'ni $4x+3y+2z-27=0$.

3. $M(2;3;-1)$ nuqtadan o‘tib, $5x-3y+2z-10=0$ tekislikka parallel tekislik tenglamasini tuzing.

Yechish. $A(x-x_0)+B(y-y_0)+C(z-z_0)=0$ formuladan

$$A(x-2)+B(y-3)+C(z+1)=0$$

Berilgan tekislikning normali $\overline{n}=(5;-3;2)$ bilan izlangan tekislikning normal vektori ustma-ust tushadi, demak, $A=5$, $B=-3$, $C=2$ va izlangan tekislik tenglamasi $5(x-2)-3(y-3)+2(z+1)=0$ yoki $5x-3y+2z+1=0$ bo‘ladi.

4. $A(5;4;3)$ nuqtadan o‘tuvchi va koordinata o‘qlaridan teng kesmalar ajratuvchi tekislik tenglamasini yozing.

Yechish. Tekislikning kesmalarga nisbatan tenglamasidan foydalanib, $(a=b=c)$ $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ ga ega bo‘lamiz. $A(5;4;3)$ nuqtaning koordinatalari izlangan tekislik tenglamasini qanoatlantiradi, shuning uchun $\frac{5}{a}+\frac{4}{a}+\frac{3}{a}=1$, bundan $a=12$. Shunday qilib, $x+y+z-12=0$ tenglamaga ega bo‘lamiz.

5. $x+y+5z-1=0$, $2x+3y-z+2=0$ tekisliklarning kesishish chizig‘idan va $M(3;2;1)$ nuqtadan o‘tuvchi tekislik tenglamasini yozing.

Yechish. Ma’lumki $A_1x+B_1y+C_1z+D_1+\lambda(A_2x+B_2y+C_2z+D_2)=0$ tenglama λ ning ixtiyoriy qiymatida $A_1x+B_1y+C_1z+D_1=0$ (I) va $A_2x+B_2y+C_2z+D_2=0$ (II) tekisliklarning kesishgan chizig‘idan o‘tuvchi tekilikni aniqlaydi.
Demak, $x+y+5z-1+\lambda(2x+3y-z+2)=0$. M nuqtaning koordinatalari bu tenglamani qanoatlantirishidan λ ni topamiz: $3+2+5-1+\lambda(6+6-1+2)=0$, bundan $\lambda=-\frac{9}{13}$. Shunday qilib, izlangan tenglama $x+y+5z-1-\frac{9}{13}(2x+3y-z+2)=0$ yoki $5x+14y-74z+31=0$ bo‘ladi.

6. $\frac{x-1}{1}=\frac{y+1}{2}=\frac{z+1}{-1}$ to‘g’ri chiziqdan va $M(2;0;1)$ nuqtadan o‘tuvchi tekislikning tenglamasini tuzing.

Yechish. Tekislik $M(2;0;1)$ nuqta orqali o‘tadi , shuning uchun

$$A(x-2)+By+C(z-1)=0.$$

To'g'ri chiziqning $\vec{s}=(1;2;-1)$ yo'naltiruvchi vektori bilan tekislikning $\vec{n}=(A;B;C)$ normal vektori perpendikulyar. Bu vektorlarning skalyar ko'paytmasi $\vec{s}\vec{n}=0$, $A+2B-C=0$.
Boshqa tomondan $A(1;-1;-1)$ nuqta to'g'ri chiqda yotadi, demak tekislikda ham, uning koordinatalari tekislik tenglamasini qanoatlantiradi.
$A(1-2)+B(-1)+C(-1-1)=0$, yoki $-A-B-2C=0$.
Quyidagi tenglamalar sistemasini yechamiz:

$$\begin{cases} A+2B-C=0, \\ -A-B-2C=0. \end{cases}$$

natijada $A=-5C$, $B=3C$.
Izlanayotgan tekislik tenglamasi $(-5(x-2)+3y+z-1)C=0$ yoki ($C\neq 0$ ga qisqartirgandan keyin) $5x-3y-z-9=0$.

7. $\frac{x-3}{1}=\frac{y-6}{1}=\frac{z+7}{-2}$ to'g'ri chiziq va $4x-2y-2z-3=0$ tekislik orasidagi burchakni toping.

Yechish. To'g'ri chiziq va tekislik orasidagi burchakni topish formulasidan

Demak, $\varphi=\frac{\pi}{6}$. $\sin\varphi=\frac{|4\cdot 1-2\cdot 1-2\cdot(-2)|}{\sqrt{1+1+4}\cdot\sqrt{16+4+4}}=\frac{6}{\sqrt{6}\sqrt{24}}=\frac{1}{2}$.

4.4. Talabaning mustaqil ishi

Topshiriq
Misollar sharti variantda berilgan.

1-variant

1. $2x+7y-3=0$ to'g'ri chiziqqa koordinatalar boshidan perpendikulyar tushiring.
2. $3x^2+3y^2-6x+8y=0$ aylananing markazi va radiusini toping.
3. $\frac{x-3}{2}=\frac{y+2}{4}=\frac{z}{1}$ to'g'ri chiziqdan va $M_0(2;-1;2)$ nuqtadan o'tuvchi tekislikning tenglamasi yozilsin.

2-variant

1. Agar uchburchak uchlarining koordinatalari $A(4;-5)$, $B(7;6)$ va $C(-7;-2)$ bo'lsa, bu uchburchak to'g'ri burchakli bo'lishi yoki bo'lmasligini tekshiring.

2. $A(-1;5)$, $B(-2;-2)$ va $C(5;5)$ nuqtalardan o'tuvchi aylananing markazi va radiusini toping.

3. $\frac{x+3}{4}=\frac{y-1}{2}=\frac{z+2}{3}$ to'g'ri chiziqdan va $M_0(2;1;-3)$ nuqtadan o'tuvchi tekislikning tenglamasi yozilsin.

3-variant

1. $M(2;3)$ nuqtadan o'tib, $P(1;7)$ va $Q(-2;-5)$ nuqtalarni tutashtiruvchi to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziqning tenglamasini tuzing to'g'ri chiziqqa koordinatalar boshidan perpendikulyar tushiring.

2. $A(5;3)$ nuqtadan o'tuvchi markazi $5x-3y-13=0$ va $x+4y+2=0$ to'g'ri kesishish nuqtasida yotuvchi aylana tenglamasini tuzing.

3. $\frac{x-3}{2}=\frac{y+2}{-7}=\frac{z+2}{-3}$ to'g'ri chiziqdan va $M_0(-1;0;2)$ nuqtadan o'tuvchi tekislikning tenglamasi yozilsin.

4-variant

1. $M(-1;7)$ va $N(3;-1)$ nuqtalarni tutashtiruvchi kesma o'rtasiga o'tkazilgan perpendikulyarning tenglamasini tuzing.

2. $x^2+y^2+4x-4y=0$ aylana bilan $x+y=0$ to'g'ri chiziqning kesishish nuqtalari va $M(4;4)$ nuqtadan o'tuvchi aylana tenglamasini tuzing.

3. $\frac{x-1}{1}=\frac{y+1}{2}=\frac{z+1}{-1}$ to'g'ri chiziqdan va $M_0(2;0;1)$ nuqtadan o'tuvchi tekislikning tenglamasi yozilsin.

5-variant

1. Rombning ikkita qarama-qarshi uchining koordinatalari berilgan $M(-3;2)$ va $N(7;-6)$. Romb diagonallarining tenglamalarini tuzing.

2. $x^2+y^2+4x+12y+15=0$ parallel to'g'ri aylananing markazidan o'tuvchi $x+y=0$ to'g'ri chiziqqa chiziq tenglamasini tuzing.

3. Ox o'qdan va $A(1;-1;3)$ nuqtadan o'tuvchi tekislik tenglamasi tuzing.

6-variant

1. $2x-5y-12=0$ to'g'ri chiziqda $A(-1;3)$ va $B(3;-5)$ nuqtalardan baravar uzoqlashgan nuqtani toping.

2. Oy o‘qiga koordinatalar boshida uringan va Ox o‘qini $M(6;0)$ nuqtada kesib o‘tuvchi aylana tenglamasini tuzing.

3. Oy o‘qdan va $B(2;1;-1)$ nuqtadan o‘tuvchi tekislik tenglamasi tuzing.

7-variant

1. $A(3;4)$ nuqtadan $2x+5y+3=0$ to‘g‘ri chiziqqa tushirilgan perpendikulyarning asosini toping.

2. $A(3;1)$ va $B(-1;3)$ nuqtalardan o‘tuvchi, markazi $3x-y-2=0$ to‘g‘ri chiziqda yotgan aylana tenglamasini tuzing.

3. $M_0(4;-4;2)$ nuqtadan o‘tuvchi va xOz tekislikka parallel tekislik tenglamasini tuzing.

8-variant

1. $A(-1;3)$ nuqtadan $3x-4y+40=0$ to‘g‘ri chiziqqacha bo‘lgan masofani toping.

2. $3x^2+4y^2-12=0$ ellipsning yarim o‘qlari, fokuslarining koordinatalarini, ekssentrisitetini toping.

3. $M_0(2;3;4)$ nuqtadan o‘tuvchi va Ox va Oy o‘qlaridan $a=1$ va $b=-1$ kesmalar ajratuvchi tekislikning tenglamasi yozilsin.

9-variant

1. Uchlarining koordinatalari $A(2;4)$, $B(-1;-2)$ va $O(11;13)$ bilan berilgan uchburchakning burchaklarini hisoblang.

2. $9x^2+4y^2=36$ ellipsning yarim o‘qlari, fokuslarining koordinatalarini, ekssentrisitetini toping.

3. $M_0(2;-3;1)$ nuqtadan o‘tuvchi $\bar{a}=(-3;2;-1)$ va $\bar{b}=(1;2;3)$ vektorlarga parallel tekislik tenglamasi yozilsin.

10-variant

1. $9x+3y-7=0$ to‘g‘ri chiziq va $A(1;-1)$ va $B(5;7)$ nuqtalardan o‘tadigan to‘g‘ri chiziq orasidagi o‘tkir burchakni toping.

2. Katta yarim o‘qi 12 ga teng, ekssentrisiteti 0,8 ga teng ellipsning kanonik tenglamasini tuzing. Ellipsning fokuslari orasidagi masofani toping.

3. $M_1(2;-15;1)$ va $M_2(3;1;2)$ nuqtalardan o‘tuvchi hamda $3x-y-4z=0$ tekislikka perpendikulyar tekislikning tenglamasi yozilsin.

11-variant

1. $M(-1;2)$ nuqtadan o'tib $x-3y+2=0$ to'g'ri chiziq bilan $45°$ li burchak tashkil qiladigan to'g'ri chiziqning tenglamasini tuzing.

2. Ellips $M_1\left(2;\sqrt{3}\right)$ va $M_2(0;2)$ nuqtalar orqali o'tadi. Ellips tenglamasini tuzing va M_1 nuqtasidan fokuslarigacha masofani toping.

3. $M_0(-2;7;3)$ nuqtadan o'tuvchi $x-4y+5z+1=0$ tekislikka parallel tekislik tenglamasi yozilsin.

12-variant

1. Uchburchakning uchlari berilgan: $A(-6;2)$, $B(10;10)$, $C(0;-10)$. A uchdan o'tkazilgan mediananing, balandlikning va bissektrissaning tenglamalarini tuzing va uzunliklarini hisoblang.

2. $9x^2+25y^2=225$ ellipsda o'ng fokusigacha bo'lgan masofa chap fokusigacha bo'lgan masofadan to'rt marta katta bo'lgan nuqtani toping.

3. $A(5;-2;3)$ va $B(6;1;0)$ nuqtalardan o'tuvchi to'g'ri chiziqqa parallel, $M_1(2;-15;1)$ va $M_2(-1;1;-1)$ nuqtadan o'tuvchi tekislik tenglamasi yozilsin.

13-variant

1. ABC uchburchakning $A(4;4)$ va $B(1;0)$ uchlari va uning medianalarining kesishgan nuqtasi $M(1;3)$ berilgan. Uchburchak tomonlarining tenglamalarini tuzing.

2. Fokuslari orasidagi masofa katta va kichik yarim o'qlari orasidagi masofaga teng ellipsning ekssentrisitetini toping.

3. $M_1(3;-1;2)$, $M_2(4;-1;-1)$ va $M_3(2;0;2)$, nuqtalardan o'tuvchi tekislik tenglamasini tuzing.

14-variant

1. ABC uchburchakda uning tomonlari o'rtalarining koordinatalari ma'lum: $M_1(-1;5)$, $M_2(3;1)$, $M_3(-5;-1)$. Uchburchak tomonlarining tenglamalarini tuzing.

2. $9x^2-16y^2=144$ giperbolaning kanonik tenglamasini tuzing. Uning uchi va fokuslari koordinatalarini, ekssentrisitetini, asimptotalarining tenglamalarini toping.

3. $M_0(-1;1;-3)$ nuqtadan o'tuvchi $\bar{a}=(1;-3;4)$ vektorga parallel to'g'ri chiziq tenglamasini tuzing.

15-variant

1. $A(-1;3)$ nuqtadan o'tuvchi, $2x+5y-1=0$ to'g'ri chiziqqa a) parallel, b) perpendikulyar bo'lgan to'g'ri chiziq tenglamalarini tuzing.

2. $A(2;1)$ va $B(-4;\sqrt{7})$ nuqtalar orqali o'tuvchi giperbolaning kanonik tenglamasini tuzing.

3. $M_1(2;-1;-1)$ va $M_2(3;3;-1)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

16-variant

1. Uchlari $A=(1;-1;2)$, $B=(5;-6;2)$, $C=(1;3;-1)$ nuqtalarda bo'lgan uchburchakning B uchidan AC tomoniga tushirilgan balandlik uzunligini toping.

2. Giperbola $M(6;-2\sqrt{2})$ nuqta orqali o'tadi va kichik yarim o'qi $b=2$. Giperbola tenglamasini tuzing va M nuqtadan giperbolagacha bo'lgan masofani toping.

3. $M(1;-5;3)$ nuqtadan o'tuvchi koordinata o'qlari bilan $\alpha=\frac{\pi}{4}$, $\beta=\frac{\pi}{3}$, $\gamma=\frac{2\pi}{3}$ burchaklar tashkil qilgan to'g'ri chiziq tenglamasini tuzing.

17-variant

1. Uchlari $A=(1;-2;3)$, $B=(0;-1;2)$, $C=(3;4;5)$ nuqtalarda bo'lgan uchburchak yuzini toping.

2. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ giperbolaning fokusidan asimptotalarigacha bo'lgan masofani va asimptotalari orasidagi burchakni toping.

3. $M(1;-3;5)$ nuqtadan o'tuvchi $\begin{cases}3x-y+2z-7=0,\\ x+3y-2z+3=0\end{cases}$ to'g'ri chiziqqa parallel to'g'ri chiziq tenglamasini tuzing.

18-variant

1. $A(5;1)$ nuqtadan o'tuvchi, $3x+2y-7=0$ to'g'ri chiziqqa a) parallel, b) perpendikulyar ikkita to'g'ri chiziq tenglamasini tuzing.

2. $\frac{x^2}{25}+\frac{y^2}{9}=1$ ellips berilgan. Uchlari ellipsning fokuslarida, fokuslari ellipsning uchlarida bo'lgan giperbola tenglamasini tuzing.

3. $M(3;-2;4)$ nuqtadan o‘tuvchi $5x+3y-7z+1=0$ tekislikka perpendikulyar to‘g‘ri chiziq tenglamasini tuzing.

19-variant

1. ABC uchburchakning $A(-7;2)$, $B(5;-3)$ va $C(8;1)$ uchlari berilgan. Uchburchakning B uchidan o‘tkazilgan medianasi, balandlik va bissektrisasi tenglamasini tuzing.

2. $x^2-4y^2=16$ giperbolaga $A(0;-2)$ nutada o‘tkazilgan urinma tenglamasini tuzing.

3. $A(4;-3;1)$ nuqtaning $x+2y-z-3=0$ tekislikdagi proeksiyasini toping.

20-variant

1. Uchburchakning $A(0;2)$ uchi va $(BM)x+y-4=0$, $(CM)y=2x$ balandliklari (bu yerda M - balandliklar kesishish nuqtasi) tenglamalari berilgan. Uchburchak tomonlari tenglamalarini tuzing.

2. Ox o‘qiga nisbatan simmetrik $(0;0)$ va $(1;-3)$ nuqtalar orqali o‘tuvchi parabola tenglamasini tuzing.

3. $A(1;2;1)$ nuqtaning $\frac{x+2}{3}=\frac{y}{-1}=\frac{z-1}{2}$ to‘g‘ri chiziqdagi proeksiyasini toping.

21-variant

1. $3x+4y-1=0$ va $4x-3y+5=0$ to‘g‘ri chiziqlar orasidagi burchak bissektrisasi tenglamasini tuzing.

2. Oy o‘qiga nisbatan simmetrik $(0;0)$ va $(2;-4)$ nuqtalar orqali o‘tuvchi parabola tenglamasini tuzing.

3. $M_0(5;2;2)$ nuqtadan o‘tuvchi va $M_1(3;4;6)$, $M_2(3;-2;-3)$ va $M_3(6;3;2)$ nuqtalardan o‘tuvchi tekislikka perpendikulyar to‘g‘ri chiziqning kanonik tenglamasini tuzing.

22-variant

1. ABC uchburchakda uchburchakning tomoni $(AB)x+7y-6=0$ va bissektrisalari $(AL)x+y-2=0$, $(BM)x-3y-6=0$ tenglamalari berilgan. Uchlarining koordinatalarini toping.

2. $y=-3x^2+12x-9$ parabolaning uchi orqali o‘tuvchi, $\frac{x}{10}+\frac{y}{8}=1$ to‘g‘ri chiziqqa parallel to‘g‘ri chiziq tenglamasini tuzing.

3. $M_0(-6;1;3)$ nuqtadan o'tuvchi va $M_1(2;3;0)$, $M_2(1;2;2)$ va $M_3(-1;0;-3)$ nuqtalardan o'tuvchi tekislikka perpendikulyar to'g'ri chiziqning kanonik tenglamasini tuzing.

23-variant

1. $5x-y+10=0$ va $8x+4y+9=0$ to'g'ri chiziqlarning kesishish nuqtasidan o'tuvchi va $x+3y=0$ to'g'ri chiziqqa parellel to'g'ri chiziq tenglamasini tuzing.

2. $y^2=4x$ parabolaning fokusidan uning $x^2+y^2=12$ aylana bilan kesishish nuqtasi orasidagi masofani toping.

3. $M_0(6;1;2)$ nuqtadan o'tuvchi va $M_1(3;4;2)$, $M_2(4;5;2)$ va $M_3(7;3;-2)$ nuqtalardan o'tuvchi tekislikka perpendikulyar to'g'ri chiziqning kanonik tenglamasini tuzing.

24-variant

1. $2x-3y+5=0$ va $3x+y-7=0$ to'g'ri chiziqlarning kesishish nuqtasidan o'tuvchi va $y=2x$ to'g'ri chiziqqa perpendikulyar to'g'ri chiziq tenglamasini tuzing.

2. $\frac{x^2}{25}+\frac{y^2}{9}=1$ giperbola berilgan. Fokuslari giperbolaning uchlarida fokuslarida, uchlari giperbolaning fokuslarida bo'lgan ellips tenglamasini tuzing.

3. $A(5;2;-1)$ nuqtaning $2x-y+3z+23=0$ tekislikdagi proeksiyasini toping.

25-variant

1. $8x+4y-3=0$ to'g'ri chiziqqa uning $x-y=0$ to'g'ri chiziq bilan kesishish nuqtasiga tushirilgan perpendikulyar tenglamasini tuzing.

2. Agar parabola $x+y=0$ to'g'ri chiziq va $x^2+y^2+4y=0$ aylananing kesishish nuqtasi orqali o'tsa va Oy o'qiga nisbatan simmetrik bo'lsa parabola va uning direktrisasi tenglamasini tuzing.

3. $M(-1;0;5)$ nuqtadan o'tuvchi koordinata o'qlari bilan $\alpha=\frac{\pi}{3}$, $\beta=\frac{\pi}{4}$, $\gamma=\frac{2\pi}{3}$ burchaklar tashkil qilgan to'g'ri chiziq tenglamasini tuzing.

V BOB. MATEMATIK TAHLILGA KIRISH

5.1. Sonli ketma-ketlik. Yaqinlashuvchi nuqtalar ketma-ketligi

Sonlar ketma-ketligi tushunchasi

1-ta'rif. Agar N to'plamdan olingan har bir x elementga biror f qoida yoki qonunga ko'ra R to'plamning bitta y elementi $(y\in R)$ mos qo'yilgan bo'lsa, N to'plamni R to'plamga akslantirish berilgan deyiladi va

$$f:N\to R \text{ yoki } x\xrightarrow{f}y, \quad (x\in N, y\in R)$$

kabi belgilanadi. Bunda N to'plam f akslantirishning aniqlanish to'plami deyiladi.

Har bir natural n songa biror haqiqiy a_n sonini mos qo'yuvchi

$$f:n\to a_n, \quad (n=1,2,3,...) \tag{1}$$

akslantirishni qaraymiz.

2-ta'rif. 1-akslantirishning akslaridan iborat ushbu

$$a_1,a_2,a_3,...,a_n,... \tag{2}$$

to'plam sonlar ketma-ketligi deyiladi. U holda a_1 bu ketma-ketlikning birinchi hadi, a_2 –ikkinchi, ..., a_n – n –hadi deyiladi. $a_1,a_2,a_3,...,a_n,...$ kema-ketlik $\{a_n\}$ kabi belgilanadi.

3-ta'rif. Ketma-ketlikning istalgan hadini shu hadning nomeri orqali ifodalaydigan formulaga (agar shunday formula mavjud bo'lsa) ketma-ketlikning umumiy n –hadi formulasi deyiladi.

Ketma-ketlik n –hadining formulasi bilan berilishi mumkin. Masalan $-1,\frac{1}{2},-\frac{1}{3},...,\frac{(-1)^n}{n},...$ ketma-ketlik $a_n=\frac{(-1)^n}{n}$ formula bilan berilgan.

4-ta'rif. Ketma-ketlikning biror hadidan boshlab ixtiyoriy hadini, bir yoki bir nechta oldingi hadlar yordamida ifoda qiladigan formula rekurrent formula deyiladi.

Masalan, $a_1=1$, $a_2=1$ va $a_{n+2}=a_n+a_{n+1}$, $n\geq 1$ rekurrent formula bilan

$$1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34,...$$

Sonlar ketma-ketligi aniqlanadi. Bu ketma-ketlikni tashkil etgan sonlar "Fibonachchi sonlari" nomi bilan yuritiladi.

5-ta'rif. Agar ketma-ketlikning har bir keyingi hadi oldingisidan katta (kichik), ya'ni $a_n < a_{n+1}$ $(a_n > a_{n+1})$ bo'lsa, bu ketma-ketlik o'suvchi (kamayuvchi) ketma-ketlik deyiladi.

Masalan, $\frac{n+1}{3n-1}$ ketma-ketlik kamayuvchidir. Haqiqatan ham, $a_n = \frac{n+1}{3n-1}$ va $a_{n+1} = \frac{n+2}{3n+2}$ bo'lgani uchun

$$a_{n+1} - a_n = \frac{n+2}{3n+2} - \frac{n+1}{3n-1} = \frac{3n^2 + 5n - 2 - 3n^2 - 5n - 2}{(3n+2)(3n-1)} = -\frac{4}{(3n+2)(3n-1)} < 0$$

$n \in N$ bo'lganda oxirgi kasr maxraji musbatdir. $a_{n+1} - a_n < 0$ dan $a_{n+1} < a_n$.

6-ta'rif. Agar ketma-ketlikning barcha hadlari uchun $a_{n+1} \geq a_n$ $(a_{n+1} \leq a_n)$ o'rinli bo'lsa, bunday ketma-ketlik kamaymaydigan (o'smaydigan) ketma-ketlik deyiladi.

O'smaydigan va kamaymaydigan ketma-ketliklar monoton ketma-ketliklar deyiladi.

7-ta'rif. Agar shunday $m(M)$ soni mavjud bo'lsaki, $\{a_n\}$ketma-ketlikning barcha hadlari uchun $a_n \geq m$ $(a_n < M)$ tengsizlik o'rinli bo'lsa, bu ketma-ketlik quyi (yuqori) dan chegaralangan deyiladi. Agar ketma-ketlik quyidan (m bilan) yuqoridan (M bilan) chegaralangan bo'lsa unga chegaralangan ketma-ketlik deyiladi (ya'ni bu holda $m \leq a_n \leq M$ bo'ladi). Masalan,

$$-3,\ -8,\ -13,\ -18,\ \ldots,(2-5n),\ldots$$

ketma-ketlik yuqoridan chegaralangan.

8-ta'rif. Agar ixtiyoriy kichik musbat ε soni uchun shunday N natural sonni ko'rsatish mumkin bo'lsaki $\{a_n\}$ketma-ketlikning N dan katta (n) nomerli barcha hadlari uchun

$$|a_n - a| < \varepsilon$$

tengsizlik o'rinli bo'lsa, o'zgarmas chekli a soniga $\{a_n\}$ketma-ketlikning limiti deyiladi va bu quyidagicha yoziladi:

$$\lim_{n\to\infty} a_n = a.$$

9-ta'rif. Agar a nuqtaning ixtiyoriy ε atrofida $\{a_n\}$ ketma-ketlikning biror hadidan keyingi barcha hadlari yotsa, u holda a soni $\{a_n\}$ketma-ketlikning *limiti* deyiladi.

Yuqorida keltirilgan ta'riflardan ko'rinadiki ε ixtiyoriy musbat son bo'lib, natural N soni esa ε ga va qaralayotgan ketma-ketlikka bog'liq ravishda topiladi.

10-ta'rif. Limitga ega bo'lgan ketma-ketlik *yaqinlashuvchi ketma-ketlik* deyiladi.

Limitga ega bo'lmagan ketma-ketlik *uzoqlashuvchi ketma-ketlik* deyiladi.

Teorema. $\{x_n\}$ ketma-ketlik yaqinlashuvchi bo'lsa, u chegaralangan bo'ladi.

Teorema. Agar $\{x_n\}$ ketma-ketlik monoton va yuqoridan chegaralangan bo'lsa, u yaqinlashuvchi bo'ladi.

Cheksiz kichik miqdorlar va ularning xossalari

Faraz qilaylik, $\{\alpha_n\}$ ketma-ketlik berilgan bo'lsin.

11-ta'rif. Agar $\{\alpha_n\}$ ketma-ketlikning limiti nolga teng, ya'ni

$$\lim_{n\to\infty} \alpha_n = 0$$

bo'lsa, $\{\alpha_n\}$ - **cheksiz kichik miqdor** deyiladi.

Masalan,

$$\alpha_n = \frac{1}{n} \quad \text{ва} \quad \alpha_n = q^n, \ (|q| < 1)$$

ketma-ketliklar cheksiz kichik miqdorlar bo'ladi.

Aytaylik, $\{x_n\}$ ketma-ketlik yaqinlashuvchi bo'lib, uning limiti a ga teng bo'lsin:

$$\lim_{n\to\infty} x_n = a.$$

U holda $\alpha_n = x_n - a$ cheksiz kichik miqdor bo'ladi. Keyingi tenglikdan topamiz: $x_n = a + \alpha_n$. Bundan esa quyidagi muhim xulosa kelib chiqadi:

$\{x_n\}$ ketma-ketlikning a $(a \in R)$ limitga ega bo'lishi uchun $\alpha_n = x_n - a$ ning cheksiz kichik miqdor bo'lishi zarur va yetarli.

Cheksiz kichiklarning ba'zi xossalari bilan tanishamiz.

1. Chekli sondagi cheksiz kichiklarning algebraik yig'indisi cheksiz kichik bo'ladi.

2. Chegaralangan ketma-ketlik bilan cheksiz kichikning ko'paytmasi cheksiz kichikdir.

3. O'zgarmas son bilan cheksiz kichikning ko'paytmasi ham cheksiz kichikdir.

4. Ikki cheksiz kichikning ko'paytmasi ham cheksiz kichikdir.

Yaqinlashuvchi ketma-ketliklar quyidagi xossalarga ega:

1) $\{x_n\}$ ketma-ketlik o'zgarmas, ya'ni $\{x_n\} = c$ bo'lsa, u holda $\lim\limits_{n \to \infty}\{x_n\} = c$;

2) $\{x_n\}$ va $\{y_n\}$ ketma-ketliklar yaqinlashuvchi bo'lib, $m-$ o'zgarmas son bo'lsa, u holda: $\{x_n + y_n\}$, $\{x_n \cdot y_n\}$, $\left\{\frac{x_n}{y_n}\right\}$, $\{mx_n\}$, $\{x_n{}^m\}$ ketma-ketliklar ham yaqinlashuvchi bo'ladi va quyidagilar o'rinli bo'ladi:

a) $\lim\limits_{k \to \infty}\{x_k + y_k\} = \lim\limits_{k \to \infty}\{x_k\} + \lim\limits_{k \to \infty}\{y_k\}$;

b) $\lim\limits_{k \to \infty}\{x_k y_k\} = \lim\limits_{k \to \infty}\{x_k\}\lim\limits_{k \to \infty}\{y_k\}$;

c) $\lim\limits_{k \to \infty}\left\{\frac{x_k}{y_k}\right\} = \frac{\lim\limits_{k \to \infty}\{x_k\}}{\lim\limits_{k \to \infty}\{y_k\}}$, $\lim\limits_{k \to \infty}\{y_k\} \neq 0$;

d) $\lim\limits_{k \to \infty}\{mx_k\} = m\lim\limits_{k \to \infty}\{x_k\}$;

e) $\lim\limits_{k \to \infty}\{x_k^m\} = \left(\lim\limits_{k \to \infty}\{x_k\}\right)^m$;

3) Agar $x_k \le y_k$ bo'lsa, u holda $\lim\limits_{k \to \infty} x_k \le \lim\limits_{k \to \infty} y_k$;

4) Agar $\lim\limits_{k \to \infty} x_k = a$, $\lim\limits_{k \to \infty} y_k = a$ va $x_k \le z_k \le y_k$ bo'lsa, u holda $\lim\limits_{k \to \infty} z_k = a$.

Cheksiz katta miqdorlar.

Cheksiz kichik va cheksiz katta miqdorlar orasidagi bog'lanish

12-ta'rif. Agar har qanday M soni olinganda ham shunday natural n_0 soni topilsaki, barcha $n > n_0$ uchun

$$|x_n| > M$$

tengsizlik bajarilsa, $\{x_n\}$ ketma-ketlikning **limiti cheksiz** deyiladi va

$$\lim_{n\to\infty} x_n = \infty$$

kabi belgilanadi.

Agar $\{x_n\}$ ketma-ketlikning limiti cheksiz bo'lsa, $\{x_n\}$ cheksiz katta miqdor deyiladi.

Masalan,

$$x_n = (-1)^n \cdot n$$

ketma-ketlik cheksiz katta miqdor bo'ladi.

Endi cheksiz kichik va cheksiz katta miqdorlar orasidagi bog'lanishni ifodalovchi tasdiqlarni keltiramiz:

1) Agar $\{x_n\}$cheksiz kichik miqdor $(x_n \neq 0)$ bo'lsa, u holda $\left\{\frac{1}{x_n}\right\}$ cheksiz katta miqdor bo'ladi.

2) Agar $\{x_n\}$cheksiz katta miqdor bo'lsa, u holda $\left\{\frac{1}{x_n}\right\}$ cheksiz kichik miqdor bo'ladi.

e soni

Ushbu $x_n = \left(1 + \frac{1}{n}\right)^n$, $(n = 1, 2, 3, ...)$ ketma-ketlikning limiti e soni deyiladi:

$$\lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n = e.$$

Bu e soni irratsional son bo'lib,

$$e = 2{,}718281828459045...$$

bo'ladi.

Faraz qilamiz, omonatchi bankda n yil muddatga S_0 so'm miqdorida jamg'arma omonatini ochdi. Bank foizlarining stavkasi esa bugungi kunda omonat pulining i foizini tashkil qiladi. U holda n yildan so'ng omonatchining hisobidagi pullar miqdori $S_n = S_0\left(1+\frac{i}{100}\right)^n$ (murakkab foizlar formulasi) ni tashkil qiladi.

Bu formuladan ko'rinib turibdiki, omonatning dastlabki pulining murakkab foizlar bo'yicha o'sishi – bu birinchi hadi S_0, maxraji esa $\left(1+\frac{i}{100}\right)$bo'lgan ***geometrik progressiya*** qonunlari bo'yicha rivojlanuvchi jarayon.

Misol. S_0 dastlabki depozit bankka $i = 100\%$ yillik foiz stavkasi bilan qo'yilgan bo'lsin, bir yildan so'ng depozit miqdori $2S_0$ ni tashkil etadi. Faraz qilamizki yarim yildan so'ng hisob $S_1 = S_0\left(1+\frac{1}{2}\right)^2 = \frac{3}{2}S_0$ natija bilan yopiladi va bu summa yana shu bankka depozit sifatida qo'yiladi. Yil yakunida depozit$S_2 = S_0\left(1+\frac{1}{2}\right)^2 = 2{,}25S_0$ ni tashkil etadi. Bankka qo'yilgan depozitni uni olgandan so'ng keyin yana qo'yish sharti bilan qo'yish vaqtini kamaytirib boramiz. Bu operatsiyalar har kvartalda takrorlanganda yil so'nggida depozit $S_3 = S_0\left(1+\frac{1}{3}\right)^3 \approx 2{,}37S_0$ni tashkil etadi. Agar olishning qo'yish operatsiyasini yil davomida xoxlagancha takrorlasak har oy manipulyatsiyalar bir yilda $S_{12} = S_0\left(1+\frac{1}{12}\right)^{12} \approx 2{,}61S_0$ summani tashkil etadi; har kungi bankka tashriflar $S_{365} = S_0\left(1+\frac{1}{365}\right)^{365} \approx 2{,}714S_0$; har soatdagida $S_{8720} = S_0\left(1+\frac{1}{8720}\right)^{8720} \approx 2{,}718S_0$ va hokazoni tashkil etadi.

$\{S_n/S_0\}$ dastlabki omonatning o'sish qiymatlarining ketma-ketligi murakkab foizlar formulasiga $S = S_0\left(1+\frac{i}{100}\right)^n$ ga ko'ra $n \to \infty$ da limiti e son bo‘lgan ketma-ketlik bilan bir xil ko‘rish qiyin emas. Shunday qilib foizlarning uzluksiz hisoblanishidan kelgan daromad bir yilda $\lim\limits_{n\to\infty}(S_n - S_0)\cdot 100\%/S_0 = (e-1)100\% \approx 172\%$ ga teng.

Ma’lumki, R^n fazoda $M(x_1, x_2, ..., x_n)$ nuqtaning δ atrofi $U_\delta(M)$ ko‘rinishda belgilanib, u markazi $M(x_1, x_2, ..., x_n)$ nuqtada bo‘lgan δ radiusli ochiq sharni anglatadi.

13-ta’rif. Agar R^n fazoda har bir $k \in N$ songa aniq bir $M_k\left(x_1^k, x_2^k, ..., x_n^k\right)$ nuqta mos qo‘yilgan bo‘lsa, u holda R^n fazoda $\{M_k\}$ nuqtalar ketma-ketligi berilgan deyiladi

Demak, R^n fazoda nuqtalar ketma-ketligi quyidagi ko‘rinishda $M_1\left(x_1^1, x_2^1, ..., x_n^1\right)$, ... , $M_k\left(x_1^k, x_2^k, ..., x_n^k\right)$, ... beriladi.

M_1 –birinchi hadi, M_2 –ikkinchi hadi, M_k – k –hadi deyiladi.

R^n fazoda qism osti nuqtalar ketma-ketligi berilgan nuqtalar ketma-ketligidan tuziladi va unda hadlarning oldinma-ketin kelish tartibi saqlanadi.

Ma’lumki, M_k, M_0 nuqtalar orasidagi masofa

$$\rho(M_k; M_0) = \sqrt{\sum_{m=1}^{n}\left(x_m^k - x_m^0\right)^2}$$

formula bilan aniqlanadi.

14-ta’rif. Agar biror bir $C \in R$ son va biror bir $M_0 \in R^n$ nuqta topilib, ixtiyoriy $k \in N$ natural son uchun $\rho(M_k; M_0) < C$ tengsizlik bajarilsa, $\{M_k\}$ nuqtalar ketma-ketligi chegaralangan, deyiladi.

R^n fazoda $\{M_k\}$ nuqtalar ketma-ketligi berilgan bo‘lsin.

15-ta’rif. Agar ixtiyoriy $\varepsilon > 0$ son uchun shunday $K(\varepsilon) \in N$ son mavjud bo‘lib, biror M_0 nuqta va barcha $m > K(\varepsilon)$ tartib raqamli hadlar uchun $M_k \in U_\varepsilon(M_0)$ bo‘lsa, u holda M_0 nuqta $\{M_k\}$ nuqtalar ketma-ketligining limiti deyiladi va $\lim\limits_{k\to\infty} M_k\left(x_1^k, x_2^k, ..., x_n^k\right) = M_0\left(x_1^0, x_2^0, ..., x_n^0\right)$ ko‘rinishda yoziladi

16-ta'rif. Agar $M \in D$ nuqtaning shunday $U_{\delta}(M)$ atrofi mavjud bo'lib, $U_{\delta}(M) \in D$ bo'lsa, u holda M to'plamning ichki nuqtasi deb ataladi.

17-ta'rif. Agar to'plamning barcha nuqtalari ichki nuqtalardan iborat bo'lsa, u holda bu to'plam ochiq to'plam deb ataladi.

18-ta'rif. Agar M nuqtaning har qanday atrofi D to'plamning hech bo'lmaganda bitta nuqtasini o'z ichiga olsa, u holda M nuqta D to'plamning urinish nuqtasi deb ataladi.

19-ta'rif. Agar to'plamning barcha urinish nuqtalari to'plamga tegishli bo'lsa, u holda bu to'plam yopiq to'plam deb ataladi.

Teorema. Agar R^n fazoda nuqtalar ketma-ketligi chekli limitga ega bo'lsa u holda bu ketma-ketlik chegaralangan bo'ladi.

20-ta'rif. Agar nuqtaning har qanday atrofida to'plamga tegishli bo'lgan nuqtalar ham, tegishli bo'lmagan nuqtalar ham mavjud bo'lsa, u holda bu nuqta chegaraviy nuqta deb ataladi.

21-ta'rif. Agar n o'lchovli nuqtalar ketma-ketligi chekli limitga ega bo'lsa, bu ketma-ketlik yaqinlashuvchi ketma-ketlik, aks holda uzoqlashuvchi ketma-ketlik deyiladi.

Har qanday yaqinlashuvchi ketma-ketlik fundamental ketma-ketlikdir va aksincha.

Yaqinlashuvchi nuqtalar ketma-ketligi uchun quyidagi xossalar o'rinli:

1) Har qanday yaqinlashuvchi ketma-ketlik chegaralangandir.

2) Har bir chegaralangan ketma-ketlikdan yaqinlashuvchi qism ketma-ketlik ajratish mumkin.

3) n o'lchovli nuqtalar ketma-ketligi M_0 nuqtaga yaqinlashsa, u holda uning har bir qism ketma-ketligi ham M_0 nuqtaga yaqinlashadi.

4) M_0 nuqta biror–bir V nuqtalar to'plamining quyuqlanish nuqtasi bo'lsa, V to'plam nuqtalaridan M_0 nuqtaga yaqinlashuvchi ketma-ketlik ajratib olish mumkin.

5) Yopiq V to'plamga tegishli nuqtalar ketma-ketligi M_0 nuqtaga yaqinlashuvchi bo'lsa, u holda $M_0 \in V$.

Nuqtalar ketma-ketligining limitini aniqlashda sonli ketma-ketlik limiti muhim ahamiyatga ega.

Masalan, nuqtalar ketma-ketligining limiti uchun quyidagi tasdiqlar o‘rinli:

1. M_k va M_0 nuqtalar orasidagi $\{\rho(M_k, M_0)\}$ masofalardan tashkil topgan sonli ketma-ketlikning limiti nolga teng bo‘lgandagina, M_0 nuqta $\{M_k\}$ nuqtalar ketma-ketligining limiti bo‘ladi.

2. R^n fazoda $\{M_k(x_1, x_2, ..., x_n)\}$ nuqtalar ketma-ketligi $M_0\left(x_1^0, x_2^0, ..., x_n^0\right)$ nuqtaga yaqinlashishi uchun $\lim\limits_{k\to\infty} x_m^k = x_m^0$, $m = \overline{1,n}$ tenglik bajarilishi zarur va yetarli.

Misollar

1. $\lim\limits_{n\to\infty} x_n = a$ ekanligi ta`rif yordamida ko`rsatilsin.

$$x_n = \frac{2n^3}{n^3 - 2}, \ a = 2$$

Yechish. $\left(\lim\limits_{n\to\infty} x_n = a\right) \Leftrightarrow \left(\forall\varepsilon > 0 \ \exists n_0 = n_0(\varepsilon) \in N : \forall n > n_0 \ |x_n - a| < \varepsilon\right).$

$$|x_n - a| = \left|\frac{2n^3}{n^3 - 3} - 2\right| = \left|\frac{2n^3 - 2n^3 + 6}{n^3 - 3}\right| = \frac{6}{|n^3 - 3|} =$$

$$= \frac{6}{\left(n - \sqrt[3]{3}\right)\left(n^2 + \sqrt[3]{3}n + \sqrt[3]{3^2}\right)} < \frac{6}{n^2 + \sqrt[3]{3}n + \sqrt[3]{9}} < \frac{6}{\sqrt[3]{3}n} <$$

$$< \frac{6}{n} < \varepsilon \Rightarrow n > \frac{6}{\varepsilon} \Rightarrow n_0 = \left[\frac{6}{\varepsilon}\right]$$

Demak, $\forall\varepsilon > 0$ son olinganda ham $n_0 = \max\left\{2, \left[\frac{6}{\varepsilon}\right]\right\}$ deb olsak, $\forall n > n_0$ uchun $|x_n - a| < \varepsilon$ bo`ladi. $\Rightarrow \lim\limits_{n\to\infty} x_n = a$

2. $\lim\limits_{n\to\infty} \frac{4n - 7}{3n + 2}$ ni hisoblang.

Yechish. Kasrning surati ham, maxraji ham chegaralanmagan ketma-ketliklar bo‘lganidan (limitga ega bo‘lmagani uchun) bo‘linmaning limiti haqidagi teoremani qo‘llanib bo‘lmaydi. Shu sababdan kasrning suratini ham, maxrajini ham n ga bo'lib, so'ngra bo'linmaning limiti haqidagi teoremadan foydalanamiz.

$$\lim_{n\to\infty}\frac{4n-7}{3n+2}=\lim_{n\to\infty}\frac{4-\frac{7}{n}}{3+\frac{2}{n}}=\frac{\lim\limits_{n\to\infty}\left(4-\frac{7}{n}\right)}{\lim\limits_{n\to\infty}\left(3+\frac{2}{n}\right)}=\frac{\lim\limits_{n\to\infty}4-\lim\limits_{n\to\infty}\frac{7}{n}}{\lim\limits_{n\to\infty}3+\lim\limits_{n\to\infty}\frac{2}{n}}=\frac{4-0}{3+0}=\frac{4}{3}.$$

3-misol. $\lim\limits_{n\to\infty}\left(\sqrt{n^2+3n}-n\right)$ ni hisoblang.

Yechish. Kamayuvchining ham ayriluvchining ham limiti mavjud bo'lmagani uchun ayirmaning limiti haqidagi teoremani qo'llanib bo'lmaydi. Shu sababdan avval berilgan ifodani qo'shmasiga ham ko'paytiramiz, ham bo'lamiz:

$$\sqrt{n^2+3n}-n=\frac{\left(\sqrt{n^2+3n}-n\right)\left(\sqrt{n^2+3n}+n\right)}{\sqrt{n^2+3n}+n}=$$

$$=\frac{n^2+3n-n^2}{\sqrt{n^2+3n}+n}=\frac{3n}{\sqrt{n^2+3n}+n}.$$

Endi kasrning surat va maxrajini n ga bo'lib, hosil bo'lgan ifodaning limitini hisoblaymiz.

$$\lim_{n\to\infty}\frac{3n}{\sqrt{n^2+3n}+n}=\lim_{n\to\infty}\frac{3}{\sqrt{1+\frac{3}{n}}+1}=\frac{\lim\limits_{n\to\infty}3}{\lim\limits_{n\to\infty}\left(\sqrt{1+\frac{3}{n}}+1\right)}=\frac{3}{\lim\limits_{n\to\infty}\sqrt{1+\frac{3}{n}}+\lim\limits_{n\to\infty}1}=$$

$$=\frac{3}{\sqrt{\lim\limits_{n\to\infty}\left(1+\frac{3}{n}\right)}+1}=\frac{3}{\sqrt{\lim\limits_{n\to\infty}1+3\lim\limits_{n\to\infty}\frac{1}{n}}+1}=\frac{3}{2}.$$

4-misol. Inflyatsiya tempi bir kunda 1% ni tashkil etadi. Yarim yildan so'ng dastlabki summa qanchaga kamayadi.

Yechish. Murakkab foizlar formulasini qo'llasak $S=S_0\left(1-\frac{1}{100}\right)^{182}$ ni hosil qilamiz, bu yerda S_0 – dastlabki summa, 182 – yarim yildagi kunlar soni. Bu ifodaning shaklini almashtirsak $S=S_0\left[\left(1-\frac{1}{100}\right)^{-100}\right]^{-\frac{182}{100}}\approx S_0/e^{1,82}$ ni hosil qilamiz, ya'ni inflyatsiya dastlabki summani taxminan 6 marta kamaytiradi.

5.2. Bir va ko'p o'zgaruvchili funksiyalar

Aytaylik, $X \subset R, Y \subset R$ to'plamlar berilgan bo'lib, x va y o'zgaruvchilar mos ravishda shu to'plamlarda o'zgarsin: $x \in X$, $y \in Y$.

1-ta'rif. Agar X to'plamdagi har bir x songa biror f qoidaga ko'ra Y to'plamdan bitta y son mos qo'yilgan bo'lsa, X to'plamda funksiya berilgan (aniqlangan) deyiladi va

$$y = f(x)$$

kabi belgilanadi.

X to'plam funksiyaning *aniqlanish sohasi*, Y esa funksiyaning *o'zgarish sohasi* deyiladi.

Shuningdek, x *erkli o'zgaruvchi* yoki *argument*, y esa *erksiz o'zgaruvchi* deyiladi.

Shuni ham alohida ta'kidlash kerakki, $y = f(x)$ funksiya:

R^1 fazoda $y = f(x)$ ko'rinishda;

R^2 fazoda $y = f(x_1, x_2)$ yoki $y = f(\vec{x}(x_1, x_2))$ ko'rinishda;

R^3 fazoda $y = f(x_1, x_2, x_3)$ yoki $y = f(\vec{x}(x_1, x_2, x_3))$ va R^n fazoda $y = f(x_1, x_2, ..., x_n)$ yoki $y = f(M)$ $(M(x_1, x_2, ..., x_n))$ ko'rinishda yoziladi.

Ko'p o'zgaruvchili funksiyalarga doir misollar keltiramiz. Ko'p o'zgaruvchili funksiyalarni $Z = f(x_1, x_2, ..., x_n)$ ko'rinishda ifoda etamiz.

1. $Z = \sqrt{R^2 - x_1^2 - x_2^2}$

Bu funksiyaning aniqlanish soxasi

$$D(z) = \{(x_1; x_2): x_1^2 + x_2^2 \le R^2\}$$

to'plamdan iborat. Bu to'plam markazi koordinata boshida $(0, 0)$, radiusi R ga $(R > 0)$ teng bo'lgan doiradir.

2. $Z = \frac{1}{x_1 \cdot x_2}$ funksiyaning aniqlanish soxasi

$$D(z) = \{(x_1; x_2): x_1 \ne 0,\ x_2 \ne 0\}$$

to'plamdan iborat. Bu to'plam X_1OX_2 tekisligidan OX_1 va OX_2 koordinata o'qlarini chiqarib tashlashdan hosil bo'ladi.

3. $Z=a_1x_1+a_2x_2+\cdots+a_nx_n$ funksiya chiziqli funksiya deyiladi, bu yerda $a_1,a_2,\cdots,a_n$ o'zgarmas sonlar.

Biz asosan, bir o'zgaruvchili funksiya grafigi va xossalari bilan tanishib chiqamiz. Asosiy tushunchalarni esa ko'p o'zgaruvchili funksiyalar uchun beramiz.

$f(x)$ funksiyaning $x=a$ nuqtadagi xususiy qiymati $f(a)$ kabi yoziladi. Masalan, $f(x)=3x^2+5x-7$ bo'lsa $f(0)=-7$, $f(1)=1$ bo'ladi.

$f(x)$ funksiyaning grafigi deb, mumkin bo'lgan $(x, f(x)), x\in D(f)\subset R^1$ juftliklarning xOy tekislikdagi geometrik o'rniga aytiladi.

Funksiyaning juftligi, toqligi va davriyligi

$f(x)$ funksiya $X\subset R$ to'plamda berilgan bo'lsin.

2-ta'rif. Agar ixtiyoriy $x\in X$ uchun $f(-x)=f(x)$ bo'lsa, u holda $f(x)$ *juft funksiya* deyiladi. Juft funksiya grafigi ordinata o'qiga nisbatan simmetrik bo'ladi.

3-ta'rif. Agar ixtiyoriy $x\in X$ uchun $f(-x)=-f(x)$ bo'lsa, u holda $f(x)$ *toq funksiya* deyiladi. Toq funksiya grafigi koordinata boshiga nisbatan simmetrik bo'ladi.

4-ta'rif. Agar shunday o'zgarmas T $(T\neq 0)$ son mavjud bo'lsaki, $\forall x\in X$ uchun

1) $x-T\in X$, $x+T\in X$

2) $f(x+T)=f(x)$

bo'lsa, $f(x)$ davriy funksiya deyiladi, T son esa $f(x)$ funksiyaning davri deyiladi.

Masalan, $f(x)=\sin x$, $f(x)=\cos x$ funksiyalar davriy funksiyalar bo'lib, ularning davri 2π ga, $f(x)=tgx$, $f(x)=ctgx$ funksiyalarning davri esa π ga teng.

Davriy funksiyalar quyidagi xossalarga ega:

a) Agar $f(x)$ davriy funksiya bo‘lib, uning davri $T\ (T\neq 0)$ bo‘lsa, u holda

$$T_n = nT \quad (n=\pm 1, \pm 2,)$$

sonlar ham shu funksiyaning davri bo‘ladi.

b) Agar T_1 va T_2 sonlar $f(x)$ funksiyaning davri bo‘lsa, u holda $T_1+T_2\neq 0$ hamda $T_1-T_2\ (T_1\neq T_2)$ sonlar ham $f(x)$ funksiyaning davri bo‘ladi.

v) Agar $f(x)$ hamda $g(x)$ lar davriy funksiyalar bo‘lib, ularning har birining davri $T\ (T\neq 0)$ bo‘lsa, u holda

$$f(x)+g(x),\quad f(x)-g(x),\quad f(x)\cdot g(x),\quad \frac{f(x)}{g(x)}\quad (g(x)\neq 0)$$

funksiyalar ham davriy funksiyalar bo‘lib, T son ularning ham davri bo‘ladi.

Murakkab funksiya. Funksiyalar kompozitsiyasi. Aytaylik, *u*=•(*x*) funksiya *X* sohada aniqlangan va qiymatlar to‘plami *E*(•) bo‘lsin. Shuningdek, *y*=*f*(*u*) funksiya *E*(•) to‘plamda aniqlangan bo‘lsa, u holda *y*=*f*(•(*x*)) funksiya *X* to‘plamda aniqlangan *murakkab funksiya* yoki • va *f* *funksiyalarning kompozitsiyasi* deyiladi va *f*.• orqali belgilanadi: *f*.•=*f*(•(*x*)).

Monoton, teskari va chegaralangan funksiya

Funksiyaning chegaralanganligi. $f(x)$ funksiya $X\subset R$ to‘plamda berilgan bo‘lsin.

5-ta’rif. Agar shunday o‘zgarmas M soni topilsaki, $\forall x\in X$ uchun $f(x)\leq M$ tengsizlik bajarilsa, $f(x)$ funksiya X to‘plamda yuqoridan chegaralangan deyiladi. Agar shunday o‘zgarmas m soni topilsaki, $\forall x\in X$ uchun $f(x)\geq m$ tengsizlik bajarilsa, $f(x)$ funksiya X to‘plamda quyidan chegaralangan deyiladi.

6-ta’rif. Agar $f(x)$ funksiya X to‘plamda ham yuqoridan, ham quyidan chegaralangan bo‘lsa, $f(x)$ funksiya X to‘plamda chegaralangan deyiladi.

7-ta'rif. Agar har qanday $M>0$ son olinganda ham shunday $x_0 \in X$ nuqta topilsaki,

$$f(x_0)>M$$

tengsizlik bajarilsa, $f(x)$ funksiya X to'plamda yuqoridan chegaralanmagan deyiladi.

Monoton funksiyalar. Aytaylik, $f(x)$ funksiya X to'plamda berilgan bo'lsin.

8-ta'rif. Agar $\forall x_1, x_2 \in X$ uchun $x_1<x_2$ tengsizlikdan $f(x_1)<f(x_2)$ tengsizlik kelib chiqsa, u holda $f(x)$funksiya X to'plamda *o'suvchi* deb ataladi.

9-ta'rif. Agar $\forall x_1, x_2 \in X$ uchun $x_1<x_2$ tengsizlikdan $f(x_1)>f(x_2)$ tengsizlik kelib chiqsa, u holda $f(x)$ funksiya X to'plamda *kamayuvchi* deb ataladi.

10-ta'rif. Agar $\forall x_1, x_2 \in X$ uchun $x_1<x_2$ tengsizlikdan $f(x_1)\leq f(x_2)$ (yoki $f(x_1)\geq f(x_2)$) tengsizlik kelib chiqsa, u holda $f(x)$ funksiya X to'plamda *kamaymaydigan* (yoki *o'smaydigan*) deyiladi.

O'smaydigan hamda *kamaymaydigan* funksiyalar umumiy nom bilan monoton funksiyalar deyiladi.

Teskari funksiya. $y=f(x)$ funksiya $X \subset R$ to'plamda berilgan bo'lib, bu funksiyaning qiymatlaridan iborat to'plam

$$Y_f=\{f(x) \mid x \in X\}$$

bo'lsin.

Faraz qilaylik, biror qoidaga ko'ra Y_f, to'plamdan olingan har bir y ga X to'plamdagi bitta x mos qo'yilgan bo'lsin. Bunday moslik natijasida funksiya hosil bo'ladi. Odatda, bu funksiya $y=f(x)$ ga nisbatan teskari funksiya deyiladi va $x=f^{-1}(y)$ kabi belgilanadi.

Iqtisodiyotda funksiyalar

Iqtisodda tez-tez uchraydigan va o'zining iqtisodiy nomiga ega bo'lgan funksiyalar qatoriga quyidagilarni keltirish mumkin:

1. Mahsulot hajmi funksiyasi. Bu funksiya ishlab chiqarishda mahsulot hajmining homashyo zaxirasi va iste'molchiga bog'liqligini aniqlaydi.

2. Sarf-xarajat funksiyasi. Bu funksiya ishlab chiqarishda sarf-xarajatlarni mahsulot hajmi bilan bog'liqligini aniqlaydi.

3. Talab, iste'mol va taklif funksiyalari. Bu funksiyalar mahsulotga bo'lgan talab, iste'mol va taklif hajmlarining turli faktorlarga (masalan, narx-navo, daromad va boshqa) bog'liqligini aniqlaydi.

Talab va taklif egri chiziqlari. Muvozanat nuqtasi.

D (dimand) talab va S (supply) taklifning P (price) tovar narxiga bog'liqligini ko'rib chiqamiz. Narx qancha kam bo'lsa, axolining doimiy sotib olish qobiliyatida talab shuncha katta bo'ladi.

Odatda D ning P ga bog'liqligi egri chiziq ko'rinishiga ega

$$D = P^a + c, (1)$$

Bu yerda $a < 0$. O'z navbatida taklif tovar narxi ortishi bilan o'sadi va shuning uchun S ning P ga bog'liqligi quyidagi ko'rinishga ega

$$S = P^b + d, (2)$$

Bu yerda $b \geq 1$. (1) va (2) formulalarda c va d ekzogen kattaliklar, ular tashqi sabablarga bog'liq. (1) va (2) formulalarga kiruvchi o'zgaruvchilar musbat shuning uchun funksiyalarning grafiklari faqat birinchi chorakda ma'noga ega.

Iqtisod uchun muvozanat sharti ya'ni talab taklifga teng bo'lgan holat katta ahamiyatga ega. Bu shart

$$D(P) = S(P) (3)$$

Tenglama bilan beriladi va D va S egri chiziqlarning kesishish nuqtaga mos keladi. Bu nuqta muvozanat nuqtasi. (3) shart bajarilgandagi P_0 narx muvozantli deyiladi.

Ma'lum iqtisodiy jarayonlar ko'p faktorlar ta'siri natijasida yuzaga kelgani uchun yuzaga keladigan funksiyalar ko'p o'zgaruvchili funksiyalar bo'ladi. Iqtisodda uchraydigan asosiy tushunchalardan biri, bu foydalilik funksiyasidir. Iqtisodiy jarayonlarni tahlil qilishda foydalilik funksiyasi tushunchasidan keng foydalaniladi. Bu funksiya iste'molchining biror bir tovarlar vektorini boshqa tovarlar vektoridan afzal ko'rishini ifodalaydi.

Deylik, iste'molchi n turdagi tovarlardan foydalansin. Bu tovarlar miqdorini bildiruvchi tovarlar vektorini X satr vektor sifatida ifodalaymiz. X va Y tovarlar orasida $X \succ Y$ afzallik munosabatini kiritamiz. Bu munosabat iste'molchining X tovarlar vektorini Y tovarlar vektoridan afzal ko'rishini ifodalaydi. Misol uchun $X > Y$ bo'lsa, u holda $X \succ Y$. Bir xil afzallikka ega bo'lgan X va Y tovarlar vektorlarini farqlanmaydigan tovarlar vektorlari deb ataymiz va $X \sim Y$ kabi belgilaymiz.

Afzallik munosabati odatda foydalilik (utility) funksiyasi deb ataluvchi $U(X)$ funksiya yordamida aniqlanadi.

11-ta'rif. Ixtiyoriy X, Y tovarlar vektorlari uchun $X \succ Y \Leftrightarrow U(X) > U(Y)$ va $X \sim Y \Leftrightarrow U(X) = U(Y)$ shartlarni qanoatlantiruvchi $U(X)$ funksiyani foydalilik funksiyasi deb ataymiz.

Odatda foydalilik funksiyasining qiymati emas, turli tovarlar vektoriga mos qiymatlari orasidagi "katta", "kichik" yoki "teng" kabi munosabatlar muhim hisoblanadi. Foydalilik funksiyasi har bir alohida o'zgaruvchisi bo'yicha (boshqa o'zgaruvchilar o'zgarmas bo'lganda) o'suvchi funksiya bo'ladi.

Muhim bo'lgan foydalilik funksiyalaridan biri CES-funksiya deb ataladi. Bu funksiya nomidagi CES (constant elasticity of substituion) qisqartmasi alternativ (bir-birining o'rnini bosuvchi) tovarlarning o'zgarmas elastiklikka egaligini bildiradi. Ikki o'zgaruvchili holda bu funksiya quyidagicha:

$$U(x_1, x_2) = \left(\alpha x_1^{1/\rho} + \beta x_2^{1/\rho}\right)^{\rho}.$$

Bu funksiyaning xususiy holatlarini qaraymiz.

1) $\rho = 1$ da chiziqli foydalilik funksiyasi hosil bo'ladi

$$u(x_1, x_2) = \alpha x_1 + \beta x_2.$$

2) $\rho \to -\infty$ da Leontev funksiyasi, deb ataluvchi foydalilik funksiyasi hosil bo'ladi

$$u(x_1,x_2)=\min\{x_1,x_2\}.$$

3) Agar $\alpha+\beta=1$ bo'lsa, $\rho\to 0$ da Kobb-Duglas funksiyasi hosil bo'ladi

$$u(x_1,x_2)=x_1^{\alpha}x_2^{\beta}.$$

Bu funksiyalarni n ta o'zgaruvchi holatiga ham umumlashtirishimiz mumkin.

Misol. Foydalilik funksiyasi $U(x_1,x_2,x_3)=0,2\lg x_1+0,3\lg x_2+0,5\lg x_3$ formula bilan aniqlangan bo'lsin. $X_1(10;100;100)$, $X_1(100;10;100)$ tovarlar vektorlarini afzallik munosabati yordamida tekshiring.

Yechish. Foydalilik funksiyasining qiymatlarini topamiz:

$$U(X_1)=U(10,100,100)=1,8;\quad U(X_2)=U(100,10,100)=1,7$$

Bundan, $U(X_1)>U(X_2)\Rightarrow X_1\succ X_2$.

Foydalilik funksiyasi umuman olganda yagona aniqlanmaydi.

Yuqoridagi misolda keltirilgan $U(x_1,x_2,x_3)=0,2\lg x_1+0,3\lg x_2+0,5\lg x_3$ foydalilik funksiyasi yordamida $10^{U(x_1,x_2,x_3)}=x_1^{0,2}x_2^{0,3}x_3^{0,5}$ Kobb-Duglas foydalilik funksiyasini hosil qilish mumkin.

Kobb-Duglas funksiyasidan ishlab chiqarish funksiyasi sifatida ham foydalaniladi.

$$Q(L,K)=A\cdot L^{\alpha}K^{\beta}$$

ishlab chiqarish funksiyasida Q – ishlab chiqarilgan mahsulot miqdori, L - mehnat resurslariga sarf xarajatni, K – ishlab chiqarishga sarflangan kapitalni, A - texnologik koeffitsiyent, α va β elastiklik koeffitsiyenlarini ifodalaydi. Misol uchun, $Q=L^{0,73}K^{0,27}$ ifodada umumiy ishlab chiqarilgan mahsulot miqdorida mehnat resurslari ulushi 73%, kapital mablag'lar ulushi 27% ni tashkil qilishini bildiradi.

Foydalilik funksiyasi yordamida bitta sodda iqtisodiy modelni qaraymiz. Faraz qilaylik iste'molchining jami mablag'i (byudjeti) S ga teng bo'lsin. U bu mablag'ni bir birligi narxi $p_1,p_2,...,p_n$ bo'lgan n xil tovar uchun sarflashi mumkin. Bu jarayondagi $U(x_1,x_2,...,x_n)$ foydalilik funksiyasi berilgan bo'lsin. Eng afzal tovarlar vektorini topish masalasini qaraymiz.

Tovarlar vektori X bo'lsin. Narxlar vektorini P kabi aniqlaymiz. Bu masalada quyidagi cheklovlar mavjud.

1) Har bir turdagi sotib olingan tovarlar miqdori nomanfiy, ya'ni $X \geq 0$.

2) Iste'molchi byudjeti cheklangan $(P,X) = p_1x_1 + ...p_nx_n \leq S$.

Bu cheklovlar byudjet to'plami $B(P,S)$ ni aniqlaydi. Demak bizdan $B(P,S)$ byudjet to'plamida $U(X)$ foydalilik funksiyasini maksimallashtirish talab qilinadi.

Ma'lumki, ikki tovar qaralgan holatda $B(P,S)$ byudjet to'plami 1-chorakda joylashgan katetlari koordinata o'qlarida yotuvchi to'g'ri burchakli uchburchakdan, uch tovar holatida uchburchakli piramidadan iborat bo'ladi.

Misollar

1. Funksiyani aniqlanish sohasini toping

a) $f(x) = \dfrac{(3x+1)}{(x^2-1)}$

Yechish. $\dfrac{3x+1}{x^2-1}$ kasrning maxraji nolga teng bo'lmasa, kasr aniqlangan. Shuning uchun funksiyaning aniqlanish sohasi $x^2-1 \neq 0, x \neq \pm 1$ shartni qanoatlantirishi kerak. Shunday qilib,

$D(f) = (-\infty;-1) \cup (-1;1) \cup (1;+\infty)$

b) $f(x) = \sqrt{5-3x}$

Yechish. $f(x) = \sqrt{5-3x}$ ildiz osti musbat bo'lganda, funksiya aniqlangan. $5-3x \geq 0$ bu yerdan $x \leq \dfrac{5}{3}$, demak, $D(f) = (-\infty; \dfrac{5}{3}]$.

v) $f(\mathrm{x}) = \ln(x+2)$

Yechish. $\ln(x+2)$ funksiya $x+2>0, x>-2$ shartda aniqlanadi. Demak, $D(f) = (-2;+\infty)$.

g) $f(x) = 2^{\frac{1}{n}} + arc\sin\dfrac{x+2}{3}$

Yechish. a^x funksiya, x ning aniq qiymatlarida $a>0$ aniqlangan, demak, $2^{\frac{1}{n}}$ funksiya ham x ning ma'lum qiymatlarida, ya'ni $\dfrac{1}{x}$, $x \neq 0$

qiymatda ma'noga ega. Ikkinchi qo'shiluvchini esa tengsizlik ko'rinishi bilan ifodalaymiz, $-1\le\frac{x+2}{3}\le 1$ bu yerdan $-3\le x+2\le 3$ va $-5\le x\le 1$. $f(x)$ funksiyaning aniqlanish sohasi $D(f)=[-5;0)\cup(0;1]$.

d) $f(x)=\frac{5}{\sqrt[3]{2x-x^2}}-7\cos 2x$

Yechish. $7cos2x$ funksiya, x ning barcha haqiqiy qiymatlarida aniqlangan, $\frac{5}{\sqrt[3]{2x-x^2}}$ funksiya esa x ning uch qiymatida, $2x-x^2\ne 0, x\ne 0, x\ne 2$.

Shunday qilib, $D(f)=(-\infty;0)\cup(0;2)\cup(2;+\infty)$.

2. $y=1+2^{x+1}$ funksiyaning qiymatlar to'plamini toping.

Yechish. $y=2^{x+1}=2\cdot 2^x$ ko'rsatkichli funksiya, uning qiymatlar to'plami $y\in(0;+\infty)$, demak berilgan funksiyaning qiymatlar to'plami $(1;+\infty)$ bo'ladi yoki berilgan funksiyaning qiymatlar to'plami uning teskari funksiyasi $x=\log_2(y-1)-1$ ning aniqlanish sohasi $y>1$ bilan ustma-ust tushadi, shuning uchun $E(y)=(1;+\infty)$.

3. $f(x)=\lg\left(x+\sqrt{x^2+1}\right)$ funksiyaning juft yoki toqligini tekshiring.

Yechish. $f(-x)=\lg\left(-x+\sqrt{(-x)^2+1}\right)=\lg\frac{\left(x+\sqrt{x^2+1}\right)\left(-x+\sqrt{x^2+1}\right)}{\left(x+\sqrt{x^2+1}\right)}=$

$$=\lg\frac{1}{\left(x+\sqrt{x^2+1}\right)}=\lg\left(x+\sqrt{x^2+1}\right)^{-1}=-\lg\left(x+\sqrt{x^2+1}\right)=-f(x).$$

4. $y=2x+1$ funksiya $(-\infty;+\infty)$ da monotonligini ko'rsating.

Yechish. $y=2x+1$ funksiya $(-\infty;+\infty)$ da o'suvchi, chunki $x_1<x_2$ bo'lsa, u holda $f(x_2)-f(x_1)=2x_2+1-(2x_1+1)=2(x_2-x_1)>0$ bo'ladi va $f(x_1)<f(x_2)$ tengsizlik kelib chiqadi.

5. Ushbu $f(x)=\frac{x}{1+x^2}$ funksiyaning $X=[1,+\infty)$ to'plamda kamayuvchi ekanligi isbotlansin.

Yechish. $[1,+\infty)$ da ixtiyoriy x_1 va x_2 nuqtalarni olib, $x_1 < x_2$ bo'lsin deylik. Unda

$$f(x_1)-f(x_2)=\frac{x_1}{1+x_1^2}-\frac{x_2}{1+x_2^2}=\frac{x_1+x_1x_2^2-x_2-x_2x_1^2}{(1+x_1^2)(1+x_2^2)}=$$

$$=\frac{x_1-x_2+x_1\cdot x_2(x_2-x_1)}{(1+x_1^2)(1+x_2^2)}=\frac{(x_1-x_2)(1-x_1\cdot x_2)}{(1+x_1^2)(1+x_2^2)}$$

bo'ladi. Keyingi tenglikda

$$x_1-x_2<0, \qquad 1-x_1\cdot x_2<0$$

bo'lishini e'tiborga olib,

$$f(x_1)-f(x_2)>0$$

ya'ni, $f(x_1)>f(x_2)$ ekanini topamiz. Demak,

$x_1<x_2 \quad \Rightarrow \quad f(x_1)>f(x_2).$

6. Ushbu $f(x)=\dfrac{1+x^2}{1+x^4}$ funksiyani qaraylik. Bu funksiya R da chegaralangan bo'ladi.

Yechish. Ravshanki, $\forall x\in R$ da $f(x)=\dfrac{1+x^2}{1+x^4}>0$.

Demak, berilgan funksiya R da quyidan chegaralangan.

Ayni paytda, $f(x)$ funksiya uchun

$$f(x)=\frac{1}{1+x^4}+\frac{x^2}{1+x^4}\le 1+\frac{x^2}{1+x^4}$$

bo'ladi. Endi

$$0\le(x^2-1)^2=x^4-2x^2+1 \quad \Rightarrow\; 2x^2\le x^4+1 \;\Rightarrow\; \frac{x^2}{x^4+1}\le\frac{1}{2}$$

bo'lishini e'tiborga olib, topamiz: $f(x)\le 1+\dfrac{1}{2}=\dfrac{3}{2}$.

Bu esa $f(x)$ funksiyaning yuqoridan chegaralanganligini bildiradi. Demak, berilgan funksiya R da chegaralangan.

7. F doimiy harajatlar (ishlab chiqarilgan mahsulotning x birligi soniga bog'liq bo'lmagan) bir oyda 125 ming pul birligini, $V(x)$

o'zgaruvchan harajatlar (x ga proporsional) mahsulotning bir birligi uchun 700 pul birligini tashkil etadi. Mahsulot birligining narxi 1200 pul birligi. Daromad a) 0 ga, b) bir oyda 105ming pul birligiga teng bo'lgandagi x mahsulot hajmini toping.

Yechish. a) x birlik mahsulotni ishlab chiqarish harajatlari $C(x)=F+V(x)=125+0,7x$ (ming pul birligi). Bu mahsulotni sotishdan keladigan daromad $R(x)=1,2x$, foyda $P(x)=R(x)-C(x)=0,5x-125$(ming pul birligi). $P(x)=0,5x-125=0$, $x=250$ (birlik).

b) $P(x)$ foyda 105 ming pul birligiga teng, $P(x)=0,5x-125=105$, $x=460$ (birlik).

5.3. Funksiya limiti va uzluksizligi

Amaliyotda funksiya tushunchasi katta ahamiyatga ega bo'lganligi sababli biz funksiyani atroflicha o'rganib chiqamiz. Bizga ma'lumki, R^1 fazoda x_0 nuqtaning δ atrofi quyidagicha aniqlanadi:

$$U_\delta(x_0)=\{x:|x-x_0|<\delta\}.$$

$y=f(x)$ funksiya biror $X\in R^1$ to'plamda aniqlangan bo'lsin.

1-ta'rif (Koshi ta'rifi). Agar ixtiyoriy $\varepsilon>0$ son uchu shunday $\delta(\varepsilon)>0$ son mavjud bo'lib, $|x-x_0|<\delta$ tengsizlikni qanoatlantiruvchi barcha x lar uchun $|f(x)-A|<\varepsilon$ tengsizlik o'rinli bo'lsa, u holda A soni $f(x)$ funksiyaning x_0 nuqtadagi limiti deyiladi.

Bu limit quyidagicha yoziladi $\lim\limits_{x\to x_0} f(x)=A$.

2-ta'rif (Geyne ta'rifi). Agar X to'plamga tegishli ixtiyoriy yaqinlashuvchi, $\lim\limits_{n\to\infty} x_n=x_0$, $x_1,x_2,...,x_n$ ketma-ketlik uchun $y=f(x)$ funksiyaning $f(x_1),f(x_2),...,f(x_n)$ qiymatlaridan tashkil topgan ketma-ketlik ham A soniga yaqinlashsa, intilsa, u holda A soni $f(x)$ funksiyaning $x\to x_0$ dagi limiti deyiladi.

Bu limit $\lim\limits_{n\to\infty} f(x_n)=A$ ko'rinishda yoziladi.

Yuqorida keltirilgan ta'riflardan birini qo'llab $\lim\limits_{x\to 0}\sin x=0$, $\lim\limits_{x\to 1}\frac{x+2}{2x-1}=3$ tengliklarni isbotlash mumkin.

X to'plamda aniqlangan limitga ega funksiyalar o'zlarining quyidagi xossalari bilan xarakterlanadi:

1. $y=f(x)$ funksiya $x\to x_0$ da chekli limitga ega bo'lsa, u holda bu limit yagonadir;

2. $y=f(x)$ funksiya $x\to x_0$ da chekli limitga ega bo'lsa, u holda x_0 nuqtaning shunday $U_\delta(x_0)$ atrofi mavjudki, $U_\delta(x_0)\cap X$ to'plamda $f(x)$ funksiya chegaralangan bo'ladi.

3. Agar $\lim\limits_{x\to x_0} f(x_n)=A\neq 0$ bo'lsa, u holda x_0 nuqtaning shunday atrofi topiladiki, bu atrofda funksiyaning ishorasi A sonning ishorasi bilan bir xil bo'ladi.

4. Agar biror $\delta>0$ son va barcha $x_0\in U_\delta(x_0)$ nuqtalar uchun $\lim\limits_{x\to x_0} f(x_n)=A$, $\lim\limits_{x\to x_0} g(x_n)=B$ bo'lib, $f(x)\leq g(x)$ bo'lsa, u holda $A\leq B$ bo'ladi.

$y=f(x)$ funksiya biror bir $X=(a,\infty)$ nurda aniqlangan bo'lsin.

3-ta'rif. Agar ixtiyoriy $\varepsilon>0$ son uchun shunday bir $K(\varepsilon)>0$ sonni ko'rsatish mumkin bo'lib, barcha $|x|>K$ munosabatni qanoatlantiruvchi x lar uchun $|f(x)-b|<\varepsilon$ tengsizlik o'rinli bo'lsa, u holda b soni $f(x)$ funksiyaning $x\to\infty$ dagi limiti deyiladi.

$y=f(x)$ funksiyaning $x\to-\infty$ limiti ham yuqoridagi kabi ta'riflanadi.

4-ta'rif. Agar ixtiyoriy $A>0$ son uchun shunday $\delta(A)>0$ son topilsaki $0<|x-x_0|<\delta$ bo'lganda $|f(x)|>A$ tengsizlik bajarilsa, u holda $f(x)$ funksiya x_0 nuqtada cheksiz limitga ega deyiladi.

5-ta'rif. Agar ixtiyoriy $\varepsilon>0$ son uchun shunday $\delta>0$ sonni topish mumkin bo'lib $x_0-\delta<x<x_0$ $(x_0<x<x_0+\delta)$ shartni qanoatlantiruvchi barcha x lar uchun $|f(x)-b|<\varepsilon$ tengsizlik bajarilsa, $b=f(x_0-0)$, $(b=f(x_0+0))$ son $f(x)$ funksiyaning x_0 nuqtadagi chap (o'ng) limiti deyiladi

Bu limit quyidagicha yoziladi

$$b=f(x_0-0)=\lim_{x\to x_0-0}f(x)\quad \left(b=f(x_0+0)=\lim_{x\to x_0+0}f(x)\right).$$

$y=f(x)$ funksiyaning x_0 nuqtada limiti mavjud bo'lishi uchun bu funksiya shu nuqtada chap va o'ng limitlarga ega bo'lib, $f(x_0-0)=f(x_0+0)$ tenglik bajarilishi zarur va yetarli.

Quyidagi teoremalar limitlar haqidagi asosiy teoremalar deb atalib, funksiya limitlarining asosiy xossalarini ifodalaydi: $\lim\limits_{x\to x_0}f(x)=A$, $\lim\limits_{x\to x_0}g(x)=B$ bo'lsin. U holda

1) $\lim\limits_{x\to x_0}\left(f(x)\pm g(x)\right)=\lim\limits_{x\to x_0}f(x)\pm\lim\limits_{x\to x_0}g(x)=A\pm B;$

2) $\lim\limits_{x\to x_0}f(x)\cdot g(x)=\lim\limits_{x\to x_0}f(x)\cdot\lim\limits_{x\to x_0}g(x)=A\cdot B;$

3) $\lim\limits_{x\to x_0}\dfrac{f(x)}{g(x)}=\dfrac{\lim\limits_{x\to x_0}f(x)}{\lim\limits_{x\to x_0}g(x)}=\dfrac{A}{B}\ (B\neq 0).$

Amaliyotda ko'p qo'llaniladigan ajoyib limitlar nomini olgan limitlarni keltirib o'tamiz:

1- ajoyib limit: $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1;$

2- ajoyib limit: $\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}}=e$.

Bu ajoyib limitlarning boshqa shakllari ham mavjud bo'lib ular quyidagilardir:

1) $\lim\limits_{x\to 0}\dfrac{tgx}{x}=1$; 2) $\lim\limits_{x\to 0}\dfrac{\arcsin x}{x}=1$; 3) $\lim\limits_{x\to 0}\dfrac{arctgx}{x}=1$;

4) $\lim\limits_{x\to 0}\log_a(1+x)^{\frac{1}{x}}=\log_a e$; 5) $\lim\limits_{x\to 0}\ln(1+x)^{\frac{1}{x}}=1$;

Uzluksizlik tushunchasi funksiyaning asosiy xarakteristikalaridan biri bo'lib, u amaliyotda muhim ahamiyatga ega.

Faraz qilamiz, $y=f(x)$ funksiya $X\subseteq R^1$ to'plamda aniqlangan bo'lib, $x_0\in X$ bo'lsin.

6-ta'rif. Agar ixtiyoriy $\varepsilon>0$ son uchun biror $\delta(\varepsilon)>0$ son topilib, $|x-x_0|<\delta$ o'rinli bo'lganda $|f(x)-f(x_0)|<\varepsilon$ tengsizlik bajarilsa, u holda $y=f(x)$ funksiya x_0 nuqtada uzluksiz deyiladi.

Funksiyaning nuqtada uzluksizligi shu nuqta atrofida argumentning cheksiz kichik orttirmasiga funksiyaning cheksiz kichik orttirmasi mos kelishidir.

Masalan, $y=\cos x$ funksiya har bir $x_0\in R^1$ nuqtada uzluksiz, haqiqatan ham

$$\lim_{\Delta x\to 0}\Delta y=\lim_{\Delta x\to 0}\Delta f(x_0)=\lim_{\Delta x\to 0}\left[\cos(x_0+\Delta x)-\cos(x_0)\right]=$$
$$=\lim_{\Delta x\to 0}\left[-2\sin\frac{\Delta x}{2}\sin\left(x_0+\frac{\Delta x}{2}\right)\right]=0.$$

Nuqtada uzluksiz funksiyalar ustida arifmetik amallar bajarish mumkin. Nuqtada uzluksiz bo'lgan funksiya shu nuqtaning kichik δ atrofida chegaralangan bo'lib o'z ishorasini saqlaydi.

Agar funksiya X to'plamning har bir nuqtasida uzluksiz bo'lsa, u holda bu funksiya X to'plamda uzluksiz deyiladi.

Agar $f(x)$ funksiya $[a,b]$ oraliqda uzluksiz bo'lsa, u holda bu funksiya shu oraliqda chegaralangan bo'ladi va o'zining eng katta va eng kichik qiymatlariga erishadi.

Uzluksiz funksiyalar uchun ba'zi teoremalarni ketirib o'tamiz.

Teorema. Agar $f(x)$ funksiya $[a,b]$ kesmada uzluksiz va kesmaning chetki nuqtalaridagi qiymatlari turli ishorali $(f(a)f(b)<0)$ bo'lsa, u holda kamida bitta shunday $c\in(a,b)$ nuqta topiladiki, bunda $f(c)=0$ tenglik bajariladi.

Teorema. Agar $f(x)$ funksiya $[a,b]$ oraliqda uzluksiz va $f(a)\neq f(b)$ bo'lsa, u holda ixtiyoriy $f(a)<C<f(b)$ uchun shunday $\xi\in[a,b]$ son topiladiki bunda $f(\xi)=C$ bo'ladi.

$y=f(x)$ funksiya x_0 nuqtada uzluksiz bo'lishi uchun $f(x_0-0)=f(x_0)=f(x_0+0)$ tenglik bajarilishi shart.

7-ta'rif. Agar $y=f(x)$ funksiya uchun $f(x_0-0)=f(x_0)=f(x_0+0)$ shartning bittasi bajarilmasa yoki u x_0 nuqtada aniqlanmagan bo'lsa, u holda x_0 nuqta $y=f(x)$ funksiyaning uzilish nuqtasi deyiladi.

8-ta'rif. Agar $y=f(x)$ funksiya x_0 nuqtada chapdan va o'ngdan limitlari mavjud bo'lib, ular o'zaro teng bo'lmasa, ya'ni $f(x_0-0)\neq f(x_0+0)$ bo'lsa, u holda x_0 nuqta $y=f(x)$ funksiyaning birinchi tur uzilish nuqtasi deyiladi.

9-ta'rif. Agar $y=f(x)$ funksiyaning x_0 nuqtada limiti mavjud, lekin bu limit funksiyaning x_0 nuqtada erishadigan $y_0=f(x_0)$ qiymatidan farq qilsa yoki $y=f(x)$ funksiya x_0 nuqtada aniqlanmagan bo'lsa, u holda x_0 nuqta bartaraf etiladigan uzilish nuqta deb ataladi.

10-ta'rif. Agar $y=f(x)$ funksiyaning x_0 nuqtada chap yoki o'ng limitlarining hech bo'lmaganda bittasi mavjud bo'lmasa yoki cheksiz bo'lsa, u holda x_0 nuqta $y=f(x)$ funksiyaning ikkinchi tur uzilish nuqtasi deyiladi.

$|f(x_0-0)-f(x_0+0)|$ ayirma $y=f(x)$ funksiyaning x_0 nuqtadagi sakrashi deyiladi.

Masalan, $f(x)=\dfrac{1}{1+2^{\frac{1}{x}}}$ funksiya $x=0$ nuqtada birinchi tur uzilishga ega, chunki $\lim\limits_{x\to 0-0} f(x)=1\neq 0=\lim\limits_{x\to 0} f(x)=\lim\limits_{x\to 0+0} f(x)$.

Masalan, $f(x)=\dfrac{\sin x}{x}$ funksiyaning $x=0$ nuqtada limiti mavjud (1-ajoyib limit). Lekin, bu funksiya $x=0$ nuqtada aniqlanmagan, birinchi tur uzilish nuqta. Bu uzilishni funksiyaga uning shu nuqtadagi limit qiymatini qo'yish orqali yo'qotish mumkin, ya'ni

$$f_1(x)=\begin{cases}\dfrac{\sin x}{x}, & x\neq 0;\\ 1, & x=0.\end{cases}$$

Bu funksiya barcha son o'qida uzluksizdir.

$y=f(M)$, $M(x_1,...,x_n)\in X\subseteq R^n$ funksiya va M_0 urinish nuqtasi berilgan bo'lsin.

11-ta'rif. Agar A nuqtaning ixtiyoriy $U(A)$ atrofi uchun M_0 nuqtaning $U(M_0)$ atrofi mavjud bo'lib, $f(M \cap U(M_0)) \subset U(A)$ munosabat o'rinli bo'lsa, u holda A nuqta $f(M)$ funksiyaning M_0 nuqtadagi limiti deb ataladi.

12-ta'rif. Agar ixtiyoriy $\varepsilon > 0$ uchun shunday $\delta(\varepsilon) > 0$ topilib, $\rho(M_0, M) < \delta$ munosabat o'rinli bo'lgan barcha $M \in X$ nuqtalar uchun $|f(M) - b| < \varepsilon$ tengsizlik bajarilsa, u holda b soni $f(M)$ funksiyaning M_0 nuqtadagi limiti deyiladi va u quyidagicha yoziladi:

$$\lim_{M \to M_0} f(M) = b, \quad \lim_{\substack{x_1 \to x_1^0 \\ x_2 \to x_2^0 \\ \cdots\cdots \\ x_n \to x_n^0}} f(x_1, x_2, \ldots, x_n) = b$$

Ma'limki, $y = f(M)$, $M \in X \subseteq R^n$ funksiyaning M_0 nuqtadagi limitini qarayotganimizda bu nuqta X to'plamga tegishli bo'lishi ham tegishli bo'lmasligi ham mumkin. Agar $M_0 \in X$ bo'lib $\lim\limits_{M \to M_0} f(M) = b$ limit mavjud bo'lsa, u holda bu limit qyidagicha yoziladi:

$$\lim_{M \to M_0} f(M) = b = f(M_0)$$

va $y = f(M)$ funksiya M_0 nuqtada uzluksiz deb ataladi.

Agar X to'plam M_0 nuqtaning qandaydir $U(M_0)$ atrofini ham o'z ichiga olsin. U holda $y = f(M)$ funksiyaning M_0 nuqtadagi barcha $U(M_0)$ atrofi bo'yicha limitiga har tomonlama limit deyiladi.

Misollar

1. $\lim\limits_{x \to 8} \dfrac{\sqrt{9+2x} - 5}{\sqrt[3]{x} - 2}$ hisoblansin.

Yechish. $\lim\limits_{x \to 8} \dfrac{\sqrt{9+2x} - 5}{\sqrt[3]{x} - 2} = \left(\dfrac{0}{0}\right) = \lim\limits_{x \to 8} \left[\dfrac{(9+2x) - 5^2}{x - 2^3} \cdot \dfrac{\sqrt[3]{x^2} + 2 \cdot \sqrt[3]{x} + 2^2}{\sqrt{9+2x} + 5}\right] =$

$$= \lim_{x \to 8} \frac{2(x-8)\left(\sqrt[3]{x^2} + 2 \cdot \sqrt[3]{x} + 4\right)}{(x-8)\left(\sqrt{9+2x} + 5\right)} = 2 \cdot \lim_{x \to \infty} \frac{\sqrt[3]{x^2} + 2 \cdot \sqrt[3]{x+4}}{\sqrt{9+2x} + 5} = 2{,}4.$$

2. $\lim\limits_{x \to \pi} \dfrac{e^{\pi} - e^{x}}{\sin 5x - \sin 3x}$ hisoblansin.

Yechish. $\lim\limits_{x \to \pi} \dfrac{e^{\pi} - e^{x}}{\sin 5x - \sin 3x} = \left(\dfrac{0}{0}\right) = \left(\begin{pmatrix} x = \pi + t \text{ almashtirish bajaramiz} \\ x \to \pi \Rightarrow t \to 0 \end{pmatrix}\right) =$

$$=\lim_{t\to 0}\frac{e^{\pi}-e^{\pi+1}}{\sin(5\pi+5t)-\sin(3\pi+3t)}=e^{\pi}\lim_{t\to 0}\frac{1-e^{t}}{-\sin 5t+\sin 3t}=$$

$$=e^{\pi}\lim_{t\to 0}\frac{e^{t}-1}{\sin 5t-\sin 3t}=e^{\pi}\lim_{t\to 0}\frac{\frac{e^{t}-1}{t}}{5\cdot\frac{\sin 5t}{5t}-3\cdot\frac{\sin 3t}{3t}}=$$

$$=\left(\left(\begin{array}{l}\lim\limits_{t\to 0}\frac{e^{t}-1}{t}=1\\ \lim\limits_{\alpha\to 0}\frac{\sin\alpha}{\alpha}=1\end{array}\right)\right)=e^{\pi}\frac{1}{5-3}=\frac{e^{\pi}}{2}.$$

3. Ushbu $\lim\limits_{x\to\frac{\pi}{2}}(\sin x)^{\frac{18\sin x}{ctgx}}$ limit hisoblansin.

Yechish. $\lim\limits_{x\to\frac{\pi}{2}}(\sin x)^{\frac{18\sin x}{ctgx}}=\left(1^{\infty}\right)=\left(\left(\begin{array}{l}x=\frac{\pi}{2}+t\\ x\to\frac{\pi}{2}\Rightarrow t\to 0 \text{ almashtirish bajaramiz}\end{array}\right)\right)=$

$$=\lim_{t\to 0}(\cos t)^{\frac{18\cos t}{-tgt}}=\lim_{t\to 0}[1+(\cos t-1)]^{\frac{18\cos t}{tgt}}=\left(\left(\lim_{\alpha\to 0}(1+\alpha)^{\frac{1}{\alpha}}=e \text{ dan foydalanamiz}\right)\right)=$$

$$e^{-\lim\limits_{t\to 0}\frac{18\cos t}{tgt}(\cos t-1)}=e^{\lim\limits_{t\to 0}\frac{18\cos^2 t}{\sin t}2\sin^2\frac{t}{2}}=e^{\lim\limits_{t\to 0}\frac{36\cos^2 t\cdot\sin^2\frac{t}{2}}{2\sin\frac{t}{2}\cdot\cos\frac{t}{2}}}=e^{\lim\limits_{t\to 0}\left[\frac{18\cos^2 t}{\cos\frac{t}{2}}\cdot\sin\frac{t}{2}\right]}=e^{0}=1.$$

4. Ushbu

$$\lim_{x\to 0}\frac{9^{x}-2^{3x}}{arctg2x-7x}$$

limit hisoblansin.

Yechish.

$$\lim_{x\to 0}\frac{9^{x}-2^{3x}}{arctg2x-7x}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{\frac{9^{x}-1}{x}-3\cdot\frac{2^{3x}-1}{3x}}{2\cdot\frac{arctg2x}{2x}-7}=\left(\left(\lim_{t\to 0}\frac{a^{t}-1}{t}=\ln a \text{ ва } \lim_{t\to 0}\frac{arctgt}{t}=1\right)\right)=$$

$$=\frac{\ln 9-3\ln 2}{2-7}=-\frac{1}{5}\ln\frac{9}{8}.$$

5. $f(x)=\lim\limits_{n\to\infty}\frac{x}{1+(2\sin x)^{2n}}$ funksiya uzluksizlikka tekshirilsin.

Yechish. $f(x)=\lim\limits_{n\to\infty}\frac{x}{1+(2\sin x)^{2n}}=\begin{cases} x, & |2\sin x|<1, \\ \frac{x}{2}, & |2\sin x|=1, \\ 0, & |2\sin x|>1 \end{cases}=\begin{cases} x, & -\frac{\pi}{6}+\pi k<x<\frac{\pi}{6}+\pi k, \\ \frac{x}{2}, & x=\pm\frac{\pi}{6}+\pi k, \\ 0, & \frac{\pi}{6}+\pi k<x<\frac{5\pi}{6}+\pi k,\ k\in Z. \end{cases}$

Bu tenglikdan ko`rinib turibdiki $f(x)$ funksiya $\left(-\frac{\pi}{6}+\pi k;\ \frac{\pi}{6}+\pi k\right)$ va $\left(\frac{\pi}{6}+\pi k;\ \frac{5\pi}{6}+\pi k\right)$, $k\in Z$ oraliqlarda uzluksiz hamda $x=\pm\frac{\pi}{6}+\pi k,\ k\in Z$ nuqtalar funksiyaning 1-tur uzilish nuqtalari bo`ladi.

5.4. Talabaning mustaqil ishi

1-topshiriq

1-misolda funksiya aniqlanish sohasini toping.
2-misolda funksiyaning qiymatlar sohasini toping.
3-misolda funksiyaning juft-toqligini tekshiring.
4-misolda berilgan funksiyalar grafigini Mathcad dasturida chizing.

1-variant

1. $f(x)=\frac{x^2+2}{x^3+1}$
2. $f(x)=x^2-8x+20$
3. $y=\frac{x^3}{x^2+1}$.
4. $y=|x-3|$

2-variant

1. $f(x)=\sin\frac{1}{|x|-2}$
2. $f(x)=3^{-x^2}$
3. 2. $y=x^4-5|x|$.
4. $y=x^2-6x+11$

3-variant

1. $f(x)=\log_3(-x)$
2. $f(x)=2\sin x-7$
3. $y=e^{x}-2e^{-x}$
4. $y=3\cos 2x$

4-variant

1. $f(x)=\sqrt[4]{x^2-7x+10}$
2. $f(x)=\frac{1}{x}+4$
3. $y=\ln\frac{1-x}{1+x}.$
4. $y=-\frac{2}{x}+1$

5-variant

1. $f(x)=x^2+tgx$
2. $f(x)=\frac{1}{\pi}arctgx$
3. $y=\frac{\sin x}{x}.$
4. $y=2^{x-1}+3$

6-variant

1. $f(x)=\sqrt{x-7}+\sqrt{10-x}$
2. $f(x)=\sqrt{5-x}+2$
3. $y=x^5+3x^3-x.$
4. $y=\log_3(-x)$

7-variant

1. $f(x)=\frac{\ln x}{\sqrt{|x^2-2|}}$
2. $f(x)=4-x^2$
3. $y=\sqrt{x}.$
4. $y=tg|x|$

8-variant

1. $f(x)=\sqrt[3]{x+2}+\frac{1}{\sqrt[6]{1-x}}$
2. $f(x)=|x|-\frac{1}{3}$
3. $y=\arcsin x.$
4. $y=\frac{x+4}{x+2}$

9-variant

1. $f(x)=e^{\sqrt{x}}\cdot\log_2(2-3x)$
2. $f(x)=2^{\frac{1}{x}}$
3. $y=\sin x+\cos x.$
4. $y=\ln x^2$

10-variant

1. $f(x)=\arccos(x-2)-\ln(x-2)$
2. $f(x)=\ln(x^2+1)$
3. $y=|x|-2.$
4. $y=||x-2|-3|$

11-variant

1. $f(x)=ctgx$
2. $f(x)=e^{x^2-2x-3}$
3. $y=\frac{3}{x^2-1}.$
4. $y=\frac{x-2}{x+3}$

12-variant

1. $f(x)=\frac{x+2}{(x+2)(x-5)}$
2. $f(x)=\frac{x}{|x|}$
3. $y=x\cdot e^{x}.$
4. $y=-\sqrt{x}+2$

13-variant

1. $f(x) = \arccos\ 3x$
2. $f(x) = \sin\ x \cdot \cos\ x$
3. $y = \dfrac{\sin x}{x^3}$.
4. $y = 1 - 3\ln x$

14-variant

1. $f(x) = \dfrac{1}{\lg x}$
2. $f(x) = \sqrt{x^2 + 4}$
3. $y = 3^{4x} \cdot x^2 + \cos x$
4. $y = x \cdot \sin\ x$

15-variant

1. $f(x) = \sqrt{x+5} - \sqrt{-8-x}$
2. $y = \left(\dfrac{1}{3}\right)^{\sin^3 x}$.
3. $y = \left(\sin^2 x + \cos x\right) \cdot x^3$.
4. $y = 2^{\frac{1}{x}}$

16-variant

1. $f(x) = \dfrac{\log_7 x}{\sqrt[5]{x-3}}$
2. $y = 5\sin x + 2\cos x$.
3. $y = \dfrac{tgx}{x^4 + x^2 + x}$.
4. $y = |x+1| + |x-2|$

17-variant

1. $f(x) = e^{\ln x}$
2. $y = e^{-\frac{x^2}{2}}$.
3. $y = x^2 \ln x$.
4. $y = \arcsin|x|$

18-variant

1. $f(x)=\arcsin(\log_3 x)$
2. $y=\frac{3x}{1+x^2}$.
3. $y=\frac{x^4}{\sin x}-x^3\ln(1+x^2)$.
4. $y=\frac{4x+5}{2x-1}$

19-variant

1. $f(x)=\sqrt{1-x^2}\cdot arctg\,\frac{1}{x}$
2. $y=\frac{3}{(\sin x+\cos x)^2+2}$.
3. $y=x+\sin x$.
4. $y=x^3-3x^2+3x-1$

20-variant

1. $f(x)=\sqrt{\frac{x}{2x+1}}-\sqrt[3]{\frac{x-2}{x+5}}$
2. $y=\log_x(\arccos x)$.
3. $y=x\cdot\sin^3 x$.
4. $y=\cos^2 x$

21-variant

1. $f(x)=\cos\frac{1}{x}+\ln(x+1)+\sqrt[10]{\pi-x}$
2. $y=\frac{2\sqrt{2}x-1}{x^2+1}$.
3. $y=\frac{\lg(1-x^2)}{\sqrt[3]{\cos x}}\cdot e^{-x^2}$.
4. $y=\sin^4 x+\cos^4 x$

22-variant

1. $y=\frac{\sqrt[5]{\lg(x+1)}}{x-1}+2^{\sqrt{10-x}}$.
2. $y=6\sin x-8\cos x$.
3. $y=\frac{x^3\cos x}{2^{x^2}}+\sin^2 x$.

4. $y = \arcsin(\sin x)$

23-variant

1. $y=\dfrac{\sqrt[6]{16-x^2}}{\lg(x-1)^2}$.

2. $y=2\cdot 5^{-2x^2}$.

3. $y=\lg\left(\dfrac{2-x^3}{2+x^3}\right)$.

4. $y = x + \sin x$

24-variant

1. $y=\sqrt{4-x^2}\cdot tgx$.

2. $y=2^{4-2x-x^2}$.

3. $y=\dfrac{3^x-1}{3^x+1}$.

4. $y = \sin(\cos x)$

25-variant

1. $y=\dfrac{\arcsin(x-1)}{\lg x}$.

2. $y=\log_{\frac{1}{2}}(x^2+1)$.

3. $y = x\cos x - x^3$.

4. $y=\left(\dfrac{1}{2}\right)^{\frac{x+1}{x}}$.

2-topshiriq

1-misolda ketma-ketlik limiti ta'rifdan foydalanib $\lim\limits_{n\to\infty} a_n = a$ tenglikni isbotlang.

2- misolda ketma-ketlikning limitini toping.

3-4-5-misollarda funksiya limitini toping.

6-misolda funksiyaning uzilish nuqtalari va ularning turini aniqlang. Grafigini chizing.

1-variant

1. $a_n=\dfrac{3n-2}{2n+1}$, $a=\dfrac{3}{2}$.

2. $\lim\limits_{n\to\infty}\frac{2n-5}{n}$.

3. $\lim\limits_{x\to\infty}\frac{5x+1}{x^3-2x+3}$.

4. $\lim\limits_{x\to 0}\frac{\sin^2 3x}{\sin^2 2x}$.

5. $\lim\limits_{x\to 0}\sqrt[2x]{1+3x}$.

6. $y=\frac{1}{4+e^{\frac{1}{x-1}}}$.

2-variant

1. $a_n=\frac{4n-1}{3n+1}$, $a=\frac{4}{3}$.

2. $\lim\limits_{n\to\infty}\frac{4-n^2}{3-n^2}$.

3. $\lim\limits_{x\to 0}\frac{x}{x^2-x}$.

4. $\lim\limits_{x\to 0}\frac{tg 2x}{\sin 5x}$.

5. $\lim\limits_{x\to\infty}\left(\frac{x-5}{x+4}\right)^x$.

6. $y=\begin{cases}3x+1, & x\geq 0 \; da,\\ -3x+1, & x<0 \; da.\end{cases}$

3-variant

1. $a_n=\frac{5-3n}{4n-1}$, $a=-\frac{3}{4}$.

2. $\lim\limits_{n\to\infty}\frac{n^4+5n^2-1}{10n^3-3n+2}$.

3. $\lim\limits_{x\to 5}\frac{\sqrt{9-x}-2}{3-\sqrt{x+4}}$.

4. $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}$.

5. $\lim\limits_{x\to 0}\left(\frac{3+5x}{3+2x}\right)^{\frac{1}{x}}$.

6. $y=\frac{4}{3+5^{\frac{3}{x-2}}}$.

4-variant

1. $a_n=\frac{7n+12}{2n-1}$, $a=\frac{7}{2}$.

2. $\lim\limits_{n\to\infty}\dfrac{7n^2-1}{5n^3+4n^2-2n+1}$.

3. $\lim\limits_{x\to 5}\dfrac{x^2-6x+5}{x^2-25}$.

4. $\lim\limits_{x\to 0}x\cdot ctgx$.

5. $\lim\limits_{x\to 2}\dfrac{e^x-e^2}{x-2}$.

6. $y=\dfrac{x}{x+2}$.

5-variant

1. $a_n=\dfrac{4n^2-2}{3n^2+2}$, $a=\dfrac{4}{3}$.

2. $\lim\limits_{n\to\infty}\dfrac{4n^3-5n^2+10n}{21n^3+7n-8}$.

3. $\lim\limits_{x\to 0}\dfrac{4x^3-3x^2+x}{2x}$.

4. $\lim\limits_{x\to 0}\dfrac{arctg2x}{x}$.

5. $\lim\limits_{x\to\infty}\left(\dfrac{5-x}{6-x}\right)^{x+2}$.

6. $y=2^{-\frac{1}{x}}$.

6-variant

1. $a_n=\dfrac{3n^3-2}{3-2n^3}$, $a=-\dfrac{3}{2}$.

2. $\lim\limits_{n\to\infty}\dfrac{(n+1)^3-(n-1)^3}{n^2+1}$.

3. $\lim\limits_{x\to -1}\dfrac{x^3+x+2}{x^3+1}$.

4. $\lim\limits_{x\to 0}\dfrac{\cos 5x-\cos 3x}{x^2}$

5. $\lim\limits_{x\to e}\dfrac{\ln x-1}{x-e}$.

6. $y=tgx$.

7-variant

1. $a_n=\dfrac{5n-7}{3n+11}$, $a=\dfrac{5}{3}$.

2. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n^2+3n}}{n+2}$.

3. $\lim\limits_{x\to-\frac{1}{2}} \dfrac{2x^2-x-1}{-6x^2+5x+4}$

4. $\lim\limits_{x\to0} \dfrac{\sin 6\pi x}{\sin \pi x}$.

5. $\lim\limits_{x\to0} (1-\sin x)^{\frac{1}{\sin x}}$.

6. $y=e^{tgx}$.

8-variant

1. $a_n=\dfrac{3-2n^2}{2n^2+1}$, $a=-1$.

2. $\lim\limits_{n\to+\infty} \dfrac{2n+1}{\sqrt[3]{n^2+n+4}}$.

3. $\lim\limits_{x\to1} \dfrac{x^3-x^2+3x-3}{2x^3-2x^2+x-1}$.

4. $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\sin 2x}{tg4x}$.

5. $\lim\limits_{x\to+\infty} \left(\dfrac{2x^2+3}{2x^2-4}\right)^{3x}$.

6. $y=\dfrac{1}{x^2-3x+2}$.

9-variant

1. $a_n=\dfrac{3n}{6n+7}$, $a=\dfrac{1}{2}$.

2. $\lim\limits_{n\to+\infty} \left(\sqrt{n^2+n}-n\right)$.

3. $\lim\limits_{a\to0} \dfrac{(x+a)^3-x^3}{a}$.

4. $\lim\limits_{x\to0} \dfrac{\sin 2x}{x}$

5. $\lim\limits_{x\to0} (1+tgx)^{ctgx}$.

6. $y=\dfrac{x^2+x}{x}$.

10-variant

1. $a_n=\dfrac{2n-12}{5n+10}$, $a=\dfrac{2}{5}$.

2. $\lim\limits_{n\to\infty} \dfrac{\sqrt{n^2+n}-\sqrt{9n^2+2n}}{\sqrt[3]{n^3+1}-\sqrt[3]{8n^3+2}}$

3. $\lim\limits_{x\to 0}\dfrac{\sqrt{x+25}-5}{x^2+2x}$.

4. $\lim\limits_{x\to 0}\dfrac{\sin^2 2x}{3x^2}$

5. $\lim\limits_{x\to\infty}\left(\dfrac{x^2+2}{x^2-2}\right)^{x^2}$.

6. $y=\dfrac{x^2+x}{|x|}$.

11-variant

1. $a_n=\dfrac{3n-2}{n+1}$, $a=3$

2. $\lim\limits_{n\to\infty}\dfrac{(n+2)^3}{5n^3}$.

3. $\lim\limits_{x\to 8}\dfrac{x^2-8x}{\sqrt{x+1}-3}$.

4. $\lim\limits_{x\to 0}\dfrac{tg^3 4x}{10x^3}$

5. $\lim\limits_{x\to\infty}\left(\dfrac{8+x}{10+x}\right)^{2x+3}$.

6. $y=\dfrac{x+1}{x+2}$.

12-variant

1. $a_n=\dfrac{3-2n^2}{2n^2+13}$, $a=-1$.

2. $\lim\limits_{n\to\infty}\left(\dfrac{3}{n+2}-\dfrac{5}{2n+1}\right)$.

3. $\lim\limits_{x\to 3}\dfrac{\sqrt{2x+3}-3}{\sqrt{x-2}-1}$.

4. $\lim\limits_{x\to 0}\dfrac{\sin 5x}{4x^2}$

5. $\lim\limits_{x\to 0}(\cos 2x)^{\frac{1}{\sin^2 x}}$.

6. $y=e^{-\frac{1}{x^2}}$.

13-variant

1. $a_n=\dfrac{4-3n^2}{n^2+1}$, $a=-3$.

2. $\lim\limits_{n\to\infty}\left(1+\dfrac{1}{2n-1}\right)^{2n-3}$.

3. $\lim\limits_{x\to 1}\dfrac{\sqrt{2-x}-1}{\sqrt{5-x}-2}$.

4. $\lim\limits_{x\to 0}\dfrac{\sin^2 6x}{2x}$

5. $\lim\limits_{x\to\infty}\left(\dfrac{2x^2-3}{2x^2+1}\right)^{-3x^2}$.

6. $y=5^{\frac{1}{2-x}}$.

14-variant

1. $a_n=\dfrac{31n-12}{21n+11}, \quad a=\dfrac{31}{21}$.

2. $\lim\limits_{n\to+\infty}\left(\sqrt[3]{n^3-4n^2-n}-n\right)$.

3. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{8-x}-2}{x}$.

4. $\lim\limits_{x\to 0}(3x\cdot ctg2x)$

5. $\lim\limits_{x\to 0}\left(\dfrac{x-1}{2x-1}\right)^{\frac{3}{x}}$.

6. $y=\dfrac{1}{1-3^{\frac{1}{x-1}}}$.

15-variant

1. $a_n=\dfrac{3n^3+7}{2n^3-5}, \quad a=\dfrac{3}{2}$.

2. $\lim\limits_{n\to+\infty}\left(\dfrac{n-1}{2n+3}\right)^n$.

3. $\lim\limits_{x\to 4}\dfrac{x^2-16}{\sqrt[2]{5-x}-\sqrt[3]{x-3}}$.

4. $\lim\limits_{x\to 0}\dfrac{6x^3}{\sin^3 2x}$

5. $\lim\limits_{x\to 0}\dfrac{\ln(x+3)-\ln 3}{5x}$.

6. $y=3^{tgx}$

16-variant

1. $a_n=\dfrac{4n-2}{2n+1}, \quad a=2$.

2. $\lim\limits_{n\to+\infty}\left(\dfrac{2n+3}{2n+1}\right)^n$.

3. $\lim\limits_{x\to\infty}\dfrac{x+5x^2-x^3}{2x^3-x^2+7x}$.

4. $\lim\limits_{x\to 0}\dfrac{arctg8x}{7x}$

5. $\lim\limits_{x\to\infty}\left(\dfrac{x+5}{x+1}\right)^x$.

6. $y=\dfrac{x^3+x}{|x|}$.

17-variant

1. $a_n=\dfrac{3n-1}{4n+1}, \quad a=\dfrac{3}{4}$.

2. $\lim\limits_{n\to\infty}\dfrac{2n^2+3n-1}{12n^2-7n-8}$.

3. $\lim\limits_{x\to\infty}\dfrac{1-3x^2}{x^2+7x-2}$.

4. $\lim\limits_{x\to 0}\dfrac{4x}{\arcsin 9x}$

5. $\lim\limits_{x\to\infty}\left(\dfrac{2x+1}{2x+5}\right)^{7x}$.

6. $y=\dfrac{1-x^2}{|x-x^3|}$.

18-variant

1. $a_n=\dfrac{5+3n}{4n-11}, \quad a=\dfrac{3}{4}$.

2. $\lim\limits_{n\to\infty}\dfrac{3n^2+7n+11}{2n^3+n-2}$

3. $\lim\limits_{x\to\infty}\dfrac{x^3+x}{x^4-3x^2+1}$.

4. $\lim\limits_{x\to 0}\dfrac{tg^2\dfrac{x}{2}}{2x}$

5. $\lim\limits_{x\to\infty}\left(\dfrac{4x^2+2}{4x^2-1}\right)^{5x^2}$.

6. $y=\begin{cases} x-2, & x<0 \text{ da}, \\ 2, & x=0 \text{ da}, \\ x^2-2, & x>0 \text{ da}. \end{cases}$

19-variant

1. $a_n=\dfrac{2n+12}{7n-1}, \quad a=\dfrac{2}{7}$.

2. $\lim\limits_{n\to\infty}\left(\dfrac{12}{n+2}-\dfrac{1}{n^2-4}\right)$

3. $\lim\limits_{x\to\infty}\dfrac{x^5-2x}{2x^3+x^2+1}$.

4. $\lim\limits_{x\to 0}\dfrac{\cos x-1}{3x^2}$

5. $\lim\limits_{x\to\infty}\left(\dfrac{x^2+x-1}{x^2-2x+5}\right)^{-2x}$.

6. $y=\begin{cases} x-2, & x<0 \text{ } da, \\ -2, & x=0 \text{ } da, \\ -x-2, & x>0 \text{ } da. \end{cases}$

20-variant

1. $a_n=-\dfrac{3n^2-2}{4n^2+2}, \quad a=-\dfrac{3}{4}$.

2. $\lim\limits_{n\to\infty}\dfrac{\sqrt{2n^3+3n-1}}{\sqrt[3]{8n^3+4n-7}}$

3. $\lim\limits_{t\to 0}\dfrac{\sqrt[3]{1+t}-1}{t}$.

4. $\lim\limits_{x\to 0}\dfrac{\sin 3x}{\sin 10x}$

5. $\lim\limits_{x\to\infty}\left(\dfrac{5x^3-2}{5x^3+1}\right)^{-6x^3}$.

6. $y=\dfrac{x-2}{x^2+2}$.

21-variant

1. $a_n=\dfrac{2n^3-1}{3-3n^3}, \quad a=-\dfrac{2}{3}$.

2. $\lim\limits_{n\to\infty}\dfrac{\sqrt{n^2+2n}-\sqrt{n^2-3n}}{5}$

3. $\lim\limits_{y\to 0}\dfrac{y-1}{\sqrt[4]{y}-1}$.

4. $\lim\limits_{x\to 0}\dfrac{tg^2 2x}{tg^2 3x}$ J $\dfrac{4}{9}$.

5. $\lim\limits_{x\to\infty}\left(\dfrac{2x^3-3x^2+x+1}{2x^3-3x^2-2x+3}\right)^{5x^2}$.

6. $y=\dfrac{x^2+2}{x-2}$.

22-variant

1. $a_n = \dfrac{4n-7}{3n+1}, \quad a = \dfrac{4}{3}.$

2. $\lim\limits_{n\to\infty} \dfrac{3n+2}{n^2+5} \sin n$

3. $\lim\limits_{x\to\frac{1}{3}} \dfrac{3x^2+5x-2}{3x-1}.$

4. $\lim\limits_{x\to 0} \dfrac{\sin^3 x}{\sin x^3}$

5. $\lim\limits_{x\to\infty} \left(\dfrac{7x^{10}-3}{7x^{10}+2} \right)^{-2x^{10}}.$

6. $y = \begin{cases} x-2, & x<2 \text{ da}, \\ x+2, & x\geq 2 \text{ da}. \end{cases}$

23-variant

1. $a_n = \dfrac{4-6n^2}{2n^2+1}, \quad a = -3.$

2. $\lim\limits_{n\to\infty} \dfrac{n-2}{3n+1} \cos \pi n$

3. $\lim\limits_{x\to\infty} \dfrac{7x^3-x^2+3x-1}{10x^2+x}.$

4. $\lim\limits_{x\to 0} \dfrac{\arcsin^3 2x}{\arcsin^3 3x}$

5. $\lim\limits_{x\to 0} \left(\dfrac{1+3x}{1+x} \right)^{\frac{5}{x}}.$

6. $y = \begin{cases} x-1, & x\geq 0 \text{ da}, \\ -x-1, & x<0 \text{ da}. \end{cases}$

24-variant

1. $a_n = \dfrac{3n}{7n+5}, \quad a = \dfrac{3}{7}.$

2. $\lim\limits_{n\to\infty} 3^{\frac{1}{n}}$

3. $\lim\limits_{x\to 1} \dfrac{x^2-2x+1}{x^3-1}.$

4. $\lim\limits_{x\to 0} \dfrac{\sin 2x + \sin 8x}{4x}$

5. $\lim\limits_{x\to 0} \left(\dfrac{4x^2-1}{3x^2-1} \right)^{\frac{3}{x^2}}.$

6. $y=\dfrac{1}{1+2^{\frac{1}{x+1}}}$.

25-variant

1. $a_n=\dfrac{5n-12}{5n+14}$, $a=1$.

2. $\lim\limits_{n\to\infty}\dfrac{e^n+e^{-n}}{e^n-e^{-n}}$

3. $\lim\limits_{x\to 0}\dfrac{x^5-3x^3+x^2}{x^4+2x^2}$.

4. $\lim\limits_{x\to 0}\dfrac{\sin 3x-\sin 7x}{\sin 5x}$.

5. $\lim\limits_{x\to 0}\left(\dfrac{5x^2+4x-3}{5x^2+x-3}\right)^{\frac{1}{x}}$.

6. $y=\begin{cases}2x-1, & x\geq 0 \ da,\\ -2x-1, & x<0 \ da.\end{cases}$

VI bob. DIFFERENSIAL HISOB

6.1. Bir o'zgaruvchili funksiya hosilasi va differensiali. Yuqori tartibli hosila va differensiallar

Hosila tushunchasi

Faraz qilaylik, $f(x)$ funksiya $(a,b)\subset R$ da berilgan bo'lib, $x_0\in(a,b)$, $x_0+\Delta x\in(a,b)$ bo'lsin.

Ma'lumki ushbu

$$\Delta f(x_0)=f(x_0+\Delta x)-f(x_0)$$

ayirma $f(x)$ funksiyaning x_0 nuqtadagi orttirmasi deyiladi.

1-ta'rif. Agar $\Delta x\to 0$ da $\frac{\Delta y}{\Delta x}$ nisbatning limiti $\lim\limits_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$ mavjud va chekli bo'lsa, bu limit $f(x)$ funksiyaning x_0 nuqtadagi hosilasi deyiladi.

$f'(x_0)$ yoki $y'(x_0)$ yoki $\frac{df(x_0)}{dx}$ orqali, ba'zan esa $f'|_{x=x_0}$ kabi belgilanadi.

Demak,

$$f'(x_0)=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}.$$

Hosila topish amali differensiallash amali deyiladi.

Faraz qilaylik, $f(x)$ funksiya $X\subset R$ to'plamda berilgan bo'lib, $(x_0-\delta,\ x_0)\subset X$ $(\delta>0)$ bo'lsin.

2-ta'rif. Agar ushbu

$$\lim_{x\to x_0-0}\frac{f(x)-f(x_0)}{x-x_0}$$

limit mavjud bo'lsa, bu limit $f(x)$ funksiyaning x_0 nuqtadagi chap hosilasi deyiladi va $f'(x_0-0)$ kabi belgilanadi:

$$f'(x_0-0)=\lim_{x\to x_0-0}\frac{f(x)-f(x_0)}{x-x_0}.$$

Aytaylik, $f(x)$ funksiya $X\subset R$ to‘plamda berilgan bo‘lib, $(x_0,\ x_0+\delta)\subset X$ $(\delta>0)$ bo‘lsin.

3-ta'rif. Agar ushbu

$$\lim_{x\to x_0+0}\frac{f(x)-f(x_0)}{x-x_0}$$

limit mavjud bo‘lsa, bu limit $f(x)$ funksiyaning x_0 nuqtadagi o‘ng hosilasi deyiladi va $f'(x_0+0)$ kabi belgilanadi:

$$f'(x_0+0)=\lim_{x\to x_0+0}\frac{f(x)-f(x_0)}{x-x_0}.$$

Masalan, $f(x)=|x|$ funksiyaning $x_0=0$ nuqtadagi o‘ng hosilasi $f'(+0)=1$, chap hosilasi $f'(-0)=-1$ bo‘ladi.

Yuqorida keltirilgan ta'riflardan quyidagi xulosalar kelib chiqadi:

1. Agar $f(x)$ funksiya x_0 nuqtada $f'(x_0)$ hosilaga ega bo‘lsa, u holda bu funksiya x_0 nuqtada o‘ng $f'(x_0+0)$ hamda chap $f'(x_0-0)$ hosilalarga ega va $f'x_0-0)=f'(x_0)=f'(x_0+0)$ tengliklar o‘rinli bo‘ladi.

2. Agar $f(x)$ funksiya x_0 nuqtada o‘ng $f'(x_0+0)$ hamda chap $f'(x_0-0)$ hosilalarga ega bo‘lib, $f'(x_0-0)=f'(x_0+0)$ bo‘lsa, u holda $f(x)$ funksiya x_0 nuqtada $f'(x_0)$ hosilaga ega va $f'(x_0-0)=f'(x_0)=f'(x_0+0)$ tengliklar o‘rinli bo‘ladi.

Hosilalar jadvali

1. $(c)'=0,\ c=const,$

2. $(x^{\alpha})'=\alpha x^{\alpha-1}$ (bu yerda $\alpha\in R$),

3. $(a^x)'=a^x\cdot\ln a,\ a>0,$ xususiy holda, $(e^x)'=e^x,$

4. $(\log_a x)'=\dfrac{1}{x\ln a},\ a>0, a\neq 1;$ xususiy holda, $(\ln x)'=\dfrac{1}{x},$

5. $(\sin x)' = \cos x$,

6. $(\cos x)' = -\sin x$,

7. $(tgx)' = \frac{1}{\cos^2 x}$,

8. $(ctgx)' = -\frac{1}{\sin^2 x}$,

9. $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$,

10. $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$,

11. $(\mathrm{arc}tgx)' = \frac{1}{1+x^2}$,

12. $(\mathrm{arc}ctgx)' = -\frac{1}{1+x^2}$,

13. $(shx)' = chx$,

14. $(chx)' = shx$,

15. $(thx)' = \frac{1}{ch^2 x}$,

16. $(cthx)' = -\frac{1}{sh^2 x}$.

Funksiyani differensiallash qoidalari

Agar $u(x)$ va $v(x)$ funksiyalar $x \in (a,b)$ nuqtada hosilaga ega bo'lsa, u holda $u(x) \pm v(x)$, $c \cdot u(x)$, $u(x) \cdot v(x)$ va $\frac{u(x)}{v(x)}$ (bu yerda $v(x) \neq 0$) funksiyalar ham x nuqtada hosilaga ega va

1. $(u \pm v)' = u' \pm v'$,
2. $(u \cdot v)' = u'v \pm uv'$, xususiy holda, $(cu)' = c \cdot u'$,
3. $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$, xususiy holda, $\left(\frac{c}{v}\right)' = -\frac{cv'}{v^2}$. tengliklar o'rinli bo'ladi.

Agar $u = \varphi(x)$ funksiya x_0 nuqtada hosilaga ega, $y = f(u)$ funksiya esa $u_0 = \varphi(x_0)$ nuqtada hosilaga ega bo'lsa, u holda $y = f(\varphi(x))$ murakkab funksiya x_0 nuqtada hosilaga ega va

$$y'(x_0) = y'(u_0) \cdot u'(x_0)$$

formula o'rinli bo'ladi.

Hosilaning iqtisodiy ma'nosi

$Q(t)$ funksiya t vaqt ichida ishlab chiqarilgan mahsulot miqdorini ifodalasin. t_0 momentda mehnat unumdorligi topilsin.

t_0 dan $t_0 + \Delta t$ vaqt oralig'ida ishlab chiqarilgan mahsulot miqdori $Q(t_0)$ qiymatdan $Q(t_0 + \Delta t)$ qiymatgacha o'zgaradi, ya'ni $\Delta Q = Q(t_0 + \Delta t) - Q(t_0)$. U holda mehnatning o‘rtacha unumdorligi shu vaqt oralig‘ida $u_{o'rt}=\dfrac{\Delta Q}{\Delta t}$ bo‘ladi. t_0 momentda mehnat unumdorligi deganda, $\Delta t\to 0$ da t_0 dan $t_0+\Delta t$ vaqt oralig‘ida o‘rtacha mehnat unumdorligining limit qiymati tushuniladi, ya’ni

$$u(t_o)=\lim_{\Delta t\to 0}Q_{o'rt}=\lim_{\Delta t\to 0}\frac{\Delta Q}{\Delta t}.$$

Shunday qilib mehnat unumdorligi – bu mahsulot hajmining o‘sish tezligidir.

Marjinal mahsulot. $Q(C)$ funksiya ishlab chiqarilgan mahsulot miqdorining C xarajatlar kattaligiga bog‘liqligini ifodalasin. $\dfrac{\Delta Q}{\Delta C}$ nisbat mahsulotning ΔC hajmdagi xarajatlar kattaligiga mos bo‘lgan o‘rtacha kattaligidir. C_0 xarajatda limit mahsulot yoki marjinal mahsulot deganda iqtisodda quyidagi limit tushuniladi:

$$MQ(C_0)=\lim_{\Delta C\to 0}Q_{o'rt}=\lim_{\Delta C\to 0}\frac{\Delta Q}{\Delta C}$$

Misol. t vaqtdagi ishlab chiqarish hajmi $Q=100t-\dfrac{1}{30}t^3$ formula yordamida bog‘langan bo‘lsin. Mehnat unumdotligini: 1) 5 vaqt birligiga mos; 2) 10 vaqt birligiga mos aniqlang.

Yechish. Bu masalaning yechimini topish uchun quyidagi ishlarni amalga oshiramiz: $u'=100-\dfrac{1}{10}t^2$, $u'(5)=100-\dfrac{1}{10}5^2=97{,}5$; $u'(10)=100-\dfrac{1}{10}10^2=90$.

Shunday qilib, mahsulotning limit qiymati, limit foyda, ishlab chiqarish limiti, samaradorlik limiti, talab limiti kabi kattaliklar hosila tushunchasi bilan uzviy bog‘liq.

Iqtisodiy nazariyada $y'(x)$ marjinal (limit) kattaliklarni $My(x)$ ko‘rinishda belgilash qabul qilingan. Bu yerda M marjinal so‘zining birinchi harfini bildiradi va limit ma’nosini beradi. Yuqorida aniqlangan limit kattaliklar iqtisodiy qonuniyatlarni isbotlashda matematik apparatlardan foydalanish imkoniyatini beradi. Buni biz differensial hisobning iqtisodiy nazariyaga ba’zi tatbiqlari sifatida ko‘rib chiqamiz.

Agar firma Q miqdorda mahsulot ishlab chiqarib uni P so'mdan sotsa, u

$$R = PQ$$

miqdordagi daromadga ega bo'ladi. Firmadagi ishlab chiqarish hajmi ΔQ miqdorga o'zgarganda uning daromadi

$$MR = \frac{dR(Q)}{dQ}$$

tezlik bilan o'zgaradi. Bu holda MR kattalik marjinal (limit) daromad deb ataladi.

Misol. Firmaning daromadi

$$R = 100Q - 2Q^2$$

funksiya ko'rinishida ifodalangan. Firmaning marjinal daromadini $Q = 15$ uchun aniqlang.

Yechish. Yuqoridagi birinchi tenglikka asosan topamiz.

$$MR = \frac{dR(Q)}{dQ} = 100 - 4Q \quad MR = 100 - 4 \cdot 15 = 40.$$

Ishlab chiqarish hajmining o'zgarishiga bog'liq ravishda xarajat funksiyasining o'zgarish tezligi marjinal (limit) xarajat deb ataladi va u quyidagi formula yordamida topiladi:

$$MC = \frac{dC(Q)}{dQ}$$

O'rtacha xarajat funksiyasi $AC = \frac{C(Q)}{Q}$.

Misol. O'rtacha xarajat funksiyasi $AC = \frac{24}{Q} + 15 + 3Q$, ko'rinishda berilgan. Marjinal xarajat funksiyasini toping.

Yechish.

$$C(Q) = AC \cdot Q = \left(\frac{24}{Q} + 15 + 3Q\right)Q = 24 + 15Q + 3Q^2.$$

$$MC=\frac{dC(Q)}{dQ}=15+6Q.$$

Funksiya elastikligi. Talab funksiyasini tahlil qilish jarayonida Al'fred Marshall tomonidan funksiya elastiklikligi tushunchasi kiritilgan. $y=f(x)$ funksiya argumentiga Δx orttirma berilgan bo'lsin. U holda

$$E_x(y)=\lim_{\Delta x\to 0}\left(\frac{\Delta y}{y}:\frac{\Delta x}{x}\right)$$

tenglik bilan aniqlanadigan kattlik $y=f(x)$ funksiyaning elastikligi deb ataladi.

Elastiklik y, x o'zgaruvchilarning nisbiy o'zgarishi orasidagi proporsionallik koeffitsiyentidir. Masalan, x ning qiymati bir foizga o'zgarsa, u holda y ning qiymati taxminan $E_x(y)$ foizga o'zgaradi.

Elastikligi o'zgarmas bo'lgan ishlab chiqarish funksiyalarining nazariy va amaliy ahamiyati alohida o'ringa ega. Bu kabi funksiyalarga **CES (Constant Elasticity Substitution)** funksiyasi misol bo'la oladi:

$$y=C_0\left[CL^{-p}+(1-C)K^{p}\right]^{-1/p}.$$

Bu yerda elastiklik $\frac{1}{1-p}\neq 1$.

Mahsulotlarga talabning elastikligini to'g'ri aniqlash davlatga yangi soliqlar va aksizlarni kiritishda katta yordam beradi. Masalan, $x-$yuvilir mahsulotlarga qo'yilgan aksiz, $y-$bu mahsulotlarga bo'lgan talab bo'lsin. Faraz qilamiz davlat bu mahsulotga qo'yilgan aksizni 10% ga oshirishni mo'ljallayotgan bo'lsin. Agar talab elastikligi $E_x(y)=-0,2$ bo'lsa, u holda mahsulotga bo'lgan talab $0,2\cdot 10\%=2\%$ kamayishini kutishimiz kerak bo'ladi. Bu mahsulotni sotishdan davlat oladigan daromad 10% ga emas, balki 8% ga ortadi.

Elastiklikni o'rganish natijasida aholi daromadining ortishi bozordagi vaziyaitning o'zgarishini baholash mumkin. Masalan, ma'lumki go'sht, yog' va tuxumlar uchun talab elastikligi aholi daromadiga nisbatan musbat, un uchun esa bu elastiklik manfiy. Demak, aholi daromadi o'sishi bilan go'sht, yog' va tuxumlarga bo'lgan talab ortadi, unga bo'lgan talab esa kamayadi.

Aholi daromadi kamayishi bilan go'sht, yog' va tuxumlarga bo'lgan talab kamayadi, unga bo'lgan talab esa ortadi.

Misol. Talab va taklif funksiyalari quyidagicha bo'lsin:

$$y = 10 - x, \quad z = 3x - 6.$$

a) talab va taklif uchun muvozanat bahoni toping;

b) muvozanat baho uchun talab va taklif funksiyalarining elastikligini toping.

Yechish. a) $y(x) = z(x) \Rightarrow 10 - x = 3x - 6 \Rightarrow x = 4$;

b) $E_x(y)$ – talab va $E_x(z)$ – taklif funksiyalarining elastiklarini quyidagicha topamiz:

$$y = 10 - x;$$

$$\Delta y = y(x + \Delta x) - y(x) = 10 - (x + \Delta x) - (10 - x) = -\Delta x$$

$$\frac{\Delta y}{y} : \frac{\Delta x}{x} = \frac{-\Delta x}{10 - x} : \frac{\Delta x}{x} = -\frac{x}{10 - x},$$

$$E_x(y) = \lim_{\Delta x \to 0}\left(-\frac{x}{10 - x}\right) = -\frac{x}{10 - x}.$$

$$z = 3x - 6;$$

$$\Delta z = z(x + \Delta x) - z(x) = 3x + 3\Delta x - 6 - (3x - 6) = 3\Delta x$$

$$\frac{\Delta z}{z} : \frac{\Delta x}{x} = \frac{3\Delta x}{3x - 6} : \frac{\Delta x}{x} = \frac{3x}{3x - 6},$$

$$E_x(z) = \frac{x}{x - 2}.$$

$$E_x(y) = -\frac{4}{10 - 4} = -\frac{2}{3}, \quad E_x(z) = \frac{4}{4 - 2} = 2.$$

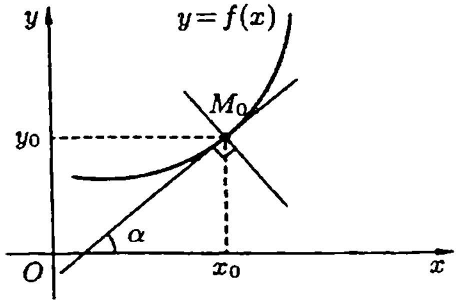
Demak, muvozanat bahosining 1% ortishi talabning (2/3) % ga kamayishiga taklifning esa 2% ga ortishiga olib keladi.

Hosilaning geometrik ma'nosi

Faraz qilaylik $y = f(x)$ funksiya x_0 nuqtada hosilaga ega, $M_0(x_0; y_0)$ funksiya grafigiga tegishli nuqta bo'lsin. U holda $y = f(x)$ funksiya grafigiga $M_0(x_0; y_0)$ nuqtasida o'tkazilgan urinma mavjud bo'lib uning tenglamasi quyidagi ko'rinishga ega:

$$y - y_0 = f'(x_0)(x - x_0)$$

Bunda $f'(x_0) = tg\alpha$, bu yerda $\alpha-$ urinmaning Ox o‘qiga og‘ish burchagi.



$y = f(x)$ funksiya grafigining $M_0(x_0;y_0)$ nuqtasidan o‘tadigan va shu nuqtadagi urinmaga perpendikulyar bo‘lgan to‘g‘ri chiziq normal deyiladi. Ma’lumki, agar $k_{urinma} \neq 0$ bo‘lsa, urinma va normalning burchak koeffitsientlari $k_{normal} \cdot k_{urinma} = -1$ shart bilan bog‘langan bo‘ladi. Bundan $y = f(x)$ funksiya grafigiga $M_0(x_0;y_0)$ nuqtasida o‘tkazilgan normal tenglamasini

$$y - y_0 = -\frac{1}{f'(x_0)}(x - x_0)$$

keltirib chiqarish mumkin.

$y = f_1(x)$ va $y = f_2(x)$ egri chiziqlar $M_0(x_0;y_0)$ nuqtada kesishsin, hamda x_0 nuqtada hosilaga ega bo‘lsin. U holda bu egri chiziqlar orasidagi burchak ularning kesishgan $M_0(x_0;y_0)$ nuqtasi orqali bu egri chiziqlarga o‘tkazilgan urinmalar orasidagi burchak kabi aniqlanadi. Bu φ burchak quyidagi formula bo‘yicha topiladi:

$$tg\varphi = \frac{f_2'(x_0) - f_1'(x_0)}{1 + f_2'(x_0) \cdot f_1'(x_0)} \quad \text{yoki}$$

$$tg\varphi = \frac{k_2 - k_1}{1 + k_2 \cdot k_1}.$$

Bu yerda $k_1 = f_1'(x_0)$ va $k_2 = f_2'(x_0)$.

Teskari funksiyaning hosilasi

Agar $y=f(x)$ funksiya x nuqtada $f'(x)\neq 0$ hosilaga ega bo'lsa, bu funksiyaga teskari $x=f^{-1}(y)$ funksiya x nuqtaga mos bo'lgan y nuqtada hosilaga ega va

$$x'_y=\frac{1}{y'_x}$$

bo'ladi.

Logarifmik hosila

Funksiya logarifmidan olingan hosilaga logarifmik hosila deyiladi.

$$(\ln y)'=\frac{y'}{y}$$

$u(x)^{v(x)}$ $(u(x)>0)$ ko'rinishdagi daraja-ko'rsatkichli funksiya berilgan va$(x)^{v(x)}$ $u(x)$, $v(x)$ funksiyalar x ning qaralayotgan qiymatlarida differensiallanuvchi bo'lsin. Bu funksiyaning hosilasini hisoblash uchun logarifmik hosiladan foydalanamiz. U holda formulaga ko'ra

$$\left(u^v\right)'=u^v v'\cdot\ln u+u^{v-1}\cdot u'\cdot v.$$

Oshkormas funksiya hosilasi

Ikkita x va y o'zgaruvchilarning qiymatlari o'zaro biror tenglama bilan bog'langan bo'lsin, biz uni simvolik tarzda bunday belgilaymiz: $F(x,y)=0$.

Agar $y=f(x)$ funksiya biror (a,b) intervalda aniqlangan bo'lib, $F(x,y)=0$ tenglamada y o'rniga $f(x)$ ifoda qo'yilganda tenglama x ga nisbatan ayniyatga aylansa, u holda $y=f(x)$ funksiya $F(x,y)=0$ tenglama bilan aniqlangan, oshkormas funksiya bo'ladi. Ammo, har qanday oshkormas berilgan funksiyani ham oshkor shaklda bermoq, ya'ni $y=f(x)$ ga qo'yish mumkin bo'lavermaydi, bu yerda $f(x)$ elementar funksiY. Masalan, $y-x-\frac{1}{4}\sin y=0$ yoki $y^6-y-x^2=0$ tenglamalar bilan berilgan funksiyalar elementar funksiyalar bilan ifodalanmaydi, ya'ni bu tenglamalarni elementar funksiyalar orqali y ga nisbatan yechish mumkin emas.

Endi oshkormas funksiyani oshkor ko‘rinishga keltirmasdan, ya’ni $y=f(x)$ shaklga almashtirmasdan, uning hosilasini topish qoidasini ko‘rsatamiz.

Funksiya ushbu $x^2+y^2-a^2=0$ tenglama bilan berilgan bo‘lsin. y ni $x$ning funksiyasi deb hisoblab, bu ayniyatning ikkala tomonini $x$bo‘yicha differensiallab (murakkab funksiyani differensiallash qoidasidan foydalangan holda), quyidagiga ega bo‘lamiz

$$2x+2yy'=0$$

bundan $y'=-\frac{x}{y}$.

Yuqori tartibli hosila

Faraz qilaylik, $f(x)$ funksiya (a,b) da berilgan bo‘lib, $\forall x\in(a,b)$ da $f'(x)$ hosilaga ega bo‘lsin. Bu $f'(x)$ funksiyani $g(x)$ orqali belgilaymiz:

$$g(x)=f'(x) \qquad (x\in(a,b)).$$

4-ta’rif. Agar $x_0\in(a,b)$ nuqtada $g(x)$ funksiya $g'(x_0)$ hosilaga ega bo‘lsa, bu hosila $f(x)$ funksiyaning x_0 nuqtadagi ikkinchi tartibli hosilasi deyiladi va $f''(x_0)$ yoki $\frac{d^2f(x_0)}{dx^2}$ kabi belgilanadi.

Xuddi shunga o‘xshash, $f(x)$ ning 3-tartibli $f'''(x)$, 4-tartibli $f^{IV}(x)$ va h.k. tartibli hosilalari ta’riflanadi.

Umuman, $f(x)$ funksiyaning $n-$tartibli hosilasi $f^{(n)}(x)$ ning hosilasi $f(x)$ funksiyaning $(n+1)-$tartibli hosilasi deyiladi:

$$f^{(n+1)}(x)=\left(f^{(n)}(x)\right)'.$$

Odatda, $f(x)$ funksiyaning $f''(x)$, $f'''(x)$, ... hosilalari uning yuqori tartibli hosilalari deyiladi. Shuni ta’kidlash lozimki, $f(x)$ funksiyaning $x\in(a,b)$ da $n-$tartibli hosilasining mavjudligi bu funksiyaning shu nuqta atrofida 1–, 2–, ..., $(n-1)-$tartibli hosilalari mavjudligini taqoza etadi. Ammo bu hosilalarning mavjudligidan $n-$tartibli hosila mavjudligi, umuman aytganda, kelib chiqavermaydi.

Parametrik berilgan funksiyaning hosilasi

Agar x ning funksiyasi y ushbu

$$\begin{cases} x = \varphi(t), \\ y = \psi(t) \end{cases}$$

parametrik tenglamalar bilan berilgan bo'lsa bu ifodaga funksiyaning parametrik ko'rinishdagi berilishi deyiladi.

Bu holda y ning x bo'yicha hosilasi y'_x

$$y'_x = y' = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{y'_t}{x'_t}$$

tenglik bilan aniqlanadi.

y'_x funksiyaning x bo'yicha hosilasi, ya'ni y ning x bo'yicha ikkinchi tartibli hosilasini quyidagicha hisoblash mumkin:

$$y''_{xx} = \frac{y''_t \cdot x'_t - x''_t \cdot y'_t}{(x'_t)^3}.$$

Differensial tushunchasi

5-ta'rif. Agar $\Delta f(x_0)$ ni ushbu

$$\Delta f(x_0) = A \cdot \Delta x + \alpha \Delta x$$

ko'rinishda ifodalash mumkin bo'lsa, $f(x)$ funksiya x_0 nuqta-da differensiallanuvchi deyiladi, bunda $A = const$, $\Delta x \to 0$, da $\alpha \to 0$.

Teorema. $f(x)$ funksiya $x \in (a, b)$ nuqtada differensiallanuvchi bo'lishi uchun uning shu nuqtada chekli $f'(x)$ hosilaga ega bo'lishi zarur va yetarli.

6-ta'rif. Funksiya orttirmasidagi $f'(x_0) \cdot \Delta x$ ifoda $f(x)$ funksiyaning x_0 nuqtadagi differensiali deyiladi va $df(x_0)$ kabi belgilanadi:

$$df(x_0) = f'(x_0) \cdot \Delta x.$$

Differensialning taqribiy hisoblashga tatbiqi

Ma'lumki, $y=f(x)$ funksiya x_0 nuqtada differensiallanuvchi bo'lsa, unda

$$\Delta f(x_0)=df(x_0)+o(\Delta x)$$

tenglik o'rinli bo'ladi. Agar $df(x_0)\neq 0$ bo'lsa, bu tenglikdan yetarlicha kichik Δx lar uchun

$$\Delta f(x_0)\approx df(x_0)$$

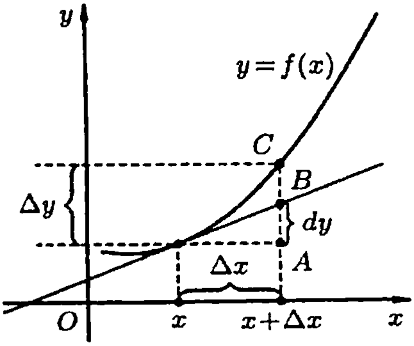
yoki

$$f(x_0+\Delta x)\approx f(x_0)+f'(x_0)\cdot\Delta x$$

taqribiy hisoblash formulasini hosil qilamiz.

Geometrik ma'nosi va differensial xossalari

Aytaylik, $x\in(a,b)$ nuqtada differensiallanuvchi $f(x)$ funksiyaning grafigi quyidagi chizmada tasvirlangan egri chiziqni ifodalasin:



$f(x)$ funksiyaning x nuqtadagi differensiali funksiya grafigiga $(x, f(x))$ nuqtada o'tkazilgan urinma orttirmasi BA ni ifodalar ekan.

Faraz qilaylik, $f(x)=x$, $x\in R$ bo'lsin. Bu funksiya differensiallanuvchi bo'lib, $df(x)=(x)'\cdot\Delta x=\Delta x$, ya'ni $dx=\Delta x$ bo'ladi. Demak, (a,b) da differensiallanuvchi $f(x)$ funksiya-ning differensialini

$$df(x)=f'(x)\cdot dx$$

ko'rinishda ifodalash mumkin.

Faraz qilaylik, $f(x)$ va $g(x)$ funksiyalari (a,b) da berilgan bo'lib, $x\in(a,b)$ nuqtada differensiallanuvchi bo'lsin. U holda $x\in(a,b)$ da

1) $d(c\cdot f(x)) = cdf(x), \quad c = const;$

2) $d(f(x)+g(x)) = df(x) + dg(x);$

3) $d(f(x)g(x)) = g(x)df(x) + f(x)dg(x);$

4) $d\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{g(x)df(x) - f(x)dg(x)}{g^2(x)}, \quad (g(x)\neq 0).$

bo'ladi.

Yuqori tartibli differensiallar. Faraz qilaylik, $f(x)$ funksiya (a,b) da berilgan bo'lib, $\forall x\in(a,b)$ nuqtada $f''(x)$ hosilaga ega bo'lsin. Ravshanki, $f(x)$ funksiyaning differensiali

$$df(x) = f'(x)dx$$

bo'lib, bunda $dx = \Delta x$ funksiya argumentning ixtiyoriy orttirmasi.

7-ta'rif. $f(x)$ funksiyaning $x\in(a,b)$ nuqtadagi differensiali $df(x)$ ning differensiali $f(x)$ funksiyaning $x\in(a,b)$ nuqtadagi ikkinchi tartibli differensiali deyi-ladi va $d^2 f(x)$ kabi belgilanadi:

$$d^2 f(x) = d(df(x)).$$

$f(x)$ funksiyaning uchinchi $d^3 f(x)$, to'rtinchi $d^4 f(x)$ va h.k. tartibdagi differensiallari huddi shunga o'xshash ta'riflanadi.

Umuman, $f(x)$ funksiyaning $n-$tartibli differensiali $d^n f(x)$ ning differensiali $f(x)$ funksiyaning $(n+1)-$tartibli differensiali deyiladi:

$$d^{n+1} f(x) = d(d^n f(x)).$$

Aniqmasliklarni ochish. Lopital qoidalari

Tegishli funksiyalarning hosilalari mavjud bo'lganda $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0\cdot\infty$, $\infty-\infty$, $1^{\infty}, 0^{0}, \infty^{0}$ ko'rinishdagi aniqmasliklarni ochish masalasi engillashadi. Odatda hosilalardan foydalanib, aniqmasliklarni ochish Lopital qoidalari deb ataladi.

1-qoida. Agar

1) *f(x)* va *g(x)* funksiyalar $(a-\delta;a)\cup(a;a+\delta)$, bu yerda $\delta>0$, to'plamda uzluksiz, differensiallanuvchi va shu to'plamdan olingan ixtiyoriy x uchun $g(x)\neq 0$, $g'(x)\neq 0$;

2) $\lim\limits_{x\to a} f(x)=\lim\limits_{x\to a} g(x)=0$;

3) $\lim\limits_{x\to a}\dfrac{f'(x)}{g'(x)}$-hosilalar nisbatining limiti (chekli yoki cheksiz) mavjud bo'lsa, u holda funksiyalar nisbatining limiti $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)}$ mavjud va

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)} \qquad (1)$$

tenglik o'rinli bo'ladi.

2-qoida. Agar

1) *f(x)* va *g(x)* funksiyalar *(a-•;a)•(a;a+•)*, bu yerda •>0, to'plamda uzluksiz, differensiallanuvchi va shu to'plamdan olingan ixtiyoriy *x* uchun *g(x)•* 0, *g'(x)•0*;

2) $\lim\limits_{x\to a} f(x)=\lim\limits_{x\to a} g(x)=\infty$,

3) $\lim\limits_{x\to a}\dfrac{f'(x)}{g'(x)}$ mavjud bo'lsa,

u holda $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)}$ mavjud va $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)}=\lim\limits_{x\to a}\dfrac{f'(x)}{g'(x)}$ bo'ladi.

Shunday qilib, funksiya hosilalari yordamida 0• •, •-•, 1˙, 0^0, •^{0}, ko'rinishdagi aniqmasliklarni ochishda, ularni $\dfrac{0}{0}$ yoki $\dfrac{\infty}{\infty}$ ko'rinishidagi aniqmaslikka keltirib, so'ng yuqoridagi qoidalar qo'llaniladi.

Eslatma. Agar *f(x)* va *g(x)* funksiyalarning *f'(x)* va *g'(x)* hosilalari ham *f(x)* va *g(x)* lar singari yuqorida keltirilgan teoremalarning barcha shartlarini qanoatlantirsa, u holda

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}=\lim_{x\to a}\frac{f''(x)}{g''(x)}$$

Tengliklar o'rinli bo'ladi, ya'ni bu holda Lopital qoidasini takror qo'llanish mumkin bo'ladi.

Misollar

1. Hosila ta'rifidan foydalanib, $y=2x^3+5x^2-7x-4$ funksiya uchun y' hosilasini toping.

Yechish. $y=2x^3+5x^2-7x-4$ funksiya orttirmasini topamiz:

$$\Delta y=(2(x+\Delta x)^3+5(x+\Delta x)^2-7(x+\Delta x)-4)-(2x^3+5x^2-7x-4)=$$
$$=6x^2\Delta x+6x\Delta x^2+2\Delta x^3+10x\Delta x+5\Delta x^2-7\Delta x$$

$\Delta x\to 0$ da quyidagi limitni topamiz:

$\lim\limits_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim\limits_{\Delta x\to 0}(6x^2+6x\Delta x+2\Delta x^2+10x+5\Delta x-7)=6x^2+10x-7$ Shunday qilib, ta'rifga ko'ra hosila $y'=6x^2+10x-7$.

2. Differensiallash qoida va formulalaridan foydalanib, $y=(x^4-x)(3tgx-1)$ funksiyaning hosilasini toping.

Yechish. Ko'paytmaning hosilasi uchun formuladan foydalanamiz:

$$y'=\left[(x^4-x)(3tgx-1)\right]'=(x^4-x)'(3tgx-1)+(x^4-x)(3tgx-1)'=$$
$$=(4x^3-1)(3tgx-1)+(x^4-x)\cdot\frac{3}{\cos^2 x}.$$

3. Oshkormas ko'rinishda berilgan $x^3+\ln y-x^2e^y=0$ funksiyaning hosilasini toping.

Yechish. $x^3+\ln y-x^2e^y=0$

$$3x^2+\frac{y'}{y}-x^2e^y y'-2xe^y=0,\quad ya'ni\quad y'=\frac{(2xe^y-3x^2)y}{1-x^2ye^y}$$

4. Parametrik ko'rinishda berilgan $\begin{cases}x=2\cos t,\\ y=3\sin t.\end{cases}$ funksiyaning hosilasini toping:

Yechish. Funksiya hosilasini $y'=\frac{y'(t)}{x'(t)}$ formuladan topamiz

$$y'(x)=\frac{(3\sin t)'}{(2\cos t)'}=-\frac{3\cos t}{2\sin t}=-1,5ctgt.$$

5. $x^2+2xy^2+3y^4=6$ egri chiziqqa M(1,-1) nuqtada o'tkazilgan urinma va normal tenglamalari yozilsin.

Yechish. Egri chiziq tenglamasidan $y`$ hosilani topamiz:

$$2x+2y^2+4xyy'+12y^3y'=0,\quad \text{ya'ni}\quad y'=-\frac{x+y^2}{2xy+6y^3}.$$

Demak $y`(-1;1)=-\frac{1+(-1)^2}{2\cdot 1(-1)+6(-1)^3}=\frac{1}{4}.$

Urinma tenglamasi $y+1=\frac{1}{4}(x-1)$

$x-4y+5=0$

Normal tenglamasi $\qquad y+1=-4(x-1)$

$4x+y-3=0$.

6. $y^2=4x$ parabola va $x^2=\frac{1}{2}y$ parabolalar orasidagi burchakni toping.

Yechish. Avvalo parabolalarning kesishish nuqtasini topamiz. Buning uchun ushbu $\begin{cases} y^2=4x, \\ x^2=\frac{1}{2}y \end{cases}$ tenglamalar sistemani yechamiz. Bu sistemaning ildizlari $x_1=0,\ y_1=0$ va $x_2=1,\ y_2=2$, demak, parabolalar $(0;0)$ va $(1;2)$ nuqtalarda kesishadi. Endi egri chiziqlarning kesishgan nuqtasidan o'tkazilgan urinmalarning burchak koeffitsientlarini topamiz. $(0;0)$ nuqtada parabolalarga urinmalar Ox va Oy o'qlardan iborat bo'ladi, binobarin, bu nuqtada parabolalar to'g'ri burchak ostida kesishadi. $y^2=4x$ parabolaga o'tkazilgan urinmaning burchak koeffitsientini topamiz. Tenglamani $y=2\sqrt{x}$ ko'rinishda qayta yozib olamiz (radikal oldida musbat ishora olamiz, chunki parabolalar birinchi chorakda kesishadi).

$$k=y'=2\cdot\frac{1}{2\sqrt{x}}=\frac{1}{\sqrt{x}};$$

$$k_{x=1}=y'_{x=1}=\frac{1}{\sqrt{1}}=1$$

$x^2=\frac{1}{2}y$ parabolaga o'tkazilgan urinmaning burchak koeffitsientini topamiz. Parabola tenglamasini $y=2x^2$ ko'rinishda qayta yozib olamiz, so'ngra

$$k=y'=4x;\ k_{x=1}=y'_{x=1}=4\cdot 1=4$$

Urinmalar orasidagi φ burchakni ularning burchak koeffitsientlari $k_1=1$ va $k_2=4$ bo'yicha $tg\varphi=\frac{k_2-k_1}{1+k_2\cdot k_1}$. formulaga ko'ra topamiz.

$$tg\varphi=\frac{4-1}{1+4\cdot 1}=\frac{3}{5}=0,6 \quad \varphi=arctg0,6=30^0 58'.$$

7. $f(x)=\sin x$ bo‘lsin. Bu funksiyaning n-tartibli hosilasi toping.

Yechish.

$$(\sin x)'=\cos x=\sin\left(x+\frac{\pi}{2}\right),$$

$$(\sin x)''=(\cos x)'=-\sin x=\sin\left(x+2\frac{\pi}{2}\right),$$

Umuman,

$$(\sin x)^{(n)}=\sin\left(x+n\frac{\pi}{2}\right)$$

bo‘ladi.

8. Ta’rifdan foydalanib, ushbu $f(x)=x-3x^2$ funksiyaning $x_0=2$ nuqtadagi differensiali topilsin.

Yechish. Bu funksiyaning $x_0=2$ nuqtadagi orttirmasini topamiz:

$$\Delta f(2)=f(2+\Delta x)-f(2)=2+\Delta x-3(2+\Delta x)^2-2+12=$$

$$=-11\cdot\Delta x-3\Delta x^2=-11\cdot\Delta x+(-3\Delta x)\cdot\Delta x.$$

Demak, $d\,f(2)=-11\cdot dx$.

9. Ushbu $\sin 29^0$ miqdor taqribiy hisoblansin.

Yechish. Agar $f(x)=\sin x$, $x_0=30^0$ deyilsa, unda (2) formulaga ko‘ra

$$\sin 29^0\approx\sin 30^0+\cos 30^0\cdot(29^0-30^0)\cdot\frac{2\pi}{360^0}=0{,}5-\frac{\sqrt{3}}{2}\cdot\frac{2\pi}{360^0}\approx 0{,}4848$$

bo‘ladi.

10. $\lim\limits_{x\to\infty} x^{\frac{1}{\sqrt{x}}}$ ni toping.

Yechish. $[\infty^0]$ ko‘rinishdagi aniqmaslikka egamiz. $\lim\limits_{x\to\infty}\left(\ln x^{\frac{1}{\sqrt{x}}}\right)$ ni topamiz: $\lim\limits_{x\to\infty}\left(\ln x^{\frac{1}{\sqrt{x}}}\right)=\lim\limits_{x\to\infty}\frac{1}{\sqrt{x}}\ln x=\left[\frac{\infty}{\infty}\right]=\lim\limits_{x\to\infty}\frac{(\ln x)'}{(\sqrt{x})'}=\lim\limits_{x\to\infty}\frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{2\sqrt{x}}\right)}=\lim\limits_{x\to\infty}\frac{2}{\sqrt{x}}=0$

Demak, $\lim\limits_{x\to\infty} x^{\frac{1}{\sqrt{x}}} = e^{\lim\limits_{x\to\infty}\ln\left(x^{\frac{1}{\sqrt{x}}}\right)} = e^0 = 1$.

11. $\lim\limits_{x\to 1}\left[\left(x-\sqrt{x}\right)\ln\ln x\right]$ ni toping.

Yechish. $x\to 1$ da $\ln x\to 0$ bo'lganligi sababli $\ln\ln x = \ln(\ln)x\to\infty$. Shunday qilib, $[0\cdot\infty]$ ko'rinishidagi aniqmaslikka ega bo'lamiz. Uni aniqmaslikning $\left[\frac{\infty}{\infty}\right]$ ko'rinishiga keltiramiz va Lopital qoidasini qo'llaymiz.

$$\lim_{x\to 0}\left(x-\sqrt{x}\right)\ln\ln x = \lim_{x\to 1}\frac{\ln\ln x}{\frac{1}{x-\sqrt{x}}} = \lim_{x\to 1}\frac{\frac{1}{\ln x}\cdot\frac{1}{x}}{-\frac{1-\frac{1}{2\sqrt{x}}}{\left(x-\sqrt{x}\right)^2}} = \lim_{x\to 1}\frac{\left(\sqrt{x}-1\right)^2 2\sqrt{x}}{(\ln x)\left(2\sqrt{x}-1\right)} =$$

$$= \lim_{x\to 1}\frac{\left(\sqrt{x}-1\right)^2}{\ln x}\lim_{x\to 1}\frac{2\sqrt{x}}{2\sqrt{x}-1} = \lim_{x\to 1}\frac{\left[\frac{2\left(\sqrt{x}-1\right)}{2\sqrt{x}}\right]}{\left[\frac{1}{x}\right]}\cdot 1 = 0$$

12. $\lim\limits_{x\to\infty}\left(x\ln^2 x-\sqrt{1+x+x^2}\right)$ ni toping.

Yechish. $[\infty-\infty]$ ko'rinishdagi aniqmaslikka egamiz. Shakl almashtiramiz:

$\lim\limits_{x\to\infty}\left(x\ln^2 x-\sqrt{1+x+x^2}\right) = \lim\limits_{x\to\infty} x\ln x^2\left(1-\frac{\sqrt{1+x+x^2}}{x\ln x^2}\right)$. Lopital qoidasini qo'llab alohida

$$\lim_{x\to\infty}\frac{\sqrt{1+x+x^2}}{x\ln x^2} = \lim_{x\to\infty}\frac{\frac{1+2x}{2\sqrt{1+x+x^2}}}{\ln^2 x+2\ln x} = \lim_{x\to\infty}\frac{\frac{1}{x}+2}{2\sqrt{\frac{1}{x^2}+\frac{1}{x}+1}\left(\ln^2 x+2\ln x\right)} = 0$$

ni topamiz.

Shunday qilib,

$$\lim_{x\to+\infty}\left(x\ln^2 x-\sqrt{1+x+x^2}\right)=\lim_{x\to+\infty}x\ln^2 x=\infty.$$

6.2. Bir o‘zgaruvchili funksiyani to‘la tekshirish

Funksiyaning monotonligi

Faraz qilaylik, $y=f(x)$ funksiya (a,b) oraliqda berilgan bo‘lsin.

1-ta’rif. $x_2>x_1$ tengsizlikni qanoatlantiruvchi $\forall x_1,x_2\in(a,b)$ uchun $f(x_2)\geq f(x_1)$ $(f(x_2)\leq f(x_1))$ bo‘lsa, $f(x)$ funksiya (a,b) oraliqda o‘suvchi (kamayuvchi) deyiladi.

Agar funksiya o‘suvchi yoki kamayuvchi bo‘lsa, bunday funksiyaga monoton funksiya deyiladi.

Teorema. $f(x)$ funksiya (a,b) intervalda chekli $f'(x)$ hosilaga ega bo‘lsin. Bu funksiya shu intervalda o‘suvchi (kamayuvchi) bo‘lishi uchun (a,b) da $f'(x)\geq 0$ $(f'(x)\leq 0)$ bo‘lishi zarur va yetarli.

Funksiyaning ekstremumlari

Aytaylik *f(x)* funksiya (a,b) intervalda aniqlangan va x_0 • *(a;b)* bo‘lsin.

2-ta’rif. Agar x_0 nuqtaning shunday (x_0-•;x_0+•) atrofi mavjud bo‘lib, shu atrofdan olingan ixtiyoriy x uchun *f(x)*•*f(x₀)* (*f(x)*•*f(x₀)*) tengsizlik o‘rinli bo‘lsa, u holda x_0 nuqta *f(x)* funksiyaning *maksimum* (*minimum*) *nuqtasi*, *f(x₀)* esa *funksiyaning maksimumi* (*minimumi*) deb ataladi.

3-ta’rif. Agar x_0 nuqtaning shunday atrofi (x_0-•;x_0+•) mavjud bo‘lib, shu atrofdan olingan ixtiyoriy x•x_0 uchun $f(x)<f(x_0)$ ($f(x)>f(x_0)$) tengsizlik o‘rinli bo‘lsa, u holda *f(x)* funksiya x_0 nuqtada qat’iy maksimumga (minimumga) ega deyiladi.

Teorema (Ekstremumning zaruriy sharti). Agar $f(x)$ funksiya x_0 nuqtada ($x_0\in(a,b)$) chekli $f'(x_0)$ hosilaga ega bo‘lib, bu nuqtada $f(x)$ funksiya ekstremumga erishsa, u holda $f'(x_0)=0$ bo‘ladi.

4-ta’rif. Funksiya hosilasini nolga aylantiradigan nuqtalar yoki hosila mavjud bo‘lmaydigan nuqtalar funksiyaning *kritik nuqtalari* deb ataladi. Funksiya hosilasi nolga teng bo‘lgan nuqtalar *statsionar nuqtalar* deb ataladi.

Har qanday kritik nuqta funksiyaning ekstremum nuqtasi bo'lavermaydi.

Ekstremum mavjud bo'lishining yetarli shartlari

Teorema. Faraz qilaylik $f(x)$ funksiya x_0 nuqtada uzluksiz va x_0 nuqta funksiyaning kritik nuqtasi bo'lsin.

a) Agar $\forall x \in (x_0-\delta; x_0)$ uchun $f'(x)>0$, $\forall x \in (x_0; x_0+\delta)$ uchun $f'(x)<0$ tengsizliklar o'rinli bo'lsa, ya'ni $f'(x)$ hosila x_0 nuqtadan o'tishida o'z ishorasini «+» dan «-» ga o'zgartirsa, u holda $f(x)$ funksiya x_0 nuqtada maksimumga ega bo'ladi.

b) Agar $\forall x \in (x_0-\delta; x_0)$ uchun $f'(x)<0$, $\forall x \in (x_0; x_0+\delta)$ uchun $f'(x)>0$ tengsizliklar o'rinli bo'lsa, ya'ni $f'(x)$ hosila x_0 nuqtadan o'tishda o'z ishorasini «-» dan «+» ga o'zgartirsa, u holda $f(x)$ funksiya x_0 nuqtada minimumga ega bo'ladi.

c) Agar $f'(x)$ hosila x_0 nuqtadan o'tishda o'z ishorasini o'zgartirmasa, u holda $f(x)$ funksiya x_0 nuqtada ekstremumga ega bo'lmaydi.

Teorema. $f(x)$ funksiya x_0 nuqtada $f', f'', ... f^{(n)}$ hosilalarga ega bo'lib,

$$f'(x_0) = f''(x_0) = ... = f^{(n-1)}(x_0) = 0, \quad f^{(n)}(x_0) \neq 0$$

bo'lsin. Unda

1) agar n juft son bo'lib,

$$f^{(n)}(x_0) < 0 \quad \left(f^{(n)}(x_0) > 0\right)$$

bo'lsa, $f(x)$ funksiya x_0 nuqtada maksimumga (minimumga) erishadi.

2) agar n toq son bo'lsa, $f(x)$ funksiya x_0 nuqtada ekstremumga erishmaydi.

$[a,b]$ kesmada uzluksiz bo'lgan $f(x)$ funksiya o'zining shu kesmadagi eng katta (eng kichik) qiymatiga kritik nuqtada yoki kesmaning chegaraviy nuqtasida erishadi.

Funksiyaning qavariqligi, egilish nuqtalari

5-ta'rif. Agar (a,b) oraliqda berilgan $y = f(x)$ funksiya grafigi $\forall [x_1, x_2] \subset (a,b)$ kesmaning chetki nuqtalarini tutashtiruvchi vatardan yuqorida (pastda) yotsa, unda $y=f(x)$ funksiya $[a,b]$ oraliqda qavariq (botiq) deb ataladi.

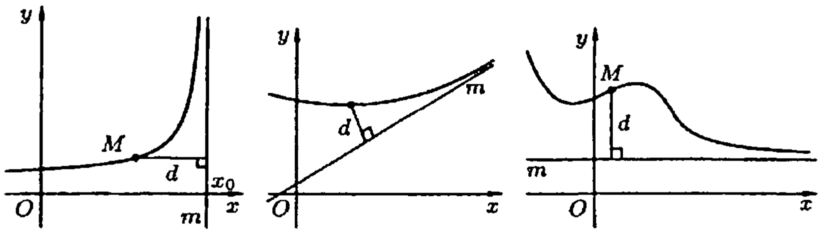
Teorema. $y=f(x)$ funksiya (a,b) intervalda aniqlangan va bu intervalda chekli $f'(x)$ hosilaga ega bo'lsin. $f(x)$ funksiyaning (a,b) da qavariq (botiq) bo'lishi uchun $f'(x)$ning (a,b)da kamayuvchi (o'suvchi) bo'lishi zarur va yetarli.

Teorema. $y=f(x)$ funksiya (a,b) intervalda aniqlangan va bu intervalda ikkinchi tartibli $f''(x)$ hosilaga ega bo'lsin. $f(x)$ ning (a,b) intervalda qavariq (botiq) bo'lishi uchun shu intervalda $f''(x)\le 0$ $(f''(x)\ge 0)$ tengsizlikning bajarilishi zarur va yetarli.

6-ta'rif. Agar $x=a$ nuqtadan o'tishda $y=f(x)$ funksiyaning grafigi qovariqligi yoki botiqligini o'zgartirsa, u holda $x=a$ nuqta funksiya grafigining egilish nuqtasi deyiladi.

Funksiya grafigining asimptotalari

7-ta'rif. Agar $y=f(x)$ egri chiziqning M nuqtasidan m to'g'ri chiziqqacha bo'lgan d masofa M nuqta cheksiz uzoqlashganda nolga intilsa, m to'g'ri chiziq $y=f(x)$ egri chiziqning asimptotasi deyiladi.



8-ta'rif. Agar $\lim\limits_{x\to x_0} f(x)=\infty$ bo'lsa, $x=x_0$ to'g'ri chiziq $y=f(x)$ funksiya grafigining vertikal asimptotasi deyiladi.

9-ta'rif. Agar $\lim\limits_{x\to\infty}\left[f(x)-(kx+b)\right]=0$ bo'lsa, $y=kx+b$ to'g'ri chiziq $y=f(x)$ funksiya grafigining og'ma asimptotasi deyiladi.

Teorema. $y=f(x)$ funksiya grafigi $x\to+\infty$ da $y=kx+b$ og'ma asimptotaga ega bo'lishi uchun

$$\lim_{x\to+\infty}\frac{f(x)}{x}=k,\ \lim_{x\to+\infty}\left[f(x)-kx\right]=b$$

bo'lishi zarur va yetarlidir.

Bu teorema $x\to-\infty$ da ham o'rinlidir.

Og'ma asimptotaning xususiy holi $(k=0)$ gorizontal asimptota bo'ladi.

10-ta'rif. Agar $\lim_{x\to\infty} f(x)=b$ bo'lsa, $y=b$ to'g'ri chiziq $y=f(x)$ funksiya grafigining gorizontal asimptotasi deyiladi.

Funksiyalarni to'liq tekshirish va grafiklarini chizish

Funksiyaning sxematik grafigini chizishning umumiy sxemasi quyidagidan iborat:

1) funksiyaning aniqlanish sohasi topiladi, so'ngra uning uzilish nuqtalari;

2) funksiyaning juft – toqligi, davriyligi. Funksiyaning asimptotalari topiladi;

3) funksiya nollari topiladi;

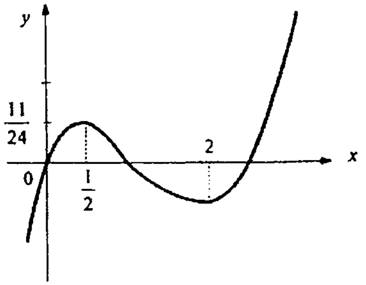
4) funksiyaning monotonlik intervallari va ekstremumlari topiladi;

5) funksiya grafigining qavariqlik yo'nalishlari va burilish nuqtalari aniqlanadi;

6) funksiya grafigining eskizi chiziladi.

Misollar

1. Ushbu $f(x)=2x^2-\ln x$ funksiyaning o'sish va kamayish intervallarini toping.

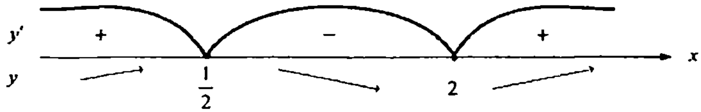


Yechish. Funksiya $(0;+\infty)$ intervalda aniqlangan. Uning hosilasi $f'(x)=4x-\frac{1}{x}$ ga teng. Agar $4x-\frac{1}{x}>0$ bo'lsa ya'ni $x>\frac{1}{2}$ da o'suvchi, agar $4x-\frac{1}{x}<0$ bo'lsa ya'ni $x<\frac{1}{2}$ da funksiya kamayuvchi bo'ladi. Demak funksiya $\left(0;\frac{1}{2}\right)$ intervalda kamayuvchi, $\left(\frac{1}{2};+\infty\right)$ intervalda o'suvchi bo'ladi.

2. Funksiyaning ekstremumlari va monotonlik intervallarini toping.

$$y=\frac{2}{3}x^3-\frac{5}{2}x^2+2x$$

Yechish. Tekshirish sxemasiga muvofiq y' ni topamiz: $y'=2x^2-5x+2$. Ko'rinib turibdiki, x ning barcha qiymatlarida hosila mavjud. Hosilani nolga tenglab, $2x^2-5x+2=0$ tenglamani olamiz, bu yerdan $x_1=\frac{1}{2}$ va $x_2=2$ kabi kritik nuqtalarni topamiz. hosila ishoralari quyidagi chizmada ko'rsatilgan:



$\left(-\infty;\frac{1}{2}\right)$ va $(2;+\infty)$ orliqlarda hosila $f'(x)>0$ va funksiya o'suvchi, $\left(\frac{1}{2};2\right)$ oraliqda hosila $f'(x)<0$ ya'ni funksiya kamayuvchi. $x=\frac{1}{2}$ — maksimum nuqta va $f_{\max}\left(\frac{1}{2}\right)=\frac{11}{24}$, $x=2$ — minimum nuqta va $f_{min}(2)=-\frac{2}{3}$.

Chunki hosila bu nuqtalardan o'tishda o'z ishorasini ($x=\frac{1}{2}$ da) «+» dan «-» ga va ($x=2$ da) «-» dan «+» ga o'zgartiradi.

Izoh: $x=\frac{1}{2}$ va $x=2$ kritik nuqtalarda ekstremum mavjudligini ikkinchi tartibli hosila yordamida aniqlasa bo'ladi: $f''(x)=4x+5$, $f''\left(\frac{1}{2}\right)=-3<0$ va $f''(2)=3>0$ bo'lganligi uchun $x=\frac{1}{2}$ — maksimum nuqta va $x=2$ — minimum nuqta.

3. $y=3x-x^3$ funksiyaning $[-2;4]$ kesmadagi eng katta va eng kichik qiymatlarini toping.

Yechish. $y' = 3 - 3x^2$ funksiya hosilasi $x = \pm 1$ nuqtalarda nolga teng. Bu nuqtalarda va kesma oxirlarida funksiya qiymatlarini topamiz: $f(-2) = 2, f(-1) = -2,\ f(1) = 2,\ f(4) = -52.$

Shunday qilib, $f_{eng\,katta} = f(-2) = f(1) = 2,\ f_{eng\,kichik} = f(4) = -52.$

4. Q mahsulot miqdoriga bog'liq bo'lgan daromad funksiyasi $R(Q) = 100Q - Q^2$ formula bilan, mahsulotni ishlab chiqarishga ketgan harajatlar funksiyasi esa $C(Q) = Q^3 - 37Q^2 + 169Q + 4000$ formulalar bilan aniqlansin. Maksimal foydani toping.

Yechish. Foyda $F(Q) = R(Q) - C(Q)$ formula bilan aniqlanadi. Bu erdan $F(Q) = -Q^3 + 36Q^2 - 69Q - 4000$. Foyda funksiyasining hosilasini nolga tenglashtirib $Q^2 - 24Q + 23 = 0$ tenglamani hosil qilamiz. Bu tenglamaning ildizlari $Q = 1$, $Q = 23$. Tekshirish shuni ko'rsatadiki, maksimal foydaga $Q = 23$ da erishiladi. $F_{\max} = 1290.$

5. Ushbu $f(x) = \dfrac{1}{x^2 + 1}$ funksiya grafigining qavariqlik oraliqlarini va burilish nuqtasini toping.

Yechish. Funksiya haqiqiy sonlar o'qida aniqlangan va ikki marta differensiallanuvchi. Funksiyaning ikkinchi tartibli hosilasini topamiz

$$f''(x) = \frac{6\left(x^2 - \frac{1}{3}\right)}{\left(x^2 + 1\right)^3}.$$

$f''(x) < 0$ da funksiya yuqoriga qavariq $x^2 - \frac{1}{3} < 0$ yoki $|x| < \frac{1}{\sqrt{3}}$. $f''(x) > 0$ da funksiya quyiga qavariq $x^2 - \frac{1}{3} > 0,\ x \in \left(-\infty; -\frac{1}{\sqrt{3}}\right) \cup \left(\frac{1}{\sqrt{3}}; +\infty\right).$

Shunday qilib funksiya grafigi $\left(-\frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}\right)$ da yuqoriga qavariq, $\left(-\infty; -\frac{1}{\sqrt{3}}\right)$ va $\left(\frac{1}{\sqrt{3}}; +\infty\right)$ da quyiga qavariq bo'ladi. Demak $x_1 = -\frac{1}{\sqrt{3}}$ va $x_2 = \frac{1}{\sqrt{3}}$ nuqtalar funksiyaning burilish nuqtalari bo'ladi.

6. $f(x)=e^{\frac{1}{x^2(1-x)}}$ funksiya grafigining vertikal asimptotasini toping.

Yechish. $x=0$ va $x=1$ - uzilish nuqtalari,

$$\lim_{x\to 0} e^{\frac{1}{x^2(1-x)}}=+\infty, \qquad \lim_{x\to 1-0} e^{\frac{1}{x^2(1-x)}}=+\infty, \qquad \lim_{x\to 1+0} e^{\frac{1}{x^2(1-x)}}=0.$$

$x=0$, $x=1$ ikkinchi tur uzilish nuqtalari, $x=0$, $x=1$ to‘g‘ri chiziqlar vertikal asimptotalar.

7. $f(x)=\dfrac{x^3+3x^2}{x^2-2}$ funksiya grafigining og‘ma asimptotasini toping.

Yechish. $k=\lim\limits_{x\to\infty}\dfrac{f(x)}{x}=\lim\limits_{x\to\infty}\dfrac{x^3+3x^2-1}{x\left(x^2-2\right)}=1,$

$$b=\lim_{x\to\infty}\left(f(x)-kx\right)=\lim_{x\to\infty}\frac{3x^2+2x-1}{x^2-2}=3.$$

$y=x+3$ –og‘ma asimptota.

8. $y=\dfrac{x^2-x+1}{x-1}$ funksiyani to‘liq tekshiring va grafigini chizing.

Yechish.

Funksiyaning aniqlanish sohasi: $D(y)=\{x\neq 1\}$

Funksiya juft ham, toq ham, davriy ham emas.

$x=1$ nuqta funktsiyaning 2-tur uzilish nuqtasi, chunki $\lim\limits_{x\to 1-0} f(x)=-\infty$ va $\lim\limits_{x\to 1+0} f(x)=+\infty$ OY o‘qi bilan kesishish nuqtasi: $y=f(0)=-1$.

OX o‘qi bilan kesishish nuqtasi: $y=0\Rightarrow x^2-x+1=0\Rightarrow x\in\varnothing\Rightarrow$ OX o‘qi bilan kesishishmaydi.

Funksiyaning ishorasi o‘zgarmaydigan oraliqlar:

X	$(-\infty;1)$	$(1;+\infty)$
Y	$-$	$+$

Endi funksiyani monotonlik va ekstremumga tekshiramiz:

$$y'=\left(\frac{x^2-x+1}{x-1}\right)'=\frac{(2x-1)\cdot(x-1)-\left(x^2-x+1\right)\cdot 1}{(x-1)^2}=\frac{2x^2-3x+1-x^2+x-1}{(x-1)^2}=\frac{x^2-2x}{(x-1)^2}=\frac{x\cdot(x-2)}{(x-1)^2}$$

Intervallar usulidan foydalanib bu ifodaning ishorasi saqlanadigan oraliqlarni topamiz va quyidagi jadvalni tuzamiz:

x	$(-\infty;0)$	0	$(0;1)$	1	$(1;2)$	2	$(2;+\infty)$
y'	+	0	-	∄	-	0	+
y	↗	max -1	↘	∄	↘	min 3	↗

Qavariqlikka tekshirish uchun y''ni hisoblaymiz:

$$y''=(y')'=\left[\frac{x^2-2x}{(x-1^2)}\right]'=\left[1-\frac{1}{(x-1)^2}\right]'=\frac{2}{(x-1)^3}\Rightarrow x\cdot 1da\cap \text{ va } x\cdot 1da\cup.$$

Funksiya asimptotalarini topamiz:

a) Vertikal asimptota: $x=1$-vertikal asimptota.

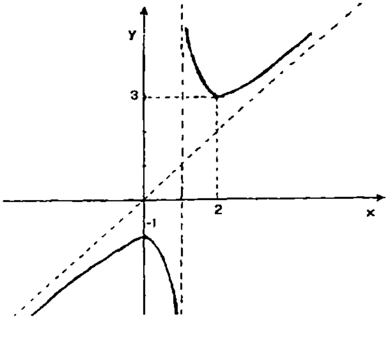
b) Gorizontal asimptota: $\lim\limits_{x\to\infty} f(x)=\lim\limits_{x\to\infty}\frac{x^2-x+1}{x-1}=\infty\Rightarrow$ gorizontal asimptota yo‘q.

v) Og‘ma asimptota: $a=\lim\limits_{x\to\infty}\frac{f(x)}{x}=\lim\limits_{x\to\infty}\frac{x^2-x+1}{x\cdot(x-1)}=1$

$$b=\lim_{x\to\infty}\left[f(x)-ax\right]=\lim_{x\to\infty}\left(\frac{x^2-x+1}{x-1}-x\right)=\lim_{x\to\infty}\frac{x^2-x+1-x^2+x}{x-1}=$$

$$=\lim_{x\to\infty}\frac{1}{x-1}=0\Rightarrow y=x \text{ og‘ma asimptota.}$$

Endi topilgan ma`lumotlardan foydalanib funksiya grafigini chizamiz



6.3. Ko‘p o‘zgaruvchili funksiya ekstremumlari

Ko‘p o‘zgaruvchili funksiyaning hosila va differensiallari

1-ta’rif. Ushbu

$$\lim_{\Delta x_k\to 0}\frac{\Delta_{x_k} f(x^0)}{\Delta x_k},\ \left(k=\overline{1,m}\right)$$

limitga $f(x)=f(x_1,...,x_m)$ funksiyaning x^0 nuqtadagi x_k o'zgaruvchi bo'yicha xususiy hosilasi deyiladi va u $\frac{\partial f(x^0)}{\partial x_k}$ kabi belgilanadi.

Xususiy hosilaning geometrik ma`nosini bilish uchun $M\subset R^2$ to'plamda aniqlangan $z=f(x,y)$ funksiyani qaraymiz. Aytaylik $(x_0,y_0)\in M$ bo'lib, bu nuqtada $\frac{\partial f(x_0,y_0)}{\partial x}$ va $\frac{\partial f(x_0,y_0)}{\partial y}$ lar mavjud bo'lsin. $z=f(x,y)$funksiya grafigi R^3 da biror sirtni aniqlaydi. $z=f(x,y_0)$ ning grafigi sirt bilan $y=y_0$ tekislikning kesishishida hosil bo'lgan Γ_1 chiziq bo'ladi. $z=f(x_0,y)$ning grafigi Γ_2 chiziq bo'ladi. Agar Γ_1 va Γ_2 chiziqlarning $(x_0,y_0,f(x_0,y_0))$ nuqtasiga o'tkazilgan urinmaning Oxy tekisligi bilan hosil qilgan burchaklarini mos ravishda α va β deb belgilasak, unda

$$\frac{\partial f(x_0,y_0)}{\partial x}=tg\alpha \text{ va } \frac{\partial f(x_0,y_0)}{\partial y}=tg\beta$$

bo'ladi. Bundan $z=f(x,y)$ sirtning (x_0,y_0,z_0) nuqtasiga o'tkazilgan urinma tekislik tenglamasi ushbu

$$z-z_0=\frac{\partial f(x_0,y_0)}{\partial x}\cdot(x-x_0)+\frac{\partial f(x_0,y_0)}{\partial y}\cdot(y-y_0)$$

ko'rinishda bo'lishi hosil qilamiz.

Teorema. Agar $f(x)$ funksiya x^0 nuqtada chekli $\frac{\partial f(x^0)}{\partial x_k},(k=\overline{1,m})$ xususiy hosilaga ega bo'lsa, unda $f(x)$ funksiya shu nuqtada mos x_k o'zgaruvchi bo'yicha xususiy uzluksiz bo'ladi.

2-ta'rif. Agar $f(x)$ funksiya x^0 nuqtadagi $\Delta f(x^0)$ orttirmasini

$$\Delta f(x^0)=A_1\cdot\Delta x_1+...+A_m\cdot\Delta x_m+\alpha_1\cdot\Delta x_1+...+\alpha_m\cdot\Delta x_m \quad (1)$$

ko'rinishda ifodalash mumkin bo'lsa, $f(x)$ funksiya x^0 nuqtada differensiallanuvchi deyiladi. Bu $A_1,...,A_m$ lar $\Delta x_1,...,\Delta x_m$ ga bog'liq bo'lmagan o'zgarmaslar va $\lim\limits_{\substack{\Delta x_1\to 0\\ \cdots\\ \Delta x_m\to 0}}\alpha_k=0,(k=\overline{1,m})$ tengliklar bajariladi.

(1)-tenglik ushbu

$$\Delta f(x_0)=A_1\cdot\Delta x_1+...+A_m\cdot\Delta x_m+o(\rho) \quad (2)$$

tenglikka ekvivalent. Bu yerda $\rho=\sqrt{(\Delta x_1)^2+...+(\Delta x_m)^2}$.

Teorema. Agar $f(x)$ funksiya x^0 nuqtada differensiallanuvchi bo'lsa, u holda bu funksiya shu nuqtada uzluksiz bo'ladi.

Teorema. Agar $f(x)$ funksiya x^0 nuqtada differensiallanuvchi bo'lsa, unda bu funksiyaning shu nuqtadagi barcha hususiy hosilalari mavjud va $\frac{\partial f(x^0)}{\partial x_1}=A_1,...,\frac{\partial f(x^0)}{\partial x_m}=A_m$ tengliklar o'rinli bo'ladi.

Izoh: teoremaning aksi har doim ham o'rinli bo'lavermaydi, ya'ni barcha xususiy hosilalari *mavjud* bo'lgan funksiya differensiallanuvchi bo'lishi shart emas.

Teorema (yetarli shart). Agar $f(x)$ funksiya x^0 nuqtaning biror atrofida barcha o'zgaruvchilari bo'yicha xususiy hosilalarga ega bo'lib, bu xususiy hosilalar x^0 nuqtada uzluksiz bo'lsa, unda $f(x)$ funksiya shu x^0 nuqtada differensiallanuvchi bo'ladi.

Ushbu

$$df(x^0)=\frac{\partial f(x^0)}{\partial x_1}dx_1+...+\frac{\partial f(x^0)}{\partial x_m}dx_m \text{ va}$$

$$d_{x_k}f(x_0)=\frac{\partial f(x^0)}{\partial x_k}dx_k,(k=\overline{1,m})$$

ifodalarga mos ravishda $f(x)$ funksiyaning x^0 nuqtadagi differensiali (to'liq differensiali) va x_k o'zgaruvchi bo'yicha xususiy differensiali deyiladi.

Endi yo'nalish bo'yicha hosila tushunchasini kiritamiz.

Ikki o'zgaruvchili $z=f(x,y)$ funksiya ochiq $M\subset R^2$ to'plamda berilgan bo'lsin. $\forall A_0(x_0,y_0)\in M$ nuqta olib, bu nuqtadan biror ℓ

to'g'ri chiziq o'tkazaylik.Bu to'g'ri chiziqning OX va OY koordinata o'qlari bilan hosil qilgan burchaklari α va β bo'lsin.

3-ta'rif. Agar A nuqta ℓ to'g'ri chiziq bo'ylab A_0 nuqtaga intilganda ushbu

$$\lim_{A\to A_0}\frac{f(A)-f(A_0)}{\rho(A_0,A)}$$

limit mavjud bo'lsa, uning qiymatiga $f(x,y)=f(A)$ funksiyaning $A_0=(x_0,y_0)$ nuqtadagi ℓ yo'nalish bo'yicha hosilasi deyiladi va $\frac{\partial f(A_0)}{\partial \ell}$ yoki $\frac{\partial f(x_0,y_0)}{\partial \ell}$ kabi belgilanadi.

Demak,

$$\frac{\partial f(A_0)}{\partial \ell}:=\lim_{A\to A_0}\frac{f(A)-f(A_0)}{\rho(A_0,A)}$$

Teorema. Agar $f(x,y)$ funksiya $A_0=(x_0,y_0)$ nuqtada differensiallanuvchi bo'lsa, u holda shu funksiya A_0 nuqtada $\forall\ell$ yo'nalish bo'yicha hosilaga ega va

$$\frac{\partial f(A_0)}{\partial \ell}:=\frac{\partial f(x_0,y_0)}{\partial x}\cos\alpha+\frac{\partial f(x_0,y_0)}{\partial y}\cos\beta$$

tenglik o'rinli.

Izoh: Funksiya biror nuqtada differensiallanuvchi bo'lmasa ham u shu nuqtada biror yo'nalish bo'yicha hosilaga ega bo'lishi mumkin.

Agar differensiallanuvchi $w=f(x,y,z)$ va $x=\varphi(u,v)$, $y=\psi(u,v)$ $z=\chi(u,v)$ funksiyalar berilgan bo'lib, ular yordamida $w=f[\varphi(u,v),\psi(u,v),\chi(u,v)]=F(u,v)$ murakkab funksiya aniqlangan bo'lsa, unda murakkab funksiya ham differensiallanuvchi bo'ladi va

$$\begin{cases}\dfrac{\partial w}{\partial u}=\dfrac{\partial w}{\partial x}\cdot\dfrac{\partial x}{\partial u}+\dfrac{\partial w}{\partial y}\cdot\dfrac{\partial y}{\partial u}+\dfrac{\partial w}{\partial z}\cdot\dfrac{\partial z}{\partial u},\\[2ex] \dfrac{\partial w}{\partial v}=\dfrac{\partial w}{\partial x}\cdot\dfrac{\partial x}{\partial v}+\dfrac{\partial w}{\partial y}\cdot\dfrac{\partial y}{\partial v}+\dfrac{\partial w}{\partial z}\cdot\dfrac{\partial z}{\partial v}.\end{cases}$$

tengliklar o'rinli bo'ladi.

Ko'p o'zgaruvchili funksiyaning ikkinchi tartibli xususiy hosilalari quyidagi tenglik yordamida aniqlanadi:

$$f''_{x_i x_k}=\frac{\partial^2 f}{\partial x_i\partial x_k}:=\frac{\partial}{\partial x_k}\left(\frac{\partial f}{\partial x_i}\right),\ \left(i,k=\overline{1,m}\right)$$

Agar $i=k$ bo'lsa, $\frac{\partial^2 f}{\partial x_k\partial x_k}=\frac{\partial^2 f}{\partial x_k^2}=f''_{x_k^2}$ kabi yoziladi.

Agar $i\neq k$ bo'lsa $\frac{\partial^2 f}{\partial x_i\partial x_k}$ - aralash hosila deb ataladi.

Yuqori tartibli xususiy hosilalar ham shu kabi aniqlanadi.

M to'plamda 1,2,3,...,k-tartibli uzluksiz xususiy hosilalarga ega bo'lgan funksiyalar sinfi $C^{(k)}(M;R)$ yoki $C^{(k)}(M)$ kabi belgilanadi.

4-ta'rif. Agar $f(x)$ funksiyaning x nuqtadagi barcha ikkinchi tartibli xususiy hosilalari mavjud bo'lsa, unda funksiyaning ikkinchi tartibli differensiali quyidagi tenglik yordamida aniqlanadi:

$$d^2 f(x) := \sum_{i,k=1}^{m} \frac{\partial^2 f(x)}{\partial x_i \partial x_k} dx_i dx_k = \left(\frac{\partial}{\partial x_1} dx_1 + \ldots + \frac{\partial}{\partial x_m} dx_m \right)^2 f(x)$$

Xuddi shunga o'xshash

$$d^n f := d\left(d^{n-1} f\right) = \left(\frac{\partial}{\partial x_1} dx_1 + \ldots + \frac{\partial}{\partial x_m} dx_m \right)^n f$$

bo'ladi.

Funksiyaning ekstremumlari

$f(x) = f(x_1, \ldots, x_m)$ funksiya ochiq $M \subset R^m$ to'plamda berilgan bo'lib, $x_0 = \left(x_1^0, \ldots, x_m^0\right) \in M$ bo'lsin.

5-ta'rif. Agar x^0 nuqtaning $\exists \bigcup_\delta (x^0) \subset M$ atrofi topilsaki, $\forall x \in \bigcup_\delta (x^0)$ uchun $f(x) \le f(x^0)$ ($f(x) \ge f(x^0)$)

bo'lsa, $f(x)$ funksiya x^0 nuqtada min (max) ga ega deyiladi. $f(x^0)$ qiymat esa $f(x)$ funksiyaning lokal (max) min qiymati deyiladi va

$$f(x^0) = \max_{x \in \bigcup_\delta (x^0)} \{f(x)\} \quad \left(f(x^0) = \min_{x \in \bigcup_\delta (x^0)} \{f(x)\} \right)$$

kabi belgilanadi.

Funksiyaning max va min qiymatlari uning ekstremumlari deb ataladi.

x^0 nuqtaning $\bigcup_\delta (x^0)$ atrofida

$$\Delta = f(x) - f(x^0)$$

ayirmani ko'raylik.

Agar bu ayirma $\bigcup_\delta (x^0)$da o'z ishorasini saqlasa ya`ni har doim $\Delta \ge 0 (\Delta \le 0)$ bo'lsa, $f(x)$ funksiya x^0 nuqtada min (max) ga erishadi. Agar Δ ayirma x^0 nuqtaning $\forall$ atrofida ham o'z ishorasini saqlamasa, unda $f(x)$ funksiya x^0 nuqtada ekstremumga ega bo'la olmaydi.

Teorema. (zaruriy shart). $f(x)$ funksiya x^0 nuqtada ekstremumga erishsa va shu nuqtada $f'_{x_1}(x_0), \ldots, f'_{x_m}(x^0)$ xususiy hosilalar mavjud bo'lsa , unda

$$f'_{x_1}(x_0) = \ldots = f'_{x_m}(x^0) = 0$$

bo'ladi.

1-izoh. Teoremaning aksi har doim ham o'rinli bo'lavermaydi. Masala, $f(x,y)=x\cdot y$ funksiya uchun $f_x'(0,0)=f_y'(0,0)=0$, lekin funksiya (0,0) nuqtada ekstremumga erishmaydi, chunki u (0,0) nuqtaning ixtiyoriy atrofida har hil ishorali qiymatlarni qabul qiladi.

2-izoh. Agar $f(x)$ funksiya x^0 nuqtada differensiallanuvchi bo'lsa, u holda funksiyaning ekstremumga erishishining zaruriy shartini $df(x^0)=0$ ko'rinishda yozish mumkin.

Teorema. (etarli shart.) $f(x)$ funksiya x^0 nuqtaning biror $\bigcup_\delta(x^0)$ atrofida berilgan bo'lib quyidagi shartlarni bajarsin:

1) $f(x)$ funksiya $\bigcup_\delta(x^0)$ da uzluksiz birinchi va ikkinchi tartibli xususiy hosilalarga ega;

2) x^0 nuqta $f(x)$ funksiyaning statsionar nuqtasi;

3) koeffitsiyentlari $a_{ik}=f''_{x_ix_k}(x^0),\ (i,k=\overline{1,m})$ bo'lgan.

$$Q(\xi_1,\dots,\xi_m)=\sum_{i,k=1}^{m}a_{ik}\xi_i\xi_k$$

kvadratik forma musbat (manfiy) aniqlangan.

U holda $f(x)$ funksiya x^0 nuqtada min (max) ga erishadi. Agar kvadratik forma noaniq bo'lsa, unda $f(x)$ funksiya x^0 nuqtada ekstremumga erishmaydi.

Bu teoremani $m=2$ bo'lgan holda alohida ko'ramiz:

$$a_{11}=\frac{\partial^2 f(x^0)}{\partial x_1^2}\quad a_{12}=\frac{\partial^2 f(x^0)}{\partial x_1 dx_2}\quad a_{22}=\frac{\partial^2 f(x^0)}{\partial x_2^2}$$

$$\Delta=\begin{vmatrix}a_{11}a_{12}\\ a_{21}a_{22}\end{vmatrix}=a_{11}a_{22}-a_{12}^2$$ bo'lsin. Unda

1) $\Delta>0, a_{11}>0$ bo'lsa, min;

2) $\Delta>0, a_{11}<0$ bo'lsa, max;

3) $\Delta<0$ bo'lsa, ekstremum mavjud emas.

4) $\Delta=0$ bo'lsa, shubhali hol bo'ladi.

Misollar

1. a) $z=x^2+2xy+3y^2$ b) $u=\dfrac{x}{x^2+y^2+z^2}$ funksiyalarning xususiy hosilalarni toping.

Yechish. a) y ni o'zgarmas deb, z'_x ni topamiz:

$$z'_x=(x^2+2xy+3y^2)'_x=(x^2)'_x+(2xy)'_x+(3y^2)'_x=2x+2y,$$

endi x ni o'zgarmas deb, $\frac{\partial z}{\partial y}$ ni topamiz:

$$z'_y=(x^2+2xy+3y^2)'_y=(x^2)'_y+(2xy)'_y+(3y^2)'_y=2x+6y.$$

b) hosila olish qoidalari va formulalaridan foydalanib quyidagilarni topamiz:

$$u'_x=\left(\frac{x}{x^2+y^2+z^2}\right)'_x=\frac{x'_x(x^2+y^2+z^2)-x(x^2+y^2+z^2)'_x}{(x^2+y^2+z^2)^2}=$$

$$=\frac{x^2+y^2+z^2-2x^2}{(x^2+y^2+z^2)^2}=\frac{-x^2+y^2+z^2}{(x^2+y^2+z^2)^2}.$$

u'_y, u'_z larni mustaqil toping.

2. Ushbu $u=x^2-2xy+4y^2+6z^2+6yz-6z$ funksiya ekstremumga tekshirilsin.

Yechish.

$$\begin{cases}\frac{\partial u}{\partial x}=2x-2y,\\ \frac{\partial u}{\partial y}=-2x+8y+6z,\\ \frac{\partial u}{\partial z}=12z+6y-6\end{cases}\qquad \begin{cases}\frac{\partial u}{\partial x}=0\\ \frac{\partial u}{\partial y}=0\\ \frac{\partial u}{\partial z}=0\end{cases}$$

sistemani yechib, $M_0(-1,-1,1)$ nuqta statsionar nuqta ekanligini topamiz. Endi ikkinchi tartibli xususiy hosilalarni hisoblab, $d^2u\,|_{M_0}$ ning ishorasini aniqlaymiz.

$$a_{11}=\frac{\partial^2 u}{\partial x^2}=2,\ a_{12}=a_{21}=\frac{\partial^2 u}{\partial x\partial y}=\frac{\partial^2 u}{\partial y\partial x}=-2,\ a_{13}=a_{31}=\frac{\partial^2 u}{\partial x\partial z}=\frac{\partial^2 u}{\partial z\partial x}=0,$$

$$a_{22}=\frac{\partial^2 u}{\partial y^2}=8,\ a_{23}=a_{32}=\frac{\partial^2 u}{\partial y\partial z}=\frac{\partial^2 u}{\partial z\partial y}=6\quad a_{33}=\frac{\partial^2 u}{\partial z^2}=12$$

$$a_{11}=2>0;\qquad \begin{vmatrix}a_{11}a_{12}\\ a_{21}a_{22}\end{vmatrix}=\begin{vmatrix}2,-2\\ -2,8\end{vmatrix}=12>0;$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} 2 & -2 & 0 \\ -2 & 8 & 6 \\ 0 & 6 & 12 \end{vmatrix} = 24 \cdot \begin{vmatrix} 1 & -1 & 0 \\ -1 & 4 & 3 \\ 0 & 1 & 2 \end{vmatrix} = 48 > 0 \Rightarrow$$

$$d^2u \,\big|_{M_0} > 0 \Rightarrow u_{\min} = u(-1;-1;\ 1) = -3.$$

6.4. Talabaning mustaqil ishi

1-topshiriq

1-2 misollarda berilgan funksiyalarning hosilalarini toping, natijani Mathcad dasturi yordamida tekshiring.

3-misolda berilgan oshkormas funksiyaning birinchi tartibli hosilalarini toping.

4-misolda parametrik ko‘rinishdagi funksiyaning birinchi tartibli hosilalarini.

5-misolda berilgan funksiyalarning n – tartibli hosilalarini toping.

1-variant

1. $y = \cos 5x.$
2. $y = 7^{3x} - 1$
3. $y^{xy} - \cos(x^2 + y^2) = 0.$
4. $x = t^3 + t, \quad y = t^2 + t + 1.$
5. $y = e^{ax}.$

2-variant

1. $y = \cos^3 x.$
2. $y = (x+1)^{100}.$
3. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1.$
4. $x = t - \sin t, \quad y = 1 - \cos t.$
5. $y = \sin ax + \cos bx.$

3-variant

1. $y = \sqrt{tgx}.$
2. $y = \arcsin \sqrt{x}$
3. $x^2 + y^2 = ln\dfrac{y}{x} + 7.$
4. $x = 3\sin t, \ y = 2\cos t.$
5. $y = xe^x.$

4-variant

1. $y = \dfrac{1}{\ln x}.$

2. $y = \ln\sin x$.

3. $x\sin y + y\sin x = 0$.

4. $x = \sin^2 t, \quad y = \cos^2 t$.

5. $y = \dfrac{1}{ax+b}$.

5-variant

1. $y = e^{ctgx}$.

2. $y = \arccos(e^x)$.

3. $x^4 - y^4 = x^2y^2$.

4. $x = 5cht, \quad y = 4sht$.

5. $y = \dfrac{x}{x^2-1}$.

6-variant

1. $y = arctg^2\frac{1}{x}$.

2. $y = \sin^9\left(\frac{x}{2}\right)$.

3. $e^y = e - xy$. y' ni $(0;1)$ nuqtada toping.

4. $x = t^3, y = 3t$.

5. $y = \dfrac{1}{ax-b}$.

7-variant

1. $y = \sqrt[3]{(1-3x)^2}$.

2. $y = \arcsin\sqrt{\dfrac{1-x}{1+x}}$.

3. $\sqrt{x} + \sqrt{y} = \sqrt{5}$.

4. $x = \cos^3 t, y = \sin^3 t$.

5. $y = e^{3x}$.

8-variant

1. $y = \ln\sqrt{\dfrac{1+tgx}{1-tgx}}$.

2. $y = (1 + tg^2 3x)\cdot e^{-\frac{x}{2}}$.

3. $\arcsin\dfrac{x}{y} = y\ln x$.

4. $x = \dfrac{t+1}{t}, y = \dfrac{t-1}{t}$.

5. $y = \dfrac{x^2+2x+3}{x}$.

9-variant

1. $y = \ln\left(x + \sqrt{x^2-1}\right)$.

2. $y = tg4x + \frac{2}{3}tg^3 4x + \frac{1}{5}tg^5 4x$

3. $x^y \cdot y^x = 1$

4. $x = t - arctg\, t, y = \frac{t^3}{3} + 1.$

5. $y = \sqrt{x}$

10-variant

1. $y = x^3 \cdot \sin(\cos x).$

2. $y = 3^{x^2} \cdot \sqrt{x^3 - 5x}.$

3. $x^2 + 3y^2 - 4xy + 10 = 0.$

4. $x = 2t + 1,\ y = t^3.$

5. $y = x^a.$

11-variant

1. $y = \log_6 \sin 4x.$

2. $y = \cos\frac{1-\sqrt{x}}{1+\sqrt{x}}.$

3. $arctgy = x^2 y.$

4. $x = \frac{1}{t+1},\ y = \frac{t}{t+1}.$

5. $y = \ln \sqrt[11]{x+1}.$

12-variant

1. $y = ln\frac{(x+1)(x+3)^3}{(x+2)^3(x+4)}.$

2. $y = arctg\,(x-2) + \frac{x-3}{x^2 - 4x + 5}.$

3. $x^2 + y^2 = 4.$ y' ni $(-\sqrt{2};\sqrt{2})$ nuqtada toping.

4. $x = a(\cos t + t\sin t),\ y = a(\sin t - t\cos t).$

5. $y = \sin ax.$

13-variant

1. $y = \sin^4\frac{x}{2} + \cos^4\frac{x}{2}.$

2. $y = e^{sh^3 5x}$

3. $2x + y - 4 = 0.$

4. $x = a\cos^2 t,\ y = a\sin^2 t.$

5. $y = \cos\beta x.$

14-variant

1. $y = \frac{x + e^{3x}}{x - e^{3x}}.$

2. $y = \arccos\sqrt{x} + \sqrt{x - x^2}.$

3. $x\ln y + y\ln x = 0.$

4. $x = a(t - \sin t),\ y = a(1 - \cos t).$

5. $y=\ln(1+x)$.

15-variant

1. $y=arctg\,\frac{x+1}{x-1}$.
2. $y=\frac{\sin^2 x}{ctgx+1}+\frac{\cos^2 x}{tgx+1}$.
3. $x\cos y-y\sin x=0$.
4. $x=e^t\sin t,\ y=e^t\cos t$.
5. $y=\sin^2 x$.

16-variant

1. $y=10^{x^2+1}$.
2. $y=tg\,4x$.
3. $\sqrt{x}+\sqrt{y}-2=0$.
4. $x=\frac{\cos^3 t}{\sqrt{\cos 2t}},\ y=\frac{\sin^3 t}{\sqrt{\sin 2t}}$.
5. $y=\ln x$.

17-variant

1. $y=ch^4\,\frac{x}{2}$.
2. $y=ln(5x^3-x)$.
3. $xy-arctg\frac{x}{y}=0$.
4. $x=\arccos\frac{1}{\sqrt{1+t^2}},\ y=\arcsin\frac{1}{\sqrt{1+t^2}}$.
5. $y=5^x$.

18-variant

1. $y=\cos^4 x-\sin 4x$.
2. $y=\sqrt{4-7x^2}$.
3. $arctg(x+y)=x$.
4. $x=e^{-2t}\sin 2t,\ y=e^{2t}\cos 2t$.
5. $y=\sin x$.

19-variant

1. $y=\sqrt[5]{1+ctg10x}$.
2. $y=(\sin 3x-\cos 3x)^2$.
3. $\ln y+\frac{x}{y}-a=0$.
4. $x=e^{2t}\sin 2t,\ y=e^{-2t}\cos 2t$.

5. $y=\frac{1}{3x+5}$.

20-variant

1. $x=ln^4 \sin 3t$.
2. $f(h)=arctg\sqrt{h}$.
3. $arctg\frac{x}{y}=\frac{1}{2}\ln(x^2+y^2)$.
4. $x=e^{3t}\sin 3t,\ y=e^{-3t}\cos 3t$.
5. $x=\ln t,\ y=\frac{1}{t}$.

21-variant

1. $y=\frac{1}{\arcsin x}$.
2. $y=\frac{\sin x}{1+tgx}$.
3. $x^y-y^x=a$.
4. $x=a\sin t,\ y=b\cos t$.
5. $y=\frac{1}{2x-3}$.

22-variant

1. $y=\frac{xlnx}{x-1}$.
2. $y=sh(ln(tg2x))$.
3. $e^x+e^y-e^{xy}-1=0$.
4. $x=2\cos t,\ y=3\sin t$.
5. $y=\frac{1}{1-3x}$.

23-variant

1. $y=x\arcsin x+\sqrt{1-x^2}$.
2. $y=3^{\sin^3 2x+4sn2x}$.
3. $x^3y^3+5xy+4=0$.
4. $x=\ln t,\ y=\sin 2t$.
5. $y=\frac{1}{5x+2}$.

24-variant

1. $y=e^{-ln\frac{x+2}{x-3}}-\frac{x-3}{x+2}$.
2. $y=x\cdot 2^{\sqrt{x}}$.
3. $x^y-y^x=0$.
4. $x=e^{-t}\sin t,\ y=e^t\cos t$.
5. $y=\cos^2 x$.

25-variant

1. $y=\frac{1}{6}ln\frac{x-3}{x+3}$.

2. $y=\frac{x^2}{2\sqrt{1-x^4}}$.

3. $x^3+y^3=\sin(x-2y)$

4. $x=2\sin t,\ y=3\cos t$.

5. $y=\frac{1}{1+2x}$.

2-topshiriq

Hosilaning tatbiqlariga doir masalalarning matematik modelini tuzing va Mathcad dasturi yordamida hisoblash ishlarini bajaring.

1-variant

1. $y=2x^3-4x^2-5x-3$, funksiya grafigiga absissasi $x_0=2$ bo'lgan nuqtada o'tkazilgan urinma va normal tenglamasini tuzing.
2. Biror firma tomonidan mahsulot ishlab chiqarishga ketgan xarajatlar funksiyasi $y(x)=0{,}1x^3-1{,}2x^2+5x+250$ ko'rinishga ega (pul bir. hisobida). Ishlab chiqarishning o'rtacha va chegaraviy xarajatlarini toping va ularning $x=10$ dagi qiymatini hisoblang.
3. Xarajatlar funksiyasi $C(x)=10+\frac{1}{10}x^2$ ko'rinishga ega. Boshlang'ich bosqichda firma ishni $A(x)$ o'rtacha xarajatlarni minimallashtirish maqsadida tashkil etdi. Keyinchalik tovarning bir birligiga 4 shartli birlikka teng bo'lgan narx belgilandi. Firma ishlab chiqarishni qancha birlika orttirishi kerak?

2-variant

1. $y=\ln(1+x)$. funksiya grafigiga absissasi $x_0=0$ bo'lgan nuqtada o'tkazilgan urinma va normal tenglamasini tuzing.
2. Biror mamlakatning iste'mol funksiyasi quyidagi ko'rinishga ega: $C(x)=15+0{,}25x+0{,}36x^{\frac{4}{3}}$, bu erda x–umumiy milliy daromad (pul. bir.). Topish kerak: a) iste'molga bo'lgan chegaraviy moyillik; b) agar milliy daromad 27 pul birligini tashkil etsa jamg'armaga bo'lgan chegaraviy moyillik.
3. Firma o'rtacha xarajatlarini minimallashtirishi natijasida xarajatlar 30 pul birlikka teng bo'ldi. Bunda chegaraviy xarajatlar qanday bo'ladi?

3-variant

1. $y=\frac{2x+3}{2x-1}$, funksiya grafigiga absissasi $x_0=0$ bo‘lgan nuqtada o‘tkazilgan urinma va normal tenglamasini tuzing.

2. Biror firma tomonidan qishki oyoq kiyimlarini ishlab chiqarish hajmi $u=\frac{1}{3}t^3-\frac{7}{2}t^2+6t+2100$ (birlik) tenglama bilan aniqlangan, bunda t–yilning tavqim oyi. Mehnat unumdorligi hamda uning o‘zgarish tempi va tezligini: a) yil boshida $(t=0)$; b) yil o‘rtasida $(t=6)$ hisoblang.

3. Agar tovarning narxi $p=14$. birlik va xarajatlar funksiyasi $C(x)=13+2x+x^3$ ko‘rinishda bo‘lsa, ishlab chiqaruvchi uchun optimal x_0 mahsulot hajmini aniqlang.

4-variant

1. $\begin{cases} x=t+3, \\ y=\sqrt{t-1}. \end{cases}$ funksiya grafigiga absissasi $M_0(5;1)$ bo‘lgan nuqtada o‘tkazilgan urinma va normal tenglamasini tuzing.

2. Agar talab funksiyasi $q=\frac{3p+14}{p+3}$ va taklif funksiyasi $s=p+2$ (bu erda q va s - mos ravishda biror vaqt birligida sotib olinayotgan va sotishga taklif etilayotgan tovar miqdori, p– bir birlik tovarning bahosi) berilgan bo‘lsa, u holda: a) muvozanat baho, ya’ni talab va taklifni baravarlashtiradigan baho; b) talab va taklif elastikligi; v) narxni muvozanat bahodan 10% ga oshirilganda daromadni o‘sishini toping.

3. Agar tovarning narxi $p=8$. birlik va xarajatlar funksiyasi $C(x)=10+x+\frac{1}{3}x\sqrt{x}$ ko‘rinishda bo‘lsa, ishlab chiqaruvchi uchun optimal x_0 mahsulot hajmini aniqlang.

5-variant

1. $\begin{cases} x=t-\sin t, \\ y=1-\cos t \end{cases}$ funksiya grafigiga absissasi $t=\frac{\pi}{2}$ bo‘lgan nuqtada o‘tkazilgan urinma va normal tenglamasini tuzing.

2. Biror firma tomonidan qishki oyoq kiyimlarini ishlab chiqarish hajmi $u=\frac{1}{3}t^3-\frac{7}{2}t^2+6t+2100$ (birlik) tenglama bilan aniqlangan, bunda t–yilning

tavqim oyi. Mehnat unumdorligi hamda uning o'zgarish tempi va tezligini yil yakunida $(t=12)$ hisoblang.

3. Agar tovarning narxi $p=1{,}85.$ birlik va xarajatlar funksiyasi $C(x)=8+\frac{1}{4}x+\frac{1}{10}2x$ ko'rinishda bo'lsa, ishlab chiqaruvchi uchun optimal x_0 mahsulot hajmini aniqlang.

6-variant

1. $\begin{cases} x=\sqrt{2}\cos^3 t, \\ y=\sqrt{2}\sin^3 t \end{cases}$ funksiya grafigiga absissasi $t=\frac{\pi}{4}$ bo'lgan nuqtada o'tkazilgan urinma va normal tenglamasini tuzing.

2. Korxonaning x mahsulot birligini ishlab chiqarishga sarflagan to'la xarajatlar funksiyasi $y=f(x)$ berilgan. To'la va o'rtacha xarajatlarning elastiklik koeffitsientlari orasidagi bog'lanishni aniqlang.

3. Agar tovarning narxi $p=10{,}5$ birlik va xarajatlar funksiyasi $C(x)=10+\frac{x}{2}+\frac{x^2}{4}$ ko'rinishda bo'lsa, ishlab chiqaruvchi firma olishi mumkin bo'lgan maksimal foydani aniqlang.

7-variant

1. $x^3+y^2+4x-17=0$ funksiya grafigiga absissasi $y_0=1$ bo'lgan nuqtada o'tkazilgan urinma va normal tenglamasini tuzing.

2. Korxonadagi y ishlab chiqarish xarajatlari va x ishlab chiqarilayotgan mahsulot hajmi orasidagi bog'liqlik $y=50x-0{,}05x^3$ funksiya bilan ifodalanadi. Ishlab chiqilgan 10 birlik hajmdagi mahsulotning o'rtacha va chegaraviy sarf-xarajatlarni aniqlang.

3. Agar tovarning narxi $p=6{,}5$ birlik va xarajatlar funksiyasi $C(x)=8+\frac{x}{2}+\frac{x^3}{8}$ ko'rinishda bo'lsa, ishlab chiqaruvchi firma olishi mumkin bo'lgan maksimal foydani aniqlang.

8-variant

1. $x^2+2xy^2+3y^4=6,$ funksiya grafigiga absissasi $M_0(1;-1)$ bo'lgan nuqtada o'tkazilgan urinma va normal tenglamasini tuzing.

2. Konfetlarni sotishdan tushgan daromad $p = 50 - 0{,}05x^2$ ni tashkil etadi, bu erda x - sotilgan mahsulot hajmi (ming birl.). Agar sotilgan mahsulot hajmi: a) 10000 birlik; b) 60000 birlik bo'lsa o'rtacha va chegaraviy daromadni aniqlang.

3. Agar tovarning narxi $p = 40$ birlik va xarajatlar funksiyasi $C(x) = 2x + \frac{1}{20}e^{\frac{x}{2}}$ ko'rinishda bo'lsa, ishlab chiqaruvchi firma olishi mumkin bo'lgan maksimal foydani aniqlang.

9-variant

1. 7.121a $y = x^2 - 5x + 8$ funksiya grafigiga absissasi $x_0 = 3$ bo'lgan nuqtada o'tkazilgan urinma tenglamasini tuzing. Urinma absissa o'qi bilan qanday burchak tashkil qiladi?
2. Korxonadagi y ishlab chiqarish xarajatlari va x ishlab chiqarilayotgan mahsulot hajmi orasidagi funksiya $y = 100x - 0{,}2x^3$ ko'rinishga ega. Ishlab chiqilgan 10 birlik hajmdagi mahsulotning o'rtacha va chegaraviy sarf-xarajatlarni aniqlang.
3. Firmaning ishlab chiqargan x birlik tovarini narxi $p(x) = 8 - \sqrt{x}$ funksiya bilan aniqlanadi. Agar xarajatlar funksiyasi $C(x) = 10 + x + \frac{x^2}{2}$ ko'rinishda bo'lsa, firma uchun optimal bo'lgan x_0 mahsulot hajmini aniqlang.

10-variant

1. $y = \ln(1 - x)$ funksiya grafigiga absissasi $x_0 = 0$ bo'lgan nuqtada o'tkazilgan urinma tenglamasini tuzing. Urinma absissa o'qi bilan qanday burchak tashkil qiladi?
2. Brigadaning mehnat unumdorligini $y = -2{,}5t^2 + 15t + 100$ tenglama bilan ifodalash mumkin, bu erda $0 \le t \le 8$- ish vaqti (soatlarda). Mehnat unumdorligining $t = 2$ va $t = 7$ bo'lgandagi o'zgarish tezligini va tempini hisoblang.
3. Firmaning ishlab chiqargan x birlik tovarini narxi $p(x) = 10 - \frac{4}{3}\sqrt{x}$ funksiya bilan aniqlanadi. Agar xarajatlar funksiyasi $C(x) = 10 + (x-1)^3$

ko'rinishda bo'lsa, firma uchun optimal bo'lgan x_0 mahsulot hajmini aniqlang.

11-variant

1. $y=\frac{2x+3}{x+4}$ funksiya grafigiga absissasi $M_0(6,2)$ bo'lgan nuqtada o'tkazilgan urinma tenglamasini tuzing.

2. Televizor ishlab chiqarishning y tannarxi (ming so'm hisobida) $y=0{,}01x^2-0{,}5x+12$ $(5\le x\le 50)$ funksiya bilan berilgan, bu erda x - bir oyda ishlab chiqarilgan mahsulot hajmi (ming.birl.). Agar korxona bir oyda 20 va 40 ming birlik mahsulot ishlab chiqargan bo'lsa, uning tannarxini o'zgarish tezligi va tempini aniqlang.

3. Firmaning ishlab chiqargan x birlik tovarini narxi $p(x)=8-\frac{x}{2}$ funksiya bilan aniqlanadi. Agar xarajatlar funksiyasi $C(x)=\frac{x}{2}+\frac{x^3}{8}$ ko'rinishda bo'lsa, firma uchun optimal bo'lgan x_0 mahsulot hajmini aniqlang.

12-variant

1. $y=x^2+5x-1$ va $y=x^2+4$ egri chiziqlar orasidagi burchakni toping.

2. Biror mamlakatning iste'mol funksiyasi quyidagi $C(x)=13+0{,}25x+0{,}37x^{\frac{4}{5}}$ ko'rinishga ega bo'lsin, bu erda $x-$umumiy milliy daromad. Agar milliy daromad 32 (shartli birlik) ni tashkil etsa: a) iste'molga bo'lgan chegaraviy moyillik; b) jamg'armaga bo'lgan chegaraviy moyillikni toping.

3. Firma tovarning x birligi uchun fiksirlangan $p=380$ narxni o'rnatdi. x birlikdagi tovarni ishlab chiqarishdagi xarajatlar $C(x)=292x+x^2$ ga teng. Bunda sotilayotgan $K(x)$ tovarning miqdori x ga quyidagicha bog'liq: $K(x)=x+\left(\sqrt{x_0}-\sqrt{x}\right)$. Firmaning maksimal foyda oladigan x ning qiymatini aniqlang.

13-variant

1. $y = x^3$ va $y = \frac{1}{x^2}$ egri chiziqlar orasidagi burchakni toping.

2. Biror mamlakatning jamg'arma funksiyasi quyidagi ko'rinishga ega: $S(x) = 25 - 0.53x - 0,41x^{\frac{2}{3}}$, bu erda x – umumiy milliy daromad. Agar milliy daromad 27 (shartli birlik) ni tashkil etsa: a) iste'molga bo'lgan chegaraviy moyillik; b) jamg'armaga bo'lgan chegaraviy moyillikni toping.

3. Firma x fiksirlangan birlikda tovar ishlab chiqaradi va tovar birligiga $p > p_0$ narxni belgilaydi. Sotilgan K tovar miqdori p ga quyidagicha bog'liq (p_0 - barcha tovarlar sotiladigan narx): $K(p) = xe^{p_0 - p}$ $(p_0 < 1)$.

Firmaning maksimal foyda oladigan p ning qiymatini aniqlang.

14-variant

1. $x^2 + 4y^2 = 9$ va $y^2 = 2x$ egri chiziqlar orasidagi burchakni toping.

2. Ishlab chiqarilayotgan mahsulotning hajmiga bog'liq bo'lgan to'la xarajatlar funksiyasi $y = x^3 - 2x^2 + 96$ munosabat bilan berilgan. Mahsulot qanday hajmda ishlab chiqarilganda chegaraviy va o'rtacha xarajatlar ustma-ust tushadi? Berilgan hajmda to'la va o'rtacha xarajatlarning elastiklik koeffitsientlarini toping.

3. Firma x fiksirlangan birlikda tovar ishlab chiqaradi va tovar birligiga $p > p_0$ narxni belgilaydi. Sotilgan K tovar miqdori p ga quyidagicha bog'liq (p_0 –barcha tovarlar sotiladigan narx): $K(p) = \frac{x}{(1 + p - p_0)^2}$ $(p_0 < \frac{1}{2})$.

Firmaning maksimal foyda oladigan p ning qiymatini aniqlang.

15-variant

1. $y = x^2$ funksiya grafigiga absissasi $M_0(1;1)$ bo'lgan nuqtada o'tkazilgan urinma tenglamasini tuzing.

2. Tayyor mahsulot ishlab chiqarish hajmi y (mln.so'm) va ishlab chiqarish fondlarning hajmi x (mln.so'm) orasidagi bog'liqlik $y = 0,6x - 4$ tenglama

bilan ifodalanadi. Agar korxona 40 mln. so‘m hajmida fondlarga ega bo‘lsa, uning mahsulot ishlab chiqarish elastikligini toping.

3. Boshlang‘ich bosqichda firma o‘rtacha xarajatlarni minimallashtirdi va u $C(x)=10+2x+\frac{5}{2}x^2$ ko‘rinishga ega bo‘ldi. Keyinchalik tovarning bir birligiga $p=37$ shartli birlikka teng bo‘lgan narx belgilandi. Firma ishlab chiqarishni qancha birlika orttirishi kerak? Bunda o‘rta xarajatlar qanchaga o‘zgaradi?

16-variant

1. $y=\ln x$ funksiya grafigiga absissasi $M_0(1;0)$ bo‘lgan nuqtada o‘tkazilgan urinma tenglamasini tuzing.

2. Mahsulot birligining tannarxi y (so‘mlarda) va mahsulotni ishlab chiqarish x (mln.so‘mda) orasidagi bog‘liqlik $y=-0,5x+80$ tenglama bilan ifodalanadi. 30 mln.so‘mlik mahsulot ishlab chiqarishdagi tannarxning elastikligini toping.

3. Xarajatlar funksiyasi $C(x)=40x+0,08x^3$ ko‘rinishga ega. Mahsulotning bir birligini sotishdan tushgan foyda 200 ga teng. Ishlab chiqaruvchi uchun optimal bo‘lgan ishlab chiqarish mahsuloti hajmini toping.

17-variant

1. $y=x^3$ funksiya grafigiga absissasi $x_0=-2$ bo‘lgan nuqtada o‘tkazilgan urinma va normal tenglamasini tuzing.

2. Quyida p narxda berilgan talab funksiyasi tannarxining elastikligini toping: $q+10p=50,\ p=3$.

3. Ishlab chiqarilgan mahsulotning V hajmi x kapital xarajatlarga bog‘liqligi $V(x)=\frac{3}{4}\ln(1+x^3)$ funksiya bilan aniqlanadi. Kapital xarajatlarni ortishi samarasiz bo‘lgan x ning intervali topilsin.

18-variant

1. $y^2=4x$ funksiya grafigiga absissasi $M_0(1;2)$ bo‘lgan nuqtada o‘tkazilgan urinma tenglamasini tuzing.

2. Berilgan $2p+3q=12$; talab funksiyasi uchun talab elastik bo'ladigan p ning qiymaini toping.

3. Firma sotayotgan mahsulotning x hajmi va uning bir birligini p narxi o'rtasidagi munosabat $x = x_0\left(\sqrt{\frac{p_0}{p}-1}\right)$, $(p < p_0)$ funksiya bilan aniqlanadi. Firma eng ko'p foyda oladigan narxning p qiymatini toping.

19-variant

1. $y=\frac{1}{x}$ egri chiziqning qaysi nuqtasida o'tkazilgan urinma $y=-\frac{1}{4}x+3$ to'g'ri chiziqqa parallel bo'ladi?

2. x narxga bog'liq bo'ladigan $q=10-x$, talab va $s=3x-6$ taklif funksiyalari berilgan. Bu funksiyalar uchun: a) muvozanat baho; b) muvozanat narx uchun talab va taklif elastikligini; v) muvozanat narx 5% ga o'zgarganda daromad qanchaga o'zgarishini aniqlang.

3. Resurslarning x birligidan foydalanib ishlab chiqarilgan mahsulotdan tushgan daromad $400\sqrt{x}$ kattalikni tashkil etadi. Resurslar birligining bahosi 10 shartli birlikni tashkil etadi. Daromad eng katta bo'lishi uchun resurslarning qanday miqdori zarur.

20-variant

1. $y=\frac{8}{x}$ va $x^2-y^2=12$ egri chiziqlar orasidagi burchakni toping.

2. Berilgan $q=50(15-\sqrt{p})$ talab funksiyasi uchun talab elastik bo'ladigan p ning qiymaini toping.

3. Xarajatlar funksiyasi $C(x)=x+0{,}1x^2$ ko'rinishga ega. Maxsulotning bir birligini sotishdan tushgan daromad 50 ga teng. Ishlab chiqaruvchi olishi mumkin bo'lgan daromadning maksimal qiymatini toping.

21-variant

1. $y=e^x$ funksiya grafigiga absissasi $x_0=0$ bo'lgan nuqtada o'tkazilgan urinma tenglamasini tuzing.

2. Berilgan $q=\frac{1}{3}(100-5p)$ talab funksiyasining talab elastik bo'ladigan p mahsulot birligi narxini toping.

3. Ishlab chiqarilayotgan mahsulotning x hajmi bilan firma daromadi o'rtasidagi munosabat $D(x)=100x-1000\sqrt{x}$ $(400 \le x \le 900)$ funksiya kabi aniqlandi. Bu oraliqdagi xarajatlar funksiyasi $C(x)=50x+\frac{4}{5}x\sqrt{x}$ ko'rinishga ega. Ishlab chiqaruvchi uchun optimal bo'lgan mahsulot hajmini aniqlang.

22-variant

1. $y=\sin x$ funksiya grafigiga absissasi $x_0=\frac{\pi}{3}$ bo'lgan nuqtada o'tkazilgan urinma tenglamasini tuzing.

2. Quyida p narxda berilgan talab funksiyasi tannarxining elastikligini toping: $5q+3p=70$, $p=10$.

3. Ishlab chiqarilgan mahsulotning narxi $p=p_0\cdot(1-0,2\sqrt{x})$ munosabatga mos holda o'rnatiladi. Mahsulot qancha hajmda ishlab chiqarilsa uni sotishdan tushgan daromad eng katta bo'ladi.

23-variant

1. $y=\ln x$ egri chiziqning qaysi nuqtasida o'tkazilgan urinma $y=2x+5$ to'g'ri chiziqqa parallel bo'ladi?

2. Quyida p narxda berilgan talab funksiyasi tannarxining elastikligini toping: $p^2+p+4q=26$, $p=2$ va $p=4$.

3. Firmaning $C(x)$ xarajatlar funksiyasi quyidagi ko'rinishga ega: $C(x)=2x$, $x\le 100$ da va $C(x)=200+p(x-100)^2$, $x>100$ bo'lganda. Hozirgi paytda mahsulot ishlab chiqarish darajasi $x=200$ ga teng. Agar mahsulot birligini sotishdan tushgan daromad 50 ga teng bo'lsa, firmaga p parametrning qanday shartida mahsulot ishlab chiqarishni kamaytirish foydali?

24-variant

1. $y=\ln x$ egri chiziqning qaysi nuqtasida o'tkazilgan urinma $y=x+\sqrt{3}$ to'g'ri chiziqqa parallel bo'ladi?

2. Berilgan $q=\frac{1}{7}(80-4p)$ talab funksiyasining talab elastik bo'ladigan p mahsulot birligi narxini toping.

3. Xalq iste'molining ba'zi tovarlariga bo'lgan q talab uning p narxi bilan $q=\frac{6000}{\sqrt{p}}-40$ munosabatta. p ning qanday qiymatida talab neytral (elastiklik birl.) bo'lishini aniqlang.

25-variant

1. $y^2=2x$ va $x^2+y^2=8$ egri chiziqlar orasidagi burchakni toping.

2. Berilgan $q=\frac{1}{5}(20-2p)$. talab funksiyasining talab elastik bo'ladigan p mahsulot birligi narxini toping.

3. Sigaret ishlab chiqarishning y xarajatlari va ulardagi x zararli moddalarning foizli tarkibi orasidagi bog'liqlik $y=\frac{10000}{x}-100$ funksiya bilan ifodalanadi. Agar zararli moddalar miqdori 10 % ni tashkil etsa, ishlab chiqarishning o'rtacha va chegaraviy xarajatlarini toping.

3-topshiriq

1-2-misollarda berilgan funksiyalarning limitini Lopital qoidasi yordamida toping.

3-misolda funksiyalarni to'la tekshiring va ularning grafigini yasang. Natijani Mathcad dasturida tekshiring.

4-misolda funksiyaning ekstremumini tekshiring.

1-variant

1. $\lim\limits_{x\to 0}\frac{x-arctgx}{x^3}$.

2. $\lim\limits_{x\to 0}\frac{\text{arctg}\,x-x}{x^3}$.

3. $y=x^3-6x$.

4. $f(x,y)=x^2-xy+y^2+9x-6y+20$.

2-variant

1. $\lim\limits_{x\to 0}\frac{e^{x^2}-1}{\cos x-1}$.

2. $\lim\limits_{x\to 0}\frac{e^{x^3}-1}{\sin^3 x}$.

3. $y=\frac{x^4}{4}-2x^2$.

4. $f(x,y)=y\sqrt{x}-y^2-x+6y$.

3-variant

1. $\lim\limits_{x\to 0}\frac{e^x-e^{-x}-2x}{x-\sin x}$.

2. $\lim\limits_{x\to 0}\frac{\operatorname{ctg}\frac{\pi x}{2}}{\ln(x-2)}$.

3. $y=\frac{x-1}{x+1}$.

4. $f(x,y)=x^3+8y^3-6xy+1$.

4-variant

1. $\lim\limits_{x\to 1}\left(\frac{1}{\ln x}-\frac{x}{\ln x}\right)$.

2. $\lim\limits_{x\to +\infty}\frac{\log_2 x}{2^x}$.

3. $y=\frac{x^2}{x^2-1}$.

4. $f(x,y)=2xy-4x-2y$.

5-variant

1. $\lim\limits_{x\to 0}\left(ctgx-\frac{1}{x}\right)$.

2. $\lim\limits_{x\to \infty}\frac{x^3-x}{5x^3+x^2-7x+3}$.

3. $y=xe^{-x}$.

4. $f(x,y)=e^{\frac{x}{2}}\left(x+y^2\right)$.

6-variant

1. $\lim\limits_{x\to 0}\left(x^2e^{1/x^2}\right)$.

2. $\lim\limits_{x\to +\infty} x\cdot\sin\frac{1}{x}$

3. $y=\frac{x^2+1}{x}$.

4. $f(x,y)=3x+6y-x^2-xy-y^2$.

7-variant

1. $\lim\limits_{x\to \frac{\pi}{2}}(tgx)^{2x-\pi}$.

2. $\lim\limits_{t\to \frac{\pi}{2}}\left(t-\frac{\pi}{2}\right)\operatorname{tg} t$.

3. $y=\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$.

4. $f(x,y)=x^2+y^2-2x-4\sqrt{xy}-2y+8$.

8-variant

1. $\lim_{x\to 0} x\left(e^{1/x^2}-1\right)$.

2. $\lim_{x\to 0} x\ln \operatorname{ctg} x$.

3. $y=\frac{x^3}{x^2+1}$.

4. $f(x,y)=2x^3-xy^2+5x^2+y^2$.

9-variant

1. $\lim_{x\to 0+0}\left(\ln\frac{1}{x}\right)^{x}$.

2. $\lim_{x\to\frac{\pi}{2}}\left(\frac{1}{\cos x}-\frac{1}{\pi-2x}\right)$.

3. $y=\frac{x^2}{x-1}$.

4. $f(x,y)=3x^2-2x\sqrt{y}+y-8x+8$.

10-variant

1. $\lim_{x\to 1-0}\ln x\cdot\ln(1-x)$.

2. $\lim_{\alpha\to 0}\left(ctg^2\alpha-\frac{1}{\alpha^2}\right)$.

3. $y=e^{-x^2}$.

4. $f(x,y)=e^{-x^2-y^2}\left(2x^2+y^2\right)$.

11-variant

1. $\lim_{x\to+\infty}\left(\frac{1}{\pi}arctgx\right)^{x}$.

2. $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\operatorname{arctg} x}\right)$.

3. $y=\frac{2x}{1+x^2}$.

4. $f(x,y)=x^2+xy+y^2-2x-3y+5\frac{2}{3}$.

12-variant

1. $\lim_{x\to 0}\left(\frac{\arcsin x}{x}\right)^{1/x^2}$.

2. $\lim_{x\to\infty}\left(1+2^{x}\right)^{\frac{1}{x}}$.

3. $y=x^2(x-4)^2$.

4. $f(x,y)=-x^2+xy-y^2-9x+3y-20$.

13-variant

1. $\lim\limits_{x\to 0}\left(\frac{arctgx}{x}\right)^{1/x^2}$.

2. $\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{3}{x}}$.

3. $y=\frac{2x}{2+x^3}$.

4. $f(x,y)=-x^2+xy-y^2-9y+6x-35$.

14-variant

1. $\lim\limits_{x\to 0}\frac{\ln\sin 2x}{\ln\sin 3x}$.

2. $\lim\limits_{x\to 0}(1-x)^{\frac{1}{x}}$.

3. $y=(x+1)e^{-x}$.

4. $f(x,y)=6x^2-7y+2y^2+6x-3y$.

15-variant

1. $\lim\limits_{x\to 2}\frac{x^3+x+10}{x^3-3x-2}$.

2. $\lim\limits_{x\to 0}\frac{e^x-1}{x}$.

3. $y=xe^{-\frac{x^2}{2}}$.

4. $f(x,y)=4x^2-5xy+3y^2-9x-8y$.

16-variant

1. $\lim\limits_{x\to 1}\frac{\ln x}{x-1}$.

2. $\lim\limits_{x\to 0}\frac{e^x-e^{-x}}{\ln(1+x)}$.

3. $y=\frac{\ln x}{x}$.

4. $f(x,y)=\frac{x^3}{3}-xy^2+\frac{x^2}{2}-3xy-2x+y^2+3y$.

17-variant

1. $\lim\limits_{x\to 0}\frac{e^x-1}{\sin x}$.

2. $\lim\limits_{x\to 0}x\ln x$.

3. $y=\frac{1}{\sin x+\cos x}$.

4. $f(x,y)=2x^3+2y^3-36x+10$.

18-variant

1. $\lim\limits_{x\to +\infty}\frac{\ln x}{x}$.

2. $\lim\limits_{x\to 1}\frac{x^3-3x^2+2}{x^3-4x^2+3}$.

3. $y=\frac{1}{\sqrt[3]{x+1}}+\frac{1}{\sqrt[3]{x-1}}$.

4. $f(x,y)=14x^3+27xy^2-69x-54y$.

19-variant

1. $\lim\limits_{x\to+\infty} x^2\cdot e^{-x}$.

2. $\lim\limits_{x\to\infty}\frac{\ln x}{x}$.

3. $y=\sqrt[3]{x+1}-\sqrt[3]{x-1}$.

4. $f(x,y)=x^4+y^4-2x^2+4xy-2y^2$.

20-variant

1. $\lim\limits_{x\to0}\left(\frac{1}{x}-\frac{1}{sinx}\right)$.

2. $\lim\limits_{x\to\infty}\frac{e^x}{x^2}$.

3. $y=\ln\left(x+\sqrt{x^2+1}\right)$.

4. $f(x,y)=x^3y^2(12-x-y)$.

21-variant

1. $\lim\limits_{x\to\infty} x\left(e^{\frac{1}{x}}-1\right)$.

2. $\lim\limits_{x\to1}\frac{\ln(x-1)}{ctg\pi x}$.

3. $y=\frac{x}{\sqrt[3]{(x^2+1)}}$.

4. $f(x,y)=x^3+y^2-6xy-39x+18y+20$.

22-variant

1. $\lim\limits_{x\to1}\left(\frac{1}{1-x^3}-\frac{1}{1-x^2}\right)$.

2. $\lim\limits_{x\to0}\frac{x-arctgx}{x^3}$.

3. $y=\sin x+\cos^2 x$.

4. $f(x,y)=2xy-3x^2-2y^2+10$.

23-variant

1. $\lim\limits_{x\to1}\frac{x^{10}-2x+1}{x^{20}-4x+3}$.

2. $\lim\limits_{x\to\frac{\pi}{6}}\frac{1-2\sin x}{\cos 3x}$.

3. $y=\frac{1}{1-e^x}$.

4. $f(x,y)=4(x-y)-x^2-y^2$.

24-variant

1. $\lim\limits_{x\to0}\frac{\sin 5x}{\sqrt{x+1-1}}$.

2. $\lim_{x\to 1}\frac{tg\left(\frac{\pi x}{2}\right)}{\ln(1-x)}$.

3. $y=\sqrt[3]{1-\ln x}$.

4. $f(x,y)=x^2+xy+y^2+x+y+1$.

25-variant

1. $\lim_{x\to 0}\frac{e\ -e^{-x}}{\sin 2x}$

2. $\lim_{x\to 0}\frac{\ln(1-x)}{\sin x}$.

3. $y=xe^{\frac{1}{x}}$.

4. $f(x,y)=4x^2y+24xy+y^2+32y-6$.

6.5. Mathcad dasturida hisoblash

Mathcad da differensiallash amali sonli va analitik shaklda amalga oshiriladi.

Analitik differensiallash

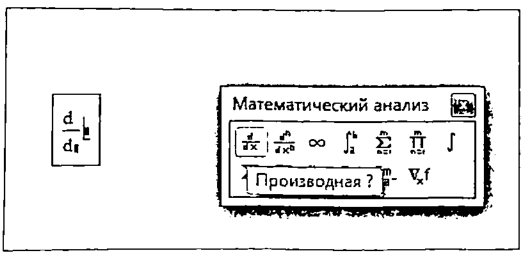
Mathcad da $f(x)$ funksiyaning hosilasini analitik topish uchun:

1. $f(x)$ funksiyani bering.

2. **Математический анализ** (Calculus) panelidan **Производная** (Derivative) tugmasini bosib yoki klaviaturadan so'roq belgisini <?> kiritib differensiallash operatorini kiriting.

3. Differensiallash operatorida o'rinto'ldirgichlarga $f(x)$ funksiyani va funksiya argumuntini kiriting.

4. Javobni olish uchun simvolli hisoblash operatori <→>ni kiriting(1-misol).



1-rasm. **Differensiallash operatori**

1-misol. Analitik differensiallash.

$f(x) := \ln(x)\cdot\cos(x)$

$$\frac{d}{dx}f(x) \rightarrow \frac{\cos(x)}{x} - \ln(x)\cdot\sin(x)$$

Funksiyaning nuqtadagi hosilasini hisoblash

Funksiyaning nuqtadagi hosilasini hisoblash uchun argumentning shu nuqtadagi qiymatini kiritish kerak. Ushbu holatda differensiallash natijasi son – hosilaning nuqtadagi qiymati bo‘ladi. Agar natija analitik ko‘rinishda bo‘lsa, sonli ifodasini olish uchun hosil bo‘lgan ifodadan so‘ng <=> simvolini kiritish etarli.

2-misol. Funksiyani nuqtada analitik differensiallash.

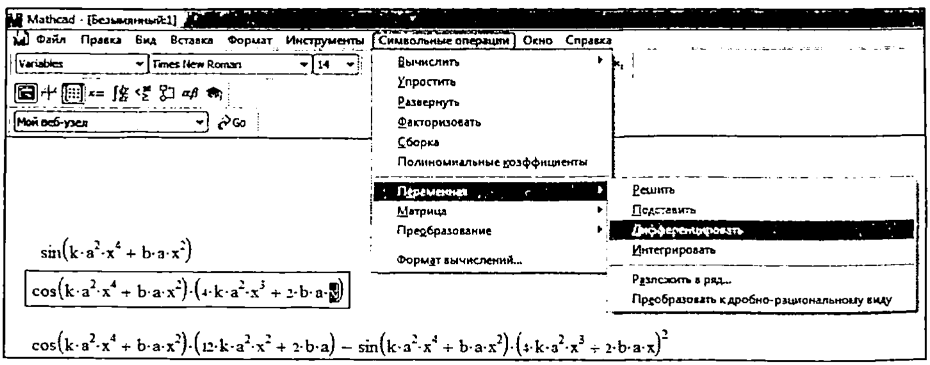
$$f(x) := \sin(x)\cdot\ln(x)$$

$$x := 2$$

$$\frac{d}{dx}f(x) \rightarrow \frac{\sin(2)}{2} + \cos(2)\cdot\ln(2) = 0.166$$

Menyu yordamida differensiallash

Biror o‘zgaruvchili ifodani analitik differensiallash uchun, o‘zgaruvchini belgilab Символьные опеpatsии / Переменная / Дифференцировать (Sumbolics / Variable / Differentiate) buyrug‘i tanlanadi. Ikkinchi tartibli hosilani topish uchun yuqoridagi amallar ketma-ketligi takroran qo‘llaniladi. (1-rasm).



1-rasm. **O‘zgaruvchi bo‘yicha analitik differensiallash**

Sonli differensiallash

Mathcad hisoblash protsessori sonli differensiallashda juda yaxshi aniqlikni ta'minlaydi.

Nuqtada differensiallash

1. Hosilani hisoblash uchun x nuqtani aniqlang, masalan, $x=0,1$.
2. Differensiallash operatorini kiriting va o'rinto'ldirgichlarga funksiya va argumentni kiriting.
3. Natijani sonli chiqarish uchun $=$operatorini kiriting.

3-misol. Funksiyani nuqtada sonli differensiallash.

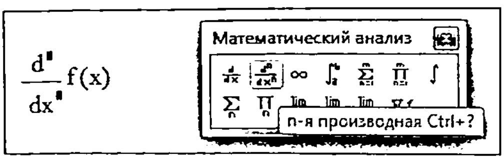
$f(x) := \sin(x)\cdot\ln(x)$

$x := 0.1$

$$\frac{d}{dx}f(x) = -1.293$$

Yuqori tatibli hosila

Mathcad yuqori tartibli hosilalarni (1- tartiblidan 5-tartibligacha) sonli aniqlaydi. Bu operator **Математический анализ** (Calculus) panelidan **Производная** (Derivative) tugmasini bosib yoki klaviaturadan <Ctrl> + <?> tugmalarini bosib kiritiladi.



Yuqori tartibli hosila operatori

4-misol. Funksiyaning nuqtadagi ikkinchi tartibli hosilani hisoblang.

$$f(x) := \frac{1}{x}$$

$$\frac{d^6}{dx^6}f(x) \rightarrow \frac{720}{x^7}$$

$$\frac{d^6}{dx^6}f(x) = \blacksquare$$

Значение должно находиться между 0 и 5.

$$f(x) := \frac{1}{x}$$

$$x := 3$$

$$\frac{d^2}{dx^2}f(x) = 0.074$$

$$\frac{d^2}{dx^2}f(x) \rightarrow \frac{2}{27}$$

5-misol. Oltinchi tartibli hosilani sonli va simvolli hisoblang.

Xususiy hosila

Xususiy hosilani topish uchun **Математический анализ** (Calculus) panelidan **Производная** (Derivative) operatori tanlanadi. 1-misolda ikki o‘zgaruvchili funksiya xususiy hosilalariga keltirilgan. Birinchi satrda funksiyaning o‘zi berilgan keyingisida x va k o‘zgaruvchilar bo‘yicha hosila olingan. Funksiyaning nuqtadagi hususiy hosilasini olish uchun barcha argumentlarning qiymatlarini berish zarur (2-misol).

6-misol. Xususiy hosilalarni analitik hisoblash.

$$f(x,k) := k \cdot \sin(x)$$

$$\frac{d}{dx}f(x,k) \rightarrow k \cdot \cos(x)$$

$$\frac{d}{dk}f(x,k) \rightarrow \sin(x)$$

7-misol. Funksiyaning nuqtadagi hususiy hosilalarini simvolli va sonli hisoblash.

$$f(x,k) := k\cdot\sin(x)$$

$$x := 10$$

$$\frac{d}{dx}f(x,k) \rightarrow k\cdot\cos(10)$$

$$k := 1$$

$$\frac{d}{dx}f(x,k) = -0.839$$

8-misol. Ikkinchi tartibli ususiy hosilani toping.

$$f(x,y) := y^2\cdot x^3 + y\cdot x^2$$

$$\frac{d^2}{dx^2}f(x,y) \rightarrow 6\cdot x\cdot y^2 + 2\cdot y$$

$$\frac{d^2}{dy^2}f(x,y) \rightarrow 2\cdot x^3$$

$$\frac{d}{dx}\left(\frac{d}{dy}f(x,y)\right) \rightarrow 6\cdot y\cdot x^2 + 2\cdot x$$

VII bob. INTEGRAL HISOB

7.1. Aniqmas integral

Boshlang'ich funksiya

Faraz qilaylik, $f(x)$va $F(x)$ funksiyalari $(a,b)\subset R$ intervalda (bu interval chekli yoki cheksiz bo'lishi mumkin) berilgan bo'lib, $F(x)$ funksiya shu $(a,b)\subset R$ da differensiallanuvchi bo'lsin.

1-ta`rif. Agar barcha $x\in[a;b]$ uchun $F'(x)=f(x)$ o'rinli bo'lsa, u holda $F(x)$ funksiya $f(x)$ funksiyaning $[a;b]$ oraliqdagi boshlang'ich funksiyasi deyiladi.

$F(x)$ funksiya $f(x)$ funksiyaning boshlang'ich funksiyasi. $F(x)$funksiyaga ixtiyoriy $C=const$ – o'zgarmas sonning qo'shilishi uning $f(x)$ – hosilasiga ta`sir qilmasligini e'tiborga olsak $f(x)$ funksiyaning boshlang'ich funksiyasini $F(x)+C$ ko'rinishda yozish mumkin.

Agar $f(x)$ funksiy berilgan intervalda uzluksiz bo'lsa, u holda shu intervalda uning boshlang'ich funksiyasi mavjud bo'ladi.

Aniqmas integral

2-ta'rif. Boshlang'ich funksiyaning $F(x)+C$ umumiy ko'rinishi berilgan $y=f(x)$ funksiyaning aniqmas integrali deyiladi.

$f(x)$ funksiyaning aniqmas integrali quyidagi ko'rinishda bo'ladi

$$\int f(x)dx=F(x)+C.$$

Bu yerda $\int$ – integral belgisi, $f(x)$ – integral osti funksiyasi, $f(x)dx$ – integral osti ifodasi deb ataladi.

Berilgan *f(x)* funksiyaning biror boshlang'ich funksiyasini va uning aniqmas integralini topish masalalari deyarli bir xil masalalardir. Shu sababli *f(x)* funksiyaning boshlang'ich funksiyasini topishni ham, aniqmas integralini topishni ham *f(x)* funksiyani integrallash deb ataymiz. Integrallash differensiallashga nisbatan teskari amaldir.

Aniqmas integral xossalari

$\int dF(x) = F(x) + C; \qquad \int F'(x)dx = F(x) + C; \qquad d\left(\int f(x)dx\right) = f(x)dx.$

$\int [f_1(x) + f_2(x)]dx = \int f_1(x)dx + \int f_2(x)dx;$

$\int Cf(x)dx = C\int f(x)dx.$

Integrallar jadvali

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1; \quad \int \frac{dx}{\sqrt{x}} = 2\sqrt{x} + C; \quad \int \frac{dx}{x^2} = -\frac{1}{x} + C.$

2. $\int \frac{dx}{x} = \ln|x| + C.$

3. $\int a^x dx = \frac{a^x}{\ln a} + C; \quad \int e^x dx = e^x + C.$

4. $\int \cos x\, dx = \sin x + C.$

5. $\int \sin x\, dx = -\cos x + C.$

6. $\int \frac{dx}{\cos^2 x} = tgx + C.$

7. $\int \frac{dx}{\sin^2 x} = -ctgx + C.$

8. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C; \ (a > 0). \int \frac{dx}{\sqrt{1 - x^2}} = \arcsin x + C.$

9. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\mathrm{arc}tg\frac{x}{a} + C; \ (a \neq 0). \int \frac{dx}{1 + x^2} = \mathrm{arc}tgx + C.$

10. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left|\frac{x - a}{x + a}\right| + C, (a > 0).$

11. $\int \frac{dx}{\sqrt{x^2 + \alpha}} = \ln\left|x + \sqrt{x^2 + \alpha}\right| + C.$

12. $\int shx\, dx = chx + C.$

13. $\int chx\, dx = shx + C.$

14. $\int tgx\, dx = -\ln|\cos x| + C.$

15. $\int ctgx\, dx = \ln|\sin x| + C.$

16. $\int \frac{dx}{\sin x} = \ln\left|tg\frac{x}{2}\right| + C.$

$$17.\int \frac{dx}{\cos x} = \ln\left|tg\left(\frac{x}{2}+\frac{\pi}{4}\right)\right| + C.$$

Integrallashning asosiy metodlari

Bevosita integrallash usuli. Bu usul integral ostidagi ifodani jadvaldagi biror integral ostidagi ifoda ko'rinishiga keltirish va aniqmas integral xossalaridan foydalanishga asoslangan.

O'zgaruvchini almashtirish usuli. Aniqmas integralni hisoblashda o'zgaruvchini almashtirish quyidagicha amalga oshiriladi: $\int f(x)dx$ integralni o'zgaruvchini alamashtirish qoidasi yordamida hisoblash kerak bo'lsin. x o'zgaruvchini t erkli o'zgaruvchining biror differensiallanuvchi funksiyasi orqali ifodalaymiz: $x=\varphi(t)$, bu yerda $t=\psi(x)$ teskari funksiya mavjud bo'lsin deb faraz qilinadi, u holda $dx=\varphi'(t)dt$ bo'lgani uchun

$$\int f(x)dx = \int f(\varphi(t))\varphi'(t)dt.$$

Bo'laklab integrallash usuli. Ma`lumki, uv ko'paytmaning differensiali

$$d(uv) = udv + vdu$$

formula bilan hisoblanadi. Bu formulaning ikkala tomonini ham integrallaymiz. U holda

$$\int d(uv) = \int udv + \int vdu \Rightarrow uv = \int vdu + \int udv \Rightarrow \int udv = uv - \int vdu.$$

bo'laklab integrallash formulasi deyiladi. Bu yerda u, v – differensiallanuvchi funksiyalar.

Bo'laklab integrallash formulasini aniqmas integralga qo'llash uchun, integral ostidagi ifoda ikki qismga ajratiladi va birinchi qismini u, ikkinchi qismini esa dv deb olinadi. So'ngra birinchi u ifodani differensiallab du ifodani, ikkinchi dv ifodani integrallab $\int vdu$ integralni hosil qilamiz.

Amaliyotda tez-tez uchrab turadigan va bo'laklab integrallash usuli bilan hisoblanadigan integrallar tiplarini keltiramiz.

$$\int R(x)\ln x dx \;\rightarrow\; u=\ln x,\; dv=R(x)dx;$$

$$\int R(x)arctgx dx \;\rightarrow\; u=arctgx,\; dv=R(x)dx;$$

$$\int R(x)e^x dx \rightarrow u = R(x),\ dv = e^x dx;$$
$$\int R(x)\sin x dx \rightarrow u = R(x),\ dv = \sin x dx;$$
$$\int R(x)\cos x dx \rightarrow u = R(x),\ dv = \cos x dx,$$

Bu yerda $R(x)$ – ratsional funksiya (ko‘phad).

Ratsional funksiyalarni integrallash. $\frac{P(x)}{Q(x)}$ ratsional funksiya integralini hisoblash talab etilsin. Bu yerda $P(x)$ va $Q(x)$ x o‘zgaruvchidan iborat bo‘lgan biror ko‘pxadlar.

Agar suratdagi $P(x)$ ko‘phadning darajasi maxrajdagi $Q(x)$ ko‘phadning darajasidan katta bo‘lsa, unda $P(x)$ ko‘phadni $Q(x)$ ko‘phadga bo‘lish bilan $\frac{P(x)}{Q(x)}$ ning butun qismini ajratib, butun ratsional funksiya hamda to‘g‘ri kasr yig‘indisi ko‘rinishida ifodalab olinadi:

$$\frac{P(x)}{Q(x)} = R(x) + \frac{P_1(x)}{Q(x)}.$$

Ravshanki,

$$\int \frac{P(x)}{Q(x)}dx = \int R(x)dx + \int \frac{P_1(x)}{Q(x)}dx.$$

Demak, $\frac{P(x)}{Q(x)}$ ratsional funksiyani integrallash ko‘phad va to‘g‘ri kasrni integrallashga keladi. To‘g‘ri kasrni integrallash uchun avval bu kasrni sodda kasrlar orqali ifodalab olinadi, so‘ngra ular integrallanadi.

$\frac{P(x)}{Q(x)}$ ni to‘g‘ri kasr deb qabul qilamiz. Suratdagi $P(x)$ ko‘phadning darajasi maxrajdagi $Q(x)$ ko‘phadning darajasidan kichik.

$Q(x)$ maxrajni chiziqli ko‘paytuvchilarga ajratamiz, ya’ni

$$Q(x) = (x-\alpha_1)^{k_1}(x-\alpha_2)^{k_2}....(x-\alpha_l)^{k_l}$$

bu yerda $i \neq j$ da $\alpha_i \neq \alpha_j$ $k_1, k_2, ..., k_l$ – musbat butun sonlar.

U holda $\frac{P(x)}{Q(x)}$ kasr sodda kasrlar yig‘indisiga keltiriladi.

$$\frac{P(x)}{Q(x)}=\frac{A_{11}}{(x-\alpha_1)}+\frac{A_{12}}{(x-\alpha_1)^2}+...+\frac{A_{1k_1}}{(x-\alpha_1)^{k_1}}+$$

$$+...+\frac{A_{l1}}{x-\alpha_l}+...+\frac{A_{lk_l}}{(x-\alpha_l)^{k_l}}.$$

bu yerda $A_{11}, A_{12}, ..., A_{lk_l}$ - biror noma'lum sonlar. Shuning uchun integrallashning ko'rib chiqilayotgan metodi aniqmas koeffitsiyentlar metodi deyiladi.

Agar $Q(x)$ ko'phad chiziqli ko'paytuvchilarga yoyilmasa ($Q(x)$ kompleks ildizlarga ega), u holda $\frac{P(x)}{Q(x)}$ kasr quyidagi sodda kasrlar yig'indisi ko'rinishida yoziladi:

$$\frac{B_1x+C_1}{x^2+px+q}+\frac{B_2x+C_2}{\left(x^2+px+q\right)^2}+...+\frac{B_mx+C_m}{\left(x^2+px+q\right)^m}.$$

Sodda kasrlarni integrallash. Sodda kasrlarning aniqmas integrallarini hisoblaymiz.

1). $\frac{A}{x-a}$ sodda kasrning aniqmas integrali.

$$\int\frac{A}{x-a}dx=A\int\frac{d(x-a)}{x-a}=A\ln|x-a|+C.$$

2). $\frac{A}{(x-a)^m}$ $(m>1)$ sodda kasrning aniqmas integrali ham tez hisoblanadi:

$$\int\frac{Adx}{(x-a)^m}=A\int\frac{d(x-a)}{(x-a)^m}=A\int(x-a)^{-m}d(x-a)=\frac{A}{1-m}\cdot\frac{1}{(x-a)^{m-1}}+C.$$

3). $\frac{Bx+C}{x^2+px+q}$ sodda kasrning (bunda x^2+px+q kvadrat uchhad haqiqiy ildizga ega emas) integrali $\int\frac{Bx+C}{x^2+px+q}dx$ ni hisoblash uchun avval kasrning mahrajida turgan x^2+px+q kvadrat uchhadni ushbu

$$x^2+px+q=\left(x+\frac{p}{2}\right)^2+q-\frac{p^2}{4}$$

ko'rinishda yozib olamiz. U holda

$$\int\frac{Bx+C}{x^2+px+q}dx=\int\frac{Bx+C}{\left(x+\frac{p}{2}\right)^2+a^2}dx$$

bo'ladi, bunda $a^2=q-\frac{p^2}{4}$. Bu integralda $x+\frac{p}{2}=t$ almashtirishni bajaramiz:

$$\int\frac{Bx+C}{x^2+px+q}dx=B\int\frac{tdt}{t^2+a^2}+(C-\frac{Bp}{2})\int\frac{dt}{t^2+a^2}=\frac{B}{2}\int\frac{d(t^2+a^2)}{t^2+a^2}+$$

$$+\left(C-\frac{Bp}{2}\right)\frac{1}{a}\int\frac{d(\frac{t}{a})}{1+(\frac{t}{a})^2}=\frac{B}{2}\ln(t^2+a^2)+\left(C-\frac{Bp}{2}\right)\frac{1}{a}arctg\frac{t}{a}+C_1=$$

$$=\frac{B}{2}\ln(x^2+px+q)+\frac{2C-Bp}{2\sqrt{q-\frac{p^2}{4}}}arctg\frac{x+\frac{p}{2}}{\sqrt{q-\frac{p^2}{4}}}+C_1\,.$$

Demak,

$$\int\frac{Bx+C}{x^2+px+q}dx=\frac{B}{2}\ln(x^2+px+q)+\frac{2C-Bp}{2\sqrt{q-\frac{p^2}{4}}}arctg\frac{x+\frac{p}{2}}{\sqrt{q-\frac{p^2}{4}}}+C_1$$

bunda C_1 ixtiyoriy o‘zgarmas.

4). $\frac{Bx+C}{(x^2+px+q)^m}$ $(m>1)$ sodda kasrning integrali $J_m=\int\frac{Bx+C}{(x^2+px+q)^m}dx$ ni hisoblash uchun 3-holdagidek o‘zgaruvchini almashtiramiz: $x+\frac{p}{2}=t$. Natijada quyidagiga ega bo‘lamiz:

$$J_m=\int\frac{Bx+C}{(x^2+px+q)^m}dx=\int\frac{Bx+C}{\left(\left(x+\frac{p}{2}\right)^2+p-\frac{p^2}{4}\right)^m}dx=\int\frac{Bt+\left(C-\frac{Bp}{t}\right)}{(t^2+a^2)^m}dt=$$

$$=\frac{B}{2}\int\frac{d(t^2+a^2)}{(t^2+a^2)^m}+\left(C-\frac{Bp}{2}\right)\int\frac{dt}{(t^2+a^2)^m}=\frac{B}{2}\cdot\frac{1}{1-m}\cdot\frac{1}{(t^2+a^2)^{m-1}}+\left(C-\frac{Bp}{2}\right)\int\frac{dt}{(t^2+a^2)^m}.$$

Bu munosabatdagi $\int\frac{dt}{(t^2+a^2)^m}$ integral rekurrent formula orqali hisoblanadi.

Sodda irratsional funksiyalarni integrallash

$\int R(x,\sqrt[n_1]{x^{m_1}},\sqrt[n_2]{x^{m_2}},...,\sqrt[n_k]{x^{m_k}})dx$ $(m_1,n_1,m_2,n_2,...,m_k,n_k$ - butun sonlar) ko‘rinishdagi integrallar

Bu integral $x=t^s$ almashtirish natijasida ratsional funksiya integraliga keltiriladi. Bu yerda $s-$ $n_1,n_2,...,n_k$ sonlarning eng kichik umumiy karralisi.

$\int R\left(x,\left(\frac{ax+b}{cx+d}\right)^{\alpha_1},...,\left(\frac{ax+b}{cx+d}\right)^{\alpha_n}\right)dx$ ko‘rinishdagi integral.

Bu integralda R-o'z argumentlarining ratsional funksiyasi, a,b,c,d lar haqiqiy sonlar va $\alpha_1, \alpha_2, ..., \alpha_n$- ratsional sonlar bo'lib, bu kasrlarning umumiy maxraji m va $ad-bc \neq 0$ bo'lsin.

Quyidagi

$$t = \sqrt[m]{\frac{ax+b}{cx+d}} \text{ yoki } t^m = \frac{ax+b}{cx+d}$$

almashtirishni kiritamiz. U holda

$$x = \frac{t^m d - b}{a - ct^m} \text{ va } dx = \frac{m(ad-bc)t^{m-1}dt}{(a-ct^m)^2}$$

bo'ladi. Natijada, berilgan t ga nisbatan ratsional funksiyani integrallashga keltiriladi.

Binomial differensiallarni integrallash. Ushbu $x^m \cdot (a+bx^n)^p dx$ differensial ifoda binomial differensial deb ataladi. Uning integrali

$\int x^m \cdot (a+bx^n)^p dx$ berilgan bo'lsin, bunda $m, n, p-$ ratsional sonlar, a va $b-$ haqiqiy sonlar.

Binomial differensialga bog'liq quyidagi teorema o'rinli.

Teorema (P.L.Chebishev). Quyidagi uch holdagina binomial differensialning integrali elementar funksiya bo'ladi:

1-*hol.* $p-$ butun son;

2-*hol.* $p = \frac{r}{s}$ kasr son, lekin $\frac{m+1}{n}$- butun son;

3-*hol.* $p = \frac{r}{s}$ va $\frac{m+1}{n}$ - kasr sonlar, lekin $\frac{m+1}{n} + p$ - butun son.

1-holda p butun son bo'lsa, m va n kasrlarning umumiy mahraji k ni topib, $x = t^k$ almashtirish bajariladi.

2-holda $\frac{m+1}{n}$ butun son bo'lsa, $a+bx^n = t^s,\ p = \frac{r}{s}, s > 0$ almashtirish bajariladi.

$\int R\left(x\sqrt{a^2-x^2}\right)dx$, $\int R\left(x\sqrt{x^2+a^2}\right)dx$, $\int R\left(x\sqrt{x^2-a^2}\right)dx$ ko'rinishidagi integrallarni topishda (bu yerda $R-$ratsional funksiya) $x = a\sin t$, $x = atgt$, $x = \frac{a}{\cos t}$ kabi o'rin almashtirishlardan foydalaniladi.

3-holda $p=\frac{r}{s}$ va $\frac{m+1}{n} + \frac{r}{s}$ butun son bo'lganda $a+bx^n = t^s x^n$

almashtirishdan foydalanamiz. U holda quyidagi tengliklar o'rinli bo'ladi:

$$x^n = a(t^s - b)^{-1},\ x = a^{\frac{1}{n}}(t^s - b)^{-\frac{1}{n}},\ dx = -\frac{s}{n}a^{\frac{1}{n}}(t^s - b)^{-\frac{n+1}{n}}\cdot t^{s-1}dt,$$

$$x^m = a^{\frac{m}{n}}(t^s - b)^{-\frac{m}{n}},\quad a + bx^n = t^s \cdot a(t^s - b)^{-1},$$

$$x^m \cdot (a + bx^n)^p dx = a^{\frac{m}{n}}(t^s - b)^{-\frac{m}{n}} \cdot a^p (t^s - b)^{-p} \cdot t^{sp} \cdot \left(-\frac{s}{n}\right) a^{\frac{1}{n}}(t^s - b)^{-\frac{n+1}{n}} \cdot t^{s-1}dt =$$

$$= -a^{\frac{m}{n}+\frac{r}{s}+\frac{1}{n}} \cdot \frac{s}{n} \cdot (t^s - b)^{-\frac{m}{n}-\frac{r}{s}-1-\frac{1}{n}} \cdot t^{r+s-1}dt = -a^{\frac{m+1}{n}+\frac{r}{s}} \cdot \frac{s}{n} \cdot t^{r+s-1} \cdot (t^s - b)^{-1-\left(\frac{m+1}{n}+\frac{r}{s}\right)}dt.$$

Teoremaning shartiga ko'ra, $\frac{m+1}{n}+\frac{r}{s}$ - butun son. Shuning uchun masala t ga nisbatan ratsional funksiyani integrallashga keltiriladi.

Trigonometrik funksiyalarni integrallash

Quyidagi ko'rinishdagi integral berilgan bo'lsin

$$\int R(\sin x, \cos x)\, dx$$

Bu integralda

$$t = tg\frac{x}{2}$$

almashtirishni bajaramiz. Unda

$$\sin x = \frac{2\mathrm{tg}\frac{x}{2}}{1 + \mathrm{tg}^2\frac{x}{2}} = \frac{2t}{1+t^2},\qquad \cos x = \frac{1 - \mathrm{tg}^2\frac{x}{2}}{1 + \mathrm{tg}^2\frac{x}{2}} = \frac{1-t^2}{1+t^2},$$

$$x = 2\,\mathrm{arctg}\,t,\qquad dx = \frac{2\,dt}{1+t^2}$$

bo'lib,

$$\int R(\sin x, \cos x)dx = \int R\left(\frac{2t}{1+t^2}, \frac{1-t^2}{1+t^2}\right)\frac{2dt}{1+t^2}$$

bo'ladi. Bunday almashtirish universal almashtirish deyiladi.

Agar quyidagi $R(-\sin x, -\cos x) = R(\sin x, \cos x)$ tenglik o'rinli bo'lsa, $t = tgx$ almashtirish qulay. Bu almashtirishda trigonometriyadan ma'lum bo'lgan

$$\sin x = \frac{\mathrm{tgx}}{\sqrt{1+\mathrm{tg}^2\mathrm{x}}} = \frac{t}{\sqrt{1+t^2}},\qquad \cos x = \frac{1}{\sqrt{1+\mathrm{tg}^2\mathrm{x}}} = \frac{1}{\sqrt{1+t^2}},$$

$$x = \mathrm{arctg}\,t,\qquad dx = \frac{dt}{1+t^2}$$

formulalardan foydalaniladi.

Agar integrallar $\int \sin x \cdot f(\cos x)dx$ va $\int \cos x \cdot f(\sin x)dx$ ko'rinishda bo'lsa, u holda $t = \cos x$, $t = \sin x$ almashtirishlar natijasida ular t ga bog'liq ratsional funksiyaga keladi.

Misollar

Bevosita integrallash usuli

1. $\int x^6 dx = \frac{x^7}{7} + C.$

2. $\int \frac{dx}{\sqrt[3]{x}} = \int x^{-\frac{1}{3}} dx = \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + C = \frac{3\sqrt[3]{x^2}}{2} + C.$

3. $\int \frac{dx}{4 + x^2} = \int \frac{dx}{2^2 + x^2} = \frac{1}{2} arctg \frac{x}{2} + C.$

4. $\int \left(2x^3 - 3\sin x + 5\sqrt{x}\right)dx = \int 2x^3 dx - 3\int \sin x dx + 5\int x^{\frac{1}{2}} dx =$

$= 2\frac{x^4}{4} + 3\cos x + 5\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{x^4}{2} + 3\cos x + \frac{10x\sqrt{x}}{3} + C.$

5. $\int \sin\left(x + \frac{\pi}{3}\right)dx = \int \sin\left(x + \frac{\pi}{3}\right)d\left(x + \frac{\pi}{3}\right) = -\cos\left(x + \frac{\pi}{3}\right) + C.$

6. $\int (2x-6)^8 dx = \int (2x-6)^8 \frac{1}{2} d(2x-6) = \frac{1}{2}\int (2x-6)^8 d(2x-6) =$

$= \frac{1}{2}\frac{(2x-6)^9}{9} + C = \frac{(2x-6)^9}{18} + C.$

7. $\int x\sqrt{2x^2 + 7}dx = \frac{1}{4}\int \sqrt{2x^2 + 7} \cdot 4xdx = \frac{1}{4}\int \sqrt{2x^2 + 7}\, d\left(2x^2 + 7\right) =$

$= \frac{1}{4} \cdot \frac{\left(2x^2 + 7\right)^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{\sqrt{\left(2x^2 + 7\right)^3}}{6} + C.$

8. $\int \frac{\ln^2 x}{x} dx = \int \ln^2 x \cdot \frac{dx}{x} = \int \ln^2 x d(\ln x) = \frac{\ln^3 x}{3} + C.$

9. $\int \frac{x^2 dx}{\sqrt{4 - x^6}} = \int \frac{x^2 dx}{\sqrt{4 - \left(x^3\right)^2}} = \frac{1}{3}\int \frac{3x^2 dx}{\sqrt{2^2 - \left(x^3\right)^2}} = \frac{1}{3}\int \frac{d\left(x^3\right)}{\sqrt{2^2 - \left(x^3\right)^2}} = \frac{1}{3}\arcsin\frac{x^3}{2} + C.$

10. $\int xe^{x^2} dx = \int e^{x^2} \cdot xdx = \frac{1}{2}\int e^{x^2} \cdot 2xdx = \frac{1}{2}\int e^{x^2} d\left(x^2\right) = \frac{1}{2}e^{x^2} + C.$

O'zgaruvchini almashtirish usuli.

11. $\int \frac{\sqrt{x}dx}{1+2\sqrt{x}}$ integralni toping.

Yechish.

($t=1+2\sqrt{x}$ almashtirish kiritamiz; bu yerdan $x=\frac{(t-1)^2}{4}$; $dx=\frac{2(t-1)}{4}dt$)

$$\int \frac{\sqrt{x}dx}{1+2\sqrt{x}}=\int \frac{\frac{t-1}{2}\cdot\frac{2(t-1)}{4}dt}{t}=\frac{1}{4}\int \frac{(t-1)^2}{t}dt=\frac{1}{4}\int \frac{t^2-2t+1}{t}dt=$$

$$=\frac{1}{4}\int\left(t-2+\frac{1}{t}\right)dt=\frac{1}{4}\int tdt-\frac{1}{2}\int dt+\frac{1}{4}\int\frac{dt}{t}=\frac{t^2}{8}-\frac{t}{2}+\frac{1}{4}\ln|t|+C=$$

$$=\frac{\left(1+2\sqrt{x}\right)^2}{8}-\frac{1+2\sqrt{x}}{2}+\frac{1}{4}\ln\left|1+2\sqrt{x}\right|+C.$$

12. $\int \frac{\sqrt{x+4}}{x}dx$ integralni toping.

($t^2=x+4$ almashtirish kiritamiz, bu yerdan $x=t^2-4$; $dx=2tdt$, shuningdek, $t=\sqrt{x+4}$)

$$\int\frac{\sqrt{x+4}}{x}dx=\int\frac{t}{t^2-4}2tdt=2\int\frac{t^2dt}{t^2-4}=2\int\frac{t^2-4+4}{t^2-4}dt=2\int\left(1+\frac{4}{t^2-4}\right)dt=$$

$$=2\int dt+8\int\frac{dt}{t^2-4}=2t+8\cdot\frac{1}{4}\ln\left|\frac{t-2}{t+2}\right|+C=2\sqrt{x+4}+2\ln\left|\frac{\sqrt{x+4}-2}{\sqrt{x+4}+2}\right|+C.$$

$$\int\frac{dx}{\sqrt{4x-4x^2+5}}=\frac{1}{\sqrt{4}}\int\frac{dx}{\sqrt{x-x^2+\frac{5}{4}}}=\frac{1}{2}\int\frac{dx}{\sqrt{-\left(x^2-x-\frac{5}{4}\right)}}=$$

$$=\frac{1}{2}\int\frac{dx}{\sqrt{-(x-0{,}5)^2+1{,}5}}=\frac{1}{2}\int\frac{d(x-0{,}25)}{\sqrt{\left(\sqrt{1{,}5}\right)^2-(x-0{,}5)^2}}=\frac{1}{2}\arcsin\frac{x-0{,}5}{\sqrt{1{,}5}}+C.$$

13. $\int\sqrt{25-x^2}dx$ integralni toping.

($x=5\sin t$ almashtirish kiritamiz, $dx=5\cos tdt$ shuningdek $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ oraliqda $\sqrt{25-x^2}=\sqrt{25-25\sin^2 t}=5\sqrt{1-\sin^2 t}=5\cos t$)

$$\int\sqrt{25-x^2}dx=\int 5\cos t\cdot 5\cos tdt=25\int\cos^2 tdt=25\int\frac{1+\cos 2t}{2}dt=\frac{25}{2}\int dt+\frac{25}{2}\int\cos 2tdt=$$

$$=\frac{25}{2}t+\frac{25}{4}\sin 2t+C.$$

Endi x o‘zgaruvchiga qaytamiz: $t=\arcsin\frac{x}{5}$, $\sin 2t=2\sin t\cos t=2\frac{x}{5}\frac{\sqrt{25-x^2}}{5}$.

$$\int\sqrt{25-x^2}dx=\frac{25}{2}\arcsin\frac{x}{5}+\frac{x\sqrt{25-x^2}}{2}+C.$$

14. $\int\frac{dx}{3+5\cos x}$ integralni toping.

($t=tg\frac{x}{2}$ u holda $dx=\frac{2dt}{1+t^2}$, $3+5\cos x=3+5\frac{1-t^2}{1+t^2}=\frac{8-2t^2}{1+t^2}$)

$$\int\frac{dx}{3+5\cos x}=\int\frac{2dt}{\left(1+t^2\right)\frac{8-2t^2}{1+t^2}}=\int\frac{dt}{4-t^2}=\frac{1}{4}\ln\left|\frac{2+t}{2-t}\right|+C=\frac{1}{4}\ln\left|\frac{2+tg\frac{x}{2}}{2-tg\frac{x}{2}}\right|+C.$$

Bo‘laklab integrallash usuli.

15. $\int arctgxdx=xarctgx-\int\frac{x}{1+x^2}dx=xarctgx-\frac{1}{2}\int\frac{2xdx}{1+x^2}=$

$$\left[\begin{array}{l|l} u=arctgx & du=\frac{dx}{1+x^2} \\ dv=dx & v=\int dx=x \end{array}\right]$$

$$=xarctgx-\frac{1}{2}\int\frac{d\left(1+x^2\right)}{1+x^2}=xarctgx-\frac{1}{2}\ln\left(1+x^2\right)+C.$$

16. $\int\ln xdx=x\ln x-\int\frac{xdx}{x}=x\ln x-\int dx=x\ln x-x+C.$

$$\left[\begin{array}{l|l} u=\ln x & du=\frac{dx}{x} \\ dv=dx & v=x. \end{array}\right]$$

Ba’zida bu metodni bir necha marta qo‘llashga to‘g‘ri keladi.

17. $\int x^2\sin xdx=-x^2\cos x+2\int x\cos xdx=$

$$\left[\begin{array}{l|l} u=x^2 & du=2xdx \\ dv=\sin xdx & v=-\cos x \end{array}\right] \quad \left[\begin{array}{l|l} u_1=x & du_1=dx \\ dv_1=\cos xdx & v_1=\sin x \end{array}\right]$$

$$=-x^2\cos x+2\left(x\sin x-\int\sin xdx\right)=-x^2\cos x+2x\sin x+2\cos x+C.$$

18. $\int e^x\sin xdx=e^x\sin x-\int e^x\cos xdx=e^x\sin x-\left(e^x\cos x+\int e^x\sin xdx\right).$

$$\left[\begin{array}{l|l} u=\sin x & du=\cos xdx \\ dv=e^xdx & v=e^x \end{array}\right] \quad \left[\begin{array}{l|l} u_1=\cos x & du_1=-\sin xdx \\ dv_1=e^xdx & v_1=e^x \end{array}\right]$$

Shuning uchun $2\int e^x \sin x dx = e^x \sin x - e^x \cos x + C_1$

$\int e^x \sin x dx = \frac{1}{2} e^x (\sin x - \cos x) + C$, bu yerda $C = \frac{C_1}{2}$.

Ratsional funksiyalarni integrallash.

Integralni toping.

19. $\int \frac{x^2 dx}{(x-1)^2(x+1)}$.

Yechish. Integral osti funksiyasini oddiy kasrlar yig'indisi ko'rinishida ifodalaymiz:

$$\frac{x^2}{(x-1)^2(x+1)} = \frac{A_1}{x-1} + \frac{A_2}{(x-1)^2} + \frac{A_3}{(x+1)}.$$

o'ng tomondagi ifodani umumiy maxrajga keltirgandan keyin:

$$\frac{x^2}{(x-1)^2(x+1)} = \frac{A_1(x^2-1) + A_2(x+1) + A_3(x-1)^2}{(x-1)^2(x+1)}.$$

hosil bo'lgan tenglik $x^2 = A_1(x^2-1) + A_2(x+1) + A_3(x-1)^2$ (1) o'rinli bo'ladi.

(1) ga $x=1$ ni qo'yib, $1 = 2A_2$ tenglikka ega bo'lamiz, bundan kelib chiqadiki,

$$A_2 = \frac{1}{2}.$$

$x=-1$da $1 = 4A_3$shuning uchun $A_3 = \frac{1}{4}$.

$x=0$ ni qo'ysak (1) da $0 = -A_1 + A_2 + A_3$ tenglik hosil bo'ladi. Oxirgi tenglikka A_2 va A_3 ning topilgan qiymatlarini qo'yib $A_1 = \frac{3}{4}$ni keltirib chiqaramiz. Natijada

$$\int \frac{x^2 dx}{(x-1)^2(x+1)} = \int \left(\frac{3}{4} \cdot \frac{1}{x-1} + \frac{1}{2} \cdot \frac{1}{(x-1)^2} + \frac{1}{4} \cdot \frac{1}{x+1} \right) dx =$$

$$\frac{3}{4} \ln|x-1| - \frac{1}{2(x-1)} + \frac{1}{4} \ln|x+1| + C.$$

20. Integralni hisoblang:

$$\int \frac{dx}{(x-1)(x^2-x+1)}.$$

Yechish. Integral ostidagi ifodani oddiy kasrlar yig'indisi ko'rinishiga keltiramiz:

$$\frac{1}{(x-1)(x^2-x+1)} = \frac{A_1}{x-1} + \frac{M_1 x + N_1}{x^2 - x + 1}.$$

Umumiy mahrajga keltirib, o'ng va chap tomonlarning suratlarini tenglashtirishdan hosil bo'ladigan tenglik quyidagicha bo'ladi:

$$1 = A_1(x^2 - x + 1) + (M_1x + N_1)(x - 1)$$

Agar $x = 1$ bo'lsa, $A_1 = 1$ agar $x = 0$ bo'lca $1 = A_1 - N_1$ tenglikka o'tamiz va natijada $N_1 = 0$ kelib chiqadi. $x = -1$ ni tenglikka qo'yib, $1 = 3A_1 + (-M_1 + N_1)(-2)$ ni hosil qilamiz, bu yerdan $M_1 = -1$. U holda

$$\int \frac{dx}{(x-1)(x^2 - x + 1)} = \int \frac{dx}{x-1} - \int \frac{xdx}{x^2 - x + 1} \text{ bo'ladi.}$$

Birinchi integral uchun differensial belgisi ostida funksiya shakllantiramiz: $dx = d(x-1)$ ikkinchisi uchun esa – maxrajida to'la kvadrat ajratamiz: $x^2 - x + 1 = (x - 1/2)^2 + 3/4$ va $t = x - \frac{1}{2}$ almashtirishdan foydalanamiz. U holda

$dt = dx,\ x = t + \frac{1}{2}$ bo'ladi va $\int \frac{dx}{(x-1)(x^2 - x + 1)} = \int \frac{d(x-1)}{x-1} - \int \frac{t + 1/2}{t^2 + 3/4} dt =$

$$= \ln|x-1| - \int \frac{tdt}{t^2 + 3/4} - \frac{1}{2}\int \frac{dt}{t^2 + 3/4} = \ln|x-1| - \frac{1}{2}\int \frac{d(t^2 + 3/4)}{t^2 + 3/4} - \frac{1}{\sqrt{3}} arctg \frac{2t}{\sqrt{3}} =$$

$$= \ln|x-1| - \frac{1}{2}\ln|t^2 + 3/4| - \frac{\sqrt{3}}{3} arctg \frac{2x-1}{\sqrt{3}} + C = \ln|x-1| - \frac{1}{2}\ln|x^2 - x + 1| - \frac{\sqrt{3}}{3} arctg \frac{2x-1}{\sqrt{3}} + C.$$

21. $\int \frac{dx}{x^4 - 1}$ ni hisoblansin.

Yechish. Integral ostidagi $\frac{1}{x^4 - 1}$ kasrni sodda kasrlarga ajratamiz:

$$\frac{1}{x^4 - 1} = \frac{1}{(x-1)(x+1)(x^2+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx + D}{x^2 + 1}$$

Bu tenglikni quyidagicha yozib olmiz:

$$\frac{1}{x^4 - 1} = \frac{A(x+1)(x^2+1) + B(x-1)(x^2+1) + (Cx + D)(x^2 - 1)}{(x-1)(x+1)(x^2+1)}$$

U holda

$$1 = A(x+1)(x^2+1) + B(x-1)(x^2+1) + (Cx + D)(x^2 - 1)$$

ya'ni

$$1 = (A + B + C)x^3 + (A - B + D)x^2 + (A + B - C)x + (A - B - D)$$

bo'ladi. Natijada A, B, C, D larni topish uchun

$$A+B+C=0,$$
$$A-B+D=0,$$
$$A+B-C=0,$$
$$A-B-D=1.$$

Sistemaga kelamiz. Bu sistemani yechib,

$$A=\frac{1}{4},\ B=-\frac{1}{4},\ C=0,\ D=-\frac{1}{2}$$

bo‘lishini topamiz. Demak,

$$\frac{1}{x^4-1}=\frac{1}{4}\cdot\frac{1}{x-1}-\frac{1}{4}\cdot\frac{1}{x+1}-\frac{1}{2}\cdot\frac{1}{x^2+1}$$

bo‘lib,

$$\int\frac{dx}{x^4-1}=\frac{1}{4}\cdot\int\frac{dx}{x-1}-\frac{1}{4}\cdot\int\frac{dx}{x+1}-\frac{1}{2}\cdot\int\frac{dx}{x^2+1}=\frac{1}{4}\ln\left|\frac{x-1}{x+1}\right|-\frac{1}{2}arctgx+C$$

bo‘ladi.

22. Ushbu

$$\int\frac{3x^2+8}{x^3+4x^2+4x}dx$$

integral hisoblansin.

Yechish. Integral ostidagi ratsional funksiyani sodda kasrlarga yoyamiz:

$$\frac{3x^2+8}{x^3+4x^2+4x}=\frac{2}{x}+\frac{1}{x+2}-\frac{10}{(x+2)^2}.$$

Demak,

$$\int\frac{3x^2+8}{x^3+4x^2+4x}dx=2\int\frac{dx}{x}+\int\frac{dx}{x+2}-10\int\frac{dx}{(x+2)^2}=$$

$$=2\ln|x|+\ln|x+2|+\frac{10}{x+2}+C.$$

23. Ushbu

$$\int\frac{x^6+2x^4+2x^2-1}{x(x^2+1)^2}dx$$

integral hisoblansin.

Yechish. Integral ostidagi funksiya ratsional funksiya bo‘lib, u noto‘g‘ri kasrdir. Bu kasrning surati $x^6+2x^4+2x^2-1$ ko‘phadni maxraji $x(x^2+1)^2$ ko‘phadga bo‘lib, uning butun qismini ajratamiz:

$$\begin{array}{r|l} -\ x^6+2x^4+2x^2-1 & x^5+2x^3+x \\ \underline{x^6+2x^4+x^2\quad\ \ } & \overline{x\qquad\qquad\quad} \\ x^2-1 & \end{array}$$

Demak,

$$\frac{x^6+2x^4+2x^2-1}{x(x^2+1)^2}=x+\frac{x^2-1}{x(x^2+1)^2}.$$

Endi

$$\frac{x^2-1}{x\left(x^2+1\right)^2}$$

to‘g‘ri kasrni sodda kasrlarga yoyamiz:

$$\frac{x^2-1}{x(x^2+1)^2}=\frac{A}{x}+\frac{Bx+C}{x^2+1}+\frac{Dx+E}{(x^2+1)^2},$$

$$x^2-1=A(x^2+1)^2+(Bx+C)x(x^2+1)+(Dx+E)x=$$

$$=(A+B)x^4+Cx^3+(2A+B+D)x^2+(C+E)x+A.$$

Keyingi tenglikdan

$$A=-1,\ B=1,\quad C=0,\quad D=2,\quad E=0$$

bo‘lishini topamiz.

Demak,

$$\frac{x^2-1}{x\left(x^2+1\right)^2}=\frac{-1}{x}+\frac{x}{x^2+1}+\frac{2x}{\left(x^2+1\right)^2}.$$

Natijada,

$$\frac{x^6+2x^4+2x^2-1}{x(x^2+1)^2}=x-\frac{1}{x}+\frac{x}{x^2+1}+\frac{2x}{\left(x^2+1\right)^2}$$

bo‘lib,

$$\int\frac{x^6+2x^4+2x^2-1}{x\left(x^2+1\right)^2}=\int xdx-\int\frac{dx}{x}+\int\frac{x}{x^2+1}dx+$$

$$+\int\frac{2x}{(x^2+1)^2}dx=\frac{x^2}{2}-\ln|x|+\frac{1}{2}\int\frac{d(x^2+1)}{x^2+1}+\int\frac{d(x^2+1)}{(x^2+1)^2}=$$

$$=\frac{x^2}{2}-\ln|x|+\frac{1}{2}\ln(x^2+1)-\frac{1}{x^2+1}+C$$

bo‘ladi.

Sodda irratsional funksiyalarni integrallash

24. Ushbu $\int\frac{dx}{\sqrt{x}+\sqrt[3]{x}}$ integralni toping.

Yechish. $x=t^6$ $dx=6t^5dt$ demak

$$\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}} = \int \frac{6t^5dt}{t^3+t^2} = 6\int \frac{t^3dt}{t+1} =$$

$$= 6\int \frac{\left(t^3+1\right)-1}{t+1}dt = 6\left[\int \frac{t^3+1}{t+1}dt - \int \frac{dt}{t+1}\right] =$$

$$= 6\left[\int \frac{(t+1)\left(t^2-t+1\right)}{t+1}dt - \ln|t+1|\right] =$$

$$= 6\left[\int \left(t^2-t+1\right)dt - \ln|t+1|\right] =$$

$$= 6\left(\frac{t^3}{3} - \frac{t^2}{2} + t - \ln|t+1|\right) + C = 2t^3 - 3t^2 + 6t - 6\ln|t+1| + C =$$

$$= 2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\ln\left|\sqrt[6]{x}+1\right| + C.$$

25. Ushbu $\int \frac{x+\sqrt{1+x}}{\sqrt[3]{1+x}}dx$ integralni toping.

Yechish. $1+x=t^6 \quad x=t^6-1, \quad dx=6t^5dt$

$$\int \frac{x+\sqrt{1+x}}{\sqrt[3]{1+x}}dx = \int \frac{\left(t^6-1\right)+t^3}{t^2}6t^5dt = 6\int\left(t^9+t^6-t^3\right)dt =$$

$$= 6\left(\frac{t^{10}}{10}+\frac{t^7}{7}-\frac{t^4}{4}\right)+C = 6t^4\left(\frac{t^6}{10}+\frac{t^3}{7}-\frac{1}{4}\right)+C =$$

$$= 6\sqrt[3]{(1+x)^2}\cdot\left(\frac{1+x}{10}+\frac{\sqrt{1+x}}{7}-\frac{1}{4}\right)+C.$$

26.. Ushbu $\int \sqrt{\frac{1-x}{1+x}}\cdot\frac{dx}{x}$ integralni toping.

Yechish. $\frac{1-x}{1+x}=t^2 \quad 1-x=(1+x)t^2, \quad x=\frac{1-t^2}{1+t^2}$

$$dx = \frac{\left(1-t^2\right)'\left(1+t^2\right)-\left(1+t^2\right)'\left(1-t^2\right)}{\left(1+t^2\right)^2}dt = -\frac{4t}{\left(1+t^2\right)^2}dt.$$

$$\int\sqrt{\frac{1-x}{1+x}}\cdot\frac{dx}{x}=\int t\cdot\frac{1+t^2}{1-t^2}\frac{(-4t)\,dt}{\left(1+t^2\right)^2}=$$

$$=4\int\frac{t^2dt}{\left(t^2-1\right)\left(t^2+1\right)}=4\int\frac{\left(t^2-1\right)+1}{\left(t^2-1\right)\left(t^2+1\right)}dt$$

$$=4\left[\int\frac{t^2-1}{\left(t^2-1\right)\left(t^2+1\right)}dt+\int\frac{dt}{\left(t^2-1\right)\left(t^2+1\right)}\right]=$$

$$=4\left[\int\frac{dt}{t^2+1}+\frac{1}{2}\int\left(\frac{1}{t^2-1}-\frac{1}{t^2+1}\right)dt\right]=$$

$$=4\int\frac{dt}{t^2+1}+2\int\frac{dt}{t^2-1}-2\int\frac{dt}{t^2+1}=$$

$$=2\left(\int\frac{dt}{t^2+1}+\int\frac{dt}{t^2-1}\right)=2\left(\operatorname{ar}ctgt+\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|\right)+C=$$

$$=2arctg\sqrt{\frac{1-x}{1+x}}+\ln\left|\frac{\sqrt{\frac{1-x}{1+x}}-1}{\sqrt{\frac{1-x}{1+x}}+1}\right|+C=$$

$$=2arctg\sqrt{\frac{1-x}{1+x}}+\ln\left|\frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1-x}+\sqrt{1+x}}\right|+C.$$

27. Integralni toping: $\int\sqrt[3]{x}\cdot\sqrt[3]{1+3\cdot\sqrt[3]{x^2}}\,dx.$

Yechish. $m=\frac{1}{3},\ n=\frac{2}{3},\ p=\frac{1}{3}$. Ushbu holatda $\frac{m+1}{n}=\frac{\frac{1}{3}+1}{\frac{2}{3}}=2$ – butun son

$1+3\cdot\sqrt[3]{x^2}=t^3$. $x=\frac{1}{3\sqrt{3}}\left(t^3-1\right)^{\frac{3}{2}}$ demak $dx=\frac{\sqrt{3}}{2}\left(t^3-1\right)^{\frac{1}{2}}\cdot t^2dt.$

$$\int\sqrt[3]{x}\cdot\sqrt[3]{1+3\cdot\sqrt[3]{x^2}}\,dx=\int\frac{\sqrt{t^3-1}}{\sqrt{3}}\cdot t\cdot\frac{\sqrt{3}}{2}\cdot\sqrt{t^3-1}\cdot t^2dt=$$

$$=\frac{1}{2}\int\left(t^3-1\right)t^3dt=\frac{1}{2}\int\left(t^6-t^3\right)dt=\frac{1}{2}\left(\frac{t^7}{7}-\frac{t^4}{4}\right)+C=$$

$$=\frac{t^7}{14}-\frac{t^4}{8}+C=\frac{1}{14}\sqrt[3]{\left(1+3\sqrt[3]{x^2}\right)^7}-\frac{1}{8}\sqrt[3]{\left(1+3\sqrt[3]{x^2}\right)^4}+C.$$

7.2. Aniq integral

Asosiy tushunchalar ва xossalari

Aytaylik, $y = f(x)$ funksiya $[a;b]$da aniqlangan bo'lsin. $[a;b]$ kesmani

$$a = x_0 < x_1 < x_2 < ... < x_n = b$$

nuqtalar bilan n ta bo'lakka bo'lamiz. $[a;b]$ ni bo'luvchi bu sonlar to'plamini $[a;b]$ ning bo'linishi deb ataymiz va τ_n bilan belgilaymiz:

$$\tau_n = \{x_0, x_1, x_2, ..., x_n | a = x_0 < x_1 < x_2 < ... < x_n = b\}$$

Har bir elementar $[x_{k-1}; x_k]$ $(k = 1,2,...,n)$ kesmada bittadan ixtiyoriy c_k nuqta tanlab, shu nuqtalarda funksiyaning $f(c_k)$ qiymatlarini hisoblaylik va quyidagi yig'indini tuzaylik:

$$S(\tau_n) = \sum_{k=1}^{n} f(c_k)\Delta x_k, \qquad (1)$$

bu yerda $\Delta x_k = x_k - x_{k-1}$ $[x_{k-1}; x_k]$ $(k = 1,2,...,n)$ kesmaning uzunligi.

Ushbu (1) yig'indi $f(x)$ funksiyaning $[a;b]$ dagi *integral yig'indisi* deb ataladi.

$[a;b]$ ning bo'linishlari τ_n va har bir $[x_{k-1}; x_k]$ kesmadan c_k nuqtalarni tanlash usullari cheksiz ko'p bo'lganligi sababli $f(x)$ ning $[a;b]$dagi (1) integral yig'indilari to'plami cheksiz to'plam bo'ladi. $\lambda = \max\limits_{1 \le k \le n} \Delta x_k$ belgilash kiritamiz.

1-ta'rif. Agar λ nolga intilganda $f(x)$ ning $[a;b]$dagi (1) integral yig'indisi chekli I limitga ega bo'lib, bu limit $[a;b]$ning τ_n bo'linishlariga va c_k nuqtalarini tanlash usuliga bog'liq bo'lmasa, o'sha I limit $f(x)$ ning $[a;b]$dagi *aniq integrali* deyiladi va u

$$\int_a^b f(x)dx$$

orqali belgilanadi:

$$\int_a^b f(x)dx = \lim_{\lambda \to 0} \sum_{k=1}^{n} f(c_k)\Delta x_k. \qquad (2)$$

Bunday holda $y=f(x)$ funksiya $[a;b]$da integrallanuvchi (yoki Riman ma'nosida integrallanuvchi) deyiladi.

Bu yerda ham aniqmas integraldagi kabi $f(x)dx$ *integral ostidagi ifoda*, $f(x)$ – *integral ostidagi funksiya*, x – *integrallash o'zgaruvchisi* deb ataladi, a va b esa mos ravishda *integrallashning quyi va yuqori chegaralari* deyiladi.

Teorema. Agar *f(x)* funksiya $[a;b]$ da integrallanuvchi bo'lsa, u holda bu funksiya $[a;b]$ da chegaralangan bo'ladi.

Teorema. Agar *f(x)* funksiya $[a;b]$ kesmada uzluksiz bo'lsa, u holda funksiya shu kesmada integrallanuvchi bo'ladi.

Teorema. Agar $[a;b]$ da chegaralangan *f(x)* funksiya shu kesmada chekli sondagi uzilish nuqtalariga ega bo'lsa, u holda *f(x)* funksiya integrallanuvchi bo'ladi.

Teorema. Agar *f(x)* funksiya $[a;b]$ kesmada monoton bo'lsa, u shu kesmada integrallanuvchi bo'ladi.

Ma'lum vaqt oralig'ida jamg'arma bankiga tushgan pul miqdori

$u=f(t)$ – funksiya t – vaqtning har bir momentida jamg'arma bankiga tushadigan pul miqdorini ifodalasin. $[0;T]$ vaqt oralig'ida bankka tushgan pulning U umumiy miqdorini topish talab etiladi.

Agar $f(t)=const$ bo'lsa, u holda $[0;T]$ vaqt oralig'ida jamg'arma bankiga tushgan U pul miqdori $U=f(c)\cdot(T-0)=f(c)\cdot T$ formula bilan topiladi, bu yerda $c\in[0;T]$. Agar $\left[0;\frac{T}{2}\right]$ vaqt oralig'ining har bir momentida bankka $f(c_1)$ pul birligi, $\left[\frac{T}{2};T\right]$ oraliqda vaqtning har bir momentida $f(c_2)$ pul birligi tushsa, u holda $[0;T]$ vaqt oralig'ida tushgan umumiy pul miqdori

$$U=f(c_1)\frac{T}{2}+f(c_2)\frac{T}{2}$$

formula bo'yicha hisoblanadi.

$f(t)$ funksiya $[0;T]$ kesmada uzluksiz funksiya bo'lsin. $0=t_0<t_1<t_2<...t_{n-1}<t_n=T$ nuqtalar yordamida $[0;T]$ kesmani kichik vaqt oraliqlariga ajratamiz. $[t_{i-1},t_i]$ vaqt oralig'ida bankka tushgan ΔU_i pul miqdori taqriban $\Delta U \approx f(c_i)\Delta t_i$ formula bilan hisoblanadi. Bu yerda $c_i \in [t_{i-1},t_i]$, $\Delta t_i = t_i - t_{i-1}$, $i=1,2,3,...,n$. U holda

$$U=\sum_{i=1}^{n}\Delta U_i \approx \sum_{i=1}^{n} f(c_i)\Delta t_i \Rightarrow U = \lim_{\max \Delta t_i \to 0}\sum_{i=1}^{n} f(c_i)\Delta t_i \Rightarrow U=\int_0^T f(t)dt.$$

$f(t)\geq 0$ bo'lgani uchun $[0;T]$ vaqt oralig'ida jamg'arma bankiga tushgan umumiy pul miqdori son jihatidan $f(t)$, $t=0$, $t=T$, Ot chiziqlar bilan chegaralangan figura yuziga teng.

Ma'lum vaqt oralig'ida ishlab chiqarilgan mahsulot hajmi

$y=f(t)$ funksiya vaqt o'tishi bilan biror ishlab chiqarishning unumdorligi o'zgarishini ifodalasin. $[0;T]$ vaqt oralig'ida ishlab chiqarilgan Q mahsulot hajmini topamiz.

$[0;T]$ kesmani $0=t_0<t_1<t_2<...<t_{n-1}<t_n=T$ nuqtalar yordamida vaqt oraliqlariga ajratamiz. $[t_{i-1},t_i]$ vaqt oralig'ida ishlab chiqarilgan ΔQ_i mahsulot hajmi taqriban $\Delta Q \approx f(c_i)\Delta t_i$ formula bilan hisoblanadi. Bu yerda $c_i \in [t_{i-1},t_i]$, $\Delta t_i = t_i - t_{i-1}$, $i=1,2,3,...,n$. U holda

$$Q=\sum_{i=1}^{n}\Delta Q_i \approx \sum_{i=1}^{n} f(c_i)\Delta t_i \Rightarrow Q = \lim_{\max \Delta t_i \to 0}\sum_{i=1}^{n} f(c_i)\Delta t_i \Rightarrow Q=\int_0^T f(t)dt.$$

Misol. Agar kun davomida mehnat unumdorligi $f(t)=-0{,}1t^2+0{,}8t+10$ empirik formula bo'yicha o'zgarsa, kunlik ish vaqti 8 soat bo'lgan Q bir kunlik ishlab chiqarilgan mahsulotni toping.

Yechish. $Q=\int_0^T f(t)dt = \int_0^8 \left(-0{,}1t^2+0{,}8t+10\right)dt = (-0{,}1\frac{t^3}{3}+0{,}8\frac{t^2}{2}+10t)/_0^8 \approx 88{,}53$

Aniq integralning xossalari:

1. $\int_a^b f(x)dx = -\int_b^a f(x)dx$

2. $\int_a^a f(x)dx = 0.$

3. $\int_a^b (f_1(x) \pm f_2(x))dx = \int_a^b f_1(x)dx \pm \int_a^b f_2(x)dx.$

4. $\int_a^b cf(x)dx = c\int_a^b f(x)dx.$

5. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, \quad a < c < b.$

6. Agar $[a;b]$ kesmada $f(x) \geq 0$ bo'lsa u holda, $\int_a^b f(x)dx \geq 0$, agar barcha $x \in [a;b]$ nuqtalar uchun $f(x) \leq 0$ bo'lsa u holda $\int_a^b f(x)dx \leq 0.$

7. Agar $[a;b]$ kesmada $f(x) \leq g(x)$ bo'lsa u holda, $\int_a^b f(x)dx \leq \int_a^b g(x)dx.$

8. Agar $f(x)$ funksiyaning $[a;b]$ kesmada $M-$ eng katta, $m-$eng kichik qiymatlari bo'lsa u holda, $m(b-a) \leq \int_a^b f(x)dx \leq M(b-a).$

9. $\int_a^b f(x)dx = f(c)\cdot(b-a), \; c \in [a;b].$

10. $\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx.$

11. $\left(\int_a^x f(t)dt\right)'_x = f(x).$

Nyuton – Leybnits formulasi

Agar $F(x)$ funksiya uzluksiz bo'lib, u $y = f(x)$ funksiyaning biror – bir boshlang'ich funksiyasi bo'lsa, u holda

$$\int_a^b f(x)dx = F(b) - F(a)$$

N'yuton – Leybnits formulasi o'rinli bo'ladi.

Juft va toq funksiyalar integralini hisoblashni osonlashtiradigan quyidagi xossalarni keltirib o'tamiz:

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_{0}^{a} f(x)\,dx, & agar\ f(x) - juft\ funksiya, \\ 0, & agar\ f(x) - toq\ funksiya. \end{cases}$$

O'zgaruvchini almashtirish usuli

Agar $f(x)$ funksiya $[a;b]$da uzluksiz, $x=\varphi(t)$ funksiya $[\alpha;\beta]$ kemada uzluksiz differensiallanuvchi, $x=\varphi(t)$funksiya qiymatlari to'plami $[a;b]$kesmadan iborat hamda $\varphi(\alpha)=a$, $\varphi(\beta)=b$ bo'lsa, u holda

$$\int_{a}^{b} f(x)dx = \int_{\alpha}^{\beta} f(\varphi(t))\varphi'(t)dt$$

tenglik o'rinli bo'ladi. Bu formula aniq integralda o'zgaruvchini almashtirish formulasi deb ataladi

Shuni ta'kidlash kerakki, aniq integralni o'zgaruvchilarni almashtirish usuli bilan hisoblaganda integral ostidagi ifoda bilan bir qatorda integrallash chegaralari ham o'zgaradi.

Bo'laklab integrallash usuli

Faraz qilaylik, $u(x)$va $v(x)$funksiyalar $[a;b]$da uzluksiz hosilalarga ega bo'lsin. U holda

$$(u\cdot v)' = u'v + uv'$$

bo'lib, $u(x)v(x)$funksiya $(u(x)\cdot v(x))' = u'(x)v(x) + u(x)v'(x)$ uzluksiz funksiyaning boshlang'ich funksiyasi bo'ladi. Nyuton-Leybnits formulasiga ko'ra

$$\int_{a}^{b} (u'v + uv')dx = (uv)\Big|_{a}^{b}.$$

Bundan

$$\int_{a}^{b} uv'dx = (uv)\Big|_{a}^{b} - \int_{a}^{b} u'vdx$$

kelib chiqadi. So‘ngra $uv'dx = udv$ va $u'vdx = vdu$ ekanligini e’tiborga olsak, natijada

$$\int_a^b udv = (uv)\Big|_a^b - \int_a^b vdu$$

aniq integralni bo‘laklab integrallash formulasi hosil bo‘ladi.

Misollar

1. $\int_1^9 \frac{dx}{5+2\sqrt{x}}$ integralni o‘zgaruvchini almashtirish usulida hisoblang.

Yechish. Bu integralda $\sqrt{x} = t$ almashtirishni bajaramiz. U holda $x = t^2$, $dx = 2tdt$. $x = 1$ da $t = 1$, $x = 9$ da $t = 3$. Demak, (1) formulaga ko‘ra

$$\int_1^9 \frac{dx}{5+2\sqrt{x}} = \int_1^3 \frac{2tdt}{5+2t} = \int_1^3 \frac{2t+5-5}{2t+5}dt =$$

$$= \int_1^3 \left(1 - \frac{5}{2t+5}\right)dt = t\Big|_1^3 - 5\cdot\frac{1}{2}\ln|2t+5|\Big|_1^3 =$$

$$= 3 - 1 - \frac{5}{2}(\ln 11 - \ln 2) = 2 - \frac{5}{2}\ln\frac{11}{7}.$$

2. $\int_{\pi/6}^{\pi/3} \frac{\cos x}{\sin^5 x}dx$ ni hisoblang.

Yechish. $\sin x = t$ deb almashtirish bajaramiz. U holda $\cos xdx = dt$, $x = \frac{\pi}{6}$ da, $t = \frac{1}{2}$, $x = \frac{\pi}{3}$ da $t = \frac{\sqrt{3}}{2}$.

$$\int_{\pi/6}^{\pi/3} \frac{\cos x}{\sin^5 x}dx = \int_{1/2}^{\sqrt{3}/2} t^{-5}dt = -\frac{1}{4t^4}\Bigg|_{1/2}^{\sqrt{3}/2} = \frac{1}{4}\left(16 - \frac{16}{9}\right) = \frac{32}{9}.$$

3. $\int_0^{\pi/2} x\cos xdx$ integralni hisoblang.

Yechish. Bunda $u = x, dv = \cos xdx$ deb olsak, $du = dx, v = \sin x$ hosil bo‘ladi.

Demak, bo‘laklab integrallash formulasiga ko‘ra

$$\int_0^{\pi/2} x\cos xdx = (x\sin x)\Big|_0^{\pi/2} - \int_0^{\pi/2} \sin xdx = \frac{\pi}{2} + \cos x\Big|_0^{\pi/2} = \frac{\pi}{2} - \cos 0 = \frac{\pi-2}{2}.$$

4. $\int_1^e (x+1)\ln xdx$ integralni hisoblang.

Yechish. Bunda $u=\ln x, dv=(x+1)dx$. U holda $du=\frac{1}{x}dx,\ v=\frac{x^2}{2}+x$ hosil bo'ladi.

Demak, bo'laklab integrallash formulasiga ko'ra

$$\int_1^e (x+1)\ln x dx=\left(\frac{x^2}{2}+x\right)\ln x\Big|_1^e-\int_1^e\left(\frac{x^2}{2}+x\right)\frac{dx}{x}=$$

$$=\frac{e^2}{2}+e-0-\left(\frac{x^2}{4}+x\right)\Bigg|_1^e=\frac{e^2}{2}+e-\frac{e^2}{4}-e+\frac{1}{4}+1=\frac{e^2+5}{4}.$$

7.3. Xosmas integral

Bizga ma'lumki, $y=f(x)$ funksiya ixtiyoriy $[a;b]$ oraliqda aniqlangan va integrallanuvchi bo'lsa, u holda

$$\int_a^b f(x)dx \qquad (1)$$

integral mavjud. Agar (1) integralning yuqori chegarasi uchun $b\to+\infty$, yoki quyi chegarasi uchun $a\to-\infty$, yoki ham yuqori ham quyi chegaralari uchun $b\to+\infty, a\to-\infty$ munosabat o'rinli bo'lsa, u holda (1) integral I tur xosmas integral deb ataladi. Shunday qilib I tur xosmas integral quyidagi ko'rinishlarda bo'lishi mumkin:

$$\int_a^{+\infty} f(x)dx,\ \int_{-\infty}^{b} f(x)dx,\ \int_{-\infty}^{+\infty} f(x)dx \qquad (2)$$

(2) xosmas integrallardagi integral osti funksiyalarning aniqlanish sohasi mos ravishda quyidagi oraliqlardan iborat bo'ladi: $[a,+\infty), (-\infty,b], (-\infty,+\infty)$. (2) integrallarni hisoblash quyidagicha amalga oshiriladi:

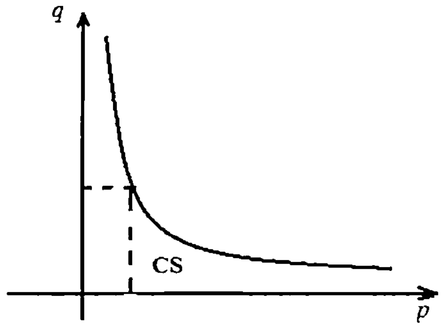
$$\int_a^{+\infty} f(x)dx=\lim_{b\to+\infty}\int_a^b f(x)dx,\ \int_{-\infty}^{b} f(x)dx=\lim_{a\to-\infty}\int_a^b f(x)dx,$$
$$\int_{-\infty}^{+\infty} f(x)dx=\lim_{a\to-\infty}\int_a^b f(x)dx+\lim_{c\to+\infty}\int_b^c f(x)dx,\ \forall c\in(-\infty,+\infty) \qquad (3)$$

Agar (3) ifodanng o'ng tomonidagi limit osti integrallar mavjud va chekli bo'lsa, u holda ifodaning chap tomonidagi xosmas integrallar yaqinlashuvchi, aks holda esa ular uzoqlashuvchi deyiladi.

Xosmas integrallarning iqtisodiyotdagi tatbiqi. Ko'pgina iqtisodiy masalalarning yechimlarini topish jarayonida xosmas integrallarni hisoblashga to'g'ri keladi. Masalan, talab elastikligi o'zgarmas bo'lgan holat uchun iste'molchining ortiqcha foydasini hisoblash masalasining talab egri chizig'ini quyidagicha yozish mumkin:

$$q = ap^{-\varepsilon},\ a > 0,\ \varepsilon > 0 \Rightarrow p = \left(\frac{q}{a}\right)^{-\frac{1}{\varepsilon}}.$$

Bu yerda ε talab elastikligining bahosi. Talab egri chzig'i p, q koordinata o'qlarida quyidagi ko'rinishga ega bo'ladi:



U holda $p = p_0$ dan boshlab iste'molchining ortiqcha foydasi quyidagi I tur xosmas integral bilan hisoblanadi:

$$CS = \int_{p_0}^{\infty} ap^{-\varepsilon} dp.$$

U holda

$$CS = \int_{p_0}^{\infty} ap^{-\varepsilon} dp = \lim_{\bar{p}\to\infty} \int_{p_0}^{\bar{p}} ap^{-\varepsilon} dp = \lim_{p\to\infty} \frac{a}{1-\varepsilon} \bar{p}^{1-\varepsilon}\Bigg|_{p_0}^{\bar{p}} = \frac{a}{1-\varepsilon}\left[\lim_{\bar{p}\to\infty} \bar{p}^{1-\varepsilon} - p_0^{1-\varepsilon}\right].$$

Bu integral $\varepsilon > 1$ holatda yaqinlashadi.

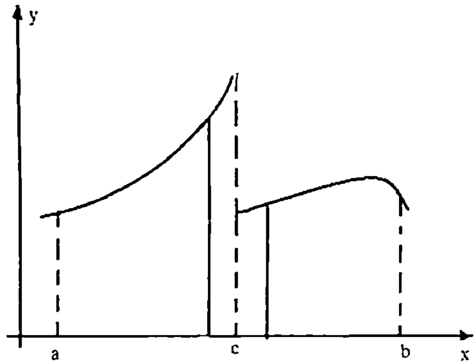
Misol. Talab funksiyasi $q = 50p^{-2}$ bo'lsa, $p = 10$ da iste'molchining ortiqcha foydasi topilsin.

Yechish. $CS = \int_{10}^{\infty} 50p^{-2} dp = \lim_{\bar{p}\to\infty} \int_{10}^{\bar{p}} 50p^{-2} dp = \frac{50}{1-2}\left[\lim_{\bar{p}\to\infty} \bar{p}^{1-2} - 10^{1-2}\right] = 5.$

Agar integral chegaralari chekli a, b sonlardan iborat bo‘lib, integral osti funksiyasi $[a,b]$ kesmaning chekli sondagi nuqtalarida aniqlanmagan bo‘lsa, bunday integral II tur xosmas integral deb ataladi. Masalan, $y=f(x)$, $x\in[a,c)\cup(c,b]$ berilgan bo‘lsin, u holda $\int_a^b f(x)dx$ integral II tur xosmas integral deb ataladi.

Xosmas integrallarni yaqinlashishga tekshirish quyidagicha amalga oshiriladi:

$$\int_a^b f(x)dx=\lim_{\varepsilon\to0}\int_a^{c-\varepsilon} f(x)dx+\lim_{\varepsilon\to0}\int_{c+\varepsilon}^{b} f(x)dx,\ \varepsilon>0. \qquad (4)$$



Agar (4) formulada qatnashayotgan limitlar mavjud va chekli bo‘lsa, xosmas integral yaqinlashuvchi deyiladi.

Agar (4) formulada qatnashayotgan limitlardan bittasi mavjud bo‘lmasa yoki cheksiz bo‘lsa, xosmas integral uzoqlashuvchi deb ataladi.

Misol. $\int_0^1\frac{dx}{\sqrt{(1-x)}}$ xosmas integralni hisoblaymiz. Integral ostidagi $y=\frac{1}{\sqrt{(1-x)}}$ funksiya $x=1$ nuqtada aniqlanmagan va nuqtadan chapda chegaralanmagan. Demak,

$$\int_0^1\frac{dx}{\sqrt{(1-x)}}=\lim_{\varepsilon\to0}\int_0^{1-\varepsilon}\frac{dx}{\sqrt{(1-x)}}=\lim_{\varepsilon\to0}\left(-2\sqrt{(1-x)}\right)\Big|_0^{1-\varepsilon}=$$

$$=-2\lim_{\varepsilon\to0}\left(\sqrt{(1-1+\varepsilon)}-\sqrt{(1-0)}\right)=-2\lim_{\varepsilon\to0}\left(\sqrt{\varepsilon}-\sqrt{1}\right)=2.$$

Bu xosmas integral yaqinlashuvchi.

Xosmas integrallarni integrallash uchun o'zgaruvchini almashtirish va bo'laklab integrallash usullaridan foydalaniladi.

Misollar

1. Xosmas integrallarni yaqinlashishga tekshiring:

$a)\int_{2}^{+\infty}\frac{dx}{x^2-1};\qquad b)\int_{-\infty}^{0}x\cos x dx\qquad c)\int_{-\infty}^{+\infty}\frac{dx}{1+x^2};$

Yechish.

a) ta'rif bo'yicha quyidagini olamiz:

$$\int_{2}^{+\infty}\frac{dx}{x^2-1}=\lim_{t\to+\infty}\int_{2}^{t}\frac{dx}{x^2-1}=\lim_{t\to+\infty}\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|\Big|_{2}^{t}=\frac{1}{2}\lim_{t\to+\infty}\left(\ln\left|\frac{t-1}{t+1}\right|-\ln\frac{1}{3}\right)=\frac{1}{2}\left(\ln\lim_{t\to+\infty}\frac{t-1}{t+1}-\ln\frac{1}{3}\right)=$$

$$=\frac{1}{2}(\ln 1+\ln 3)=0.5\ln 3$$

b) $\int_{-\infty}^{0}x\cos x dx$

$$\int_{-\infty}^{0}x\cos x dx=\lim_{a\to-\infty}\int_{a}^{0}x\cos x dx=$$

$$\begin{array}{r|l} u=x & du=dx \\ dv=\cos x dx & v=\sin x \end{array}$$

$$=\lim_{a\to-\infty}\left(x\sin x\Big|_{a}^{0}+\cos x\Big|_{a}^{0}\right)=0-\lim_{a\to-\infty}a\sin a+1-\lim_{a\to-\infty}\cos a,$$

$\lim\limits_{a\to-\infty}a\sin a$; $\lim\limits_{a\to-\infty}\cos a$ mavjud emas.

Demak, bu xosmas integral uzoqlashuvchi.

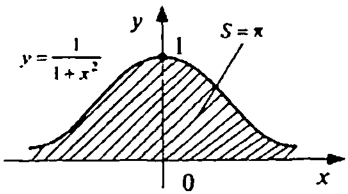
c) $c=0$ integral osti funksiyasi juft funksiya bo'lgani uchun quyidagiga ega bo'lamiz:

$$\int_{-\infty}^{+\infty}\frac{dx}{1+x^2}=\int_{-\infty}^{0}\frac{dx}{1+x^2}+\int_{0}^{+\infty}\frac{dx}{1+x^2}=2\int_{0}^{+\infty}\frac{dx}{1+x^2}=2\lim_{t\to+\infty}\int_{0}^{t}\frac{dx}{1+x^2}=2\lim_{t\to+\infty}(arctgt-arctgo)=$$

$$=2\left(\frac{\pi}{2}-0\right)=\pi.$$

Geometrik nuqtai nazardan yaqinlashuvchi $\int_{a}^{+\infty}f(x)dx$ xosmas integral $y=f(x)\geq 0$ egri chiziq, $x=a, y=0$ to'g'ri chiziqlar bilan chegaralangan va Ox o'qi yo'nalishida cheksiz cho'zilgan figuraning chekli S yuzaga ega ekanligini anglatadi. Shunga o'xshash, $\int_{-\infty}^{b} f(x)dx$ va $\int_{-\infty}^{+\infty} f(x)dx$ yaqinlashuvchi xosmas integrallarga ham geometrik talqin berish mumkin.

Misolning *c* variantida natija $(-\infty;+\infty)$ intervalda $\frac{dx}{1+x^2}$ egri chiziq ostidagi yuza π(bir.^{2}) ga tengligini bildiradi.



2. Integrallarni hisoblang:

$a)\ \int_0^1 \frac{dx}{\sqrt{1-x^2}}$; $\qquad b)\int_{-7}^{2} \frac{dx}{\sqrt[3]{(x-1)^2}}$

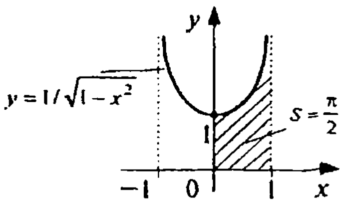
agar ular yaqinlashuvchi bo'lsa.

Yechish:

a) bunda $x=1$ nuqta integral ostidagi funksiyaning maxsus nuqtasidir. Bu holda

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}} = \lim_{t\to 1-0}\int_0^t \frac{dx}{\sqrt{1-x^2}} = \lim_{t\to 1-0} \arcsin x\Big|_0^t = \lim_{t\to 1-0} \arcsin t = \frac{\pi}{2}.$$

Geometrik nuqtai nazardan chegaralanmagan funksiyaning xosmas integrali $y=f(x)$ egri chiziq, $x=a, x=b$ to'g'ri chiziqlar bilan chegaralangan va $x\to b-0$ da $(x\to a+0, x\to c\pm 0)$ Oy o'qi yo'nalishida cheksiz cho'zilgan figuraning chekli yuzga ega ekanligini anglatadi.



Misolning a) variantida olingan natijaning geometrik ma'nosi $[0,1)$ yarimintervalda $y=\frac{dx}{\sqrt{1-x^2}}$ egri chiziq ostidagi yuza $\frac{\pi}{2}$ (bir.^{2}) ga tengligini bildiradi.

b) integral osti $\frac{dx}{\sqrt[3]{(x-1)^2}}$ funksiya $[-7;2]$kesmadagi $x=1$ ichki nuqtada aniqlanmagan va bu nuqta atrofida chegaralanmagan. Aniq integral xossalaridan foydalangan holda, bu integralni ikkita integralning yig'indisi ko'rinishda ifodalaymiz

$$\int_{-7}^{2}\frac{dx}{\sqrt[3]{(x-1)^2}}=\int_{-7}^{1}\frac{dx}{\sqrt[3]{(x-1)^2}}+\int_{1}^{2}\frac{dx}{\sqrt[3]{(x-1)^2}}$$

$$\int_{-7}^{2}\frac{dx}{\sqrt[3]{(x-1)^2}}=\lim_{t\to 1-0}\int_{-7}^{t}(x-1)^{-2/3}dx+\lim_{\tau\to 1+0}\int_{\tau}^{2}(x-1)^{-2/3}dx=\lim_{t\to 1-0}3(x-1)^{1/3}\Big|_{-7}^{t}+\lim_{\tau\to 1+0}3(x-1)^{1/3}\Big|_{\tau}^{2}=$$

$$=\lim_{t\to 1-0}3t^{1/3}+6+3-\lim_{t\to 1+0}3(-\tau)^{1/3}=9.$$

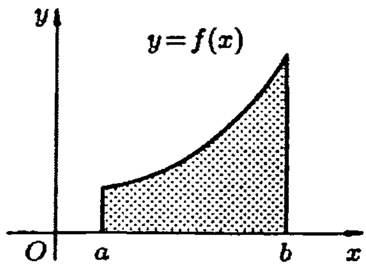
7.4. Aniq integralning tatbiqlari

Aniq integralning geometrik tatbiqlari

Yassi sirt yuzini hisoblash

Bizga $y=f(x)$ egri chiziq, $x=a, x=b$ to'g'ri chiziqlar va Ox o'qi bilan chegaralangan egri chiziqli trapetsiyaning yuzi

$$S=\int_{a}^{b}f(x)dx=\int_{a}^{b}ydx$$



formula bo'yicha hisoblanishi ma'lum.

Agar $x\in[a;b]$ da $f(x)\le 0$ bo'lsa, u holda

$$S=-\int_a^b f(x)\,dx.$$

Yuqoridagi ikkita formulani bittaga birlashtirish mumkin

$$S=\int_a^b |f(x)|\,dx.$$

$y=f_1(x)$ va $y=f_2(x)$ egri chiziqlar bilan chegeralangan yassi sirt yuzi quyidagi formula bilan topiladi:

$$S=\int_a^b [f_2(x)-f_1(x)]\,dx.$$

Bu yerda a va b sonlar yuqoridagi egri chiziqlar kesishish nuqtalarining absissalari. Bu yerda $f_2(x)\ge f_1(x)$.

$x=\varphi(y)$ egri chiziq, $y=c$, $y=d$ to'g'ri chiziqlar va Oy o'qi bilan chegaralangan egri chiziqli trapetsiyaning yuzi quyidagi formula bo'yicha hisoblanadi:

$$S=\int_c^d \varphi(y)\,dy.$$

Egri chiziqli trapetsiyadagi egri chiziq parametik tenglamasi $\begin{cases} x=\varphi(t), \\ y=\psi(t) \end{cases}$

$(\alpha\le t\le\beta)$ bilan berilgan bo'lsin, bunda $\varphi(\alpha)=a$, $\varphi(\beta)=b$, $[\alpha;\beta]$ kesmada $\psi(t)$ uzluksiz, $\varphi(t)$ esa monoton va uzluksiz $\varphi'(t)$ hosilaga ega deb faraz qilamiz. O'zgaruvchini almashtirish qoidasiga asosan quyidagiga ega bo'lamiz:

$$S=\int_a^b f(x)dx=\int_\alpha^\beta \psi(t)\varphi'(t)dt$$

Egri chiziq yoyi uzunligini hisoblash

Tekislikda egri chiziq $y=f(x)$ yoki $x=\varphi(y)$ tenglamasi bilan berilgan. Egri chiziqda $A(a;c)$ va $B(b;d)$ nuqtalarni tanlaymiz. Egri chiziqning A nuqtasidan B nuqtasigacha bo‘lgan l yoyi uzunligi quyidagi formulalar bo‘yicha hisoblanadi:

$$l=\int_a^b\sqrt{1+(y')^2}\,dx;$$

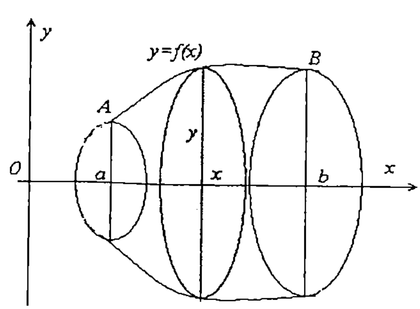
$$l=\int_c^d\sqrt{1+(x')^2}\,dy.$$

Agar egri chiziq parametrik tenglamasi bilan berilgan bo‘lsa $\begin{cases}x=x(t),\\ y=y(t),\end{cases}$ $t_1\le t\le t_2$, egri chiziq yoyi uzunligi quyidagi formula bo‘yicha hisoblanadi.

$$l=\int_{t_1}^{t_2}\sqrt{(x')^2+(y')^2}\,dt.$$

Aylanma jism hajmi va sirtini hisoblash

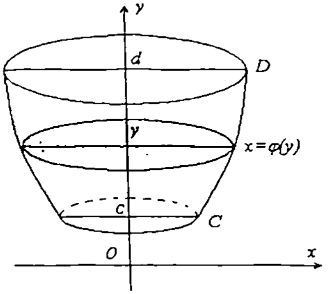
Uzluksiz $y=f(x)$ egri chiziq, absissalar o‘qi hamda $x=a$, $x=b$ $(a<b)$ to‘g‘ri chiziqlar bilan chegaralangan egri chiziqli trapetsiyani Ox o‘q atrofida aylantirishdan hosil bo‘lgan jism hajmini $V=\pi\int_a^b[f(x)]^2dx$



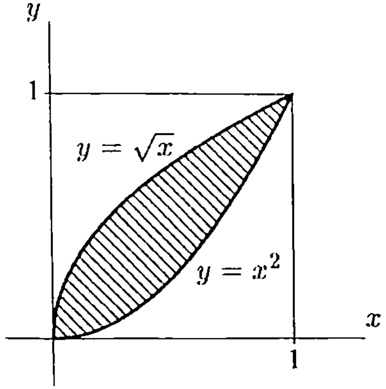
formula bilan hisoblaymiz.

Xuddi shunga o‘xshash, uzluksiz $x=\varphi(y)$ egri chiziq, ordinatalar o‘qi va $y=c, y=d$ $(c<d)$ to‘g‘ri chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning Oy o‘q atrofida aylanishidan hosil bo‘lgan jism hajmini

$$V=\pi\int_c^d [\varphi(y)]^2 dy$$



formula bilan hisoblaymiz.



Xuddi shunga o‘xshash, uzluksiz $x=\varphi(y)$ egri chiziq, ordinatalar o‘qi va $y=c, y=d$ $(c<d)$ to‘g‘ri chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning Oy o‘q atrofida aylanishidan hosil bo‘lgan jism hajmi quyidagi formula bilan hisoblanadi:

$$V=\pi\int_c^d x^2 dy=\pi\int_c^d (\varphi(y))^2 dy, \text{ bu yerda } x=\varphi(y), \quad y\in[c,d]$$

Misollar

1. $y=\sqrt{x}$ va $y=x^2$ chiziqlar bilan chegaralanagan figuraning yuzini toping.

Yechish. Ushbu

$$\begin{cases} y=x^2, \\ y=\sqrt{x} \end{cases}$$

tenglamalar sistemani yyechib egri chiziqlar kesishish nuqtalarining koordinatalarini topamiz.

$$x_1=0, \ x_2=1, \ y=0, \ y_2=1.$$

Berilgan figura yuqoridan $y=\sqrt{x}$, $0\le x\le 1$ chiziq bilan, quyidan esa $y=x^2$, chiziq bilan chegaralangan. Shuning uchun

$$S=\int_0^1(\sqrt{x}-x^2)dx=\frac{2x^{\frac{3}{2}}}{3}\Bigg|_0^1-\frac{1}{3}x^3\Bigg|_0^1=\frac{2}{3}-\frac{1}{3}=\frac{1}{3}.$$

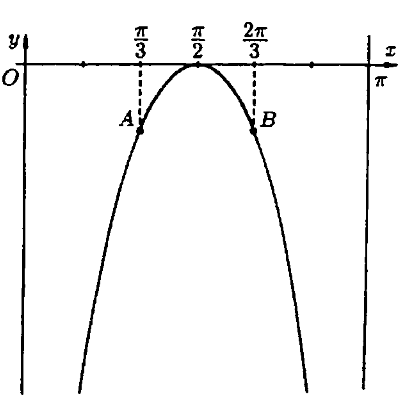
2. Ox o'qi va $\begin{cases}x=a(t-\sin t),\\ y=a(1-\cos t),\end{cases}$ $0\le t\le 2\pi$ sikloidaning bir arkasi bilan chegaralangan figura yuzini hisoblang.

Yechish. Yuqoridagi formulaga ko'ra

$$S=\int_0^{2\pi}a(1-\cos t)a(1-\cos t)dt=a^2\int_0^{2\pi}(1-\cos t)^2$$

$$=a^2(\int_0^{2\pi}dt-2\int_0^{2\pi}\cos tdt+\int_0^{2\pi}\cos^2 tdt)=a^2((t-2\text{s}$$

$$+\frac{1}{2}\int_0^{2\pi}(1+\cos 2t)dt)=a^2(2\pi+\frac{1}{2}(t+\frac{1}{2}\sin 2t)\Bigg|_0^2$$



3. $y=\ln\sin x$ egri chiziqning $x_1=\frac{\pi}{3}$ dan $x_2=\frac{2\pi}{3}$ gacha bo'lgan yoyining uzunligini hisoblang.

Yechish. $y=\ln\sin x$ funksiyaning $x\in(0;\pi)$ da grafigini tasvirlaymiz.

$y=\ln\sin x$, $y'=\frac{\cos x}{\sin x}$, $\quad\sqrt{1+(y')^2}=\sqrt{1+\frac{\cos^2 x}{\sin^2 x}}=\frac{1}{\sin x}$, $\quad x\in\left[\frac{\pi}{3};\frac{2}{3}\pi\right]$. $\quad AB$

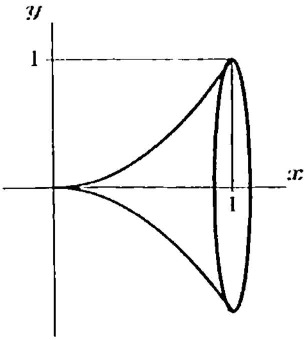
yoyning l uzunligini hisoblaymiz:

$$l=\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}}\frac{dx}{\sin x}=\ln\left|tg\frac{x}{2}\right|\Bigg|_{\frac{\pi}{3}}^{\frac{2\pi}{3}}=\ln\sqrt{3}-\ln\frac{1}{\sqrt{3}}=2\ln\sqrt{3}.$$

4. $\begin{cases}x=a(t-\sin t),\\ y=a(1-\cos t)\end{cases}$ $0\le t\le 2\pi$ sikloida arkasi uzunligini hisoblang.

Yechish. $l=\int_0^{2\pi}\sqrt{a^2(1-\cos t)^2+a^2\sin^2 t}dt=a\int_0^{2\pi}\sqrt{2(1-\cos t)}dt=$

$$=a\int_0^{2\pi}\sqrt{4\sin^2\frac{t}{2}}dt=2a\int_0^{2\pi}\sin\frac{t}{2}dt=-4a\cos\frac{t}{2}\Big|_0^{2\pi}=8a.$$



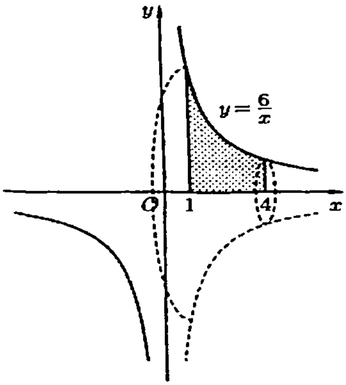
5. $y=x^2$ parabola va $y=0$, $x=1$ to‘g‘ri chiziqlar bilan chegaralangan sohani Ox o‘qi atrofida aylatirishdan hosil bo‘lgan jism hajmini toping.

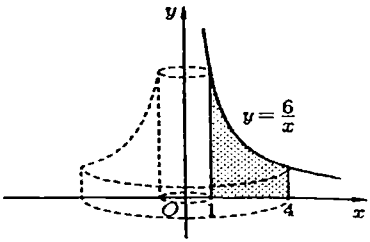
Yechish. $a=0, b=1, f(x)=x^2$, bu yerdan

$$V_x=\pi\int_a^b f^2(x)dx=\pi\int_0^1\left(x^2\right)^2dx=\pi\frac{x^2}{5}=\frac{1}{5}\pi.$$

6. $xy=6, x=1, x=4, y=0$ chiziqlar bilan chegaralangan shaklni Ox va Oy o‘qi atrofida aylantirishdan hosil bo‘lgan jism hajmini toping.

Yechish.





$$V_{Ox}=\pi\int_1^4\left(\frac{6}{x}\right)^2dx=36\pi\left(-\frac{1}{x}\right)\Big|_1^4=27\pi.$$

$$V_{Oy}=2\pi\int_1^4 x\cdot\frac{6}{x}dx=2\pi\cdot 6x\Big|_1^4=36\pi.$$

Iqtisodiyotda integral tushunchasidan foydalanish

Marjinal miqdorlar

Agar firmaning marjinal daromad funksiyasi $MR(Q)$ berilgan bo‘lsa, ya’ni

$$MR(Q)=f(Q)$$

funksiya ma'lum bo'lsa, u holda firmaning yalpi daromad funksiyasi quyidagi aniqmas integral yordamida topiladi:

$$R(Q)=\int MR(Q)dQ=\int f(Q)dQ=F(Q)+C.$$

Bu yerda $F(Q)$ – boshlang'ich funksiya.

Misol. Firmaning marjinal daromad funksiyasi

$$MR(Q)=150-15Q$$

ko'rinishida berilgan. Agar $Q=10$ birlik mahsulot ishlab chiqarilganda firmaning umumiy daromad $R(Q)=1000$ p.b. ni tashkil etsa, u holda yalpi daromad funksiyasi qanday ko'rinishda bo'ladi?

Yechish. Umumiy daromad funksiyasi $R(Q)$ ni quyidagi integraldan foydalanib topamiz:

$$R(Q)=\int(150-15Q)dQ=150Q-\frac{15Q^2}{2}+C.$$

$Q=10$, $R(Q)=1000$ bo'lganidan foydalanib, $C=250$ ekanligini aniqlaymiz. Demak, firmaning yalpi daromad funksiyasi

$$R=150Q-\frac{15Q^2}{2}+250$$

ko'rinishda bo'ladi.

Xuddi shuningdek, marjinal xarajat va foyda funksiyalari ma'lum bo'lganda umumiy xarajat va yalpi foyda funksiyalarini ham aniqmas integraldan foydalanib topish mumkin.

Misol. Firmaning marjinal xarajat funksiyasi

$$MC=Q^2+3Q+8$$

ko'rinishga ega. Agar $Q=0$ da firmaning xarajati 250 p.b. ni tashkil etsa, u holda uning umumiy xarajat funksiyasini toping.

Yechish. Umumiy xarajat funksiyasi $C(Q)$ ni quyidagi integraldan foydalanib topamiz: $\frac{d}{dQ}(C)=MC=Q^2+3Q+8.$

$$C(Q)=\int\left(Q^2+3Q+8\right)dQ.$$

$$C(Q)=\int Q^2dQ+3\int QdQ+8\int dQ=\frac{1}{3}Q^3+3\left(\frac{1}{2}Q^2\right)+8Q+k,$$

$C(Q)=250,\ Q=0$ da $k=250.$

Demak, umumiy xarajatlar funksiyasi quyidagi ko'rinishda bo'ladi:

$$C(Q)=\frac{1}{3}Q^3+\frac{3}{2}Q^2+8Q+250.$$

$Q(L)$ – umumiy mahsulot funksiyasi, $MP(L)$ – marjinal mahsulot funksiyasi bo'lsin. Marjinal mahsulot funksiyasi umumiy mahsulot hosilasidan iborat bo'lganligi sababli quyidagi tenglik o'rinli bo'ladi:

$$MP(L)=\frac{dQ(L)}{dL}\Rightarrow Q(L)=\int MP(L)dL$$

Faraz qilamiz $MP(L)=a=const>0$ bo'lsin. U holda

$$Q(L)=\int adL=aL+C.$$

Iqtisodiy nuqtai nazardan $L=0$ bo'lganda umumiy mahsulot $Q=0$ bo'ladi deb faraz qilamiz. U holda $C=0\Rightarrow Q=aL$.

Misol. Marjinal mahsilot funksiyasi $MP(L)=3(3L+2)$ ko'rinishda bo'lgan firmaning umumiy mahsulot funksiyasini topnig.

$$Q(L)=\int MP(L)dL=3\int(3L+2)dL=\frac{9}{2}L^2+6L+C.$$

Kobb-Duglas funksiyasi asosida ishlab chiqarish hajmini aniqlash

Agar Kobb-Duglas funsiyasida mehnat sarfi vaqtga chiziqli bog'liq bo'lib kapital sarfi o'zgarmas bo'lsa, u holda bu funksiya

$$Q(t)=(\alpha t+\beta)e^{\gamma t}$$

ko'rinishiga ega bo'ladi. U holda $t=0,1,2,...T$ yil ichida ishlab chiqariilgan mahsulot miqdori quyidagi aniq integral yordamida topiladi.

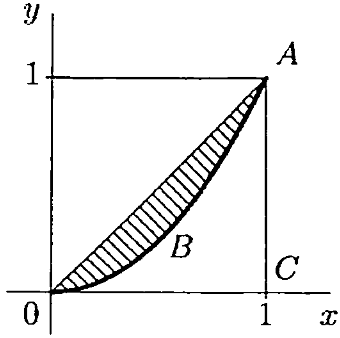
$$Q=\int_0^T(\alpha t+\beta)e^{\gamma t}\,dt,$$

Misol. Agar Kobb-Duglas funksiyasi

$$Q(t)=(1+t)e^{3t}$$

ko'rinishga ega bo'lsa, u holda 4 yilda ishlab chiqariladigan mahsulot miqdorini aniqlang.

Yechish. Yuqoridagi formulaga asosan 4 yilda ishlab chiqariladigan mahsulot miqdorini topamiz.



$$Q = \int_0^4 (1+t)e^{3t}\,dt$$

Ushbu integralni bo'laklab integrallaymiz.

$$Q = \int_0^4 (1+t)e^{3t}\,dt = \begin{Bmatrix} u = t+1, \\ du = dt, \\ dv = e^{3t}dt, \\ v = \frac{1}{3}e^{3t}. \end{Bmatrix} = \frac{1}{3}(t+1)e^{3t} - \frac{1}{3}\int e^{3t}dt = \frac{1}{3}(t+1)e^{3t} - e^{3t} = \frac{1}{3}(t+1)e^{3t}\Big|_0^4 -$$

$$-\frac{1}{9}e^{3t}\Big|_0^4 = \frac{5}{3}e^{12} - \frac{1}{3} - \frac{1}{9}e^{12} + \frac{1}{9} = 2{,}53\cdot 10^5.$$

Daromadning aholi o'rtasidagi taqsimotining notekislik darajasini (Jini koeffitsiyentini) aniqlash

Lorens egri chizig'i bo'yicha tengsizlik darajasini aniqlash uchun Djini koeffitsiyentini qo'llashga doir misolni keltiramiz. Lorens egri chizig'i daromad foizining, bu daromadga ega bo'lgan aholi foiziga bog'lanishni ifodalaydi. *Oy* o'qi bo'yicha ma'lum daromadga ega bo'lgan aholini, *Ox* o'qi bo'yicha aholining ma'lum qismiga to'g'ri keladigan daromadlar ulushi qo'yiladi. Lorens egri chizig'i yordamida aholi daromadini taqsimlashda tengsizlik darajasini aniqlash mumkin. Daromad bir tekis taqsimlanganda Lorens egri chizig'i chiziqli funksiya - *OA* bissektrisa bo'ladi, notekis taqsimlanganda - *OBA* ko'rinishdagi egri chiziq bo'ladi. Djini koeffitsiyenti deb *OA* bissektrisa va Lorens egri chizig'i orasidagi shakl yuzining *OAC* uchburchak yuziga nisbatiga aytiladi. Koeffitsiyent nolga teng bo'lganda aholi daromadlarida to'liq tenglik ko'rinib turibdi, koeffitsiyent qiymati 0,3 dan kichik bo'lganda –kuchsiz zaif tengsizlik, 0,3 – 0,7 da undan kuchliroq tengsizlik, 0,7 – 1 da kuchli tengsizlik ko'rinib turibdi.

Misol. Mamlakatlarning biri uchun Lorens egri chizig'i $y = x^2$ tenglama bilan ifodalanishi mumkin, bu yerda x–aholi qismi (ulushi), y–aholi daromadining ulushi. k Djini koeffitsiyentini hisoblang.

Yechish. $S_{\Delta OAC} = \frac{1}{2}$, $\quad S_{OAB} = \int\limits_0^1 (x - x^2)dx = \left(\frac{x^2}{2} - \frac{x^3}{3}\right)/_0^1 = \frac{1}{6}$

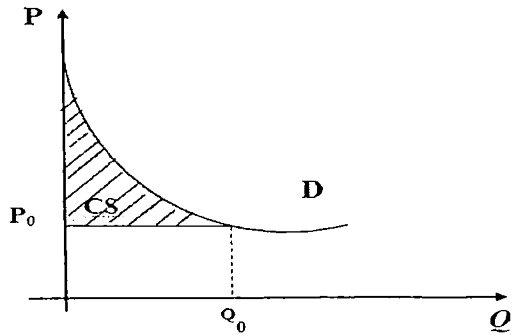
bo'lgani uchun, $k = \frac{S_{OAB}}{S_{OAC}} = \frac{1}{6} \div \frac{1}{2} = \frac{1}{3} > 0{,}3$

$k = 0{,}3$, $(0{,}3;0{,}7)$ intervalga teng bo'lgani uchun o'rganilayotgan mamlakatda daromadlarda kattagina tengsizlik borligi kuzatiladi.

Iste'molchilarning ortiqcha foydasi. Ishlab chiqaruvchi (ta'minotchilar)ning ortiqcha foydasi

Bozorda muvozanat narx o'rnatilgandan so'ng mahsulotni yuqoriroq narxda sotib olmoqchi bo'lgan iste'molchilar uni muvozanat bahosida sotib olish oqibatida qandaydir yutuqqa ega bo'ladilar. Ana shunday iste'molchilar yutug'larining yig'indisi iste'molchilarning ortiqcha foydasi deb ataladi.

Grafik ma'noda istemolchilarning ortiqcha foydasi talab egri chizig'i, ordinatalar o'qi va absissalar o'qiga parallel va bozor muvozanati nuqtasidan o'tuvchi to'g'ri chiziq bilan chegaralangan figura yuzasiga teng deb tasavvur qilish mumkin.



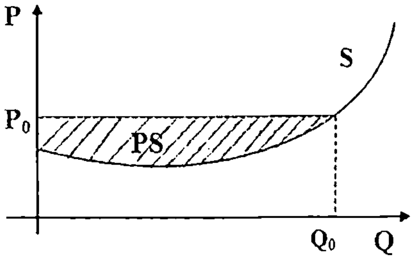
Iste'molchilarning ortiqcha foydasi $S.S.$ bilan belgilanadi va quyida aniq integral yordamida topiladi.

$$CS = \int_0^{Q_0} P(Q)dQ - P_0Q_0$$

Ishlab chiqaruvchi (ta'minotchilar)ning ortiqcha foydasi ta'minotchilar o'z tovarini bozordagi muvozanat narxda sotganda hosil bo'ladigan umumiy foydalarini ifodalaydi va u quyidagi formula yordamida topiladi:

$$PS = P_0Q_0 - \int_0^{Q_0} P(Q)\, dQ$$

Geometrik ma'noda ta'minotchilarning ortiqcha foydasini taklif egri chizig'i, ordinatalar o'qi va absissalar o'qiga parallel va bozor muvozanati nuqtasidan o'tuvchi to'g'ri chiziq bilan chegaralangan figura yuzasiga teng deb tasavvur qilish mumkin.



Misol. Biror tovarga talab $P = \dfrac{50}{Q+1}$ funksiya bilan berilgani ma'lum. Bu yerda $Q-$ mahsulot miqdori (dona), $P-$ bir birlik mahsulot narxi, taklif esa $P = Q+6$ funksiya bilan beriladi.

a) ushbu mahsulotni sotib olishdan iste'molchilar yutuqlari miqdori;

b) ushbu mahsulotni sotishdan ishlab chiqaruvchining (ta'minotchining) ortiqcha foydasini hisoblang.

Yechish. a) $\begin{cases} p = \dfrac{50}{Q+1} \\ p = Q+6 \end{cases}$ sistemani yechish orqali narxning va berilgan mahsulot miqdorining muvozanat qiymatlarini aniqlash zarur.

Sistemaning yechimi $Q^* = 4,\ P^* = 10$ juftlik hisoblanadi.

Iste'molchilar ortiqcha foydasi formulasidan foydalanib topamimz:

$$CS = \int_0^4 \frac{50}{Q+1} dQ - 10 \cdot 4 = 50 \ln|Q+1| /_0^4 - 40 =$$
$$= 50 \ln 5 - 50 \ln 1 - 40 = 50 \ln 5 - 40 = 40 (пул / бирл).$$

b) Modomiki taklif funksiyasi chiziqli funksiya bilan berilganligi uchun, ta'minotchining ortiqcha foydasi kattaligini turli usullarda hisoblash mumkin.

Birinchi usul PS ni hisoblash formulasiga asoslanadi.

$$PS = 10 \cdot 4 - \int_0^4 (Q+6) dQ = 40 - \left(\frac{Q^2}{2} + 6Q \right) /_0^4 = 40 - 8 - 24 = 8 (пул / бирл).$$

7.5. Talabaning mustaqil ishi

1-topshiriq

1-2 misollarda bevosita integrallash usuli bilan aniqmas integrallarni toping. Natijani differensiallash orqali tekshiring.

3-4 misollarda o'zgaruvchini almashtirish usuli bilan aniqmas integrallarni toping.

5- misolda bo'laklab integrallash usuli bilan aniqmas integrallarni toping.

6-misolda ratsional kasrlarni integrallash usuli bilan aniqmas integrallarni toping.

1-variant

1. $\int \frac{dx}{x^3}$
2. $\int \frac{dx}{\sqrt{x^3}}$
3. $\int (7x-1)^{23} dx;$
4. $\int x^2 \cdot \sin(x^3+1) dx;$
5. $\int x \ln x dx.$
6. $\int \frac{6x-7}{x^2+4x+13}.$

2-variant

1. $\int 2^x dx;$
2. $\int \frac{dx}{\sqrt{5-x^2}};$
3. $\int \frac{xdx}{x^2+1}.$
4. $\int \frac{dx}{\sqrt{e^x-1}}$
5. $\int (2x+3) \cdot \cos xdx.$
6. $\int \frac{4dx}{x+3}.$

3-variant

1. $\int (3tgx - 2ctgx)^2 dx$
2. $\int \frac{4\sqrt{1-x^2}+3x^2}{x^2-1} dx.$
3. $\int \sqrt{4x-5dx}.$
4. $\int \frac{dx}{(3x+2)^4}.$

5. $\int x \cdot sh5x dx.$

6. $\int \frac{dx}{(x-1)^5}.$

4-variant

1. $\int x^{10} dx.$
2. $\int \frac{dx}{x^7}.$
3. $\int \sin^3 x \cdot \cos x dx.$
4. $\int e^{x^3} \cdot x^2 dx.$
5. $\int \frac{x \cdot \cos x dx}{\sin^3 x}.$
6. $\int \frac{11 dx}{(x+2)^3}.$

5-variant

1. $\int \sqrt[4]{x} dx.$
2. $\int \frac{dx}{x^2+9}.$
3. $\int \frac{\ln^5 x dx}{x}.$
4. $\int \frac{\sin x dx}{\cos x + 1}.$
5. $\int x^2 \ln x dx.$
6. $\int \frac{dx}{x^2+10x+29}.$

6-variant

1. $\int \frac{dx}{x^2 - \frac{1}{2}}.$
2. $\int \frac{dx}{\sqrt{x^2+3}}.$
3. $\int \frac{x^2 dx}{x^3+1}.$
4. $\int \frac{arctgx dx}{x^2+1}.$
5. $\int (x^2-4x+1)e^{-x} dx.$
6. $\int \frac{(x+6)dx}{x^2-2x+17}.$

7-variant

1. $\int (3 \cdot 5^x - \frac{2}{\sqrt[3]{x}} + 7) dx;$
2. $\int \frac{x^2-3x+5}{\sqrt{x}} dx.$
3. $\int \frac{x - \sin\frac{1}{x}}{x^2} dx.$
4. $\int \frac{5x-1}{\sqrt{4-x^2}} dx.$
5. $\int x^3 e^x dx.$
6. $\int \frac{(4x-1)dx}{x^2+x+1}.$

8-variant

1. $\int \frac{x^4+x^2-6x}{x^3} dx.$
2. $\int (\frac{5}{x} - \frac{10}{\sqrt[4]{x^3}} - \frac{3}{x^2+7}) dx.$
3. $\int \frac{4x+3}{\sqrt{x^2-5}} dx.$
4. $\int e^{\sin^2 x} \cdot \sin 2x dx.$
5. $\int \frac{\arccos x dx}{\sqrt{1+x}}.$
6. $\int \frac{8x+5}{(x^2-2x+17)}.$

9-variant

1. $\int\sqrt{x}(x^2+1)dx.$
2. $\int\frac{3+\sqrt{4-x^2}}{\sqrt{4-x^2}}dx.$
3. $\int\frac{1-2\sin x}{\cos^2 x}dx.$
4. $\int\frac{3x-4}{x^2-4}dx.$
5. $\int\frac{\arcsin\sqrt{x}}{\sqrt{1-x}}dx.$
6. $\int\frac{dx}{\left(x^2+1\right)^3}.$

10-variant

1. $\int\frac{(x^3+2)^2}{\sqrt{x}}dx.$
2. $\int(4\sin x+8x^3-\frac{11}{\cos^2 x})dx$
3. $\int\frac{\sqrt{1-x^2}}{x^2}dx.$
4. $\int\frac{dx}{\sqrt{x}\left(1+\sqrt{x}\right)}.$
5. $\int\frac{x^2dx}{\left(x^2-1\right)^2}.$
6. $\int\frac{dx}{\left(x^2-4x+29\right)^2}$

11-variant

1. $\int\frac{dx}{\sqrt{16-9x^2}}.$
2. $\int\cos 2xdx.$
3. $\int\sqrt{9-x^2}dx.$
4. $\int\frac{dx}{x\sqrt{x+1}}.$
5. $\int\cos\ln xdx.$
6. $\int\frac{3x-2}{\left(x^2+6x+10\right)^2}dx.$

12-variant

1. $\int(9x+2)^{17}dx.$
2. $\int\frac{dx}{8x-1}.$
3. $\int x\sqrt{2-x}dx.$
4. $\int\frac{\sqrt{x}dx}{x+16}.$
5. $\int e^{3x}\cdot\cos^2 xdx.$
6. $\int\frac{7x+4}{(x-3)(x+2)}dx.$

13-variant

1. $\int 4^{3-5x}dx.$
2. $\int\sqrt{3x+4dx}.$
3. $\int\cos(6x+1)dx.$
4. $\int\frac{dx}{\sqrt[3]{(5x-2)^4}}.$
5. $\int x^2e^xdx.$
6. $\int\frac{x^2+5x-2}{\left(x^2-1\right)(x+1)}dx.$

14-variant

1. $\int tg^2 x dx.$
2. $\int \frac{4x+1}{x-5} dx.$
3. $\int \frac{\sqrt{tgx}dx}{\cos^2 x}.$
4. $\int \frac{e^x dx}{e^{2x}+9}.$
5. $\int x^3 \cos x dx.$
6. $\int \frac{x^5-1}{x^3+x^2+x} dx.$

15-variant

1. $\int \sin^2 x dx;$
2. $\int \frac{x^2}{x^2+1} dx;$
3. $\int \frac{x^5 dx}{\sqrt{x^6+7}}.$
4. $\int \frac{dx}{\arccos x \cdot \sqrt{1-x^2}}.$
5. $\int x \cdot 2^x dx.$
6. $\int \frac{2x-3}{(x-5)(x+2)} dx.$

16-variant

1. $\int \cos^2 x dx$
2. $\int \frac{x-2}{x+3} dx.$
3. $\int \frac{(2x+3)dx}{(x^2+3x-1)^4}.$
4. $\int \cos^{11} 2x \cdot \sin 2x dx.$
5. $\int \frac{x}{\cos^2 x} dx.$
6. $\int \frac{x+2}{x^2-6x+5} dx.$

17-variant

1. $\int \frac{x^2 dx}{x^2-9}.$
2. $\int \frac{5+\sin^3 x}{\sin^2 x} dx.$
3. $\int \frac{7^{\sqrt{x}} dx}{\sqrt{x}}.$
4. $\int \frac{e^{\frac{1}{x}}}{x^2} dx.$
5. $\int \frac{\ln x}{\sqrt{x}} dx.$
6. $\int \frac{dx}{x^4+x^2}.$

18-variant

1. $\int \frac{dx}{x^2\sqrt{x}};$
2. $\int \frac{dx}{x^2+3}.$
3. $\int \frac{\ln 5x dx}{x}.$
4. $\int ctgx dx.$
5. $\int x arctgx dx.$
6. $\int \frac{x^5+x^4-8}{x^3-4x} dx.$

19-variant

1. $\int \frac{1}{5^x} dx.$
2. $\int \frac{dx}{\sqrt{4-x^2}}.$
3. $\int 4x \cdot \sqrt[3]{x^2+8} dx.$

4. $\int \frac{\cos x dx}{\sin^2 x}$.

5. $\int \frac{\arcsin\sqrt{x}}{\sqrt{1-x}} dx$.

6. $\int \frac{dx}{x^3 - 8}$.

20-variant

1. $\int \frac{dx}{\sqrt{x^2 - 1}}$.
2. $\int \frac{dx}{x^2 - 25}$.
3. $\int tg 2x dx$.
4. $\int \frac{x dx}{x^4 + 1}$.
5. $\int e^x (\cos x - \sin x) dx$.
6. $\int \frac{7x^3 - 10x^2 + 50x - 77}{(x^2 + 9)(x^2 + x - 2)} dx$.

21-variant

1. $\int \left(x + \frac{2}{x}\right)^2 dx$
2. $\int \frac{dx}{4x^2 + 1}$.
3. $\int e^{-x^3} \cdot x^2 dx$.
4. $\int \frac{x^2 dx}{\sqrt{x^6 - 4}}$.
5. $\int \sqrt{x^2 + 4} dx$.
6. $\int \frac{5dx}{x + \sqrt{2}}$.

22-variant

1. $\int \left(7^x - \frac{8}{x} + 4\cos x\right) dx$.
2. $\int \left(\frac{\sqrt{3}}{\cos^2 x} - \sqrt[3]{x} - \frac{2}{x^4}\right) dx$.
3. $\int \left(8\cos\frac{x}{3} - 5\right)^2 \sin\frac{x}{3} dx$.
4. $\int \frac{(3x^2 - 2x + 7)dx}{\sqrt{x^3 - x^2 + 7x - 2}}$.
5. $\int arctg\sqrt{x - 1} dx$.
6.. $\int \frac{4dx}{\left(x - \frac{1}{2}\right)^3}$.

23-variant

1. $\int \frac{\sqrt{x} - 3\sqrt[5]{x^2} + 1}{\sqrt[4]{x}} dx$.
2. $\int (0,7 \cdot x^{-0,1} + 0,2 \cdot (0,5)^x) dx$.
3. $\int x(2x+1)^{35} dx$.
4. $\int (x-2)\sqrt{x+4} dx$.
5. $\int e^{\arcsin x} dx$.
6. $\int \frac{7dx}{(x+3)^6}$.

24-variant

1. $\int (5shx - 7chx + 1) dx$.
2. $\int (x^2 - 1)(\sqrt{x} + 4) dx$.
3. $\int \frac{3\sqrt{x} - 2\cos\frac{1}{x^2}}{x^3} dx$.

4. $\int \frac{7x+2}{\sqrt{x^2+10}}dx.$

5. $\int x^2 \ln^2 x dx.$

6. $\int \frac{dx}{(3x+2)^4}.$

25-variant

1. $\int \frac{7-\sqrt{x^2+\pi}}{\sqrt{x^2+\pi}}dx.$

2. $\int \left(\frac{\sqrt{x-5}}{x}\right)^3 dx.$

3. $\int \frac{dx}{e^x+e^{-x}}.$

4. $\int \frac{x+8}{x^2+3}dx.$

5. $\int \frac{dx}{\left(x^2+1\right)^3}.$

6. $\int \frac{dx}{x^2-4x+8}.$

2-topshiriq

1-misolda o'zgaruvchini almashtirish usulida aniq integralni hisoblang

2- misolda bo'laklab integrallash usuli bilan aniq integralni hisoblang.

3-4-misollarda xosmas integrallarni hisoblang (agar ular yaqinlashuvchi bo'lsa).

1-variant

1. $\int_0^3 \frac{x+2}{\sqrt{1+x}}dx.$

2. $\int_1^e \ln^2 x dx.$

3. $\int_1^\infty \frac{dx}{x}.$

4. $\int_0^3 \frac{dx}{\sqrt{9-x^2}}.$

2-variant

1. $\int_4^9 \frac{\sqrt{x}}{\sqrt{x}-1}dx.$

2. $\int_0^{\frac{\pi}{2}} x^2 \sin x dx.$

3. $\int_1^\infty \frac{dx}{x^2}.$

4. $\int_1^5 \frac{dx}{\sqrt{5-x}}.$

3-variant

1. $\int_0^{\ln 5} \frac{e^x\sqrt{e^x-1}}{e^x+3}dx.$

2. $\int_1^e \frac{\ln x}{x^2}dx.$

3. $\int_1^\infty \frac{dx}{\sqrt{x}}.$

4. $\int_{-1}^0 \frac{dx}{(x+1)^2}.$

4-variant

1. $\int\limits_{1}^{2}\frac{\sqrt{x^2-1}}{x}dx.$

2. $\int\limits_{0}^{1}\frac{\arcsin x}{\sqrt{1+x}}dx.$

3. $\int\limits_{-\infty}^{\infty}\frac{2xdx}{1+x^2}.$

4. $\int\limits_{0}^{2}\frac{dx}{\sqrt[3]{1-x}}.$

5-variant

1. $\int\limits_{0}^{2}x^2\sqrt{4-x^2}dx.$

2. $\int\limits_{0}^{1}xe^x dx.$

3. $\int\limits_{-\infty}^{\infty}\frac{dx}{x^2+2x+5}.$

4. $\int\limits_{\frac{3\pi}{4}}^{\pi}\frac{dx}{1+\cos x}.$

6-variant

1. $\int\limits_{0}^{1}\frac{dx}{\left(1+x^2\right)^3}.$

2. $\int\limits_{0}^{\pi}e^x\sin xdx.$

3. $\int\limits_{1}^{\infty}\frac{dx}{x\sqrt{x^2-1}}.$

4. $\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{2}}tgxdx.$

7-variant

1. $\int\limits_{0}^{\sqrt{3}}\sqrt{x^2+1}dx.$

2. $\int\limits_{0}^{\frac{\pi}{4}}\frac{x}{\cos^2 x}dx.$

3. $\int\limits_{0}^{\infty}e^{-2x}dx.$

4. $\int\limits_{-1}^{2,5}\frac{dx}{x^2-5x+6}.$

8-variant

1. $\int\limits_{0}^{\frac{\pi}{2}}\frac{\cos x}{6-5\sin x+\sin^2 x}dx.$

2. $\int\limits_{0}^{1}\frac{x\arcsin x}{\sqrt{1-x^2}}dx.$

3. $\int\limits_{0}^{\infty}x\sin xdx.$

4. $\int\limits_{-2}^{2}\frac{dx}{x^2-1}.$

9-variant

1. $\int\limits_0^{\frac{\pi}{2}} \frac{1-\cos x}{1+\cos x}dx.$

2. $\int\limits_0^1 x^3 arctgx dx.$

3. $\int\limits_0^{\infty} e^{-\sqrt{x}}dx.$

4. $\int\limits_{-\frac{1}{\ln 2}}^{0} \frac{e^{\frac{1}{x}}dx}{x^3}.$

10-variant

1. $\int\limits_0^{\frac{\pi}{4}} tg^4 x dx.$

2. $\int\limits_0^{16\pi} \cos^8 x dx.$

3. $\int\limits_1^{\infty} \frac{arctgx dx}{1+x^2}.$

4. $\int\limits_0^{\frac{1}{\ln 2}} \frac{e^{\frac{1}{x}}dx}{x^3}.$

11-variant

1. $\int\limits_{\frac{1}{2}}^{1} x\sqrt[4]{2x-1}dx.$

2. $\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{x\,dx}{\sin^2 x}.$

3. $\int\limits_1^{+\infty} \frac{dx}{(x+1)^4}.$

4. $\int\limits_1^2 \frac{dx}{x\sqrt{3x^2-2x-1}}.$

12-variant

1. $\int\limits_1^9 \frac{xdx}{\sqrt{2x+7}}.$

2. $\int\limits_0^{0,2} xe^{5x}dx.$

3. $\int\limits_{-1}^{+\infty} \frac{dx}{\sqrt[3]{(2x+1)^2}}.$

4. $\int\limits_1^2 \frac{2dx}{\sqrt{(x-1)(2-x)}}.$

13-variant

1. $\int\limits_{-0,4}^{0} (2+5x)^4 dx.$

2. $\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{3}} 4x tg^2 x dx.$

3. $\int\limits_1^{+\infty} \frac{\ln x}{x^3}dx.$

4. $\int\limits_0^{\frac{\pi}{4}} \frac{dx}{1-\cos 2x}.$

14-variant

1. $\int\limits_{0}^{\pi} \sin\left(\frac{5}{4}x - \frac{\pi}{4}\right)dx.$

2. $\int\limits_{1}^{e^2} \ln^2 x dx.$

3. $\int\limits_{-\infty}^{+\infty} \frac{dx}{x^2 + 4x + 9}.$

4. $\int\limits_{0}^{1} x \ln x dx.$

15-variant

1. $\int\limits_{0}^{1} \frac{x^2}{(x+1)^3} dx.$

2. $\int\limits_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{x \cos x}{\sin^2 x} dx.$

3. $\int\limits_{0}^{+\infty} e^{-x} \sin x dx.$

4. $\int\limits_{0}^{\frac{1}{4}} \frac{dx}{x \ln x}.$

16-variant

1. $\int\limits_{0}^{3} x^2 \sqrt{9 - x^2} dx.$

2. $\int\limits_{0}^{2} \frac{x^3}{\sqrt{1 + x^2}} dx.$

3. $\int\limits_{0}^{+\infty} arctgx dx.$

4. $\int\limits_{0}^{1} \ln x dx.$

17-variant

1. $\int\limits_{\frac{3}{4}}^{1} \frac{2dx}{x\sqrt{x^2 + 1}}.$

2. $\int\limits_{0}^{\sqrt{3}} \frac{x^2}{\left(1 + x^2\right)^2} dx.$

3. $\int\limits_{-\infty}^{+\infty} x e^{2x} dx.$

4. $\int\limits_{-1}^{2} \frac{dx}{x}.$

18-variant

1. $\int\limits_{0}^{\ln 2} \frac{dz}{e^z + 1}.$

2. $\int\limits_{0}^{\frac{\pi^2}{4}} \sin \sqrt{x} dx.$

3. $\int\limits_{-\infty}^{+\infty} xe^{-x^2}dx.$

4. $\int\limits_{0}^{1} \frac{dx}{\sqrt{1-x^2}}.$

19-variant

1. $\int\limits_{-1}^{1} \frac{xdx}{\sqrt{5-4x}}.$

2. $\int\limits_{0}^{9} e^{\sqrt{x}}dx.$

3. $\int\limits_{\frac{1}{2}}^{+\infty} \frac{dx}{x\sqrt{1+x^2}}.$

4. $\int\limits_{2}^{3} \frac{xdx}{\sqrt{x-2}}.$

20-variant

1. $\int\limits_{1}^{16} \frac{dx}{x+\sqrt[4]{x}}.$

2. $\int\limits_{-1}^{0} xe^{-x}dx.$

3. $\int\limits_{1}^{+\infty} \frac{dx}{(1+x)\sqrt{x}}.$

4. $\int\limits_{0}^{2} \frac{dx}{\sqrt[3]{(x-1)^2}}.$

21-variant

1. $\int\limits_{-1}^{7} \frac{dx}{1+\sqrt[3]{x+1}}.$

2. $\int\limits_{0}^{2} \ln\left(x^2+4\right)dx.$

3. $\int\limits_{0}^{+\infty} 2x\sin xdx.$

4. $\int\limits_{0}^{1} \frac{dx}{x\ln^2 x}.$

22-variant

1. $\int\limits_{\frac{1}{2}}^{1} \frac{\sqrt{1-x^2}}{x^2}dx.$

2. $\int\limits_{1}^{e} \frac{\ln^3 x}{x^2}dx.$

3. $\int\limits_{-\infty}^{0} xe^{x}dx.$

4. $\int\limits_{0}^{e^3} \frac{dx}{x\sqrt{\ln x}}.$

23-variant

1. $\int\limits_{5}^{13} \sqrt{2x-1}dx.$

2. $\int\limits_{-1}^{0} 9x^2\ln(x+2)dx.$

3. $\int\limits_{-\infty}^{+\infty}\frac{dx}{x^2+6x+12}$.

4. $\int\limits_{0}^{1}\frac{dx}{x^2+x^4}$.

24-variant

1. $\int\limits_{0}^{1}\left(2x^3+1\right)^4 x^2dx$.

2. $\int\limits_{0}^{1}4x\arcsin x dx$.

3. $\int\limits_{0}^{+\infty}2e^{-\sqrt{x}}dx$.

4. $\int\limits_{1}^{3}\frac{dx}{\sqrt{3-x}}$.

25-variant

1. $\int\limits_{3}^{8}\frac{xdx}{\sqrt{1+x}}dx$.

2. $\int\limits_{0}^{1}\left(\arcsin x\right)^2 dx$.

3. $\int\limits_{0}^{+\infty}e^{-4x}dx$.

4. $\int\limits_{2}^{5}\frac{dx}{\left(x-4\right)^2}$.

3-topshiriq

1- misolda berilgan chiziq bilan chegaralangan shaklning yuzini toping.

2- misolda berilgan chiziq yoyi uzunligini toping.

3-misolda tenglamasi bilan berilgan chiziqlar bilan chegaralangan F shaklning ko'rsatilgan koordinata o'qi atrofida aylanishidan hosil bo'lgan jismning hajmini toping.

4-misolda iqtisodiy mazmundagi masalalarning matematik modelini tuzing va hisoblash ishlarini bajaring.

1-variant

1. $y=-x^3,\ y=-9x$.

2. $y=2\sqrt{x},\ \ x=0$ dan $x=1$ gacha.

3. $y=1-x^2,\ \ y=0,\ x=0,\ Ox$.

4. Yangi texnologiyani joriy qilgandan boshlab ishlab chiqarish unumdorligining o'zgarishi $z=32-2^{-0,5t+5}$ funksiya bilan beriladi, bunda t - vaqt (oylar hisobida). Joriy qilingan texnologiya bo'yicha birinchi oyda ishlab chiqarilgan mahsulot hajmini toping.

2-variant

1. $y=\arccos x,\ x=-1,\ x=0,\ y=0.$

2. $y=\ln x,\ \ x=\sqrt{3}$ dan $x=\sqrt{8}$ gacha.

3. $y=e^{x},\ \ x=0,\ x=1,\ y=0,\ Ox.$

4. Yangi texnologiyani joriy qilgandan boshlab ishlab chiqarish unumdorligining o'zgarishi $z=32-2^{-0,5t+5}$ funksiya bilan beriladi, bunda t – vaqt (oylar hisobida). Joriy qilingan texnologiya bo'yicha uchinchi oyda ishlab chiqarilgan mahsulot hajmini toping.

3-variant

1. $y=tg^{2}x,\ x=\dfrac{\pi}{4},\ y=0.$

2. $x=t-\sin t,\ \ y=1-\cos t,\ t=0$ dan $t=2\pi$ gacha.

3. $y=4x-x^{2},\ \ y=x,\ Ox.$

4. Yangi texnologiyani joriy qilgandan boshlab ishlab chiqarish unumdorligining o'zgarishi $z=32-2^{-0,5t+5}$ funksiya bilan beriladi, bunda t – vaqt (oylar hisobida). Joriy qilingan texnologiya bo'yicha oltinchi oyda ishlab chiqarilgan mahsulot hajmini toping.

4-variant

1. $y=\sin x,\ y=2\sin x,\ x=0,\ x=\dfrac{7}{4}\pi.$

2. $y=\ln\sin x,\ \ x=\dfrac{\pi}{3}$ dan $x=\dfrac{\pi}{2}$ gacha.

3. $y=\sqrt{x},\ \ y=x,\ Ox.$

4. Yangi texnologiya joriy qilgandan boshlab ishlab chiqarish unumdorligining o'zgarishi $z=32-2^{-0,5t+5}$ funktsiya bilan beriladi, bunda t – vaqt (oylar hisobida). Joriy qilingan texnologiya bo'yicha yilning oxirgi oyida ishlab chiqarilgan mahsulot hajmini toping.

5-variant

1. $y=x^{2},\ y=\dfrac{1}{x^{2}},\ y=0,\ x=0,\ x=3.$

2. $y=\dfrac{2}{5}x\sqrt[4]{x}-\dfrac{2}{3}\sqrt[4]{x^{8}}$ Ox o'qi bilan kesishish nuqtalari orasidagi.

3. $y = x^2,\quad y = 1, x = 0, Ox.$

4. Kobb-Duglas $A(t) = e^t,\ L(t) = (1+t)^2,\ K(t) = \left(100 - 3t^2\right)$ funksiyasida $a_0 = 1,\ \alpha = 1,$ $\beta = \gamma = 0{,}5,$ t – vaqt (yillarda) bo‘lsa, besh yil ichida ishlab chiqarilgan mahsulot hajmini aniqlang.

6-variant

1. $y^2 = 2x + 1,\ y = x - 1.$

2. $y = \frac{1}{2}x^2,\quad x = 0$ dan $x = 1$ gacha.

3. $y = x^3,\quad y = x^2,\ Ox.$

4. Biror mamlakatning daromadlarni taqsimlashni tadqiq qilish bo‘yicha olingan ma’lumotlar natijasida Lorens egri chizig‘i $y = \frac{x}{3-2x}$ tenglama bilan berilishi mumkinligi aniqlandi, bu erda $x \in [0,1]$. Djini koeffitsiyenti k ni hisoblang.

7-variant

1. $y = -\frac{1}{2}x^2 + 3x + 6,\ y = \frac{1}{2}x^2 - x + 1.$

2. $y = 1 - \ln\cos x,\quad x = 0$ dan $x = \frac{\pi}{6}$ gacha.

3. $y = x^3,\quad y = 4x,\ Ox.$

4. Agar talab va taklif qonunlari $p = 186 - x^2,\quad p = 20 + \frac{11}{6}x$ ko‘rinishga ega bo‘lsa, bozor muvozanatini o‘rnatish bo‘yicha taklifdan iste’molchi va ta’minotchining yutug‘ini toping.

8-variant

1. $y = x^2,\ y = 2x,\ y = x.$

2. $y = -x^2 + 2x$ parabola uchidan absissasi $x = 2$ bo‘lgan nuqtagacha.

3. $y = \sin x,\quad y = 0,\ 0 \le x \le \pi,\ Ox.$

4. Agar mehnat unumdorligi $f(t) = 11{,}3e^{-0{,}417t}$ formula bo‘yicha berilgan bo‘lsa, dastlabki besh soatda ishlab chiqarilgan mahsulot hajmini aniqlang, bu erda t – vaqt (soatlarda).

9-variant

1. $y = x^3 - 3x,\quad y = x.$

2. $y = \ln x,\quad x = \sqrt{8}$ dan $x = \sqrt{15}$ gacha.

3. $y=\frac{4}{x}$, $x=1$, $x=4, y=0$, Ox.

4. Korxonaning kunlik unumdorligi $f(t)=-0{,}0033t^2-0{,}089t+20{,}96$ funksiya bilan berilgan bo‘lsa, uning bir yilda (258 ish kuni) ishlab chiqargan mahsulot hajmini toping, bu erda $1\le t\le 8$, t – vaqt (soatlarda).

10-variant

1. $y=x^2-2x+3$, $y=3x-1$.

2. $y=\frac{1}{2}x^2-4x+\frac{15}{2}$, Ox o‘qi bilan kesishish nuqtalari orasidagi.

3. $y=\frac{1}{2}x^2-2x$, $y=0$, Ox.

4. Kimyoviy tola uzluksiz ishlab chiqarilganda $f(t)$ mehnat unumdorligi ish boshlashdan boshlab 10 soat davomida o‘sadi, so‘ngra bir tekis davom etadi. Agar $f(t)=e^{\frac{t}{5}}-1$, $t\in[0{,}10]$ bo‘lsa, apparat ishga tushirilgandan so‘ng birinchi sutkada qancha tola beradi.

11-variant

1. $y=4-x^2$, $y=0$.

2. $\frac{3}{2}x=y^{\frac{3}{2}}$, $O(0;0)$ nuqtadan $A\left(2\sqrt{3};3\right)$ gacha.

3. $y=x^2$, $xy=8$, $y=0$, $x=4$, Ox.

4. Agar Kobb-Duglas funksiyasida $A(t)=e^{3t}$, $L(t)=(t+1)$, $K(t)=10$, $a_0=\alpha=\beta=\gamma=1$ bo‘lsa, to‘rt yilda ishlab chiqarilgan mahsulot hajmini toping.

12-variant

1. $y=x^2$, $y=1$.

2. $y=\frac{a}{2}\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)$ absissasi 0 va a ga teng bo‘lgan nuqtalar orasidagi.

3. $x=\sqrt{y-1}$, $x=0$, $y=5$, Ox.

4. Aytaylik, biror mamlakatda daromadni taqsimlashning Lorens egri chizig‘i $y=0{,}85x^2+0{,}15x$ tenglama bilan berilgan bo‘lsin. Eng kam ish haqi oladigan 10% aholi daromadning qanday ulushini oladi? Mamlakat uchun Djini koeffitsiyentini aniqlang.

13-variant

1. $y=\ln x$, $y=e$, $y=0$.

2. $y^2=16x,\ x=4$ to‘g‘ri chiziq bilvn kesishish nuqtalari orasidagi.

3. $y=\ln x,\quad y=0,\ x=e,\ Ox.$

4. Aytaylik, biror mamlakatda daromadni taqsimlashning Lorens egri chiziqg‘i $y=2^x-1$ tenglama bilan berilgan bo‘lsin. Eng kam ish haqi oladigan 10% aholi daromadning qanday ulushini oladi? Mamlakat uchun Djini koeffitsiyentini aniqlang.

14-variant

1. $y=\sqrt{x},\quad y=x.$

2. $y^2=9-x,\ y=-3,\ y=0.$

3. $y=-x^2+4,\quad y=x^2,\ x=0,\ Ox.$

4. Aytaylik, biror mamlakatda daromadni taqsimlashning Lorens egri chiziqg‘i $y=0{,}7x^3+0{,}3x^2$ tenglama bilan berilgan bo‘lsin. Eng kam ish haqi oladigan 10% aholi daromadning qanday ulushini oladi? Mamlakat uchun Djini koeffitsiyentini aniqlang.

15-variant

1. $y=tgx,\quad x=0,\ x=\dfrac{\pi}{3}.$

2. $\begin{cases} x=\cos^3 t \\ y=\sin^3 t \end{cases}$ $t=0$ dan $t=\dfrac{\pi}{2}$ gacha.

3. $y=\sqrt{6x},\quad y=\sqrt{16-x^2},\ x=0,\ Ox.$

4. Biror tovarga bo‘lgan talab tenglamasi $p=134-x^2$ ko‘rinishga ega. Agar muvozanat narx 70 ga teng bo‘lsa, iste’molchi yutug‘ini aniqlang.

16-variant

1. $y=\sin x,\quad y=x^2-\pi x.$

2. $\begin{cases} x=e^t\cos t \\ y=e^t\sin t \end{cases}$ $t=0$ dan $t=1$ gacha.

3. $y=x^2+1,\quad x=y^2+1,\ y=0,\ x=0,\ x=2,\ Ox.$

4. Biror tovarga bo‘lgan talab tenglamasi $p=\dfrac{100}{x+15}$ ko‘rinishga ega. Agar tovarning muvozanat miqdori 10 ga teng bo‘lsa, iste’molchi foydasini aniqlang.

17-variant

1. $y=x^4-2x^2,\quad y=0.$

2. $\begin{cases} x=\dfrac{1}{6}t^6, \\ y=2-\dfrac{t^4}{4} \end{cases}$ Ox va Oy o‘qlari bilan kesishish nuqtalari orasidagi.

3. $y=x^3,\quad y=4x,\ Oy.$

4. Mahsulotga bo‘lgan taklab va taklif qonunlari $p=250-x^2$ va $p=\dfrac{1}{3}x+20.$ ko‘rinishda bo‘lsa, iste’molchi hamda ta’minotchining yutug‘ini aniqlang.

18-variant

1. $y=3+2x-x^2,\quad y=x+1.$

2. $\begin{cases} x=\cos t+t\sin t, \\ y=\sin t-t\cos t \end{cases}$ $t=0$ dan $t=\dfrac{\pi}{4}$ gacha.

3. $y=\sin x,\quad y=0,\ 0\le x\le\pi,\ Oy.$

4. Mahsulotga bo‘lgan taklab va taklif qonunlari $p=240-x^2$ va $p=x^2+2x+20.$ ko‘rinishda bo‘lsa, tovar iste’molchi hamda etkazib beruvchining yutug‘ini aniqlang.

19-variant

1. $y=x^2+3,\quad xy=4,\ y=2,\ x=0.$

2. $\begin{cases} x=4(t-\sin t), \\ y=4(1-\cos t) \end{cases}$ $\dfrac{\pi}{2}\le t\le\dfrac{2\pi}{3}.$

3. $y=\dfrac{4}{x},\quad x=1,\ x=4,\ y=0,\ Oy.$

4. Agar kun davomida mehnat unumdorligi $f(t)=-0{,}1t^2+0{,}8t+10$ empirik formula bo‘yicha o‘zgarsa, kunlik ish vaqti 8 soat bo‘lgan Q bir kunlik ishlab chiqarilgan mahsulotni toping.

20-variant

1. $y=\sqrt{1-x},\quad y=x+1.$

2. $y=\ln\dfrac{e}{\cos x}$ $x=0$ dan $x=\dfrac{\pi}{6}$ gacha.

3. $y=\dfrac{1}{2}x^2-2x,\quad y=0,\ Oy.$

4. Agar unumdorlik $f(t)=0{,}0033t^2-0{,}089t+20{,}96$, $0<t<8$ funksiya bilan berilgan bo‘lsa va kunlik ish soati 8s bo‘lgan bo‘lsa bir yilda (258 ish kuni) ishlab chiqarilgan mahsulot hajmini toping.

21-variant

1. $y=\cos 2x,\quad y=0,\ x=0,\ x=\frac{\pi}{4}.$

2. $y=\sqrt{x-1}$ $A(1;0)$ nuqtadan $B(2;1)$gacha.

3. $y=x^2,\ xy=8,\quad y=0,\ x=4, Oy.$

4. Agar funksiya elastikligi $E_q=2p^2$ berilgan bo‘lsa, $q=q(p)$. $q(1)=e$ funksiyani toping.

22-variant

1. $y=2-x^4,\quad y=x^2.$

2. $y=\ln(1-x^2)$ $x=0$dan $x=\frac{3}{4}$gacha.

3. $x=\sqrt{y-1},\quad x=0,\ y=5, Oy.$

4. Agar funksiya elastikligi $E_q=p^2e^{-p}$berilgan bo‘lsa, $q=q(p)$. $q(1)=e^{-2e^{-1}}$ funksiyani toping.

23-variant

1. $xy=1,\quad y=x^2, x=3, y=0.$

2. $2y-x^2+3=0,\quad Ox$ o‘qi bilan kesishish nuqtalari orasidagi.

3. $y=\ln x,\quad y=0,\ x=e, Oy.$

4. Chegaraviy daromadi funksiya $MR(q)=\frac{10}{(1+q)^2}$. Agar $R(0)=0$ ma’lum bo‘lsa, $R(q)$ daromad funksiyasini toping va uning $q=9$ dagi qiymatini hisoblang.

24-variant

1. $x=0,\quad x=2, y=2^x, y=2x-x^2.$

2. $x^2=(y-1)^3$, $x=2$ to‘g‘ri chiziq bilvn kesishish nuqtalari orasidagi.

3. $y=-x^2+4,\quad y=x^2,\ x=0, Oy.$ J 4π.

4. Chegaraviy xarajat funksiyasi $MC(q)=\frac{100}{\pi}arctgq$ berilgan. Agar $C(0)=1000$ ma’lum bo‘lsa, $C(q)$xarajat funksiyasi uchun ifodani va uning $q=100$ dagi qiymatini toping.

25-variant

1. $y=\arcsin 2x,\quad x=0,\ y=-\frac{\pi}{2}.$

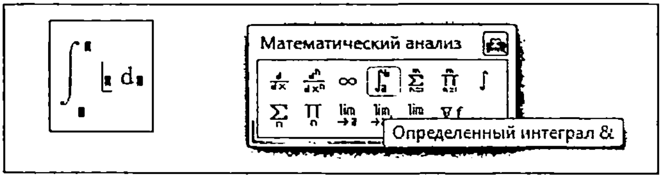
2. $\begin{cases} x = 8\sin t + 6\cos t \\ y = 6\sin t - 8\cos t \end{cases}$ $t = 0$ dan $t = \dfrac{\pi}{2}$ gacha.

3. $y = \sqrt{6x}$, $\quad y = \sqrt{16 - x^2}, x = 0, Oy.$

4. Chegaraviy xarajat funksiyasi $MC(y) = 0{,}8 + \dfrac{0{,}2}{\sqrt{y}}$ berilgan. Agar $C(100) = 100$ ma'lum bo'lsa, $C(y)$ iste'mol funksiyasi uchun ifodani va uning $y = 400$ dagi qiymatini toping.

7.6. Mathcad dasturida hisoblash
Aniq integral. Integrallash operatori

1. **Математический анализ** (Calculus) panelidan **integral belgisi tugmasini bosing** yoki klaviaturadan so'roq belgisini <Shift>+<7> klavishlalarini birgalikda bosing (yoki "&" simvolini) integrallash operatorini kiriting. Integral simvoli kiritilganda bir necha o'rinto'ldirgichlar hosil bo'ladi ularga quyi va yuqori integrallash chegaralarini, integral osti funkцiyasini, integrallash o'zgaruvchisini kiritish kerak.



1-rasm. **Integrallash operatori**

Integrallash natijasini olish uchun tenglik yoki simvolli tenglik belgisini kiritish mumkin. Birinchi holda integrallash sonli metodda o'tkaziladi, ikkinchi holda Mathcad simvolli protsessori yordamida integralning aniq qiymati topiladi.

1-misol. Aniq integralni sonli va simvolli hisoblang.

$$\int_0^{\pi} \exp\left(-x^2\right) dx = 0.886$$

$$\int_0^{\pi} \exp\left(-x^2\right) dx \to \frac{\sqrt{\pi}\cdot \mathrm{erf}(\pi)}{2} = 0.886$$

2-misol. Xosmas integralni hisoblang.

$$\int_{-\infty}^{\infty} \exp\left(-x^2\right) dx \rightarrow \sqrt{\pi}$$

3-misol. Yoy uzunligini hisoblang.

$$f(x) := x^2 - \frac{x^3}{2}$$

$$a := 0 \qquad b := 2$$

$$\int_a^b \sqrt{1 + \left(\frac{d}{dx} f(x)\right)^2}\, dx = 2.42$$

Aniqmas integral. Simvolli integrallash

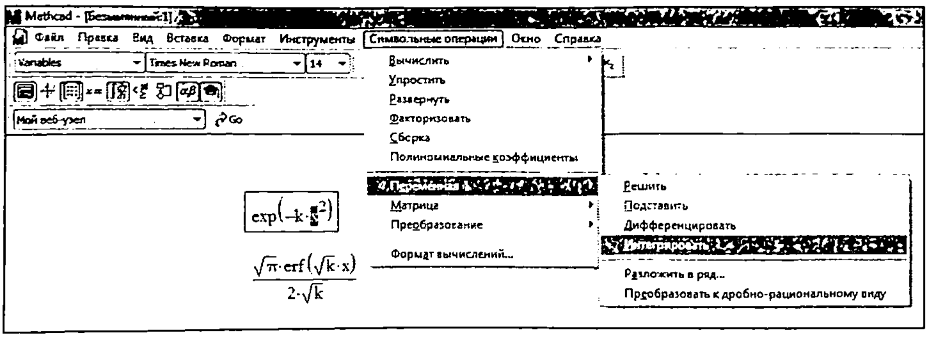
Biror funksiyani analitik integrallash uchun **Математический анализ** (Calculus) panelidan aniqmas integral simvolini kiriting.

4-misol. Aniqmas integralni toping.

$$\int \exp\left(-a \cdot x^2\right) dx \rightarrow \frac{\sqrt{\pi} \cdot \operatorname{erf}\left(\sqrt{a} \cdot x\right)}{2 \cdot \sqrt{a}}$$

Menyu yordamida integrallash

Biror o‘zgaruvchili ifodani analitik integrallash uchun, o‘zgaruvchini belgilab Символные операции / Переменная / Integrirovat (Sumbolics / Variable / Integrate) buyrug‘i tanlanadi. Ikkinchi tartibli hosilani topish uchun yuqoridagi amallar ketma-ketligi takroran qo‘llaniladi (1-rasm).



1-rasm. **Menyu yordamida o‘zgaruvchi bo‘yicha ifodani integrallash.**

VIII bob. DIFFERENSIAL TENGLAMALAR VA QATORLAR

8.1. Birinchi tartibli differensial tenglamalar

Asosiy tushunchalar. O'zgaruvchilari ajraladigan differensial tenglama

Iqtisodda dinamik jarayonlarni tadqiq qilish muhim masalalardan hisoblanadi. Bunda miqdorning vaqtga bog'liq o'zgarishi tahlil qilinadi. Ma'lumki, biror miqdorning o'sish yoki kamayish tezligi (o'zgarish tezligi) shu miqdorni ifodalovchi vaqtga bog'liq funksiyaning hosilasiga teng. Shu sabab iqtisodiy dinamik jarayonlarning modellarida noma'lum miqdorni ifodalovchi noma'lum funksiya bilan bir qatorda uning hosilalari ham ishtirok qiladi.

Masalan, ota o'zining 7 yoshdagi o'g'lining 12 yildan keyingi universitetda o'qishi bilan bog'liq 35 000 shartli pul birligini qoplash uchun 10% foiz stavkasi bilan bankka pul qo'ymoqchi. Bank foizini stavkasining ulushini uzluksiz ravishda hisoblansin. Bu masalada bizdan otaning hozirda bankka qancha miqdorda pul qo'yishi kerakligini topish talab qilinadi.

Deylik x vaqtdan keyin bank depozitidagi pul miqdori $y(x)$ ga teng bo'lsin. Masala shartiga ko'ra bu pul miqdorining o'zgarish tezligi $y'(x)$ shu vaqtdagi pul miqdori $y(x)$ ning 10% ga teng, ya'ni

$$y'(x) = 0{,}1y. \qquad (1)$$

Masala shartiga ko'ra

$$y(12) = 35000 \qquad (2)$$

bo'lib, $y(0)$ ni topish talab qilinadi.

Izoh. Dinamik masalalarda odatda vaqtni ifodalovchi o'zgaruvchi x emas, balki t bilan belgilanadi. Bu holatda vaqt bo'yicha hosila $\dot{y}$ kabi belgilanadi. Masalan, (1) tenglama bu belgilashlarda $\dot{y} = 0{,}1y$ kabi yoziladi.

Biz sodda misolda dinamik model tuzdik. Xuddi shuningdek, yetarlicha ma'lumot mavjud bo'lganda, boshqa murakkab iqtisodiy jarayonlar uchun ham matematik modellar tuzish mumkin. Bu modellar odatda "differensial tenglamalar" bilan ifodalanadi.

1-ta'rif. Erkli o'zgaruvchi x ni, noma'lum $y(x)$ funksiyani va uning hosilalarini bog'lovchi tenglamaga differensial tenglama deyiladi. Bu tenglamada ishtirok etgan hosilaning eng katta tartibi differensial tenglamaning tartibi deyiladi.

n – tartibli differensial tenglamaning umumiy ko'rinishi:

$$F(x,y,y',y'',...y^{n})=0\ . \qquad (3)$$

Yuqoridagi misoldagi (1) differensial tenglama 1- tartibli tenglamadir.

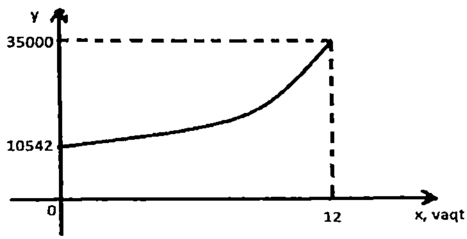
2-ta'rif. (3) tenglamani ayniyatga aylantiruvchi va kamida n marta differensiallanuvchi har qanday $y=f(x)$ funksiyaga (3) differensial tenglama yechimi deyiladi.

Masalan, $y=e^{0,1x}$ funksiya $y'=0,1y$ differensial tenglamaning yechimi bo'lib, u tenglamaning cheksiz ko'p yechimlaridan biridir. Har qanday $y=c\cdot e^{0,1x}$ funksiya ham, bu yerda, c – ixtiyoriy o'zgarmas son, tenglamani qanoatlantiradi. Bu funksiya tenglamaning umumiy yechimi bo'ladi. Umumiy yechimda ixtiyoriy o'zgarmas c qatnashgani uchun, tenglama yechimlari to'plami yagona ixtiyoriy c o'zgarmasga bog'liq deyiladi.

O'zgarmas c ga turli son qiymatlar berilganda, uning konkret yoki xususiy yechimlari kelib chiqadi. Misol uchun (1) tenglamaning (2) shartni qanoatlantiruvchi yechimini topaylik. (2) shartdan

$$c\cdot e^{0,1\cdot 12}=35000 \text{ yoki } c=35000\cdot e^{-1,2}\approx 10542.$$

Demak, ota bankka 10542 shartli p.b. miqdorida pul qo'yishi kerak ekan.



Boshqa misol ko'raylik.

Misol. $y'''=0$ differensial tenglama yechimlarini bevosita qurish mumkin:

$$y''=c_1,\quad y'=c_1x+c_2,\quad y=c_1x^2/2+c_2x+c_3.$$

Bu yerda, c_1, c_2 va c_3 ixtiyoriy o'zgarmaslar bo'lib, ularning har qanday qiymatlarida $y = c_1x^2/2 + c_2x + c_3$ funksiya differensial tenglamani qanoatlantiradi va shu sababli $y = c_1x^2/2 + c_2x + c_3$ umumiy yechim bo'lib hisoblanadi. $y''' = 0$ differensial tenglama umumiy yechimi uch ixtiyoriy o'zgarmasga bog'liq va har birining konkret qiymatlarida xususiy yechim hosil bo'ladi.

Yuqoridagi misollardan differensial tenglama umumiy yechimida o'zgarmaslar soni tenglamaning tartibiga teng ekanligini va uning xususiy yechimlari umumiy yechim o'zgarmaslarining konkret qiymatlarida kelib chiqishini xulosa qilish mumkin.

Differensial tenglama yechimlarini qurish jarayoniga differensial tenglamani integrallash deb yuritiladi. Differensial tenglamani integrallab, masalaning qo'yilishiga qarab, uning yoki umumiy yechimi yoki xususiy yechimi topiladi.

Oddiy differensial tenglamalar. Birinchi tartibli sodda differensial tenglamalar

Birinchi tartibli differensial tenglama umumiy

$$F(x;y;y') = 0$$

yoki y' hosilaga nisbatan yechilgan

$$y' = f(x;y)$$

ko'rinishda yozilishi mumkin. Ushbu tenglama ham, odatda, cheksiz ko'p yechimga ega bo'lib, ulardan biror – bir xususiy yechimni ajratib olish uchun qo'shimcha shartni talab etadi. Ko'p hollarda ushbu shart Koshi masalasi shaklida qo'yiladi.

Koshi masalasi:

$$y' = f(x;y) \qquad (4)$$

differensial tenglamaning

$$y|_{x=x_0} = y_0 \qquad (5)$$

boshlang'ich shartni qanoatlantiruvchi yechimini topishdan iborat.

(4), (5) masala yechimining mavjudlik va yagonalik sharti quyidagi teoremadan aniqlanadi.

Teorema. Agar $f(x;y)$ funksiya $(x_0;y_0)$ nuqtaning biror atrofida aniqlangan, uzluksiz va $\partial f/\partial y$ – uzluksiz xususiy hosilaga ega bo‘lsa, u holda $(x_0;y_0)$ nuqtaning shunday atrofi mavjudki, bu atrofda $y'=f(x;y)$ differensial tenglama uchun $y|_{x=x_0}=y_0$ boshlang‘ich shartli Koshi masalasi yechimi mavjud va yagonadir.

Differensial tenglamaning umumiy va xususiy yechimlari tushunchalariga aniqlik kiritamiz.

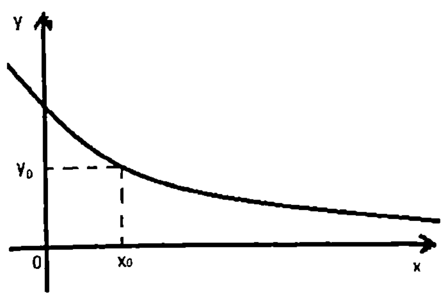
Agar boshlang‘ich $(x_0;y_0)$ nuqtaning berilishi (4) tenglama yechimining yagonaligini aniqlasa, u holda ushbu yagona yechim xususiy yechim deyiladi.

Differensial tenglamaning barcha xususiy yechimlari to‘plamiga uning umumiy yechimi deyiladi.

Odatda, umumiy yechim oshkor $y=\varphi(x,c)$ yoki oshkormas $\Phi(x,y,c)=0$ ko‘rinishda yoziladi. c o‘zgarmas $(x_0;y_0)$ boshlang‘ich shart asosida $y_0=\varphi(x_0;c)$ tenglamadan topiladi.

3-ta’rif. Tenglamaning umumiy integrali (yoki yechimi), deb c o‘zgarmasning turli qiymatlarida barcha xususiy yechimlari aniqlanadigan $\Phi(x,y,c)=0$ munosabatga aytiladi

Masalan, yechimning mavjudlik va yagonalik shartlari (teoremadagi) yuqorida ko‘rilgan $y'=-y$ tenglama uchun xOy tekislikning har bir nuqtasida bajariladi. Tenglama umumiy yechimi $y=c\cdot e^{-x}$ formuladan iborat bo‘lib, har qanday boshlang‘ich $y|_{x=x_0}=y_0$ shart mos c o‘zgarmas tanlanganda qanoatlantiriladi. c o‘zgarmas $y_0=c\cdot e^{-x_0}$ tenglamadan topiladi: $c=y_0\cdot e^{x_0}$.



Differensial tenglamani shartlarsiz yechish uning umumiy yechimini (yoki umumiy integralini) topishni anglatadi.

Differensial tenglama yechimi mavjudligi va yagonaligini ta'minlaydigan muhim shartlardan biri $\partial f/\partial y$ xususiy hosilaning uzluksizligidir. Ba'zi bir nuqtalarda ushbu shart bajarilmasligi va ular orqali birorta ham integral chiziq o'tmasligi yoki, aksincha, bir nechta integral chiziqlar o'tishini bildiradi. Bunday nuqtalar differensial tenglamaning maxsus nuqtalari deyiladi.

Differensial tenglamaning integral chizig'i faqat uning maxsus nuqtalaridan iborat bo'lishi mumkin. Ushbu egri chiziqlar tenglamaning maxsus yechimlari, deb yuritiladi.

$$y' = f(x)$$

ko'rinishga tenglamani oddiy integrallash yo'li bilan yechiladi. Natijada, $y = \int f(x)dx$. Agar $f(x)$ funksiyaning boshlang'ich funksiyalaridan biri $F(x)$ bo'lsa, u holda umumiy yechim $y = F(x) + c$ ko'rinishda yoziladi.

$$y' = p(x)q(y) \qquad (6)$$

(6) ko'rinishidagi tenglama o'zgaruvchilari ajraladigan differensial tenglama deb yuritiladi. (6) tenglamani yechish uchun noma'lum y funksiyaning qaralayotgan o'zgarish sohasida $q(y) \neq 0$ shart bajariladi deb, (6) tenglamani $dy/q(y) = p(x)dx$ shaklda yozamiz va ikkala qismini integrallab,

$$\int dy/q(y) = \int p(x)dx$$

tenglikni olamiz. Agar $Q(y)$ funksiya $1/q(y)$ funksiyaning, $P(x)$ esa $p(x)$ ning boshlang'ich funksiyalaridan biri bo'lsa, (6) tenglamaning umumiy integrali:

$$Q(y) = P(x) + c$$

ko'rinishdan iborat bo'ladi.

Bir jinsli differensial tenglamalar

Birinchi tartibli bir jinsli differensial tenglama deb,

$$dy/dx = f(y/x) \qquad (7)$$

ko'rinishdagi tenglamaga aytiladi.

(7) tenglamani yechish uchun noma‘lum $y(x)$ funksiyadan $u(x)=y(x)/x$ funksiyaga o‘tamiz. U holda

$$y=xu, \qquad dy/dx=u+x\,du/dx$$

tengliklar o‘rinli bo‘lib, (7) tenglama:

$$u+x\,du/dx=f(u) \qquad \text{yoki} \qquad du/(f(u)-u)=dx/x$$

ko‘rinishga keltiriladi. Oxirgi tenglama o‘zgaruvchilari ajralgan differensial tenglamadir va ma‘lum usulda yechiladi. Natijada,

$$\int\frac{du}{f(u)-u}=\ln|x|+c.$$

$u(x)$ funksiya topilgandan so‘ng, $y(x)=x\cdot u(x)$ funksiyaga qaytiladi.

Chiziqli differensial tenglamalar

Noma‘lum funksiya va uning hosilasiga nisbatan chiziqli bo’lgan

$$\frac{dy}{dx}+P(x)y=Q(x) \qquad (8)$$

ko’rinishdagi tenglama *chiziqli differensial tenglama* deyiladi. Bu yerda $P(x)$ va $Q(x)$ biror oraliqda berilgan uzluksiz funksiyalar. Agar $Q(x)=0$ bo’lsa, (8) tenglama *bir jinsli*, aks holda *bir jinsli bo’lmagan* chiziqli differensial tenglama deyiladi.

Dastlab

$$y'+P(x)y=0$$

bir jinsli chiziqli differensial tenglamani yechish bilan shug’ullanamiz.

Ravshanki, bu tenglama o’zgaruvchilari ajraladigan tenglama bo’ladi. Uni integrallaymiz:

$$\frac{dy}{y}=-P(x)dx \Leftrightarrow \ln|y|=-\int P(x)dx+\ln|C| \Leftrightarrow \ln\left|\frac{y}{C}\right|=-\int P(x)dx.$$

Bundan $y=Ce^{-\int P(x)dx}$ umumiy yechimga ega bo’lamiz .

Bir jinsli bo’lmagan chiziqli differensial tenglama asosan 2 ta usul bilan yechilishi mumkin. Bu usullar mos ravishda *Bernulli va Lagranj usullari* deb yurutiladi.

Bernulli usuli. Bu usulda noma‘lum funksiya $y=uv$ ko’rinishda ifodalaniladi, bu yerda u funksiya

$$\frac{du}{dx} + P(x)u = 0 \tag{9}$$

tenglamani qanoatlantiradi, ya'ni

$$u = C_1 e^{-\int P(x)dx}. \tag{10}$$

$y' = u\frac{dv}{dx} + v\frac{du}{dx}$ hosilani berilgan (8) tenglamaga qo'yib, quyidagilarga ega bo'lamiz:

$$u\frac{dv}{dx} + v\frac{du}{dx} + P(x)uv = Q(x)$$

$$u\frac{dv}{dx} + v\left(\frac{du}{dx} + P(x)u\right) = Q(x).$$

Bundan (9) va (10) ni inobatga olsak, noma'lum v funksiya uchun

$$u\frac{dv}{dx} = Q(x), \quad C_1 e^{-\int P(x)dx}\frac{dv}{dx} = Q(x); \qquad C_1 dv = Q(x)e^{\int P(x)dx}dx$$

munosabatlarga ega bo'lamiz.

Integrallab v ni topamiz :

$$C_1 v = \int Q(x)e^{\int P(x)dx}dx + C_2; \qquad v = \frac{1}{C_1}\left(\int Q(x)e^{\int P(x)dx}dx + C\right)$$

Natijada $y = uv = C_1 e^{-\int P(x)dx} \cdot \frac{1}{C_1}\left(\int Q(x)e^{\int P(x)dx}dx + C\right)$, yani

$$y = e^{-\int P(x)dx} \cdot \left(\int Q(x)e^{\int P(x)dx}dx + C\right).$$

Lagranj usuli. Dastlab bir jinsli

$$y' + P(x)y = 0$$

tenglamaning $y = Ce^{-\int P(x)dx}$ yechimi topiladi.

Bundan keyin C parametrni x o'zgaruvchining funksiyasi deb o'linadi va (8) tenglamaning yechimi

$$y = C(x)e^{-\int P(x)dx} \tag{11}$$

ko'rinishda qidiriladi.

Ravshanki,

$$y' = \frac{dy}{dx} = \frac{dC(x)}{dx}e^{-\int P(x)dx} + C(x)e^{-\int P(x)dx} \cdot (-P(x)).$$

(8) ga qo'yamiz:

$$\frac{dC(x)}{dx}e^{-\int P(x)dx} - C(x)P(x)e^{-\int P(x)dx} + P(x)C(x)e^{-\int P(x)dx} = Q(x)$$ va natijada $C(x)$ ga nisbatan tenglamaga kelamiz:

$$\frac{dC(x)}{dx}e^{-\int P(x)dx} = Q(x)$$

Bundan $dC(x) = Q(x)e^{\int P(x)dx}dx$ va $C(x) = \int Q(x)e^{\int P(x)dx}dx + C$ ni topamiz.

$C(x)$ ni (11) ga qo'yib

$$y = e^{-\int P(x)dx}\left(\int Q(x)e^{\int P(x)dx}dx + C\right)$$

umumiy yechimga ega bo'lamiz. Kutilganidek, ikkala usul ham bir xil natijaga olib keldi.

Bernulli tenglamasi

Endi biz chiziqli tenglamaga olib kelinadigan muhim tenglamani o'rganamiz.

$n \neq 0$ va $n \neq 1$ bolsin

$$y' + P(x)y = Q(x) \cdot y^n, \quad n \neq 0,1 \qquad (12)$$

ko'rinishdagi tenglama *Bernulli tenglamasi* deb yuritiladi.

$z = \frac{1}{y^{n-1}}$ almashtirish yordamida Bernulli tenglamasi chiziqli tenglamaga keltirishini ko'rsatamiz.

Buning uchun (12) tenglamaning ikkala tarafini y^n ga bo'lamiz:

$$\frac{y'}{y^n} + P\frac{1}{y^{n-1}} = Q\ .$$

Bundan $z' = -\frac{(n-1)y^{n-2}}{y^{2n-2}} \cdot y' = -\frac{(n-1)y'}{y^n}$ ni inobatga olib, z ga nisbatan chiziqli tenglamaga ega bo'lamiz:

$$-\frac{z'}{n-1} + Pz = Q, \quad z' - (n-1)Pz = -(n-1)Q$$

Misollar

1. $yy' = \frac{-2x}{\cos y}$ differensial tenglamani yeching.

Yechish. $yy' = \frac{-2x}{\cos y}$ tenglamani soddalashtiramiz:

$$y\cos y \cdot \frac{dy}{dx} = -2x \Leftrightarrow y\cos y dy = -2xdx$$

Oxirgi tenglama o'zgaruvchilari ajralgan, uni integrallaymiz:

$$\int y\cos y dy = -2\int xdx$$

Chap tarafdagi integral bo'laklab integrallash usuli yordamida hisoblanadi:

$$\int y\cos y dy = \begin{Bmatrix} u = y; & dv = \cos y dy; \\ du = dy; & v = \sin y \end{Bmatrix} = y\sin y - \int \sin y dy = y\sin y + \cos y$$

Natijada

$$y\sin y + \cos y + x^2 = C$$

umumiy integralni hosil qilamiz .

2. Differensial tenglamaning berilgan boshlang'ich shartni qanoatlantiradigan yechimlarini toping:

$$\frac{y}{y'} = \ln y \ , \ y|_{x=2} = 1 .$$

Yechish. Berilgan $\frac{y}{y'} = \ln y$ tenglamani $\frac{ydx}{dy} = \ln y$ ko'rinishda yozib, undan o'zgaruvchilari ajralgan

$$dx = \frac{\ln y dy}{y}$$

tenglamani hosil qilamiz. Bu tenglamani integrallaymiz:

$$\int dx = \int \frac{\ln y dy}{y}, \quad x + C = \int \ln y d(\ln y), \quad x + C = \frac{\ln^2 y}{2}.$$

Endi $u(2) = 1$ boshlang'ich shartdan foydalanib, C ning qiymatini topamiz:

$$2 + C = \frac{\ln^2 1}{2}; \quad \Rightarrow \quad 2 + C = 0; \quad \Rightarrow \quad C = -2;$$

Bundan $2(x-2) = \ln^2 y$ yani $y = e^{\pm\sqrt{2x-4}}$ ko'rinishdagi xususiy yechimlarga ega bo'lamiz.

3. Quyidagi tenglamarning umumiy yechimini toping:

$$\left(y^2 - 2xy\right)dx + x^2 dy = 0.$$

Yechish. $(y^2-2xy)dx+x^2dy=0$ tenglama tarkibidagi $P=y^2-2xy,\quad Q=x^2$ funksiyalar ikkalasi ham ikkinchi tartibli bir jinsli funksiyalar bo'lgani uchun bu tenglama bir jinsli tenglama bo'ladi.

Shuning uchun $y=xu$ almashtirishni qo'llaymiz. U holda $dy=xdu+udx$ va tenglama $x^2(u^2-2u)dx+x^2(xdu+udx)=0$ yoki $(u^2-u)dx+xdu=0$ ko'rinishda bo'ladi.

O'zgaruvchilarni ajratamiz: $\frac{dx}{x}=\frac{du}{u(1-u)}$ va hosil qilingan tenglamani integrallaymiz:

$$\int\frac{dx}{x}=\int\frac{du}{u(1-u)}$$

O'ng tomondagi integralni topamiz:

$$\int\frac{du}{u(1-u)}=\int\left(\frac{1}{u}+\frac{1}{1-u}\right)du=\int\frac{du}{u}+\int\frac{du}{1-u}=\ln|u|-\ln|1-u|+\ln|C|=\ln\left|\frac{Cu}{1-u}\right|.$$

Demak,

$\ln|x|=\ln\left|\frac{Cu}{1-u}\right|$, yani $x=\frac{Cu}{1-u}$ yoki $u=\frac{x}{C+x}$ ga ega bo'lamiz.

So'ngi ifodadagi u o'rniga $\frac{y}{x}$ ni qo'yib, $y=\frac{x^2}{C+x}$ umumiy yechimni topamiz.

4. Quyidagi tenglamaning umumiy yechimini toping:

$y'+2xy=2xe^{-x^2}$

Yechish. $y'+2xy=2xe^{-x^2}$ tenglama chiziqli differensial tenglama.

Bernulli usulidan foydalanamiz. $y=uv$ deylik. U holda $y'=vu'+uv'$ bo'ladi va bularni berilgan tenglamaga qo'ysak, u quyidagi

$vu'+u(v+2xv)=2xe^{-x^2}$ ko'rinishga keladi.

$v'+2xv=0$ bo'lishini talab qilamiz. O'zgaruvchilarni ajratib, $\frac{dv}{v}=-2xdx$ ni hosil qilamiz, bu yerdan $\ln|v|=-x^2+\ln|C|,\quad v=Ce^{-x^2}$.

$C=1$ deb $v=e^{-x^2}$ xususiy yechim bilan cheklanish mumkin. v ning ifodasini almashtirilgan $vu'=2e^{-x^2}$ tenglamaga qo'yamiz:

$e^{-x^2}u' = 2xe^{-x^2}$, $du = 2xdx$ Bu yerdan: $u = x^2 + C$ ma'lumki, $y = uv$, u holda umumiy yechim $y = e^{-x^2}(x^2 + C)$ ko'rinishda hosil bo'ladi.

5. Quyidagi tenglamaning umumiy yechimini toping:

$xy' - 2y = x^3 \cos x.$

Yechish. $xy' - 2y = x^3 \cos x.$ tenglama

$$y' - \frac{2y}{x} = x^2 \cos x$$

chiziqli differensial tenglamaga olib kelinadi $(x \neq 0)$.

Bu tenglamani Lagranj usuli yordamida yechamiz:

Dastlab bir jinsli

$$y' - \frac{2y}{x} = 0$$

tenglamaning yechimini topamiz.

$$\frac{dy}{dx} = \frac{2y}{x} \Leftrightarrow \frac{dy}{y} = \frac{2dx}{x} \Leftrightarrow y = Cx^2.$$

Bundan keyin C parametr x o'zgaruvchining funksiyasi deb o'linadi va tenglamaning yechimi

$$y = C(x)x^2$$

ko'rinishda izlanadi.

Ravshanki,

$$y' = \frac{dC(x)}{dx}x^2 + 2xC(x).$$

$y' - \frac{2y}{x} = x^2 \cos x$ ga qo'yamiz:

$$y' - \frac{2y}{x} = \frac{dC(x)}{dx}x^2 + 2xC(x) - \frac{2C(x)x^2}{x} = x^2 \cos x$$ va natijada $C(x)$ ga

nisbatan tenglamaga kelamiz:

$$\frac{dC(x)}{dx} = \cos x$$

Bundan $C(x) = \sin x + C$ ni topamiz.

$C(x)$ ni $y = C(x)x^2$ ga qo'yib

$$y=(\sin x+C)x^2$$

umumiy yechimga ega bo'lamiz.

6. $y'=\frac{1}{2}y^2+\frac{1}{2x^2}$ tenglamani yeching.

Yechish. $y=\frac{z}{x}$ almashtirishdan foydalansak,

$$\frac{z'x-z}{x^2}=\frac{z^2}{2x^2}+\frac{1}{2x^2}, \qquad 2z'x=(z+1)^2, \qquad \frac{2dz}{(z+1)^2}=\frac{dx}{x},$$

$$-\frac{2}{z+1}=\ln|x|+C, \qquad z+1=\frac{2}{C-\ln|x|}, \qquad y=-\frac{1}{x}+\frac{2}{x(C-\ln|x|)}.$$

7. Agar ishlab chiqarish hajmi (investitsiyalar normasi $0{,}6$, narxi $0{,}15$ (shart.bir) va $l=0{,}4$ shartlarda) vaqtning boshlang'ich momentida $Q_0=Q(0)=24$ (shart.bir) ni tashkil etgan bo'lsa, to'yinmagan bozor sharoitda 6 oyda ishlab chiqarilgan mahsulot hajmini toping.

Yechish. $m=0{,}6$, $P=0{,}15$, $l=0{,}4$ qiymatlarni (5) tenglamaga qo'yib tenglamani yechamiz

$$Q'=0{,}4\cdot 0{,}6\cdot 0{,}15Q$$

$$Q=Ce^{0{,}216t}$$

$Q(0)=24$ boshlang'ich shartdan C doimiyning qiymatini topamiz: $C=24$. Quyidagi funksiyani olamiz

$$Q=24e^{0{,}216t}$$

U holda $Q(6)=24e^{0{,}216\cdot 6}\approx 29{,}8$.

8.2. Ikkinchi tartibli differensial tenglamalar

Ikkinchi tartibli o'zgarmas koeffitsientli chiziqli differensial tenglama

Ikkinchi tartibli o'zgarmas koeffitsientli chiziqli differensial tenglama

$$y''+py'+qy=f(x) \qquad (1)$$

ko'rinishga ega bo'lib, tenglamada p va q o'zgarmas sonlar, $f(x)$ esa uzluksiz funksiyadir.

Agar (1) tenglamada $f(x)=0$ bo'lsa, u holda

$$y'' + py' + qy = 0 \qquad (2)$$

tenglamaga (1) tenglamaning bir jinsli tenglamasi deyiladi.
O'zgarmas koeffitsientli bir jinsli (2) tenglama fundamental yechimlari sistemasini qurishning sodda usuli mavjud.

(2) tenglama xususiy yechimini $y=e^{\lambda x}$ ko'rsatkichli funksiya ko'rinishida qidiramiz. Funksiyani ikki marta differensiallab,

$$y' = \lambda \cdot e^{\lambda x}, \qquad y'' = \lambda^2 e^{\lambda x}$$

tengliklarni olamiz. Funksiya va uning hosilalarini (2) tenglamaga qo'ysak,

$$\left(\lambda^2 + p\lambda + q\right) \cdot e^{\pi x} = 0$$

tenglama hosil bo'ladi. $e^{\lambda x} \neq 0$ (har doim musbat) ekanligini hisobga olsak, oxirgi tenglamaga teng kuchli

$$\lambda^2 + p\lambda + q = 0 \qquad (3)$$

tenglamani olamiz.

(3) algebraik tenglamaga (2) differensial tenglamaning xarakteristik tenglamasi deyiladi.

(2) tenglamaning fundamental yechimlari sistemasini qurishning navbatdagi qadami quyidagicha: (3) kvadrat tenglama ikki λ_1 va λ_2 haqiqiy yoki kompleks ildizlarga ega bo'lsin. Unda $y_1 = e^{\lambda_1 x}$, $y_2 = e^{\lambda_2 x}$ funksiyalarning har biri (2) tenglamaning yechimi bo'ladi. Agar ushbu funksiyalar chiziqli erkli bo'lsa, tenglamaning umumiy yechimi $c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$ ko'rinishda yoziladi.

Agar funksiyalar chiziqli bog'liq bo'lsa, umumiy yechimni qurish jarayoni qo'shimcha mulohazalarni talab etadi.

Umumiy yechimni tuzishning xarakteristik tenglama yechimlari bilan bog'liq barcha hollarini qaraymiz:

1) λ_1 va λ_2 ildizlar haqiqiy va turlicha bo'lsin. Ularga mos $y_1 = e^{\lambda_1 x}$, $y_2 = e^{\lambda_2 x}$ yechimlar chiziqli erkli, chunki

$$W(y_1;y_2)=\begin{vmatrix} e^{\lambda_1 x} & e^{\lambda_2 x} \\ \lambda_1 e^{\lambda_1 x} & \lambda_2 e^{\lambda_2 x} \end{vmatrix}=(\lambda_2-\lambda_1)e^{(\lambda_1+\lambda_2)x}\neq 0.$$

Demak, y_1 va y_2 fundamental yechimlar sistemasini tashkil etadi.

Misol. $y''-8y'+7y=0$ tenglama umumiy yechimini quring.

Yechish. Xarakteristik tenglama $\lambda^2-8\lambda+7=0$ ko‘rinishga ega va uning ildizlar $\lambda_1=1, \lambda_2=7$. Natijada, chiziqli erkli $y_1=e^x$; $y_2=e^{7x}$ xususiy yechimlarni olamiz. Tenglama umumiy yechimi

$$y=c_1e^x+c_2e^{7x}.$$

2) λ_1 va λ_2 ildizlar o‘zaro qo‘shma $\lambda_1=\alpha+\beta i$; $\lambda_2=\alpha-\beta i$ kompleks sonlar bo’lsin, bu yerda $-\beta\neq 0$.

Ildizlarga mos kompleks yechimlarni $z_1;z_2$ deb belgilaymiz:

$$z_1=e^{\alpha+\beta i},\qquad z_2=e^{\alpha-\beta i}.$$

$\lambda_1\neq\lambda_2$ bo‘lganidan, ular chiziqli erkli.

Eyler formulasidan foydalanib,

$$z_1=e^{\alpha x}(\cos\beta x+i\sin\beta x),\quad z_2=e^{\alpha x}(\cos\beta x-i\sin\beta x)$$

funksiyalarni olamiz. Funksiyalarning quyidagi chiziqli kombinatsiyalarini tuzamiz:

$$y_1=\frac{1}{2}(z_1+z_2)=e^{\alpha x}\cos\beta x,\qquad y_2=\frac{1}{2i}(z_1-z\ )=e^{\alpha x}\sin\beta x.$$

y_1; y_2 funksiyalar (2) tenglamaning haqiqiy yechimlari bo’lib, chiziqli erklidir. Natijada, umumiy yechim

$$y=c_1e^{\alpha x}\cos\beta x+c_2e^{\alpha x}\sin\beta x=e^{\alpha x}(c_1\cos\beta x+c_2\sin\beta x)$$

ko‘rinishda yoziladi.

Misol. $y''-6y'+10y=0$ tenglama umumiy yechimini toping.

Yechish. Xarakteristik tenglama

$$\lambda^2-6\lambda+10=0$$

bo‘lib, uning ildizlari $\lambda_1=3+i$, $\lambda_2=3-i$. Shunday qilib, xususiy yechimlar

$$y_1=e^{3x}\cos x,\quad y_2=e^{3x}\sin x.$$

Umumiy yechim:

$$y=e^{3x}(c_1\cos x+c_2\sin x).$$

3) λ_1 va λ_2 ildizlar o`zaro teng va haqiqiy. $\lambda_1=\lambda_2=\lambda$ ildizlarga xususiy: $e^{\lambda x}$; $xe^{\lambda x}$ chiziqli erkli (tekshirib ko'ring) yechimlarni mos qo'yish mumkin. Shunday qilib, umumiy yechim

$$y=c_1e^{\lambda x}+c_2xe^{\lambda x}=e^{\lambda x}(c_1+c_2x).$$

Misol. $y''+4y'+4y=0$ tenglama umumiy yechimini toping.

Xarakteristik tenglama $\lambda^2+4\lambda+4=0$; $\lambda_1=\lambda_2=-2$.

Umumiy yechim

$$y=e^{-2x}(c_1+c_2x).$$

Teorema. Bir jinslimas (1) differensial tenglamaning umumiy yechimi ushbu tenglama biror $y_0(x)$ xususiy yechimi va mos bir jinsli (2) tenglama umumiy yechimlari yig'indisiga teng.

(1) tenglamaning biror – bir xususiy yechimini ixtiyoriy o'zgarmasni variatsiyalash usulida qurish mumkin.

Agar (1) tenglamaning o'ng tomoni $f(x)=P(x)e^{\alpha x}$ ko'rinishda bo'lsa, bu yerda, $P(x)$ – ko'phad, u holda tenglamaning xususiy yechimini qurishning oddiy usuli mavjud.

I. Agar a (3) xarakteristik tenglamaning ildizlaridan biri bo'lmasa, xususiy yechim $y=Q(x)e^{\alpha x}$ ko'rinishda qidiriladi. Bu yerda: $Q(x)$– darajasi $P(x)$ ning darajasiga teng aniqmas koeffitsiyentli ko'phad. $y=Q(x)e^{\alpha x}$ ifodani (1) tenglamaga qo'yiladi, $e^{\alpha x}$ ga qisqartirilgandan so'ng ko'phadlar tengligidan, $Q(x)$ ko'phadning aniqmas koeffitsiyentlari aniqlanadi.

Misol. $y''-6y'+8y=(3x-1)e^x$ tenglamaning xususiy yechimini toping.

Yechish. Ushbu holda $a=1$, xarakteristik tenglama ildizlari esa 2 va 4 ga teng. Masala yechimini $y=(ax+b)e^x$ ko'rinishda qidiramiz. Funksiya hosilalarini aniqlaymiz:

$$y'=ae^x+(ax+b)e^x=(ax+a+b)e^x$$

$$y''=ae^x+(ax+a+b)e^x=(ax+2a+b)e^x$$

y, y', y'' ifodalarni tenglamaga qo'yiladi va e^x ga qisqartirilgandan so'ng:

$$(ax+2a+b)-6(ax+a+b)+8\,(ax+b)=x-1 \text{ yoki}$$

$$3ax-4a+3b=3x-1.$$

Mos koeffitsiyentlarni tenglab, $a=1, b=-1$ natijani olamiz. Izlanayotgan xususiy yechim:

$$y=(x-1)e^{x};$$

II. Agar a xarakteristik tenglamalardan biriga teng bo'lib, ikkinchisidan, farq qilsa, xususiy yechim $y=xQ(x)e^{ax}$ ko'rinishida izlanadi.

III. Agarda a xarakteristik tenglama ikki karrali ildizlariga teng bo'lsa, u holda xususiy yechim $y=x^2Q(x)e^{ax}$ ko'rinishida qidiriladi.

Misollar

1. Quyidagi bir jinsli tenglamalarning umumiy yechimini toping.

$a) y''-5y'+6y=0;$

b) $y''-10y'+25y=0;$

c) $y''+2y'+5y=0.$

Yechish. $a) y''-5y'+6y=0$ tenglama uchun $k^2-5k+6=0$ xarakteristik tenglama $k_1=2$, $k_2=3$ ildizlarga ega, shuning uchun umumiy yechim ushbu ko'rinishda bo'ladi: $y=C_1e^{2x}+C_2e^{3x}$.

b) $y''-10y'+25y=0$ tenglamaga mos xarakteristik tenglama $k^2-10k+25=0$ ikki karrali $k=5$ ildizga ega, binobarin, umumiy yechim quyidagicha bo'ladi:

$$y=(C_1+C_2x)e^{5x}.$$

c) $y''+2y'+5y=0$ tenglamaga mos xarakteristik tenglama $k^2+2k+5=0$ ning ildizlari $k_{1,2}=-1\pm 2i$ demak, tenglamaning umumiy yechimi:

$$y=e^{-x}(C_1\cos 2x+C_2\sin 2x).$$

2. Quyidagi bir jinslimas tenglamalarning umumiy yechimini toping.

$a)$ $7y''-y'=14x.$

$b)$ $y''+4y'-2y=8\sin 2x.$

$c)$ $y''+y=4x\cos x.$

$d)$ $y''+2y'+5y=e^{-x}\cos 2x$

Yechish.

a) $7y''-y'=14x$ tenglamaga mos bir jinsli tenglamaning umumiy yechimi: $\bar{y}=C_1+C_2e^{\frac{x}{7}}$, chunki xarakteristik tenglamaning ildizlari $k_1=0, k_2=\frac{1}{7}$. 0 soni xarakteristik tenglamaning oddiy ildizi bo'lgani uchun xususiy yechimni $\tilde{y}=x(Ax+B)$ ko'rinishda izlash kerak. Tegishli algebraik tenglamalardan A, B larni topamiz: A=-7, B=-98.

Demak, xususiy yechim: $\tilde{y}=C_1+C_2e^{\frac{x}{7}}-7x^2-98x$.

b) $y''+4y'-2y=8\sin 2x$ tenglamaga mos bir jinsli tenglamaning umumiy yechimi:

$$\bar{y}=C_1e^{(-2-\sqrt{6})x}+C_2e^{(-2+\sqrt{6})x}$$

Berilgan tenglamaning o'ng tomoni $f(x)=e^{0x}P_0(x)\sin 2x$ ko'rinishida bo'lib, $a+bi=2i$ xarakteristik tenglamaning ildizi bo'lmagani uchun xususiy yechimni $\tilde{y}=A\cos 2x-B\sin 2x$ shaklda izlaymiz. Bu ifodani berilgan tenglamaga qo'ysak,

$$(-6A+8B)\cos 2x-(6B+8A)\sin 2x=8\sin 2x$$

cos2x va sin2x oldidagi koeffitsientlarni tenglab, A va B larni topamiz: $A=-\frac{16}{25}, B=-\frac{12}{25}$. Demak, xususiy yechim $\tilde{y}=-\frac{16}{25}\cos 2x-\frac{12}{25}\sin 2x$, umumiy yechim $y=C_1e^{-(\sqrt{6}+2)x}+C_2e^{(\sqrt{6}+2)x}-\frac{16\cos 2x+12\sin 2x}{25}$.

c) $y''+y=4x\cos x$ tenglamaga mos bir jinsli tenglamaning umumiy yechimi: $\bar{y}=C_1\cos x+C_2\sin x$. $a+bi=i$ xarakteristik tenglamaning oddiy ildizi bo'lgani uchun xususiy yechimni $\tilde{y}=x((Ax+B)\cos x+(Cx+D)\sin x)$ ko'rinishida izlaymiz. A, B, C, D lar uchun mos tenglamalarni yechib, A=0, B=1, C=1, D=1 larni topamiz. Demak, xususiy yechim: $\tilde{y}=x\cos x+x^2\sin x$, umumiy yechim:

$$y=C_1\cos x+C_2\sin x+x\cos x+x^2\sin x.$$

d) $y''+2y'+5y=e^{-x}\cos 2x$ tenglamaga mos $y''+2y'+5y=0$ tenglama uchun $k^2+2k+5=0$ xarakteristik tenglama $k_{1,2}=-1\pm 2i$ ildizlarga ega.

Shuning uchun, mos bir jinsli tenglamaning umumiy yechimi: $\bar{y}=(C_1\cos 2x+C_2\sin 2x)e^{-x}$, $a+bi=-1+2i$ son xarakteristik tenglamaning oddiy ildizi bo'lgani uchun xususiy yechimni $\tilde{y}=x(A\cos 2x+B\sin 2x)e^{-x}$ ko'rinishda izlaymiz. Noma'lum A va B koeffitsientlarni topish uchun $\tilde{y}$ ni va uning hosilalarini tenglamaga qo'yib va e^{-x} ga qisqartirib olamiz, bu yerdan A=0, $B=\frac{1}{4}$. Demak, $\tilde{y}=\frac{1}{4}xe^{-x}\sin 2x$. Shunday qilib, umumiy yechim:

$$y=(C_1\cos 2x+C_2\sin 2x)e^{-x}+\frac{1}{4}xe^{-x}\sin 2x.$$

8.3. Differensial tenglamalar sistemasi

Umumiy holda differensial tenglamalar sistemasi

$$\begin{cases} \dfrac{dy_1}{dx}=f_1(x,y_1,y_2,...,y_n), \\ \dfrac{dy_2}{dx}=f_2(x,y_1,y_2,...,y_n), \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ \dfrac{dy_n}{dx}=f_n(x,y_1,y_2,...,y_n). \end{cases} \qquad (1)$$

ko'rinishga ega bo'ladi.

Agar differensial tenglamalar sistemasida noma'lum funksiyaning hosilasi differensial tenglamaning chap tomonida bo'lib, o'ng tomonida hosilalar qatnashmasa bunday differensial tenglamalar sistemasiga normal differensial tenglamalar sistemasi deyiladi. (1) normal differensial tenglamalar sistemasidir. (1) tenglamalar sistemasining yechimi, deb 1-tartibli uzluksiz hosilaga ega bo'lib, (1) tenglamalar sistemasini ayniyatga aylantiradigan har qanday $y_1=\varphi_1(x), y_2=\varphi_2(x),...,y_n=\varphi_n(x)$ funksiyalarga aytiladi.

(1) differensial tenglamalar sistemasi uchun Koshi masalasi, deb

$$y_1(x_0)=y_1^0, y_2(x_0)=y_2^0,...,y_n(x_0)=y_n^0$$

boshlang'ich shartlarni qanoatlantiruvchi (1) tenglamalar sistemasining $y_1=\varphi_1(x), y_2=\varphi_2(x),...,y_n=\varphi_n(x)$ yechimiga aytiladi.

(1) differensial tenglamalar sistemasining yechimini topish quyidagicha amalga oshiriladi.

Biz o'zgarmas koeffitsiyentli normal differensial tenglamalar sistemasi:

$$\begin{cases} \frac{dy_1}{dx} = a_{11}y_1 + ... + a_{1n}y_n + f_1(x), \\ \frac{dy_2}{dx} = a_{21}y_1 + ... + a_{2n}y_n + f_2(x), \\ .... \\ \frac{dy_n}{dx} = a_{n1}y_1 + ... + a_{nn}y_n + f_n(x) \end{cases} \quad (2)$$

bilan to'liqroq tanishib chiqamiz. Bu tenglamalar sistemasini matritsali shakldа $y' = Ay + f(x)$ kabi yozishimiz mumkin, bu yerda y noma'lum funksiyalar vektori, A koeffitsiyentlar matritsasi, $f(x)$ tashqi ta'sirni ifodalovchi vektor.

Agar $f_1(x) = 0,\ f_2(x) = 0, ..., f_n(x) = 0$ bo'lsa, (2) sistema bir jinsli chiziqli differensial tenglamalar sistemasi deb ataladi. (2) sistemaning umumiy yechimi bir jinsli differensial tenlamalar sistemasining umumiy yechimi va bir jinsli bo'lmagan sistemaning xususiy yechimi yig'indisi shaklida ifodalanadi, ya'ni

$$y = y_{bir\ jinsli} + y_{xususiy}.$$

Bir jinsli differensial tenglamalar sistemasining yechimini $y_1 = \alpha_1 e^{\lambda x}, ..., y_n = \alpha_n e^{\lambda x}$ ko'rinishda izlaymiz. Bu yerda $\alpha_1, ..., \alpha_n$, λ – noma'lum sonlar. $\frac{dy_i}{dx} = \alpha_i \lambda e^{\lambda x}$ $(i = 1, 2, ..., n)$ bo'lgani uchun (2) sistema quyidagi ko'rinishga ega bo'ladi:

$$\begin{cases} (a_{11} - \lambda)\alpha_1 + a_{12}\alpha_2 + ... + a_{1n}\alpha_n = 0, \\ a_{21}y_1 + (a_{22} - \lambda)\alpha_2 + ... + a_{2n}\alpha_n = 0, \\ .... \\ a_{n1}\alpha_1 + a_{n2}\alpha_2 + ... + (a_{nn} - \lambda)\alpha_n = 0. \end{cases} \quad (3)$$

Bu sistema noldan farqli yechimga ega bo'lishi uchin

$$\begin{vmatrix} a_{11} - \lambda & ... & a_{1n} \\ ... & ... & ... \\ a_{n1} & ... & a_{nn} - \lambda \end{vmatrix} = 0 \quad (4)$$

bo'lishi kerak. Agar determinant hisoblab chiqilsa biz bu yerdan $n-$ tartibli tenglamani hosil qilamiz. Bu tenglama (2) differensial tenglamalar sistemasining xarakteristik tenglamasi, deb ataladi. Bu tenglamadan $\lambda_1,...,\lambda_n$ −(2) sistema matritsasining xos sonlarini, so'ngra esa λ_i $(i=1,2,...,n)$ sonlarga mos $\alpha^{(j)}=\{\alpha_1^{(j)},....,\alpha_n^{(j)}\}$ − xos vektorlarni topamiz. Natijada chiziqli erkli $\{\alpha_i^{(j)}e^{\lambda_j x}\}$ $i,j=1,2,3,...,n$ −vektorlar sistemasini, ya'ni fundamental yechimlar sistemasini hosil qilamiz.

U holda (2) sistemaning umumiy yechimi quyidagicha bo'ladi:

$$\begin{cases} y_1(x)=\sum_{j=1}^{n} C_j\alpha_1^{(j)}\exp(\lambda_j x), \\ .... \\ y_n(x)=\sum_{j=1}^{n} C_j\alpha_n^{(j)}\exp(\lambda_j x). \end{cases}$$

Ikkita tenglamadan iborat bir jinsli avtonom sistemani alohida tahlil qilib chiqamiz

$$\begin{aligned} y_1' &= a_{11}y_1+a_{12}y_2+b_1, \\ y_2' &= a_{21}y_1+a_{22}y_2+b_2. \end{aligned} \qquad (5)$$

Bu sistemaga mos bir jinsli tenglamalar sistemasi quyidagicha bo'ladi:

$$\begin{aligned} y_1' &= a_{11}y_1+a_{12}y_2, \\ y_2' &= a_{21}y_1+a_{22}y_2. \end{aligned}$$

Bu sistemaning asosiy matritsasi

$$A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

ga teng. Xarakteristik tenglamani tuzamiz

$$\det\begin{pmatrix} a_{11}-\lambda & a_{12} \\ a_{21} & a_{22}-\lambda \end{pmatrix}=\lambda^2-(a_{11}+a_{22})\lambda+(a_{11}a_{22}-a_{12}a_{21}),$$

$$\lambda^2-(a_{11}+a_{22})\lambda+(a_{11}a_{22}-a_{12}a_{21})=0. \qquad (6)$$

Bu tenglamaning ildizlarini (xarakteristik sonlarni yoki matritsa xos sonlarini) topamiz. Maktab kursidan bizga ma'lumki, bunda uch holat ro'y berishi mumkin.

1) $D=(a_{11}+a_{22})^2-4(a_{11}a_{22}-a_{12}a_{21})>0$. Bu holatda (6) tenglama ikkita bir-biridan farqli ildizlarga ega:

$$\lambda_{1,2} = \frac{a_{11} + a_{22}}{2} \pm \frac{1}{2}\sqrt{D}$$

Ularga mos xos vektorlarni $(A - \lambda_k E)\alpha_k = 0$ tenglamalar sistemasidan topamiz:

$$\alpha_1 = \begin{pmatrix} a_{12} \\ \lambda_1 - a_{11} \end{pmatrix}, \quad \alpha_2 = \begin{pmatrix} a_{12} \\ \lambda_2 - a_{11} \end{pmatrix}.$$

Demak, bir jinsli tenglamalar sistemasining umumiy yechimi

$$y_1 = C_1 a_{12} e^{\lambda_1 x} + C_2 a_{12} e^{\lambda_2 x},$$
$$y_2 = C_1(\lambda_1 - a_{11})e^{\lambda_1 x} + C_2(\lambda_2 - a_{11})e^{\lambda_2 x}.$$

2) $D = (a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21}) = 0$. Bu shart ostida (6) tenglama ikkita bir xil yechimga ega $\lambda_1 = \lambda_2 = \lambda = \frac{a_{11} + a_{22}}{2}$. Bir jinsli tenglamalar sistemasining yechimi quyidagicha bo‘ladi:

$$y_1 = (C_1 + C_2 x)e^{\lambda x},$$
$$y_2 = \left[\frac{\lambda - a_{11}}{a_{12}}(C_1 + C_2 x) + \frac{C_2}{a_{12}}\right]e^{\lambda x}.$$

3) $D = (a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21}) < 0$. Bu shart ostida (6) tenglama haqiqiy sonlarda yechimga ega emas. Uning ildizlari kompleks qo‘shma sonlardan iborat bo‘ladi. $\lambda = h \pm iv$ bo‘lsin. Bu yerda

$$h = \frac{a_{11} + a_{22}}{2}, \quad v = \sqrt{|D|}.$$

U holda differensial tenglamalar sistemasining yechimi quyidagicha ifodalanadi:

$$y_1 = e^{hx}(C_1 \cos vx + C_2 \sin vx),$$
$$y_2 = e^{hx}\left(\frac{(h - a_{11})C_1 + vC_2}{a_{12}}\cos vx + \frac{(h - a_{11})C_2 - vC_1}{a_{12}}\sin vx\right).$$

Bunda C_1 va C_2 ixtiyoriy o‘zgarmaslar. Bu o‘zgarmaslarni aniqlash uchun odatda bizga ikkita shart zarur bo‘ladi: $y_1(0) = a_1$, $y_2(0) = a_2$. Bu ko‘rinishdagi shartlarni boshlang‘ich shartlar, deb ataymiz. Tenglamalar sistemasining boshlang‘ich shartlarni qanoatlantiruvchi yechimini topish masalasi Koshi masalasi deb ataladi.

Misol. $\begin{cases} \dot{x}(t) = x + 2y, \\ \dot{y}(t) = y + 2x \end{cases}$ sistemaning yechimini toping.

Yechish. Xarakteristik ildizlarni topamiz:

$$\begin{vmatrix} 1-\lambda & 2 \\ 2 & 1-\lambda \end{vmatrix} = (\lambda - 1)^2 - 4 = 0 \Rightarrow \lambda_1 = 3,\ \lambda_2 = -1.$$

Demak, sistemaning yechimini quyidagi ko‘rinishda izlash mumkin:

$$x(t) = C_1 e^{3t} + C_2 e^{-t}, \quad y(t) = a_1 e^{3t} + a_2 e^{-t}.$$

U holda sistemadagi 1-tenglamadan quyidagini hosil qilamiz:

$$3C_1 e^{3t} - C_2 e^{-t} = C_1 e^{3t} + C_2 e^{-t} + 2a_1 e^{3t} + 2a_2 e^{-t} \Rightarrow$$

$$2C_1 - 2a_1 = 0,\ -2C_2 - 2a_2 = 0 \Rightarrow a_1 = C_1,\ a_2 = -C_2.$$

Shunday qilib sistemaning umumiy yechimi:

$$\begin{cases} x(t) = C_1 e^{3t} + C_2 e^{-t}, \\ y(t) = C_1 e^{3t} - C_2 e^{-t}. \end{cases}$$

Sistemani yuqori darajali tenglamaga keltirib uning yechimini topish ham mumkin. Bu usul bilan biz tanishib chiqamiz.

Misol. $\begin{cases} y_1' = 2y_1 + 2y_2 \\ y_2' = y_1 + 3y_2 \end{cases}$ sistemaning umumiy yechimni toping.

Yechish. Bu tenglamalar sistemasining birinchi tenglamasidan x argument bo‘yicha hosila olamiz $y_1'' = 2y_1' + 2y_2'$. Ikkinchi tenglamadan foydalanib quyidagini hosil qilamiz: $y_1'' = 2y_1' + 2y_1 + 6y_2$. $y_2(x)$ funksiyani $y_1(x)$ va $y_1'(x)$ orqali ifodalaymiz: $y_2 = \frac{1}{2}y_1' - y_1$. So‘ngra quyidagi $y_1'' = 5y_1' - 4y_1$ tenglamaga ega bo‘lamiz. Bu tenglamaning yechimi $y_1(x) = C_1 e^{x} + C_2 e^{4x}$. Bu yechimdan hosila olib $y_2(x) = -\frac{1}{2}C_1 e^{x} + C_2 e^{4x}$ ga ega bo‘lamiz.

Misol. $\begin{cases} y_1' = -7y_1 + y_2 \\ y_2' = -2y_1 - 5y_2 \end{cases}$ sistemani $y_1|_{x=0} = 1$, $y_2|_{x=0} = 0$ boshlang‘ich shartlarni qanoatlantiruvchi yechimini toping.

Yechish. Bu yerda ham sistemaning birinchi tenglamasini differensiallab, so‘ng $y_2 = y_1' + 7y_1$ dan foydalanib $y_1'' + 12y_1' + 37 = 0$

tenglamaga ega bo‘lamiz. Bu tenglamaning umumiy yechimi: $y_1 = e^{-6x}\left(C_1 \cos x + C_2 \sin x\right)$. Bundan

$$y_2(x) = c_2 e^{-6x}(\cos x - \sin x) + c_2 e^{-6x}(\cos x + \sin x)$$

yechimni aniqlaymiz. Agar $y_1|_{x=0} = 1$, $y_2|_{x=0} = 0$ boshlang‘ich shartlardan foydalanilsa, tenglamalar sistemasining yechimi

$$y_1(x) = e^{-6x}(\cos x - \sin x)$$
$$y_2(x) = -2e^{-6x} \sin x$$

ko‘rinishda bo‘ladi.

Bir jinsli bo‘lmagan tenglamalar sistemasining yechimini topamiz. Buning uchun bu sistemaning xususiy yechimi topib, uni bir jinsli tenglamalar sistemasi yechimiga qo‘shib qo‘yish yetarli. Xususiy yechimni topishni ko‘rib chiqamiz.

Ta’rif. (5) tenglamalar sistemasining turg‘un yechimi, deb $y_1' = 0$, $y_2' = 0$ shartni qanoatlantiruvchi yechimga aytiladi

Turg‘un yechim quyidagi algebraik tenglamalar sistemasini yechib topiladi:

$$a_{11}y_1 + a_{12}y_2 = -b_1,$$
$$a_{21}y_1 + a_{22}y_2 = -b_2.$$

Agar $a_{11}a_{22} - a_{12}a_{21} \neq 0$ bo‘lsa,

$$\overline{y}_1 = \frac{a_{21}b_1 - a_{22}b_2}{a_{11}a_{22} - a_{12}a_{21}},$$
$$\overline{y}_2 = \frac{a_{12}b_1 - a_{11}b_2}{a_{11}a_{22} - a_{12}a_{21}}.$$

Bu holatda turg‘un yechim (5) sistemaning xususiy yechimi bo‘ladi.

Misol. $\begin{cases} \dot{x}(t) = x + 2y - 5, \\ \dot{y}(t) = y + 2x - 4 \end{cases}$ tenglamalar sistemasining umumiy yechimini toping.

Yechish. Xususiy yechimni topamiz.

$$x + 2y - 5 = 0,$$
$$y + 2x - 4 = 0$$

tenglamalar sistemasidan $\bar{x}=1$, $\bar{y}=2$ turg'un yechimni hosil qilamiz. Oldingi misollarda bu sistemaga mos bir jinsli tenglamalar sistemasining yechimi quyidagicha topilgan edi:

$$\begin{cases} x_b(t)=C_1e^{3t}+C_2e^{-t}, \\ y_b(t)=C_1e^{3t}-C_2e^{-t}. \end{cases}$$

Demak, sistemating umumiy yechimi

$$\begin{cases} x(t)=C_1e^{3t}+C_2e^{-t}+1, \\ y(t)=C_1e^{3t}-C_2e^{-t}+2. \end{cases}$$

8.4. Sonli qatorlar

Yaqinlashuvchi qatorlar va ularning xossalari

Ushbu

$$a_1, a_2, \ldots, a_n, \ldots$$

haqiqiy sonlar ketma-ketligi berilgan bo`lsin.

1-ta`rif. Quyidagi

$$a_1+a_2+\ldots+a_n+\ldots \qquad (1)$$

ifodaga qator (sonli qator) deyiladi va u $\sum_{n=1}^{\infty} a_n$ kabi belgilanadi.

Shunday qilib,

$$\sum_{n=1}^{\infty} a_n = a_1+a_2+\ldots+a_n+\ldots \qquad (2)$$

ekan. $\{a_n\}$ ketma-ketlikning $a_1, a_2, \ldots, a_n, \ldots$ elementlari qatorning hadlari deyiladi, a_n esa qatorning umumiy hadi deb ataladi. Ushbu

$$S_n=\sum_{k=1}^{n} a_k, \; n=1,2,\ldots. \qquad (3)$$

yig`indilar esa (2)-qatorning qismiy yig`indilari deyiladi.

**2-ta`rif.** Agar $\{S_n\}$ketma-ketlik chekli limitga ega, ya`ni

$$\lim_{n\to\infty} S_n = S$$

bo`lsa, unda qator yaqinlashuvchi deyiladi va bu limitning qiymati $S$ (2)-qatorning yig`indisi deb ataladi hamda u

$$S = a_1 + a_2 + \ldots + a_n + \ldots = \sum_{n=1}^{\infty} a_n$$

kabi yoziladi.

Agar $\{S_n\}$ ketma-ketlik yaqinlashuvchi bo`lmasa, u holda uzoqlashuvchi deyiladi.

3-ta`rif. Ushbu

$$\sum_{n=m+1}^{\infty} a_n = a_{m+1} + a_{m+2} + \ldots \qquad (4)$$

qator (2)-qatorning (m-hadidan keyingi) qoldig`i deyiladi.

Teorema. Agar (2)-qator yaqinlashuvchi bo`lsa, uning istalgan (4)-qoldig`i ham yaqinlashuvchi bo`ladi va aksincha, (4)-qoldiqning yaqinlashuvchi bo`lishidan berilgan (2)-qatorning yaqinlashuvchi bo`lishi kelib chiqadi.

1-natija. Agar (2)-qator yaqinlashuvchi bo`lsa, uning qoldig`i

$$r_m = a_{m+1} + a_{m+2} + \ldots$$

$m \to \infty$ da nolga intiladi.

Teorema. Agar (2)-qator yaqinlashuvchi bo`lib, uning yig`indisi S bo`lsa, u holda $\sum_{n=1}^{\infty} ca_n$ qator ham yaqinlashuvchi bo`lib, uning yig`indisi $c \cdot S$ bo`ladi , ya`ni

$$\sum_{n=1}^{\infty} ca_n = c \cdot \sum_{n=1}^{\infty} a_n$$

tenglik bajariladi.

Teorema. Agar $\sum_{n=1}^{\infty} a_n$ va $\sum_{n=1}^{\infty} b_n$ qatorlar yaqinlashuvchi bo`lsa, unda $\sum_{n=1}^{\infty}(a_n + b_n)$qator ham yaqinlashuvchi bo`lib,

$$\sum_{n=1}^{\infty}(a_n + b_n) = \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n$$

bo`ladi.

Yuqoridagi 2 ta teoremalardan quyidagi natija kelib chiqadi.

2-natija. Agar $\sum_{n=1}^{\infty} a_n$ va $\sum_{n=1}^{\infty} b_n$ qatorlar yaqinlashuvchi bo`lsa, $\sum_{n=1}^{\infty}(ca_n + d_0 b_n)$ $(c, d - const)$ qator ham yaqinlashuvchi bo`lib,

$$\sum_{n=1}^{\infty}(c \cdot a_n + d \cdot b_n) = c \cdot \sum_{n=1}^{\infty} a_n + d \cdot \sum_{n=1}^{\infty} b_n$$

bo`ladi.

Teorema. (Qator yaqinlashishining zaruriy sharti).
Agar (2)-qator yaqinlashuvchi bo`lsa, u holda

$$\lim_{n\to\infty} a_n = 0 \qquad (5)$$

bo`ladi.

Izoh. Teoremaning aksi har doim ham o`rinli bo`lavermaydi. Masalan, $\sum_{n=1}^{\infty}\frac{1}{n}$ uchun $\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{1}{n} = 0$, lekin bu qator yaqinlashuvchi emas.

Musbat hadli qatorlar va ularning yaqinlashishi

Aytaylik,

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + ... + a_n + ... \qquad (6)$$

qator berilgan bo`lsin. Agar $\forall n \in N$ uchun $a_n \geq 0$ bo`lsa, unda (6)-qatorga musbat hadli qator yoki qisqacha musbat qator deb ataladi.
Bu punktda biz musbat hadli qatorlar uchun yaqinlashish alomatlarini keltiramiz.

Faraz qilaylik, (6)-qator va ushbu

$$\sum_{n=1}^{\infty} b_n = b_1 + b_2 + ... + b_n + ... \qquad (7)$$

qatorlar berilgan bo`lsin. Unda quyidagi taqqoslash teoremalari o`rinli bo`ladi.

Teorema. (Birinchi taqqoslash alomati) Agar n ning biror $n_0 (n_0 \geq 1)$ qiymatidan boshlab barcha $n \geq n_0$ lar uchun

$$a_n \leq b_n$$

tengsizlik o`rinli bo`lsa, unda (7)-qatorning yaqinlashuvchi bo`lishidan (6) qatorning yaqinlashuvchi bo`lishi va (6)-qatorning uzoqlashuvchi bo`lishidan (7)-qatorning uzoqlashuvchi bo`lishi kelib chiqadi.

Teorema. Agar

$$\lim_{n\to\infty}\frac{a_n}{b_n}=k \quad (0\le k\le\infty)$$

bo`lsa,

a) $k<\infty$ bo`lganda, (7)-qatorning yaqinlashuvchi bo`lishidan (6)-qatorning yaqinlashuvchi bo`lishi;

b) $k>0$ bo`lganda, (7)-qatorning uzoqlashuvchi bo`lishidan (6)-qatorning uzoqlashuvchi bo`lishi kelib chiqadi.

Natija. Agar $n\to\infty$ da $a_n=0^*(b_n)$ bo`lsa $(y'ni\ \ 0<k<\infty\ bo'lsa)$ unda (6)-qatorning yaqinlashishi (7)-qatorning yaqinlashishiga ekvivalent bo`ladi.

Teorema. (Ikkinchi taqqoslash alomati) Agar n ning biror $n_0(n_0\ge 1)$ qiymatidan boshlab barcha $n\ge n_0$ lar uchun

$$\frac{a_{n+1}}{a_n}\le\frac{b_{n+1}}{b_n}$$

tengsizlik bajarilsa , unda

1) (7)-qator yaqinlashuvchi bo`lsa, (6)-qator yaqinlashuvchi;

2) (6)-qator uzoqlashuvchi bo`lsa, (7)-qator uzoqlashuvchi bo`ladi.

Endi musbat hadli (6)-qator uchun yaqinlashish alomatlarini keltiramiz.

Teorema (Dalamber alomati). $\sum\limits_{n=1}^{\infty}a_n$ qator uchun $\lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=m$ limit mavjud bo'lsin. U holda,

1. Agar $m<1$ bo'lsa, qator yaqinlashadi;
2. Agar $m>1$ bo'lsa, qator uzoqlashadi.

Teorema (Koshi alomati). $\sum\limits_{n=1}^{\infty}a_n$ qator uchun $\lim\limits_{n\to\infty}\sqrt[n]{a_n}=l$ limit mavjud bo'lsin. U holda,

1. Agar $l<1$ bo'lsa, qator yaqinlashadi;
2. Agar $l>1$ bo'lsa, qator uzoqlashadi.

Izoh. Dalamber va **Koshi alomatlarida** $m = l = 1$ bo`lsa, qator uzoqlashuvchi ham, yaqinlashuvchi ham bo`lishi mumkin.

Teorema. (Koshining integral alomati). Faraz qilaylik, $f(x)$ funksiya $[1;+\infty)$ oroliqda aniqlangan bo`lib, $f(x) > 0$ va monoton kamayuvchi bo`lsin. U holda

$$\sum_{n=1}^{\infty} f(n)$$

qatorning yaqinlashuvchi bo`lishi uchun

$$\int_{1}^{+\infty} f(x)dx$$

integralning yaqinlashuvchi bo`lishi zarur va yetarli.

Ishoralari navbatlashuvchi qatorlar

4-ta'rif. Ushbu

$$u_1 - u_2 + u_3 - u_4 + \ldots + (-1)^n u_n + \ldots = \sum_{n=1}^{\infty} (-1)^{n+1} u_n \qquad (8)$$

bu yerda $u_1, u_2, u_3, \ldots, u_n, \ldots$ musbat sonlar, qator ishoralari navbatlashuvchi qator deyiladi.

Isharalari navbatlashuvchi qatorlar uchun quyidagi teorema o'rinli:

Teorema (Leybnits teoremasi). Agar ishoralari navbatlashuvchi

$$u_1 - u_2 + u_3 - u_4 + \ldots + (-1)^n u_n + \ldots$$

qatorda

1) qator hadlarining absolyut qiymatlari kamayuvchi, ya'ni

$$u_1 > u_2 > u_3 > u_4 > \ldots > u_n > \ldots \qquad (9)$$

bo'lsa,

2) qatorning u_n umumiy hadi $n \to \infty$ da nolga intilsa:

$$\lim_{n\to\infty} u_n = 0 \qquad (10)$$

u holda (8) qator yaqinlashuvchi bo'ladi.

Absolyut yaqinlashuvchi va shartli yaqinlashuvchi qatorlar

Teorema. Agar ixtiyoriy hadli

$$u_1 + u_2 + u_3 + u_4 + \ldots + u_n + \ldots \qquad (11)$$

qator hadlarining absolyut qiymatlaridan tuzilgan

$$|u_1| + |u_2| + |u_3| + |u_4| + \ldots + |u_n| + \ldots \qquad (12)$$

qator yaqinlashsa, u holda berilgan qator ham yaqinlashuvchi bo'ladi.

5-ta'rif. Ixtiyoriy hadli (11) qator hadlari absolyut qiymatlaridan tuzilgan (12) qator yaqinlashuvchi bo'lsa, (11) qator absolyut yaqinlashuvchi qator deyiladi.

6-ta'rif. Agar ixtiyoriy hadli (11) qator yaqinlashuvchi bo'lib, bu qator hadlarining absolyut qiymatlaridan tuzilgan (12) qator uzoqlashuvchi bo'lsa, u holda (11) qator shartli yaqinlashuvchi deyiladi.

Ixtiyoriy hadli qatorni absolyut yaqinlashishga tekshirganda musbat qatorlar uchun isbotlangan taqqoslash, Dalamber, Koshi alomatlaridan foydalanish mumkin.

Misollar

1. Birinchi taqqoslash alomatidan foydalanib

$\frac{2}{3} + \frac{1}{2}\left(\frac{2}{3}\right)^2 + \frac{1}{3}\left(\frac{2}{3}\right)^3 + \ldots + \frac{1}{n}\left(\frac{2}{3}\right)^n + \ldots$ qatorni yaqinlashishga tekshiring.

Yechish. Ushbu qatorni qaraymiz: $\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \ldots + \left(\frac{2}{3}\right)^n + \ldots$.

Ravshanki, $a_n = \frac{1}{n}\left(\frac{2}{3}\right)^n \le \left(\frac{2}{3}\right)^n = b_n$. Mahraji $q = \frac{2}{3}$ bo'lgan $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$ geometrik qator yaqinlashuvchi, demak 1-teoremaga ko'ra berilgan $\sum_{n=1}^{\infty}\frac{1}{n}\cdot\left(\frac{2}{3}\right)^n$ qator ham yaqinlashuvchi bo'ladi.

2. Qatorni yaqinlashishga tekshiring:

$$\frac{2}{1^2} + \frac{2^2}{2^2} + \frac{2^3}{3^2} + \ldots + \frac{2^n}{n^2} + \ldots$$

Yechish. Ravshanki, $a_n = \frac{2^n}{n^2}$, $a_{n+1} = \frac{2^{n+1}}{(n+1)^2}$. Dalamber alomatidan quyidagini topamiz:

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \lim_{n\to\infty}\frac{\frac{2^{n+1}}{(n+1)^2}}{\frac{2^n}{n^2}} = 2\lim_{n\to\infty}\frac{n^2}{(n+1)^2} = 2 > 1.$$

Demak, qator uzoqlashuvchi.

3. $\sum_{n=1}^{\infty}\frac{1}{n^2}$ qatorning yaqinlashuvchi ekanligi ko‘rsatilsin.

Yechish. Qator umumiy hadi.

$a_n = f(n) = \frac{1}{n^2}$ ko‘rinishda.

Qatorga mos keluvchi xosmas integralni hisoblaymiz

$$\int_1^{\infty} f(x)dx = \int_1^{\infty}\frac{1}{x^2}dx = \lim_{b\to\infty}\int_1^{b}\frac{dx}{x^2} = \lim_{b\to\infty}\left[-\frac{1}{x}\Big|_1^b\right] = \lim_{b\to\infty}\left[-\frac{1}{b}+1\right] = 1$$

Demak: $\sum_{n=1}^{\infty}\frac{1}{n^2}$ yaqinlashuvchi qatordir.

4. Berilgan qatorni yaqinlashishga tekshiring:

$$\frac{1}{ln2} + \frac{1}{ln^2 3} + \ldots + \frac{1}{ln^n(n+1)} + \ldots$$

Yechish. $\lim_{n\to\infty}\sqrt[n]{a_n} = \lim_{n\to\infty}\sqrt[n]{\frac{1}{\ln^n(n+1)}} = \frac{1}{\ln(n+1)} = 0 < 1$

Demak, qator yaqinlashuvchi.

5. Ushbu

$$\frac{\sin\frac{\pi}{4}}{1!} + \frac{2^2\sin\frac{3\pi}{4}}{2!} + \frac{3^2\sin\frac{5\pi}{4}}{3!} + \ldots + \frac{n^2\sin\frac{(2n-1)\pi}{4}}{n!} + \ldots$$

qatorni yaqinlashishga tekshiring.

Yechish. Berilgan qator hadlarining absolyut qiymatlaridan tuzilgan ushbu

$$\sum_{n=1}^{\infty}\left|\frac{n^2\sin\frac{(2n-1)\pi}{4}}{n!}\right|$$

qatorni qaraymiz. Bu qatorni Dalamber alomati yordamida tekshiramiz. U holda

$$\lim_{n\to\infty}\left|\frac{(n+1)^2 n!\sin\frac{(2n+1)\pi}{4}}{(n+1)!n^2\sin\frac{(2n-1)\pi}{4}}\right| = \lim_{n\to\infty}\left(\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}\cdot\frac{n+1}{n^2}\right) = 0 < 1.$$ Dalamber alomatiga ko‘ra

$\sum_{n=1}^{\infty}\left|\frac{n^2\sin\frac{(2n-1)\pi}{4}}{n!}\right|$ qator yaqinlashuvchi. Demak, berilgan qator absolyut yaqinlashadi.

6. Ushbu

$$\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{n}{3n+1}\right)^n$$

qatorni yaqinlashishga tekshiring.

Yechish. Qator hadlarining absolyut qiymatlaridan tuzilgan musbat hadli $\sum_{n=1}^{\infty}\left(\frac{n}{3n+1}\right)^n$ qatorga Koshining radikal alomatini tatbiq etamiz:

$\lim_{n\to\infty}\sqrt[n]{\left|\left(\frac{n}{3n+1}\right)^n\right|} = \lim_{n\to\infty}\frac{n}{3n+1} = \frac{1}{3} < 1$. Koshi alomatiga ko‘ra $\sum_{n=1}^{\infty}\left(\frac{n}{3n+1}\right)^n$ qator yaqinlashuvchi, demak, berilgan qator absolyut yaqinlashuvchi bo‘ladi.

8.5.Talabaning mustaqil ishi

1-topshiriq
1-misolda berilgan o‘zgaruvchilari ajraladigan differensial tenglamaning umumiy yechimini (umumiy integralini) toping, 2-misolda differensial tenglamaning berilgan boshlang’ich shartni qanoatlantiruvchi xususiy yechimini (xususiy integralini), 3- misolda berilgan bir jinsli differensial tenglamaning umumiy yechimini, 4-misolda chiziqli differensial tenglamaning umumiy yechimini toping.

1-variant

1. $(1+y)dx-(1-x)dy=0.$

2. $x^2dy-y^2dx=0,\ \ y\left(\frac{1}{2}\right)=\frac{1}{3}$

3. $(y+2)dx=(2x+y-4)dy.$

4. $y'+2y=3e^x$

2-variant

1. $\sqrt{1-y^2}\,dx+y\sqrt{1-x^2}\,dy=0.$

2. $1+y^2=xyy',\ \ y(2)=1$

3. $y'=\dfrac{1-3x-3y}{1+x+y}$

4. $(1+x^2)y'+2xy=3x^2$

3-variant

1. $xyy'=1-x^2.$

2. $(x+xy^2)dx+(x^2y-y)dy=0,\ \ y(0)=1.$

3. $(4x^2+3xy+y^2)dx+(4y^2+3xy+x^2)dy=0.$

4. $2(x+y^4)y'-y=0$

4-variant

1. $y'(1+y)=xysinx.$

2. $y'(x^2-2)=2xy,\ \ y(2)=2.$

3. $xdy-ydx=\sqrt{x^2+y^2}\,dx.$

4. $y^2dx+(xy-1)dy=0$

5-variant

1. $e^y(1+y')=1.$

2. $cosxsinydy=cosysinxdx,\ \ y(\pi)=\pi$

3. $y-xy'=x+yy'.$

4. $xy'+y=\dfrac{y^2}{2}lnx$

6-variant

1. $y'-xy^2=0.$

2. $y'=1{,}5\sqrt[3]{y},\ \ y(-2)=1.$

3. $xy'=y\ln\dfrac{x}{y}.$

4. $y'+2xy=2xy^3$

7-variant

1. $(\sqrt{xy}+\sqrt{x})y'-y=0.$

2. $y'=2^{x+y}+2^{x-y},\ \ y(0)=0.$

$3. xy^2dy=\left(x^3+y^3\right)dx.$

$4. y'+y\cos x=\sin 2x.$

8-variant

$1. y'=3^{x-y}.$

$2. xy'-\frac{y}{\ln x}=0,\ \ y(e)=1$

$3. \left(xy-x^2\right)y'=y^2.$

$4. x\frac{dy}{dx}+y=4x^3$

9-variant

$1. y'=\frac{y+1}{x+1}.$

$2. y'\sin x-(2y+1)\cos x=0,\ \ y\left(\frac{\pi}{3}\right)=1$

$3. y-xy'=y\ln\frac{x}{y}.$

$4. y'e^{x^2}-\left(xe^{x^2}-y^2\right)y=0$

10-variant

$1. ds+s\,tgt\,dt=0.$

$2. \left(e^x+8\right)dy-ye^xdx=0,\ \ y(0)=1$

$3. y'=-\frac{x+y}{x}.$

$4. x^3y^2y'+x^2y^3=1$

11-variant

$1. \frac{yy'}{x}+e^y=0.$

$2. ydx+ctgx\,dy=0,\ \ y\big|_{x=\frac{\pi}{3}}=-1.$

$3. y'=\frac{y}{x}-1.$

$4. y'x^3\sin y-xy'+2y=0$

12-variant

$1. x+xy+y'\left(y+xy\right)=0.$

$2. 2\sqrt{y}dx-dy=0,\ \ y(0)=1$

$3. \left(y^2-2xy\right)dx+x^2dy=0$

$4. y'-y=\left(x+\frac{1}{x}\right)e^x$

13-variant

$1. y'+y=5.$

$2. y'=8\sqrt{y},\ \ y(0)=4.$

3. $(x^2+y^2+xy)dx-x^2dy=0$

4. $y'+\frac{x}{1-x^2}y=2$

14-variant

1. $v'-4tv=0.$

2. $y'sinx-ylny=0,\quad y\left(\frac{\pi}{2}\right)=1$

3. $y\left(x^2+y^2\right)dx-x^3dy=0$

4. $y'-\frac{y}{sinx}=tg\frac{x}{2}$

15-variant

1. $dy-ycos^2xdx=0.$

2. $\left(1+y^2\right)dx+\left(1+x^2\right)dy=0,\quad y(1)=2.$

3. $xy'-y=(x+y)\,ln\frac{x+y}{x}$

4. $xcos^2xy'+2ycos^2x=2x\sqrt{y}$

16-variant

1. $y'=sin\frac{x-y}{2}-sin\frac{x+y}{2}.$

2. $y'-1=e^{\frac{y}{x}}+\frac{y}{x},\quad y(1)=0$

3. $\frac{xy'-y}{x}=tg\frac{y}{x}.$

4. $ydx+(4lny-2x-y)dy=0$

17-variant

1. $\left(e^x+1\right)e^yy'+e^x\left(1+e^y\right)=0.$

2. $xdy=(x+y)dx,\quad y(1)=0$

3. $\left(2x^3y-y^4\right)dx+\left(2xy^3-x^4\right)dy=0$

4. $y=x(y'-x\cos x).$

18-variant

1. $y'+\frac{xsinx}{ycosy}=0.$

2. $y^2+x^2y'=xyy',\quad y(1)=1$

3. $\left(3x^2-y^2\right)y'=2xy$

4. $y'+2y=x^2+2x.$

19-variant

1. $y'=\cos(y-x).$

2. $\left(y'-\frac{y}{x}\right)arctg\frac{y}{x}=1,\quad y\left(\frac{1}{2}\right)=0$

$3.\frac{dy}{dx}-\frac{y}{x}(1+lny-lnx)=0$

$4.x^2y'+xy+1=0.$

20-variant

$1.(xy+x)\frac{dx}{dy}=1.$

$2.x^2y'+xy-x^2-y^2=0,\ y(1)=0.$

$3.xy'+xtg\frac{y}{x}=y$

$4.y'+ytgx=\frac{1}{\cos x}.$

21-variant

$1.6xdx-6ydy-2x^2ydy+3xy^2dx=0.$

$2.x^2-3y^2+2xyy'=0,\ y(-2)=2$

$3.y'cos\frac{y}{x}-\frac{y}{x}cos\frac{y}{x}+1=0$

$4.y'+2xy=2xe^{-x^2}.$

22-variant

$1.x^2dy+(y-a)dx=0.$

$2.y-xy'=2(x+yy'),\ y(1)=0$

$3.y'=\frac{x+y}{x-y}$

$4.(xy'-1)\ln x=2y.$

23-variant

$1.y'tgx-y=a.$

$2.y'=\frac{y}{x}ln\frac{y}{x},\ y(1)=e$

$3.\sqrt{y}(2\sqrt{x}-\sqrt{y})dx+xdy=0$

$4.(2x+y)dy=ydx+4\ln ydy.$

24-variant

$1.y'cosx-(y+1)sinx=0.$

$2.xy'-y-x^3=0,\ y(2)=4$

$3.x^2+y^2=2xyy'$

$4.x\ln x\cdot y'-y=x^3(3\ln x-1).$

25-variant

$1.y'-2yctgx=ctgx.$

$2.y'sinx-ycosx=1,\ y\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$

$3.ss'-2s+t=0$

$4.xy'-2y=2x^4.$

2-topshiriq

1-misolda o'zgarmas koeffitsientli chiziqli bir jinsli bo'lgan differensial tenglamalarning umumiy yechimini; 2- misolda o'zgarmas koeffitsientli chiziqli bir jinsli bo'lmagan differensial tenglamalarning umumiy yechimini toping; 3- iqtisodiy mazmundagi masalalarning matematik modelini quring va yeching.

1-variant

1. $y''-5y'+6y=0$.
2. $y''-4y'+4y=x^2$.
3. Agar elastiklik $E_p=-\frac{1}{2}$ o'zgarmas va talabning $y=2$ qiymatida $p=5$ narx berilgan bo'lsa, $y=y(p)$ talab funksiyasini toping.

2-variant

1. $y''-3y'+2y=0$.
2. $y''+8y'=8x$.
3. Agar elastiklik $E_p=-3$ o'zgarmas va talabning $y=27$ qiymatida $p=2$ narx berilgan bo'lsa, $y=y(p)$ talab funksiyasini toping.

3-variant

1. $y''-4y'+4y=0$.
2. $7y''-y'=14x$.
3. Agar talabning $y=10$ qiymatida narx $p=90$ berilgan hamda elastiklik $E_p=\frac{y-100}{y}$, $0<y<100$ ko'rinishda bo'lsa, talab funksiyasini toping.

4-variant

1. $y''-8y'+25y=0$.
2. $y''-2y'-3y=e^{4x}$.
3. Tog' ruda posyolkasi aholisining soni vaqt o'tishi bilan o'zgarishi $y'=0{,}3y(2-10^{-4}y)$ tenglama bilan ifodalanadi, bu erda $y=y(t)$, t–vaqt (yillarda). Vaqtning boshlang'ich momentida posyolka aholisi 500 odamni tashkil etgan. Uch yildan so'ng u qanday bo'ladi.

5-variant

1. $y''-2y'+2y=0$.
2. $y''+4y'+3y=9e^{-3x}$.
3. Agar $E_p=-2=const$ va $y(3)=\frac{1}{6}$ bo'lsa, u holda talab funksiyasini toping.

6-variant

1. $y''+4y'=0$.

2. $y''+4y'+4y=8e^{-2x}$.

3. Talab va taklif funksiyalari mos ravishda $y=25-2p+3\frac{dp}{dt}$ va $x=15-p+4\frac{dp}{dt}$ ko'rinishiga ega. Agar boshlang'ich moment $p=9$ bo'lsa, muvozanat narx bilan vaqt o'rtasidagi bog'liqlikni toping.

7-variant

1. $y''+3y'+2y=0$.
2. $y''-3y'+2y=\sin x$.
3. Agar ishlab chiqarish hajmi (investitsiyalar normasi 0,6, sotilish bahosi 0,15 va $l=0{,}4$ shartlarda) vaqtning boshlang'ich momentida $y_0=y(0)=24$ (shart.bir) ni tashkil etgan bo'lsa, to'yinmagan bozor sharoitda 6 oyda ishlab chiqarilgan mahsulot hajmini toping.

8-variant

1. $y''+2y'+5y=0$.
2. $y''+y=4\sin x$.
3. Tovar narxi $p(y)=(5+3e^{-y})y^{-1}$, $m=0{,}6$, $l=0{,}4$, $y(0)=1$ funksiya bilan beriladi deb faraz qilib, sotilayotgan mahsulot hajmi bilan vaqt orasidagi $y=y(t)$ bog'liqlikni toping.

9-variant

1. $y''-y=0$.
2. $y''+3y'-4y=e^{-4x}+xe^{-x}$.
3. Biror tuman aholisining o'sish ($y=y(t)$) soni $\frac{dy}{dt}=\frac{0{,}2y}{m}(m-y)$ tenglama bilan ifolanadi, bu erda $m-$mazkur tuman aholisining mumkin bo'lgan maksimal soni. Aholi soni vaqtning boshlang'ich momentida maksimal sonning 1% ni tashkil etgan. Necha yildan so'ng aholi soni maksimal sonning 80% ni tashkil etadi.

10-variant

1. $y''+y=0$.
2. $y''-5y'=3x^2+\sin 5x$.
3. Axolisi 3000 odam bo'lgan mahallada gripp epidemiyasining tarqalishi $\frac{dy}{dt}=0{,}001y(3000-y)$ tenglama bilan ifodalangan, bu erda $y-t$ vaqt momentida kasal bo'lganlar soni $t-$haftalar soni. Agar vaqtning boshlang'ich momentida kasallar soni 3 ta bo'lgan bo'lsa, 2 haftadan so'ng mahalladagi kasallar sonini aniqlang.

11-variant

1. $y''-5y'+6y=0$.
2. $y''+2y'+5y=e^{-x}\cdot\sin 2x$.

3. Agar talabning $y=1$ qiymatida narx $p=18$ berilgan hamda elastiklik $E_p=\frac{p}{p-20}$, $0<p<20$, ko'rinishda bo'lsa, talab funksiyasini toping.

12-variant

1. $2y''+5y'-7y=0$.
2. $y''+y'=\cos^2 x+e^x+x^2$.
3. Biror tovarga bo'lgan talab va taklif funksiyalari mos ravishda $y=50-2p-4\frac{dp}{dt}$, $x=70+2p-5\frac{dp}{dt}$ ko'rinishda berilgan. Agar $p(0)=10$ bo'lsa muvozanat narx bilan vaqt orasidagi bog'liqlikni toping.

13-variant

1. $y''+4y'-3y=0$.
2. $y''+2y'+y=e^x$.
3. Agar biror tovarga bo'lgan talab va taklif funksiyalari mos ravishda $y=50-2p-4\frac{dp}{dt}$, $x=70+2p-5\frac{dp}{dt}$ ko'rinishda berilgan bo'lsa, muvozanat narx barqaror bo'la oladimi?

14-variant

1. $3y''+y'-2y=0$.
2. $y''+y'-2y=-4$.
3. Biror tovarga bo'lgan talab va taklif funksiyalari mos ravishda $y=30-p-4\frac{dp}{dt}$, $x=20+p+\frac{dp}{dt}$ ko'rinishda berilgan. Muvozanat narx bilan vaqt orasidagi bog'liqlikni toping.

15-variant

1. $y''+25y'=0$.
2. $y''+3y=9x$.
3. Agar biror tovarga bo'lgan talab va taklif funksiyalari mos ravishda $y=30-p-4\frac{dp}{dt}$, $x=20+p+\frac{dp}{dt}$ ko'rinishda berilgan bo'lsa, muvozanat narx barqaror bo'la oladimi?

16-variant

1. $4y''-9y'=0$.
2. $y''-5y'+6y=6x$.
3. Faraz qilamiz, mahsulot ishlab chiqarish hajmi $q(t)$ bo`lib, kichik $\Delta t$ oralig`ida mahsulot hajmining o`zgarishi $p(t)$ narxga proportsional bo`lsin: $\Delta q=\alpha pq\Delta t$. Boshqacha aytganda $q'=\alpha pq$. Bu yerda $p=p(q)$ deb hisoblaymiz va mahsulot hajmining ortishi mahsulot narxining pasayishiga olib keladi: $\frac{\Delta p}{\Delta q}<0$.

Agar E narxning elastikligi bo`lsin. $E_1 = -0,7$; $E_2 = -1,2$ holatlar uchun ishlab chiqarish o`sishining tezlashishini ($q = q(t)$ funksiyaning qavariqliligini) yoki sekinlashishini ($q = q(t)$ funksiyaning botiqliligini) aniqlang.

17-variant

1. $y'' - 6y' + 9y = 0.$

2. $y'' + y' - 2y = 2e^{2x}.$

3. Faraz qilamiz, mahsulot ishlab chiqarish hajmi $q(t)$ bo`lib, kichik $\Delta t$ oralig`ida mahsulot hajmining o`zgarishi $p(t)$ narxga proportsional bo`lsin: $\Delta q = \alpha pq\Delta t$. Boshqacha aytganda $q' = \alpha pq$. Bu yerda $p = p(q)$ deb hisoblaymiz va mahsulot hajmining ortishi mahsulot narxining pasayishiga olib keladi: $\frac{\Delta p}{\Delta q} < 0$.

Agar $p = 10 - q$ bo`lsa, u holda $q = q(t)$ funksiyani qavariqlikka tekshiring.

18-variant

1. $y'' - 4y' + 4y = 0.$

2. $y'' - 5y' + 6y = e^{2x}.$

3. Faraz qilamiz, mahsulot ishlab chiqarish hajmi $q(t)$ bo`lib, kichik $\Delta t$ oralig`ida mahsulot hajmining o`zgarishi $pq - c$ foydaga proportsional bo`lsin: $\Delta q = \alpha(pq - c)\Delta t$. Bu yerda p – mahsulot narxi, c – xarajat. Agar $p = 10 - q$, $c = \beta q + 4$, $\beta < 10$ bo`lsa: ishlab chiqarish $\beta$ ning qanday qiymatlarida $q_0 = q(0)$ – boshlang`ich qiymatga bog`liq bo`lmagan kamayadi;

19-variant

1. $4y'' - 12y' + 9y = 0.$

2. $y'' + 3y' - 4y = (x+1)e^x.$

3. Faraz qilamiz, mahsulot ishlab chiqarish hajmi $q(t)$ bo`lib, kichik $\Delta t$ oralig`ida mahsulot hajmining o`zgarishi $pq - c$ foydaga proportsional bo`lsin: $\Delta q = \alpha(pq - c)\Delta t$. Bu yerda p – mahsulot narxi, c – xarajat. Agar $p = 10 - q$, $c = \beta q + 4$, $\beta < 10$ bo`lsa: $\beta = 5$ bo`lganda ishlab chiqarihsning $q_0 = q(0)$ – boshlang`ich qiymatga bog`liqligini tekshiring.

20-variant

1. $9y'' + 12y' + 4y = 0.$

2. $y'' - 2y' + y = (x+1)e^x.$

3. $y = y(t)$ ishlab chiqarishning intensivligi $y' = ky,\ k = const$ bo`lsin. Agar ishlab chiqarishning I kvartaldagi o`sishi 3% bo`lsa, u holda uning yillik o`sishini toping;

21-variant

1. $y''+4y=0.$

2. $y''+2y'+y=(x+3)e^{-x}.$

3. $y = y(t)$ ishlab chiqarishning intensivligi $y' = ky,\ k = const$ bo`lsin. Agar ishlab chiqarishda yillik o`sish 25% bo`lsa, u holda har oylik o`sish qanday bo`ladi?

22-variant

1. $4y''+9y=0.$

2. $y''+4y'-5y=1.$

3. $p = p(t)$ narxning o`zgarish tezligi talab va taklif farqiga proportsional bo`lsin: $p' = \gamma(d - s)$. Agar talab va taklif uchun mos ravishda $d = 100 - 10p,\ s = 10 + 20p$ munosabatlar o`rinli bo`lsa, u holda $p(t)$ ning $t \to \infty$ dagi xususiyatini $p_0 = p(0)$ boshlang`ich shartga bog`liq holda tekshiring.

23-variant

1. $y''+y'+y=0.$

2. $y''+y=ctgx.$

3. Faraz qilamiz, mahsulot ishlab chiqarish o`zgarishining tezligi foydaga proportsional bo`lsin, $q' = \alpha(pq - c)$. Bu yerda p – mahsulot narxi, c – xarajatlar. $\alpha = 0,2,\ p = 10 - q,\ c = \beta q + 4,\ \beta = const$ bo`lsin. Agar $\beta = 6$ bo`lsa, $q = q(t)$ funksiyani toping va uni tekshiring.

24-variant

1. $y''-y'+6y=0.$

2. $y''-3y'+2y=10e^{-x}.$

3. Faraz qilamiz, mahsulot ishlab chiqarish o`zgarishining tezligi foydaga proportsional bo`lsin, $q' = \alpha(pq - c)$. Bu yerda p – mahsulot narxi, c – xarajatlar. $\alpha = 0,2,\ p = 10 - q,\ c = \beta q + 4,\ \beta = const$ bo`lsin. Agar $\beta = 10$ bo`lsa, $q = q(t)$ funksiyani toping va uni tekshiring.

25-variant

1. $2y''-3y'+5y=0.$

2. $y''-6y'+9y=2x^2-x+3.$

3. Faraz qilamiz, mahsulot ishlab chiqarish o`zgarishining tezligi foydaga proportsional bo`lsin, $q' = \alpha(pq - c)$. Bu yerda p – mahsulot narxi, c – xarajatlar. $\alpha = 0,2,\ p = 10 - q,\ c = \beta q + 4,\ \beta = const$ bo`lsin. Agar $\beta = 5$ bo`lsa, $q = q(t)$ funksiyani toping va uni tekshiring.

3-topshiriq

1-misolda Dalamber alomatini qo‘llab qatorni yaqinlashishga tekshiring. $\lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}$ ni ko‘rsating.

2-misolda qatorlarni yaqinlashishga tekshiring. Qo‘llanilgan alomatni ko‘rsating.

3-misolda 2-taqqoslash alomatini qo‘llab qatorni yaqinlashishga tekshiring.

4-misolda Ishora almashinuvchi qatorlarni yaqinlashishga tekshiring. Qo‘llanilgan alomatni ko‘rsating. Qo‘shimcha ko‘rsating:

1) $\lim\limits_{n\to\infty} a_n$ zaruriy alomat uchun;

2) 1- va 2- taqqoslash alomati uchun;

3) $\lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}$ – Dalamber alomati uchun;

4) $\lim\limits_{n\to\infty}\sqrt[n]{|a_n|}$ – Koshi alomati uchun.

1-variant

1. $\sum\limits_{n=1}^{\infty}\frac{n^5}{3^{n+1}}$.
2. $\sum\limits_{n=1}^{\infty}\left(\frac{n+2}{2n+1}\right)^{3n+1}$.
3. $\sum\limits_{n=1}^{\infty}\frac{1}{3n-\sqrt{n}}$.
4. $\sum\limits_{n=1}^{\infty}\frac{(-1)^{n-1}}{e^{n+1}}$

2-variant

1. $\sum\limits_{n=1}^{\infty}\frac{n^n}{n!}$.
2. $\sum\limits_{n=1}^{\infty}\frac{1}{(n+1)\ln(n+1)}$.
3. $\sum\limits_{n=1}^{\infty}\ln\left(\frac{n^2+3}{n^2}\right)$.
4. $\sum\limits_{n=1}^{\infty}(-1)^n\frac{3n+1}{3n-1}$.

3-variant

1. $\sum\limits_{n=1}^{\infty}\frac{2^n}{n^2}$.

2. $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}$.

3. $\sum_{n=1}^{\infty}\frac{2n+1}{n(n+2)}$.

4. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(2n)!}{4^n n!}$.

4-variant

1. $\sum_{n=1}^{\infty}\frac{3^n}{n!}$.

2. $\sum_{n=1}^{\infty}\left(\frac{n-1}{2n+1}\right)^n$.

3. $\sum_{n=1}^{\infty}\frac{n+3}{n^2+n}$.

4. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{3n-1}$.

5-variant

1. $\sum_{n=1}^{\infty}\frac{(n!)^2}{(2n)!}$.

2. $\sum_{n=1}^{\infty}\left(\frac{2n-1}{3n+1}\right)^{\frac{n}{2}}$.

3. $\sum_{n=1}^{\infty}tg\frac{1}{n\sqrt{n}}$.

4. $\sum_{n=1}^{\infty}(-1)^n\ln^n\left(\frac{2n}{n+2}\right)$.

6-variant

1. $\sum_{n=1}^{\infty}\frac{n^n}{n!2^n}$.

2. $\sum_{n=1}^{\infty}\frac{1}{(2n+1)\ln(2n+1)}$.

3. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n+2}}$.

4. $\sum_{n=1}^{\infty}(-1)^n\frac{\sin n}{n^2}$.

7-variant

1. $\sum_{n=1}^{\infty}\frac{1\cdot 4\cdot\ldots\cdot(3n-2)}{n!2^n}$.

2. $\sum_{n=1}^{\infty}\left(\arcsin\frac{1}{n}\right)^{n}$.

3. $\sum_{n=1}^{\infty}\ln\left(\frac{n^3+1}{n^3}\right)$.

4. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n\sqrt{\ln n}}$.

8-variant

1. $\sum_{n=1}^{\infty}\frac{1\cdot3\cdot5\cdot\ldots\cdot(2n-1)}{2\cdot7\cdot12\cdot\ldots\cdot(5n-3)}$.

2. $\sum_{n=1}^{\infty}\frac{1}{n\ln n(\ln\ln n)^2}$.

3. $\sum_{n=1}^{\infty}\frac{n+5}{n^2-2}$.

4. $\sum_{n=1}^{\infty}(-1)^n\ln 2$.

9-variant

1. $\sum_{n=1}^{\infty}\frac{n^2}{3^n}$.

2. $\sum_{n=1}^{\infty}\left(1-\frac{1}{n}\right)^{n^2}$.

3. $\sum_{n=1}^{\infty}\frac{2-n}{n^3+n-1}$.

4. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n^2}{2^n}$.

10-variant

1. $\sum_{n=1}^{\infty}\frac{3^n n^3}{5^{\frac{n}{2}}}$.

2. $\sum_{n=1}^{\infty}\left(\frac{2n-1}{5n+2}\right)^{3n-2}$.

3. $\sum_{n=1}^{\infty}\frac{n^2+2}{3n+1}$.

4. $\sum_{n=1}^{\infty}\frac{n}{2n+i\sqrt{n}}$.

11-variant

1. $\sum_{n=2}^{\infty}\frac{1\cdot 4\cdot\ldots\cdot(3n-2)}{n!}$.

2. $\sum_{n=1}^{\infty}\left(\frac{3n+1}{2n+1}\right)^{n+1}$.

3. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n^2+3}}$.

4. $\sum_{n=1}^{\infty}\left(\frac{2+i}{3}\right)^{n}$.

12-variant

1. $\sum_{n=1}^{\infty}\frac{n^7}{5^{\frac{n}{2}}}$.

2. $\sum_{n=1}^{\infty}\frac{1}{(3n-1)\ln(3n-1)}$.

3. $\sum_{n=1}^{\infty}\frac{n-1}{\sqrt{n^3+3n-1}}$.

4. $\sum_{n=1}^{\infty}\frac{1}{n(2+i)^n}$.

13-variant

1. $\sum_{n=1}^{\infty}\frac{3^{n+1}}{2^n\cdot n^4}$.

2. $\sum_{n=1}^{\infty}\left(\frac{n+1}{3n-1}\right)^{n-1}$.

3. $\sum_{n=1}^{\infty}\frac{\sqrt{n}+\sqrt[3]{n}}{n+\sqrt[3]{n^5}}$.

4. $\sum_{n=1}^{\infty}\frac{i^{2n}}{\sqrt{n}}$.

14-variant

1. $\sum_{n=1}^{\infty}\frac{n^3}{n!}$.

2. $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{n^2}$.

3. $\sum_{n=1}^{\infty}\ln\left(\frac{n^2+1}{n^2}\right)$.

4. $\sum_{n=1}^{\infty}\left(\frac{2n+i}{3ni-2}\right)^{n}$.

15-variant

1. $\sum_{n=1}^{\infty}\frac{(n+1)!}{5^{n}}$.

2. $\sum_{n=1}^{\infty}\frac{1}{\ln^{n}(n+1)}$.

3. $\sum_{n=1}^{\infty}\arcsin^{2}\frac{1}{\sqrt{n}}$.

4. $\sum_{n=1}^{\infty}\left(\frac{3-i}{2}\right)^{n}$.

16-variant

1. $\sum_{n=1}^{\infty}\frac{n!3^{n}}{n^{n}}$.

2. $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{n^{2}}$.

3. $\sum_{n=1}^{\infty}\sqrt{n}\cdot\sin\frac{\pi}{n^{2}}$.

4. $\sum_{n=1}^{\infty}\frac{\cos n+i\sin n}{n^{2}}$.

17-variant

1. $\sum_{n=1}^{\infty}\frac{1\cdot3\cdot5\cdot\ldots\cdot(2n-1)}{n^{2}\cdot3^{n}}$.

2. $\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{2n-1}{3n+1}\right)^{n}$.

3. $\sum_{n=1}^{\infty}n^{5}\cdot tg^{3}\frac{2}{n^{2}}$.

4. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n+2}{2n+5}$.

18-variant

1. $\sum_{n=1}^{\infty}\frac{2\cdot5\cdot8\cdot\ldots\cdot(3n-1)}{1\cdot5\cdot9\cdot\ldots\cdot(4n-3)}$.

2. $\sum_{n=1}^{\infty}\frac{1}{3^{n}}\left(1+\frac{1}{n}\right)^{n}$

3. $\sum_{n=1}^{\infty}\frac{2^{n}+3}{5^{n}+2}$.

4. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}n}{(2n+1)\cdot 3^n}$.

19-variant

1. $\sum_{n=1}^{\infty}\frac{3^n}{n!2^{n+1}}$.
2. $\sum_{n=1}^{\infty}\frac{1}{n^n}$.
3. $\sum_{n=1}^{\infty}\frac{2+n}{n^2-3}$.
4. $\sum_{n=1}^{\infty}(-1)^n\frac{n+2}{3n^2\sqrt{n}+1}$.

20-variant

1. $\sum_{n=1}^{\infty}\frac{(3n)!}{(n!)^3 2^{3n}}$.
2. $\sum_{n=1}^{\infty}\sqrt{n}\cdot\left(\frac{5n-3}{3n+2}\right)^{n+1}$.
3. $\sum_{n=1}^{\infty}\frac{2n+3}{3n-2}$.
4. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3^{n^2}}{n^n}$.

21-variant

1. $\sum_{n=1}^{\infty}\frac{3\cdot 5\cdot\ldots\cdot(2n+1)}{2\cdot 5\cdot\ldots\cdot(3n-1)}\cdot 2^n$.
2. $\sum_{n=2}^{\infty}\frac{1}{n\sqrt{\ln n}}$.
3. $\sum_{n=1}^{\infty}\frac{n^3+3n^2-2}{2n+5-n^5}$.
4. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3n}{n(n+1)}$.

22-variant

1. $\sum_{n=1}^{\infty}\frac{n!}{5^n+n^2}$.
2. $\sum_{n=1}^{\infty}n\cdot\left(1-\frac{1}{n}\right)^{n^2}$.
3. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n^4+n^2-1}}$.

4. $\sum_{n=1}^{\infty}(-1)^n \frac{\cos(n+2)}{3^n}$.

23-variant

1. $\sum_{n=1}^{\infty}\frac{(n+1)!}{n^n+2^n}$.

2. $\sum_{n=1}^{\infty}\frac{1}{(n+1)\ln^2(n+1)}$.

3. $\sum_{n=1}^{\infty}\frac{2+3\sqrt{n}}{2n-5}$.

4. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n(2+\ln n)^3}$.

24-variant

1. $\sum_{n=1}^{\infty}\left(\frac{2n+1}{5n+4}\right)^n$.

2. $\sum_{n=2}^{\infty}\frac{\ln n}{n}$.

3. $\sum_{n=1}^{\infty}\frac{\sqrt[3]{n^2}+\sqrt{n^2}}{\sqrt{n^4+\sqrt{n^3}}}$.

4. $\sum_{n=1}^{\infty}\frac{(-1)^n n^2}{n\sqrt{n}+3n}$.

25-variant

1. $\sum_{n=1}^{\infty}\frac{n^3}{3^n}$.

2. $\sum_{n=1}^{\infty}\left(\frac{n-1}{n+1}\right)^{n(n-1)}$.

3. $\sum_{n=1}^{\infty}\ln\left(\frac{n+2}{n}\right)$.

4. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\ln(n+1)}{n}$.

JAVOBLAR

I BOB

1-topshiriq javoblari

1-variant. **1.** $\begin{pmatrix} -4 & 13 & 6 \\ 6 & 5 & 1 \end{pmatrix}$. **2.** $AB=\begin{pmatrix} 36 & 7 & 25 \\ -1 & -1 & -10 \end{pmatrix}$, BA mavjud emas. **3.** $\begin{pmatrix} 0 & 6 \\ 9 & -3 \end{pmatrix}$. 2-variant. **1.** $\begin{pmatrix} -1 & -1 & 2 \\ 5 & -6 & 3 \\ -1 & 0 & -5 \end{pmatrix}$. **2.** $AB=\begin{pmatrix} 5 & 2 \\ 7 & 0 \end{pmatrix}$. $BA=\begin{pmatrix} 29 & -22 \\ 31 & -24 \end{pmatrix}$.

3. $\begin{pmatrix} 6 & 95 \\ 0 & -70 \end{pmatrix}$. 3-variant. **1.** $\begin{pmatrix} -7 & -9 & -10 \\ 22 & 11 & -23 \\ -12 & -6 & 40 \end{pmatrix}$. **2.** $AB=(31)$,

$BA=\begin{pmatrix} 12 & 0 & -6 & 9 & 3 \\ 4 & 0 & -2 & 3 & 1 \\ -4 & 0 & 2 & -3 & -1 \\ 20 & 0 & -10 & 15 & 5 \\ 8 & 0 & -4 & 6 & 2 \end{pmatrix}$. **3.** $\begin{pmatrix} -24 & 12 \\ -12 & -12 \end{pmatrix}$. 4-variant. **1.** $\begin{pmatrix} 29 & -10 & -3 & -8 \\ 28 & 2 & 3 & 13 \\ 17 & 1 & 10 & 20 \end{pmatrix}$.

2. $AB=\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, $BA=\begin{pmatrix} 20 & 40 \\ -10 & -20 \end{pmatrix}$. **3.** $\begin{pmatrix} 0 & 8 & -6 \\ 6 & 1 & -13 \\ -20 & 1 & 27 \end{pmatrix}$. 5-variant. **1.** $\begin{pmatrix} 3 & 4 \\ 7 & 16 \end{pmatrix}$.

2. $AB=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $BA=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. **3.** $A=\begin{pmatrix} 8 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$. 6-variant. **1.** $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

2. $AB=\begin{pmatrix} -4 & 8 \\ -24 & 31 \end{pmatrix}$, $BA=\begin{pmatrix} 14 & -6 \\ -19 & 13 \end{pmatrix}$. **3.** $\begin{pmatrix} 0 & 0 \\ 0 & 6 \end{pmatrix}$. 7-variant. **1.** $\begin{pmatrix} 0 & 3 \\ 3 & -4 \end{pmatrix}$.

2. $AB=\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$, $BA=\begin{pmatrix} -3 & -4 \\ 7 & 10 \end{pmatrix}$. **3.** $\begin{pmatrix} 21 & -60 \\ 0 & 61 \end{pmatrix}$. 8-variant.

1. $\begin{pmatrix} 4 & -22 & -29 & 47 \\ 64 & -7 & -33 & 4 \\ -8 & -18 & 14 & -19 \end{pmatrix}$. **2.** $AB=(-1)$, $BA=\begin{pmatrix} 5 & -10 & 15 & 0 \\ -3 & 6 & -9 & 0 \\ -4 & 8 & -12 & 0 \\ 1 & -2 & 3 & 0 \end{pmatrix}$. **3.** $\begin{pmatrix} 18 & -20 \\ 30 & -2 \end{pmatrix}$.

9-variant. **1.** $\begin{pmatrix} -10 & -13 & 6 \\ 24 & 19 & -16 \\ -17 & 10 & 23 \end{pmatrix}$. **2.** $AB=\begin{pmatrix} 7 \\ 3 \end{pmatrix}$, BA mavjud emas

3. $\begin{pmatrix} 0 & 0 & -3 \\ 3 & -3 & 1 \\ 0 & -12 & -3 \end{pmatrix}$. 10-variant. **1.** $\begin{pmatrix} 1 & 12 \\ 19 & -1 \end{pmatrix}$.**2.** $AB=\begin{pmatrix} -14 & 11 \\ 10 & 8 \end{pmatrix}$, $BA=\begin{pmatrix} 14 & 2 & -2 \\ -9 & -15 & 3 \\ 17 & 23 & -5 \end{pmatrix}$.**3.** $\begin{pmatrix} -102 & 105 \\ 35 & -32 \end{pmatrix}$. 11-variant. **1.** $\begin{pmatrix} 9 & -17 & -1 \\ -3 & 1 & -10 \end{pmatrix}$.

2. $AB=\begin{pmatrix} 2 & -9 & 14 \\ 21 & -38 & 9 \\ -18 & 19 & -32 \end{pmatrix}$. $BA=\begin{pmatrix} -18 & -2 & 8 \\ -13 & -13 & -17 \\ -18 & -9 & -37 \end{pmatrix}$.**3.** $\begin{pmatrix} -25 & 60 & -6 \\ 60 & -18 & 44 \\ 70 & 23 & -63 \end{pmatrix}$.

12-variant. **1.** $\begin{pmatrix} -5 & -8 & 7 \\ -19 & 6 & -7 \\ -5 & -15 & 3 \end{pmatrix}$.**2.** $AB=\begin{pmatrix} 5 & -1 \\ -5 & 1 \end{pmatrix}$, $BA=\begin{pmatrix} 2 & -2 \\ -4 & 4 \end{pmatrix}$.

3. $\begin{pmatrix} 5 & 0 & 4 \\ 3 & 3 & 3 \\ 0 & 0 & 9 \end{pmatrix}$. 13-variant. **1.** $\begin{pmatrix} 10 & -3 \\ -1 & 2 \end{pmatrix}$.**2.** $AB=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $BA=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

3. $\begin{pmatrix} -100 & -256 & 145 \\ 38 & 251 & -79 \\ -44 & 35 & -59 \end{pmatrix}$. 14-variant. **1.** $\begin{pmatrix} 13 & -6 \\ 1 & -45 \end{pmatrix}$.**2.** $AB=\begin{pmatrix} 3 & -13 & 3 \end{pmatrix}$. BA mavjud emas **3.** $\begin{pmatrix} 9 & -4 \\ -1 & 11 \end{pmatrix}$. 15-variant. **1.** $\begin{pmatrix} -5 & 10 & -8 \\ 8 & 7 & 15 \\ -16 & -10 & 2 \end{pmatrix}$.**2.** $AB=\begin{pmatrix} -3 & 9 \\ 6 & -2 \\ 20 & 3 \\ 5 & 14 \end{pmatrix}$.

BA mavjud emas **3.** $\begin{pmatrix} 21 & -23 & 15 \\ -13 & 34 & 10 \\ -9 & 22 & 25 \end{pmatrix}$. 16-variant. **1.** $\begin{pmatrix} 7 & -3 \\ -1 & -16 \end{pmatrix}$.

2. $AB=\begin{pmatrix} 4 & 5 \\ -17 & -9 \end{pmatrix}$, $BA=\begin{pmatrix} -6 & 5 & 15 \\ -2 & 1 & 3 \\ -3 & 0 & 0 \end{pmatrix}$. **3.** $\begin{pmatrix} -12 & -12 & 8 \\ -4 & -4 & 2 \\ -4 & -8 & -4 \end{pmatrix}$. 17-variant.

1. $\begin{pmatrix} 6 & 10 & 7 \\ -3 & 0 & -14 \end{pmatrix}$.**2.** $AB=\begin{pmatrix} -6 & 6 \\ 23 & 27 \end{pmatrix}$, $BA=\begin{pmatrix} -4 & 8 \\ 25 & 25 \end{pmatrix}$.**3.** $\begin{pmatrix} 6 & -1 & 14 \\ 2 & 14 & 6 \\ 6 & 5 & 12 \end{pmatrix}$.

18-variant. **1.** $\begin{pmatrix} 7 & -12 & 2 \\ -1 & 4 & 0 \end{pmatrix}$.**2.** $AB=\begin{pmatrix} 13 & 6 \\ 20 & 11 \end{pmatrix}$, $BA=\begin{pmatrix} 14 & 9 & 23 \\ 5 & 4 & 9 \\ 3 & 3 & 6 \end{pmatrix}$.

3. $\begin{pmatrix} 6 & -1 & 14 \\ 2 & 14 & 6 \\ 6 & 5 & 12 \end{pmatrix}$. 19-variant. **1.** $\begin{pmatrix} -5 & 7 & 6 \\ 3 & 8 & 8 \end{pmatrix}$. **2.** $AB=\begin{pmatrix} 4 & 5 \\ 17 & 9 \end{pmatrix}$, $BA=\begin{pmatrix} 10 & 11 & -7 \\ 10 & 11 & -7 \\ 6 & 11 & -8 \end{pmatrix}$. **3.** $\begin{pmatrix} 3 & 8 & -2 \\ 5 & 9 & -3 \\ 1 & 4 & -2 \end{pmatrix}$. 20-variant. **1.** $\begin{pmatrix} 6 & 7 \\ 7 & 16 \\ 19 & 22 \end{pmatrix}$. **2.** $AB=\begin{pmatrix} -6 & 15 \\ 2 & 23 \end{pmatrix}$, $BA=\begin{pmatrix} 24 & 25 \\ 0 & -7 \end{pmatrix}$. **3.** $\begin{pmatrix} 23 & 35 & 33 \\ 11 & 57 & 36 \\ 58 & 79 & 67 \end{pmatrix}$. 21-variant. **1.** $\begin{pmatrix} 11 & 13 & 7 \\ 2 & 2 & 24 \end{pmatrix}$. **2.** $AB=\begin{pmatrix} -4 & 8 \\ -24 & 31 \end{pmatrix}$, $BA=\begin{pmatrix} 14 & -6 \\ -19 & 13 \end{pmatrix}$. **3.** $\begin{pmatrix} 18 & 20 & 21 \\ 33 & 17 & 30 \\ 80 & 49 & 69 \end{pmatrix}$. 22-variant. **1.** $\begin{pmatrix} -1 & 2 & -5 \\ 1 & 3 & -2 \\ 0 & -4 & 9 \end{pmatrix}$. **2.** $AB=\begin{pmatrix} 4 & 5 \\ -17 & -9 \end{pmatrix}$, $BA=\begin{pmatrix} -6 & 5 & 15 \\ -2 & 1 & 3 \\ -3 & 0 & 0 \end{pmatrix}$. **3.** $\begin{pmatrix} -1 & 0 & 0 \\ 4 & -1 & 0 \\ 4 & 4 & -1 \end{pmatrix}$.

23-variant. **1.** $\begin{pmatrix} 10 & 0 & 2 \\ 0 & 2 & -1 \\ 2 & -1 & 3 \end{pmatrix}$. **2.** $AB=\begin{pmatrix} -6 & 15 \\ 2 & 23 \end{pmatrix}$, $BA=\begin{pmatrix} 24 & 25 \\ 0 & -7 \end{pmatrix}$. **3.** $\begin{pmatrix} 5 & -6 & 1 \\ 0 & 5 & -6 \\ 0 & 0 & 5 \end{pmatrix}$. 24-variant. **1.** $\begin{pmatrix} 8 & 6 & 1 \\ 5 & 6 & 4 \\ 8 & 12 & 13 \end{pmatrix}$. **2.** $AB=\begin{pmatrix} -4 & 7 \\ 10 & -27 \end{pmatrix}$, $BA=\begin{pmatrix} -29 & -4 \\ -5 & -2 \end{pmatrix}$. **3.** $\begin{pmatrix} 6 & 7 & 0 \\ 13 & 5 & 10 \\ 17 & 3 & 18 \end{pmatrix}$. 25-variant. **1.** $\begin{pmatrix} -39 & -15 & -14 \\ 2 & 9 & -42 \end{pmatrix}$. **2.** $AB=\begin{pmatrix} 4 & 5 \\ -17 & -9 \end{pmatrix}$, $BA=\begin{pmatrix} -2 & 1 & -1 \\ -8 & -6 & -18 \\ -1 & 3 & 3 \end{pmatrix}$. **3.** $A=\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$.

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1-variant. **1.** $rang(A)=2$. **2.** $A^{-1}=\begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$. **3.** $\begin{pmatrix} 2 & -23 \\ 0 & 8 \end{pmatrix}$. 2-variant. **1.** $rang(A)=2$. **2.** $A^{-1}=\begin{pmatrix} 0,333 & 0,333 & -0,667 \\ 1,333 & -0,667 & 0,333 \\ -1 & 0 & 1 \end{pmatrix}$. **3.** $\begin{pmatrix} -27 & 4 \\ 17 & -2 \end{pmatrix}$. 3-variant.

1. $rang(A)=3$ **2.** $A^{-1}=\begin{pmatrix}1&0&0\\-5&1&0\\38&-9&1\end{pmatrix}$.**3.** $\begin{pmatrix}2&1{,}6&1{,}6\\0&0{,}4&0{,}4\\-5&-4{,}4&-3{,}4\end{pmatrix}$. 4-variant.

1. $rang(A)=2$ **2.** $A^{-1}=-\frac{1}{9}\begin{pmatrix}-1&-2&-2\\-2&-1&2\\-2&2&-1\end{pmatrix}$. **3.** $\begin{pmatrix}1&-2&7\\0&1&-2\\0&0&1\end{pmatrix}$. 5-variant.

1. $rang(A)=2$ **2.** $A^{-1}=\frac{1}{5}\begin{pmatrix}-3&13&9\\0&5&5\\2&-7&-6\end{pmatrix}$.**3.** $\begin{pmatrix}-1&-1\\2&3\end{pmatrix}$. 6-variant. **1.** $rang(A)=2$.

2. $A^{-1}=-\frac{1}{29}\begin{pmatrix}-1&-11&-2\\-11&-5&7\\-2&7&-4\end{pmatrix}$.**3.** $\begin{pmatrix}6&4&5\\2&1&2\\3&3&3\end{pmatrix}$. 7-variant. **1.** $rang(A)=3$.

2. $A^{-1}=\begin{pmatrix}-2{,}333&2&-0{,}333\\1{,}667&-1&-0{,}333\\-2&1&1\end{pmatrix}$.**3.** $\begin{pmatrix}-7&-1&5\\15&2&-3\end{pmatrix}$. 8-variant. **1.** $rang(A)=3$.

2. $A^{-1}=\begin{pmatrix}3&1&1\\1&1&0\\1&0&1\end{pmatrix}$.**3.** $\begin{pmatrix}-22&-25\\17&18\end{pmatrix}$. 9-variant. **1.** $rang(A)=3$.

2. $A^{-1}=\begin{pmatrix}\frac{1}{8}&\frac{1}{4}&-\frac{1}{8}\\\frac{1}{26}&-\frac{2}{13}&\frac{3}{26}\\-\frac{7}{104}&\frac{1}{52}&\frac{31}{104}\end{pmatrix}$. **3.** $\begin{pmatrix}-0{,}143&0{,}571&1\\0{,}286&-0{,}143&-1\\0{,}571&-0{,}286&-1\end{pmatrix}$. 10-variant. **1.** $rang(A)=2$.

2. A^{-1} mavjud emas. **3.** $\begin{pmatrix}1&0\\0&1\end{pmatrix}$. 11-variant. **1.** $rang(A)=3$.

2. $A^{-1}=\begin{pmatrix}\frac{11}{41}&\frac{6}{41}&-\frac{4}{41}\\-\frac{5}{41}&\frac{1}{41}&\frac{13}{41}\\\frac{14}{41}&-\frac{11}{41}&-\frac{20}{41}\end{pmatrix}$.**3.** $\begin{pmatrix}7&23\\6&19\end{pmatrix}$. 12-variant. **1.** $rang(A)=1$.

2. $A^{-1}=\begin{pmatrix}-1&-\frac{1}{3}&\frac{2}{3}\\0&-\frac{2}{3}&\frac{1}{3}\\1&\frac{1}{2}&-\frac{1}{2}\end{pmatrix}$. **3.** $\begin{pmatrix}2&-2\\1&3\end{pmatrix}$. 13-variant. **1.**

2. $A^{-1}=\begin{pmatrix}-3 & 2 & 3\\ 0 & 1 & 2\\ -4 & 3 & 5\end{pmatrix}$. **3.** $\begin{pmatrix}2\\0\end{pmatrix}$. 14-variant. **1.** $rang(A)=3$ **2.** A^{-1} mavjud emas. **3.** $\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$. 15-variant. **1.** $rang(A)=2$ **2.** $A^{-1}=\begin{pmatrix}3 & 0 & -1\\ 2 & 1 & 0\\ -4 & -1 & 1\end{pmatrix}$. **3.** X mavjud emas.

16-variant. **1.** $rang(A)=2$ **2.** $\begin{pmatrix}\frac{4}{3} & \frac{7}{3} & -\frac{5}{3}\\ -\frac{2}{3} & -\frac{1}{6} & \frac{1}{3}\\ \frac{1}{3} & -\frac{2}{3} & \frac{1}{3}\end{pmatrix}$. **3.** $\begin{pmatrix}\frac{15}{7}\\ -\frac{16}{7}\\ -\frac{11}{7}\end{pmatrix}$. 17-variant.

1. $rang(A)=2$. **2.** $\begin{pmatrix}3 & -1 & -1\\ -1 & 1 & 0\\ -1 & 0 & 1\end{pmatrix}$. **3.** $\begin{pmatrix}0 & 0 & \frac{1}{3}\\ 0 & 1 & 0\\ 3 & 0 & 0\end{pmatrix}$. 18-variant. **1.** $rang(A)=3$

2. $A^{-1}=\begin{pmatrix}5 & -1 & -1{,}5\\ -3 & 1 & 1\\ -1 & 0 & 0{,}5\end{pmatrix}$. **3.** $\begin{pmatrix}4 & 3\\ -5 & -4\end{pmatrix}$. 19-variant. **1.** $rang(A)=3$

2. $\begin{pmatrix}\frac{1}{19} & -\frac{1}{19} & -\frac{3}{19}\\ \frac{9}{19} & \frac{10}{19} & \frac{11}{19}\\ -\frac{13}{19} & -\frac{25}{19} & -\frac{18}{19}\end{pmatrix}$. **3.** $\begin{pmatrix}-2 & 4\\ -1 & -1\\ -1 & 6\end{pmatrix}$. 20-variant. **1.** $rang(A)=2$. **2.** A^{-1} mavjud emas. **3.** $\begin{pmatrix}20 & -15 & 13\\ -17 & 13 & -10\\ -8 & 5 & -4\end{pmatrix}$. 21-variant. **1.** $rang(A)=3$. **2.**

$$\begin{pmatrix}\frac{2}{3} & -\frac{5}{12} & -\frac{1}{12}\\ -\frac{1}{3} & \frac{7}{12} & -\frac{1}{12}\\ -\frac{1}{3} & \frac{1}{12} & \frac{5}{12}\end{pmatrix}.$$

3. X mavjud emas. 22-variant. **1.** $rang(A)=3$. **2.** $\begin{pmatrix}-1 & 2 & -1\\ -2 & 1 & 0\\ -3 & -1 & 2\end{pmatrix}$. **3.** $\begin{pmatrix}6\\ -5\\ -3\end{pmatrix}$.

23-variant. **1.** $rang(A)=1$ **2.** $\begin{pmatrix}-4 & 3 & -2\\ -8 & 6 & -5\\ -7 & 5 & -4\end{pmatrix}$. **3.** $\begin{pmatrix}-2 & 2\\ 1 & 2\end{pmatrix}$. 24-variant. **1.** $rang(A)=3$. **2.** $\begin{pmatrix}\frac{1}{3} & \frac{2}{3} & \frac{2}{3}\\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3}\\ \frac{2}{3} & -\frac{2}{3} & \frac{1}{3}\end{pmatrix}$. **3.** X mavjud emas. 25-variant. **1.** $rang(A)=3$. **2.** A^{-1} mavjud emas. **3.** $\begin{pmatrix}-4 & 1\\ -3 & 2\end{pmatrix}$.

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1-variant. **1.** $\sin(\alpha+\beta)\sin(\alpha-\beta)$. **2.** $3abc-(a^3+b^3+c^3)$. **3.** $x_1=2; x_2=3$. **4.** 1. 2-variant. **1.** -2. **2.** $4a$. **3.** $x_1=0; x_2=-2$. **4.** -16. 3-variant. **1.** 18. **2.** $-2b^2$. **3.** $x_1=0; x_2=1$. **4.** 460. 4-variant. **1.** $2a$. **2.** $(x-y)(y-z)(x-z)$. **3.** $x=1$. **4.** $8a+15b+12c-19d$. 5-variant. **1.** $4ab$. **2.** 0. **3.** $\frac{\pi n}{5}, n\in Z$. **4.** $-8a+2b+4c-14d$. 6-variant. **1.** 2. **2.** 0. **3.** $x_1=1; x_2=-3$. **4.** -150. 7-variant. **1.** -10. **2.** 0. **3.** $(-1;2)$. **4.** -3. 8-variant. **1.** -5. **2.** 0. **3.** $(-\sqrt{23};\sqrt{23})$. **4.** -10. 9-variant. **1.** 5. **2.** 0. **3.** 4,5. **4.** $8a+15b+12c-19d$. 10-variant. **1.** $6a^2b+2b^3$. **2.** 1. **3.** 5. **4.** $(be-cd)^2$. 11-variant. **1.** $8ab(a^2+b^2)$. **2.** 1. **3.** $1;-\frac{1}{2}$. **4.** 223. 12-variant. **1.** 7. **2.** $\sin(\beta-\gamma)+\sin(\gamma-\alpha)+\sin(\alpha-\beta)$. **3.** $(2;-1)$ **4.** 48. 13-variant. **1.** 1. **2.** 4. **3.** 1. **4.** 0. 14-variant. **1.** 1. **2.** -8. **3.** $\left(-\infty;-\frac{36}{5}\right]$. **4.** $61a+55b-31c-21d$. 15-variant. **1.** $\cos 2\varphi$. **2.** -9. **3.** $-\frac{5}{3};2$. **4.** $2a-8b+c+5d$. 16-variant. **1.** 0. **2.** 8. **3.** 2. **4.** $8a+15b+12c-19d$. 17-variant. **1.** 0. **2.** 0. **3.** $x\geq-\frac{41}{21}$. **4.** 150. 18-variant. **1.** 0. **2.** $-xyz$. **3.** $\frac{\pi n}{2}, \pm\frac{\pi}{6}+\pi n, n\in Z$. **4.** -6. 19-variant. **1.** $\frac{1}{\cos^2\varphi}$. **2.** 6. **3.** $(2;-3)$. **4.** $2a-8b+c+5d$. 20-variant. **1.** $\sin(\alpha-\beta)$. **2.** -6. **3.** $-4;1;2$. **4.** 60. 21-variant. **1.** 0. **2.** 6. **3.** 5. **4.** 17. 22-variant. **1.** 1. **2.** -12. **3.** $-3;-\frac{5}{2}$. **4.** 16. 23-variant. **1.**

$ad-bc$. **2.** 40. **3.** 1;2. **4.** 100. 24-variant. **1.** 0. **2.** 0. **3.** 1;5. **4.** $(be-cd)^2$. 25-variant. **1.** 0. **2.** −3 **3.** 13. **4.** 63.

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1. a) $A=\begin{pmatrix}5&3&9\\3&6&3\\8&4&7\\7&3&7\end{pmatrix}$. b) $D=\begin{pmatrix}1&-3&-5\\1&2&-1\\0&2&-3\\3&1&1\end{pmatrix}$. **2.** $C=\begin{pmatrix}600&1300&700&1300\end{pmatrix}$. **3.** $S=\begin{pmatrix}930\\960\\450\\690\end{pmatrix}$. **4.** 3990 *pul.birl.* **5.** $X_2=(0{,}629;\quad 0{,}371)$, $X_3=(0{,}634;\quad 0{,}366)$.

6. $a)\begin{pmatrix}5&12&6&5\\10&9&8&4\\13&13&12&9\end{pmatrix}$; $b)\,B_1=\begin{pmatrix}-1&1&1&0\\-1&1&1&1\\-1&1&0&1\end{pmatrix}$; $B_2=\begin{pmatrix}1&1&1&-1\\0&1&0&-1\\0&-1&0&1\end{pmatrix}$.

7. $C=(250;\quad 180;\quad 150)$; 1 − *region*. **8.** $(680\quad 2040\quad 540\quad 1020)'$. **9.** 1) $S=(700\quad 1000\quad 900)'$; 2) $C=63000$. **10.**

$$\begin{pmatrix}0&0&0&0&0&0\\-100&100&100&100&100&100\\-200&0&200&200&200&200\\-300&-100&100&300&300&300\\-400&-200&0&200&400&400\\-500&-300&-100&100&300&500\end{pmatrix}.$$

11. 1) $\begin{pmatrix}5&9\\13&7\\4&9\end{pmatrix}$; 2) $\begin{pmatrix}1&-1\\-1&1\\0&1\end{pmatrix}$. **12.** 1) $\begin{pmatrix}4&6\\11&7\\5&6\end{pmatrix}$; 2) $\begin{pmatrix}-2&-2\\-1&-1\\1&0\end{pmatrix}$. **13.** 1) $\begin{pmatrix}3&4\\6&8\\8&3\end{pmatrix}$; 2) $\begin{pmatrix}-1&0\\0&0\\0&1\end{pmatrix}$. **14.** 1) $\begin{pmatrix}50\\80\end{pmatrix}$; 2) 410. **15.** 1) $\begin{pmatrix}40\\60\end{pmatrix}$; 2) 320. **16.** 1) $\begin{pmatrix}80\\60\end{pmatrix}$; 2) 260. **17.** 41. **18.** $S=570\,kg$, $T=1220\,c$, $P=3500$ *pul.birl.* **19.** $\begin{pmatrix}575\\550\\835\\990\end{pmatrix}$. **20.**

$$A_{yil}=\begin{pmatrix}800 & 750 & 510 & 720 & 980\\ 0 & 300 & 680 & 360 & 0\\ 1600 & 2250 & 0 & 480 & 840\\ 600 & 1500 & 1190 & 600 & 560\end{pmatrix}$$ **21.**

$$\begin{pmatrix}11000 & 18900 & 9010 & 7440 & 8120\\ 13600 & 24750 & 14450 & 10680 & 10780\\ 14800 & 25050 & 13260 & 11040 & 11480\end{pmatrix}.$$ **22.**

$P=(2008000 \quad 3496500 \quad 1878500 \quad 1494000 \quad 1552600)$. **23.**

$$\begin{pmatrix}162.63 & 187.2 & 239.85 & 397.8 & 503.1\\ 142.74 & 152.1 & 169.65 & 189.54 & 177.84\end{pmatrix}$$ **24.** $\begin{pmatrix}850\\ 870\\ 920\end{pmatrix}$. **25.** a) $(102 \quad 204 \quad 81 \quad 144 \quad 116)$, b) 3607.

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1-variant. **1.** $X=(1,1,1)$. **2.** $X=(14,12,-4,5)$. **3.** Birgalikda va aniqmas sistema; umumiy yechim $(3-t_1-t_2;t_1;t_2)$; xususiy yechim $(3;0;0)$. 2-variant. **1.** $X=(1,2,3)$. **2.** $X=(1;-1;-1;1)$. **3.** Birgalikda va aniqmas sistema; umumiy yechim $(2-2\alpha;-2+\alpha;1+\alpha;\alpha)$; xususiy yechim $(0;-1;2,1)$. 3-variant. **1.** $X=(3,0,-2)$. **2.** $X=\left(\frac{15}{4};\frac{3}{2};-\frac{13}{4};2\right)$ **3.** Birgalikda va aniqmas sistema; umumiy yechim $(-1-2\alpha+\beta;1-2\alpha+\beta;\alpha;\beta)$; xususiy yechim $(-1;1;1,2)$ 4-variant. **1.** $X=(1,1,1)$ **2.** $X=(2;-1;3;1)$. **3.** Birgalikda va aniqmas sistema; umumiy yechim $(1+2t_1+t_2-3t_3;t_1;1;t_2;t_3)$; xususiy yechim $(1;0;1;0;0)$ 5-variant. **1.** $X=(1,3,5)$ **2.** $X=(-2;3;5;2)$. **3.** Birgalikda va aniqmas sistema; umumiy yechim $\left(\frac{1-4t_1-t_2}{3};t_1;t_2;1\right)$; xususiy yechim $(-1;1;0;1)$ 6-variant. **1.** $X=(3,1,-1)$. **2.** $X=(1;2;3;4)$ **3.** Birgalikda va aniq sistema; $(2;-2;3)$ 7-variant. **1.** $X=(-3,2,1)$. **2.** Cistema birgalikda emas **3.** Birgalikda va aniqmas sistema; umumiy yechim $(11t;2t;7t)$; xususiy yechim $(11;2;7)$ 8-variant. **1.** $X=(-2,2,1)$. **2.** $X=(0;1;2;-3)$ **3.** Birgalikda va aniq sistema; $(-1;2)$. 9-variant. **1.** $X=(1;2,-3)$. **2.** $X=(2;-3;2;-1)$ **3.** Cistema birgalikda emas. 10-variant. **1.** $X=(1;1,1)$ **2.** $X=(-2;2;-3;1)$ **3.** Birgalikda va aniqmas sistema; umumiy yechim $\left(\frac{1+t_1-t_2\sqrt{5}}{2\sqrt{5}};t_1;t_2\right)$; xususiy yechim $(-1;-1;2)$.

11-variant. **1.** $X=(-3;3,0)$ **2.** $X=(0;0;0;0)$ **3.** Birgalikda va aniqmas sistema; umumiy yechim $(4-t;2;t;-7-2t)$; xususiy yechim $(3;2;1;-9)$ 12-variant. **1.** $X=(-1;1,3)$ **2.** $X=(-2;0;1;-1)$. **3.** Cistema birgalikda emas. 13-variant. **1.** $X=(2;-3,2)$ **2.** $X=(2t-1;t+1;t)$; $t\in R$ **3.** Birgalikda va aniqmas sistema; umumiy yechim $(1+t\sqrt{3};t)$; xususiy yechim $(1;0)$ 14-variant. **1.** $X=(-1;2,-3)$ **2.** $X=(2;-1,3;1)$ **3.** Birgalikda va aniq sistema; $(2;-1;3)$ 15-variant. **1.** $X=(-4;1,2)$ **2.** $X=(1;1;1;1)$ **3.** Birgalikda va aniq sistema; $(0;0;0)$ 16-variant. **1.** $X=(-2;1;-1)$ **2.** $X=(1;-1,-1,1)$ **3.** Birgalikda va aniq sistema; $(3;0;-5;11)$ 17-variant. **1.** $X=(0;1;-2)$ **2.** $X=(1;-1;2,0)$ **3.** Birgalikda va aniqmas sistema; umumiy yechim $(2+t_1-t_2;3-2t_1+t_2;t_1;t_2)$; xususiy yechim $(2;2;1;1)$ 18-variant. **1.** $X=(1;1,1)$ **2.** $X=(-2;0;1;-1)$ **3.** Cistema birgalikda emas. 19-variant. **1.** $X=(1;1;-1)$. **2.** $X=(1;-1;0;1)$. **3.** Birgalikda va aniqmas sistema; umumiy yechim $(-3t;t;5t+1)$; xususiy yechim $(0;0;1;)$. 20-variant. **1.** Cistema birgalikda emas **2.** $X=(2;0;-1;3)$ **3.** Birgalikda va aniq sistema; $(0;5;1)$. 21-variant. **1.** $X=(-1;3;-2;2)$. **2.** $X=(2;1;-1)$. **3.** Birgalikda va aniq sistema; $(2;3;5)$. 22-variant. **1.** $X=\left(0;2;\frac{1}{3};-\frac{3}{2}\right)$. **2.** $X=(5;-4;1)$ **3.** Birgalikda va aniqmas sistema; umumiy yechim $(-3t;t;5t)$; xususiy yechim $(0;0;0;)$. 23-variant. **1.** $X=(1;-1;-1;1)$ **2.** $X=(1;2;-1)$ **3.** Birgalikda va aniq sistema; $(0;0;0)$. 24-variant. **1.** $X=(4;-3;2;-1)$ **2.** $X=(1;1;1;1)$. **3.** Cistema birgalikda emas. 25-variant. **1.** $X=(2;-1;3;-4)$. **2.** $X=(1;2;3;4)$ **3.** Cistema birgalikda emas.

III BOB

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1-variant. **1.** 120°. **2.** $(0,1,3,0;0)$, $(0;-2;0;0;3)$. 2-variant. **1.** $(28;34;-44;10)$ **2.** $(8;-6;1;0)$, $(-7;5;0;1)$. 3-variant. **1.** 90°. **2.** $(\sqrt{3}t;t)$,$(\sqrt{3};1)$. 4-variant. **1.** 20. **2.** $(t;-2t;t)$,$(1;-2;1)$. 5-variant. **1.** a) $\alpha=4$, $\beta=-1$; b) $\alpha=c$, $\beta=c+d$, bu yerda c − ixtiyoriy haqiqiy son. **2.** $(2t_1-3t_2;t_1;t_2)$,$(2;1;0)$,$(-3,0,1)$. 6-variant. **1.** $\vec{c}=2\vec{b}-2\vec{a}$. **2.** Umumiy yechim $(0;0;0;0;0;0)$. Fundamental yechimlar tizimi yo'q. 7-variant. **1.** $\vec{a}=\dfrac{3\vec{b}-\vec{c}+\vec{d}}{2}$. **2.** $(t_1;t_2;t_2-2t_1)$,$(1;0;-2)$,$(0,1,1)$. 8-variant. **1.** $|\vec{a}|=7$; $\cos\alpha=\frac{2}{7}$, $\cos\beta=\frac{3}{7}$, $\cos\gamma=-\frac{6}{7}$. **2.** Umumiy yechim $(0;0;0)$. Fundamental yechimlar tizimi yo'q. 9-variant. **1.** 45° yoki 135°. **2.** $(8t_1-7t_2;-6t_1+5t_2;t_1;t_2)$, $(8;-6;1;0)$,$(-7;5;0;1)$. 10-variant. **1.** 45° **2.** Sistema faqat nol yechimga ega. 11-variant. **1.** $m=-6$. **2.**

$(-9,3,4,0,0)$, $(-3,1,0,2,0)$, $(-2;1;0;0;1)$. 12-variant. **1.** $\frac{4\sqrt{2}}{3}$. **2.** $(-7,5,1,0)$, $(7,-5,0,2)$ 13-variant. **1.** $\frac{5}{\sqrt{89}}$. **2.** $(-9,-3;11,0,0)$, $(3,1,0,11,0)$, $(-10;4;0;0;11)$. 14-variant. **1.** $\vec{d}=-\frac{3}{2}\vec{i}+\frac{3}{4}\vec{j}+\frac{3}{2}\vec{k}$. **2.** $(8,-6,1,0)$, $(-7,5,0,1)$ 15-variant. **1.** $|\vec{a}|=\sqrt{6}$; $|\vec{b}|=\sqrt{2}$; $\varphi=\frac{\pi}{6}$. **2.** $(11,7,1,0)$, $(0;-2;0,1)$. 16-variant. **1.** $|\overline{M_1M_2}|=7$; $\cos\alpha=\frac{2}{7}$, $\cos\beta=-\frac{6}{7}$, $\cos\gamma=\frac{3}{7}$. **2.** $(-3;-5;1,0)$, $(-2;1;0;1)$ 17-variant. **1.** $M(-4;4;4\sqrt{2})$. **2.** $(4;1,-2,0)$, $(-4,0;0,1)$ 18-variant. **1.** $\varphi=\arccos\frac{2}{7}$. **2.** $(-2t;7t;0;9t)$, $(-2;7;0;9)$. 19-variant. **1.** $m=4$. **2.** $(2t_1-3t_2;t_1;t_2)$, $(2;1;0)$, $(-3;0;1)$. 20-variant. **1.** $\vec{a}=4\vec{b}-5\vec{c}+3\vec{d}$. **2.** $(0;0;0)$. Fundamental yechimlar tizimi yo'q. 21-variant. **1.** $\alpha=-1$, $\beta=4$. **2.** $(14,11,-2;1;0;0)$, $(-4,-3,0,0;1;0)$, $(1;0,1,0;0;1)$. 22-variant. **1.** $\vec{c}=5\vec{a}+2\vec{b}$. **2.** $\left(1;0;-\frac{5}{2};\frac{7}{2}\right)$, $(0;1;5;-7)$. 23-variant. **1.** $\alpha=-2$. **2.** $(2;1;0;0)$, $(-4,0;1,0)$, $(1;0;0;1)$. 24-variant. **1.** $|\vec{a}|=\sqrt{3}$; $|\vec{b}|=\sqrt{2}$; $\varphi=\arccos\sqrt{\frac{2}{3}}$. **2.** $\left(\frac{1}{7};\frac{3}{7};1;0\right)$, $\left(\frac{1}{7};-\frac{4}{7};0;1\right)$. 25-variant. **1.** $\sqrt{7}$. **2.** $\left(\frac{1}{7};\frac{3}{7};1\right)$.

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1-variant. **1.** $e_3'=(3;4;-5)$. **2.** $y=(-4;7;7)$. **3.** $L=2y_1^2-2y_2^2-3y_3^2$, agar $y_1=x_1-x_2+x_3$, $y_2=x_2+x_3$, $y_3=x_3$. 2-variant. **1.** $x=(-4;-8;8)$. **2.** $\tilde{A}=\begin{pmatrix}-85 & -59 & 18\\ 121 & 84 & -25\\ -13 & -9 & 3\end{pmatrix}$. **3.** $L=y_1^2-y_2^2+y_3^2$, agar $y_1=x_1+x_2$, $y_2=x_2-x_3$, $y_3=x_3$. 3-variant. **1.** $m=1$. **2.** $y=(2;3)$. **3.** $L=y_1^2-4y_2^2+y_3^2$, agar $y_1=x_1$, $y_2=x_2$, $y_3=-2x_2+x_3$. 4-variant. **1.** $d=2a_1-2a_2+a_3$. **2.** $2, (\alpha;0;0)^T$; $\alpha\neq 0$ **3.** $L=y_1^2+2y_2^2+y_3^2$, agar $y_1=x_1+x_2+x_3$, $y_2=x_2+x_3$, $y_3=x_3$. 5-variant. **1.** $e_1=\left(1;\frac{1}{3};0\right)$, $e_2=\left(-\frac{1}{2};-\frac{1}{3};\frac{1}{2}\right)$, $e_3=\left(0;\frac{1}{3};0\right)$. **2.** $3, (\alpha;0;0)^T$; $4, (0;0;\alpha)^T$ $\alpha\neq 0$. **3.** Musbat aniqlangan. 6-variant. **1.** $\varphi=\arccos\sqrt{\frac{6}{13}}$. **2.** $\begin{pmatrix}-2 & 11 & 7\\ -4 & 14 & 8\\ 5 & -15 & -8\end{pmatrix}$. **3.** Manfiy aniqlangan.

7-variant. **1.** Ha. **2.** $y=(-3;3)$. **3.** Umumiy ko'rinishda. 8-variant. **1.** $A=\begin{pmatrix}0 & -1 & 1\\ 1 & 0 & 1\\ 1 & 2 & 0\end{pmatrix}$. **2.** $7,(2;6;11)^T\alpha$; $5(0;0;1)^T\alpha, 0, (-10;5;1)^T\alpha, \alpha\neq 0$. **3.** Umumiy ko'rinishda. 9-variant. **1.** $e_2' = (2;1;0)$. **2.** $\begin{pmatrix} -2 & 3 \\ 1 & -2 \end{pmatrix}$. **3.** 2. 10-variant. **1.** Ixtiyoriy m **2.** $0,(4;0;-1)^T\alpha;\ 7(2;7;3)^T\alpha,-2,(2;-2;3)^T\alpha,\alpha\neq 0$. **3.** 2. 11-variant. **1.** $a_3 = \left(\frac{8}{5};-\frac{1}{5}\right)$. **2.** $-1,(1;-1;1)^T\alpha;\ 0(1;0;-1)^T\alpha,2,(1;2;1)^T\alpha,\alpha\neq 0$. **3.** 2. 12-variant. **1.** $m=\frac{5}{3}$. **2.** $3,(1;2;0)^T\alpha;(0;-1;1)^T\alpha,6,(2;1;0)^T\alpha,\alpha\neq 0$. **3.** m ning hech qanday qiymatlarida manfiy aniqlanmagan; $m>4$ da musbat aniqlangan. 13-variant. **1.** Ixtiyoriy m da **2.** $\alpha_1=1,\alpha_2=3,\alpha_3=-3;(-2c_1;c_1;c_1);(0;c_2;c_2),\ (6c_3;-7c_3;5c_3),c_1\neq 0,c_2\neq 0,c_3\neq 0$. **3.** m ning bunday qiymatlari yo'q. 14-variant. **1.** Chiziqli bog'liq. **2.** $\alpha_1=-2,\alpha_2=3;(-2c_1;0;c_1);(0;c_2;0),\ c_1\neq 0,c_2\neq 0$. **3.** $m>1$. 15-variant. **1.** $e_1=\left(\frac{5}{13};-\frac{3}{13}\right),e_2=\left(\frac{1}{13};\frac{2}{13}\right)$. **2.** $\alpha_1=2,\alpha_2=3,\alpha_3=-3;(3c_1;-5c_1;c_1);(4c_2;0;c_2),\ (2c_3;0_3;-c_3),c_1\neq 0,c_2\neq 0,c_3\neq 0$. **3.** $m>0,5$. 16-variant. **1.** $\arccos\sqrt{\frac{5}{42}}\approx 70^0$. **2.** $\alpha_1=3,\alpha_2=4,\alpha_3=-1;(0;c_1;0);(3c_2;5c_2;-c_2),\ (2c_3;0;c_3),c_1\neq 0,c_2\neq 0,c_3\neq 0$. **3.** $m>1$. 17-variant. **1.** Chiziqli erkli. **2.** $(-c_1;-c_1;c_1);(c_3;-c_3;c_3),\ c_1^2+c_2^2\neq 0,c_3\neq 0$. **3.** $m$ning bunday qiymatlari yo'q. 18-variant. **1.** Yo'q. **2.** $(-c_1;-c_1;c_1);(c_3;-c_2;c_2),\ c_2^2+c_3^2\neq 0,c_1\neq 0$ **3.** $m$ning bunday qiymatlari yo'q. 19-variant. **1.** $m\neq 12$. **2.** $(c_1;c_1;c_1);(c_3;-c_2;c_2),\ c_2^2+c_3^2\neq 0,c_1\neq 0$ **3.** $m<-2$. 20-variant. **1.** Ha. **2.** $2,(0;-1;1)^T\alpha;\ 9(0;4;3)^T\alpha,1,(-8;4;1)^T\alpha,\alpha\neq 0$. **3.** $m$ning bunday qiymatlari yo'q. 21-variant. **1.** $b=-a_1+4a_2+3a_3$. **2.** $0,(0;a;b)^T$, $a\neq 0$ *yoki* $b\neq 0$. **3.** $m<-2,5$. 22-variant. **1.** $b=a_1+2a_2+5a_3$. **2.** $y=(1;3;4)$. **3.** Musbat aniqlangan. 23-variant. **1.** $b=2a_1-2a_2+a_3$. **2.** $\begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}$. **3.** $L=y_1^2+y_2^2+5y_3^2$, agar $y_1=x_1+x_2+x_3$, $y_2=x_2+x_3, y_3=x_3$. 24-variant. **1.** Chiziqli bog'liq. **2.** $1,(\alpha;0;0)^T;\ 2(0;\alpha;0)^T,3,(0;0;\alpha)^T,\alpha\neq 0$. **3.** Musbat aniqlangan. 25-variant. **1.** $(x,y)=-90;\ |x|=2\sqrt{26},\ |y|=\sqrt{105}$. **2.** $1,(\alpha;\alpha;0)^T;\ -1(\alpha;-\alpha;0)^T$, $,2,(0;0;\alpha)^T,\alpha\neq 0$. **3.** Musbat aniqlangan.

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1. 2:4:3. **2.** 1400:1460:2200:1210. **3.** $X=\begin{pmatrix} 483 \\ 192 \end{pmatrix}$, $x_{11}=144,9\ x_{12}=38,4$; $x_{21}=72,5\ x_{22}=19,2$; 534,6 *shartli pul birligi*; 221,9 *shartli pul birligi*; **4.** $X=(1000;1000)'$, $\Delta X=(184;132)'$. **5.** 134, 201, 67. **6.** 198, 114, 90. **7.** $A=\begin{pmatrix} 0,2 & 0,4 \\ 0,55 & 0,1 \end{pmatrix}$. **8.** $A=\begin{pmatrix} 0,16 & 0,4 \\ 0,14 & 0,1 \end{pmatrix}$, $S=\begin{pmatrix} 1,29 & 0,57 \\ 0,2 & 1,2 \end{pmatrix}$, $X=\begin{pmatrix} 622,5 \\ 430 \end{pmatrix}$. **9.**

90, 114, 198. 90, 114, 198. **10.** $\Delta Y=(120;10)'$. **11.** 134, 67, 201. **12.** $\Delta X=(23;\ 56;\ 27)'$. $\Delta Y=(3,6;\ -11,6;\ 18,6)'$. **13.** $a)\ S=\begin{pmatrix}2,5 & 1,25\\ \frac{5}{6} & \frac{25}{12}\end{pmatrix}$; $b)\ X=\begin{pmatrix}4050\\2750\end{pmatrix}$; $c)\ \Delta Y=\begin{pmatrix}450\\300\end{pmatrix}$. **14.** 24:7:21. **15.** $\Delta Y=(44;36)'$. **16.** 15:11:9. **17.** $y_1=270,\ y_2=470$. **18.** $x_1=100,\ x_2=100$. **19.** 45:29:23. **20.** $\Delta Y=(86;10)'$. **21.** 23:11:9. **22.** $\Delta X=(130;180)'$. **23.** $\Delta X=(160;230)'$. **24.** 1000, 1200. **25.** $\Delta X=(250;360)'$.

IV BOB

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1-variant. **1.** $7x-2y=0$. **2.** $\left(1;-\frac{4}{3}\right);R=\frac{5}{3}$. **3.** $7x-5y+6z-31=0$. 2-variant. **1.** *Ha, Aburchak to'g'ri burchak.* **2.** $(2;1);R=5$. **3.** $2x-19y+10z+45=0$. 3-variant. **1.** $x+4y-14=0$. **2.** $(x-2)^2+(y+1)^2=25$. **3.** $11x-2y+12z-13=0$. 4-variant. **1.** $x-2y+5=0$. **2.** $x^2+(y-4)^2=16$. **3.** $5x-3y-z-9=0$. 5-variant. **1.** $4x+5y+2=0,\ 5x-4y-18=0$. **2.** $x+y+8=0$. **3.** $3y+z=0$. 6-variant. **1.** $(-9;-6)$. **2.** $(x-3)^2+y^2=9$. **3.** $x+2z=0$. 7-variant. **1.** $(1;-1)$. **2.** $(x-2)^2+(y-4)^2=10$. **3.** $y+4=0$. 8-variant. **1.** 5. **2.** $a=2,b=\sqrt{3};F_1(-1;0),\ F_2(1;0);\varepsilon=0,5$. **3.** $\frac{x}{1}-\frac{y}{1}+\frac{z}{2}=1$ yoki $2x-2y+z-2=0$. 9-variant. **1.** $A=180^0-arctg\frac{1}{3},\ B=arctg\frac{3}{14},C=arctg\frac{1}{9}$. **2.** $a=3,b=2;F_1\left(0;-\sqrt{5}\right),\ F_2\left(0;\sqrt{5}\right);\varepsilon=\frac{\sqrt{5}}{3}$. **3.** $x+y-z+2=0$. 10-variant. **1.** 45^0. **2.** $\frac{x^2}{144}+\frac{y^2}{108}=1;\ 2c=12$. **3.** $9x-y+7z-40=0$. 11-variant. **1.** $2x-y+4=0$ yoki $x+2y-3=0$ **2.** $\frac{x^2}{16}+\frac{y^2}{4}=1;\ \varepsilon=\frac{\sqrt{3}}{2};\ z_1=4-\sqrt{3},\ z_2=4+\sqrt{3}$. **3.** $x-4y+5z+15=0$. 12-variant. **1.** $m_A:2x+11y-10=0,\ 5\sqrt{5};\ h_A:x+2y+2=0,\ \frac{24}{25}\sqrt{5};\quad l_A:x+3y=0,\ \frac{24\sqrt{10}}{7}$. **2.**

$\left(-\frac{15}{4};\pm\frac{\sqrt{63}}{4}\right)$. **3.** $3x-4y-3z+4=0$. 13-variant. **1.** $x+6y-28=0; 5x+3y-5=0; 4x-3y-4=0$. **2.** $\varepsilon=\sqrt{0,4}$. **3.** $3x+3y+z-8=0$. 14-variant. **1.** $x+y+6=0(AB)$, $x-4y+21=0(BC)$, $3x-2y-7=0(AC)$. **2.** $\frac{x^2}{16}-\frac{y^2}{9}=1$; $F_1(-5;0), F_2(5;0)$, $A_1(-4;0), A_2(4;0)$, $B_1(0;-3), B_2(0;3)$, $\varepsilon=\frac{5}{4}$, *asimptotalari* $y=\pm\frac{3x}{4}$. **3.** $\frac{x+1}{1}=\frac{y-1}{-3}=\frac{z+3}{4}$. 15-variant. **1.** $a) 2x+5y-13=0$; $b) 5x-2y+11=0$. **2.** $\frac{x^2}{2}-\frac{y^2}{1}=1$. **3.** $\frac{x-2}{1}=\frac{y+1}{4}=\frac{z+1}{0}$. 16-variant. **1.** 5. **2.** $\frac{x^2}{12}-\frac{y^2}{4}=1$; $r_1=6\sqrt{3}, r_2=2\sqrt{3}$. **3.** $\frac{x-1}{\sqrt{2}}=\frac{y+5}{1}=\frac{z-3}{-1}$. 17-variant. **1.** $4\sqrt{2}$. **2.** $d=b$; $\varphi=2arctg\left(\frac{b}{a}\right)$. **3.** $\frac{x-1}{-2}=\frac{y+3}{4}=\frac{z-5}{5}$. 18-variant. **1.** a) $3x+2y-17=0$;; b) $2x-3y-7=0$. **2.** $\frac{x^2}{16}-\frac{y^2}{9}=1$. **3.** $\frac{x-3}{5}=\frac{y+2}{3}=\frac{z-4}{-7}$. 19-variant. **1.** BD mediana tenglamasi $x+y-2=0$. BE balandlik tenglamasi $15x-y-78=0$. BF bissektrisa tenglamasi $11x+3y-46=0$. **2.** $y+2=\pm\frac{\sqrt{2}}{2}x$. **3.** $(5;-1;0)$. 20-variant. **1.** $x-y+2=0$, $x+2y-4=0$, $2x+y-8=0$. **2.** $y^2=9x$. **3.** $\left(-\frac{1}{2};-\frac{1}{2};2\right)$. 21-variant. **1.** $x-7y+6=0$ va $7x+y+4=0$. **2.** $x^2=-y$. **3.** $\frac{x-5}{5}=\frac{y-2}{-9}=\frac{z-2}{6}$. 22-variant. **1.** $A\left(\frac{4}{3};\frac{2}{3}\right)$, $B(6;0)$, $C(2;-4)$. **2.** $8x+10y-46=0$. **3.** $\frac{x+6}{1}=\frac{y-1}{-1}=\frac{z-3}{0}$. 23-variant. **1.** $x+3y-2=0$. **2.** 3. **3.** $\frac{x-6}{4}=\frac{y-1}{-4}=\frac{z-2}{5}$. 24-variant. **1.** $11x+22y-74=0$. **2.** $\frac{x^2}{4}+\frac{y^2}{8}=1$. **3.** $B(1;4;-7)$. 25-variant. **1.** $4x-8y+1=0$. **2.** $y=-\frac{x^2}{2}$; $y=0,5$. **3.** $\frac{x+1}{\frac{1}{2}}=\frac{y}{\frac{\sqrt{2}}{2}}=\frac{z-5}{-\frac{1}{2}}$.

V BOB

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1-variant. **1.** $(-\infty;-1)\cup(-1;+\infty)$. **2.** $[4;+\infty)$. **3.** Toq. 2-variant. **1.** $(-\infty;-2)\cup(-2;2)\cup[2;+\infty)$. **2.** $(0;1]$. **3.** Juft. 3-variant. **1.** $(-\infty;0)$. **2.** $[-9;-5]$. **3.** Juft ham emas, toq ham emas. 4-variant. **1.** $(-\infty;2]\cup[5;+\infty)$. **2.** $(-\infty;4)\cup(4;+\infty)$. **3.** Toq. 5-variant.

1. $x \neq \frac{\pi}{2}+\pi n, n\in Z$. **2.** $\left(-\frac{1}{2};\frac{1}{2}\right)$. **3.** Juft. 6-variant. **1.** $[7;10]$. **2.** $[2;+\infty)$. **3.** Toq. 7-variant. **1.** $(0;\sqrt{2})\cup(\sqrt{2};+\infty)$. **2.** $(-\infty;4]$. **3.** Juft ham emas, toq ham emas. 8-variant. **1.** $[-2;1)$. **2.** $\left[-\frac{1}{3};+\infty\right)$. **3.** Toq. 9-variant. **1.** $\left[0;\frac{2}{3}\right)$. **2.** $(0;1)\cup(1;+\infty)$. **3.** Juft ham emas, toq ham emas. 10-variant. **1.** $(2;3]$. **2.** $(0;+\infty)$. **3.** Juft. 11-variant. **1.** $x\neq \pi n, n\in Z$. **2.** $[e^{-1};+\infty)$. **3.** Juft. 12-variant. **1.** $(-\infty;2)\cup(-2;5)\cup(5;+\infty)$. **2.** $\{-1;1\}$. **3.** Juft ham emas, toq ham emas. 13-variant. **1.** $\left[-\frac{1}{3};\frac{1}{3}\right]$. **2.** $\left[-\frac{1}{2};\frac{1}{2}\right]$. **3.** Juft. 14-variant. **1.** $(0;1)\cup(1;+\infty)$. **2.** $[2;+\infty)$. **3.** Juft ham emas, toq ham emas. 15-variant. **1.** $\varnothing$. **2.** $\left[\frac{1}{3};3\right]$. **3.** Toq. 16-variant. **1.** $(0;3)\cup(3;+\infty)$. **2.** $[-\sqrt{29};\sqrt{29}]$. **3.** Juft ham emas, toq ham emas. 17-variant. **1.** $(0;+\infty)$. **2.** $(0;1]$. **3.** Juft ham emas, toq ham emas. 18-variant. **1.** $\left[\frac{1}{3};3\right]$. **2.** $[-1,5;1,5]$. **3.** Toq. 19-variant. **1.** $[-1;0)\cup(0;1]$. **2.** $[0,75;1,5]$. **3.** Toq. 20-variant. **1.** $(-\infty;-5)\cup\left(-5;-\frac{1}{2}\right)\cup[0;+\infty)$. **2.** $(-\infty;1]$. **3.** Juft. 21-variant. **1.** $(-1;0)\cup(0;\pi)$. **2.** $[-2;1]$. **3.** Juft. 22-variant. **1.** $(-1;1)\cup(1;10]$. **2.** $[-10;10]$. **3.** Juft ham emas, toq ham emas. 23-variant. **1.** $[-4;0)\cup(0;1)\cup(1;2)\cup(2;4]$. **2.** $[0;2]$. **3.** Toq. 24-variant. **1.** $\left[-2;-\frac{\pi}{2}\right)\cup\left(-\frac{\pi}{2};\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2};2\right]$. **2.** $(0;23]$. **3.** Toq. 25-variant. **1.** $(0;1)\cup(1;2)$. **2.** $(-\infty;0]$. **3.** Toq.

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1-variant. **2.** 2. **3.** 3. **4.** $2,25$. **5.** $e^{1,5}$. **6.** $x=1$, birinchi tur. 2-variant. **2.** 1. **3.** -1. **4.** $0,4$. **5.** e^{-9}. **6.** Funksiya uzluksiz. 3-variant. **2.** ∞. **3.** $1,5$. **4.** $0,5$. **5.** e. **6.** $x=2$, birinchi tur. 4-variant. **2.** 0. **3.** $0,4$. **4.** 1. **5.** e^2. **6.** $x=-2$, ikkinchi tur. 5-variant. **2.** $\frac{4}{21}$. **3.** $0,5$. **4.** 2. **5.** e. **6.** $x=0$, ikkinchi tur. 6-variant. **2.** 6. **3.** $\frac{4}{3}$. **4.** -8. **5.** e^{-1}. **6.** $x=\frac{\pi}{2}+\pi n, n\in Z$. ikkinchi tur. 7-variant. **2.** 1. **3.** $-\frac{3}{11}$. **4.** -6. **5.** e^{-1}. **6.** $x=\frac{\pi}{2}+\pi n, n\in Z$. ikkinchi tur. 8-variant. **2.** ∞. **3.** $\frac{4}{3}$. **4.** $-0,5$. **5.** 1. **6.** $x=1, x=2$ larda ikkinchi tur. 9-variant. **2.** $\frac{1}{2}$. **3.** $3x^2$. **4.** 2. **5.** e. **6.** $x=0$ bartaraf etilishi mumkin bo'lgan uzilish, $x\neq 0$ bo'lsa $y=x+1$. 10-variant. **2.** 2. **3.** $0,05$. **4.** $\frac{4}{3}$. **5.** e^4. **6.** $x=0$ birinchi tur $y=\begin{cases}x<0, y=-x-1,\\ x>0, y=x+1.\end{cases}$ 11-variant. **2.** $\frac{1}{5}$. **3.** 48. **4.** $6,4$. **5.** e^{-4}. **6.** $x=-2$, ikkinchi tur. 12-variant. **2.** 0. **3.** $\frac{2}{3}$. **4.** ∞. **5.** e^{-2}. **6.** $x=0$ bartaraf etilishi mumkin bo'lgan uzilish, $\lim\limits_{x\to 0} e^{-\frac{1}{x^2}}=0$. 13-variant. **2.** e **3.** 2 **4.** 0 **5.** e^6 **6.** $x=2$ ikkinchi tur. 14-variant. **2.** $-\frac{4}{3}$. **3.** $-\frac{1}{12}$. **4.** 1,5. **5.** e **6.** $x=1$, birinchi tur. 15-variant. **2.** 0 **3.** −12. **4.** 0,75. **5.** e **6.** $x=\frac{\pi}{2}+\pi n, n\in Z$. ikkinchi tur. 16-variant. **2.** e **3.** $-\frac{1}{2}$. **4.** $\frac{8}{7}$. **5.** e^4. **6.** $x=0$ bartaraf etilishi mumkin bo'lgan uzilish. 17-variant. **2.** $\frac{1}{6}$ **3.** −3. **4.** $\frac{4}{9}$. **5.** e^{-14}. **6.** $x=0$ ikkinchi tur, $x=\pm 1$, birinchi tur. 18-variant. **2.** 0 **3.** 0 **4.** 0,125. **5.** $e^{1,75}$ **6.** $x=0$ birinchi tur. 19-variant. **2.** 1 **3.** ∞. **4.** $-\frac{1}{6}$ **5.** e^{-6}. **6.** Funksiya uzluksiz. 20-variant. **2.** ∞ **3.** $\frac{1}{3}$. **4.** 0,3. **5.** $e^{3,6}$. **6.** Funksiya uzluksiz. 21-variant. **2.** $\frac{1}{2}$ **3.** 4. **4.** $\frac{4}{9}$. **5.** $e^{7,5}$. **6.** $x=0$, ikkinchi tur. 22-variant. **2.** 0 **3.** $2\frac{1}{3}$. **4.** 1. **5.** $e^{\frac{10}{7}}$. **6.** $x=2$, birinchi tur. 23-variant. **2.** Mavjud emas **3.** ∞. **4.** $\frac{8}{27}$. **5.** e^{10}. **6.** Funksiya uzluksiz. 24-variant. **2.** 1 **3.** 0. **4.** 1,25. **5.** e^{-3}. **6.** $x=-1$, birinchi tur. 25-variant. **2.** 1. **3.** 0,5. **4.** −0,4. **5.** $\frac{1}{e}$. **6.** Funksiya uzluksiz.

VI BOB

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1-variant. **1.** $-5\sin 5x$. **2.** $3\ln 7\cdot 7^{3x-1}$. **3.** $-\dfrac{2x\sin(x^2+y^2)+ye^{xy}}{2y\sin(x^2+y^2)+xe^{xy}}$. **4.** $\dfrac{2t+1}{3t^2+1}$. **5.** $y^{(n)}=a^n e^{ax}$.

2-variant. **1.** $-3\cos^2 x\sin x$. **2.** $100(x+1)^{99}$. **3.** $-\dfrac{b^2x}{a^2y}$. **4.** $\dfrac{\sin t}{1-\cos t}$ yoki $ctg\frac{t}{2}$ **5.** $y^{(n)}=a^n\sin\left(ax+\dfrac{\pi n}{2}\right)+b^n\cos\left(bx+\dfrac{\pi n}{2}\right)$. 3-variant. **1.** $\dfrac{1}{2\sqrt{tgx}\cos^2 x}$. **2.** $\dfrac{1}{2\sqrt{x-x^2}}$. **3.** $\dfrac{(2x^2+1)y}{x(1-2y^2)}$. **4.** $y'_x=-\dfrac{2}{3}tgt$. **5.** $y^{(n)}=(x+n)e^x$. 4-variant. **1.** $-\dfrac{1}{x\ln^2 x}$. **2.** $ctgx$. **3.** $-\dfrac{y\cos x+\sin y}{x\cos y+\sin x}$. **4.** −1. **5.** $y'=\dfrac{(-1)^n a^n n!}{(ax+b)^{n+1}}$. 5-variant. **1.** $-\dfrac{e^{ctgx}}{\sin^2 x}$. **2.** $\dfrac{-e^x}{\sqrt{1-e^{2x}}}$. **3.** $\dfrac{x(2x^2-y^2)}{y(2y^2+x^2)}$. **4.** $0{,}8ctht$. **5.** $y'=\dfrac{(-1)^n n!}{2}\left(\dfrac{1}{(x+1)^{n+1}}+\dfrac{1}{(x-1)^{n+1}}\right)$. 6-variant. **1.** $-\dfrac{2arctg\frac{1}{x}}{1+x^2}$. **2.** $4{,}5\sin^8\dfrac{x}{2}\cos\dfrac{x}{2}$. **3.** $y'=-\dfrac{y}{x+e^y}$, $y(0)=-\dfrac{1}{e}$. **4.** $\dfrac{1}{t^2}$. **5.** $y^{(n)}=(-1)^n\dfrac{a^n n!}{(ax-b)^{n+1}}$. 7-variant. **1.** $\dfrac{2}{\sqrt[3]{3x-1}}$. **2.** $-\dfrac{1}{\sqrt{2(x-x^2)}(1+x)}$. **3.** $-\sqrt{\dfrac{y}{x}}$. **4.** $-tgt$. **5.** $y^{(n)}=3^n e^{3x}$. 8-variant. **1.** $\sec 2x$. **2.**

$\frac{e^{-\frac{x}{2}}(12\sin 3x-\cos 3x)}{2\cos^3 3x}$. **3.** $\frac{y\left(x-y\sqrt{y^2-x^2}\right)}{x\left(y\ln x\cdot\sqrt{y^2-x^2}+x\right)}$. **4.** -1. **5.** $y^{(n)}=(-1)^n 3\frac{n!}{x^{n+1}}$ 9-variant. **1.** $\frac{1}{\sqrt{x^2-1}}$. **2.** $\frac{4}{\cos^6 4x}$. **3.** $-\frac{y(y+x\ln y)}{x(x+y\ln x)}$. **4.** t^2+1. **5.** $y^{(n)}=(-1)^{n-1}\frac{1\cdot 3\cdot 5\cdots(2n-3)}{2^n\sqrt{x^{2n-1}}}$. 10-variant. **1.** $x^2(3\sin(\cos x)-x\sin x\cos(\cos x))$. **2.** $\frac{3^{x^2}}{2\sqrt{x^3-5x}}(4x^4\ln 3+x^2(3-20\ln 3)-5)$. **3.** $\frac{x-2y}{2x-3y}$. **4.** $\frac{3t^2}{2}$. **5.** $y^{(n)}=a(a-1)\ldots(a-n+1)x^{a-n}$. 11-variant. **1.** $\frac{4ctg4x}{\ln 6}$. **2.** $\frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}\cdot\sin\frac{1-\sqrt{x}}{1+\sqrt{x}}$. **3.** $\frac{2xy(1+y^2)}{1-x^2-x^2y^2}$. **4.** -1. **5.** $y^{(n)}=\frac{1}{33}(-1)^{n+1}\frac{(n-1)!}{x^n}$. 12-variant. **1.** $\frac{6}{(x+1)(x+2)(x+3)(x+4)}$. **2.** $\frac{2x-2}{\left(x^2-4x+5\right)^2}$. **3.** $y'=-\frac{x}{y}$, $y'\left(-\sqrt{2}\right)=1$. **4.** tgt. **5.** $y^{(n)}=a^n\sin\left(ax+\frac{\pi}{2}n\right)$. 13-variant. **1.** $-\frac{1}{2}\sin 2x$. **2.** $5e^{sh^2 5x}sh10x$. **3.** -2. **4.** -1. **5.** $y^{(n)}=\beta^t\cos\left(ax+\frac{\pi}{2}n\right)$. 14-variant. **1.** $\frac{2e^{3x}(3x-1)}{\left(x-e^{3x}\right)^2}$. **2.** $-\sqrt{\frac{x}{1-x}}$. **3.** $-\frac{y}{x}\frac{x\ln y+y}{x+\ln x}$. **4.** $ctg\frac{t}{2}$. **5.** $y'=\frac{(-1)^{n-1}(n-1)!}{(1+x)^n}$. 15-variant. **1.** $-\frac{1}{x^2+1}$. **2.** $-\cos 2x$. **3.** $\frac{\cos y-y\cos x}{\sin x+x\sin y}$. **4.** $tg\left(t-\frac{\pi}{4}\right)$. **5.** $-2^{n-1}\cos\left(2x+n\frac{\pi}{2}\right)$. 16-variant. **1.** $2x\cdot 10^{x^2+1}\ln 10$. **2.** $\frac{4}{\cos^2 4x}$. **3.** $-\frac{\sqrt{y}}{\sqrt{x}}$. **4.** $-tg3t$. **5.** $\frac{(-1)^{n-1}(n-1)!}{x^n}$. 17-variant. **1.** $2ch^3\frac{x}{2}sh\frac{x}{2}$. **2.** $\frac{15x^2-1}{5x^3-x}$. **3.** $\frac{y}{x}\frac{1-x^2-y^2}{1+x^2+y^2}$. **4.** $\begin{cases}1, \text{agar } t<0;\\ -1, \text{agar } t>0.\end{cases}$ **5.** $\left(5^x+(-1)^n 5^{-x}\right)\ln^n 5$. 18-variant. **1.** $-2\sin 2x$. **2.** $\frac{-7x}{\sqrt{4-7x^2}}$. **3.** $(x+y)^2$. **4.** e^{4t}. **5.** $\sin\left(x+\frac{\pi}{2}n\right)$. 19-variant. **1.** $\frac{-2}{\sin^2 10x\cdot\sqrt[5]{(1+ctg10x)^4}}$. **2.** $-6\cos 6x$. **3.** $\frac{y}{x-y}$. **4.** $-e^{-4t}$. **5.** $\frac{(-1)^n\cdot n!3^n}{(3x+5)^{n+1}}$. 20-variant. **1.** $12\ln^3\sin 3t\cdot ctg3t$. **2.** $\frac{1}{2\sqrt{h}(1+h)}$ **3.** $\frac{x+y}{x-y}$. **4.** $-e^{-6t}$. **5.** $\frac{(-1)^n}{t}$. 21-variant. **1.** $-\frac{1}{\sqrt{1-x^2}\arcsin^2 x}$. **2.** $\frac{\cos^3 x-\sin^3 x}{(\cos x+\sin x)^2}$. **3.** $\frac{y}{x}\frac{x\ln y-y}{y\ln x-x}$. **4.** $y'_x=-\frac{b}{a}tgt$. **5.** $\frac{(-1)^n\cdot n!2^n}{(2x-3)^{n+1}}$. 22-variant. **1.** $\frac{x-\ln x-1}{(x-1)^2}$. **2.** $\frac{4ch(\ln tg2x)}{\sin 4x}$. **3.** $-\frac{e^x-ye^{xy}}{e^y-xe^{xy}}$. **4.** $y'_x=-1,5\,ctgt$. **5.** $\frac{n!3^n}{(1-3x)^{n+1}}$. 23-variant. **1.** $\arcsin x$. **2.** $2\ln 3\cdot 3^{\sin^3 2x+4\sin 2x}\cdot\cos 2x\cdot\left(3\sin^2 2x+4\right)$. **3.** $y'=-\frac{3x^2y^2+5y}{2x^3y+5x}$. **4.** $y'=2t\cos 2t$. **5.** $\frac{(-1)^n\cdot n!5^n}{(5x+2)^{n+1}}$. 24-variant. **1.** 0. **2.**

$2^{\sqrt{x}}\left(1+\frac{\sqrt{x}}{2}\ln 2\right)$. 3. $y'=\frac{y(y-x\ln y)}{x(x-y\ln x)}$. 4. $y'=e^{2t}$. 5. $2^{n-1}\cos\left(2x+n\frac{\pi}{2}\right)$. 25-variant. 1. $-\frac{1}{x^2-9}$. 2. $\frac{x}{\sqrt{(1-x^4)^3}}$. 3. $y'=\frac{\cos(x-2y)-3x^2}{3y^2+2\cos(x-2y)}$. 4. $y'_x=-1,5tgt$ 5. $\frac{(-1)^n\cdot n!2^n}{(1+2x)^{n+1}}$.

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1-variant. 1. $y-3x+19=0;\ 3y+x+37=0$. 2. $y'(x)=0,3x^2-2,4x+5;\quad y'(10)=11$. $y_1(x)=0,1x^2-1,2x+5+\frac{250}{x};\quad y_1(10)=28$. 3. 10. 2-variant. 1. $y-x=0,\ y+x=0$. 2. $C'(x)=0,25+0,48x^{\frac{1}{3}};\ C'(27)=1,69$.

$S'(x)=1-C'(x)=0,75-0,48x^{\frac{1}{3}};\ S'(27)=-0,69$. 3. 30. 3-variant. 1. $y+8x+3=0;\ 8y-x+24=0$. 2. $z(0)=6\ z(0)=6; z'(0)=-7;\ T_z(0)=-0,857$. $z(6)=0; z'(6)=5;\ T_z(6)=0$. 3. 2. 4-variant. 1. $2y-x-9=0;\ y+2x-7=0$. 2. $a)p=2;\quad b)E_{p=2}(q)=-0,1;\ E_{p=2}(s)=0,5;\quad v)9\%$ 3. 196. 5-variant. 1. $y-x-2+\frac{\pi}{2}=0;\ y+x-\frac{\pi}{2}=0$. 2. $z(12)=66;\ z'(12)=16;\ T_z(12)=4,125$. 3. $4\log_2\ln 2$. 6-variant. 1. $y+x-1=0;\ y-x=0$. 2. $E_x(y_1)=E_x(y)-1$. 3. 90. 7-variant. 1. $y+7x-15=0;\quad 7y-x-5=0$. 2. 45; 35. 3. 0. 8-variant. 1. $4y-x+5=0;\ y+4x-3=0$. 2. $a)95;90\ b)70;40$ 3. $76(\ln 1920-1)$. 9-variant. 1. $\frac{\pi}{4};\ y-x+1=0$. 2. 80;40. 3. 4. 10-variant. 1. $\frac{3\pi}{4};\ y+x=0$. 2. $y'(2)=5;\ T_{x=2}\approx 0,04$. $y'(7)=-20;\ T_{x=7}\approx -0,24$. 3. 4. 11-variant. 1. $5y-x-4=0$. 2. $y'(20)=-0,1;\ T_{x=20}\approx -0,0167$. $y'(40)=0,3;\ T_{x=40}\approx 0,0375$. 3. 4. 12-variant. 1. $arctg\left(\frac{1}{3}\right)$. 2. $a)0,398;\ b)0,602$. 3. 5. 13-variant. 1. $\frac{\pi}{4}$. 2. $a)\approx -1,62;\ b)\approx -0,62$. 3. $p=1$. 14-variant. 1. $arctg\left(\frac{5\sqrt{2}}{7}\right)$. 2. $x=4, E_0=1, E_2=0$. 3. $p=1-p_0$. 15-variant. 1. $y=2x-1$. 2. 1,2. 3. 5*birlik*; O'rtacha xarajatlar $\frac{125}{14}$ga ko'payadi. 16-variant. 1. $y=x-1$. 2. $\approx -0,23$ 3. 1000. 17-variant. 1. $y=12x+16;\ y=-\frac{1}{12}x-8\frac{1}{6}$. 2. $-1,5$. 3. $[2;+\infty)$. 18-variant. 1. $y=x+1$. 2. $(3;6)$. 3. $\frac{p_0}{2}$. 19-variant. 1. $x=\pm 2$. 2. $a)4;\ b)-\frac{2}{3};2\ v)\approx 1,5\%$. 3. 400. 20-variant. 1. $\varphi=\frac{\pi}{2}$. 2. $(100;225)$. 3. 245. 21-variant. 1. $y=x+1;\ y=-x+1$. 2. $(10;20)$. 3. 625. 22-variant. 1. $y=\frac{1}{2}x+\frac{3\sqrt{3}-\pi}{6};\ y=-2x+\frac{4\pi+3\sqrt{3}}{6}$. 2. $-0,75$. 3. $33\frac{1}{3}$.

23-variant. **1.** $x_0=0,5$ **2.** $-0,5;-6$. **3.** $p>\frac{1}{4}$. 24-variant. **1.** $x_0=1$. **2.** $(10;20)$. **3.** 5625. 25-variant. **1.** $arctg3$. **2.** $(5;10)$. **3.** $90;-100$.

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1-variant. **1.** $\frac{1}{3}$. **2.** $-\frac{1}{3}$. **4.** $x=-4, y=1$ *da* $z_{min}=-1$. 2-variant. **1.** -2 **2.** 1. **4.** $x=y=4$ *da* $z_{max}=12$. 3-variant. **1.** 2 **2.** ∞. **4.** $x=1,\ y=-\frac{1}{2}$ *da* $z_{min}=0$. 4-variant. **1.** -1 **2.** 0. **4.** Ekstremum yo'q. 5-variant. **1.** 0 **2.** 0,2. **4.** $x=-2,\ y=0$ *da* $z_{min}=-\frac{2}{e}$. 6-variant. **1.** ∞. **2.** 1. **4.** $x=0, y=3$ *da* $z_{min}=9$. 7-variant. **1.** 1. **2.** -1. **4.** $x=y=2$ *da* $z_{min}=0$. 8-variant. **1.** 1. **2.** 0. **4.** $x=0, y=0$ *da* $z_{min}=0$. 9-variant. **1.** 1 **2.** 1. **4.** $x=2, y=4$ *da* $z_{min}=0$. 10-variant. **1.** 0. **2.** $-\frac{2}{3}$. **4.** $(0,0)-\min$, $(0,\pm1)$ *nuqtada ekstremum yo'q*, $(\pm1;0)-\max$. 11-variant. **1.** $e^{-2/\pi}$. **2.** 0. **4.** $x=\frac{1}{3},\ y=\frac{4}{3}$ *da* $z_{min}=\frac{10}{3}$. 12-variant. **1.** $\sqrt[6]{e}$. **2.** 2. **4.** $x=-5, y=-1$ *da* $z_{max}=1$. 13-variant. **1.** $\frac{1}{\sqrt[3]{e}}$. **2.** 1. **4.** $x=1, y=-4$ *da* $z_{max}=-14$. 14-variant. **1.** 1. **2.** 1. **4.** Ekstremum yo'q. 15-variant. **1.** $1\frac{4}{9}$. **2.** 1. **4.** $\left(\frac{94}{23},\frac{109}{23}\right)-\min$. 16-variant. **1.** 1. **2.** 2. **4.** Ekstremum yo'q. 17-variant. **1.** 1. **2.** 0. **4.** $x=6, y=6$ *da* $z_{min}=-422$. 18-variant. **1.** 0. **2.** $\frac{3}{5}$. **4.** $x=1, y=1$ *da* $z_{min}=-82$. $x=-1, y=-1$ *da* $z_{max}=82$. 19-variant. **1.** 0. **2.** 0. **4.** $x=\pm\sqrt{2},\ y=\mp\sqrt{2}$ *da* $z_{min}=-8$. 20-variant. **1.** 0. **2.** 0. **4.** $x=6, y=4$ *da* $z_{max}=6912$. 21-variant. **1.** 1. **2.** 0 **4.** $x=5, y=6$ *da* $z_{min}=-86$. 22-variant. **1.** ∞. **2.** $\frac{1}{3}$. **4.** $(0,0)-\max$. $x=0, y=-5$ *da* $z_{max}=41$. 23-variant. **1.** 0,5. **2.** $\frac{1}{\sqrt{3}}$. **4.** $(2,-2)-\max$. 24-variant. **1.** 10. **2.** 1. **4.** $(-1,1)-\min$. 25-variant. **1.** 1. **2.** 1. **4.** $(-3,2)-\min$. $(-4,0);(-2,0)-$*ekstremum yo'q*.

VII BOB

1-topshiriq javoblari

1-variant. **1.** $-\frac{1}{2x^2}+C$. **2.** $-\frac{2}{\sqrt{x}}+C$. **3.** $\frac{(7x-1)^{24}}{168}+C$. **4.** $-\frac{1}{3}\cos\left(x^3+1\right)+C$. **5.** $\frac{x^2}{4}(2\ln x-1)+C$. **6.** $3\ln\left(x^2+4x+13\right)-\frac{19}{3}arctg\frac{x+2}{3}+C$. 2-variant. **1.** $\frac{2^x}{\ln 2}+C$. **2.** $\arcsin\frac{x}{\sqrt{5}}+C$. **3.** $\frac{1}{2}\ln\left(x^2+1\right)+C$. **4.** $2arctg\sqrt{e^x-1}+C$

5. $(2x+3)\sin x+2\cos x+C$. 6. $4\ln|x-3|+C$. 3-variant. 1. $9tgx-4ctgx-25x+C$. 2. $3x+\frac{3}{2}\ln\left|\frac{x-1}{x+1}\right|-4\arcsin x+C$. 3. $\frac{1}{6}\sqrt{(4x-5)^3}+C$. 4. $-\frac{1}{9(3x+2)^3}+C$.

5. $\frac{1}{5}xch5x-\frac{1}{25}sh5x+C$. 6. $-\frac{1}{(x-4)^4}+C$. 4-variant. 1. $\frac{x^{11}}{11}+C$. 2. $-\frac{1}{6x^6}+C$. 3. $\frac{1}{4}\sin^4 x+C$. 4. $\frac{1}{3}e^{x^3}+C$. 5. $-\frac{x}{2\sin^2 x}-\frac{1}{2}ctgx+C$. 6. $-\frac{11}{2(x+2)^2}+C$. 5-variant.

1. $\frac{4}{5}x^{\frac{5}{4}}+C$. 2. $\frac{1}{3}arctg\frac{x}{3}+C$. 3. $\frac{1}{6}\ln^6 x+C$. 4. $-\ln|\cos x+1|+C$. 5. $\frac{x^3}{9}(3\ln x-1)+C$. 6. $\frac{1}{2}arctg\frac{x+5}{2}+C$. 6-variant. 1. $\frac{\sqrt{2}}{2}\ln\left|\frac{\sqrt{2}x-1}{\sqrt{2}x+1}\right|+C$. 2. $\ln\left|x+\sqrt{x^2+3}\right|+C$. 3. $\frac{1}{3}\ln|x^3+1|+C$. 4. $e^{-x}(1+2x-x^2)+C$. 5. $\frac{1}{2}arctg^2x+C$. 6. $\frac{1}{2}\ln(x^2-2x+17)+\frac{7}{4}arctg\frac{x-1}{4}+C$. 7-variant. 1. $\frac{3\cdot 5^x}{\ln 5}-3\sqrt[3]{x^2}+7x+C$. 2. $\frac{2}{5}x^{\frac{5}{2}}-2x^{\frac{3}{2}}+10x^{\frac{1}{2}}+C$. 3. $\ln|x|-\cos\frac{1}{x}+C$. 4. $-5\sqrt{4-x^2}-\arcsin\frac{x}{2}+C$. 5. $e^x(x^3-3x^2+6x-6)+C$. 6. $2\ln(x^2+x+1)-2\sqrt{3}arctg\frac{2x+1}{\sqrt{3}}+C$. 8-variant. 1. $\frac{x^2}{2}+\ln|x|+\frac{6}{x}+C$. 2. $5\ln|x|-40\sqrt[4]{x}-\frac{3\sqrt{7}}{7}arctg\frac{x}{\sqrt{7}}+C$. 3. $4\sqrt{x^2-5}+3\ln\left|x+\sqrt{x^2-5}\right|+C$. 4. $e^{\sin^2 x}+C$. 5. $2\left(\sqrt{1+x}\arccos x-2\sqrt{1-x}\right)+C$. 6. $\frac{13}{32}\left(\frac{x-1}{x^2-2x+17}+\frac{1}{4}arctg\frac{x-1}{4}\right)-\frac{4}{x^2-2x+17}+C$. 9-variant. 1. $\frac{2}{7}x^3\cdot\sqrt{x}+\frac{2}{3}x\sqrt{x}+C$. 2. $3\arcsin\frac{x}{2}+x+C$. 3. $\frac{\sin x-2}{\cos x}+C$. 4. $\frac{\ln|x-2|+5\ln|x+2|}{2}+C$. 5. $2\left(\sqrt{x}-\sqrt{1-x}\arcsin\sqrt{x}\right)+C$. 6. $\frac{x}{4(x^2+1)^2}+\frac{3}{8}\left(\frac{x}{x^2+1}+arctgx\right)+C$. 10-variant. 1. $\frac{2}{13}x^6\cdot\sqrt{x}+\frac{8}{7}x^3\cdot\sqrt{x}+8\sqrt{x}+C$. 2. $-4\cos x+2x^4-11tgx+C$. 3. $-\frac{\sqrt{1-x^2}}{x}-\arcsin x+C$. 4. $2\ln(\sqrt{x}+1)+C$. 5. $\frac{1}{4}\left(\ln\left|\frac{x-1}{x+1}\right|-\frac{2x}{x^2-1}\right)+C$. 6. $\frac{1}{250}\left[\frac{5(x-2)}{x^2-4x+29}+arctg\frac{x-2}{5}\right]+C$. 11-variant. 1. $\frac{1}{3}\arcsin\frac{3x}{4}+C$. 2.

$\frac{1}{2}\sin 2x+C.$ 3. $\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\arcsin\frac{x}{3}+C.$ 4. $\left|\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right|+C.$ 5. $\frac{x}{2}(\sin\ln x+\cos\ln x)+C.$ 6. $-\frac{11x+36}{2(x^2+6x+10)}-\frac{11}{2}arctg(x+3)+C.$ 12-variant. 1. $\frac{(9x+2)^{18}}{162}+C.$ 2. $\frac{1}{8}\ln|8x-1|+C.$ 3. $\frac{2}{5}\sqrt{(2-x)^5}-\frac{4}{3}\sqrt{(2-x)^3}+C.$ 4. $2\sqrt{x}-8arctg\frac{\sqrt{x}}{4}+C.$ 5. $\frac{e^{3x}}{13}(2\sin 2x+3\cos 2x)+C.$ 6. $5\ln|x-3|+2\ln|x+2|+C.$

13-variant. 1. $-\frac{4^{3-5x}}{5\ln 4}+C.$ 2. $\frac{2}{9}\sqrt{(3x+4)^3}+C.$ 3. $\frac{1}{6}\sin(6x+1)+C.$ 4. $-\frac{3}{5\sqrt[3]{5x-2}}+C.$ 5. $x^2e^x-2xe^x+2e^x+C.$ 6. $\ln|x-1|-\frac{3}{x+1}+C.$ 14-variant. 1. $tgx-x+C.$ 2. $4x+21\ln|x-5|+C.$ 3. $\frac{2}{3}\sqrt{tg^3x}+C.$ 4. $\frac{1}{3}arctg\frac{e^x}{3}+C.$ 5. $x^3\sin x+3x^2\cos x-6x\sin x-6\cos x+C.$ 6. $\frac{x^3}{3}-\frac{x^2}{2}+\ln\left|\frac{x^2+x+1}{x}\right|+C.$ 15-variant. 1. $\frac{x}{2}-\frac{\sin 2x}{4}+C.$ 2. $x-arctgx+C.$ 3. $\frac{1}{3}\sqrt{x^6+7}+C.$ 4. $-\ln\arccos+C.$ 5. $\frac{x}{\ln 2}2^x-\frac{1}{\ln^2 2}2^x+C.$ 6. $\ln|(x-5)(x+2)|+C.$ 16-variant. 1. $\frac{x}{2}+\frac{\sin 2x}{4}+C.$ 2. $x-5\ln|x+3|+C.$ 3. $-\frac{1}{3(x^2+3x-1)^3}+C.$ 4. $-\frac{\cos^{12}2x}{24}+C.$ 5. $xtgx+\ln(\cos x)+C.$ 6. $\frac{7}{4}\ln|x-5|-\frac{3}{4}\ln|x-1|+C.$ 17-variant. 1. $x+\frac{3}{2}\ln\left|\frac{x-3}{x+3}\right|+C.$ 2. $-5ctgx-\cos x+C.$ 3. $\frac{2}{\ln 7}\cdot 7^{\sqrt{x}}+C.$ 4. $-e^{\frac{1}{x}}+C.$ 5. $2\sqrt{x}\ln x-4\sqrt{x}+C.$ 6. $-\frac{1}{x}-arctgx+C.$ 18-variant. 1. $-\frac{2}{3x\sqrt{x}}+C.$ 2. $\frac{1}{\sqrt{3}}arctg\frac{x}{\sqrt{3}}+C.$ 3. $\frac{1}{2}\ln^2 5x+C.$ 4. $\ln|\sin x|+C.$ 5. $\frac{1}{2}x^2arctgx+\frac{1}{2}arctgx-\frac{1}{2}x+C.$ 6. $\frac{x^3}{3}+\frac{x^2}{2}+4x+2\ln|x|+5\ln|x-2|-3\ln|x+2|+C.$ 19-variant. 1. $-\frac{1}{5^x\ln 5}+C.$ 2. $\arcsin\frac{x}{2}+C.$ 3. $\frac{3}{2}\sqrt[3]{(x^2+8)^4}+C.$ 4. $-\frac{1}{\sin x}+C.$ 5. $-2\sqrt{1-x}\arcsin\sqrt{x}+2\sqrt{x}+C.$

$6.\frac{1}{12}\ln|x-2|-\frac{1}{24}\ln\left(x^2+2x+4\right)-\frac{\sqrt{3}}{12}\operatorname{arctg}\frac{x+1}{\sqrt{3}}+C.$ 20-variant. **1.** $\ln\left|x+\sqrt{x^2-1}\right|+C.$ **2.** $\frac{1}{10}\ln\left|\frac{x-5}{x+5}\right|+C.$ **3.** $-\frac{1}{2}\ln|\cos 2x|+C.$ **4.** $\frac{1}{2}arctgx^2+C.$ **5.** $e^x\cos x+C.$ **6.** $\frac{1}{2}\ln\left(x^2+9\right)-\ln|x-1|+7\ln|x+2|-\frac{2}{3}arctg\frac{x}{3}+C.$ 21-variant. **1.** $\frac{x^3}{3}+4x-\frac{4}{x}+C.$ **2.** $\frac{1}{2}arctg2x+C.$ **3.** $-\frac{1}{3}e^{-x^3}+C.$ **4.** $\frac{1}{3}\ln\left|x^3+\sqrt{x^6-4}\right|+C.$ **5.** $\frac{1}{2}x\sqrt{x^2+4}+2\ln\left|x+\sqrt{x^2+4}\right|+C.$ **6.** $5\ln\left|x+\sqrt{2}\right|+C.$ 22-variant. **1.** $\frac{7^x}{\ln 7}-8\ln|x|+4\sin x+C.$ **2.** $\sqrt{3}tgx-\frac{3}{4}x\sqrt[3]{x}+\frac{2}{3x^3}+C.$ **3.** $-\frac{1}{8}\left(8\cos\frac{x}{3}-5\right)^3+C.$ **4.** $2\sqrt{x^3-x^2+7x-2}+C.$ **5.** $xarctg\sqrt{x-1}-\sqrt{x-1}+C.$ **6.** $-\frac{2}{\left(x-\frac{1}{2}\right)^2}+C.$ 23-variant. **1.** $\frac{4}{5}x\sqrt[4]{x}-2\frac{14}{23}x\cdot\sqrt[20]{x^3}+\frac{4}{3}\sqrt[4]{x^3}+C.$ **2.** $\frac{7}{9}\cdot x^{0,9}-\frac{1}{2^x\cdot 5\ln 2}+C.$ **3.** $\frac{1}{4}\left(\frac{(2x+1)^{37}}{37}-\frac{(2x+1)^{36}}{36}\right)+C.$ **4.** $\frac{2}{5}\sqrt{(x+4)^5}-4\sqrt{(x+4)^3}+C.$ **5.** $\frac{1}{2}e^{\arcsin x}\left(x+\sqrt{1-x^2}\right)+C.$ **6.** $-\frac{7}{5(x+3)^5}+C.$ 24-variant. **1.** $5chx-7shx+x+C.$ **2.** $\frac{2}{7}x^3\sqrt{x}+\frac{4}{3}x^3-\frac{2}{3}x\sqrt{x}-4x+C.$ **3.** $\sin\frac{1}{x^2}-\frac{2}{\sqrt{x^3}}+C.$ **4.** $7\sqrt{x^2+10}+2\ln\left|x+\sqrt{x^2+10}\right|+C.$ **5.** $\frac{1}{3}x^3\ln^2 x-\frac{2}{9}x^3\ln x+\frac{2}{27}x^3+C.$ **6.** $-\frac{1}{9(3x+2)^3}+C.$ 25-variant. **1.** $7\ln\left|x+\sqrt{x^2+\pi}\right|-x+C.$ **2.** $\frac{125}{2x^2}-\frac{50}{x\sqrt{x}}+\frac{15}{x}-\frac{1}{2\sqrt{x}}+C.$ **3.** $arctge^x+C.$ **4.** $\frac{1}{2}\ln\left(x^2+3\right)+\frac{8}{\sqrt{3}}arctg\frac{x}{\sqrt{3}}+C.$ **5.** $\frac{1}{2}arctgx+\frac{1}{2}\frac{x}{x^2+1}+C.$ **6.** $\frac{1}{2}arctg\frac{x-2}{2}+C.$

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1-variant. **1.** $\frac{20}{3}$. **2.** $e-2$. **3.** ∞ **4.** $\frac{\pi}{2}$. 2-variant. **1.** $7+2\ln 2$. **2.** $\pi-2$. **3.** 1. **4.** 4. 3-variant. **1.** $4-\pi$. **2.** $1-\frac{2}{e}$. **3.** ∞ **4.** Uzoqlashuvchi. 4-variant. **1.** $\sqrt{3}-\frac{1}{3}\pi$. **2.**

$\pi\sqrt{2}-4$. **3.** ∞ **4.** 0. 5-variant. **1.** π. **2.** 1. **3.** $\frac{\pi}{2}$. **4.** Uzoqlashuvchi. 6-variant. **1.** $\frac{1}{4}+\frac{1}{8}\pi$. **2.** $\frac{1}{2}e^{\pi}+\frac{1}{2}$. **3.** $\frac{\pi}{2}$. **4.** Uzoqlashuvchi. 7-variant. **1.** $\sqrt{3}+\frac{1}{2}\ln\left(2+\sqrt{3}\right)$. **2.** $\frac{1}{4}\pi-\frac{1}{2}\ln 2$. **3.** $\frac{1}{2}$. **4.** Uzoqlashuvchi. 8-variant. **1.** $2\ln 2-\ln 3$. **2.** 1. **3.** Uzoqlashuvchi. **4.** Uzoqlashuvchi. 9-variant. **1.** $2-\frac{1}{2}\pi$. **2.** $\frac{1}{6}$. **3.** 2. **4.** $-\frac{1}{2}\ln 2e$. 10-variant. **1.** $-\frac{2}{3}+\frac{1}{4}\pi$. **2.** $\frac{35}{8}\pi$. **3.** $\frac{3}{32}\pi^2$. **4.** Uzoqlashuvchi. 11-variant. **1.** $\frac{14}{45}$. **2.** $\ln 2+\frac{\sqrt{3}\pi}{6}$. **3.** $\frac{1}{24}$. **4.** $\frac{\pi}{2}-arctg\sqrt{7}$. 12-variant. **1.** $\frac{28}{3}$. **2.** $\frac{1}{25}$. **3.** Uzoqlashuvchi. **4.** 2π. 13-variant. **1.** $\frac{32}{25}$. **2.** $-\frac{7\pi^2}{72}+\frac{4\sqrt{3}\pi}{3}-\pi-2\ln 2$. **3.** $\frac{1}{4}$. **4.** Uzoqlashuvchi. 14-variant. **1.** $\frac{4+2\sqrt{2}}{5}$. **2.** $2e^2-2$. **3.** $\frac{\pi}{\sqrt{5}}$. **4.** $-\frac{1}{4}$. 15-variant. **1.** $\ln 2-\frac{5}{8}$. **2.** $\frac{\ln 3-\pi}{2}+\frac{2\sqrt{3}\pi}{9}$. **3.** 0,5. **4.** Uzoqlashuvchi. 16-variant. **1.** $\frac{81}{16}\pi$. **2.** $\frac{2\sqrt{5}+2}{3}$. **3.** Uzoqlashuvchi. **4.** -1. 17-variant. **1.** $2\ln\left(3\sqrt{2}-3\right)$. **2.** $\frac{\pi}{6}-\frac{\sqrt{3}}{8}$. **3.** Uzoqlashuvchi. **4.** Uzoqlashuvchi. 18-variant. **1.** $\ln\frac{4}{3}$. **2.** 2. **3.** 1. **4.** $\frac{1}{2}\pi$. 19-variant. **1.** $\frac{1}{6}$. **2.** $4e^3+2$. **3.** $\ln\left(2+\sqrt{5}\right)$. **4.** $\frac{14}{3}$. 20-variant. **1.** $\frac{4}{3}\ln\frac{9}{2}$. **2.** -1. **3.** $\frac{\pi}{2}$. **4.** 6. 21-variant. **1.** $3\ln 3$. **2.** $\pi-4+6\ln 2$. **3.** Uzoqlashuvchi. **4.** ∞ 22-variant. **1.** $\sqrt{3}-\frac{\pi}{3}$. **2.** $\frac{6e-16}{e}$. **3.** -1. **4.** 4. 23-variant. **1.** $32\frac{2}{3}$. **2.** $24\ln 2-16$. **3.** $\frac{\pi}{\sqrt{3}}$. **4.** ∞ 24-variant. **1.** $8\frac{1}{15}$. **2.** $\frac{\pi}{2}$. **3.** 4. **4.** $2\sqrt{2}$. 25-variant. **1.** $10\frac{2}{3}$. **2.** $\frac{\pi^2-8}{4}$. **3.** $\frac{1}{4}$. **4.** Uzoqlashuvchi.

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1-variant. **1.** $\frac{81}{2}$. **2.** $\sqrt{2}+\ln\left(1+\sqrt{2}\right)$. **3.** $\frac{8\pi}{15}$. **4.** 4,95. 2-variant. **1.** $\pi-1$. **2.** $1+0{,}5\ln 1{,}5$. **3.** $\frac{\pi\left(e^2-1\right)}{2}$. **4.** 18,48. 3-variant. **1.** $1-\frac{\pi}{4}$. **2.** 8. **3.** $\frac{108\pi}{5}$. **4.** 27,22. 4-variant. **1.** $\frac{6+\sqrt{2}}{2}$. **2.** $\frac{1}{2}\ln 3$ **3.** $\frac{\pi}{6}$. **4.** 31,4. 5-variant. **1.** 1. **2.**

$\frac{20}{9}\sqrt{\frac{5}{3}}$. 3. $0,8\pi$. 4. 64825 6-variant. 1. $\frac{16}{3}$. 2. $\frac{\sqrt{2}+\ln\left(1+\sqrt{2}\right)}{2}$ 3. $\frac{2\pi}{35}$. 4. $0,352$ 7-variant. 1. 36. 2. $\frac{1}{2}\ln 3$ 3. $\frac{1024\pi}{21}$. 4. $C=11,52;\ P=132\left(pul\,birligi\right)$ 8-variant. 1. $\frac{7}{6}$. 2. $\frac{\sqrt{5}}{2}-\frac{1}{4}\ln\left(\sqrt{5}-2\right)$. 3. $\frac{\pi^2}{2}$. 4. 40. 9-variant. 1. 8. 2. $1+\frac{1}{2}\ln\frac{6}{5}$. 3. 12π. 4. 42381. 10-variant. 1. $\frac{9}{2}$. 2. $\sqrt{2}+\frac{1}{2}\ln\frac{\sqrt{2}+1}{\sqrt{2}-1}$. 3. $\frac{128\pi}{15}$. 4. $11,392t$. 11-variant. 1. $\frac{32}{3}$. 2. $\frac{14}{3}$. 3. $\frac{112\pi}{5}$. 4. $2,529\cdot 10^6$. 12-variant. 1. $\frac{4}{3}$. 2. $\frac{a}{2}\left(e-e^{-1}\right)$. 3. $\frac{544\pi}{15}$. 4. $0,0235;\ 0,283$ 13-variant. 1. 1. 2. $8\left(\sqrt{2}+\ln\left(\sqrt{2}+1\right)\right)$. 3. $\pi(e-2)$. 4. $0,073;0,114$. 14-variant. 1. $\frac{1}{6}$. 2. $\frac{1}{4}\left(6\sqrt{37}-\ln\left(\sqrt{37}-6\right)\right)$. 3. $\frac{32\sqrt{2}\pi}{3}$. 4. $0,0037;0,45$. 15-variant. 1. $\ln 2$. 2. $\frac{3}{2}$. 3. $\frac{52\pi}{3}$. 4. $341,3$. 16-variant. 1. $2+\frac{\pi^3}{6}$. 2. $\sqrt{2}(e-1)$. 3. $\frac{397\pi}{30}$. 4. $112,8$. 17-variant. 1. $\frac{16\sqrt{2}}{15}$. 2. $\frac{13}{3}$. 3. $\frac{256\pi}{15}$. 4. $C=3250;P=17,5$. 18-variant. 1. $4,5$. 2. $\frac{\pi^2}{32}$. 3. $2\pi^2$. 4. $C=667;P=767$. 19-variant. 1. $4\ln 2-\frac{2}{3}$. 2. $8\sqrt{2}-1$. 3. 24π 4. 88,53 birlik. 20-variant. 1. $\frac{7}{6}$. 2. $\frac{1}{2}\ln 3$. 3. $\frac{64\pi}{3}$. 4. 42381 birlik. Ko‘rsatma. Avval 8 soatda ishlab chiqarilgan mahsulot hajmini topib, so‘ng 258 ga ko‘paytirish kerak. 21-variant. 1. $0,5$. 2. $\frac{1}{2}\left(\sqrt{5}+\frac{1}{2}\ln\left(2+\sqrt{5}\right)\right)$. 3. 40π 4. e^{p^2}. 22-variant. 1. $\frac{44}{15}$. 2. $\ln 7-\frac{3}{4}$. 3. 8π 4. $\exp\left(-e^{-p}(p+1)\right)$. 23-variant. 1. $\frac{1}{3}+\ln 3$. 2. $2\sqrt{3}+\ln\left(\sqrt{3}+2\right)$. 3. $\frac{\pi\left(e^2+1\right)}{2}$. 4. $\frac{10q}{1+q}$;9. 24-variant. 1. $\frac{3}{\ln 2}-\frac{4}{3}$. 2. $\frac{2}{27}\left(13\sqrt{13}-8\right)$. 3. 4π. 4. $\frac{100}{\pi}\left(q\,arctg\,q-\frac{1}{2}\ln\left(1+q^2\right)\right)+1000;5822$. 25-variant. 1. $0,5$. 2. 5π. 3. $\frac{112\sqrt{3}\pi}{5}$. 4. $0,8y+0,4\sqrt{y}+16;344$.

VIII BOB

1-topshiriq javoblari

1-variant. **1.** $(1-x)(1+y)=C$. **2.** $y=\frac{x}{x+1}$. **3.** $(y+2)^2=C(x+y-1),\ y=1-x$. **4.** $y=Ce^{-2x}+e^x$. 2-variant. **1.** $\sqrt{1-y^2}=\arcsin x+C$. **2.** $x^2-2y^2=2$. **3.** $3x+y+2\ln|x+y-1|=C$. **4.** $y=\frac{x^3+C}{x^2+1}$. 3-variant. **1.** $x^2+y^2=\ln Cx^2$. **2.** $1+y^2=\frac{2}{1-x^2}$. **3.** $(x^2+y^2)^3(x+y)^2=C$. **4.** $x=y^4+Cy^2,\ y=0$. 4-variant. **1.** $y+\ln|y|=\sin x-x\cos x+C$ **2.** $y=x^2-2$. **3.** $y=\frac{Cx^2}{2}-\frac{1}{2C},\ x=0$. **4.** $x=\frac{\ln y+C}{y},\ y=0$. 5-variant. **1.** $y=\ln(1+Ce^{-x})$. **2.** $y=x,\ y=2\pi-x$. **3.** $0{,}5\ln(x^2+y^2)+arctg\frac{y}{x}=C$. **4.** $y=\frac{2}{\ln x+Cx+1}$. 6-variant. **1.** $y=-\frac{2}{C+x^2}$. **2.** $y^2=(x+3)^3$. **3.** $y=xe^{(x+1}$. **4.** $y=\frac{1}{\sqrt{Ce^{2x^2}+1}},\ y=0$. 7-variant. **1.** $2\sqrt{y}+\ln|y|-2\sqrt{x}=C,\ y=0$. **2.** $2^x arctg\,2^y=1-\frac{\pi}{4}$. **3.** $y^3=3x^3\ln|Cx|$. **4.** $y=Ce^{-\sin x}+2\sin x-2$. 8-variant. **1.** $y=\log_3(C+3^x)$. **2.** $y=\ln x$. **3.** $y=Ce^{\frac{y}{x}}$. **4.** $y=x^3+\frac{C}{x}$.

9-variant. **1.** $y=-1+C(x+1)$. **2.** $2y+1=4\sin^2 x$. **3.** $y=xe^{Cx}$. **4.** $y^2(2x+C)=e^{x^2},\ y=0$. 10-variant. **1.** $x=C\cos t$. **2.** $y=\frac{e^x+8}{9}$. **3.** $y=\frac{C}{x}-\frac{x}{2}$. **4.** $y=\frac{\sqrt[3]{3x+C}}{x}$. 11-variant. **1.** $\frac{x^2}{2}-e^{-y}(y+1)=C$. **2.** $y=-2\cos x$. **3.** $y=x\ln\frac{C}{x}$. **4.** $x^2=\frac{y}{C-\cos y},\ y=0$. 12-variant. **1.** $x+y=\ln(C(x+1)(y+1)),\ y=-1$. **2.** $y=(x+1)^2$. **3.** $y=\frac{x^2}{C+x}$ **4.** $y=e^x\left(C+\ln x+\frac{x^2}{2}\right)$.

13-variant. **1.** $y=5+Ce^{-x}$. **2.** $y=(4x+2)^2$. **3.** $y=xtg(\ln Cx)$. **4.** $y=\sqrt{1-x^2}\,(2\arcsin x+C)$.

14-variant. **1.** $v=Ce^{2r^2}$. **2.** $y=1$ **3.** $xe^{\frac{x^2}{2y^2}}=C$. **4.** $y=(x+C)tg\frac{x}{2}$. 15-variant. **1.** $y=Ce^{\frac{2x+\sin 2x}{4}}$. **2.** $\frac{x+y}{1-xy}=-3$. **3.** $\ln\frac{x+y}{x}=Cx$. **4.** $y=\left(\frac{C+xtgx+\ln\cos x}{x}\right)^2,\ y=0$. 16-variant. **1.** $\ln\left|tg\frac{y}{4}\right|=C-2\sin\frac{x}{2}$. **2.** $e^{\frac{y}{x}}=\frac{x}{2-x}$. **3.** $x^3+y^3=Cxy$. **4.** $x=Cy^2+\ln y^2-y+1,\ y=0$. 17-variant. **1.** $(1+e^x)(1+e^y)=C$. **2.** $y=x\ln x$. **3.** $y=x\ln x$ **4.** $y=x(c+\sin x)$. 18-variant. **1.** $y\sin y-x\cos x+\cos y+\sin x=C$. **2.** $y=x\ln ey$ **3.** $y^2-x^2=Cy^3$. **4.** $y=Ce^{-2x}+\frac{2x^2+2x-1}{4}$. 19-variant. **1.** $ctg\frac{y-x}{2}=x+C$. **2.** $\ln 2\sqrt{x^2+y^2}=\frac{y}{x}arctg\frac{y}{x}$. **3.** $y=xe^{Cx}$. **4.** $y=\frac{C-\ln|x|}{x}$. 20-variant. **1.** $y=Ce^{\frac{x^2}{2}}-1$. **2.** $y=x-\frac{x}{\ln ex}$. **3.** $x\sin\frac{y}{x}=C$. **4.** $y=\sin x+C\cos x$. 21-variant. **1.** $\frac{\sqrt{(3+x^2)^3}}{2+y^2}=C$. **2.**

$y=-x$ **3.** $\sin\frac{y}{x}+\ln x=C$. **4.** $y=\left(x^2+C\right)e^{-x^2}$. 22-variant. **1.** $y=a+Ce^{\frac{1}{x}}$. **2.** $\ln\left(x^2+y^2\right)+arctg\frac{y}{x}=0$. **3.** $arctg\frac{y}{x}=\ln C\sqrt{x^2+y^2}$. **4.** $y=C\ln^2x-\ln x$. 23-variant. **1.** $y=C\sin x-a$ **2.** $y=cx$ **3.** $y=x\ln^2\frac{C}{x}$. **4.** $x=2\ln y-y+1+Cy^2$. 24-variant. **1.** $y=\frac{C}{\cos x}-1$. **2.** $y=\frac{1}{2}x^3$. **3.** $x^2-y^2=Cx$. **4.** $y=C\ln x+x^3$. 25-variant. **1.** $y=C\sin^2x-\frac{1}{2}$. **2.** $y=2\sin x-\cos x$ **3.** $\frac{t}{s-t}=\ln C(s-t)$. **4.** $y=Cx^2+x^4$.

2-topshiriq javoblari

1-variant. **1.** $y=C_1e^{\dots}+C_2e^{\dots}$ **2.** $y=(C_1+C_2x)e^{2x}+\frac{1}{8}(2x^2+4x+3)$. 2-variant. **1.** $y=C_1e^{\dots}+C_2e^{\dots}$ **2.** $y=C_1+C_2e^{-8x}+\frac{x^2}{2}-\frac{x}{8}$. 3-variant. **1.** $y=e^{2x}(C_1x+C_2)$. **2.** $y=C_1+C_2e^{\frac{x}{7}}-7x^2-98x$. 4-variant. **1.** $y=e^{4x}(C_1\cos3x+C_2\sin3x)$. **2.** $y=C_1e^{-x}+C_2e^{3x}+\frac{e^{4x}}{5}$. 5-variant. **1.** $y=e^x(C_1\cos x+C_2\sin x)$. **2.** $y=C_1e^{-\dots}+C_2e^{\dots}-4{,}5xe^{-\dots}$ 6-variant. **1.** $y=C_1+C_2e^{\dots}$ **2.** $y=(C_1+C_2x)e^{-2x}+4x^2e^{-2x}$. 7-variant. **1.** $y=C_1e^{\dots}+C_2e^{\dots}$ **2.** $y=C_1e^{\dots}+C_2e^{\dots}+0{,}1\sin x+0{,}3\cos x$ 8-variant. **1.** $y=e^{-x}(C_1\cos2x+C_2\sin2x)$. **2.** $y=C_1\cos x+C_2\sin x-2x\cos x$. 9-variant. **1.** $y=C_1e^{\dots}+C_2e^{\dots}$ **2.** $y=C_1e^x+C_2e^{-4x}-0{,}2xe^{-4x}-\left(\frac{x}{6}+\frac{e^{-x}}{36}\right)$. 10-variant. **1.** $y=C_1\cos x+C_2\sin x$. **2.** $y=C_1+C_2e^{5x}-0{,}2x^3-0{,}12x^2-0{,}048x+0{,}02(\cos5x-\sin5x)$. 11-variant. **1.** $y=C_1e^{\dots}+C_2e^{\dots}$ **2.** $y=e^{-x}(C_1\cos2x+C_2\sin2x)-0{,}5xe^{-x}\cos2x$. 12-variant. **1.** $y=C_1e^x+C_2e^{-\frac{7}{2}x}$. **2.** $y=C_1+C_2e^{-x}+\left(\frac{x^3}{3}-x^2+2x\right)+0{,}5e^x+\frac{1}{20}\sin2x-\frac{1}{10}\cos2x$. 13-variant. **1.** $y=C_1e^{(-2+\sqrt{7})x}+C_2e^{(-2-\sqrt{7})x}$. **2.** $y=e^{-x}(C_1x+C_2)+\frac{1}{4}e^x$. 14-variant. **1.** $y=C_1e^{-x}+C_2e^{\frac{2}{3}x}$. **2.** $y=C_1e^{\dots}+C_2e^{-\dots}+2$ 15-variant. **1.** $y=C_1+C_2e^{\dots}$ **2.** $y=C_1\cos\sqrt{3}x+C_2\sin\sqrt{3}x+3x$. 16-variant. **1.** $y=C_1+C_2e^{\frac{9}{4}x}$. **2.** $y=C_1e^{2x}+C_2e^{3x}+x-\frac{5}{6}$. 17-variant. **1.** $y=(C_1+C_2x)e^{3x}$. **2.** $y=C_1e^x+C_2e^{-2x}+\frac{1}{2}e^{2x}$. 18-variant. **1.** $y=(C_1+C_2x)e^{2x}$. **2.** $y=C_1e^{\dots}+C_2e^{\dots}-xe^{\dots}$ 19-variant. **1.** $y=(C_1+C_2x)e^{\frac{3}{2}x}$. **2.** $C_1e^x+C_2e^{-4x}+\left(\frac{1}{10}x^2+\frac{4}{25}x\right)e^x$. 20-variant. **1.** $y=(C_1+C_2x)e^{-\frac{2}{3}x}$. **2.** $(C_1+C_2x)e^x+\left(\frac{1}{6}x^3+\frac{1}{2}x^2\right)e^x$. 21-variant. **1.** $y=C_1\cos2x+C_2\sin2x$. **2.** $\left(C_1+C_2x+\frac{1}{6}x^3+\frac{3}{2}x^2\right)e^x$. 22-variant. **1.** $y=C_1\cos\frac{3}{2}x+C_2\sin\frac{3}{2}x$. **2.** $C_1e^x+C_2e^{-5x}-\frac{1}{5}$. 23-variant. **1.**

$y=\left(C_1\cos\frac{\sqrt{3}}{2}x+C_2\sin\frac{\sqrt{3}}{2}x\right)e^{-\frac{x}{2}}$ **2.** $C_1\cos x+C_2\sin x+\sin x\ln\left|tg\frac{x}{2}\right|$. 24-variant. **1.** $y=\left(C_1\cos\frac{5}{2}x+C_2\sin\frac{5}{2}x\right)e^{\frac{x}{2}}$. **2.** $C_1e^{x}+C_2e^{2x}+\frac{5}{3}e^{-x}$. 25-variant. **1.** $y=\left(C_1\cos\frac{\sqrt{31}}{4}x+C_2\sin\frac{\sqrt{31}}{4}x\right)e^{\frac{1}{4}x}$. **2.** $(C_1+C_2x)e^{3x}+\frac{2}{9}x^2+\frac{5}{27}x+\frac{11}{27}$.

3-topshiriq javoblari

1-variant. **1.** Yaqinlashuvchi; $\frac{1}{3}$. **2.** Yaqinlashuvchi; $\frac{1}{8}$. **3.** Uzoqlashuvchi; $\frac{1}{3n}$. **5.** Absolyut yaqinlashuvchi; 2-taqqoslash alomati; $\frac{1}{e^{n+1}}$. 2-variant. **1.** Uzoqlashuvchi; e. **2.** Uzoqlashuvchi; integral alomati; $\ln\ln(x+1);+\infty$. **3.** Yaqinlashuvchi; $\frac{3}{n^2}$. **4.** Uzoqlashuvchi; zaruriy alomat 1. 3-variant. **1.** Uzoqlashuvchi; 2 **2.** Uzoqlashuvchi; Koshi alomati; e. **3.** Uzoqlashuvchi; $\frac{2}{n}$. **4.** Uzoqlashuvchi; Dalamber alomati $+\infty$. 4-variant. **1.** Yaqinlashuvchi; 0. **2.** Yaqinlashuvchi; Koshi alomati; $\frac{1}{2}$. **3.** Uzoqlashuvchi; $\frac{1}{n}$. **4.** Shartli yaqinlashuvchi; 2-taqqoslash alomati; $\frac{1}{n}$. 5-variant. **1.**Yaqinlashuvchi; $\frac{1}{4}$. **2.** Yaqinlashuvchi; Koshi alomati; $\sqrt{\frac{2}{3}}$. **3.** Yaqinlashuvchi; $\frac{1}{n^{\frac{3}{2}}}$. **4.** Absolyut yaqinlashuvchi; Koshi alomati, $\ln 2$. 6-variant. **1.** Uzoqlashuvchi; $\frac{e}{2}$. **2.** Uzoqlashuvchi; integral alomati; $\frac{1}{2}\ln\ln(2x+1);+\infty$ **3.** Uzoqlashuvchi; $\frac{1}{n^{\frac{1}{3}}}$. **4.** Absolyut yaqinlashuvchi; 1-taqqoslash alomati; $\frac{1}{n^2}$. 7-variant. **1.** Uzoqlashuvchi; $\frac{3}{2}$ **2.** Yaqinlashuvchi; Koshi alomati; 0. **3.** Yaqinlashuvchi; $\frac{1}{n^3}$. **4.** Shartli yaqinlashuvchi; integral alomati, $2\sqrt{\ln n};+\infty$ 8-variant. **1.** Yaqinlashuvchi; $\frac{2}{5}$. **2.** Yaqinlashuvchi; integral alomati; $-\frac{1}{\ln\ln x};\frac{1}{\ln\ln 2}$. **3.** Uzoqlashuvchi; $\frac{1}{n}$. **4.** Uzoqlashuvchi; zaruriy alomat, $\ln 2$. 9-variant. **1.** Yaqinlashuvchi; $\frac{1}{3}$. **2.** Yaqinlashuvchi; Koshi alomati; $\frac{1}{e}$. **3.** Yaqinlashuvchi; $\frac{1}{n^2}$. **4.** Absolyut yaqinlashuvchi; Dalamber alomati, $\frac{1}{2}$. 10-

variant. **1.** Uzoqlashuvchi; $\frac{3}{\sqrt{5}}$. **2.** Yaqinlashuvchi; Koshi alomati; $\frac{8}{125}$. **3.** Uzoqlashuvchi; $\frac{n}{3}$. **4.** Uzoqlashuvchi; zaruriy alomat, $\frac{1}{2}$. 11-variant. **1.** Uzoqlashuvchi; 3 **2.** Uzoqlashuvchi; Koshi alomati; $\frac{3}{2}$. **3.** Uzoqlashuvchi; $\frac{1}{n}$. **4.** Absolyut yaqinlashuvchi; Koshi alomati, $\frac{\sqrt{5}}{3}$. 12-variant. **1.** Yaqinlashuvchi; $\frac{1}{\sqrt{5}}$. **2.** Uzoqlashuvchi; integral alomati; $\frac{1}{3}\ln\ln(3x-1)$; $+\infty$. **3.** Uzoqlashuvchi; $\frac{1}{n^{\frac{1}{2}}}$. **4.** Absolyut yaqinlashuvchi; Dalamber alomati, $\frac{1}{\sqrt{5}}$. 13-variant. **1.** Uzoqlashuvchi; $\frac{3}{2}$. **2.** Yaqinlashuvchi; Koshi alomati; $\frac{1}{3}$. **3.** Yaqinlashuvchi; $\frac{1}{n^{\frac{7}{6}}}$. **4.** Shartli yaqinlashuvchi; 2-taqqoslash alomati, $\frac{1}{\sqrt{n}}$. 14-variant. **1.** Yaqinlashuvchi; 0 **2.** Yaqinlashuvchi; Koshi alomati; $\frac{1}{9}$. **3.** Yaqinlashuvchi; $\frac{1}{n^2}$. **4.** Absolyut yaqinlashuvchi; Koshi alomati, $\frac{2}{3}$. 15-variant. **1.** Uzoqlashuvchi; $+\infty$ **2.** Yaqinlashuvchi; Koshi alomati; 0. **3.** Uzoqlashuvchi; $\frac{1}{n}$. **4.** Uzoqlashuvchi; Koshi alomati; $\frac{\sqrt{10}}{2}$. 16-variant. **1.** Uzoqlashuvchi; $\frac{3}{e}$. **2.** Yaqinlashuvchi; Koshi alomati; $\frac{1}{e}$. **3.** Yaqinlashuvchi; $\frac{1}{n^{\frac{3}{2}}}$. **4.** Absolyut yaqinlashuvchi; 2-taqqoslash alomati, $\frac{1}{n^2}$.

17-variant. **1.** Uzoqlashuvchi; $+\infty$. **2.** Yaqinlashuvchi; Koshi alomati; $\frac{2}{3}$. **3.** Uzoqlashuvchi; $\frac{8}{n}$. **4.** Uzoqlashuvchi; zaruriy alomat, $\frac{1}{2}$. 18-variant. **1.** Yaqinlashuvchi; $\frac{3}{4}$. **2.** Yaqinlashuvchi; Koshi alomati; $\frac{e}{3}$. **3.** Yaqinlashuvchi; $\left(\frac{2}{5}\right)^n$. **4.** Absolyut yaqinlashuvchi; Dalamber alomati, $\frac{1}{3}$.

19-variant. **1.** Yaqinlashuvchi; 0 **2.** Yaqinlashuvchi; Koshi alomati; 0 **3.** Uzoqlashuvchi; $\frac{1}{n}$. **4.** Absolyut yaqinlashuvchi; 2-taqqoslash alomati $\frac{1}{3n\sqrt{n}}$. 20-

variant. 1. Uzoqlashuvchi; $\frac{27}{8}$. 2. Uzoqlashuvchi; Koshi alomati; $\frac{5}{3}$. 3. Uzoqlashuvchi; $\frac{2}{3}$. 4. Uzoqlashuvchi; Koshi alomati; $+\infty$. 21-variant. 1. Uzoqlashuvchi; $\frac{4}{3}$. 2. Uzoqlashuvchi; integral alomati; $2\sqrt{\ln x}; +\infty$.

3. Yaqinlashuvchi; $\frac{1}{n^2}$. 4. Shartli yaqinlashuvchi; 2-taqqoslash alomati, $\frac{3}{n}$. 22-variant. 1. Uzoqlashuvchi; ∞ 2. Yaqinlashuvchi; Koshi alomati; $\frac{1}{e}$.

3. Yaqinlashuvchi; $\frac{1}{n^2}$. 4. Absolyut yaqinlashuvchi; 1-taqqoslash alomati, $\frac{1}{3^n}$.

23-variant. 1. Yaqinlashuvchi; e^{-1}. 2. Yaqinlashuvchi; integral alomati; $-\frac{1}{\ln(x+1)}; \frac{1}{\ln 2}$. 3. Uzoqlashuvchi; $\frac{3}{2\sqrt{n}}$. 4. Absolyut yaqinlashuvchi; integral alomati, $-\frac{(2+\ln n)^{-2}}{2}; \frac{1}{8}$. 24-variant. 1. Yaqinlashuvchi; $\frac{2}{5}$. 2. Uzoqlashuvchi; integral alomati; $\frac{1}{2}\ln^2 x; +\infty$ 3. Uzoqlashuvchi; $\frac{1}{n}$. 4. Uzoqlashuvchi; zaruriy alomat, $+\infty$. 25-variant. 1. Yaqinlashuvchi; $\frac{1}{3}$. 2. Yaqinlashuvchi; Koshi alomati; $\frac{1}{e^2}$. 3. Uzoqlashuvchi; $\frac{2}{n}$. 4. Shartli yaqinlashuvchi; 1-taqqoslash alomati, $\frac{1}{n}$.

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Nashr. lits. AI № 305. 22.06.2017.
Bosishga ruxsat 19.02.2019-yilda berildi. Bichimi $60x84/_{16}$.
Ofset qog'ozi. «New Times Roman» garniturasi.
Shartli b.t. 23,2. Nashr hisob t. 23,6
Adadi 100 dona. 7-buyurtma.

"IQTISOD-MOLIYA" nashriyoti.
100000, Toshkent, Amir Temur 60«A».

"DAVR MATBUOT SAVDO" MChJ
bosmaxonasida chop etildi.
100198, Toshkent, Qo'yliq 4-mavze, 46.