MACROECONOMICS

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Week 8

Intertemporal model

RBC I

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Two period model

- All decisions discussed so far (consumption-saving, consumption-leissure) are to be combined into a model in what behavior of participants is consistent and depends on each other
- One additional decision: firm’s investment
- Lots of variables, we ought to simplify

Representative consumer

- Chooses consumption and leisure (labor supply) optimally in both time periods. Intertemporally optimizes taking decision on saving-consumption taking the interest rate into consideration
- Given h, T, T', w, w', π, π' and r for her, she decides about optimal values of c c', N^s, $N^{s}{}'$.

Calculus

$$U = \ln C + \ln(h - N^{s}) + \ln C' + \ln(h - N^{s}{}') $$

$$C + \frac{C'}{(1 + \mathrm{r})} = wN^{S} + \Pi - T + \frac{1}{(1 + \mathrm{r})}(w'N^{S'} + \Pi' - T')$$

Lagrange:

$$L = 1\mathrm{n}\ C + 1\mathrm{n}(h - N^{3}) + 1\mathrm{n}C' + 1\mathrm{n}(h - N^{3'}) +$$

$$\lambda\left[wN^{s} + \pi - T + \frac{1}{(1+r)}(w'N^{s'} + \pi' - T') - \left(C + \frac{C'}{(1+r)}\right)\right]$$

Solution

$$\frac{1}{C} = \lambda \ , \quad \frac{1}{C'} = \frac{\lambda}{(1+r)} \quad \rightarrow \quad C = \frac{C'}{(1+r)} \qquad 1.$$

2. $\dfrac{C}{h - N^s} = w$ 3. $\dfrac{C'}{h - N^{s'}} = w'$ $\longrightarrow$ $\dfrac{h - N^{S'}}{h - N^S} = \dfrac{w(1+r)}{w'}$

4. $$C + \frac{C'}{(1+r)} = wN^s + \pi - T + \frac{1}{1+r}(w'N^{s'} + \pi' - T')$$

Four equations to be solved for four variables C,C' N^s, N^{s}', to be expressed as functions of w, w', π, π' T, T and the exogenous parameter h

Consumer

- Solution of the consumer's problem results in four functions (behavioral equations)
- $N^s(w, r, w', \pi, \pi', T, T')$
- N^{s}'(……………………….)
- C(……………………….)
- C'(……………………..)
- Consumer makes decisions reacting to the variables listed according to these functions

Producer

- The producer uses a standard neoclassical technology
- $Y = zF(K,N^d)$, z and K are given parameters
- $Y' = z'F(K',N^{d\prime})$, z' is given
- Given w, w' and r, the producer decides about labor supply in both periods and on investments (K')
- Production and provit in both periods are results of the decisions described above

Investment

- $I = K' - (1 - d)K$
- Profit in current period:

  $\Pi = Y - wN^d - I$
- Investment is carried out by the firm via witholding fininancing from the profit. Profit in the second period:

  $\Pi' = Y' - w'N^{d'} + (1 - d)K'$
- The second period is the last. Left over capital in this period is transferred to the consumer as profit

Profitmaximization

- The firm maximises the present value of the profit in the two periods

$$\pi + \frac{\pi'}{(1+r)} = zF(K,N^d) - wN^d - K' + (1-d)K + \frac{1}{1+r}\left[z'F(K',N^{d'}) - w'N^{d'} + (1-d)K'\right]$$

The firm determines the level of capital in the second period. To invest, it witholds profit from the current period. The decision is optimal at that level of K’ where the derivative of the intertemporal profit function with respect to K’ is zero.

Demand for capital (solution)

$$-1 + \frac{1}{(1+r)}[MPK' + (1-d)] = 0$$

MPK' – d = r

Marginal product of capital is a negative function of K'. The higher r is, the lower the optimal capital level is going to be. Az **I = K' – (1 – d)K,** given K, investment depends on K' only. This defines an investment function which depends on r, K' and z'.

1. $\mathrm{I} = \mathrm{I}(\mathrm{r}, \mathrm{K}, \mathrm{z}'), \quad \frac{\partial \mathrm{I}}{\partial \mathrm{r}} < 0$

Demand for labor

- Taking the derivative of the profit function with respect to current and future labor gives us labor demand in the two periods
- $MPL(N^d, K, z) = w$
- $MPL'(N^{d'}, K', z') = w'$
- These three equations would determine three unknows: investment and labor demand in the two periods

Government

- The government observes her intertemporal budget constraint. If time path for G is given, this determines the present value of taxes

$$G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$$

Equilibrium

- On the labor market $N^d = N^s = N$ for both periods (two equations)
- Labor market equilibrium determines the level of output and the profit
- Goods market, GDP identity: $Y = C + I + G$
- If it holds for the current period, then equilibrium in the future period is given by the consumer's budget constraint

Equilibrium

- Together with the government budget constraint this gives four additional equations to be solved for four variables: wages in the current and future periods, r, and the present value of taxes
- Given those, we can determine all the ramining variables (quantities), like investment, capital consumption etc.

Solution

- The problem is set in a form of system of equations. Due to the complicated functions in them, tere is no way of solving it manually – see the one period version – however a solution exists
- Due to the high number of variables, parameters etc. graphical representation is also quite difficult. To be able to draw diagrams, we have to simplify a lot

Simplifications for drawing diagrams

- We concentrate on the present period only, take the future as exogenous
- Take two of the markets, goods and labor, only
- Current period labor market will include the consumer's labor supply and the firm's labor demand
- Goods market will show the firm's supply and consumer demand, firm's investment demand as well as the government demand for the output

Supply of labor

- Consumer’s labor supply in the present

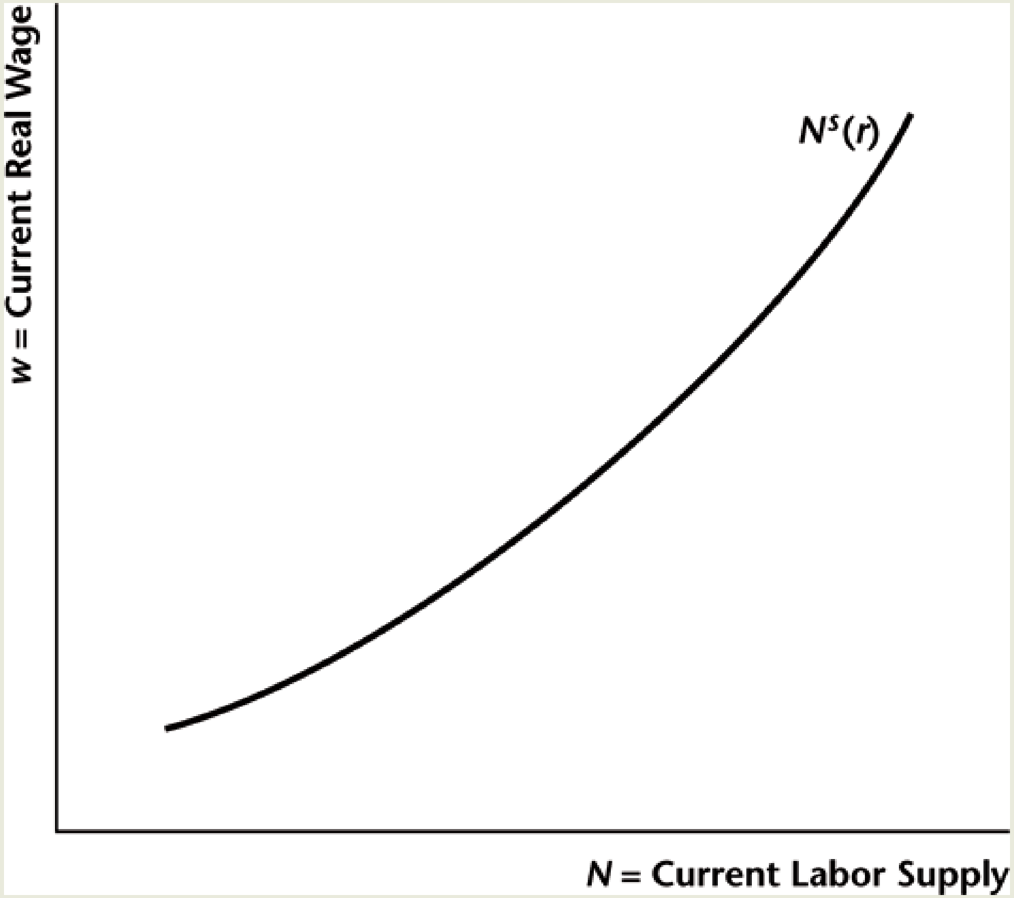
  $N^s(w, r, w', \pi, \pi', T, T')$

- We concntrate on w and r, the rest is pulled together into the present value of income

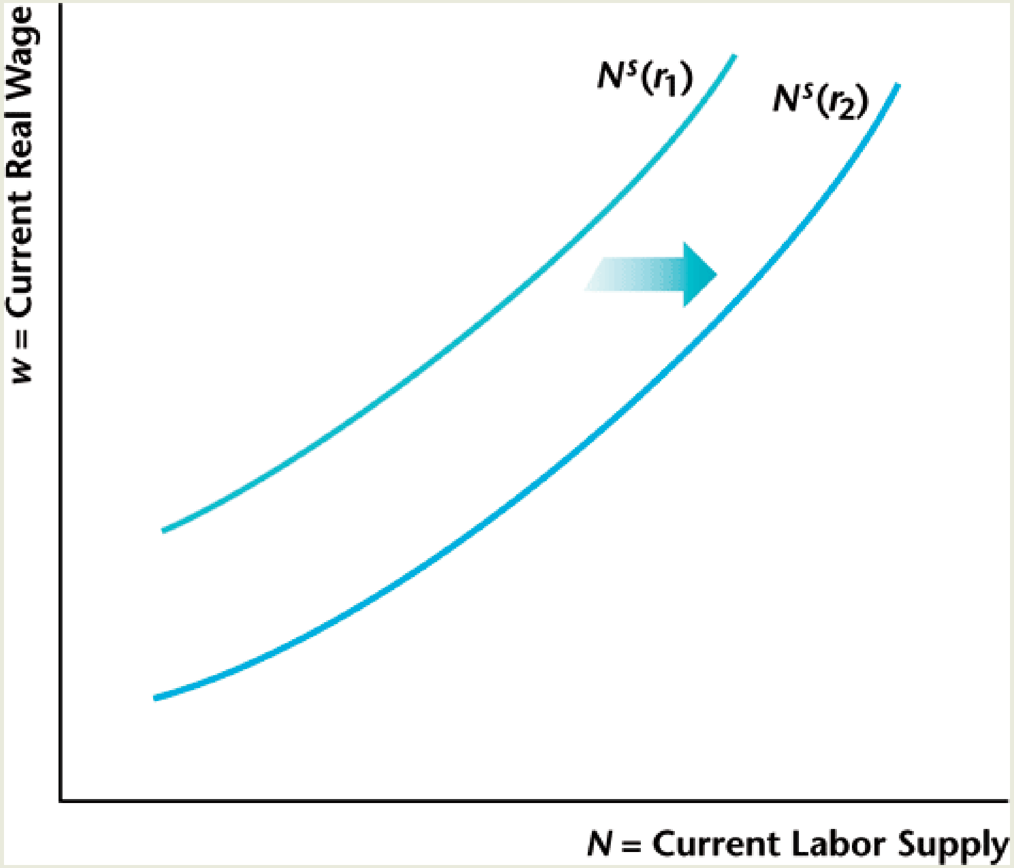
  $N^s(w, r, wc\{ .....T, T'\})$

- Increase in w and r increases labor supply through intertemporal substitution, an increase in the present value of income decreases labor supply through the increase of demand for leisure

Supply of labor

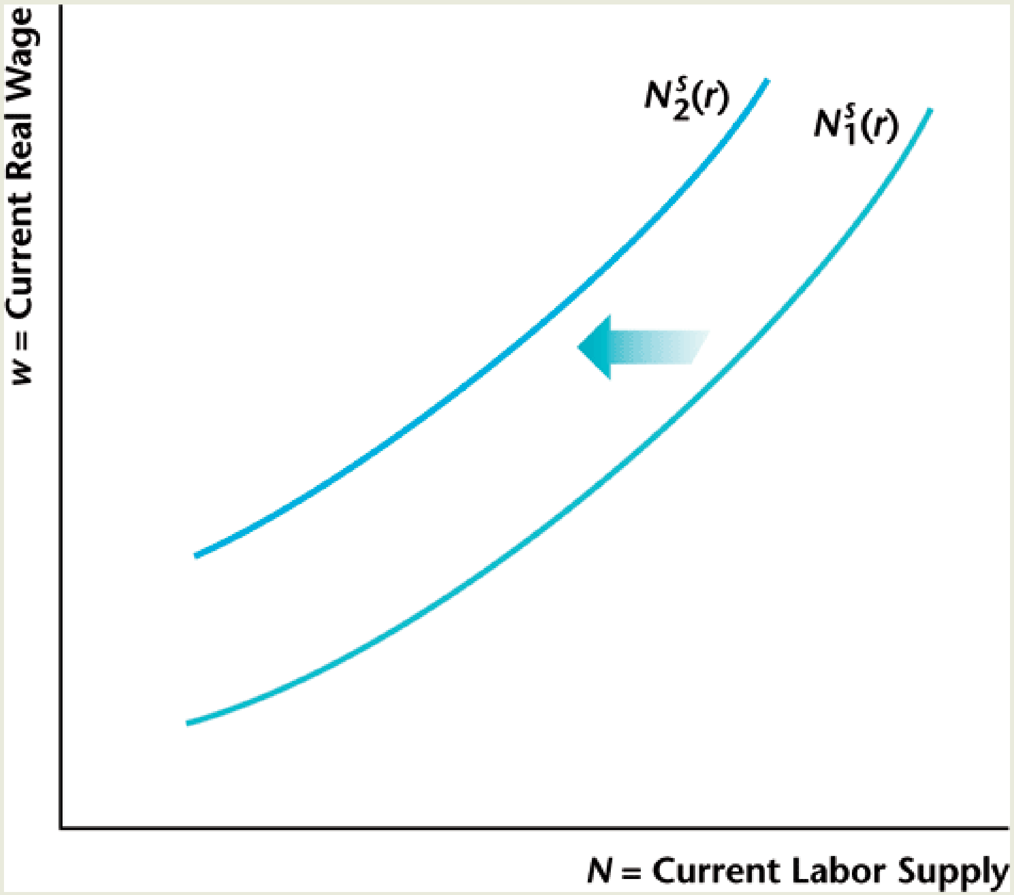


Labor supply



An increase in the rate of interest shifts the labor supply curve out due to intertemporal substitution. People prefer to work more now, and enjoy leissure later

Labor supply



A decrease in taxes (an exogenous increase in life time income) shifts the curve backwards. People want to buy more leissure now, they offer to work less

Consumer demand

$C(w, r, w', \pi, \pi', T, T')$

- For drawing we ignore the distribution of consumer’s income into wage and other, so that we can show consumer demand as a function of income and the interest rate

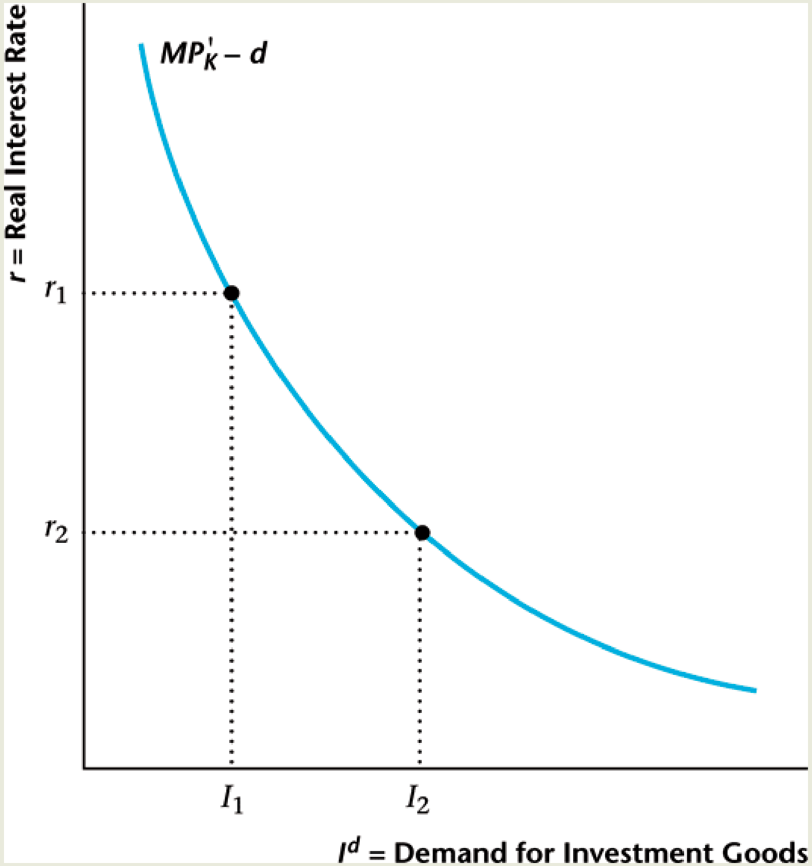
$C(Y, r, Y', T, T')$

- An increase in income reulrs in an increase of C. The effect is dampened, consumers smooth consumption

Consumer demand

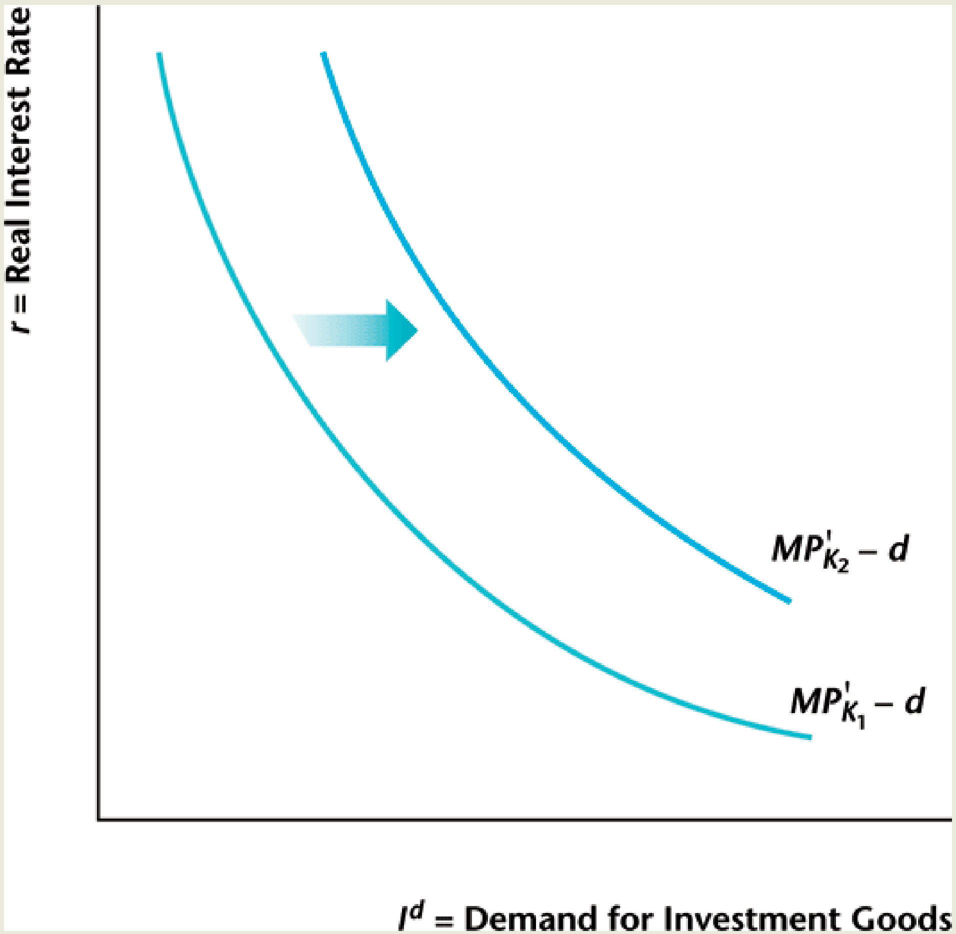
- An increase in r induces an intertemporal substitution. Consumers prefer to postpone consumption to the second period, because it is cheaper
- An increase in future income results in increasing consumption in the present
- Changes in taxes influence consumption throgh changes in disposable income

The firm's investment function



Investment is a function of the rate of interest

Shifts in the investment function

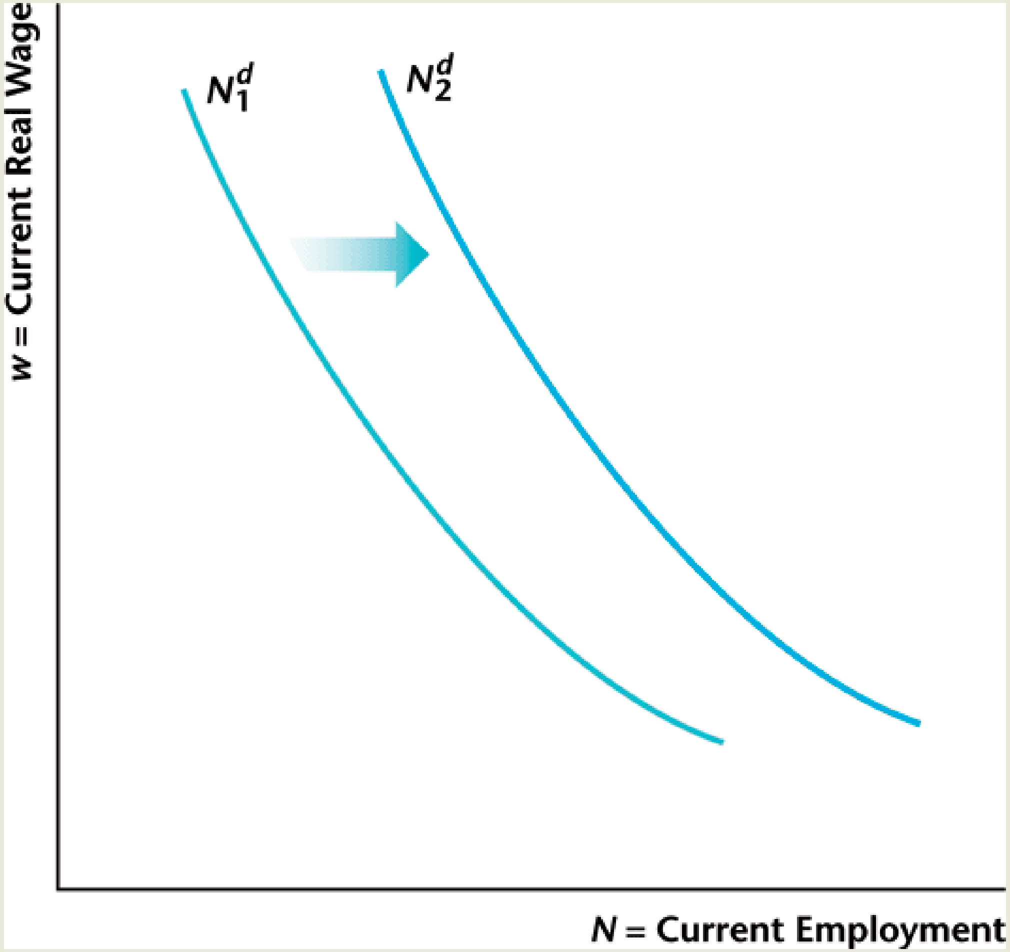


An expected increase in TFP in the future or a decrease in K in the present shift the investment funbction outward. In both cases a larger capitel in the future (more investment) is needed to make the marginal product of capital equal to the rate of interest

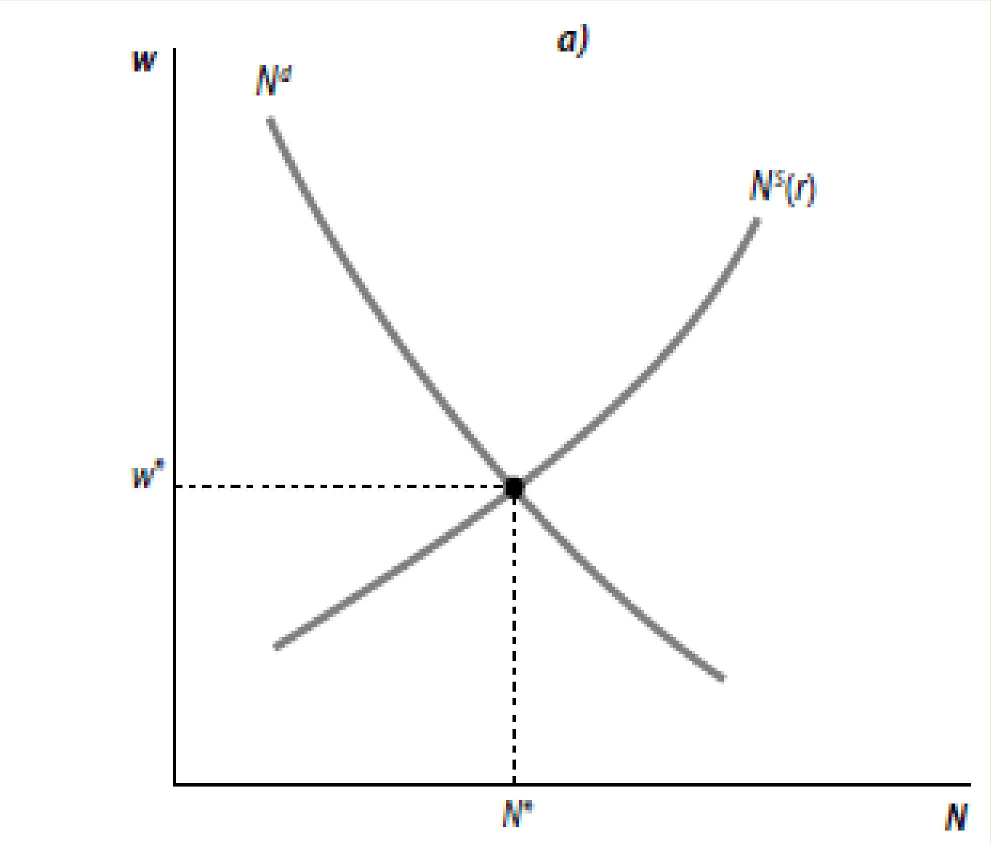
Demand for labor

- $N^d(w, z, K)$
- An increase in TFP or an increase in K both increase the marginal product of labor. Therefore, both events would shift the labor demand curve outward, representing higher demand for labor

Demand for labor



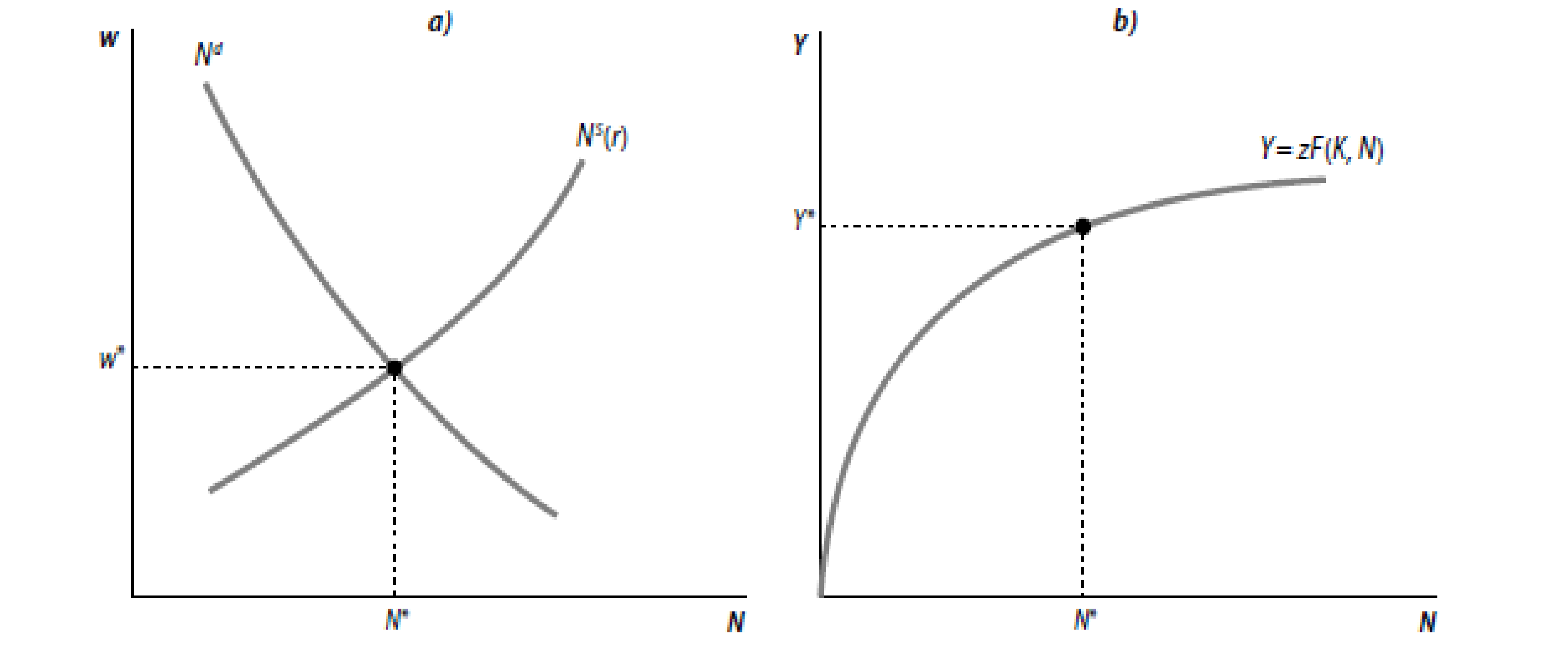
The labor market



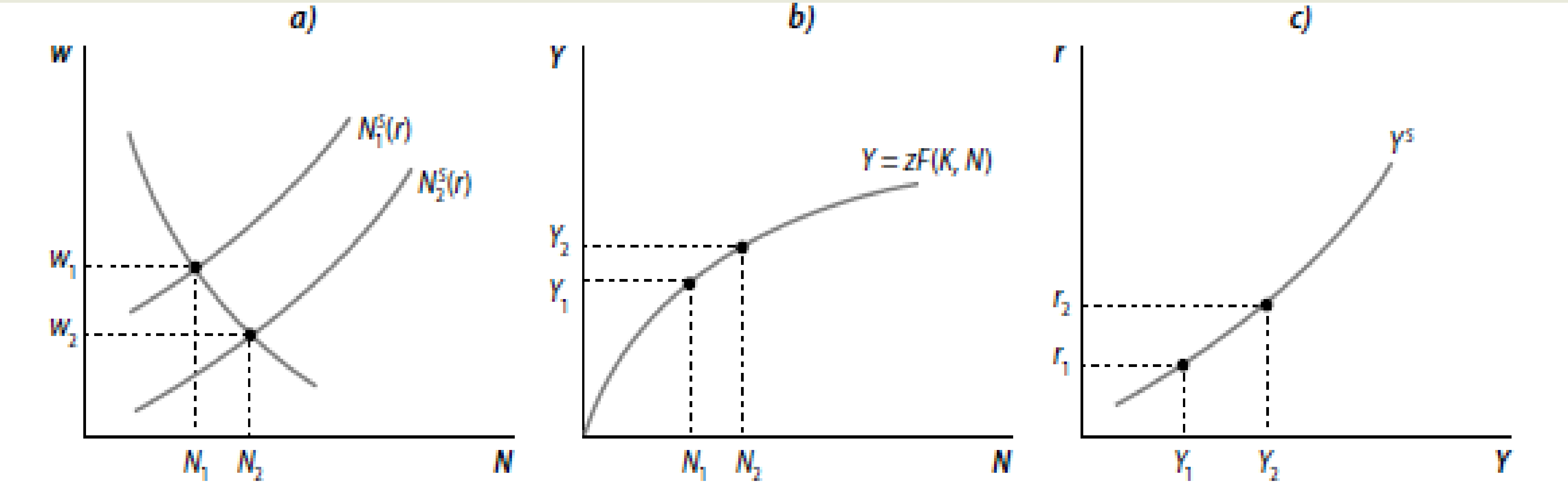
Current output supply

- Equilibrium on the labor market determines employment
- In the present period capital is given, output deppends on employment only
- Equilibrium on the labor market depends on the rate of interest (among other factors). The supply of output as a function of r is determined by labor market equilibrium

Determination of output suply



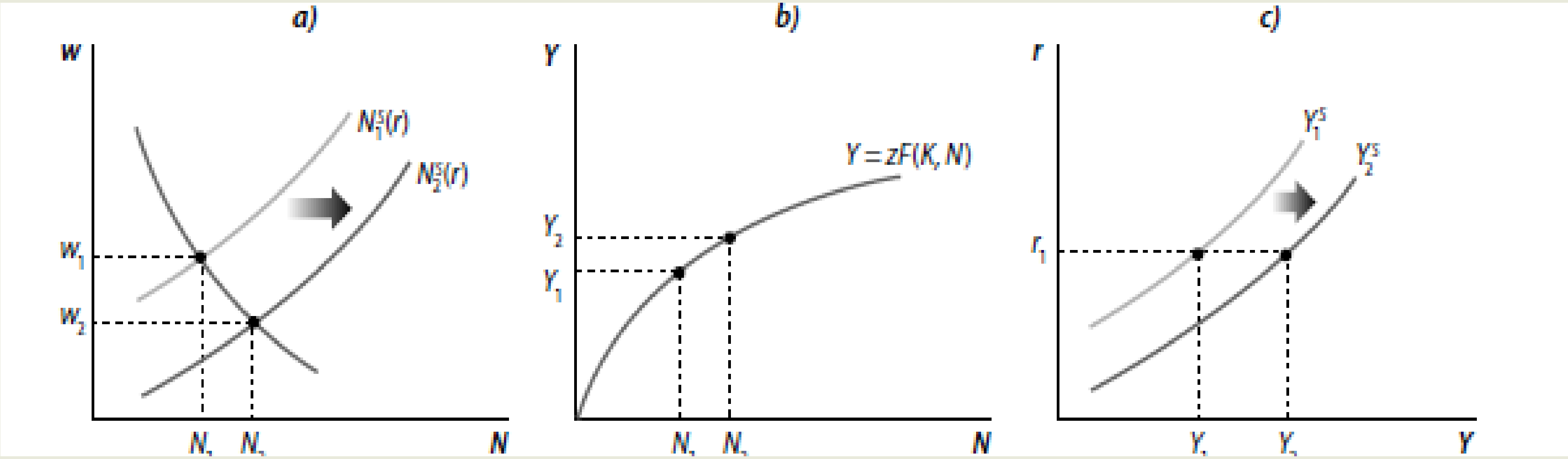
The output supply curve



Shifts in the output supply curve

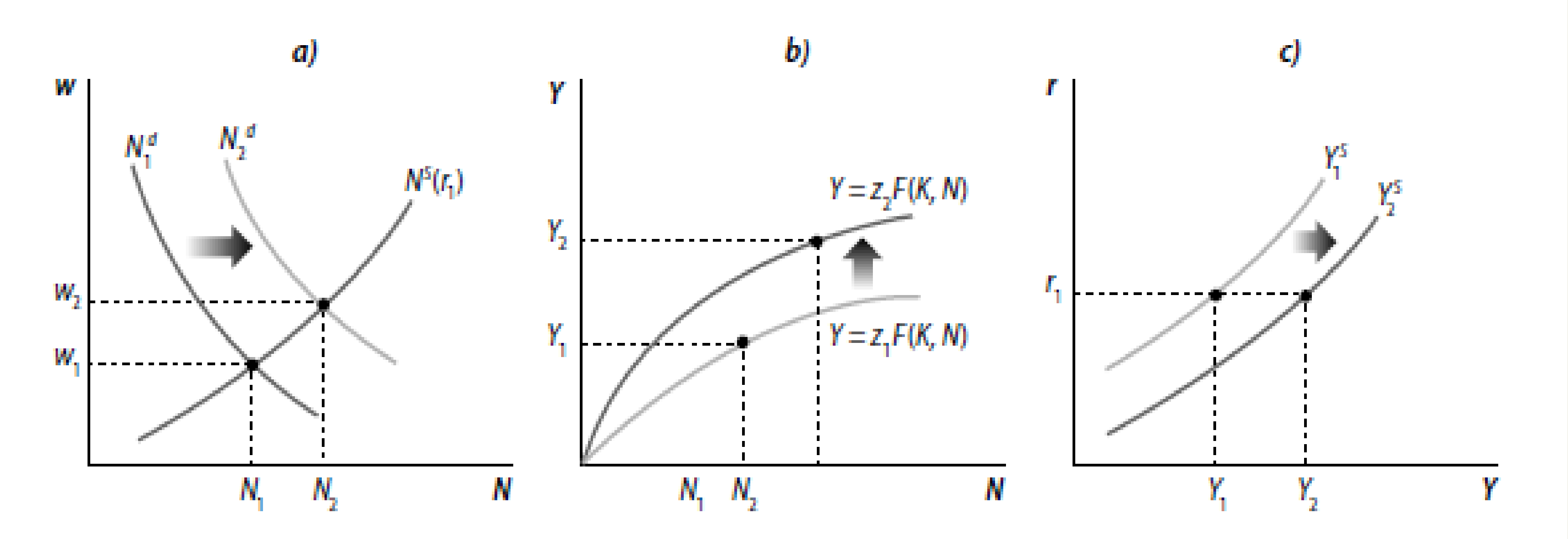
- The output supply curve is shifted by any factor that causes a shift in either the labor supply or the labor demand curve (apart from the rate of interest, as the latter is already included into the supply curve
- The most imprtant factors are TFP, and factors influencing life time income (taxes etc.)

Shifts in the output supply curve



An increase in government expenditures can be carried out only with a parallel increase in taxes, either now or in the future. An increase of taxes reduces the life time income of the consumer, therefore results in an increase of the labor suply. The output supply curve shifts outward

Shifts in the output supply curve



An increase in TFP increases the marginal product of labor, therefore demand for labor increaes. More labor and higher productivity result in higher aoutput at any level of employment. The output supply curve shifts outward

Demand for output

- Consumer demand, a function of income and the rate of interest
- Investment demand, a function of the rate of interest
- Government demand, exogenous
- Output demand is the sum of these.
- We represent it as a function of income and the rate of interest

Demand for output

$$Y^d = C^d(Y^d, r) + I^d(r) + G.$$

- Spending on goods depend on several other factors, (For example, C depends on future income and taxes, I depends on the level of capital and on productivity.
- It is the Income/expenditure identity. It signifies, that in an exchange economy income can be earned only if someone else spends. It is an identity only, does not say anithing about which causes what, or whether the individual items are interrelated or not

The multiplier effect

$$Y^d = C^d(Y^d, r) + I^d(r) + G.$$

- Income influeces consumption. Therefore, any exogenous increase on the demand side has a direct,
  as well as an indirect effect on output demanded
  (on income to be spent)
- Examples: I increases because an expected increase in z in the future
- C increeases because they expect Y' (not Y) to be higher
- Direct effect: Y^d increases because spending increases
- Indirect effect: C is a function of Y^d, C will also increase, causing Y to increase further and so on

The multiplier effect

- How big the direct and indirect effects together are going to be?
- $\Delta Y^d = \Delta C + \Delta E$
- $\Delta C = \partial C / \partial Y^d \times \Delta Y^d = MPC \times \Delta Y^d$
- MPC, marginal propensity to consume, the derivative of C with respect to Y

$$\frac{\partial c}{\partial y} = MPC < 1 \qquad \Delta Y^d = \frac{1}{1 - MPC} \Delta E$$

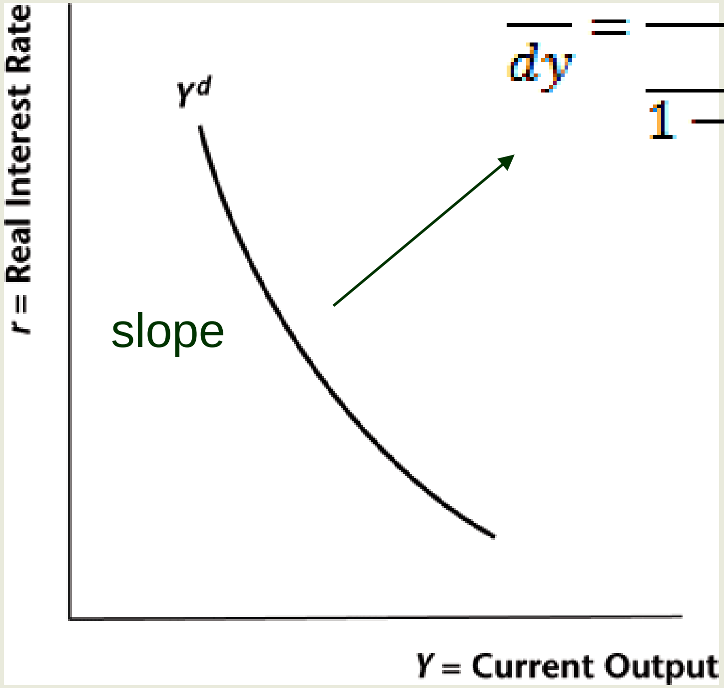
multiplier

The output demand curve

- The relationship between Y^d and r
- Has a negative slope as both C and I are negative functions of r
- Let ΔE stand for the direct increase in the spending caused by the increase in r
- $\Delta E = (\partial C/\partial r + \partial I/\partial r) \times \Delta r$,
- substituting into the multiplier formula

$$\frac{\Delta Y^{d}}{\Delta r} = \frac{1}{1 - MPC}\left(\frac{\Delta C}{\Delta r} + \frac{\Delta I}{\Delta r}\right) < 0$$

The output demand curve



$$\frac{dr}{dy} = \frac{1}{\frac{1}{1 - MPC}\left(\frac{dc}{dr} + \frac{dI}{dr}\right)}$$

The larger the multiplier, the flatter the output demand curve is

Shifts in the output demand curve

- Moving along the curve we learn how r influences Y^d
- Any factor changing C, I or G independently of Y and r (that is, exogenously) will shift the output demand curve.
- There are several factors like this, and they may be interrelated. C is a function of Y, reactions to exogenous changes will imply the multiplier process as well

Shifts in the output demand curve

- An increase in government spending

$$\frac{\partial y}{\partial G} = \frac{1}{(1 - MPC)}$$

An increase in the government spending will shift output demand outward. Due to the multiplier effect, size of the shit is larger then proportional

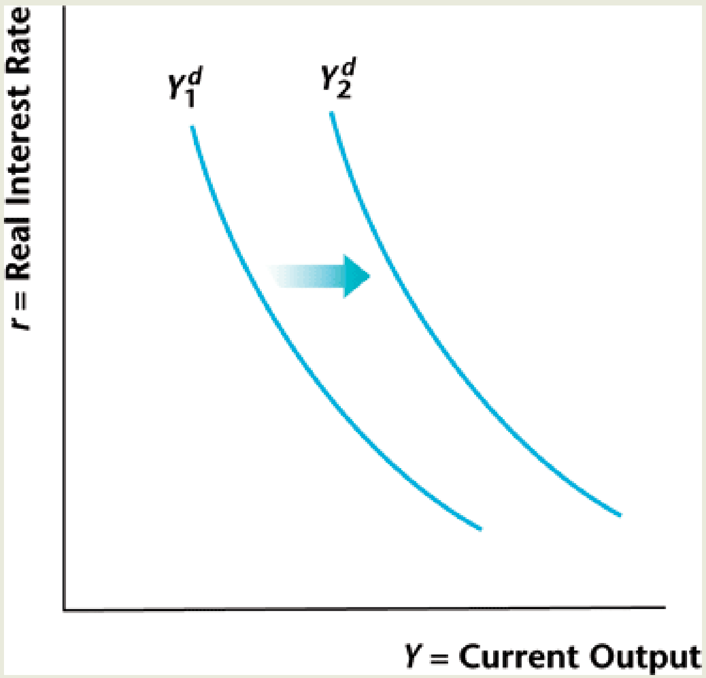
However, due to the government's budget constraint, taxes should be raised too, either in the present, or in the future. Tax increases should also be taken into consideration, their effect is negative through reducing consumption

Shifts in the output demand curve

- Reduction in taxes

$$\frac{dy}{dT} = \frac{-MPC}{(1 - MPC)}$$

$$\frac{dy}{dT} = \frac{-MPC}{(1 + r)(1 - MPC)}$$



A decrease in taces increases life time income, therefore increases consumption in the present. Output demand curve shifts outward. Tax increses have the opposite effect

Shifts in the output demand curve

- An expected increase in future income

$$\frac{dy}{dy'} = \frac{MPC}{(1 + r)\,(1 - MPC)}$$

Due to consumption smoothing it induces an increase in the present consumption. The output demand curve shifts to the right.

Expecred improvements in the technology or expected increases in government spending in the fututre can cause such kind of optimistic expectations in the incomes.

Shifts in the output demand curve

- An expected increase in the TFP

$$\frac{dy}{dz'} = \frac{\frac{\partial I}{\partial z'}}{(1 - MPC)}$$

Expected increases in the productivity would cause a positive shift in the demand for investment goods. Due to the related increase in incomes (multiplier efect) consumption demand also increases, the output demand curve shifts out quite well.

An increase in K depresses investments, the demand for output shifts backwards

$$\frac{dy}{dK} = \frac{-1}{(1 - MPC)}$$

The complete model