

$$\begin{aligned} \Rightarrow & \begin{vmatrix} + & - & + & - \\ 1 & 4 & 9 & 16 \\ 4 & 9 & 16 & 25 \\ 9 & 16 & 25 & 36 \\ 16 & 25 & 36 & 49 \end{vmatrix} = 1 \cdot \begin{vmatrix} 9 & 16 & 25 \\ 16 & 25 & 36 \\ 25 & 36 & 49 \end{vmatrix} - 4 \begin{vmatrix} 4 & 16 & 25 \\ 9 & 25 & 36 \\ 16 & 36 & 49 \end{vmatrix} + \end{aligned}$$

$$+ 9 \begin{vmatrix} 4 & 9 & 25 \\ 9 & 16 & 36 \\ 16 & 25 & 49 \end{vmatrix} - 16 \begin{vmatrix} 4 & 9 & 16 \\ 9 & 16 & 25 \\ 16 & 25 & 36 \end{vmatrix} =$$

$$= \begin{vmatrix} 9 & 16 & 25 \\ 16 & 25 & 36 \\ 25 & 36 & 49 \end{vmatrix} - \begin{vmatrix} 16 & 64 & 100 \\ 36 & 100 & 144 \\ 64 & 144 & 196 \end{vmatrix} + \begin{vmatrix} 36 & 81 & 225 \\ 81 & 144 & 324 \\ 144 & 225 & 341 \end{vmatrix}$$

$$- \begin{vmatrix} 64 & 144 & 256 \\ 144 & 256 & 400 \\ 256 & 400 & 576 \end{vmatrix} =$$

$$= 9 \cdot 25 \cdot 49 + 16 \cdot 36 \cdot 25 + 16 \cdot 36 \cdot 25 - (25^3 + 36^2 \cdot 9 + 16^2 \cdot 49) - ((1600 \cdot 196 + 36 \cdot 14400 + 64^2 \cdot 144) - (100^2 \cdot 64 + 144^2 \cdot 16 + 64 \cdot 36 \cdot 196))$$

$$+ ((36 \cdot 144 \cdot 341 + 81 \cdot 225^2 + 81 \cdot 324 \cdot 144) - (144^2 \cdot 225 + 81^2 \cdot 341 + 36 \cdot 324 \cdot 144)) - ((64 \cdot 256 \cdot 576 + 144 \cdot 400 \cdot 256 + 144 \cdot 400 \cdot 256) - (256^3 + 400^2 \cdot 64 + 144^2 \cdot 576)) =$$

$$= 0$$

Matritalar

$$\textcircled{1} \quad A = \begin{pmatrix} 2 & 1 & -1 \\ 0 & 1 & -4 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & 1 & 0 \\ -3 & 2 & 3 \end{pmatrix}$$

$$\begin{aligned} \text{a) } 3A + 2B &= 3 \cdot \begin{pmatrix} 2 & 1 & -1 \\ 0 & 1 & -4 \end{pmatrix} + 2 \cdot \begin{pmatrix} -2 & 1 & 0 \\ -3 & 2 & 3 \end{pmatrix} = \\ &= \begin{pmatrix} 6 & 3 & -3 \\ 0 & 3 & -12 \end{pmatrix} + \begin{pmatrix} -4 & 2 & 0 \\ -6 & 4 & 6 \end{pmatrix} = \begin{pmatrix} 2 & 5 & -3 \\ -6 & 7 & -6 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \text{b) } -A + 3B &= -1 \cdot \begin{pmatrix} 2 & 1 & -1 \\ 0 & 1 & -4 \end{pmatrix} + 3 \cdot \begin{pmatrix} -2 & 1 & 0 \\ -3 & 2 & 3 \end{pmatrix} = \\ &= \begin{pmatrix} -2 & -1 & 1 \\ 0 & -1 & 4 \end{pmatrix} + \begin{pmatrix} -6 & 3 & 0 \\ -9 & 6 & 9 \end{pmatrix} = \begin{pmatrix} -8 & 2 & 1 \\ -9 & 5 & 13 \end{pmatrix} \end{aligned}$$

$$\text{v) } 2A + kB = \begin{pmatrix} 4 & 2 & -2 \\ 0 & 2 & -8 \end{pmatrix} + \begin{pmatrix} -8 & 4 & 0 \\ -12 & 8 & 12 \end{pmatrix} = \begin{pmatrix} -4 & 6 & -2 \\ -12 & 10 & 4 \end{pmatrix}$$

$$\text{g) } \frac{1}{2}A + 1,5B = \begin{pmatrix} 1 & 0,5 & -0,5 \\ 0 & 0,5 & -2 \end{pmatrix} + \begin{pmatrix} -3 & 1,5 & 0 \\ -4,5 & 3 & 4,5 \end{pmatrix} = \begin{pmatrix} -2 & 2 & -0,5 \\ -4,5 & 3,5 & 2,5 \end{pmatrix}$$

$$2) \text{ Q)} \quad A = \begin{pmatrix} 3 & 5 & 7 \\ 2 & -1 & 0 \\ 4 & 3 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 3 & -2 \\ -1 & 0 & 1 \end{pmatrix}$$

$$A \times B = \begin{pmatrix} 3 & 5 & 7 \\ 2 & -1 & 0 \\ 4 & 3 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 4 \\ 2 & 3 & -2 \\ -1 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 3+10-7 & 6+15+0 & 12-10+7 \\ 2-2+0 & 4-3+0 & 8+2+0 \\ 4+6-2 & 2+9+0 & 16-6+2 \end{pmatrix} =$$

$$= \begin{pmatrix} 6 & 21 & 9 \\ 0 & 1 & 10 \\ 8 & 11 & 12 \end{pmatrix}$$

$$B \times A = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 3 & -2 \\ -1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 3 & 5 & 7 \\ 2 & -1 & 0 \\ 4 & 3 & 2 \end{pmatrix} =$$

$$= \begin{pmatrix} 3+4+16 & 5-2+12 & 7+0+8 \\ 6+6-8 & 10-3-6 & 14+0+4 \\ -3+0+4 & -5+0+3 & -7+0+2 \end{pmatrix} =$$

$$= \begin{pmatrix} 23 & 15 & 15 \\ 4 & 1 & 10 \\ 1 & -2 & -5 \end{pmatrix}$$

$$2) \quad A = \begin{pmatrix} 3 & -1 & 2 \\ 2 & 4 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 \\ 4 & 3 \\ 7 & 0 \end{pmatrix}$$

$$b) \quad A \cdot B = \begin{pmatrix} 3 & -1 & 2 \\ 2 & 4 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 4 & 3 \\ 7 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 3 - 4 + 14 & 6 - 3 + 0 \\ 2 + 16 + 7 & 4 + 12 + 0 \end{pmatrix} = \begin{pmatrix} 13 & 3 \\ 25 & 16 \end{pmatrix}$$

$$B \cdot A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \\ 7 & 0 \end{pmatrix} \cdot \begin{pmatrix} 3 & -1 & 2 \\ 2 & 4 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 3 + 4 & -1 + 8 & 2 + 2 \\ 12 + 6 & -4 + 12 & 2 + 3 \\ 21 + 0 & -7 + 0 & 14 + 0 \end{pmatrix} = \begin{pmatrix} 7 & 7 & 4 \\ 18 & 8 & 11 \\ 21 & -7 & 14 \end{pmatrix}$$

$$c) \quad \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix} \cdot \begin{pmatrix} 3 & 4 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 9 - 4 & 12 - 10 \\ 15 - 8 & 20 - 20 \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ 7 & 0 \end{pmatrix}$$

~~$$d) \quad \begin{pmatrix} 4 & 3 \\ 7 & 5 \end{pmatrix} \cdot \begin{pmatrix} -28 & 93 \\ 38 & -126 \end{pmatrix} \cdot \begin{pmatrix} 7 & 3 \\ 2 & 1 \end{pmatrix} =$$~~

~~$$= \begin{pmatrix} 28 + 6 & 12 + 3 \\ 49 + 10 & 21 + 5 \end{pmatrix} \cdot \begin{pmatrix} -28 & 93 \\ 38 & -126 \end{pmatrix} = \begin{pmatrix} 34 & 15 \\ 59 & 26 \end{pmatrix} \cdot \begin{pmatrix} -28 & 93 \\ 38 & -126 \end{pmatrix}$$~~

~~$$= \begin{pmatrix} 952 & 570 \end{pmatrix}$$~~

$$d) \quad \begin{pmatrix} 4 & 3 \\ 7 & 5 \end{pmatrix} \cdot \begin{pmatrix} -28 & 93 \\ 38 & -126 \end{pmatrix} \cdot \begin{pmatrix} 7 & 3 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} -112 + 114 & 372 - 378 \\ -196 + 190 & 651 - 630 \end{pmatrix} \cdot \begin{pmatrix} 7 & 3 \\ 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -6 \\ -6 & 21 \end{pmatrix} \cdot \begin{pmatrix} 7 & 3 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 14 - 12 & 6 - 6 \\ -42 + 42 & -18 + 21 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$$

$$e) \begin{pmatrix} 5 & 0 & 2 & 3 \\ 4 & 1 & 5 & 3 \\ 3 & 1 & -1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -2 \\ 4 \\ 4 \end{pmatrix} = \begin{pmatrix} 30 + 0 + 14 + 12 \\ 24 - 2 + 35 + 12 \\ 18 - 2 - 4 + 8 \end{pmatrix} = \begin{pmatrix} 56 \\ 69 \\ 14 \end{pmatrix}$$

$$f) (4 \ 0 \ -2 \ 3) \begin{pmatrix} 3 \\ 1 \\ -1 \\ 5 \end{pmatrix} = 12 + 0 + 2 + 15 = 29.$$

$$h) = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix}^2 = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix} =$$

$$= \begin{pmatrix} 1-6 & -2+4 \\ 3+9 & -6+16 \end{pmatrix} = \begin{pmatrix} -5 & 2 \\ 12 & 10 \end{pmatrix}$$

$$i) \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & na \\ 0 & 1 \end{pmatrix}$$

$$j) \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix}^3 = \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix} =$$

$$= \begin{pmatrix} 1-6 & -2+4 \\ 3+9 & -6+16 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix} = \begin{pmatrix} -5 & 2 \\ 12 & 10 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 3 & -4 \end{pmatrix} =$$

$$= \begin{pmatrix} -5+6 & 10-8 \\ 12+30 & -24-40 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 42 & -64 \end{pmatrix}$$

$$k) \begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix}^2 = \begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix} \begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix} = \begin{pmatrix} a^2+0 & a+a \\ 0+0 & 0+a^2 \end{pmatrix} =$$

$$= \begin{pmatrix} a^2 & 2a \\ 0 & a^2 \end{pmatrix}$$

$$l) \begin{pmatrix} \cos d & -\sin d \\ \sin d & \cos d \end{pmatrix}^n = \begin{pmatrix} \cos nd & -\sin nd \\ \sin nd & \cos nd \end{pmatrix}$$

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a) $A = \begin{pmatrix} 2 & 2 & 9 & -19 & -1 \\ -1 & 0 & 3 & -5 & 1 \\ 2 & 1 & -1 & 0 & -2 \\ 1 & 1 & -3 & 4 & -2 \end{pmatrix} \sim$

$$\begin{pmatrix} 1 & 1 & 4,5 & -9,5 & -0,5 \\ -1 & 0 & 3 & -5 & 1 \\ 2 & 1 & -1 & 0 & -2 \\ 1 & 1 & -3 & 4 & -2 \end{pmatrix} \sim$$

$$\begin{pmatrix} 1 & 1 & 4,5 & -9,5 & -0,5 \\ 0 & 1 & 7,5 & -14,5 & 0,5 \\ 0 & -1 & -10 & 19 & -1 \\ 0 & 0 & -7,5 & 13,5 & -1,5 \end{pmatrix} \sim$$

$$\begin{pmatrix} 1 & 1 & 4,5 & -9,5 & -0,5 \\ 0 & 1 & 7,5 & -14,5 & 0,5 \\ 0 & 0 & 1 & -1,8 & 0,2 \\ 0 & 0 & -7,5 & 13,5 & -1,5 \end{pmatrix} \sim$$

$$\begin{pmatrix} 1 & 1 & 7,5 & -8,5 & -0,5 \\ 0 & 1 & 7,5 & -14,5 & 0,5 \\ 0 & 0 & 1 & -1,8 & 0,2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \sim$$

$\text{rang}(A) = 3$

b) $B = \begin{pmatrix} 2 & -1 & 3 & -2 & 4 \\ 4 & -2 & 5 & 1 & 7 \\ 2 & -1 & 1 & 8 & 2 \end{pmatrix} \sim$

$$\begin{pmatrix} 1 & -0,5 & 1,5 & -1 & 2 \\ 4 & -2 & 5 & 1 & 7 \\ 2 & -1 & 1 & 8 & 2 \end{pmatrix} \sim$$

$$\begin{pmatrix} 1 & -0,5 & 1,5 & -1 & 2 \\ 0 & 0 & -1 & 5 & -1 \\ 0 & 0 & -2 & 10 & -2 \end{pmatrix} \sim$$

$$\begin{pmatrix} 1 & -0,5 & 1,5 & -1 & 2 \\ 0 & 0 & 1 & -5 & 1 \\ 0 & 0 & -2 & 10 & -2 \end{pmatrix} \sim$$

$$\begin{pmatrix} 1 & -0,5 & 1,5 & -1 & 2 \\ 0 & 0 & 1 & -5 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \sim$$

$\text{rang}(B) = 2$

$$s) C = \begin{pmatrix} 0 & 2 & -4 \\ -1 & -4 & 5 \\ 3 & 1 & 7 \\ 0 & 5 & -10 \\ 2 & 3 & 0 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} -1 & -4 & 5 \\ 0 & 2 & -4 \\ 3 & 1 & 7 \\ 0 & 5 & -10 \\ 2 & 3 & 0 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 4 & -5 \\ 0 & 2 & -4 \\ 3 & 1 & 7 \\ 0 & 5 & -10 \\ 2 & 3 & 0 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 4 & -5 \\ 0 & 2 & -4 \\ 0 & -11 & 22 \\ 0 & 5 & -10 \\ 0 & -5 & 10 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 4 & -5 \\ 0 & 1 & -2 \\ 0 & -11 & 22 \\ 0 & 5 & -10 \\ 0 & -5 & 10 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 4 & -5 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\text{rang}(C) = 2$$

$$d) A = \begin{pmatrix} 1 & 3 & 5 & -1 \\ 2 & -1 & -3 & 4 \\ 5 & 1 & -1 & 7 \\ 7 & 7 & 9 & 1 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 3 & 5 & -1 \\ 0 & -7 & -13 & 6 \\ 0 & -14 & -26 & 22 \\ 0 & -14 & -26 & 8 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 3 & 5 & -1 \\ 0 & 1 & \frac{13}{7} & -\frac{6}{7} \\ 0 & -14 & -26 & 22 \\ 0 & -14 & -26 & 8 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 3 & 5 & -1 \\ 0 & 1 & \frac{13}{7} & -\frac{6}{7} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 3 & 5 & -1 \\ 0 & 1 & \frac{13}{7} & -\frac{6}{7} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 \end{pmatrix}$$

$$\text{rang}(A) = 3$$

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$$a) \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$A_{11} = (-1)^2 \cdot 4 \quad A_{21} = (-1)^3 \cdot 2$$

$$A_{12} = (-1)^3 \cdot 3 \quad A_{22} = (-1)^4 \cdot 1$$

$$\det A = 1 \cdot 4 - 3 \cdot 2 = -2$$

$$A^{-1} = \frac{1}{\det A} \cdot \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \left(-\frac{1}{2}\right) \cdot \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 1,5 & -0,5 \end{pmatrix}$$

$$b) \begin{pmatrix} 3 & 4 \\ 5 & 7 \end{pmatrix}$$

$$A_{11} = (-1)^2 \cdot 7$$

$$A_{21} = (-1)^3 \cdot 5$$

$$A_{12} = (-1)^3 \cdot 4$$

$$A_{22} = (-1)^4 \cdot 3$$

$$\det A = 3 \cdot 7 - 4 \cdot 5 = 1$$

$$A^{-1} = 1 \cdot \begin{pmatrix} 7 & -5 \\ -4 & 3 \end{pmatrix} = \begin{pmatrix} 7 & -5 \\ -4 & 3 \end{pmatrix}$$

$$c) \begin{pmatrix} 2 & 5 & 7 \\ 6 & 3 & 4 \\ 5 & -2 & -3 \end{pmatrix} \quad \det A = \begin{vmatrix} 2 & 5 & 7 & 5 \\ 6 & 3 & 4 & 6 \\ 5 & -2 & -3 & 5 \end{vmatrix}$$

$$\det A = (2 \cdot 3 \cdot (-3) + 5 \cdot 4 \cdot 5 + 6 \cdot (-2) \cdot 7 - (5 \cdot 3 \cdot 4 + 6 \cdot 5 \cdot (-3) + (-2) \cdot 4 \cdot 2)) = (-1)$$

$$A_{11} = (-1)^0 \begin{vmatrix} 3 & 4 \\ -2 & -3 \end{vmatrix} = -1$$

$$A_{31} = (-1)^4 \begin{vmatrix} 5 & 7 \\ 3 & 4 \end{vmatrix} = -24$$

$$A_{12} = (-1)^3 \begin{vmatrix} 6 & 4 \\ 5 & -3 \end{vmatrix} = 38$$

$$A_{32} = (-1)^5 \begin{vmatrix} 2 & 7 \\ 6 & 3 \end{vmatrix} = 34$$

$$A_{13} = (-1)^4 \begin{vmatrix} 6 & 3 \\ 5 & -2 \end{vmatrix} = -27$$

$$A_{33} = (-1)^6 \begin{vmatrix} 2 & 5 \\ 6 & 3 \end{vmatrix} = -24$$

$$A_{21} = (-1)^3 \begin{vmatrix} 5 & 7 \\ -2 & -3 \end{vmatrix} = 1$$

$$A_{22} = (-1)^4 \begin{vmatrix} 2 & 7 \\ 5 & -3 \end{vmatrix} = -41$$

$$A_{23} = (-1)^5 \begin{vmatrix} 2 & 5 \\ 5 & -2 \end{vmatrix} = 29$$

$$A^T = \begin{pmatrix} -1 & 1 & -1 \\ 38 & -41 & 34 \\ -27 & 29 & -24 \end{pmatrix}$$

$$A^{-1} = \frac{1}{(-1)} \begin{pmatrix} -1 & 1 & -1 \\ 38 & -41 & 34 \\ -27 & 29 & -24 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 \\ -38 & 41 & -34 \\ 27 & -29 & 24 \end{pmatrix}$$

$$d) \begin{pmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{pmatrix} \det A = \begin{vmatrix} 3 & -4 & 5 & 3 & -4 \\ 2 & -3 & 1 & 2 & -3 \\ 3 & -5 & -1 & 3 & -5 \end{vmatrix}$$

$$\det A = 3 \cdot (-3) \cdot (-1) + 2 \cdot (-5) \cdot 5 + (-4) \cdot 1 \cdot 3 -$$

$$- (5 \cdot (-3) \cdot 3 + (-4) \cdot 2 \cdot (-1) + (-5) \cdot 1 \cdot 3) = -1$$

$$A_{11} = (-1)^2 \cdot \begin{vmatrix} -3 & 1 \\ -5 & -1 \end{vmatrix} = 8 \quad A_{12} = (-1)^3 \cdot \begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} = 5$$

$$A_{13} = (-1)^4 \cdot \begin{vmatrix} 2 & -3 \\ 3 & -5 \end{vmatrix} = -1 \quad A_{21} = (-1)^1 \cdot \begin{vmatrix} -4 & 5 \\ -5 & -1 \end{vmatrix} = -29$$

$$A_{22} = (-1)^4 \cdot \begin{vmatrix} 3 & 5 \\ 3 & -1 \end{vmatrix} = -18 \quad A_{23} = (-1)^5 \cdot \begin{vmatrix} 3 & -4 \\ 3 & -5 \end{vmatrix} = 3$$

$$A_{31} = (-1)^5 \cdot \begin{vmatrix} -4 & 5 \\ -3 & 1 \end{vmatrix} = 11 \quad A_{32} = (-1)^6 \cdot \begin{vmatrix} 3 & 5 \\ 2 & 1 \end{vmatrix} = 7$$

$$A_{33} = (-1)^6 \cdot \begin{vmatrix} 3 & -4 \\ 2 & -3 \end{vmatrix} = -1$$

$$A^T = \begin{pmatrix} 8 & -29 & 11 \\ 5 & -18 & 7 \\ -1 & 3 & -1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{(-1)} \cdot \begin{pmatrix} 8 & -29 & 11 \\ 5 & -18 & 7 \\ -1 & 3 & -1 \end{pmatrix} = \begin{pmatrix} -8 & 29 & -11 \\ -5 & 18 & -7 \\ 1 & -3 & 1 \end{pmatrix}$$

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$$a) \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot X = \begin{pmatrix} 3 & 5 \\ 5 & 9 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ 5 & 9 \end{pmatrix}$$

$$\begin{cases} 1 \cdot x_1 + 2 \cdot x_3 = 3 \\ 1 \cdot x_2 + 2 \cdot x_4 = 5 \\ 3 \cdot x_1 + 4 \cdot x_3 = 5 \\ 3 \cdot x_2 + 4 \cdot x_4 = 9 \end{cases}$$

$$\begin{cases} x_1 + 2x_3 = 3 & x_1 = \\ 3x_1 + 4x_3 = 5 & x_3 = \\ x_2 + 2x_4 = 5 & x_2 = - \\ 3x_2 + 4x_4 = 9 & x_4 = \end{cases}$$

$$Y: \begin{pmatrix} -1 & -1 \\ 2 & 3 \end{pmatrix}$$

$$b) X \cdot \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ -5 & 6 \end{pmatrix}$$

$$\begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix} \cdot \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ -5 & 6 \end{pmatrix}$$

$$\begin{cases} x_1 \cdot 3 + x_2 \cdot 5 = -1 \\ x_1 \cdot (-2) + x_2 \cdot (-4) = 2 \\ x_2 = -2 \quad x_1 = 3 \end{cases}$$

$$\begin{cases} x_3 \cdot (-2) + (-4)x_4 = 6 \\ x_3 \cdot 3 + 5x_4 = -5 \\ x_4 = -1 \quad x_3 = 5 \end{cases}$$

$$Y: X = \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix}$$

$$3) \begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -4 \\ 2 & -1 & 0 \end{pmatrix} \cdot X = \begin{pmatrix} 1 & -3 & 0 \\ 10 & 2 & 7 \\ 10 & 7 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & -3 \\ 3 & 2 & -4 \\ 2 & -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} X_1 & X_2 & X_3 \\ X_4 & X_5 & X_6 \\ X_7 & X_8 & X_9 \end{pmatrix} = \begin{pmatrix} 1 & -3 & 0 \\ 10 & 2 & 7 \\ 10 & 7 & 8 \end{pmatrix}$$

$$\begin{cases} 1 \cdot X_1 + 2 \cdot X_4 + (-3) \cdot X_7 = 1 \\ 3 \cdot X_1 + 2 \cdot X_4 + (-4) \cdot X_7 = 10 \\ 2 \cdot X_1 + (-1) \cdot X_4 + 0 \cdot X_7 = 10 \end{cases} \quad \begin{cases} 1 \cdot X_2 + 2 \cdot X_5 + (-3) \cdot X_8 = -3 \\ 3 \cdot X_2 + 2 \cdot X_5 + (-4) \cdot X_8 = 2 \\ 2 \cdot X_2 + (-1) \cdot X_5 + 0 \cdot X_8 = 7 \end{cases}$$

$$\begin{cases} X_1 + 2X_4 - 3X_7 = 1 & (1) \\ 3X_1 + 2X_4 - 4X_7 = 10 & (2) \\ 2X_1 - X_4 = 10 \end{cases} \quad \boxed{X_4 = 2X_1 - 10} \quad \begin{cases} X_2 + 2X_5 - 3X_8 = -3 \\ 3X_2 + 2X_5 - 4X_8 = 2 \\ 2X_2 - X_5 = 7 \end{cases} \quad \boxed{X_5 = 2X_2 - 7}$$

$$\begin{cases} X_1 + 2X_4 - 3X_7 = 1 \\ 4X_4 - 5X_7 = -7 \\ X_4 = 2X_1 - 10 \end{cases} \quad \begin{cases} 3X_2 + 2(2X_2 - 7) - 4X_8 = 2 \\ 7X_2 = 4X_8 + 16 \\ \boxed{X_2 = \frac{4X_8 + 16}{7}} \end{cases} \quad \boxed{X_8 = \frac{7X_2 - 16}{4}}$$

$$4(2X_1 - 10) - 5X_7 = -7$$

$$8X_1 - 5X_7 = 33$$

$$\boxed{X_7 = \frac{8X_1 - 33}{5}}$$

$$X_1 + 2 \cdot (2X_1 - 10) - 3 \cdot \left(\frac{8X_1 - 33}{5} \right) = 1$$

$$X_1 + 4X_1 - 20 - \frac{24X_1 - 99}{5} = 1$$

$$5X_1 - 100 + 99 = 1$$

$$X_1 = 2$$

$$X_4 = -6$$

$$X_7 = -\frac{17}{5}$$

$$X_2 + 2(2X_2 - 7) - 3$$

$$\frac{4X_8 + 16}{7} + 2(2)$$

$$X_2 + 2(2X_2 - 7) - 3 \cdot \left(\frac{4X_8 + 16}{7} \right) = -3$$

$$5X_2 - 14 - \frac{21X_2 - 48}{7} = -3$$

$$20X_2 - 56 - 21X_2 + 48 = -12$$

$$-X_2 = -4$$

$$X_2 = 4 \quad X_5 = 1$$

$$X_8 = 3$$

$$Z: X = \begin{pmatrix} 2 \\ -6 \\ -\frac{17}{5} \\ 4 \\ 1 \\ 3 \end{pmatrix}$$

$$\begin{cases} 1 \cdot x_3 + 2 \cdot x_6 + (-3) \cdot x_9 = 0 \\ 3 \cdot x_3 + 2 \cdot x_6 + (-4) \cdot x_9 = 7 \\ 2 \cdot x_3 + (-1) \cdot x_6 + 0 \cdot x_9 = 8 \end{cases}$$

$$\begin{cases} x_3 + 2x_6 - 3x_9 = 0 \\ 3x_3 + 2x_6 - 4x_9 = 7 \\ 2x_3 - x_6 = 8 \end{cases}$$

$$\boxed{x_6 = 2x_3 - 8}$$

$$3x_3 + 2(2x_3 - 8) - 4x_9 = 7$$

$$\boxed{x_9 = \frac{7x_3 - 23}{4}}$$

$$x_3 + 2(2x_3 - 8) - 3 \cdot \left(\frac{7x_3 - 23}{4} \right) = 0$$

$$20x_3 - 64 - 21x_3 + 69 = 0$$

$$-x_3 = -5$$

$$x_3 = 5 \quad x_6 = 2 \quad x_9 = 3$$

$$J : X = \begin{pmatrix} 2 & -4 & 5 \\ -6 & 1 & 2 \\ -\frac{17}{5} & 3 & 3 \end{pmatrix}$$

$$d) X^* \begin{pmatrix} 5 & 3 & 1 \\ 1 & -3 & -2 \\ -5 & 2 & 1 \end{pmatrix} = \begin{pmatrix} -8 & 3 & 0 \\ -5 & 9 & 0 \\ -2 & 15 & 0 \end{pmatrix}$$

$$\begin{pmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{pmatrix}^* \begin{pmatrix} 5 & 3 & 1 \\ 1 & -3 & -2 \\ -5 & 2 & 1 \end{pmatrix} = \begin{pmatrix} -8 & 3 & 0 \\ -5 & 9 & 0 \\ -2 & 15 & 0 \end{pmatrix}$$

$$\begin{cases} x_1 \cdot 5 + x_2 \cdot 1 + x_3 \cdot (-5) = -8 \\ x_1 \cdot 3 + x_2 \cdot (-3) + x_3 \cdot 2 = 3 \\ x_1 \cdot 1 + x_2 \cdot (-2) + x_3 \cdot 1 = 0 \end{cases}$$

$$\begin{cases} 5x_1 + x_2 - 5x_3 = -8 \\ 3x_1 - 3x_2 + 2x_3 = 3 \quad | \cdot (-3) \\ x_1 - 2x_2 + x_3 = 0 \quad | \cdot 5 \end{cases}$$

$$\begin{cases} 5x_1 + x_2 - 5x_3 = -8 \\ 3x_1 - 3x_2 + 2x_3 = 3 \\ -7x_2 + 5x_3 = 9 \end{cases}$$

$$x_3 = \frac{7x_2 + 9}{5}$$

$$5x_1 + x_2 - 5 \left(\frac{7x_2 + 9}{5} \right) = -8$$

$$25x_1 - 30x_2 - 45 = -40$$

$$x_1 = \frac{30x_2 + 5}{25}$$

$$3 \left(\frac{6x_2 + 1}{5} \right) - 3x_2 + 2 \left(\frac{7x_2 + 9}{5} \right) = 3 \quad \frac{40 - 7x_5}{15} - 2x_5 + \frac{5 + 11x_5}{10} = 0$$

$$18x_2 + 3 - 15x_2 + 14x_2 + 18 = 15$$

$$17x_2 = -6$$

$$x_2 = -\frac{6}{17} \quad x_3 = \frac{111}{85}$$

$$x_1 = -\frac{19}{85}$$

$$\begin{cases} x_4 \cdot 6 + x_5 \cdot 1 + x_6 \cdot (-5) = -5 \\ x_4 \cdot 3 + x_5 \cdot (-3) + x_6 \cdot 2 = 9 \\ x_4 \cdot 1 + x_5 \cdot (-2) + x_6 \cdot 1 = 0 \end{cases}$$

$$\begin{cases} 5x_4 + x_5 - 5x_6 = -5 \\ 3x_4 - 3x_5 + 2x_6 = 9 \\ x_4 - 2x_5 + x_6 = 0 \quad | \cdot 5 \end{cases}$$

$$\begin{cases} x_4 - 2x_5 + x_6 = 0 \\ 3x_4 - 3x_5 + 2x_6 = 9 \\ -11x_5 + 10x_6 = 5 \end{cases}$$

$$x_6 = \frac{5 + 11x_5}{10}$$

$$3x_4 - 3x_5 + \frac{5 + 11x_5}{5} = 9$$

$$15x_4 - 4x_5 + 5 = 45 - 11x_5$$

$$x_4 = \frac{40 - 7x_5}{15}$$

$$80 - 14x_5 - 60x_5 + 15 + 33x_5 = 0$$

$$-41x_5 = -95$$

$$x_5 = \frac{95}{41}$$

$$x_4 = \frac{65}{41}$$

$$x_6 = \frac{250}{123}$$

$$\begin{cases} X_7 \cdot 5 + X_8 \cdot 1 + X_9 \cdot (-5) = -2 \\ X_7 \cdot 3 + X_8 \cdot (-3) + X_9 \cdot 2 = 15 \\ X_7 \cdot 1 + X_8 \cdot (-2) + X_9 \cdot 1 = 0 \end{cases}$$

$$\begin{cases} 5X_7 + X_8 - 5X_9 = -2 \\ 3X_7 - 3X_8 + 2X_9 = 15 \\ X_7 - 2X_8 + X_9 = 0 \quad (3) \end{cases}$$

$$\begin{cases} X_7 - 2X_8 + X_9 = 0 \\ -3X_8 + X_9 = -15 \\ \cancel{5X_7 - 3X_8 + 2X_9 = 15} \\ 5X_7 + X_8 - 5X_9 = -2 \end{cases}$$

$$\boxed{X_9 = 3X_8 - 15}$$

$$5X_7 + X_8 - 5(3X_8 - 15) = -2$$

$$5X_7 + X_8 - 15X_8 + 75 = -2$$

$$\boxed{X_7 = \frac{14X_8 - 77}{5}}$$

$$\frac{14X_8 - 77}{5} - 2X_8 + 3X_8 - 15 = 0.$$

$$19X_8 = 152$$

$$X_8 = \frac{152}{19}$$

$$X_9 = \frac{171}{19}$$

$$X_7 = \frac{133}{19}$$

$$J: X = \begin{pmatrix} -\frac{19}{85} & -\frac{6}{17} & \frac{111}{85} \\ \frac{65}{41} & \frac{95}{41} & \frac{250}{123} \\ \frac{133}{19} & \frac{152}{19} & \frac{133}{19} \end{pmatrix}$$

Jeng lamalah sistem
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$$\begin{cases} 3x_1 + 2x_2 + x_3 = 5 \\ 2x_1 + 3x_2 + x_3 = 1 \\ 2x_1 + x_2 + 3x_3 = 11 \end{cases}$$

$$D = \begin{vmatrix} 3 & 2 & 1 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{vmatrix} = \begin{vmatrix} 3 & 2 & 1 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 27 + 4 + 2 - (6 + 3 + 12) =$$

$$= 33 - 21 = 12 \neq 0$$

$$D_1 = \begin{vmatrix} 5 & 2 & 1 \\ 1 & 3 & 1 \\ 11 & 1 & 3 \end{vmatrix} = 45 + 22 + 1 - (33 + 5 + 6) =$$

$$= 68 - 44 = 24$$

$$D_2 = \begin{vmatrix} 3 & 5 & 1 \\ 2 & 1 & 1 \\ 2 & 11 & 3 \end{vmatrix} = 9 + 22 + 10 - (2 + 33 + 30) = 41 - 65 = -24$$

$$D_3 = \begin{vmatrix} 3 & 2 & 5 \\ 2 & 3 & 1 \\ 2 & 1 & 11 \end{vmatrix} = 99 + 10 + 4 - (30 + 3 + 44) = 113 - 77 = 36$$

$$x_1 = \frac{D_1}{D} = \frac{24}{12} = 2 \quad x_3 = \frac{D_3}{D} = \frac{36}{12} = 3$$

$$x_2 = \frac{-24}{12} = -2$$

Gaus sa terkari matritra umli:

$$\begin{cases} 3x + 4y - 6z = 16 & /4 \\ 4x + y - z = 24 & /-3 \\ x - 3y - 2z = 1 & /-3 \end{cases}$$

$$\begin{cases} 3x + 4y - 6z = 16 & 3x + 4 \cdot 1 - 6 \cdot 1 = 16 \\ 13y - 21z = -2 & x = 6 \\ 13y = 13 & \\ y = 1 & \\ 13 - 21z = -8 & \\ z = 1 & \end{cases}$$

$$A = \begin{pmatrix} 3 & 4 & -6 \\ 4 & 1 & -1 \\ 1 & -3 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 16 \\ 24 \\ 1 \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$A X = B$$

$$X = B \cdot A^{-1}$$

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{-5}{91} & \frac{26}{91} & \frac{2}{91} \\ \frac{4}{91} & 0 & \frac{-21}{91} \\ \frac{-13}{91} & \frac{13}{91} & \frac{-13}{91} \end{pmatrix} \cdot \begin{pmatrix} 16 \\ 24 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \\ 1 \end{pmatrix}$$

$$d = \begin{vmatrix} 3 & 4 & -6 \\ 4 & 1 & -1 \\ 1 & -3 & -2 \end{vmatrix} = 91 \neq 0$$

$$A_{11} = -5 \quad A_{12} = 4 \quad A_{13} = -13$$

$$A_{21} = 26 \quad A_{22} = 0 \quad A_{23} = 13$$

$$A_{31} = 2 \quad A_{32} = -21 \quad A_{33} = -13$$

$$A^{-1} = \frac{1}{91} \begin{pmatrix} -5 & 26 & 2 \\ 4 & 0 & -21 \\ -13 & 13 & -13 \end{pmatrix} = \begin{pmatrix} \frac{-5}{91} & \frac{26}{91} & \frac{2}{91} \\ \frac{4}{91} & 0 & \frac{-21}{91} \\ \frac{-13}{91} & \frac{13}{91} & \frac{-13}{91} \end{pmatrix}$$