MATEMATIKA

YAKKABOG

ALGEBRA

Sonlarga doir ba'zi ma'lumotlar

Bo'linish alomatlari

2 ga; oxirgi raqami 0,2,4,6,8 bo'lsa
3 ga; raqamlarining yig'ndisi 3 ga bo'linsa
4 ga; oxirgi ikkita raqam 0 bo'lsa, yoki 4 ga bo'linsa
5 ga; oxirgi raqam 0 yoki 5 bilan tugasa
6 ga; $6=2\cdot3$. sonning o'zi ham 2 ga ham 3 ga bo'linsa
7 ga; sondagi o'nlar sonidan birlar xonasidagi raqamning ikkilangani ayirmasi 7 ga bo'linsa

M-n; 1)91 $9-2\cdot1=7$ 2)1127 $112-2\cdot7=98$ 98 soni 7 ga bo'linadi.
8 ga; oxirgi 3 ta raqami 0bo'lsa, yoki 8 ga bo'linsa
9 ga; raqamlar yig'indisi 9 ga bo'linsa
10 ga; oxirgi raqami 0 bo'lsa
11 ga; toq o'rindagi raqamlar yig'indisi bilan juft o'rindagi raqamlar yig'indisining ayirmasi 0 bo'lsa yoki 11 ga bo'linsa
M-n; 9873424 soni uchun; $(9+7+4+4)-(3+3+2)=11$.
25 ga; oxirgi ikkita raqami 0 bo'lsa yoki 25 ga bo'linsa
Qolgan sonlarga bo'linish belgilarini bevosita ularni tub ko'paytuvchilarga ajratish yo'li bilan topiladi.
M-n; 12 uchun; $12=3\cdot2^2=3\cdot4$. Demak 3 ga va 4ga bo'linadigan sonlar 12 ga bo'linadi

Sonlar to'plami

1) Natural; N 1,2,3,4,...
2) Butun; Z ...-3,-2,-1,0,1,2,3,...
3) Ratsional; Q $\frac{m}{n}$, ($m\in Z$, $n\in N$) ko'rinishdagi sonlar
4) Irratsional : cheksiz,davriy bo'lmagan o'nli kasr
M-n; $\pi=3,14...$, $\sqrt{3}$, $\sqrt{5}$,..
5) Tub sonlar; faqat 1 ga va o'ziga bo'linadigan sonlar:
M-n: 2,3,5,7,11,...
6) O'zaro tub; 1 dan boshqa umumiy bo'luvchilarga ega bo'lmagan sonlar; 16 va 27, 10 va 21, 8 va 23, 15 va 16,...

EKUB va EKUK haqida tushuncha

• Ikki sonning eng katta umumiy bo'luvchisi (EKUB) va eng kichik umumiy karralisi (EKUK) ni topish. Ikki son har birining bo'luvchilari sonini va ularning umumiy bo'luvchilari sonini topishga doir misol

594 va 378 sonlari uchun

594	2	378	2
297	3	189	3
99	3	63	3
33	3	21	3
11	11	7	7
1		1	

$594 = 2 \cdot 3^3 \cdot 11 \qquad 378 = 2 \cdot 3^3 \cdot 7$

$EKUB(594, 378) = 2 \cdot 3^3$

$EKUK(594, 378) = 2 \cdot 3^3 \cdot 11 \cdot 7$

• 594 sonining bo'luvchilari soni;

$594 = 2^1 \cdot 3^3 \cdot 11^1$

$(1+1) \cdot (3+1) \cdot (1+1) = 16$ ta

• 378 sonining bo'luvchilari soni;

$378 = 2^1 \cdot 3^3 \cdot 7^1$

$(1+1) \cdot (3+1) \cdot (1+1) = 16$ ta

• 594 va 378 sonlarining umumiy bo'luvchilari soni;

$EKUB(594, 378) = 2^1 \cdot 3^3$

$(1+1) \cdot (3+1) = 8$ ta

Kasr sonlar uchun *EKUB* va *EKUK* ni hisoblash;

$$EKUB\left(\frac{n_1}{m_1}, \frac{n_2}{m_2}, \ldots, \frac{n_k}{m_k}\right) = \frac{EKUB(n_1, n_2, \ldots n_k)}{EKUK(m_1, m_2, \ldots m_k)}$$

$$EKUK\left(\frac{n_1}{m_1}, \frac{n_2}{m_2}, \ldots, \frac{n_k}{m_k}\right) = \frac{EKUK(n_1, n_2, \ldots n_k)}{EKUB(m_1, m_2, \ldots m_k)}$$

Davriy kasrlar va ularni oddiy kasrga aylantirishga doir misollar

1) $0,(3) = 0,333\ldots = \frac{3}{9} = \frac{1}{3}$

2) $0,(17) = 0,1717\ldots = \frac{17}{99}$

3) $0,(813) = 0,813813\ldots = \frac{813}{999}$

4) $1,(7) = 1\frac{7}{9}$ yoki $\frac{17-1}{9}$

5) $2,(83) = 2\frac{83}{99}$ yoki $\frac{283-2}{99}$

8) $0,3(25) = \frac{325-3}{990}$

9) $0,3021(23) = \frac{302123-3021}{990000}$

10) $7,30(8901) = 7\frac{308901-30}{999900}$

Proporsiya

a,b,c,d-proportsiya hadlari; $a:b=c:d$ yoki $\frac{a}{b}=\frac{c}{d}$

a sonini $m:n:k$ nisbatda proporsional bo'laklarga ajratish;

To'g'ri proporsional; $\frac{a}{m+n+k}\cdot m$; $\frac{a}{m+n+k}\cdot n$; $\frac{a}{m+n+k}\cdot k$;

Teskari proporsional; $\frac{a}{\frac{1}{m}+\frac{1}{n}+\frac{1}{k}}\cdot\frac{1}{m}$; $\frac{a}{\frac{1}{m}+\frac{1}{n}+\frac{1}{k}}\cdot\frac{1}{n}$; $\frac{a}{\frac{1}{m}+\frac{1}{n}+\frac{1}{k}}\cdot\frac{1}{k}$

O'rta qiymatlar

$x_1, x_2, x_3, \ldots x_n$ sonlari uchun

1) O'rta arifmetik; $\frac{x_1+x_2+x_3+\ldots+x_n}{n}$

2) O'rta geometrik; $\sqrt[n]{x_1\cdot x_2\cdot x_3\cdot\ldots\cdot x_n}$

a va b sonlari uchun

1) O'rta arifmetik; $\frac{a+b}{2}$

2) O'rta geometrik; $\sqrt{a\cdot b}$

Protsentlar

1) a sonning P% i; $\frac{P}{100}a$

2) P% i a ga teng bo'lgan son

$\frac{100\cdot a}{P}$, yoki, $\frac{P}{100}x=a$

tenglamadan topiladi.

3) a va b sonlarining protsent nisbati, ya'ni a soni b sonining necha protsenti ekanligi $\frac{a}{b}\cdot 100\%$

4) a soni P% ga oshganda;

$100\%+P\%=\frac{100+P}{100}$ va

$\frac{100+P}{100}a$ yoki $\left(1+\frac{P}{100}\right)a$

5) a soni P% ga kamayganda;

$100\%-P\%=\frac{100-P}{100}$ va

$\frac{100-P}{100}a$ yoki $\left(1-\frac{P}{100}\right)a$

6) a soni ketma-ket P% dan n marta oshganda;

$a\left(1+\frac{P}{100}\right)^n$ ga teng bo'ladi.

7) a soni ketma-ket P% dan n marta kamayganda;

$a\left(1-\frac{P}{100}\right)^n$ ga teng bo'ladi

Protsentga doir qo'shimcha misollar ($a > b$ shart uchun)

1) a soni b sonining necha foizi?

b — 100%

a — x

$$x = \frac{a}{b} \cdot 100\%$$

2) a soni b sonidan necha foizga ortiq? $\left(\frac{a}{b} - 1\right) \cdot 100\%$

3) b soni a sonidan necha foizga kam? $\left(1 - \frac{b}{a}\right) \cdot 100\%$

4) a ning 20% i

$20\% = 0,2$ va $0,2a$

5) a soni 20% ga oshdi

$100\% + 20\% = 120\% = 1,2$

va $1,2a$

6) a soni 20% ga kamaydi

$100\% - 20\% = 80\% = 0,8$

va $0,8a$

7) a soni avval 20% ga oshdi, so'ngra hosil bo'lgan son yana 30% ga oshdi.

$100\% + 20\% = 120\% = 1,2$

$100\% + 30\% = 130\% = 1,3$

va $1,2 \cdot 1,3a$

8) a soni avval 30% ga oshdi, so'ngra hosil bo'lgan son yana 40% ga kamaydi

$100\% + 30\% = 130\% = 1,3$

$100\% - 40\% = 60\% = 0,6$

va $1,3 \cdot 0,6a$

9) a sonining 20% ga oshirilgani, b va c sonlari yig'indisining 60% iga teng.

$$1,2a = 0,6(b + c)$$

Qismlarga doir misollar

1) a ning $\frac{3}{4}$ qismi: $\frac{3}{4}a$

2) a soni o'zining $\frac{3}{4}$ qismiga ortdi:

$$a + \frac{3}{4}a \text{ yoki } \left(1 + \frac{3}{4}\right)a$$

3) a soni o'zining $\frac{3}{4}$ qismiga kamaydi:

$$a - \frac{3}{4}a \text{ yoki } \left(1 - \frac{3}{4}\right)a$$

Munosabatlar

1) a soni

a) musbat; $a > 0$ b) manfiy; $a < 0$

c) nomusbat; $a \le 0$

d) nomanfiy; $a \ge 0$

e) 5 dan katta; $a > 5$

f) 5 dan kichik; $a < 5$

g) 5 dan katta emas; $a \le 5$

h) 5 dan kichik emas; $a \ge 5$

2) a ning $\frac{4}{5}$ qismi va b ning ikkilangani yig'indisi musbat: $\frac{4}{5}a + 2b > 0$

3) a sonining 30% ga orttirilgani b sonining 20% idan 10 ga ortiq;

$$1,3a = 0,2b + 10$$

4) a soni b soniga

a) to'g'ri proportsional: $a = kb$

b) teskari proportsional: $a = \frac{k}{b}$

k – proportsionallik koeffitsiyenti

Ikki xonali son

$\overline{xy} = 10x + y$ masalan; $75 = 7 \cdot 10 + 5$

Raqamlarining o'rni almashsa;

$\overline{yx} = 10y + x$

Raqmlari:

1) yig'indisi; $x + y$
2) ko'paytmasi; $x \cdot y$
3) kvadratlari yig'indisi; $x^2 + y^2$

Darajaning xossalari

1) $a^0 = 1$
2) $a^1 = a$
3) $a^{-n} = \frac{1}{a^n}, a \neq 0$
4) $a^{\frac{p}{q}} = \sqrt[q]{a^p}$
5) $a^{-\frac{p}{q}} = \frac{1}{\sqrt[q]{a^p}}$
6) $a^p \cdot a^q = a^{p+q}$
7) $a^p : a^q = a^{p-q}$
8) $(a^p)^q = a^{pq}$
9) $(a \cdot b)^p = a^p \cdot b^p$
10) $(a^m \cdot b^n)^k = a^{mk} \cdot b^{nk}$
11) $\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$
12) $\left(\frac{a^m}{b^n}\right)^k = \frac{a^{mk}}{b^{nk}}$

Ildiz hossalari

1) $\sqrt[2n]{a^{2n}} = |a|$
2) $\sqrt[2n+1]{a^{2n+1}} = a$
3) $\sqrt[nk]{a^{mk}} = \sqrt[n]{a^m}$
4) $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
5) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
6) $\sqrt[n]{\sqrt[k]{a}} = \sqrt[nk]{a}$
7) $\left(\sqrt[n]{a^m}\right)^k = \sqrt[n]{a^{mk}}$
8) $\left(\sqrt[n]{a}\right)^k = \sqrt[n]{a^k}$
9) $a \cdot \sqrt[n]{b} = \sqrt[n]{a^n b}$
10) $a^k \cdot \sqrt[n]{b} = \sqrt[n]{a^{kn} b}$

Qo'shimcha;

1) $\sqrt[n]{a \cdot \sqrt[n]{a \cdot \sqrt[n]{a \cdot \ldots}}} = \sqrt[n-1]{a}$

yoki: $\sqrt[n]{a \cdot \sqrt[n]{a \cdot \sqrt[n]{a \cdot \ldots}}} = x$

$\sqrt[n]{a \cdot x} = x$

$a \cdot x = x^n$

$x^{n-1} = a$

$x = \sqrt[n-1]{a}$

2) $\sqrt[n]{a : \sqrt[n]{a : \sqrt[n]{a : \ldots}}} = \sqrt[n+1]{a}$

3) $\sqrt{a + \sqrt{a + \sqrt{a + \ldots}}} = \frac{1 + \sqrt{1 + 4a}}{2}$

4) $\sqrt{a - \sqrt{a - \sqrt{a - \ldots}}} = \frac{\sqrt{1 + 4a} - 1}{2}$

Murakkab ildiz formulalari

1) $\sqrt{A \pm \sqrt{B}} = \sqrt{\frac{A + \sqrt{A^2 - B}}{2}} \pm \sqrt{\frac{A - \sqrt{A^2 - B}}{2}}$

2) $\sqrt{a + b\sqrt{c}} = \sqrt{\frac{a + \sqrt{a^2 - b^2 c}}{2}} + \sqrt{\frac{a - \sqrt{a^2 - b^2 c}}{2}}$

3) $\sqrt{a - b\sqrt{c}} = \sqrt{\frac{a + \sqrt{a^2 - b^2 c}}{2}} - \sqrt{\frac{a - \sqrt{a^2 - b^2 c}}{2}}$

4) $\sqrt{a + b\sqrt{c}} + \sqrt{a - b\sqrt{c}} = \sqrt{\left(\sqrt{a + b\sqrt{c}} + \sqrt{a - b\sqrt{c}}\right)^2}$

5) $\sqrt{a + b\sqrt{c}} - \sqrt{a - b\sqrt{c}} = \sqrt{\left(\sqrt{a + b\sqrt{c}} - \sqrt{a - b\sqrt{c}}\right)^2}$

6) $\sqrt{a - b\sqrt{c}} + \sqrt{a + b\sqrt{c}} = \sqrt{\left(\sqrt{a - b\sqrt{c}} + \sqrt{a + b\sqrt{c}}\right)^2}$

7) $\sqrt{a - b\sqrt{c}} - \sqrt{a + b\sqrt{c}} = -\sqrt{\left(\sqrt{a - b\sqrt{c}} - \sqrt{a + b\sqrt{c}}\right)^2}$

Qisqa ko'paytirish formulalari

1) $(a+b)^2 = a^2 + 2ab + b^2 = a^2 + b^2 + 2ab$

2) $(a-b)^2 = a^2 - 2ab + b^2 = a^2 + b^2 - 2ab$

3) $(a+b)^3 = a^3 + 3a^2 b + 3ab^2 + b^3 = a^3 + 3ab(a+b) + b^3$

4) $(a-b)^3 = a^3 - 3a^2 b + 3ab^2 - b^3 = a^3 - 3ab(a-b) - b^3$

5) $a^2 - b^2 = (a-b)\cdot(a+b)$

6) $a^3 + b^3 = (a+b)\cdot(a^2 - ab + b^2)$

7) $a^3 - b^3 = (a-b)\cdot(a^2 + ab + b^2)$

Qo'shimcha ma'lumot

$$(a-b)^3 + (b-c)^3 + (c-a)^3 = 3(a-b)(b-c)(c-a)$$

yoki; $(a-b)^3 - (c-b)^3 - (a-c)^3 = 3(a-b)(b-c)(c-a)$

1) Chiziqli tenglamalar

$ax+b=0$ yoki $ax=-b$

1) $a=0, b\neq 0$ – ildizga ega emas

2) $a=0, b=0$ – cheksiz ko'p ildiz

$0\cdot x=0$

3) $a\neq 0, b\in R$ da $x=-\frac{b}{a}$ ga teng bo'lgan yagona ildizga ega.

Kvadrat tenglama

$ax^2+bx+c=0,\ a\neq 0$

$D=b^2-4ac;\quad x_{1,2}=\frac{-b\pm\sqrt{D}}{2a}$

yoki $x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

• To'la kavadratga ajratish usuli

$ax^2+bx+c=$

$=a\left(x+\frac{b}{2a}\right)^2-\frac{b^2-4ac}{4a}$

• Ko'paytuvchilarga ajratish:

1) $D>0$ da 2 ta har xil haqiqiy ildizlarga ega bo'ladi

$ax^2+bx+c=a(x-x_1)(x-x_2)$

2) $D<0$ da haqiqiy ildizlari yo'q va ko'paytuvchilarga ajralmaydi.

3) $D=0$ da $x_1=x_2=-\frac{b}{2a}$

a) ildizlari teng b) yagona ildiz

$ax^2+bx+c=a\left(x+\frac{b}{a}\right)^2$

c) agar a biror natural sonning kvadrati bo'lsa, ya'ni $\sqrt{a}\in N$;

$ax^2+bx+c=\left(\sqrt{a}\left(x+\frac{b}{2a}\right)\right)^2$

$D=0$ va $\sqrt{a}\in N$ to'la kavadrat bo'lish sharti.

*** Umumiy holda:**

$ax^2+bx+c=0, a\neq 0$ tenglama ildizga ega bo'lishi uchun $D\geq 0$ bo'lishi kerak

*** Qo'shimcha ma'lumot**

$ax^2+bx+c=0$ tenglama uchun

1) ildizlari orasidagi masofa

$|x_1-x_2|=\frac{\sqrt{b^2-4ac}}{a}$

2) Ildizlari qarama-qarshi sonlardan iborat bo'lsa;

$\begin{cases}D>0\\ x_1+x_2=0\end{cases}$ yoki $\begin{cases}c<0\\ b=0\end{cases}$

3) Ildizlari $ax^2+bx+c=0$ kvadrat tenglamaning ildizlariga teskari bo'lgan kvadrat tenglama

$cx^2+bx+a=0$

ko'rinishda bo'ladi.

Viyet teoremasi

I) $ax^2+bx+c=0$ ko'rinishdagi kvadrat tenglama uchun

$ax^2+bx+c=0$ yoki $x^2+\frac{b}{a}x+\frac{c}{a}=0$ va $D>0$ bo'lgan holda

$$\begin{cases} x_1+x_2=-\frac{b}{a} \\ x_1\cdot x_2=\frac{c}{a} \end{cases}$$

* Ba'zida uchraydigan hollar

1) $x_1^2+x_2^2=(x_1+x_2)^2-2x_1x_2=\left(-\frac{b}{a}\right)^2-2\frac{c}{a}=\frac{b^2-2ac}{a^2}$

2) $x_1^3+x_2^3=x_1^3+3x_1x_2(x_1+x_2)+x_2^3-3x_1x_2(x_1+x_2)=(x_1+x_2)^3-$
$-3x_1x_2(x_1+x_2)=\left(-\frac{b}{a}\right)^3-3\frac{c}{a}\left(-\frac{b}{a}\right)=-\frac{b^3}{a^3}+3\frac{bc}{a^2}=\frac{3abc-b^3}{a^3}$

3) $\frac{1}{x_1^2}+\frac{1}{x_2^2}=\frac{x_1^2+x_2^2}{x_1^2x_2^2}=\frac{(x_1+x_2)^2-2x_1x_2}{(x_1x_2)^2}=\frac{\frac{b^2-2ac}{a^2}}{\left(\frac{c}{a}\right)^2}=\frac{b^2-2ac}{c^2}$

4) $\frac{1}{x_1^3}+\frac{1}{x_2^3}=\frac{x_1^3+x_2^3}{x_1^3x_2^3}=\frac{x_1^3+x_2^3}{(x_1x_2)^3}=\frac{\frac{3abc-b^3}{a^3}}{\left(-\frac{c}{a}\right)^3}=\frac{b^3-3ac}{c^3}$

5) $|x_1-x_2|=\sqrt{(x_1-x_2)^2}=\sqrt{x_1^2-2x_1x_2+x_2^2}=\sqrt{x_1^2+2x_1x_2+x_2^2-4x_1x_2}=$
$=\sqrt{(x_1+x_2)^2-4x_1x_2}=\sqrt{\left(-\frac{b}{a}\right)^2-4\frac{c}{a}}=\frac{\sqrt{b^2-4ac}}{|a|}=\frac{\sqrt{D}}{|a|}$

II) Keltirilgan kvadrat tenglama $x^2+px+q=0$, $D>0$ **bo'lganda**

Viyet teoremasi; $\begin{cases} x_1+x_2=-p \\ x_1\cdot x_2=q \end{cases}$. Bundan; $\begin{cases} p=-(x_1+x_2) \\ q=x_1\cdot x_2 \end{cases}$

q – ozod had.

- **Ildizlari x_1' va x_2' bo'lgan keltirilgan kvadrat tenglama tuzish**

Tuzilishi kerak bo'lgan kvadrat tenglama; $x^2 + p'x + q' = 0$

p' va q' koeffiysiyentlar quyidagidan topiladi; $\begin{cases} p' = -(x_1' + x_2') \\ q' = x_1' \cdot x_2' \end{cases}$

- **Ba'zan uchraydigan shartlar**

1) 2 ta har - xil musbat ildiz

$$\begin{cases} D > 0 \\ x_1 + x_2 > 0 \\ x_1 \cdot x_2 > 0 \end{cases}$$

2) 2 ta har - xil manfiy ildiz

$$\begin{cases} D > 0 \\ x_1 + x_2 < 0 \\ x_1 \cdot x_2 > 0 \end{cases}$$

3) Ildizlari har - xil ishorali

$$\begin{cases} D > 0 \\ x_1 \cdot x_2 < 0 \end{cases}$$

4) Ildizlari o'zaro qarama - qarshi sonlardan iborat

$$\begin{cases} D > 0 \\ x_1 + x_2 = 0 \end{cases} \text{ yoki } \begin{cases} p = 0 \\ q < 0 \end{cases}$$

5) Ildizlari o'zaro teskari sonlardan iborat

$$\begin{cases} D > 0 \\ x_1 \cdot x_2 = 1 \end{cases}$$

6) To'la kvadrat (yoki yagona ildizga ega) bo'lish sharti

$$D = 0$$

- <u>Ba'zida uchraydigan hollar</u>

1) $x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2 = (-p)^2 - 2q = p^2 - 2q$

2) $x_1^3 + x_2^3 = x_1^3 + 3x_1x_2(x_1 + x_2) + x_2^3 - 3x_1x_2(x_1 + x_2) = (x_1 + x_2)^3 -$
$- 3x_1x_2(x_1 + x_2) = (-p)^3 - 3q(-p) = 3pq - p^3$

3) $\dfrac{1}{x_1^2} + \dfrac{1}{x_2^2} = \dfrac{x_1^2 + x_2^2}{x_1^2 x_2^2} = \dfrac{(x_1 + x_2)^2 - 2x_1x_2}{(x_1x_2)^2} = \dfrac{(-p)^2 - 2q}{q^2} = \dfrac{p^2 - 2q}{q^2}$

4) $\dfrac{1}{x_1^3} + \dfrac{1}{x_2^3} = \dfrac{x_1^3 + x_2^3}{x_1^3 x_2^3} = \dfrac{x_1^3 + x_2^3}{(x_1x_2)^3} = \dfrac{3pq - p^3}{q^3}$

5) $|x_1 - x_2| = \sqrt{(x_1 - x_2)^2} = \sqrt{x_1^2 - 2x_1x_2 + x_2^2} = \sqrt{x_1^2 + 2x_1x_2 + x_2^2 - 4x_1x_2} =$
$= \sqrt{(x_1 + x_2)^2 - 4x_1x_2} = \sqrt{(-p)^2 - 4q} = \sqrt{p^2 - 4q} = \sqrt{D}$

Bikvadrat tenglama. Kvadrat tenglamaga keltiriladigan tenglamalar

$ax^4 + bx^2 + c = 0$ tenglamada $x^2 = t$ deb belgilashdan

$at^2 + bt + c = 0$ ko'rinishdagi kvadrat tenglama hosil qilinadi.

@ Ildizlarining yig'indisi nolga teng.

@ Ildizlarining ko'paytmasi $\frac{c}{a}$ ga teng.

@ Haqiqiy ildizlarining soni ko'pi bilan 4 ta bo'ladi.

@ Eng katta ildizining eng kichik ildiziga nisbati (-1) ga teng.

@ $D = 0$ va $\sqrt{a} \in N$ bo'lganda to'la kvadrat bo'ladi.

* Qo'shimcha misol;

m ning qanday qiymatlarida $x(x+a)(x+2b)(x+a+2b)+m^2$ ifoda to'la kvadrat bo'ladi?

Yechish;

$x(x+a)(x+2b)(x+a+2b)+m^2 = x(x+a+2b)(x+a)(x+2b)+m^2 =$

$= (x^2+ax+2bx)(x^2+ax+2bx+2ab)+m^2 = \{x^2+ax+2bx=t\} =$

$= t(t+2ab)+m^2 = t^2+2abt+m^2$

Ma'lumki to'la kvadrat bo'lishi D = 0 bo'lishi kerak;

$D = 4a^2b^2 - 4m^2 = 0$ dan $m = \pm ab$.

* Bu misol uchun ushbu formula ham o'rinli;

$x(x+p)(x+q)(x+p+q)+k$ ifoda to'la kvadrat bo'lsa, $k = \frac{p^2q^2}{4}$ o'rinli.

Ko'phadga doir qo'shimcha ma'lumot

$P(x) = a_1x^n + a_2x^{n-1} + a_3x^{n-2} + ... + a_nx + a_0$ ko'rinishdagi ko'phad uchun

1) Ozod hadini topish; $x = 0$, $P(0) = a_0$

2) Barcha koeffitsiyentlari yig'indisini topish;

$x = 1$, $P(1) = a_1 + a_2 + a_3 + ... + a_0$

3) x ning barcha toq nomerli darajali hadlari oldidagi koeffitsiyentlari yig'indisini topish; $\frac{P(1) - P(-1)}{2}$

4) x ning barcha juft nomerli darajali hadlari oldidagi koeffitsiyentlari yig'indisini topish; $\frac{P(1) + P(-1) - 2P(0)}{2}$ yoki $\frac{P(1) + P(-1) - 2a_0}{2}$

Kvadratik tengsizlik

I) Noqat'iy tengsizlik. ($x_1 < x_2$ bo'lgan hol uchun)

$ax^2 + bx + c \geq 0$

1) $a > 0, D > 0$ da
$x \in (-\infty; x_1] \cup [x_2; \infty)$

2) $a > 0, D = 0$ da $x \in (-\infty; \infty)$
$a > 0, D < 0$ da $x \in (-\infty; \infty)$
Umumiy holda $a > 0, D \leq 0$ da tengsizlik x ning ixtiyoriy qiymatlarida o'rinli bo'ladi.

3) $a < 0, D > 0$ da $x \in [x_1; x_2]$

4) $a < 0, D = 0$ da $x = x_1 = x_2$

5) $a < 0, D < 0$ da $x \in \emptyset$,
ya'ni $a < 0, D < 0$ da tengzislik yechimga ega bo'lmaydi.

$ax^2 + bx + c \leq 0$

1) $a > 0, D > 0$ da $x \in [x_1; x_2]$

2) $a > 0, D = 0$ da $x = x_1 = x_2$

3) $a > 0, D < 0$ da $x \in \emptyset$, ya'ni
$a > 0, D < 0$ da tengsizlik yechimga ega bo'lmaydi

4) $a < 0, D > 0$ da
$x \in (-\infty; x_1] \cup [x_2; \infty)$

5) $a < 0, D = 0$ da $x \in (-\infty; \infty)$
$a < 0, D < 0$ da $x \in (-\infty; \infty)$
Umumiy holda $a < 0, D \leq 0$ da tengsizlik x ning istalgan qiymatida o'rinli bo'ladi.

II) Qat'iy tengsizlik. ($x_1 < x_2$ bo'lgan hol uchun)

$ax^2 + bx + c > 0$

1) $a > 0, D > 0$ da
$x \in (-\infty; x_1) \cup (x_2; \infty)$

2) $a > 0, D = 0$ da $x \neq x_1$

3) $a > 0, D < 0$ da $x \in (-\infty; \infty)$
ya'ni $a > 0, D < 0$ da tengzislik x ning ixtiyoriy qiymatlarida o'rinli bo'ladi.

4) $a < 0, D > 0$ da $x \in (x_1; x_2)$

5) $a < 0, D = 0$ da $x \in \emptyset$
$a < 0, D < 0$ da $x \in \emptyset$
Umumiy holda $a < 0, D \leq 0$ da tengsizlik yechimga ega bo'lmaydi.

$ax^2 + bx + c < 0$

1) $a > 0, D > 0$ da $x \in (x_1; x_2)$

2) $a > 0, D < 0$ da $x \in \emptyset$
$a > 0, D = 0$ da $x \in \emptyset$
Umumiy holda $a > 0, D \leq 0$ da tengsizlik yechimga ega bo'lmaydi

3) $a < 0, D > 0$ da
$x \in (-\infty; x_1) \cup (x_2; \infty)$

4) $a < 0, D = 0$ da $x \neq x_1$

5) $a < 0, D < 0$ da $x \in (-\infty; \infty)$
ya'ni tengsizlik x ning ixtiyoriy qiymatida bajariladi.

Ikki noma'lumli chiziqli tenglamalar sistemasi

I) $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$ tenglamalar sistemasi uchun ba'zi shartlar

1) Sistemaning yechimga ega bo'lmaslik sharti; $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$

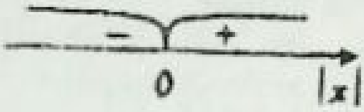
2) Sistemaning yagona yechimga ega bo'lish sharti; $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$

3) Sistemaning cheksiz ko'p yechimga ega bo'lish sharti; $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

Ba'zida uchraydigan tenglamalar sistemasi

I) $\begin{cases} a_1|x| + b_1y = c_1 \\ a_2|x| + b_2y = c_2 \end{cases}$

Yechilishi;



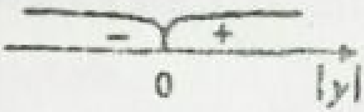
uchun

$\begin{cases} x \le 0 \\ -a_1x + b_1y = c_1 \\ -a_2x + b_2y = c_2 \end{cases}$ ⋮ $\begin{cases} x \ge 0 \\ a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$

$\dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$ va $x = 0$ da yagona yechimga ega bo'ladi.

II) $\begin{cases} a_1x + b_1|y| = c_1 \\ a_2x + b_2|y| = c_2 \end{cases}$

Yechilishi;



uchun

$\begin{cases} y \le 0 \\ a_1x - b_1y = c_1 \\ a_2x - b_2y = c_2 \end{cases}$ ⋮ $\begin{cases} y \ge 0 \\ a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$

$\dfrac{a_1}{a_2} = \dfrac{c_1}{c_2}$ va $y = 0$ da yagona yechimga ega bo'ladi.

Arifmetik progressiya

$a_1, a_2, a_3, \ldots, a_{n-1}, a_n, a_{n+1}, \ldots$

$d = a_{n+1} - a_n$ yoki $d = a_2 - a_1$ $\quad a_{o'r} = \dfrac{a_1 + a_n}{2}$

$a_n = a_1 + (n-1)d \quad d = \dfrac{a_n - a_1}{n-1} \quad n = \dfrac{a_n - a_1}{d} + 1$

d – arifmetik progressiyaning ayirmasi

a_n – arifmetik progressiyaning n chi hadi yoki umumiy hadi

$a_{o'r}$ – arifmetik progressiyaning dastlabki n ta hadi uchun o'rta hadi

Qo'shimcha ma'lumotlar

1) $a_n = a_m + kd$, agar $n = m + k$ bo'lsa. M - n; $a_{42} = a_{20} + 22d$

2) $a_n = a_m - kd$, agar $n = m - k$ bo'lsa. M - n; $a_{15} = a_{32} - 17d$

3) $a_n - a_m = (n-m)d$. M - n; $a_{24} - a_{10} = 14d$

4) $a_n = \dfrac{a_{n-1} + a_{n+1}}{2}$ yoki $2a_n = a_{n-1} + a_{n+1}$ M - n; $2a_8 = a_7 + a_9$

5) $a_m + a_n = 2a_k$, agar $m + n = 2k$ bo'lsa. M - n; $a_{20} + a_{42} = 2a_{31}$

6) $a_m + a_n = a_k + a_l$, agar $m + n = k + l$ bo'lsa. M - n; $a_{15} + a_9 = a_5 + a_{19}$

7) a, b, c – sonlarining arifmetik progressiyaning ketma-ket hadlari bo'lish sharti; $2b = a + c$ yoki $b = \dfrac{a+c}{2}$

Arifmetik progressiya hadlarining yig'indisi

$S_1 = a_1 \quad S_2 = a_1 + a_2 \quad S_3 = a_1 + a_2 + a_3 \quad S_n = a_1 + a_2 + a_3 + \ldots + a_n$

$S_n = \dfrac{a_1 + a_n}{2} n \quad S_n = \dfrac{2a_1 + (n-1)d}{2} n \quad S_n = a_{o'r} \cdot n$

$S_n = \dfrac{a_1 + a_n}{2} n = \dfrac{a_k + a_l}{2} n$, agar $1 + n = k + l$ bo'lsa

- Yig'indiga oid qo'shimcha ma'lumot ($m > n$ bo'lgan hol uchun)

1) $S_n - S_{n-1} = a_n$ M - n; $S_{20} - S_{19} = a_{20}$

2) $S_m - S_n = \underbrace{a_{n+1} + a_{n+2} + \ldots + a_m}_{m-n \text{ ta}} = \dfrac{a_{n+1} + a_m}{2}(m-n) = a_{\frac{m+n+1}{2}} \cdot (m-n)$

• Agar arifmetik progessiya hadlarining soni (n) juft bo'lsa

1) Barcha juft nomerli hadlarining yig'indisi;

$$\underbrace{a_2+a_4+a_6+\dots+a_n}_{\frac{n}{2}}=\frac{a_2+a_n}{2}\cdot\frac{n}{2}=\frac{a_2+a_n}{4}n$$

2) Barcha toq nomerli hadlarining yig'indisi;

$$\underbrace{a_1+a_3+a_5+\dots+a_{n-1}}_{\frac{n}{2}}=\frac{a_1+a_{n-1}}{2}\cdot\frac{n}{2}=\frac{a_1+a_{n-1}}{4}n$$

Geometrik progressiya

$b_1, b_2, b_3, \dots, b_{n-1}, b_n, b_{n+1}, \dots$

$$b_n=b_1q^{n-1}\quad b_n^2=b_{n-1}\cdot b_{n+1}\quad q=\frac{b_{n+1}}{b_n}\quad q=\frac{b_2}{b_1}\quad b_{o'r}=\sqrt{b_1\cdot b_n}$$

q – geometrik progressiyaning maxraji

b_n – geometrik progressiyaning n chi hadi yoki umumiy hadi

$b_{o'r}$ – geometrik progressiyaning dastlabki n ta hadi uchun o'rta had

Qo'shimcha ma'lumotlar

1) $b_m q^n = b_{m+n}$. M-n; $b_5 q^7 = b_{12}$

2) $\frac{b_m}{q^n} = b_{m-n}$. M-n; $\frac{b_{25}}{q^{10}} = b_{15}$

3) $b_m b_n = b^2_{\frac{m+n}{2}}$ M-n; $b_4 b_{10} = b_7^2$

4) $\frac{b_m}{b_n} = q^{m-n}$. M-n; $\frac{b_{25}}{b_{15}} = q^{10}$

5) $b_m b_n = b_k b_l$, agar $m+n=k+l$

M-n; $b_3 b_7 = b_2 b_8$

6) a, b va c – sonlarining geometrik progressiyaning ketma-ket hadlari bo'lish sharti; $b^2=a\cdot c$

• Geometrik progressiya hadlarining yig'indisi

$$S_1=b_1\quad S_2=b_1+b_2\quad S_3=b_1+b_2+b_3\quad S_n=b_1+b_2+b_3+\dots+b_n$$

$$S_n=\frac{b_1(1-q^n)}{1-q}\quad S_n=\frac{b_1(q^n-1)}{q-1}\quad S_n=\frac{b_1-b_nq}{1-q}\quad S_n=\frac{b_nq-b_1}{q-1},\ q\neq 1$$

@ $q=1$ bo'lgan hol uchun yig'indi $S_n=n\cdot b_1$ formuladan topiladi.

• Yig'indiga oid qo'shimcha ma'lumot ($m>n$ bo'lgan hol uchun)

1) $S_n - S_{n-1} = b_n$ M-n; $S_{20}-S_{19}=b_{20}$

2) $$S_m-S_n=\underbrace{b_{n+1}+b_{n+2}+\dots+b_m}_{m-n}=\frac{b_{n+1}(1-q^{m-n})}{1-q}=\frac{b_1q^n(1-q^{m-n})}{1-q}$$

• Agar geometrik progressiya hadlarining soni (n) juft bo'lsa

1) Barcha juft nomerli hadlarining yig'indisi;

$$\underbrace{b_2+b_4+b_6+...+b_n}_{\frac{n}{2}\text{ ta}}=\frac{b_2\left(1-\left(q^2\right)^{\frac{n}{2}}\right)}{1-q^2}=\frac{b_2\left(1-q^n\right)}{1-q^2}=\frac{b_1q\left(1-q^n\right)}{1-q^2}$$

2) Barcha toq nomerli hadlarining yig'indisi;

$$\underbrace{b_1+b_3+b_5+....+b_{n-1}}_{\frac{n}{2}\text{ ta}}=\frac{b_1\left(1-\left(q^2\right)^{\frac{n}{2}}\right)}{1-q^2}=\frac{b_1\left(1-q^n\right)}{1-q^2}$$

Cheksiz kamayuvchi geometrik progressiya

$b_1, b_2, b_3, ..., b_{n-1}, b_n, b_{n+1}, ...\quad |q|<1$

Barcha hadlarining yig'indisi; $S=b_1+b_2+...+b_n+...=\frac{b_1}{1-q}$

Dastlabki n ta hadi yig'indisi (n-kichik); $S_n=b_1+b_2+...+b_n=\frac{b_1\left(1-q^n\right)}{1-q}$

Barcha toq nomerli hadlar yig'indisi; $b_1+b_3+b_5+...=\frac{b_1}{1-q^2}$

Barcha juft nomerli hadlar yig'indisi; $b_2+b_4+b_6+...=\frac{b_2}{1-q^2}=\frac{b_1q}{1-q^2}$

Barcha hadlar kvadratlari yig'indisi; $b_1^2+b_2^2+...=\frac{b_1^2}{1-q^2}=\frac{b_1}{1-q}\cdot\frac{b_1}{1+q}$

Ba'zida uchraydigan yig'indilar

1) $1^2+2^2+...+n^2=\frac{n(n+1)(2n+1)}{6}$; 2) $1^3+2^3+...+n^3=\left(\frac{n(n+1)}{2}\right)^2$

3) Agar $a_1, a_2, a_3, ..., a_n$ sonlar arifmetik progressiyaning hadlari bo'lsa;

$$\frac{1}{a_1\cdot a_2}+\frac{1}{a_2\cdot a_3}+...+\frac{1}{a_{n-1}\cdot a_n}=\frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_n}\right)=\frac{a_n-a_1}{d\cdot a_1\cdot a_n}=\frac{n-1}{a_1\cdot a_n}$$

Misol.

1) $\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\ldots+\frac{1}{n\cdot(n+1)}=\{d=2-1=1\}=\frac{1}{1}\left(\frac{1}{1}-\frac{1}{n+1}\right)=\frac{n}{n+1}$

2) $\frac{1}{1\cdot3}+\frac{1}{3\cdot5}+\ldots+\frac{1}{(2n-1)(2n+1)}=\{d=2\}=\frac{1}{2}\left(\frac{1}{1}-\frac{1}{2n+1}\right)=\frac{n}{2n+1}$

Logarifmlar

$\log_a b$ $a>0,\ a\neq1$ va $b>0$

1) $\log_a a=1$	5) $\log_a b^p=p\log_a b$	8) $\log_a b^{\frac{m}{n}}=\frac{m}{n}\log_a b$
2) $\log_a 1=0$	6) $\log_{a^q} b=\frac{1}{q}\log_a b$	
3) $a^{\log_a b}=b$		9) $\log_{\sqrt[n]{a^m}} b=\frac{n}{m}\log_a b$
4) $a^{\log_b c}=c^{\log_b a}$	7) $\log_{a^q} b^p=\frac{p}{q}\log_a b$	10) $\log_{\sqrt[m]{a}} b=m\log_a b$

• O'nli logarifm;

$\lg a=\log_{10} a$, M - n; $\lg10=1$, $\lg100=2$, $\lg0{,}001=-3$, $10^{\lg a}=a$

• Natural logarifm; $\ln a=\log_e a$, $\ln e=1$, $\ln e^2=2$, $e^{\ln a}=a$.

Bu yerda e – eksponenta: $e\approx2{,}71\ldots$

Logarifmning ba'zi xossalari

1) $\log_a(bc)=\log_a b+\log_a c$ $\qquad \log_a\left(\frac{b}{c}\right)=\log_a b-\log_a c$

$\log_a b=\frac{\log_c b}{\log_c a}$ $\qquad \frac{\log_c b}{\log_c a}=\log_a b$ $\qquad \log_a b=\frac{1}{\log_b a}$

II) Musbat va manfiyligi

1) $a>1, b>1$ bo'lsa, $\log_a b>0$ (+) bo'ladi

2) $0<a<1, 0<b<1$ bo'lsa, $\log_a b>0$ (+) bo'ladi

3) $a>1, 0<b<1$ bo'lsa, $\log_a b<0$ (–) bo'ladi

4) $0<a<1, b>1$ bo'lsa, $\log_a b<0$ (–) bo'ladi

• Qo'shimcha: $a_1, a_2, a_3, \ldots, a_n$ arifmetik progressiyaning ketma-ket hadlari bo'lsa: $\log_{a_1} a_2\cdot\log_{a_2} a_3\cdot\log_{a_3} a_4\cdot\ldots\cdot\log_{a_{n-1}} a_n=\log_{a_1} a_n$

Funktsiya

x – argument, erkli o'zgaruvchi, abstsissa

y – funktsiya, erksiz o'zgaruvchi, ordinata

• **Juftligi va toqligi**

$y(-x) = y(x)$ bo'lganda funktsiya juft. Grafigi OY o'qiga nisbatan va $y(-x) = -y(x)$ bo'lganda funktsiya toq. Grafigi koordinatalar boshiga nisbatan simmetrik bo'ladi.

M – n:

1) $y(x) = x^4 + 2$; $y(-x) = (-x)^4 + 2 = x^4 + 2 = y(x)$ juft.

2) $y(x) = x^3 + x$; $y(-x) = (-x)^3 + (-x) = -x^3 - x = -(x^3 + x) = y(x)$ toq.

3) $y(x) = x^5 + 1$; $y(-x) = (-x)^5 + 1 = -x^5 + 1 = -(x^5 - 1)$ juft ham, toq ham emas.

• **Aniqlanish sohasi** (A.s) – x ning qabul qiladigan qiymatlari to'plami, yoki x ning shunday qiymatlari to'plamiga aytiladiki, bu to'plamda funktsiyaning ma'nosi buzilmaydi.

• **Aniqlanish sohasini topishga doir ba'zi holatlar**

$f(x)$, $g(x)$ va $p(x)$ – a.s lari $x \in R$ (yoki $x \in (-\infty;\infty)$) bo'lgan funktsiyalar.

1) $y = \dfrac{f(x)}{g(x)}$

A.s. $g(x) \neq 0$

2) $y = \sqrt{f(x)}$

A.s. $f(x) \geq 0$

3) $y = \dfrac{g(x)}{\sqrt{f(x)}}$

A.s. $f(x) > 0$

4) $y = \sqrt[2n]{f(x)}$

A.s. $f(x) \geq 0$

5) $y = \sqrt[2n+1]{f(x)}$

A.s. $y = f(x)$ b-n bir xil

6) $y = \sqrt{f(x)} \pm \sqrt{g(x)}$

A.s. $\begin{cases} f(x) \geq 0 \\ g(x) \geq 0 \end{cases}$

7) $y = \dfrac{f(x)}{\sqrt{g(x)} - \sqrt{p(x)}}$

A.s. $\begin{cases} g(x) \geq 0 \\ p(x) \geq 0 \\ \sqrt{g(x)} \neq \sqrt{p(x)} \end{cases}$

8) $y = \dfrac{\sqrt{f(x)}}{g(x)}$

A.s. $\begin{cases} f(x) \geq 0 \\ g(x) \neq 0 \end{cases}$

9) $y = \sqrt[2n]{f(x)} \pm \dfrac{g(x)}{p(x)}$

A.s. $\begin{cases} f(x) \geq 0 \\ p(x) \neq 0 \end{cases}$

10) $y = \dfrac{\sqrt{f(x)}}{\sqrt{g(x)}}$

A.s. $\begin{cases} f(x) \geq 0 \\ g(x) \neq 0 \end{cases}$

11) $y = \dfrac{\sqrt{f(x)} \pm \sqrt{g(x)}}{\sqrt{p(x)}}$

A.s. $\begin{cases} f(x) \geq 0 \\ g(x) \geq 0 \\ p(x) > 0 \end{cases}$

*** Ba'zi elementar funksiyalar va ular qatnashgan hollar uchun**

aniqlanish sohasini topish usullari

1) $f(x)=\log_a f(x)$; A.s. $f(x)>0$

2) $y=\log_{g(x)} b$; A.s. $g(x)>0, g(x)\neq 1$

3) $y=\log_{g(x)} f(x)$

A.s. $\begin{cases} f(x)>0 \\ g(x)>0, g(x)\neq 1 \end{cases}$

4) $y=\sqrt{\log_a f(x)}$; A.s. $\begin{cases} \log_a f(x)\geq 0 \\ f(x)>0 \end{cases}$

5) $y=\dfrac{f(x)}{\log_a g(x)}$

A.s. $g(x)>0, g(x)\neq 1$

6) $y=\dfrac{\log_a f(x)}{\sqrt{g(x)}}$; A.s. $\begin{cases} f(x)>0 \\ g(x)>0 \end{cases}$

7) $y=\arcsin f(x)$; A.s. $-1\leq f(x)\leq 1$

8) $y=\arccos f(x)$;

A.s. $-1\leq f(x)\leq 1$

9) $y=\arcsin f(x)\pm\log_a g(x)$

A.s. $\begin{cases} -1\leq f(x)\leq 1 \\ g(x)>0 \end{cases}$

10) $y=\dfrac{\arccos f(x)}{\log_a g(x)}$

A.s. $\begin{cases} -1\leq f(x)\leq 1 \\ g(x)>0, g(x)\neq 1 \end{cases}$

11) $y=\dfrac{\arccos f(x)\pm\sqrt{g(x)}}{\log_a p(x)}$

A.s. $\begin{cases} -1\leq f(x)\leq 1 \\ g(x)\geq 0 \\ p(x)>0, p(x)\neq 1 \end{cases}$

Ba'zi funksiyalarning qiymatlari sohasini topish usullari

I) $y=ax^2+bx+c$, $y_0=\dfrac{4ac-b^2}{4a}$

Q.s. 1) $a>0$ bo'lsa; $y\geq y_0$ 2) $a<0$ bo'lsa; $y\leq y_0$

II) $y=\sqrt{ax^2+bx+c}\Rightarrow y\geq 0$ va $y^2=ax^2+bx+c\Rightarrow$

$\Rightarrow ax^2+bx+c-y^2=0.$ $D=b^2-4a(c-y^2)$

$\begin{cases} y\geq 0 \\ D\geq 0 \end{cases}$ yoki $\begin{cases} y\geq 0 \\ b^2-4a(c-y^2)\geq 0 \end{cases}$ dan y (yani Q.s.) aniqlanadi.

Umumiy holda; $y=\sqrt{ax^2+bx+c}$ funksiya uchun:

1) $y_0>0, a>0$ da, $E(y)=[\sqrt{y_0};\infty)$

2) $y_0>0, a<0$ da, $E(y)=[0;\sqrt{y_0}]$

3) $y_0<0, a>0$ da, $E(y)=[0;\infty)$

4) $y_0<0, a<0$ da, $E(y)=\emptyset$

IV) $y = a^x$, $a>0, a\neq 1$; $E(y)=(0;\infty)$

V) $y = x^2 + \frac{1}{x^2}$ funktsiya uchun;

$$y = x^2 - 2 + \frac{1}{x^2} + 2 = \left(x - \frac{1}{x}\right)^2 + 2 \Rightarrow y - 2 = \left(x - \frac{1}{x}\right)^2 \Rightarrow$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 \geq 0 \Rightarrow y - 2 \geq 0 \Rightarrow \boxed{y \geq 2} \quad \text{Q.s; } E(y) \in [2;\infty)$$

VI) $y = ax^2 + \frac{b}{x^2}$, $(a>0, b>0)$ funktsiya uchun;

$$y = (\sqrt{a}x)^2 + \left(\frac{\sqrt{b}}{x}\right)^2 = (\sqrt{a}x)^2 - 2\sqrt{a}x \cdot \frac{\sqrt{b}}{x} + \left(\frac{\sqrt{b}}{x}\right)^2 + 2\sqrt{a}x \cdot \frac{\sqrt{b}}{x} =$$

$$= \left(\sqrt{a}x - \frac{\sqrt{b}}{x}\right)^2 + 2\sqrt{ab} \Rightarrow y - 2\sqrt{ab} = \left(\sqrt{a}x - \frac{\sqrt{b}}{x}\right)^2$$

$$\left(\sqrt{a}x - \frac{\sqrt{b}}{x}\right)^2 \geq 0 \Rightarrow y - 2\sqrt{ab} \geq 0 \Rightarrow \boxed{y \geq 2\sqrt{ab}}$$

Q.s. $E(y) = [2\sqrt{ab};\infty)$

VII) $y = \log_a(\arcsin x)$

$$\begin{cases} -1 \leq x \leq 1 \\ -\frac{\pi}{2} \leq \arcsin x \leq \frac{\pi}{2} \\ \arcsin x > 0 \end{cases} \Rightarrow 0 < \arcsin x \leq \frac{\pi}{2}$$

$a^y = \arcsin x \Rightarrow 0 < a^y \leq \frac{\pi}{2} \Rightarrow a > 0$ bo'lgani uchun; $a^y \leq \frac{\pi}{2}$

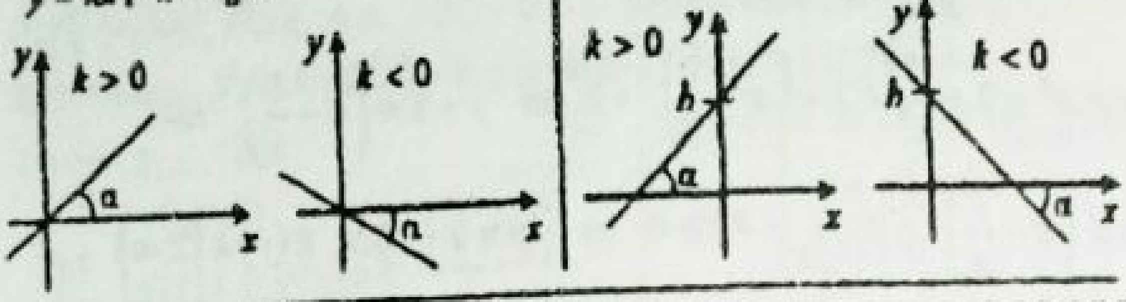
1-hol. $0 < a < 1 \Rightarrow \boxed{y \geq \log_a \frac{\pi}{2}}$ | 2-hol; $a > 1 \Rightarrow \boxed{y \leq \log_a \frac{\pi}{2}}$

Q.s. $E(y) \in \left[\log_a \frac{\pi}{2};\infty\right)$ | Q.s; $E(y) \in \left(-\infty;\log_a \frac{\pi}{2}\right]$

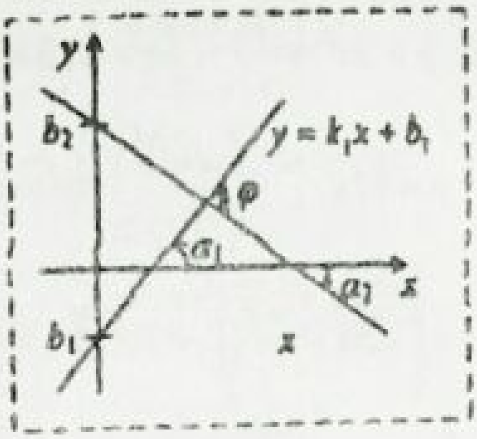
Chiziqli funktsiya. To'g'ri chiziq

1) To'g'ri chiziqning burchak koeffitsiyentli tenglamasi

$y = kx,\ k = tg\alpha$ $\qquad$ $y = kx + b,\ k = tg\alpha$



k – burchak koeffitsiyenti, b – ozod had, ya'ni to'g'ri chiziqning OY o'qi bilan kesishish nuqtasining ordinatasi.



- Ikki to'g'ri chiziqning kesishish nuqtasini topish; $\begin{cases} y = k_1x + b_1 \\ y = k_2x + b_2 \end{cases}$
- Ikki to'g'ri chiziq orasidagi burchakni topish; $k_1 = tg\alpha_1$ va $k_2 = tg\alpha_2$

$$tg\varphi = \left|\frac{k_2 - k_1}{1 + k_1 \cdot k_2}\right| \quad \text{yoki} \quad tg\alpha = \left|\frac{tg\alpha_2 - tg\alpha_1}{1 + tg\alpha_1 \cdot tg\alpha_2}\right|$$

Ba'zi shartlar

a) $y = k_1x + b_1$ va $y = k_2x + b_2$ to'g'ri chiziqlar uchun

1) Parallellik sharti; $k_1 = k_2$
2) Perpendikulyrlik sharti; $k_1 \cdot k_2 = -1$
3) Kesishish sharti; $k_1 \neq k_2$
4) Ustma-ust tushush sharti; $k_1 = k_2$ va $b_1 = b_2$

b) $y = kx + b$ to'g'ri chiziqqa simmetrik bo'lgan to'g'ri chiziqlar

1) OX o'qiga nisbatan; $y = -kx - b$
2) OY o'qiga nisbatan; $y = -kx + b$
3) $y = b$ to'g'ri chiziqqa nisbatan; $y = -kx + b$
4) $x = c$ to'g'ri chiziqqa nisbatan; $y = -kx + 2kc + b$
5) $y = x$ to'g'ri chiziqqa nisbatan. $y = \frac{1}{k}x - \frac{b}{k}$

10) To'g'ri chiziqning umumiy tenglamasi

$Ax+By+C=0$; A, B, C – to'g'ri chiziqning koeffitsiyentlari.

$A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ to'g'ri chiziqlar uchun;

1) Parallellik sharti: $\dfrac{A_1}{A_2}=\dfrac{B_1}{B_2}$

2) Perpendikulyarlik sharti;

$$\frac{A_1}{B_1}\cdot\frac{A_2}{B_2}=-1$$

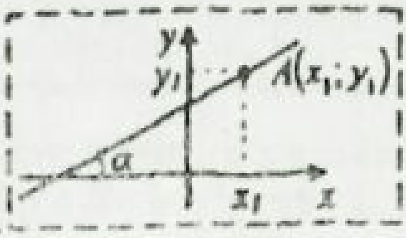
3) Kesishish sharti: $\dfrac{A_1}{A_2}\neq\dfrac{B_1}{B_2}$

4) Ustma-ust tushish sharti;

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}$$

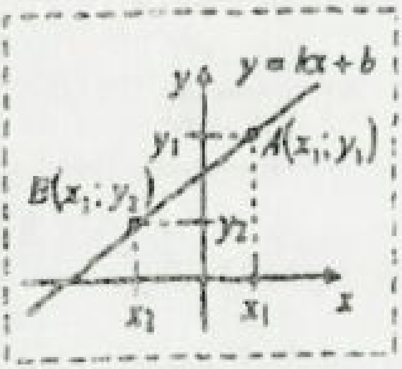
- $A(x_1;y_1)$ nuqtalardan o'tuvchi va OX o'qi bilan α burchak hosil qiluvchi to'g'ri chiziq tenglamasi;

$$y-y_1=k(x-x_1),\ \text{bu yerda } k=tg\alpha.$$



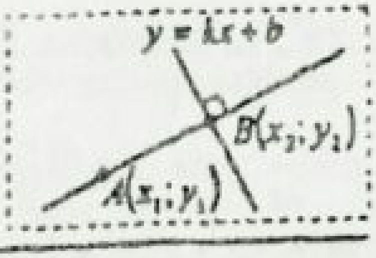
- $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi; $\dfrac{y-y_1}{y_2-y_1}=\dfrac{x-x_1}{x_2-x_1}$,

yoki $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalar izlanayotgan $y=kx+b$ to'g'ri chiziqqa tegishli bo'lgani uchun; $\begin{cases} y_1=kx_1+b \\ y_2=kx_2+b \end{cases}$ dan k va b topiladi.



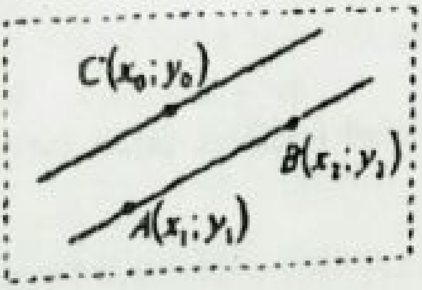
- $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalardan o'tuvchi to'g'ri chiziqqa perpendikulyar, va $B(x_2;y_2)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasi;

$$(x_1-x_2)\cdot(x-x_2)+(y_1-y_2)\cdot(y-y_2)=0$$



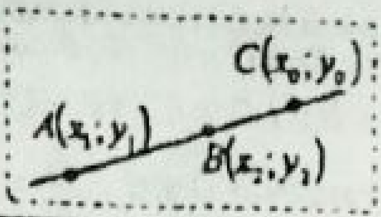
- $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalardan o'tuvchi to'g'ri chiziqqa parallel, hamda $C(x_0;y_0)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasi;

$$\frac{y-y_0}{y_2-y_1}=\frac{x-x_0}{x_2-x_1} \quad \text{yoki} \quad \frac{y-y_0}{y_1-y_2}=\frac{x-x_0}{x_1-x_2}$$



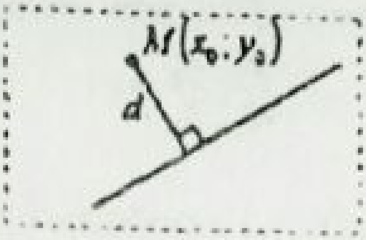
- $A(x_1;y_1)$, $B(x_2;y_2)$ va $C(x_3;y_3)$ nuqtalarning bir to'g'ri chiziqda yotish sharti;

$$\frac{x_3 - x_1}{x_2 - x_1} = \frac{y_3 - y_1}{y_2 - y_1}$$



- $M(x_0;y_0)$ nuqtadan $y = kx + b$ yoki $Ax + By + C = 0$ to'g'ri chiziqqacha masofa;

$$d = \frac{|kx_0 + b - y_0|}{\sqrt{k^2+1}} \quad \text{yoki} \quad d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2+B^2}}$$

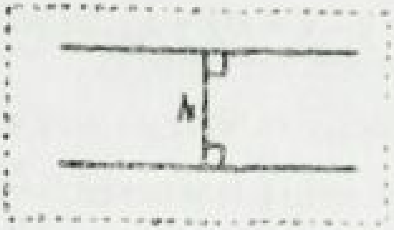


- Parllel to'g'ri chiziqlar orasidagi masofa;

1) $Ax + By + C_1 = 0$ 2) $y = kx + b_1$

$Ax + By + C_2 = 0$ $\quad y = kx + b_2$

$$h = \frac{|C_1 - C_2|}{\sqrt{A^2+B^2}} \qquad h = \frac{|b_1 - b_2|}{\sqrt{k^2+1}}$$



Kvadratik funktsiya

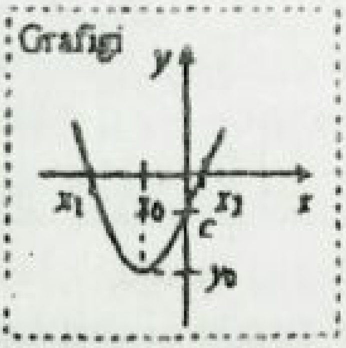
Umumiy ko'rinishi; $y = ax^2 + bx + c$, $a \neq 0$.

A.s. $D(y) = (-\infty;\infty)$, Q.s $\begin{cases} E(y) = [y_0;\infty) \text{, agar } a > 0 \text{ bo'lsa} \\ E(y) = (-\infty;y_0] \text{, agar } a < 0 \text{ bo'lsa} \end{cases}$

- **Grafigi paraboladan iborat**

x_0 – parabola uchining abstsissasi;

$$x_0 = -\frac{b}{2a} \quad \text{yoki} \quad x_0 = \frac{x_1 + x_2}{2} \; , \; x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$



x_1 va x_2 lar $y = ax^2 + bx + c$ kvadrat funktsiyaning OX o'qi bilan kesishish nuqtalari, ya'ni $ax^2 + bx + c = 0$ tenglamaning ildizlari $(x_1;0)$ va $(x_2;0)$ parabolaning OX o'qini kesish nuqtalarining koordinatalari bo'ladi.

y_0 - parabola uchining ordinasi; $y_0 = \frac{4ac - b^2}{4a}$ yoki $y_0 = ax_0^2 + bx_0 + c$

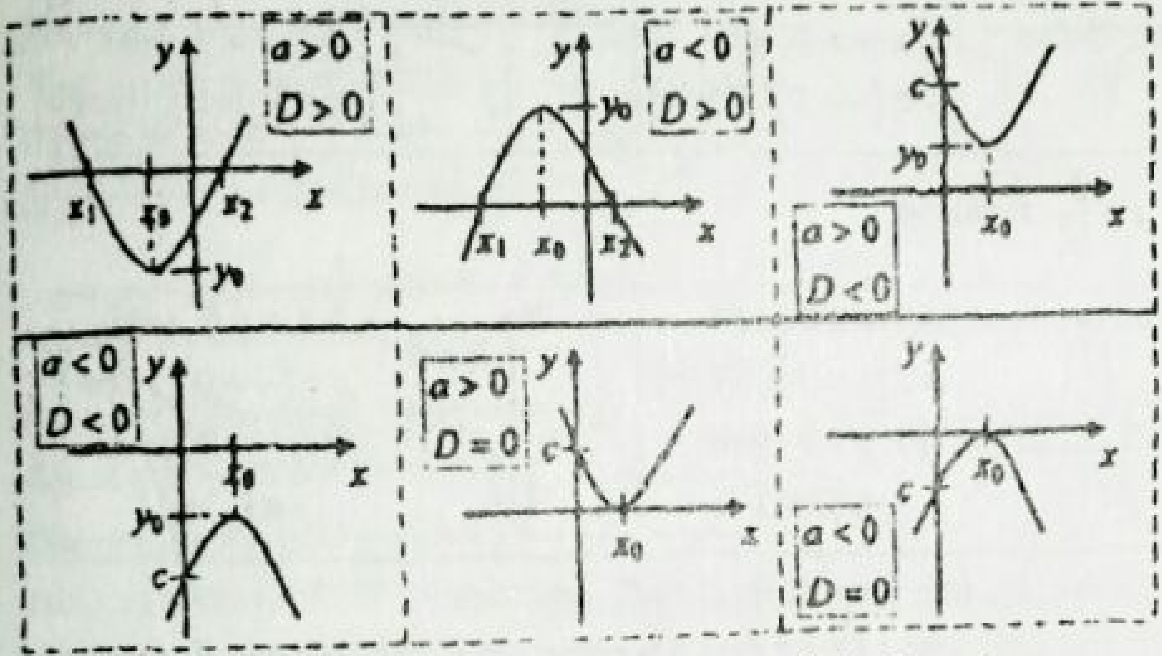
$(x_0 ; y_0)$ – parabola uchining koordinatasi.

c – ozod had. Parabolaning OY o'qini kesish nuqtasi bo'ladi.

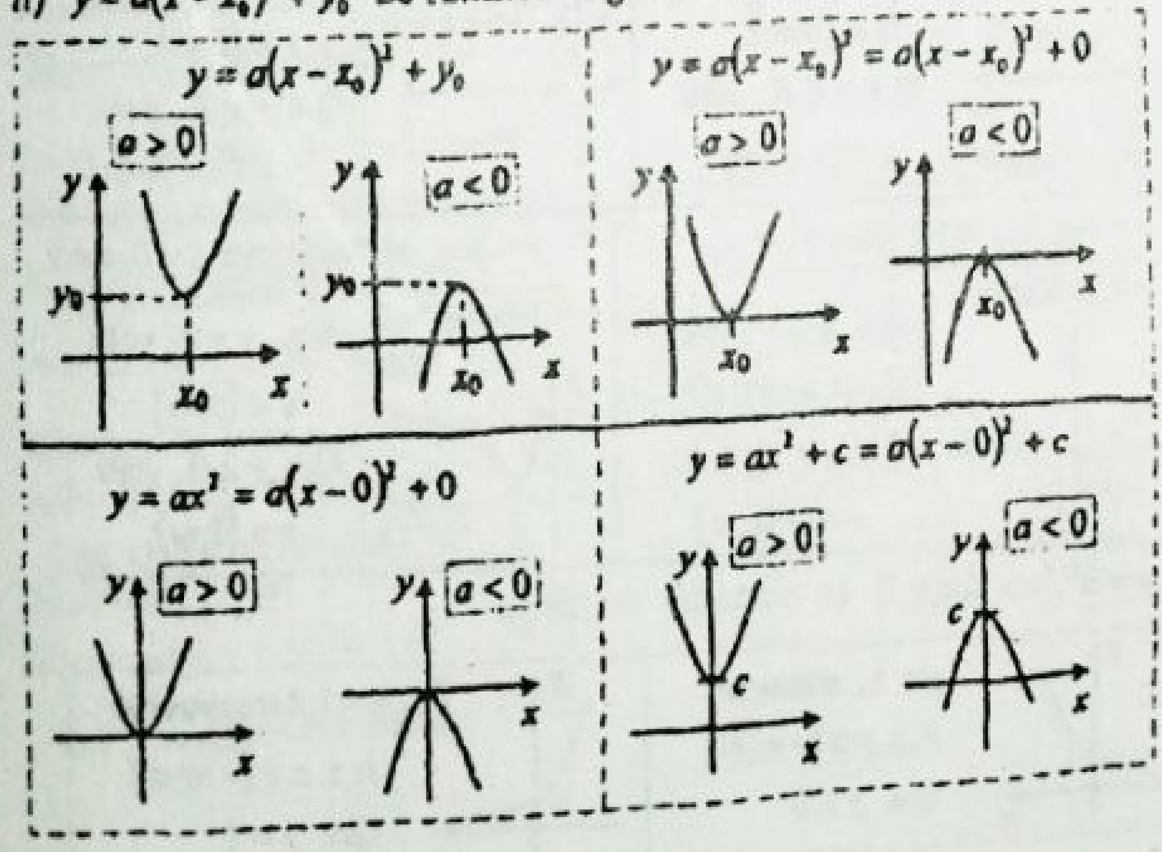
Kvadratik funksiyaning grafiklari

$$y = ax^2 + bx + c = a(x - x_1)(x - x_2) = a(x - x_0)^2 + y_0$$

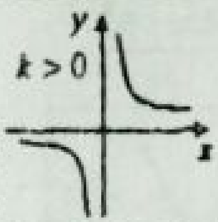
I) $y = ax^2 + bx + c$; $x_0 = -\frac{b}{2a} = \frac{x_1 + x_2}{2}$, $y_0 = ax_0^2 + bx_0 + c = \frac{4ac - b^2}{4a}$



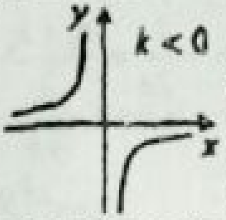
II) $y = a(x - x_0)^2 + y_0$ ko'rinishda kelgan kvadrat funktsiya uchun



- $y = \frac{k}{x}$ funktsiya

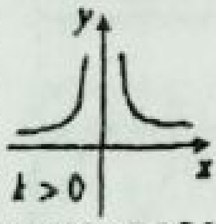


A.s. $x \neq 0$, yoki
$x \in (-\infty;0) \cup (0;\infty)$
Q.s. $y \neq 0$, yoki
$y \in (-\infty;0) \cup (0;\infty)$

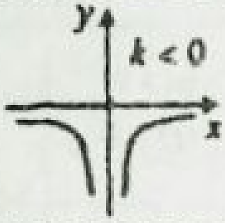


A.s. $x \neq 0$, yoki
$x \in (-\infty;0) \cup (0;\infty)$
Q.s. $y \neq 0$, yoki
$y \in (-\infty;0) \cup (0;\infty)$

- $y = \frac{k}{|x|}$ funktsiya

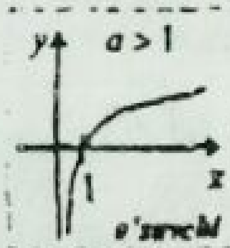


A.s. $x \neq 0$, yoki
$x \in (-\infty;0) \cup (0;\infty)$
Q.s. $y > 0$, yoki
$y \in (0;\infty)$

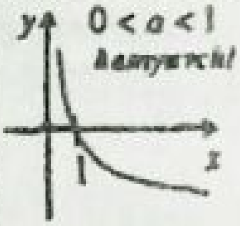


A.s. $x \neq 0$, yoki
$x \in (-\infty;0) \cup (0;\infty)$
Q.s. $y < 0$, yoki
$y \in (-\infty;0)$

- $y = \log_a x$, $(a > 0, a \neq 1)$ logarifmik funktsiya



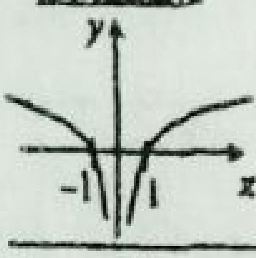
A.s. $x > 0$, yoki
$x \in (0;\infty)$
Q.s. $y \in R$, yoki
$y \in (-\infty;\infty)$



A.s. $x > 0$, yoki
$x \in (0;\infty)$
Q.s. $y \in R$, yoki
$y \in (-\infty;\infty)$

- $y = \log_a |x|$, $a > 1$ funktsiya

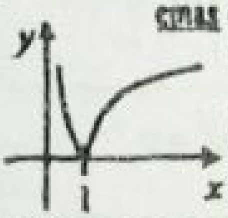
juft funktsiya



A.s. $x \neq 0$, yoki
$x \in (-\infty;0) \cup (0;\infty)$
Q.s. $y \in R$, yoki
$y \in (-\infty;\infty)$

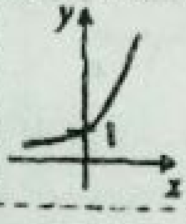
- $y = |\log_a x|$ funktsiya, $a > 0, a \neq 1$

juft ham toq ham emas

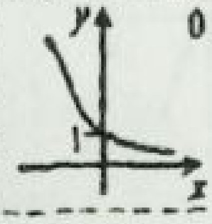


A.s. $x > 0$, yoki
$x \in (0;\infty)$
Q.s. $y \geq 0$, yoki
$y \in [0;\infty)$

- $y = a^x$, $(a > 0, a \neq 1)$ ko'rsatkichli funktsiya



$a > 1$, o'suvchi
A.s. $x \in (-\infty;\infty)$
Q.s. $y > 0$



$0 < a < 1$, kamayuvchi
A.s. $x \in (-\infty;\infty)$
Q.s. $y > 0$

Trigonometrik funktsiyalar

• $y = Sinx$ funktsiya

A.s. $x \in R$ yoki $x \in (-\infty;\infty)$

Q.s. $-1 \le y \le 1$ yoki $y \in [-1;1]$

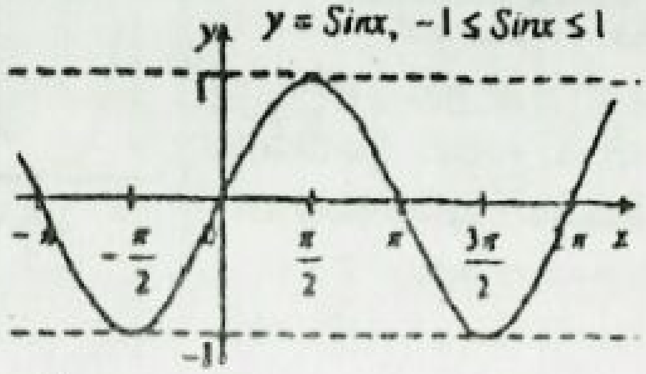
$Sin(-x) = -Sinx$ toq funksiya

Eng kichik musbat davri:

1) $y = Sinx$ uchun $T = 2\pi$

2) $y = Sinkx$, $y = Sin(kx + b)$

va $y = Sin(b - kx)$ uchun; $T = \frac{2\pi}{k}$



• $y = \cos x$ funktsiya

A.s. $x \in R$ yoki $x \in (-\infty;\infty)$

Q.s. $-1 \le y \le 1$ yoki $y \in [-1;1]$

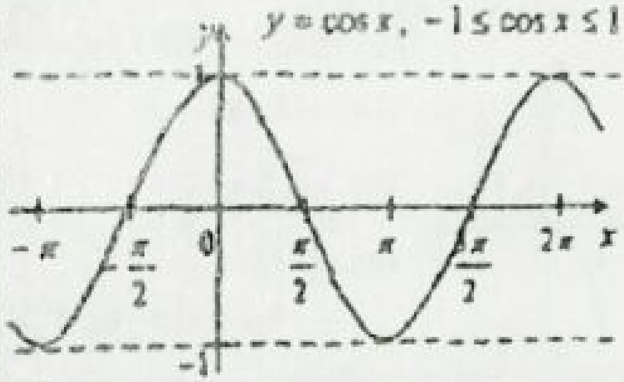
$\cos(-x) = \cos x$ juft funksiya.

Eng kichik musbat davri:

1) $y = \cos x$ uchun $T = 2\pi$

2) $y = \cos kx$, $y = \cos(kx + b)$

va $y = \cos(b - kx)$ uchun; $T = \frac{2\pi}{k}$



• $y = tgx$ funktsiya

A.s. $x \ne \frac{\pi}{2} + \pi n, n \in Z$

Q.s. $y \in R$, yoki $y \in (-\infty;\infty)$

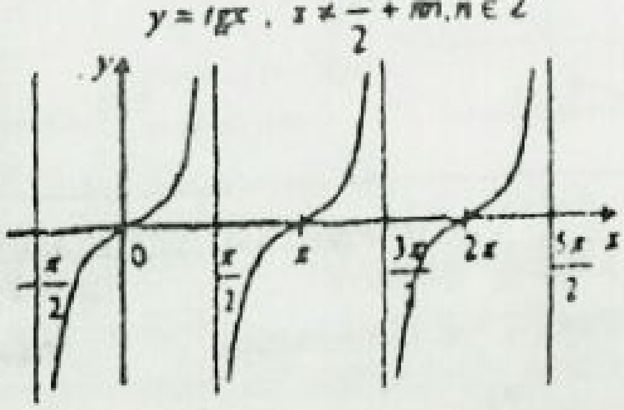
$tg(-x) = -tgx$ toq funksiya.

Eng kichik musbat davri:

1) $y = tgx$ uchun $T = \pi$

2) $y = tgkx$, $y = tg(kx + b)$

va $y = tg(b - kx)$ uchun; $T = \frac{\pi}{k}$



- $y = ctgx$ funktsiya

A.s. $x \neq \pi n, n \in Z$

Q.s. $y \in R$, yoki $y \in (-\infty; \infty)$

$ctg(-x) = -ctgx$ toq funksiya.

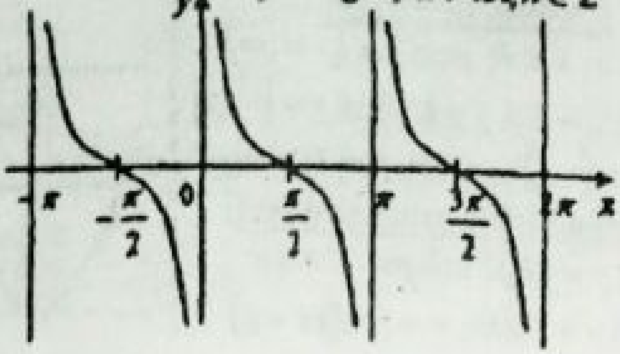
Eng kichik musbat davri:

1) $y = ctgx$ uchun $T = \pi$

2) $y = ctgkx, y = ctg(kx+b)$

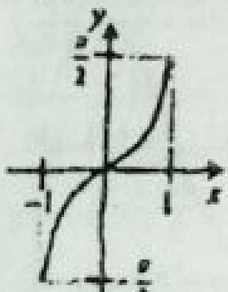
va $y = ctg(b-kx)$ uchun; $T = \frac{\pi}{k}$

$y = ctgx$, $x \neq \pi n, n \in Z$



Teskari trigonometrik funktsiyalar

I) $y = \arcsin x$ funktsiya



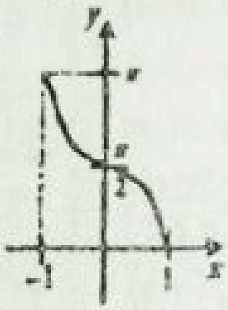
A.s $-1 \le x \le 1$

Q.s $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$\arcsin(-x) = -\arcsin x$

toq funktsiya.

II) $y = \arccos x$ funktsiya



A.s $-1 \le x \le 1$

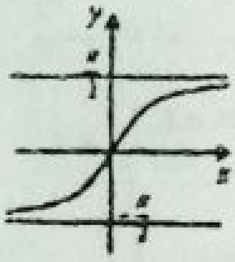
Q.s $0 \le y \le \pi$

$\arccos(-x) =$

$= \pi - \arccos x$

toq ham juft ham emas.

III) $y = arctgx$ funktsiya



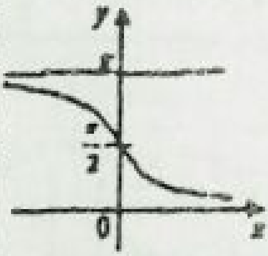
A.s $x \in R$

Q.s $-\frac{\pi}{2} < y < \frac{\pi}{2}$

$arctg(-x) = -arctgx$

toq funktsiya.

IV) $y = arcctgx$ funktsiya



A.s $x \in R$

Q.s $0 < y < \pi$

$arcctg(-x) =$

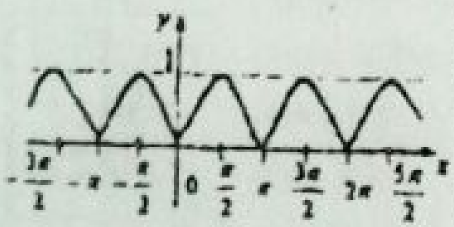
$= \pi - arcctgx$

toq ham, juft ham emas.

Trigonometrik funktsiyalarga qo'shimcha

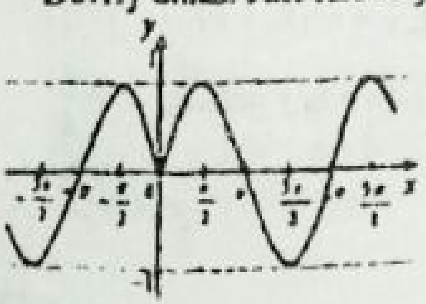
I) $y = |Sinx|$ funktsiya

Davri; $T = \pi$. Juft funktsiya

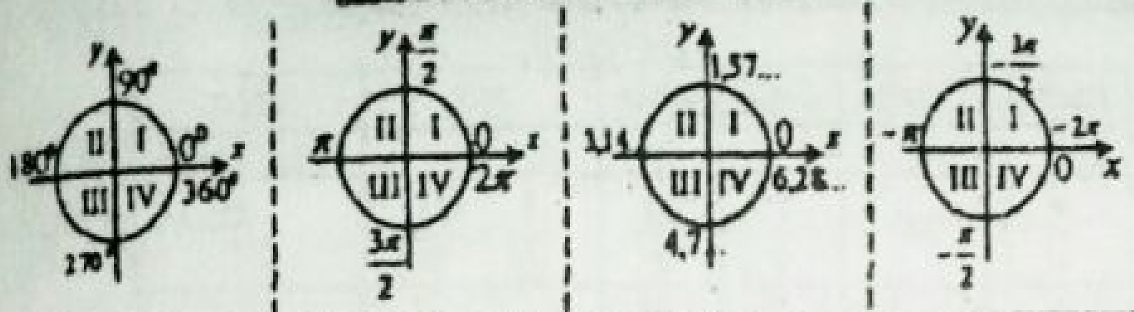


II) $y = Sin|x|$ funktsiya

Davriy emas. Juft funktsiya.

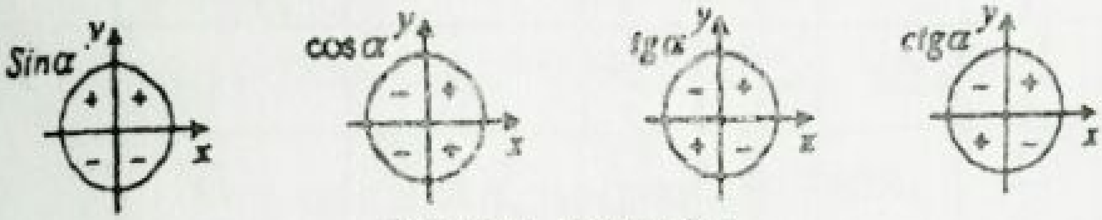


Trigonometriya formulalari



I - chorak	II - chorak	III - chorak	IV - chorak
$0^0 < \alpha < 90^0$	$90^0 < \alpha < 180^0$	$180^0 < \alpha < 270^0$	$270^0 < \alpha < 360^0$
$0 < \alpha < \frac{\pi}{2}$	$\frac{\pi}{2} < \alpha < \pi$	$\pi < \alpha < \frac{3\pi}{2}$	$\frac{3\pi}{2} < \alpha < 2\pi$
$0 < \alpha < 1{,}57...$	$1{,}57... < \alpha < 3{,}14...$	$3{,}14... < \alpha < 4{,}7...$	$4{,}7... < \alpha < 6{,}28...$

Trigonometrik funksiyalarning ishoralari



Keltirish formulalari

x	$\frac{\pi}{2}-\alpha$	$\frac{\pi}{2}+\alpha$	$\pi-\alpha$	$\pi+\alpha$	$\frac{3\pi}{2}-\alpha$	$\frac{3\pi}{2}+\alpha$	$2\pi-\alpha$
$Sinx$	$\cos x$	$\cos x$	$Sinx$	$-Sinx$	$-\cos x$	$-\cos x$	$-Sinx$
$\cos x$	$Sinx$	$-Sinx$	$-\cos x$	$-\cos x$	$-Sinx$	$Sinx$	$\cos x$
tgx	$ctgx$	$-ctgx$	$-tgx$	tgx	$ctgx$	$-ctgx$	$-tgx$
$ctgx$	tgx	$-tgx$	$-ctgx$	$ctgx$	tgx	$-tgx$	$-ctgx$

© Keltirish formulalariga qo'shimcha:

$2m =$ juft $\cdot \pi = j\pi$ hamda $(2n+1)\pi =$ toq $\cdot \pi = t\pi$,, $n \in N$ bo'lsin.

$Sin(j\pi \pm \alpha) = Sin(\pm\alpha)$	$\cos(j\pi \pm \alpha) = \cos\alpha$	$tg(n\pi \pm \alpha) = tg(\pm\alpha)$
$Sin(\alpha \pm j\pi) = Sin\alpha$	$\cos(\alpha \pm j\pi) = \cos\alpha$	$tg(\alpha \pm n\pi) = tg\alpha$
$Sin(t\pi \pm \alpha) = Sin(\pi \pm \alpha)$	$\cos(t\pi \pm \alpha) = \cos(\pi \pm \alpha)$	$ctg(n\pi \pm \alpha) = ctg(\pm\alpha)$
$Sin(\alpha \pm t\pi) = Sin(\alpha \pm \pi)$	$\cos(\alpha \pm t\pi) = \cos(\alpha \pm \pi)$	$ctg(\alpha \pm n\pi) = ctg\alpha$

M - n, $Sin\frac{71\pi}{3} = Sin\left(23\pi + \frac{2\pi}{3}\right) = Sin\left(\pi + \frac{2\pi}{3}\right) = -Sin\frac{2\pi}{3} = -\frac{\sqrt{3}}{2}$

Trigonometrik funktsiyalar uchun ba'zi burchaklarning qiymatlari

Radian	Gradus	$\sin\alpha$	$\cos\alpha$	$tg\alpha$	$ctg\alpha$
0	0°	0	1	0	mav. emas
$\frac{\pi}{6}$	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
$\frac{\pi}{3}$	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$
$\frac{\pi}{2}$	90°	1	0	mav. emas	0
$\frac{2\pi}{3}$	120°	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}=-\frac{\sqrt{3}}{3}$
$\frac{3\pi}{4}$	135°	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	-1
$\frac{5\pi}{6}$	150°	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}=-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$
π	180°	0	-1	0	mav. emas
$\frac{7\pi}{6}$	210°	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{5\pi}{4}$	225°	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	1
$\frac{4\pi}{3}$	240°	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$
$\frac{3\pi}{2}$	270°	-1	0	mav. emas	0
$\frac{5\pi}{3}$	300°	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}=-\frac{\sqrt{3}}{3}$
$\frac{7\pi}{4}$	315°	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	-1
$\frac{11\pi}{6}$	330°	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}=-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$
2π	360°	0	1	0	mav. emas

Trigonometrik funktsiyalarning ba'zi kichik burchaklardagi qiymatlari

Burchak grad.(rad)	$Sin x$	$\cos x$	$tg x$	$ctg x$
$15^{\circ}\left(\frac{\pi}{2}\right)$	$\frac{\sqrt{3}-1}{2\sqrt{2}}=\frac{\sqrt{2-\sqrt{3}}}{2}$	$\frac{\sqrt{3}+1}{2\sqrt{2}}=\frac{\sqrt{2+\sqrt{3}}}{2}$	$2-\sqrt{3}$	$2+\sqrt{3}$
$22{,}5^{\circ}\left(\frac{\pi}{8}\right)$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\sqrt{2}-1$	$\sqrt{2}+1$

Burchak o'lchovlari orasidagi munosabatlar

$\alpha\, rad = \frac{180^{0}}{\pi}\cdot\alpha$ – gradusga o'tish. M-n; $\frac{\pi}{9} rad = \frac{180^{0}}{\pi}\cdot\frac{\pi}{9} = 20^{0}$.

$\alpha^{\circ} = \frac{\pi}{180}\cdot\alpha\, rad$ – radianga o'tish. M-n; $45^{0} = \frac{\pi}{180}\cdot 45 = \frac{\pi}{4} rad$.

Asosiy trigonometrik formulalar

$Sin^2 x + \cos^2 x = 1$ $\quad Sin^2 x = 1 - \cos^2 x$ $\quad \cos^2 x = 1 - Sin^2 x$

$Sin x = \pm\sqrt{1-\cos^2 x}$ $\quad \cos x = \pm\sqrt{1 - Sin^2 x}$

$tg x = \frac{Sin x}{\cos x}$ $\quad Sin x = \cos x \cdot tg x$ $\quad \cos x = \frac{Sin x}{tg x}$

$ctg x = \frac{\cos x}{Sin x}$ $\quad \cos x = Sin x \cdot ctg x$ $\quad Sin x = \frac{\cos x}{ctg x}$

$tg x \cdot ctg x = 1$ $\quad tg x = \frac{1}{ctg x}$ $\quad ctg x = \frac{1}{tg x}$

$1 + tg^2 x = \frac{1}{\cos^2 x}$ $\quad \cos x = \pm\frac{1}{\sqrt{1+tg^2 x}}$ $\quad tg x = \pm\sqrt{\frac{1}{\cos^2 x} - 1}$

$1 + ctg^2 x = \frac{1}{Sin^2 x}$ $\quad Sin x = \pm\frac{1}{\sqrt{1+ctg^2 x}}$ $\quad ctg x = \pm\sqrt{\frac{1}{Sin^2 x} - 1}$

$\sec x = \frac{1}{\cos x}$ $\quad \operatorname{cosec} x = \frac{1}{Sin x}$

Burchaklar yig'indisi va ayirmasining formulalari

$Sin(x+y)=Sinx\cos y+\cos xSiny$	$\cos(x+y)=\cos x\cos y-SinxSiny$
$Sin(x-y)=Sinx\cos y-\cos xSiny$	$\cos(x-y)=\cos x\cos y+SinxSiny$
$tg(x+y)=\dfrac{tgx+tgy}{1-tgx\cdot tgy}$	$ctg(x+y)=\dfrac{ctgx\cdot ctgy-1}{ctgx+ctgy}$
$tg(x-y)=\dfrac{tgx-tgy}{1+tgx\cdot tgy}$	$ctg(x-y)=\dfrac{ctgx\cdot ctgy+1}{ctgy-ctgx}$

Yarim burchak formulalari

$$Sin\frac{x}{2}=\pm\sqrt{\frac{1-\cos x}{2}}\qquad \cos\frac{x}{2}=\pm\sqrt{\frac{1+\cos x}{2}}$$

$$tg\frac{x}{2}=\pm\sqrt{\frac{1-\cos x}{1+\cos x}}\qquad tg\frac{x}{2}=\frac{Sinx}{1+\cos x}=\frac{1-\cos x}{Sinx}$$

$$ctg\frac{x}{2}=\pm\sqrt{\frac{1+\cos x}{1-\cos x}}\qquad ctg\frac{x}{2}=\frac{Sinx}{1-\cos x}=\frac{1+\cos x}{Sinx}$$

Darajani pasaytirish formulalari

$$Sin^2x=\frac{1-\cos 2x}{2}\qquad 2Sin^2x=1-\cos 2x\qquad 1-\cos 2x=2Sin^2x$$

$$\cos^2x=\frac{1+\cos 2x}{2}\qquad 2\cos^2x=1+\cos 2x\qquad 1+\cos 2x=2\cos^2x$$

$$tg^2x=\frac{1-\cos 2x}{1+\cos 2x}\qquad ctg^2x=\frac{1+\cos 2x}{1-\cos 2x}$$

Daraja pasaytirishga qo'shimcha

$$Sin^3x=\frac{3Sinx-Sin3x}{4}\qquad Sin^4x=\frac{1}{8}(\cos 4x-4\cos 2x+3)$$

$$\cos^3x=\frac{3\cos x+\cos 3x}{4}\qquad \cos^4x=\frac{1}{8}(\cos 4x+4\cos 2x+3)$$

Karrali burchaklar

$Sin2x = 2Sinx\cos x \qquad 2Sinx\cos x = Sin2x \qquad Sinx\cos x = \frac{1}{2}Sin2x$

$\cos 2x = \cos^2 x - Sin^2 x \qquad \cos 2x = 2\cos^2 x - 1 \qquad \cos 2x = 1 - 2Sin^2 x$

Universal almashtirish formulalari

$$Sin2x = \frac{2tgx}{1+tg^2 x} \qquad \frac{2tgx}{1+tg^2 x} = Sin2x \qquad \frac{tgx}{1+tg^2 x} = \frac{1}{2}Sin2x$$

$$\cos 2x = \frac{1-tg^2 x}{1+tg^2 x} \qquad \frac{1-tg^2 x}{1+tg^2 x} = \cos 2x$$

$$tg2x = \frac{2tgx}{1-tg^2 x} \;\Big|\; \frac{2tgx}{1-tg^2 x} = tg2x \;\Big|\; \frac{tgx}{1-tg^2 x} = \frac{1}{2}\cdot\frac{2tgx}{1-tg^2 x} = \frac{1}{2}tg2x$$

$$ctg2x = \frac{ctg^2 x - 1}{2ctgx} \;\Big|\; \frac{ctg^2 x - 1}{2ctgx} = ctg2x \;\Big|\; \frac{ctg^2 x - 1}{ctgx} = 2\cdot\frac{ctg^2 x - 1}{2ctgx} = 2ctg2x$$

$$Sin3x = 3Sinx - 4Sin^3 x \qquad Sin4x = \cos x\left(4Sinx - 8Sin^3 x\right)$$

$$\cos 3x = 4\cos^3 x - 3\cos x \qquad \cos 4x = 8\cos^4 x - 8\cos^2 x + 1$$

$$tg3x = \frac{3tgx - tg^3 x}{1 - 3tg^2 x} \qquad ctg3x = \frac{ctg^3 x - 3ctgx}{3ctg^2 x - 1}$$

Qo'shimcha ma'lumot

$$1 - Sin2x = \left(Sinx - \cos x\right)^2 \qquad \left(Sinx - \cos x\right)^2 = \left(\cos x - Sinx\right)^2 = 1 - Sin2x$$

$$1 + Sin2x = \left(Sinx + \cos x\right)^2 \qquad \left(Sinx + \cos x\right)^2 = \left(\cos x + Sinx\right)^2 = 1 + Sin2x$$

$$\cos^4 x - Sin^4 x = \cos^2 x - Sin^2 x = \cos 2x$$

$$Sin^4 x + \cos^4 x = \frac{1+\cos^2 2x}{2} = \frac{2 - Sin^2 2x}{2} = \frac{3+\cos 4x}{4}$$

$$\cos^6 x + Sin^6 x = \frac{1}{8}\left(5 + 3\cos 4x\right) = \frac{1}{4}\left(1 + 3\cos^2 2x\right)$$

$$\cos^6 x - Sin^6 x = \frac{1}{16}\left(15\cos 2x + \cos 6x\right)$$

$$\cos^8 x - Sin^8 x = \frac{1}{4}\cos 2x\left(3 + 4\cos 4x\right)$$

$$tg\left(x+y+z\right) = \frac{tgx + tgy + tgz - tgx\cdot tgy\cdot tgz}{1 - \left(tgx\cdot tgy + tgy\cdot tgz + tgx\cdot tgz\right)}$$

Yig'indini ko'paytmaga keltirish

1. $\sin x+\sin y=2\sin\frac{x+y}{2}\cdot\cos\frac{x-y}{2}$

$\sin x-\sin y=2\sin\frac{x-y}{2}\cdot\cos\frac{x+y}{2}$

$\cos x+\cos y=2\cos\frac{x+y}{2}\cdot\cos\frac{x-y}{2}$

$\cos x-\cos y=-2\sin\frac{x-y}{2}\cdot\sin\frac{x+y}{2}$

$tgx+tgy=\frac{\sin(x+y)}{\cos x\cdot\cos y}$

$tgx-tgy=\frac{\sin(x-y)}{\cos x\cdot\cos y}$

$ctgx+ctgy=\frac{\sin(x+y)}{\sin x\cdot\sin y}$

$ctgx-ctgy=-\frac{\sin(x-y)}{\sin x\cdot\sin y}$

$tgx+ctgy=\frac{\cos(x-y)}{\cos x\cdot\sin y}$ $\qquad$ $ctgx-tgy=\frac{\cos(x+y)}{\sin x\cdot\cos y}$

Qo'shimcha:

$$\sin x+\cos y=\sin x+\sin\left(\frac{\pi}{2}-y\right)=2\sin\frac{x+\left(\frac{\pi}{2}-y\right)}{2}\cdot\cos\frac{x-\left(\frac{\pi}{2}-y\right)}{2}$$

$$\sin x+\cos y=\cos\left(\frac{\pi}{2}-x\right)+\cos y=2\cos\frac{\left(\frac{\pi}{2}-x\right)+y}{2}\cdot\cos\frac{\left(\frac{\pi}{2}-x\right)-y}{2}$$

$\sin x-\cos y=\sin x-\sin\left(\frac{\pi}{2}-y\right)$ yoki $\sin x-\cos y=\cos\left(\frac{\pi}{2}-x\right)-\cos y$

2. $\cos x+\sin x=\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)$

$\cos x-\sin x=-\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)=\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)$

$\sin x-\cos x=\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)=-\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)$

Qo'shimcha: $a\sin kx\pm b\cos kx=\sqrt{a^2+b^2}\sin(kx\pm\varphi)$

Bu yerda; $tg\varphi=\frac{a}{b}$, $\varphi=arctg\frac{a}{b}$.

3. $Sinx+\sqrt{3}\cos x=2Sin\left(x+\frac{\pi}{3}\right)=2\cos\left(x-\frac{\pi}{6}\right)$

$$Sinx-\sqrt{3}\cos x=2Sin\left(x-\frac{\pi}{3}\right)=-2\cos\left(x+\frac{\pi}{6}\right)$$

$$\sqrt{3}Sinx+\cos x=2Sin\left(x+\frac{\pi}{6}\right)=2\cos\left(x-\frac{\pi}{3}\right)$$

$$\sqrt{3}Sinx-\cos x=2Sin\left(x-\frac{\pi}{6}\right)=-2\cos\left(x+\frac{\pi}{3}\right)$$

$$\cos x-\sqrt{3}Sinx=-2Sin\left(x-\frac{\pi}{6}\right)=2\cos\left(x+\frac{\pi}{3}\right)$$

$$\sqrt{3}\cos x-Sinx=-2Sin\left(x-\frac{\pi}{3}\right)=2\cos\left(x+\frac{\pi}{6}\right)$$

Ko'paytmani yig'indiga keltirish

$$Sinx\cdot Siny=\frac{1}{2}\left(\cos(x-y)-\cos(x+y)\right)$$

$$\cos x\cdot\cos y=\frac{1}{2}\left(\cos(x-y)-\cos(x+y)\right)$$

$$Sinx\cdot\cos y=\frac{1}{2}\left(Sin(x-y)+Sin(x+y)\right)$$

$$Sin(x+y)\cdot Sin(x-y)=\cos^2 y-\cos^2 x$$

$$\cos(x-y)\cdot\cos(x+y)=\cos^2 y-Sin^2 x$$

$$tgx\cdot tgy=\frac{tgx+tgy}{ctgx+ctgy}$$

$$ctgx\cdot ctgy=\frac{ctgx+ctgy}{tgx+tgy}$$

$$\cos x\cdot\cos 2x=\frac{1}{2^2}\frac{Sin4x}{Sinx}$$

$$\cos x\cdot\cos 2x\cdot\cos 4x=\frac{1}{2^3}\frac{Sin8x}{Sinx}$$

$$\cos x\cdot\cos 2x\cdot\cos 4x\cdot\ldots\cdot\cos 2^n x=\frac{1}{2\cdot 2^n}\frac{Sin(2\cdot 2^n x)}{Sinx}$$

$$\cos x+\cos 2x+\cos 3x+\ldots+\cos nx=\frac{Sin\frac{nx}{2}\cdot\cos\frac{(n+1)x}{2}}{Sin\frac{x}{2}}$$

$$Sinx+Sin2x+Sin3x+\ldots+Sinkx=\frac{Sin\frac{kx}{2}\cdot Sin\frac{(k+1)x}{2}}{Sin\frac{x}{2}}$$

Teskari trigonometrik funksiyalar formulalari

$\arcsin a = \alpha$

$$-1 \le a \le 1 \quad \begin{cases} -\frac{\pi}{2} \le \alpha \le \frac{\pi}{2} \\ -90^\circ \le \alpha \le 90^\circ \\ -1{,}57... \le \alpha \le 1{,}57... \end{cases}$$

$\arcsin(-a) = -\arcsin a$

$\arccos a = \alpha$

$$-1 \le a \le 1 \quad \begin{cases} 0 \le \alpha \le \pi \\ 0^\circ \le \alpha \le 180^\circ \\ 0 \le \alpha \le 3{,}14... \end{cases}$$

$\arccos(-a) = \pi - \arccos a$

$\mathrm{arctg}\, a = \alpha$

$$a \in (-\infty; \infty) \quad \begin{cases} -\frac{\pi}{2} < \alpha < \frac{\pi}{2} \\ -90^\circ < \alpha < 90^\circ \\ -1{,}57... < \alpha < 1{,}57... \end{cases}$$

$\mathrm{arctg}(-a) = -\mathrm{arctg}\, a$

$\mathrm{arcctg}\, a = \alpha$

$$a \in (-\infty; \infty) \quad \begin{cases} 0 < \alpha < \pi \\ 0^\circ < \alpha < 180^\circ \\ 0 < \alpha < 3{,}14... \end{cases}$$

$\mathrm{arcctg}(-a) = \pi - \mathrm{arcctg}\, a$

$$\arcsin a + \arccos a = \frac{\pi}{2} \qquad \mathrm{arctg}\, a + \mathrm{arcctg}\, a = \frac{\pi}{2}$$

a	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\arcsin a$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\arccos a$	π	$\frac{5\pi}{6}$	$\frac{3\pi}{4}$	$\frac{2\pi}{3}$	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$	0

a	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$	0	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$
$\mathrm{arctg}\, a$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\mathrm{arcctg}\, a$	$\frac{5\pi}{6}$	$\frac{3\pi}{4}$	$\frac{2\pi}{3}$	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$

Teskari trigonometrik funksiyalarga qo'shimcha ma'lumot

Eslatma belgilar: $j\pi$ – $jufi \cdot \pi$ va $t\pi$ – $toq \cdot \pi$ deb hisoblang.

1. $\arcsin(Sin\alpha)=\alpha$, agar $\begin{cases} -\frac{\pi}{2}\le\alpha\le\frac{\pi}{2} \\ -90^\circ\le\alpha\le90^\circ \\ -1,57...\le\alpha\le1,57... \end{cases}$ bo'lsa

Agar α ko'rsatilgan oraliqqa tushmasa, quyidagi o'rinli bo'ladi:
$\arcsin(Sin\alpha)=\arcsin(Sin(\alpha\pm j\pi))$ yoki $\arcsin(Sin(t\pi-\alpha))$

$$Sin(\arccos x)=\sqrt{1-x^2}$$

$$Sin(arctgx)=\frac{x}{\sqrt{1+x^2}}$$

$$Sin(arcctgx)=\frac{1}{\sqrt{1+x^2}}$$

$$Sin\left(\frac{1}{2}\arccos x\right)=\sqrt{\frac{1-x}{2}}$$

$$Sin(2\arcsin x)=2x\sqrt{1-x^2}$$

$$Sin(2\arccos x)=2x\sqrt{1-x^2}$$

$$Sin(2arctgx)=\frac{2x}{1+x^2}$$

$$Sin(2arcctgx)=\frac{2x}{1+x^2}$$

$$\arcsin(\cos x)=\frac{\pi}{2}-x$$

---

2. $\arccos(\cos\alpha)=\alpha$, agar $\begin{cases} 0\le\alpha\le\pi \\ 0^\circ\le\alpha\le180^\circ \\ 0\le\alpha\le3,14... \end{cases}$ bo'lsa.

Agar α ko'rsatilgan oraliqqa tushmasa, quyidagi o'rinli bo'ladi:
$\arccos(\cos\alpha)=\arccos(\cos(\alpha\pm j\pi))$ yoki $\arccos(\cos(j\pi-\alpha))$

$$\cos(\arcsin x)=\sqrt{1-x^2}$$

$$\cos(arctgx)=\frac{1}{\sqrt{1+x^2}}$$

$$\cos(arcctgx)=\frac{x}{\pm\sqrt{1+x^2}}$$

$$\cos\left(\frac{1}{2}\arccos x\right)=\sqrt{\frac{1+x}{2}}$$

$$\cos(2\arccos x)=2x^2-1$$

$$\cos(2\arcsin x)=1-2x^2$$

$$\cos(2arctgx)=\frac{1-x^2}{1+x^2}$$

$$\cos(2arcctgx)=\frac{x^2-1}{x^2+1}$$

$$\arccos(\sin x)=\frac{\pi}{2}-x$$

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3. $arctg(tg\alpha)=\alpha$, agar $\begin{cases} -\frac{\pi}{2}<\alpha<\frac{\pi}{2} \\ -90^0<\alpha<90^0 \\ -1{,}57...<\alpha<1{,}57... \end{cases}$ bo'lsa

Agar α ko'rsatilgan oraliqqa tushmasa, quyidagi o'rinli bo'ladi:
$arctg(tg\alpha)=arctg(tg(\alpha\pm n\pi))$, $n\in N$

$$tg(\arcsin x)=\frac{x}{\sqrt{1-x^2}} \qquad tg(2arcctgx)=\frac{2x}{1-x^2}$$

$$tg(\arccos x)=\frac{\sqrt{1-x^2}}{x} \qquad tg(2\arcsin x)=\frac{2x\sqrt{1-x^2}}{1-2x^2}$$

$$tg(arcctgx)=\frac{1}{x} \qquad tg(2\arccos x)=\frac{2x\sqrt{1-x^2}}{2x^2-1}$$

$$arctg(ctgx)=\frac{\pi}{2}-x \qquad tg(2arcctgx)=\frac{2x}{x^2-1}$$

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4. $arcctg(ctg\alpha)=\alpha$, agar $\begin{cases} 0<\alpha<\pi \\ 0^0<\alpha<180^0 \\ 0<\alpha<3{,}14... \end{cases}$ bo'lsa

Agar α ko'rsatilgan oraliqqa tushmasa, quyidagi o'rinli bo'ladi:
$arcctg(ctg\alpha)=arcctg(ctg(\alpha\pm n\pi))$, $n\in N$

$$ctg(\arcsin x)=\frac{\sqrt{1-x^2}}{x} \qquad ctg(2arcctgx)=\frac{x^2-1}{2x}$$

$$ctg(\arccos x)=\frac{x}{\sqrt{1-x^2}} \qquad ctg(2\arcsin x)=\frac{2x\sqrt{1-x^2}}{1-2x^2}$$

$$ctg(arctgx)=\frac{1}{x} \qquad ctg(2\arccos x)=\frac{2x^2-1}{2x\sqrt{1-x^2}}$$

$$arcctg(tgx)=\frac{\pi}{2}-x \qquad ctg(2arctgx)=\frac{1-x^2}{2x}$$

Trigonometrik tenglamalar

I) $\sin x = a$, $-1 \le a \le 1$

$x = (-1)^n \arcsin a + \pi n$, $n \in Z$

b musbat son bo'lganda $(0 < b < 1)$

a) $\sin x = b$

$x = (-1)^n \arcsin b + \pi n$, $n \in Z$

b) $\sin x = -b$

$x = (-1)^{n+1} \arcsin b + \pi n$, $n \in Z$

c) $\sin x = \pm b$

$x = \pm \arcsin b + \pi n, n \in Z$

d) $\sin x = \pm 1 \Leftrightarrow \cos x = 0$

$x = \frac{\pi}{2} + \pi n$, $n \in Z$

Ba'zi xususiy hollar:

1) $\sin x = -1$

$x = -\frac{\pi}{2} + 2\pi n, n \in Z$

2) $\sin x = 0$

$x = \pi n, n \in Z$

3) $\sin x = 1$

$x = \frac{\pi}{2} + 2\pi n, n \in Z$

II) $\cos x = a$, $-1 \le a \le 1$

$x = \pm \arccos a + 2\pi n$, $n \in Z$

b musbat son bo'lganda $(0 < b < 1)$

a) $\cos x = b$

$x = \pm \arccos b + 2\pi n$, $n \in Z$

b) $\cos x = -b$

$x = \pm(\pi - \arccos b) + 2\pi n$, $n \in Z$

c) $\cos x = \pm b$

$x = \pm \arccos b + \pi n$, $n \in Z$

d) $\cos x = \pm 1 \Leftrightarrow \sin x = 0$

$x = \pi n$, $n \in Z$

Ba'zi xususiy hollar:

1) $\cos x = -1$

$x = \pi + 2\pi n, n \in Z$

2) $\cos x = 0$

$x = \frac{\pi}{2} + \pi n, n \in Z$

3) $\cos x = 1$

$x = 2\pi n, n \in Z$

III) $tgx = a$, $a \in R$

$x = arctga + \pi n, n \in Z$

$tgx = \pm b$, $b > 0$

$x = \pm arctgb + \pi n, n \in Z$

IV) $ctgx = a$, $a \in R$

$x = arcctga + \pi n, n \in Z$

$ctgx = \pm b, b > 0$

$x = \pm arcctgb + \pi n, n \in Z$

Eng sodda trigonometrik tengsizliklar

$y = \cos x$ funksiya uchun

I) $\cos x < a$, $-1 \le a \le 1$, $\arccos a + 2\pi n < x < 2\pi - \arccos a + 2\pi n$, $n \in Z$

$\cos x > a$, $-1 \le a \le 1$, $-\arccos a + 2\pi n < x < \arccos a + 2\pi n$, $n \in Z$

Ba'zi xususiy hollar

1) $\cos x \ge 1$

$-1 \le \cos x \le 1$

$\cos x = 1$

$x = 2\pi n, n \in Z$

2) $\cos x > 1$

$-1 \le \cos x \le 1$

$x \in \phi$

3) $\cos x \le 1$

$-1 \le \cos x \le 1$

$x \in R$

4) $\cos x < 1$

$-1 \le \cos x \le 1$

$\cos x \ne 1$

$x \ne 2\pi n, n \in Z$

5) $\cos x \ge -1$

$-1 \le \cos x \le 1$

$x \in R$

6) $\cos x > -1$

$-1 \le \cos x \le 1$

$\cos x \ne -1$

$x \ne \pi + 2\pi n, n \in Z$

7) $\cos x \le -1$

$-1 \le \cos x \le 1$

$\cos x = -1$

$x = \pi + 2\pi n, n \in Z$

8) $\cos x < -1$

$-1 \le \cos x \le 1$

$x \in \phi$

9) $\cos x > 2$

$-1 \le \cos x \le 1$

$x \in \phi$

10) $\cos x < 2$

$-1 \le \cos x \le 1$

$x \in R$

11) $\cos x > -2$

$-1 \le \cos x \le 1$

$x \in R$

12) $\cos x < -2$

$-1 \le \cos x \le 1$

$x \in \phi$

II) a) $a < \cos x < b$, $-1 \le a < b \le 1$

$-\arccos a + 2\pi n < x < -\arccos b + 2\pi n$ va

$\arccos b + 2\pi n < x < \arccos a + 2\pi n$, $n \in Z$

b) $\cos x < a$, $\cos x > b$, $-1 \le a < b \le 1$

$-\arccos b + 2\pi n < x < \arccos b + 2\pi n$ va

$\arccos a + 2\pi n < x < 2\pi - \arccos a + 2\pi n$, $n \in Z$

III) a) $-a < \cos x < a$, $0 < a < 1$

$\arccos a + \pi n < x < \pi - \arccos a + \pi n$, $n \in Z$

b) $\cos x < -a$, $\cos x > a$

$-\arccos a + \pi n < x < \arccos a + \pi n$, $n \in Z$

$y = \sin x$ funksiya uchun

I) $\sin x < a$, $-1 \le a \le 1$, $-\pi - \arcsin a + 2\pi n < x < \arcsin a + 2\pi n$, $n \in Z$

$\sin x > a$, $-1 \le a \le 1$, $\arcsin a + 2\pi n < x < \pi - \arcsin a + 2\pi n$, $n \in Z$

Ba'zi xususiy hollar

1) $\sin x \ge 1$

$-1 \le \sin x \le 1$

$\sin x = 1$

$x = \frac{\pi}{2} + 2\pi n, n \in Z$

2) $\sin x > 1$

$-1 \le \sin x \le 1$

$x \in \phi$

3) $\sin x \le 1$

$-1 \le \sin x \le 1$

$x \in R$

4) $\sin x < 1$

$-1 \le \sin x \le 1$

$\sin x \ne 1$

$x \ne \frac{\pi}{2} + 2\pi n, n \in Z$

5) $\sin x \ge -1$

$-1 \le \sin x \le 1$

$x \in R$

6) $\sin x > -1$

$-1 \le \sin x \le 1$

$\sin x \ne -1$

$x \ne -\frac{\pi}{2} + 2\pi n, n \in Z$

7) $\sin x \le -1$

$-1 \le \sin x \le 1$

$\sin x = -1$

$x = -\frac{\pi}{2} + 2\pi n, n \in Z$

8) $\sin x < -1$

$-1 \le \sin x \le 1$

$x \in \phi$

9) $\sin x > 2$

$-1 \le \sin x \le 1$

$x \in \phi$

10) $\sin x < 2$

$-1 \le \sin x \le 1$

$x \in R$

11) $\sin x > -2$

$-1 \le \sin x \le 1$

$x \in R$

12) $\sin x < -2$

$-1 \le \sin x \le 1$

$x \in \phi$

II) a) $a < \sin x < b$, $-1 \le a < b \le 1$

$\arcsin a + 2\pi n < x < \arcsin b + 2\pi n$ va

$\pi - \arcsin b + 2\pi n < x < \pi - \arcsin a + 2\pi n$, $n \in Z$

b) $\sin x < a$, $\sin x > b$, $-1 \le a < b \le 1$

$-\pi - \arcsin a + 2\pi n < x < \arcsin a + 2\pi n$ va

$\arcsin b + 2\pi n < x < \pi - \arcsin b + 2\pi n$, $n \in Z$

III) a) $-a < \sin x < a$, $0 < a < 1$

$-\arcsin a + \pi n < x < \arcsin a + \pi n$, $n \in Z$

b) $\sin x < -a$, $\sin x > a$

$\arcsin a + \pi n < x < \pi - \arcsin a + \pi n$, $n \in Z$

$y = tgx$ funktsiya uchun

I) $tgx \ge a,\ a \in R \Rightarrow arctga + \pi n \le x < \frac{\pi}{2} + \pi n,\ n \in Z$

$tgx \le a,\ a \in R \Rightarrow -\frac{\pi}{2} + \pi n < x \le arctga + \pi n,\ n \in Z$

II) a) $a \le arctgx \le b,\ a < b$

$arctga + \pi n \le x \le arctgb + \pi n, n \in Z$

b) $tgx \le a,\ tgx \ge b,\ a < b$

$-\frac{\pi}{2} + \pi n < x \le arctga + \pi n$ va $arctgb + \pi n \le x < \frac{\pi}{2} + \pi n,\ n \in Z$

$y = ctgx$ funktsiya uchun

I) $ctgx \ge a,\ a \in R \Rightarrow \pi n < x \le arcctga + \pi n,\ n \in Z$

$ctgx \le a,\ a \in R \Rightarrow arcctga + \pi n \le x < \pi + \pi n,\ n \in Z$

II) a) $a \le ctgx \le b,\ a < b$

$arcctgb + \pi n \le x \le arcctga + \pi n, n \in Z$

b) $ctgx \le a,\ ctgx \ge b,\ a < b$

$\pi n < x \le arcctgb + \pi n$ va $arcctga + \pi n \le x < \pi + \pi n,\ n \in Z$

Teskari trigonometrik funktsiyalarga doir qo'shimcha ma'lumot

$$arctgx + arctgy = \begin{cases} arctg\frac{x+y}{1-xy}, & \text{agar } xy < 1 \\ \pi + arctg\frac{x+y}{1-xy}, & \text{agar } x > 0,\ xy > 1 \\ -\pi + arctg\frac{x+y}{1-xy}, & \text{agar } x < 0,\ xy > 1 \end{cases}$$

$$arctgx - arctgy = \begin{cases} arctg\frac{x-y}{1+xy}, & \text{agar } xy > -1 \\ \pi + arctg\frac{x-y}{1+xy}, & \text{agar } x > 0,\ xy < -1 \\ -\pi + arctg\frac{x-y}{1+xy}, & \text{agar } x < 0,\ xy < -1 \end{cases}$$

Ba'zi murakkabroq ko'rinishdagi trigonometrik tenglamarni yechishga yordam

I) $Sin^m x + cos^n x = 1$; $m > 2$, $n > 2$; $m, n \in N$

1) m – juft, n – juft

$\begin{cases} Sinx = \pm 1 \\ cos x = 0 \end{cases}$ va $\begin{cases} Sinx = 0 \\ cos x = \pm 1 \end{cases}$

$x = \frac{\pi k}{2}$, $k \in Z$

2) m – juft, n – toq

$\begin{cases} Sinx = \pm 1 \\ cos x = 0 \end{cases}$ va $\begin{cases} Sinx = 0 \\ cos x = 1 \end{cases}$

$x_1 = \frac{\pi}{2} + \pi k$; $x_1 = 2\pi k$, $k \in Z$

3) m – toq, n – juft

$\begin{cases} Sinx = 1 \\ cos x = 0 \end{cases}$ va $\begin{cases} Sinx = 0 \\ cos x = \pm 1 \end{cases}$

$x_1 = \frac{\pi}{2} + 2\pi k$; $x_2 = \pi k$, $k \in Z$

4) m – toq, n – toq

$\begin{cases} Sinx = 1 \\ cos x = 0 \end{cases}$ va $\begin{cases} Sinx = 0 \\ cos x = 1 \end{cases}$

$x_1 = \frac{\pi}{2} + 2\pi k$; $x_1 = 2\pi k$, $k \in Z$

II) $a \in N, b \in N, a \neq b$

1) $aSinmx + b\cos nx = a + b$

$Sinmx = 1$ | $\cos nx = 1$

2) $aSinmx - b\cos nx = a + b$

$Sinmx = 1$ | $\cos nx = -1$

III) $m \in N, n \in N, m \neq n$

1) $Sinmx \cdot \cos nx = 1$

$\begin{cases} Sinmx = 1 \\ \cos nx = 1 \end{cases}$ yoki $\begin{cases} Sinmx = -1 \\ \cos nx = -1 \end{cases}$

2) $Sinmx \cdot \cos nx = -1$

$\begin{cases} Sinmx = 1 \\ \cos nx = -1 \end{cases}$ yoki $\begin{cases} Sinmx = -1 \\ \cos nx = 1 \end{cases}$

IV) $\cos x \cdot \cos 2x \cdot \cos 4x = 1$ uchun $\cos x = 1$, $x = 2\pi n, n \in Z$

Trigonometrik tenglama va tengsizliklarning ko'rsatilgan oraliqlardagi yechimlari

I) $Sinx = a$ tenglamaning $[b;c]$ oraliqdagi yechimlarini topish uchun, $x_1 = \arcsin a + 2\pi n$ | $x_2 = \pi - \arcsin a + 2\pi n$, $n \in Z$ deb olinadi.

II) $\cos x = a$ tenglamaning $[b;c]$ oraliqdagi yechimlarini topish uchun, $x_1 = -\arccos a + 2\pi n$ | $x_2 = \arccos a + 2\pi n$, $n \in Z$ deb olinadi.

III) $tgx = a$, $[b;c]$ uchun $x = arctga + \pi n$, $n \in Z$

IV) $ctgx = a$, $[b;c]$ uchun $x = arcctga + \pi n$, $n \in Z$

Misollar:

1) $Sin2x = \frac{\sqrt{3}}{2}$ tenglamaning $[-\pi;\pi]$ oraliqdagi barcha ildizlarini toping.

$$2x = \arcsin\frac{\sqrt{3}}{2} + 2\pi n \qquad\qquad 2x = \pi - \arcsin\frac{\sqrt{3}}{2} + 2\pi n$$

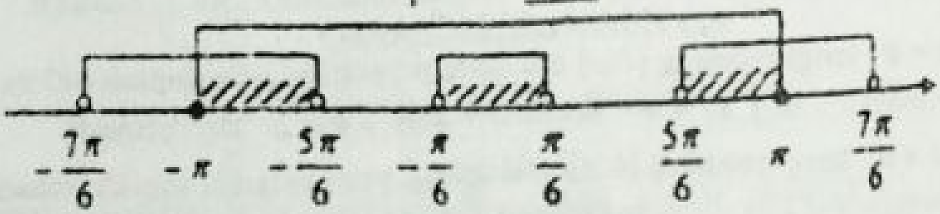
	n	x		n	x
$2x = \frac{\pi}{3} + 2\pi n$	0	$\frac{\pi}{6}$	$2x = \pi - \frac{\pi}{3} + 2\pi n$	0	$\frac{\pi}{3}$
$2x = \frac{\pi + 6\pi n}{3}$	1	$\frac{7\pi}{6}\phi$	$2x = \frac{2\pi + 6\pi n}{3}$	1	$\frac{4\pi}{3}\phi$
$2x = \frac{(6n+1)\pi}{3}$	-1	$-\frac{5\pi}{6}$	$x = \frac{\pi + 3\pi n}{3}$	-1	$-\frac{2\pi}{3}$
$x = \frac{(6n+1)\pi}{6}$	-2	$-\frac{11\pi}{6}\phi$	$x = \frac{(3n+1)\pi}{3}$	-2	$-\frac{5\pi}{3}\phi$

Javob. $-\frac{5\pi}{6}, -\frac{2\pi}{3}, \frac{\pi}{6}, \frac{\pi}{3}$

2) $\cos 2x > \frac{1}{2}$ tengsizlikning $[-\pi;\pi]$ oraliqdagi yechimlarini toping.

$$-\arccos\frac{1}{2} + 2\pi n < 2x < \arccos\frac{1}{2} + 2\pi n, \ n \in Z$$

	n	x
$-\frac{\pi}{3} + 2\pi n < 2x < \frac{\pi}{3} + 2\pi n$	0	$-\frac{\pi}{6} < x < \frac{\pi}{6}$
$\frac{-\pi + 6\pi n}{3} < 2x < \frac{\pi + 6\pi n}{3}$	1	$\frac{5\pi}{6} < x < \frac{7\pi}{6}$
$\frac{(6n-1)\pi}{6} < x < \frac{(6n+1)\pi}{6}$	-1	$-\frac{7\pi}{6} < x < -\frac{5\pi}{6}$



Javob. $-\pi \le x < -\frac{5\pi}{6}$, $-\frac{\pi}{6} < x < \frac{\pi}{6}$, $\frac{5\pi}{6} < x \le \pi$

Yoki. $\left[-\pi; -\frac{5\pi}{6}\right) \cup \left(-\frac{\pi}{6}; \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}; \pi\right]$

HOSILA

$y = f(x)$ funktsiyaning hosilasi; $y' = f'(x) = \lim\limits_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$f(x+\Delta x) - f(x)$ – funktsiya orttirmasi; Δx – argument orttirmasi

Hosilalar jadvali

$C' = 0$, $x' = 1$, $(Cx)' = C$, $C \neq 0$	$(\sin x)' = \cos x$
$(x^p)' = p \cdot x^{p-1}$	$(\sin(g(x)))' = \cos(g(x)) \cdot g'(x)$
$(g^p(x))' = p \cdot g^{p-1}(x) \cdot g'(x)$	$(\cos x)' = -\sin x$
$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$	$(\cos(g(x)))' = -\sin(g(x)) \cdot g'(x)$
$\left(\frac{1}{g(x)}\right)' = -\frac{1}{g^2(x)} g'(x)$	$(tgx)' = \frac{1}{\cos^2 x}$
$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$	$(tg(g(x)))' = \frac{1}{\cos^2(g(x))} \cdot g'(x)$
$(\sqrt{g(x)})' = \frac{1}{2\sqrt{g(x)}} \cdot g'(x)$	$(ctgx)' = -\frac{1}{\sin^2 x}$
$(a^x)' = a^x \cdot \ln a$	$(ctg(g(x)))' = -\frac{1}{\sin^2(g(x))} \cdot g'(x)$
$(a^{g(x)})' = a^{g(x)} \cdot \ln a \cdot g'(x)$	$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$
$(e^x)' = e^x$	$(\arcsin(g(x)))' = \frac{1}{\sqrt{1-g^2(x)}} \cdot g'(x)$
$(e^{g(x)})' = e^{g(x)} \cdot g'(x)$	$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$
$(\log_a x)' = \frac{1}{x \cdot \ln a}$	$(\arccos(g(x)))' = -\frac{1}{\sqrt{1-g^2(x)}} \cdot g'(x)$
$(\log_a g(x)) = \frac{1}{g(x) \cdot \ln a} \cdot g'(x)$	$(arctgx)' = \frac{1}{1+x^2}$
$(\ln x)' = \frac{1}{x}$	$(arctg(g(x)))' = \frac{1}{1+g^2(x)} g'(x)$
$(\ln g(x))' = \frac{1}{g(x)} g'(x)$	

$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$

$(\operatorname{arcctg}(g(x)))' = -\frac{1}{1+g^2(x)}\cdot g'(x)$

Agar $F'(x) = f(x)$ bo'lsa;

$F'(kx) = k\cdot f(kx)$

M: $F'(-2x) = f(-2x)\cdot(-2x)' = -2f(-2x)$

• Hosila olish qoidalari

$(C\cdot f(x))' = C\cdot f'(x)$, $C\neq 0$

$(f(x)\cdot g(x))' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

$(f(x)\pm g(x))' = f'(x)\pm g'(x)$

$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{g^2(x)}$

• Hosila olishga doir misollar

1) $(\cos(x^3+2))' = -\sin(x^3+2)\cdot(x^3+2)' = -3x^2\sin(x^3+2)$

2) $(\ln^4(3x^5+7))' = 4\ln^3(3x^5+7)\cdot(\ln(3x^5+7))' = 4\ln^3(3x^5+7)\times$

$\times\frac{1}{3x^5+7}\cdot(3x^5+7)' = 4\ln^3(3x^5+7)\cdot\frac{1}{3x^5+7}\cdot 15x^4 = \frac{60x^4\cdot\ln^3(3x^5+7)}{3x^5+7}$

• Hosila yordamida funktsiyaning ba'zi xossalarini aniqlash

I. $y = f(x)$ funktsiyaning o'sish va kamayish oraliqlarini topish

a) $f'(x)\geq 0$ da o'sadi b) $f'(x)\leq 0$ da kamayadi

II. $y = f(x)$ funktsiyaning statsionar (kritik) nuqtalarini topish

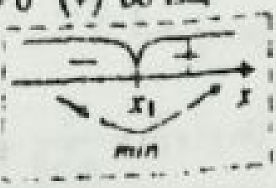
x ning funktsiya hosilasi nolga teng bo'ladigan qiymatlariga aytiladi, ya'ni; $f'(x) = 0$ tenglamaning ildizlariga aytiladi.

III. $y = f(x)$ funktsiyaning ekstremum (*max* yoki *min*) nuqtalari

$f'(x) = 0$ tenglamaning ildizi x_1 bo'lsin.

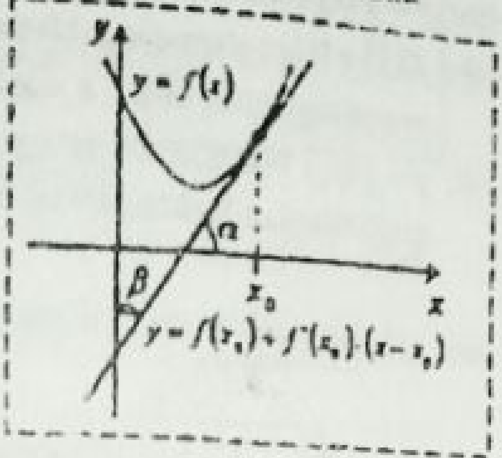
a) Agar; $x < x_1$ da $f'(x) > 0$ (+) va $x > x_1$ da $f'(x) < 0$ (–) bo'lsa, $x = x_1$ funktsiyaning maksimum (*max*) nuqtasi deyiladi; $y = f(x_1)$ funktsiyaning maksimumi, yoki funktsiyaning maksimum nuqtasidagi qiymati deyiladi.

b) Agar; $x < x_1$ da $f'(x) < 0$ (–) va $x > x_1$ da $f'(x) > 0$ (+) bo'lsa, $x = x_1$ funktsiyaning minimum (*min*) nuqtasi deyiladi. $y = f(x_1)$ funktsiyaning minimumi, yoki funktsiyaning minimum nuqtasidagi qiymati deyiladi.



Hosilaning geometrik ma'nosi. Urinma tenglamasi

$y=f(x)$ funktsiyaga x_0 – abstsissali nuqtada o'tkazilgan urinma tenglamasi: $y=f(x_0)+f'(x_0)\cdot(x-x_0)$



- Urinmaning burchak koeffitsiyenti:
  $k=f'(x_0)$
- Urinma va OX o'qi orasidagi burchak:
  $k=tg\alpha$ | $tg\alpha=k=f'(x_0)$
  $\alpha=arctgk$ | $\alpha=arctg(f'(x_0))$
- Urinma OY o'qi orasidagi burchak:
  $\beta=90^0-|\alpha|=90^0-|arctg(f'(x_0))|$
- x ning qanday qiymatida $y=f(x)$ funktsiyaga o'tkazilgan urinma $y=kx+b$ to'g'ri chiziqqa parallel bo'ladi? $f'(x)=k$
- $y=f(x)$ va $y=g(x)$ funktsiyalarning qanday burchak ostida kesishishini aniqlash;
  1) $f(x)=g(x)$ dan $x=x_0$, ya'ni funktsiyalar kesishish nuqtasining abstsissasi topiladi.
  2) $k_1=f'(x_0)$ va $k_2=g'(x_0)$ burchak koeffitsiyentlar aniqlanadi.
  3) $tg\varphi=\left|\frac{k_1-k_2}{1+k_1\cdot k_2}\right|$ formuladan funktsiyalar (ya'ni $x=x_0$ abstsissali nuqtadagi urinmalar) orasidagi φ burchakning tangensi topiladi.

Ba'zi xususiy hollar

- $k_1\cdot k_2=-1$ yoki $f'(x_0)\cdot g'(x_0)=-1$ perpendikulyarlik sharti.
  Ya'ni $y=f(x)$ va $y=g(x)$ funktsiyalar 90° burchak ostida kesishadi.
  Savol: x ning qanday qiymatida $y=f(x)$ va $y=g(x)$ funktsiyalarga o'tkazilgan urinmalar perpendikulyar bo'lad? $f'(x)\cdot g'(x)=-1$
- $k_1=k_2$ yoki $f'(x_0)=g'(x_0)$ bo'lganda $y=f(x)$ va $y=g(x)$ funktsiyalarga $x=x_0$ nuqtada o'tkazilgan urinmalar parallel bo'ladi.
  Savol: x ning qanday qiymatida $y=f(x)$ va $y=g(x)$ funktsiyalarga o'tkazilgan urinmalar parallel bo'ladi? $f'(x)=g'(x)$

Ba'zi funksiyalarning boshlang'ich funktsiyalarini topish usullari

$f(x)$	$F(x)$
a	$ax+C$
$x^p,\ p\neq -1$	$\frac{x^{p+1}}{p+1}+C$
$(kx+b)^p$	$\frac{(kx+b)^{p+1}}{k(p+1)}+C$
$(b-kx)^p$	$-\frac{(b-kx)^{p+1}}{k(p+1)}+C$
$g^p(x)\cdot g'(x)$	$\frac{g^{p+1}(x)}{p+1}+C$
$\frac{1}{x}$	$\ln x+C$
$\frac{1}{kx}$	$\frac{1}{k}\ln x+C$
$\frac{1}{kx+b}$	$\frac{1}{k}\ln(kx+b)+C$
$\frac{1}{b-kx}$	$-\frac{1}{k}\ln(b-kx)+C$
$\frac{1}{g(x)}\cdot g'(x)$	$\ln g(x)+C$
a^x	$\frac{a^x}{\ln a}+C$
a^{kx+b}	$\frac{a^{kx+b}}{k\ln a}+C$
a^{b-kx}	$-\frac{a^{b-kx}}{k\ln a}+C$
$a^{g(x)}\cdot g'(x)$	$\frac{a^{g(x)}}{\ln a}+C$
e^x	e^x+C
e^{kx+b}	$\frac{1}{k}e^{kx+b}+C$
e^{b-kx}	$-\frac{1}{k}e^{b-kx}+C$
$e^{g(x)}\cdot g'(x)$	$e^{g(x)}+C$

$f(x)$	$F(x)$
$\sin x$	$-\cos x+C$
$\sin(kx+b)$	$-\frac{1}{k}\cos(kx+b)+C$
$\sin(b-kx)$	$\frac{1}{k}\cos(b-kx)+C$
$\sin(g(x))\cdot g'(x)$	$-\cos(g(x))+C$
$\cos x$	$\sin x+C$
$\cos(kx+b)$	$\frac{1}{k}\sin(kx+b)+C$
$\cos(b-kx)$	$-\frac{1}{k}\sin(b-kx)+C$
$\cos(g(x))\cdot g'(x)$	$\sin(g(x))+C$
$\operatorname{tg}x$	$-\ln\cos x+C$
$\operatorname{tg}(kx+b)$	$-\frac{1}{k}\ln\cos(kx+b)+C$
$\operatorname{tg}(b-kx)$	$\frac{1}{k}\ln\cos(b-kx)+C$
$\operatorname{tg}(g(x))\cdot g'(x)$	$\ln\cos(g(x))+C$
$\operatorname{ctg}x$	$\ln\sin x+C$
$\operatorname{ctg}(kx+b)$	$\frac{1}{k}\ln\sin(kx+b)+C$
$\operatorname{ctg}(b-kx)$	$-\frac{1}{k}\ln\sin(b-kx)+C$
$\operatorname{ctg}(g(x))\cdot g'(x)$	$\ln\sin(g(x))+C$
$\frac{1}{\sin^2 x}$	$-\operatorname{ctg}x+C$
$\frac{1}{\sin^2(kx+b)}$	$-\frac{1}{k}\operatorname{ctg}(kx+b)+C$
$\frac{1}{\sin^2(b-kx)}$	$\frac{1}{k}\operatorname{ctg}(b-kx)+C$
$\frac{1}{\sin^2(g(x))}\cdot g'(x)$	$-\operatorname{ctg}(g(x))+C$

Yakka mat-2

Boshlang'ich funktsiyalar jadvalining davomi

$f(x)$	$F(x)$
$\frac{1}{\cos^2 x}$	$tgx+C$
$\frac{1}{\cos^2(kx+b)}$	$\frac{1}{k}tg(kx+b)+C$
$\frac{1}{\cos^2(b-kx)}$	$-\frac{1}{k}tg(b-kx)+C$
$\frac{g'(x)}{\cos^2(g(x))}$	$tg(g(x))+C$
$\cos^2 x$	$\frac{1}{2}x+\frac{1}{4}Sin2x+C$
$Sin^2 x$	$\frac{1}{2}x-\frac{1}{4}Sin2x+C$
$tg^2 x$	$tgx-x+C$
$ctg^2 x$	$-ctgx-x+C$
$\frac{1}{\sqrt{1-x^2}}$	$\arcsin x+C$
$\frac{g'(x)}{\sqrt{1-g^2(x)}}$	$\arcsin(g(x))+C$

$f(x)$	$F(x)$
$-\frac{1}{\sqrt{1-x^2}}$	$\arccos x+C$
$-\frac{g'(x)}{\sqrt{1-g^2(x)}}$	$\arccos(g(x))+C$
$\frac{1}{1+x^2}$	$arctgx+C$
$\frac{g'(x)}{1+g^2(x)}$	$arctg(g(x))+C$
$-\frac{1}{1+x^2}$	$arcctgx+C$
$-\frac{g'(x)}{1+g^2(x)}$	$arcctg(g(x))+C$
$\frac{1}{x^2-1}$	$\frac{1}{2}\ln\frac{x-1}{x+1}+C$
$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\ln\frac{x-a}{x+a}+C$
$\frac{1}{a^2x^2-b^2}$	$\frac{1}{2ab}\ln\frac{ax-b}{ax+b}+C$

$f(x)$	$F(x)$
$Sin^4 x$	$\frac{1}{32}Sin4x-\frac{1}{4}Sin2x+\frac{3}{8}x+C$
$Sin^4(g(x))\cdot g'(x)$	$\frac{1}{32}Sin(4\cdot g(x))-\frac{1}{4}Sin(2\cdot g(x))+\frac{3}{8}\cdot g(x)+C$
$\cos^4 x$	$\frac{1}{32}Sin4x+\frac{1}{4}Sin2x+\frac{3}{8}x+C$
$\cos^4(g(x))\cdot g'(x)$	$\frac{1}{32}Sin(4\cdot g(x))+\frac{1}{4}Sin(2\cdot g(x))+\frac{3}{8}\cdot g(x)+C$

Qo'shimcha: Agar $Y=F(x)$ funktsiya $y=f(x)$ funktsiyaning boshlang'ich funktsiyasi bo'lsa, u holda $y=f(kx)$ funktsiyaning boshlang'ich funktsiyasi $Y=\frac{1}{k}F(kx)$ bo'ladi.

Aniq integral

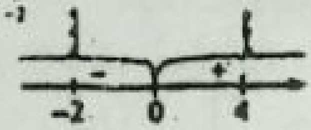
Nyuton – Leybnits formulasi: $\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$

1) $-\int_a^b f(x)dx = \int_b^a f(x)$ 2) $-F(x)\Big|_a^b = F(x)\Big|_b^a$

Bo'laklab integrallash: $\int_a^b f(x)\cdot g'(x)dx = f(x)\cdot g(x)\Big|_a^b - \int_a^b g(x)\cdot f'(x)dx$

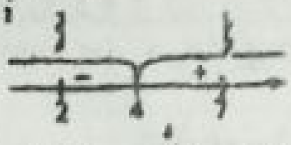
Modulli integrallarni yechishga doir boshlang'ich qadam

1) $\int_{-2}^{4}|x|dx$



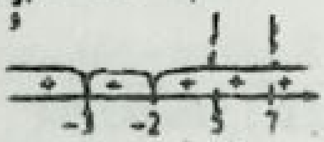
$\int_{-2}^{4}|x|dx = -\int_{-2}^{0}xdx + \int_{0}^{4}xdx$

2) $\int_{2}^{7}|x-4|dx$



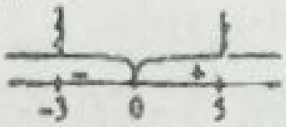
$\int_{2}^{7}|x-4|dx = -\int_{2}^{4}(x-4)dx + \int_{4}^{7}(x-4)dx$

3) $\int_{5}^{7}|x^2+5x+6|dx$



$\int_{5}^{7}|x^2+5x+6|dx = \int_{5}^{7}(x^2+5x+6)dx$

4) $\int_{-3}^{5}(|x|+2)dx$



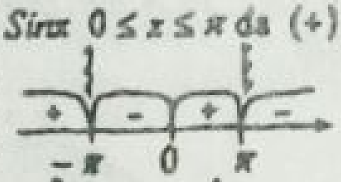
$\int_{-3}^{5}(|x|+2)dx = \int_{-3}^{0}(-x+2)dx + \int_{0}^{5}(x+2)dx$

Trigonometrik funktsiyalar qatnashgan modulli integrallar

1) $\int_{-\pi}^{\pi}|\sin x|dx$

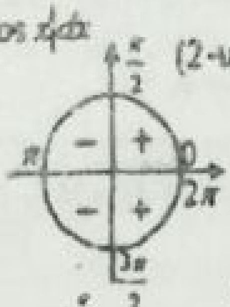
$\sin x = 0 \Rightarrow x = \pi n, n \in Z$

n	x	$x \in [-\pi, \pi]$
0	0	$\sin x$ $0 \le x \le \pi$ da (+)
−1	$-\pi$	
1	π	



$\int_{-\pi}^{\pi}|\sin x|dx = -\int_{-\pi}^{0}\sin x dx + \int_{0}^{\pi}\sin x dx$

2) $\int_{0}^{\frac{3\pi}{2}}|\cos x|dx$ (2-usulda ishlash)



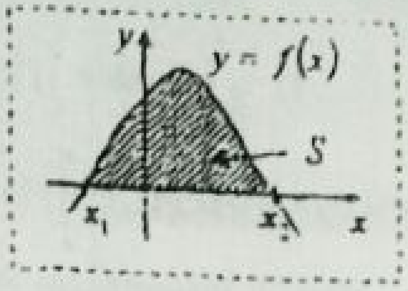
$\int_{0}^{2\pi}|\cos x|dx = \int_{0}^{\frac{\pi}{2}}\cos x dx - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\cos x dx$

Aniq integral yordamida yuzalarni hisoblash

I) $y=f(x)$ funktsiya va OX o'qi orasidagi yuza

$f(x)=0$ dan x_1 va x_2 lar topiladi

$$S=\int_{x_1}^{x_2} f(x)dx$$



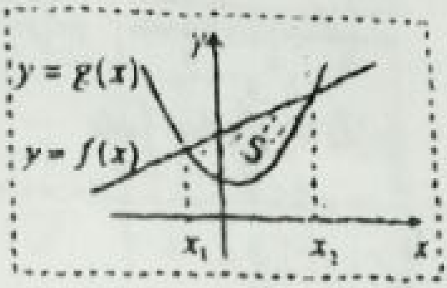
-usul; Grafik chizmasdan yechish usuli;
$f(x)=0$ dan x_1 va x_2 lar topiladi

$$S=\int_{x_1}^{x_2} |f(x)|dx$$

II) $y=f(x)$ va $y=g(x)$ funktsiyalar orasidagi yuza

$f(x)=g(x)$ dan x_1 va x_2 lar topiladi,

$$S=\int_{x_1}^{x_2} (f(x)-g(x))dx$$



-usul; Grafik chizmasdan yechish usuli;
$f(x)=g(x)$ dan x_1 va x_2 lar topiladi,

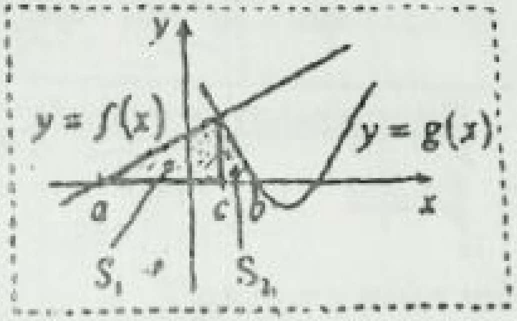
$$S=\int_{x_1}^{x_2} |f(x)-g(x)|dx$$

III) $y=f(x)$, $y=g(x)$ funktsiyalar va OX o'qi (yoki $y=0$) orasidagi yuza

$f(x)=0$ dan $x=a$, $g(x)=0$ dan $x=b$ va $f(x)=g(x)$ dan $x=c$ topiladi

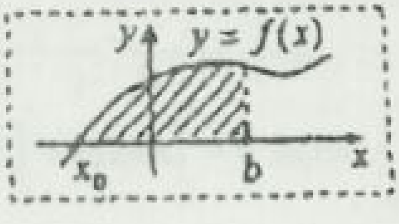
$$S_1=\int_a^c f(x)dx \qquad S_2=\int_c^b g(x)dx$$

$$S=S_1+S_2$$



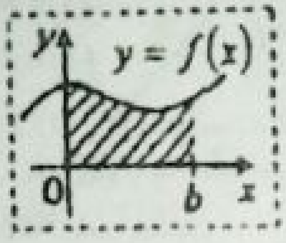
IV) $y=f(x)$ funktsiya , OX o'qi (yoki $y=0$) va $x=b$ to'g'ri chiziq orasidagi yuza

$f(x)=0$ dan x_0 topiladi; $S=\int_{x_0}^{b} f(x)dx$



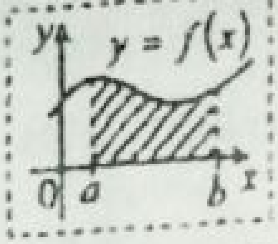
V) $y=f(x)$ funksiya, OX o'qi ($y=0$), OY o'qi ($x=0$) va $x=b$ chiziq

$$S=\int_0^b f(x)dx$$



VI) Egri chiziqli trapetsiyaning yuzi

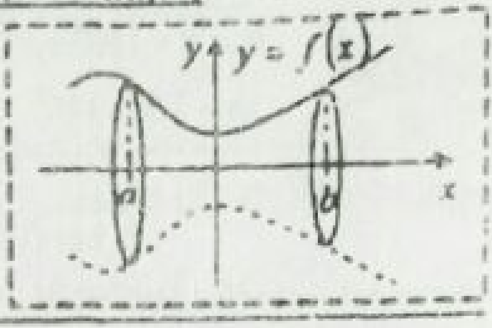
$$S=\int_a^b f(x)dx$$



IV) $y=f(x)$ funktsiyani OX o'qi atrofida aylantirishdan hosil bo'lgan jismning sirti va hajmi

Sirti; $S=2\pi\int_a^b f(x)\sqrt{1+[f'(x)]^2}\,dx$

Hajmi; $V=\pi\int_a^b [f(x)]^2\,dx$



Jismning harakati

t – vaqt, $\upsilon(t)$ – tezlik, $S(t)$ – masofa, $a(t)$ – tezlanish

$$\upsilon(t)=S'(t) \qquad a(t)=\upsilon'(t)$$

$$a(t)=S''(t) \quad \upsilon(t)=\int_{t_1}^{t_2} a(t)dt$$

- Tezlik $\upsilon(t)$ bo'lganda dastlabki t_1 vaqtda bosib o'tilgan yo'l

$$S=\int_0^{t_1}\upsilon(t)dt$$

- Tezlik $\upsilon(t)$ bo'lganda t_1 va t_2 vaqt oralig'ida o'tilgan yo'l (masofa)

$$S=\int_{t_1}^{t_2}\upsilon(t)dt$$

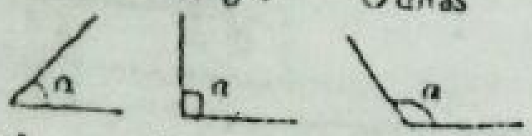
- Jism to'xtaganda
  $\upsilon(t)=0$ yoki $S'(t)=0$
- Tezlik eng katta yoki eng kichik bo'lganda
  $\upsilon'(t)=0$ yoki $S''(t)=0$
  yoki $a(t)=0$
- Tezlanish eng katta yoki eng kichik bo'lganda
  $a'(t)=0$, $\upsilon''(t)=0$
  yoki $S'''(t)=0$
- Agar $\upsilon(t)=f(t)$ bo'lsa, $S(t)=F(t)$ bo'ladi.

M; $\upsilon(t)=3t^2+2t+3$ bo'lsa
$S(t)=t^3+t^2+3t+C$

GEOMETRIYA
PLANIMETRIYA

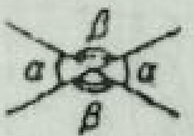
Burchaklar

O'tkir   To'g'ri   O'tmas



$0^\circ < \alpha < 90^\circ$ $\alpha = 90^\circ$ $90^\circ < \alpha < 180^\circ$

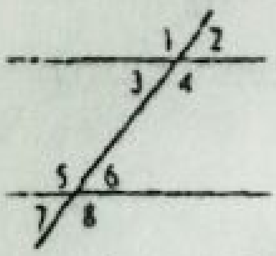
Qo'shni va vertikal burchaklar



$\alpha + \beta = 180^\circ$

$2\alpha + 2\beta = 360^\circ$

Ikki parallel to'g'ri chiziq va kesuvchi



Mos burchaklar

$\angle 1$ va $\angle 5$

$\angle 2$ va $\angle 6$

$\angle 3$ va $\angle 7$

$\angle 4$ va $\angle 8$

Ichki almashinuvchi burchaklar

$\angle 3$ va $\angle 6$, $\angle 4$ va $\angle 5$

Ichki bir tomonli burchaklar

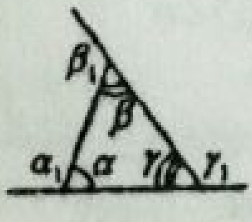
$\angle 4$ va $\angle 6$, $\angle 4 + \angle 6 = 180^\circ$

$\angle 3$ va $\angle 5$, $\angle 3 + \angle 5 = 180^\circ$

Uchburchaklar

Uchburchaklar burchaklari

$\alpha + \beta + \gamma = 180^\circ$



$\alpha_1 + \beta_1 + \gamma_1 = 360^\circ$

$\alpha + \beta = \gamma_1$

$\alpha + \gamma = \beta_1$

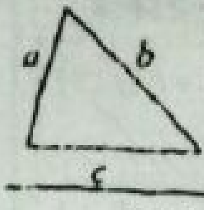
$\beta + \gamma = \alpha_1$

Ixtiyoriy uchburchak uchun;

$$\cos\alpha + \cos\beta + \cos\gamma \le \frac{3}{2}$$

munosabat o'rinli.

Uchburchak tengsizligi



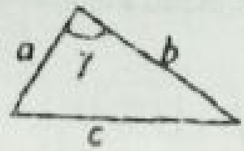
$$\begin{cases} a < b + c \\ b < a + c \\ c < b + c \end{cases} \qquad \begin{cases} a > |b - c| \\ b > |a - c| \\ c > |a - b| \end{cases}$$

Uchburchak turlari

c – eng katta tomon

γ – eng katta burchak

I) O'tmas burchakli uchburchak

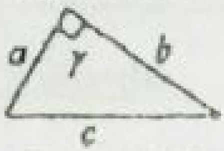


$90^\circ < \gamma < 180^\circ$

$c^2 > a^2 + b^2$

Tashqi chizilgan aylana markazi uchburchak tashqarisida bo'ladi.

II) To'g'ri burchakli uchburchak

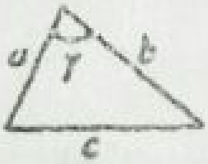


$\gamma = 90^\circ$

$c^2 = a^2 + b^2$

Tashqi chizilgan aylana markazi gipotenuzaning o'rtasida bo'ladi.

III) O'tkir burchakli uchburchak

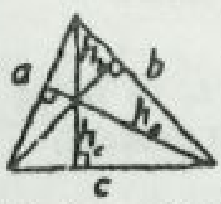


$0^\circ < \gamma < 90^\circ$

$c^2 < a^2 + b^2$

Tashqi chizilgan aylana markazi uchburchakning ichida bo'ladi.

Ixtiyoriy uchburchak



$$S = \frac{ah_a}{2} = \frac{bh_b}{2} = \frac{ch_c}{2}$$

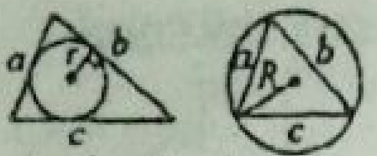
$ah_a = bh_b = ch_c$

Perimetr; $P = a + b + c$

Yarim perimetr; $p = \dfrac{a + b + c}{2}$

Geron formulasi ;

$$S = \sqrt{p(p - a)(p - b)(p - c)}$$



$$\frac{1}{r}=\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_c}$$

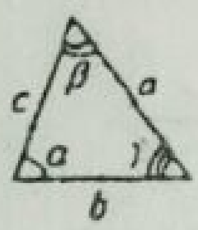
$$P=a+b+c \qquad p=\frac{a+b+c}{2}$$

$$r=\frac{2S}{a+b+c} \qquad r=\frac{2S}{P} \qquad r=\frac{S}{p}$$

$$S=p\cdot r \qquad S=\frac{Pr}{2} \qquad R=\frac{abc}{4S}$$

Ixtiyoriy uchburchakka ichki va tashqi chizilgan aylanalar markazlari orasidagi masofa

$$x^2=R^2-2Rr$$



$$S=\frac{a^2\cdot Sin\beta\cdot Sin\gamma}{2Sin\alpha}$$

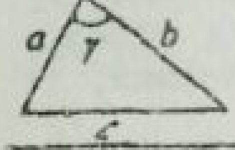
$$S=\frac{b^2\cdot Sin\alpha\cdot Sin\gamma}{2Sin\beta}$$

$$S=\frac{c^2\cdot Sin\alpha\cdot Sin\beta}{2Sin\gamma}$$

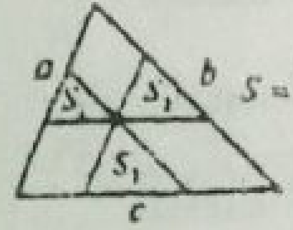
$$S=\frac{abSin\gamma}{2}=\frac{bcSin\alpha}{2}=\frac{acSin\beta}{2}$$

$$S=2R^2Sin\alpha\cdot Sin\beta\cdot Sin\gamma$$

- Xususan a va b tomonlar uchun

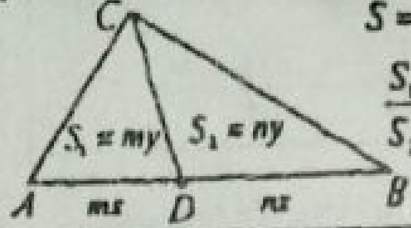


$$S=\frac{abSin\gamma}{2}$$



$$S=\left(\sqrt{S_1}+\sqrt{S_2}+\sqrt{S_3}\right)^2$$

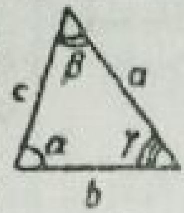
Uchburchakning biror uchidan chiqqan to'g'ri chiziq qarshisidagi tomonni qanday nisbatda bo'lsa, uchburchak yuzini ham shunday nisbatda bo'ladi.



$$S=S_1+S_2$$

$$\frac{S_1}{S_2}=\frac{AD}{DB}$$

Kosinuslar va sinuslar teoremasi



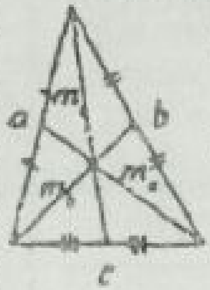
$$a^2=b^2+c^2-2bc\cdot\cos\alpha$$

$$b^2=a^2+c^2-2ac\cdot\cos\beta$$

$$c^2=a^2+b^2-2ab\cdot\cos\gamma$$

$$\frac{a}{Sin\alpha}=\frac{b}{Sin\beta}=\frac{c}{Sin\gamma}=2R$$

Uchburchak medianalari



$$m_a=\frac{1}{2}\sqrt{2\left(b^2+c^2\right)-a^2}$$

$$m_b=\frac{1}{2}\sqrt{2\left(a^2+c^2\right)-b^2}$$

$$m_c=\frac{1}{2}\sqrt{2\left(a^2+b^2\right)-c^2}$$

Har bir mediana medianalar kesishish nuqtasida uchburchak uchidan boshlab hisoblaganda 2:1 nisbatda bo'linadi

Uchburchak tomonlarini uning medianalari orqali ifodalanishi

$$a=\frac{2}{3}\sqrt{2\left(m_b^2+m_c^2\right)-m_a^2}$$

$$b=\frac{2}{3}\sqrt{2\left(m_a^2+m_c^2\right)-m_b^2}$$

$$c=\frac{2}{3}\sqrt{2\left(m_a^2+m_b^2\right)-m_c^2}$$

Uchburchak yuzini medianalar orqali ifodalanishi

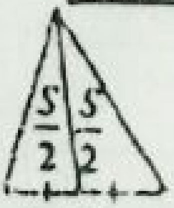
$$S=\frac{4}{3}\sqrt{m(m-m_a)(m-m_b)(m-m_c)}$$

Bu yerda; $m=\dfrac{m_a+m_b+m_c}{2}$

• S_Δ – biror uchburchakning yuzi, S_m – esa shu uchburchak medianalaridan iborat uchburchakning yuzi bo'lsa, quyidagi o'rinli:

$$\frac{S_\Delta}{S_m}=\frac{4}{3} \quad \text{yoki} \quad S_\Delta=\frac{4}{3}S_m$$

Medianalarga qo'shimcha



• *Bitta mediana uchburchakning yuzini teng ikkiga bo'ladi*

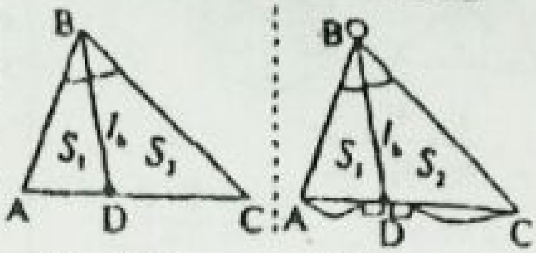
• *Uchta mediana uchburchakning yuzini teng olti qismga bo'ladi.*

• Uchlarining koordinatalari $A(x_1;y_1;z_1)$, $B(x_2;y_2;z_2)$ va $C(x_3;y_3;z_3)$ nuqtalardan iborat bo'lgan uchburchak og'irlik markazining ya'ni medianalari kesishgan nuqtasi $O(x;y;z)$ ning koordinatalarini topish:

$$x=\frac{x_1+x_2+x_3}{3};\quad y=\frac{y_1+y_2+y_3}{3};$$

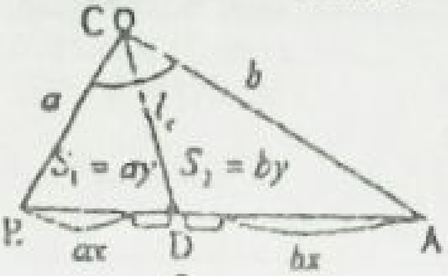
va $z=\dfrac{z_1+z_2+z_3}{3}$

Uchburchak bissektrisasi



$$\frac{AB}{AD}=\frac{BC}{CD} \quad \text{yoki} \quad \frac{AB}{BC}=\frac{AD}{DC}=\frac{S_1}{S_2}$$

• *Bissektrisalarning kesishish nuqtasi uchburchakka ichki chizilgan aylananing markazida bo'ladi.*



$$ax+bx=c \qquad x=\frac{c}{a+b}$$

$$BD=\frac{c}{a+c}\cdot a \qquad AD=\frac{c}{a+b}\cdot b$$

$$S_1+S_2=S \;\Big|\; ay+by=S \;\Big|\; y=\frac{S}{a+b}$$

$$S_1=\frac{a}{a+b}\cdot S \qquad S_2=\frac{b}{a+b}\cdot S$$

Uchburchak bissektrisasi bissektrisalarning kesishish nuqtasida uchburchak uchidan boshlab hisoblaganda quyidagi nisbatda bo'linadi;

l_a bissektrisa $\dfrac{b+c}{a}$ nisbatda

l_b bissektrisa $\dfrac{a+c}{b}$ nisbatda

l_c bissektrisa $\dfrac{a+b}{c}$ nisbatda

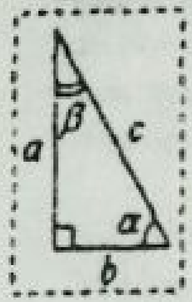
Uchburchak bissektrisalarini topish formulalari

$$l_a = \frac{\sqrt{bc(a+b+c)(b+c-a)}}{b+c}$$

$$l_b = \frac{\sqrt{ac(a+b+c)(a+c-b)}}{a+c}$$

$$l_c = \frac{\sqrt{ab(a+b+c)(a+b-c)}}{a+b}$$

To'g'ri burchakli uchburchak

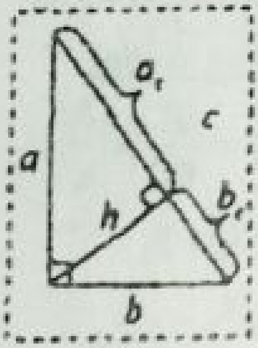


$\alpha + \beta = 90^\circ$ | $a^2 + b^2 = c^2$

$$\sin\alpha = \frac{\text{qarsh. katet}}{\text{gipotenuza}} = \frac{a}{c}$$

$$\cos\alpha = \frac{\text{yopish. kat.}}{\text{gipotenuza}} = \frac{b}{c}$$

$$tg\alpha = \frac{\text{qarshisidagi katet}}{\text{yopishgan katet}} = \frac{a}{b}$$



$$a_c + b_c = c$$

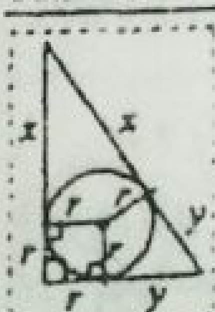
$$h = \sqrt{a_c \cdot b_c}$$

$a^2 = a_c \cdot c$ | $b^2 = b_c \cdot c$

$$\frac{a_c}{b_c} = \frac{a^2}{b^2}$$

$S = \frac{ab}{2}$ $\quad S = \frac{ch}{2}$ $\quad ab = ch$

• *To'g'ri burchakning bissektrisasi gipotenuzani m : n nisbatda bo'lsa, balandlik $m^2 : n^2$ nisbatda bo'ladi.*



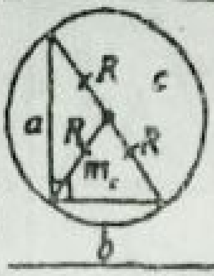
$a = r + x \quad b = r + y$

$$c = x + y$$

$$P = a + b + c$$

$$r = \frac{a+b-c}{2} = \frac{P-2c}{2}$$

$S = x \cdot y$ | $S = \frac{P \cdot r}{2} = \frac{(a+b+c)\,r}{2}$

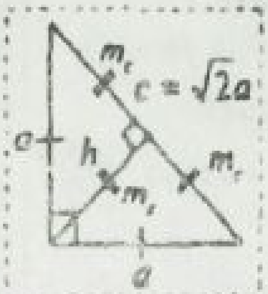


$m_c = R \quad c = d$

$c = 2R$ | $c = 2m_c$

$R = \frac{c}{2}$ | $m_c = \frac{c}{2}$

Teng yonli to'g'ri burchakli uchburchak

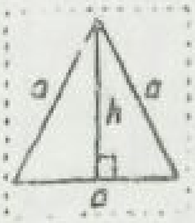


$$h = m_c = l_c = R = \frac{c}{2}$$

$a = \frac{c}{\sqrt{2}}$ | $R = \frac{a}{\sqrt{2}}$

$S = \frac{a^2}{2} \quad S = \frac{c^2}{4}$

Teng tomonli uchburchak



$m = l = h$ | $S = \frac{a^2\sqrt{3}}{4}$

$S = \frac{3\sqrt{3}R^2}{4}$ | $S = 3\sqrt{3} \cdot r^2$

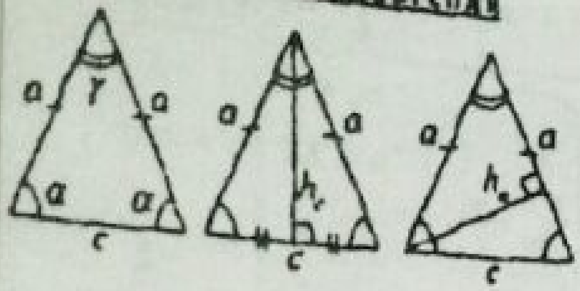
$$r = \frac{a}{2\sqrt{3}} = \frac{a\sqrt{3}}{6} = \frac{h}{3} = \frac{R}{2}$$

$$R = \frac{a}{\sqrt{3}} = \frac{a\sqrt{3}}{3} = 2r = \frac{2}{3}h$$

$$h = \frac{a\sqrt{3}}{2} = 3r = \frac{3R}{2}$$

• *Teng tomonli uchburchak ichidagi ixtiyoriy nuqtadan uning tomonlarigacha bo'lgan masofalar yig'indisi 3r ga yoki h ga teng*

Teng yonli uchburchak



$$2\alpha+\gamma=180^\circ \quad \alpha=90^\circ-\frac{\gamma}{2}$$

$$ch_c=ah_a \qquad h_c=m_c=l_c$$

Teng yonli uchburchak formulalarining davomi

$$r=\frac{c(2a-c)}{4h_c}=\frac{c}{2}tg\frac{\alpha}{2}=a\cos\alpha\cdot tg\frac{\alpha}{2}$$

$$R=\frac{a}{2Sin\alpha}=\frac{c}{2Sin\gamma}=\frac{a^2}{2h_c}$$

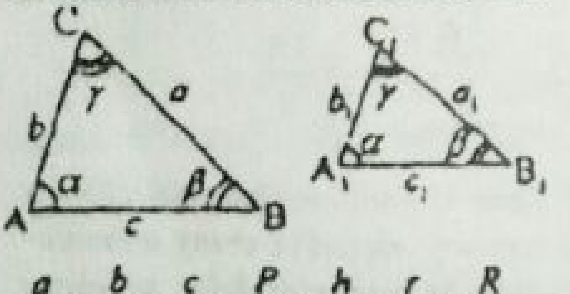
$$S=\frac{ah_a}{2}=\frac{ch_c}{2}=\frac{c\sqrt{4a^2-c^2}}{4}$$

$$\frac{c^2}{4}+h_c^2=a^2 \quad h_c=\frac{\sqrt{4a^2-c^2}}{2}$$

$$h_c=aSin\alpha=\frac{c}{2}tg\alpha=a\cos\frac{\gamma}{2}$$

$$h_a=aSin\gamma=cSin\alpha=\frac{c\sqrt{4a^2-c^2}}{2a}$$

Uchburchaklarning o'xshashligi



$$\frac{a}{a_1}=\frac{b}{b_1}=\frac{c}{c_1}=\frac{P}{P_1}=\frac{h}{h_1}=\frac{r}{r_1}=\frac{R}{R_1}$$

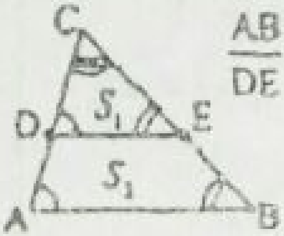
$$\frac{S}{S_1}=\frac{a^2}{a_1^2}=\frac{b^2}{b_1^2}=\frac{c^2}{c_1^2}=\frac{P^2}{P_1^2}=\frac{h^2}{h_1^2}=$$

$$=\frac{r^2}{r_1^2}=\frac{R^2}{R_1^2}$$

O'xshashlik alomatlari

- Agar bir uchburchakning ikkita burchagi ikkinchi uchburchakning ikkita burchagiga mos ravishda teng bo'lsa, bu uchburchaklar o'xshash deyiladi.
- Agar bir uchburchakning ikki tomoni ikkinchi uchburchakning ikki tomoniga proportsional, hamda bu tomonlar orasidagi burchaklar mos ravishda teng bo'lsa, bunday uchburchaklar o'xshash deyiladi.
- Bir uchburchakning tomonlari ikkinchi uchburchakning tomonlari uzunliklariga mos ravishda proportsional bo'lsa, bu uchburchaklar o'xshash deyiladi.

O'xshashlikka keladigan hollar



$$\frac{AB}{DE}=\frac{AC}{DC}=\frac{BC}{EC}=\frac{h}{h_1}$$

$$S=S_1+S_2$$

$$\frac{S}{S_1}=\frac{AB^2}{DE^2}=\frac{AC^2}{DC^2}=\frac{BC^2}{EC^2}=\frac{h^2}{h_1^2}$$

h – ABC uchburchakning balandligi

h_1 – DEC uchburchakning balandligi

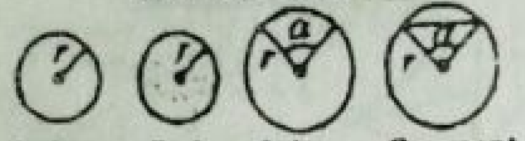
- Agar DE – uchburchakning o'rta chizig'i bo'lsa;

$\frac{S}{S_1}=\frac{4}{1}$ yoki $S_1=x$ va $S_2=3x$

$S=S_1+S_2$ va $S=4x$

munosabat o'rinli bo'ladi.

Aylana va doira



Aylana Doira Sektor Segment

Aylana	Doira
 $d = 2r$, $l = 2\pi r$, $l = \pi d$	 $d = 2r$, $l = 2\pi r = \pi d$, $S = \pi r^2$, $S = \frac{\pi d^2}{4}$

Sektor

α - gradusda	α - radianda
$l = \frac{\pi r \alpha^0}{180^0}$	$l = r\alpha$
$S = \frac{\pi r^2 \alpha^0}{360^0}$	$S = \frac{1}{2}\pi r^2 \alpha$

l – α burchakka mos yoy uzunligi.

Segment

α – radianlarda

$$S_{segm} = S_{sekt} - S_{\Delta}$$

$$S_{\Delta} = \frac{r^2 Sin\alpha}{2}$$

$$S_{segm} = \frac{1}{2} r^2 (\alpha - Sin\alpha)$$

Katta segment yuzi ya'ni doiraning ikkinchi qismning yuzi;

$$S_{katta\ segm} = S_{katta\ sekt} + S_{\Delta} =$$

$$= \frac{1}{2} r^2 (\beta - Sin\beta) = \frac{1}{2} r^2 (\beta + Sin\alpha)$$

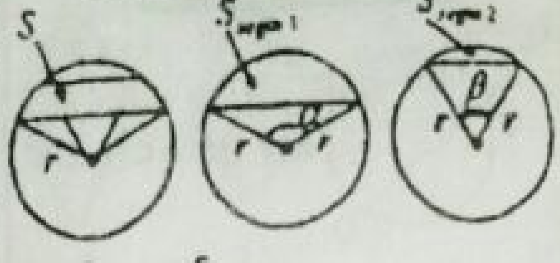
Halqa

$x = R - r \mid S = S_R - S_r$

$$S = \pi(R^2 - r^2)$$

x – halqa kengligi

Doira kesimining yuzi



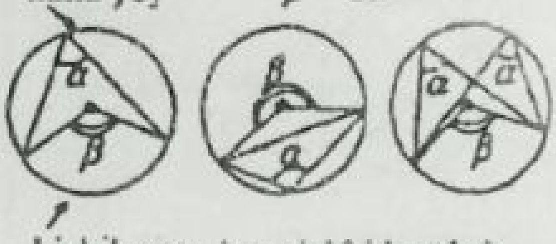
$$S = S_{segm1} - S_{segm2}$$

$$S = \frac{\pi r^2}{360^0}(\alpha - \beta) - \frac{1}{2} r^2 (Sin\alpha - Sin\beta)$$

$$S = \frac{1}{2} r^2 (\alpha - \beta) - \frac{1}{2} r^2 (Sin\alpha - Sin\beta)$$

Aylana burchaklari

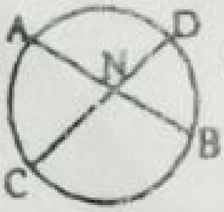
katta yoy $\beta = 2\alpha$



kichik yoy | α – ichki burchak

β – α ga mos markaziy burchak

Kesishuvchi vatarlar

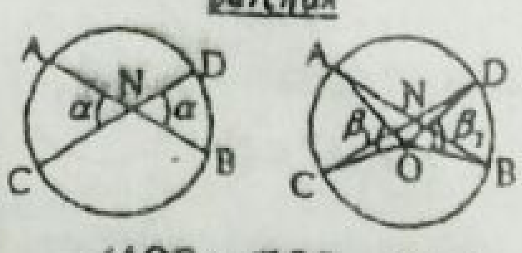


$$AN \cdot NB = CN \cdot ND = R^2 - q^2$$

R – aylana radiusi

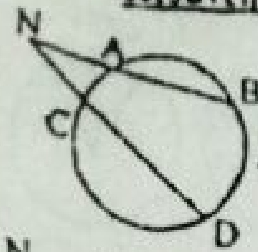
q – aylana markazidan vatarlarning kesishish nuqtasi(N) gacha masofa

Kesishuvchi vatarlar orasidagi burchak

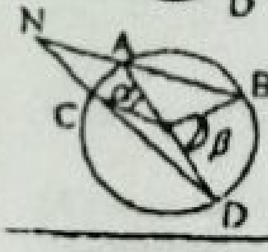


$$\alpha = \frac{\angle AOC + \angle BOD}{2} = \frac{\beta_1 + \beta_2}{2}$$

Kesuvchilar hossasi



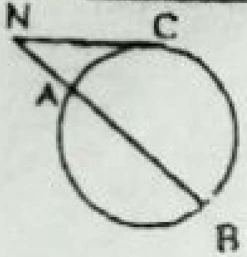
$AN \cdot BN = CN \cdot DN$



Kesuvchilar orasidagi burchak

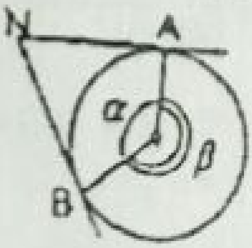
$\angle N = \dfrac{\beta - \alpha}{2}$

Kesuvchi va urinma



$CN^2 = AN \cdot BN$

Urinmalar orasidagi burchakni topish



$\angle N = \dfrac{\beta - \alpha}{2}$

Markazi C(a;b) nuqtada bo'lgan R radiusli

a) aylana tenglamasi

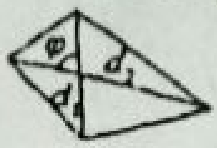
$(x-a)^2 + (y-b)^2 = R^2$

b) doira tenglamasi

$(x-a)^2 + (y-b)^2 \le R^2$

* Aylana tenglamasiga doir to'liqroq ma'lumotni betda berilgan

Ixtiyoriy to'rtburchak



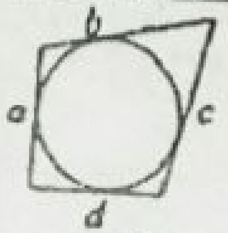
$S = \dfrac{d_1 d_2 \sin\varphi}{2}$

Aylanaga ichki chizilish sharti

$\alpha + \gamma = \beta + \varphi = 180^\circ$

$AC \cdot BD = AB \cdot CD + AD \cdot BC$

Aylanaga tashqi chizilish sharti



$a + c = b + d$

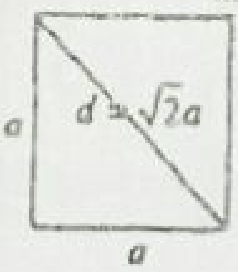
$P = a + b + c + d$

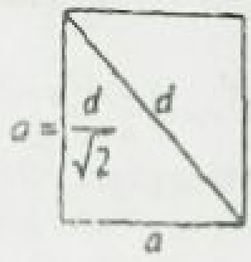
$P = 2(a + c)$

$P = 2(b + d)$

$S = \dfrac{P \cdot r}{2} = (a + c)r = (b + d)r$

Kvadrat



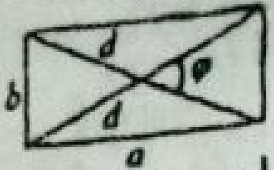


$r = \dfrac{a}{2}$ $\quad a = 2r$ $\quad r = \dfrac{R}{\sqrt{2}}$

$R = \dfrac{a}{\sqrt{2}}$ $\quad a = \sqrt{2}R$ $\quad d = 2R$

$S = a^2$ $\quad S = \dfrac{d^2}{2}$ $\quad S = 2R^2 = 4r^2$

To'g'ri to'rtburchak

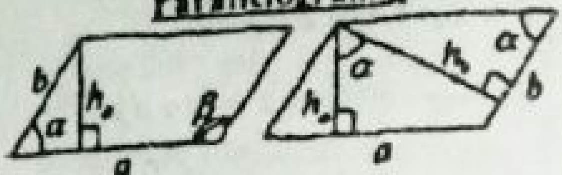


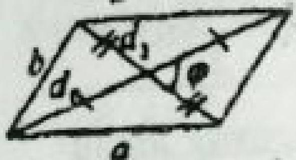
$a^2 + b^2 = d^2$

$S = \frac{d^2 \sin\varphi}{2}$

$S = ab$ | $R = \frac{d}{2}$ | $R = \frac{\sqrt{a^2 + b^2}}{2}$

Parallelogramm





$\alpha + \beta = 180^\circ$

$P = 2(a + b)$

• O'tkir burchakdan tushirilgan balandliklar orasidagi burchak;

$180^\circ - \alpha$ ga teng bo'ladi.

$h_a = b \cdot \sin\alpha \quad h_b = a \cdot \sin\alpha$

$2(a^2 + b^2) = d_1^2 + d_2^2$

$a^2 = \frac{1}{4}\left(d_1^2 + d_2^2 + 2d_1 d_2 \cos\varphi\right)$

$b^2 = \frac{1}{4}\left(d_1^2 + d_2^2 - 2d_1 d_2 \cos\varphi\right)$

$d_1 = \sqrt{a^2 + b^2 + 2ab\cos\alpha}$

$d_2 = \sqrt{a^2 + b^2 - 2ab\cos\alpha}$

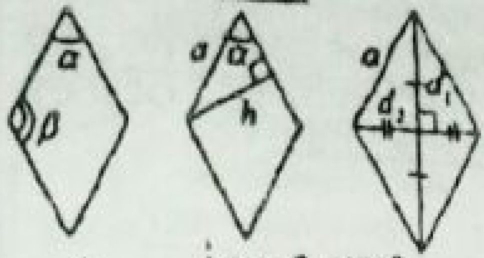
$S = ah_a = bh_b$ | $S = ab\sin\alpha$

$S = \frac{d_1 d_2 \sin\varphi}{2}$ | $S = \frac{h_a h_b}{\sin\alpha}$

$ah_a = bh_b = ab\sin\alpha = \frac{d_1 d_2 \sin\varphi}{2}$

$\sin\varphi = 1$ ya'ni $\varphi = 90^\circ$ da parallelogrammning yuzi eng katta bo'ladi.

Romb



$\alpha + \beta = 180^\circ$

$h = a \cdot \sin\alpha \quad h = 2r$

$r = \frac{h}{2} \quad r = \sqrt{x \cdot y}$

$a = x + y$

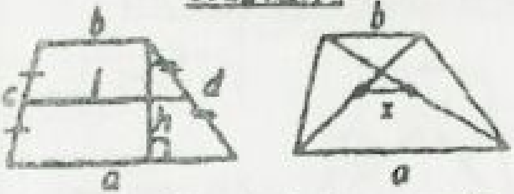
$4a^2 = d_1^2 + d_2^2$

$S = ah \quad S = a^2 \sin\alpha$

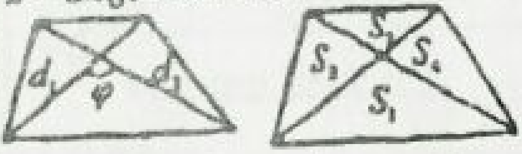
$S = \frac{d_1 d_2}{2} \quad S = 2ar$

$\frac{d_1 d_2}{2} = a^2 \sin\alpha = ah$

Trapetsiya



x – diagonallar o'rtalari or. masofa.

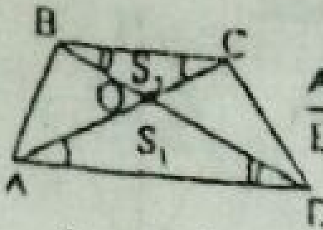


$l = \frac{a + b}{2}$ | $x = \frac{a - b}{2}$ | $S = \frac{a + b}{2} h$

$S = lh$ | $S = \frac{d_1 d_2 \sin\varphi}{2}$ | $S_1 S_2 = S_3 S_4$

$S_3 = S_4$ | $\frac{S_1}{S_2} = \frac{a^2}{b^2}$ | $S = \left(\sqrt{S_1} + \sqrt{S_2}\right)^2$

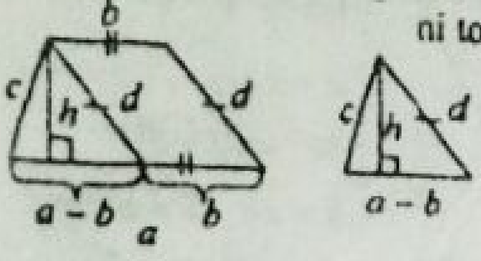
$d_1 \perp d_2$ da trapetsiya yuzi eng katta



$$\frac{AD}{BC}=\frac{AO}{OC}=\frac{DO}{OB}$$

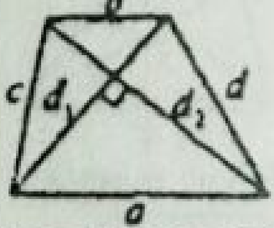
$$\frac{S_1}{S_2}=\frac{AD^2}{BC^2}=\frac{AO^2}{OC^2}=\frac{DO^2}{OB^2}$$

* Trapetsiyaning a, b, c va d tomonlari berilganda uning balandligini topish



$$S_\Delta=\frac{(a-b)h}{2} \quad \text{dan} \quad h=\frac{2S_\Delta}{a-b}$$

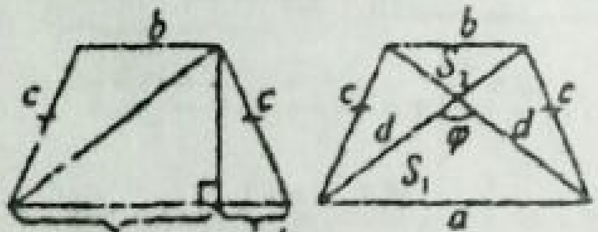
Trapetsiyaning diagonallari o'zaro perpendikulyar bo'lgan hol



$$S=\frac{d_1d_2}{2}$$

$$h=\frac{d_1d_2}{\sqrt{d_1^2+d_2^2}}$$

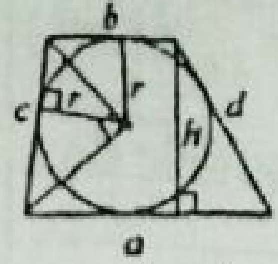
Teng yonli trapetsiya



$$l=\frac{a+b}{2} \qquad \frac{a-b}{2} \qquad S=l\cdot h$$

$$S=\frac{d^2\sin\varphi}{2} \qquad S=\left(\sqrt{S_1}+\sqrt{S_2}\right)^2$$

Trapetsiyaning aylanaga tashqi chizilish shartlari



$$a+b=c+d$$

$$h=2r \quad r=\frac{h}{2}$$

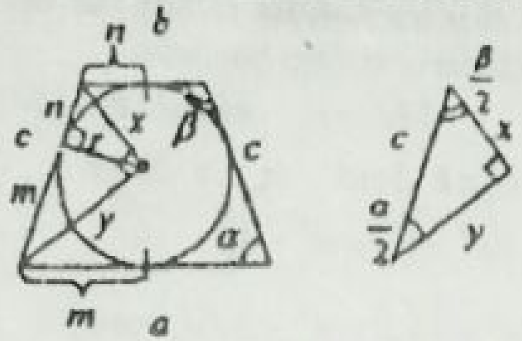
$$l=\frac{a+b}{2}=\frac{c+d}{2}$$

$$P=2(a+b)=2(c+d)$$

$$S=\frac{P\cdot r}{2}=(a+b)r=(c+d)r$$

$$S=lh=\frac{a+b}{2}h=\frac{c+d}{2}h$$

Teng yonli trapetsiya uchun



$$a+b=2c \quad c=\frac{a+b}{2}=l \quad c=m+n$$

$$m=\frac{a}{2} \quad n=\frac{b}{2} \quad c^2=x^2+y^2$$

$$r=\sqrt{mn} \quad c\cdot r=x\cdot y$$

$$S=lh=ch=2cr$$

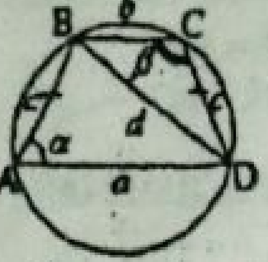
$$\sin\frac{\alpha}{2}=\frac{x}{c} \;\Big|\; \cos\frac{\alpha}{2}=\frac{y}{c} \;\Big|\; tg\frac{\alpha}{2}=\frac{x}{y}$$

x, y – aylana markazidan trapetsiya asoslarining uchlarigacha bo'lgan masofalar.

Trapetsiyaning aylanaga ichki chizilish shartlari.

Trapetsiya teng yonli bo'ladi.

$\alpha + \beta = 180^\circ$



$$R = \frac{acd}{4S_{ABD}}$$

$$R = \frac{bcd}{4S_{BCD}}$$

$$\frac{d}{Sin\alpha} = 2R \quad \text{yoki} \quad \frac{d}{Sin\beta} = 2R$$

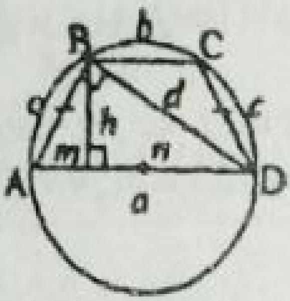
Trapetsiya diagonali uning yon tomoniga perpendikulyar bo'lgan holda

$a = 2R \quad R = \frac{a}{2}$

$h = \sqrt{m \cdot n}$

$a \cdot h = c \cdot d$

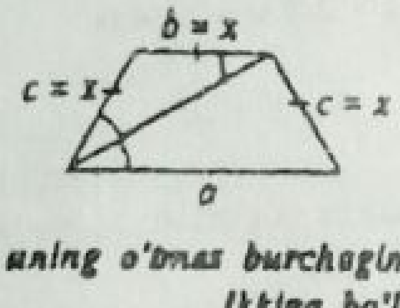
$l = \frac{a+b}{2} = n$



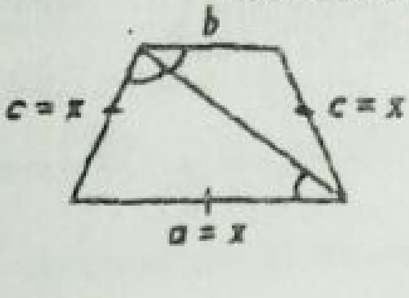
Qo'shimcha ma'lumot

Teng yonli trapetsiya diagonali

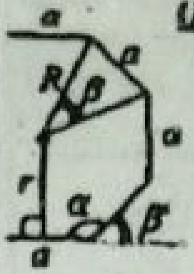
a) uning o'tkir burchagini teng ikkiga bo'lganda



b) uning o'tmas burchagini teng ikkiga bo'lganda



Muntazam qavariq ko'pburchak (Muntazam n burchak)



α – ichki burchak

β – markaziy burchak

β' – tashqi burchak

n – tomonlar soni

m – diagonallari soni

$$m = \frac{n(n-3)}{2} \quad \Big| \quad \beta = \beta' = \frac{360^\circ}{n} = \frac{2\pi}{n}$$

$$\alpha = 180^\circ - \frac{360^\circ}{n} = \frac{180^\circ(n-2)}{n}$$

$$\alpha + \beta = 180^\circ \qquad \alpha + \beta' = 180^\circ$$

° *Ichki burchaklar yig'indisi:*

$$n\alpha = 180^\circ(n-2) = \pi(n-2)$$

° *n burchakning bittadan olingan tashqi burchaklarining yig'indisi:*

360° yoki 2π ga teng.

$$R = \frac{a}{2Sin\frac{180^\circ}{n}} = \frac{\sqrt{4r^2 + a^2}}{2}$$

$$r = \frac{a}{2tg\frac{180^\circ}{n}} = \frac{\sqrt{4R^2 - a^2}}{2}$$

$$a = 2RSin\frac{180^\circ}{n} \qquad a = 2r \cdot tg\frac{180^\circ}{n}$$

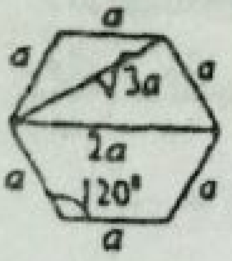
$P = na \qquad P$ – perimetr

$$S = \frac{P \cdot r}{2} \quad S = \frac{nar}{2} \quad S = \frac{na^2}{4tg\frac{180^\circ}{n}}$$

$$S = \frac{nR^2 Sin\frac{360^\circ}{n}}{2} \quad S = nr^2 tg\frac{180^\circ}{n}$$

* *Muntazam n burchak ichidagi ixtiyoriy nuqtadan uning tomonlarigacha bo'lgan masofalar yig'indisi nr ga teng.*

Muntazam oltiburchak



$n = 6$ ta

$\alpha = 120^\circ$

$n\alpha = 720^\circ$

$\beta = \beta' = 60^\circ$

$R = a \;\Big|\; r = \dfrac{a\sqrt{3}}{2} \;\Big|\; S = \dfrac{3\sqrt{3}a^2}{2}$

$S = 2\sqrt{3}\cdot r^2 \qquad S = \dfrac{3\sqrt{3}R^2}{2}$

* Diagonallari soni ; 9 ta.

Muntazam beshburchak

$$a = \frac{R}{2}\sqrt{10 - 2\sqrt{5}} = 2r\sqrt{5 - 2\sqrt{5}}$$

$$R = \frac{a}{10}\sqrt{50 + 10\sqrt{5}} = r\left(\sqrt{5} - 1\right)$$

$$r = \frac{R}{4}\left(\sqrt{5} + 1\right) = \frac{a}{10}\sqrt{25 + 10\sqrt{5}}$$

$$d = \frac{1 + \sqrt{5}}{2}a \text{ , } d \text{ - diagonal}$$

$$S = \frac{5}{8}R^2\sqrt{10 + 2\sqrt{5}} =$$

$$= \frac{a^2}{4}\sqrt{25 + 10\sqrt{5}} = 5r^2\sqrt{5 - 2\sqrt{5}}$$

$\alpha = 108^\circ$; $\beta = 72^\circ$; $\beta' = 72^\circ$

Ichki burchaklar yi'g'indisi;

$n\alpha = 540^\circ$

Bittadan olingan tashqi burchaklarining yig'indisi; $n\beta' = 360^\circ$

* Diagonallari soni; 5 ta.

Muntazam sakkizburchak

$R = \dfrac{a}{\sqrt{2 - \sqrt{2}}} \qquad r = \dfrac{a}{2\left(\sqrt{2} - 1\right)}$

$S = \dfrac{2a^2}{\sqrt{2} - 1}$; diag.soni = 20 ta

$\alpha = 135^\circ \qquad \beta = 45^\circ$

Ichki burchaklar yi'g'indisi;

$n\alpha = 1080^\circ$

Bittadan olingan tashqi burchaklarining yig'indisi; $n\beta' = 360^\circ$

Muntazam o'nikkiburchak

$R = \dfrac{\sqrt{2}a}{\sqrt{3} - 1} \qquad r = \dfrac{a}{2\left(2 - \sqrt{3}\right)}$

$S = \dfrac{3a^2}{2 - \sqrt{3}}$; dag.soni = 54 ta

$\alpha = 150^\circ \qquad \beta = \beta' = 30^\circ$

Ichki burchaklar yi'g'indisi,

$n\alpha = 1800^\circ$

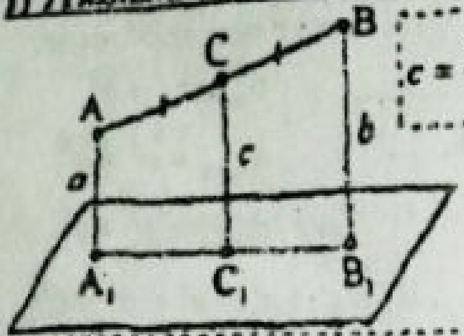
Bittadan olingan tashqi burchaklarining yig'indisi, $n\beta' = 360^\circ$

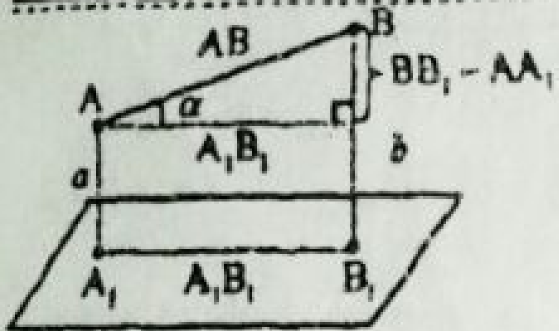
STEREOMETRIYA

Kesma va tekislik

I) Tekislikni kesib o'tmagan hol

$c = \frac{a+b}{2}$





A_1B_1 – chiziq AB kesmaning tekislikdagi proyektsiyasi

α – AB kesma yotuvchi to'g'ri chiziq va tekislik (yoki AB kesma va tekislik) orasidagi burchak

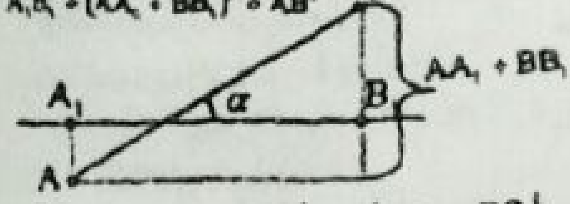
$A_1B_1^2 + (BB_1 - AA_1)^2 = AB^2$

$\sin\alpha = \frac{|BB_1 - AA_1|}{AB}$ | $\cos\alpha = \frac{A_1B_1}{AB}$

$tg\alpha = \frac{|BB_1 - AA_1|}{A_1B_1}$

II) Tekislikni kesib o'tgan hol

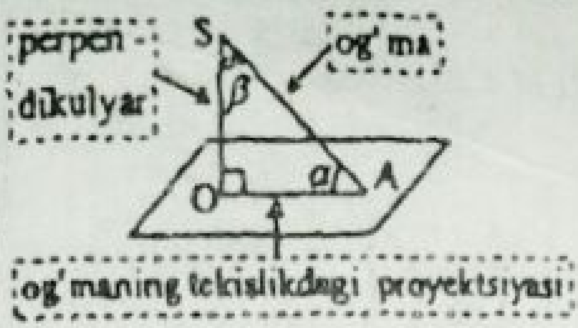
$A_1B_1^2 + (AA_1 + BB_1)^2 = AB^2$



$\sin\alpha = \frac{AA_1 + BB_1}{AB}$ | $c = \frac{|AA_1 - BB_1|}{2}$

c – kesma o'rtasidan tek-cha masofa

Og'ma, perpendikulyar va tekislik



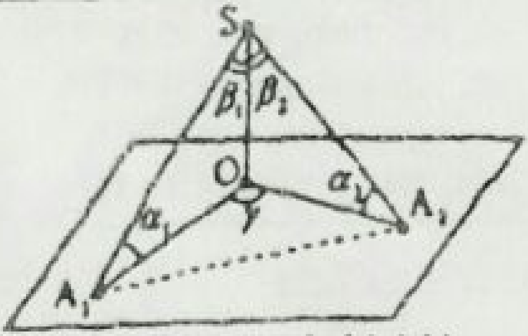
α – og'ma va tekislik orasidagi burchak

β – og'ma va perpendikulyar orasidagi burchak

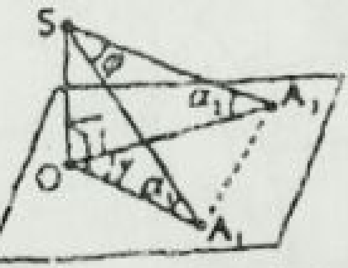
$SO^2 + OA^2 = SA^2$ | $\alpha + \beta = 90^\circ$

$\sin\alpha = \frac{SO}{SA}$ | $\cos\alpha = \frac{OA}{SA}$ | $tg\alpha = \frac{SO}{OA}$

Perpendikulyar va ikkita og'ma



Yon tomondan ko'rinishi;



$SO^2 + OA_1^2 = SA_1^2$ | $SO^2 + OA_2^2 = SA_2^2$

$SA_1^2 - OA_1^2 = SA_2^2 - OA_2^2$

$A_1A_2^2 = SA_1^2 + SA_2^2 - 2SA_1 \cdot SA_2 \cdot \cos\varphi$

γ – og'malar proyektsiya-ri or. burchak

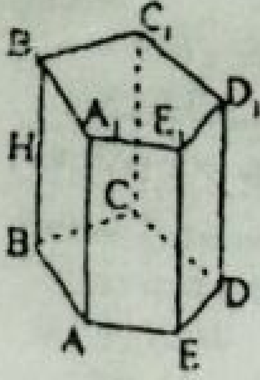
φ – og'malar orasidagi burchak

A_1A_2 – og'malar asoslari orasidagi mas.

Ko'pyoqlar

Prizma

I) To'g'ri prizma



* Yon sirti

$$S_{yon} = P_{asos} \cdot H$$

* To'la sirti

$$S_t = 2S_{asos} + S_{yon}$$

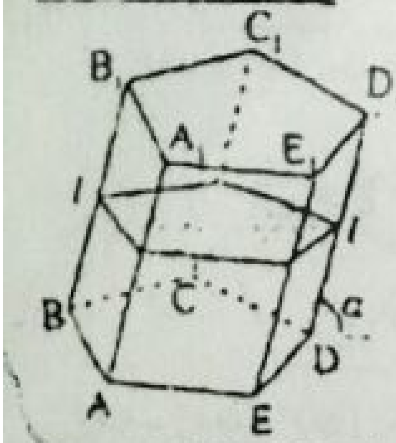
* Hajmi

$$V = S_{asos} \cdot H$$

n – asosining tomonlari soni bo'lsa;

* Asosi diagonallari soni; $\frac{n(n-3)}{2}$
* Prizma diagonallari soni; $n(n-3)$
* Yon yoqlarining soni, n ta
* Jami yoqlarining soni, $n+2$ ta
* Jami qirralarining soni, $3n$ ta
* Jami uchlarining soni, $2n$ ta

II) Og'ma prizma



$$H = l \cdot Sin\alpha$$

φ – yon qirra va asos tek. or. burchak

$$S_{yon} = P_{\perp} \cdot l \quad | \quad S_t = 2S_{asos} + S_{yon}$$

$$V = S_{asos} \cdot H \quad | \quad V = S_{\perp} \cdot l$$

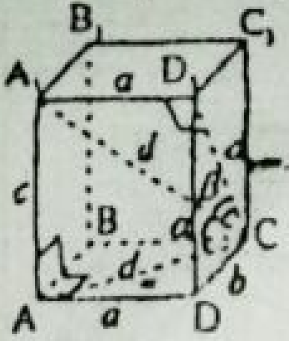
$P_{\perp}$ – perpendikulyar kesim perimetri

$S_{\perp}$ – perpendikulyar kesim yuzi

PARALLELEPIPED

I) To'g'ri burchakli parallelepiped

* Asos to'g'ri to'rtburchak va yon qirra asosga perpendikulyar.



$$d_{as} = a^2 + b^2$$

$$d_{yon1} = a^2 + c^2$$

$$d_{yon2} = b^2 + c^2$$

$$d = a^2 + b^2 + c^2$$

$$Sin\alpha = \frac{c}{d}$$

$$S_{as} = ab \qquad S_{yon} = 2(a+b)c$$

$$S_t = 2S_{as} + S_{yon} = 2(ab + ac + bc)$$

$$V = S_{as} \cdot H \qquad V = abc$$

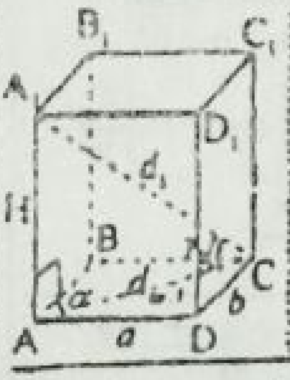
α – par. diagonali va asos tek. or. bur.

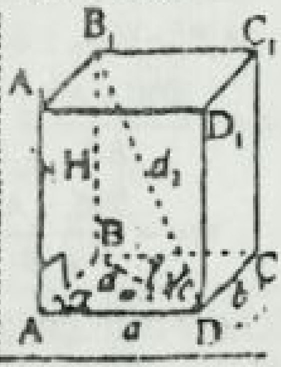
β – par. diagonali va yon yoq or. bur.

* 5 ta simmetriya tekisligiga ega

II) To'g'ri parallelepiped

* Asos parallelogramm va yon qirra asosga perpendikulyar.





d_{as1}, d_{as2} – parallelepiped asosining katta va kichik diagonallari

d_1, d_2 – parallelepipedning katta va kichik diagonallari

γ_1, γ_2 – parallelepipedning katta va kichik diagonallarining asos tekisligi bilan hosil qilgan burchaklari

$V = S_{as}H$ α - o'tkir burchak.

$S_{as} = abSin\alpha = \frac{d_{as1}d_{as2}}{2}Sin\varphi$

φ - parallelogramm diag. or. bur.

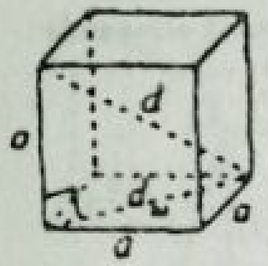
$H^2 + d_{as1}^2 = d_1^2$ $H^2 + d_{as2}^2 = d_2^2$

$Sin\gamma_1 = \frac{H}{d_1}$ | $\cos\gamma_1 = \frac{d_{as1}}{d_1}$ | $tg\gamma_1 = \frac{H}{d_{as1}}$

$d_{as1}^2 = a^2 + b^2 + 2ab\cos\alpha$

$d_{as2}^2 = a^2 + b^2 - 2ab\cos\alpha$

Kub



$S_{as} = a^2$ $S_{yon} = 4a^2$

$S_T = 6a^2$ $V = a^3$

$d_{as} = \sqrt{2}a$ $d = \sqrt{3}a$

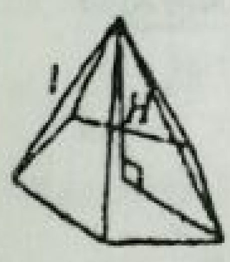
$r = \frac{a}{2}$ $R = \frac{a\sqrt{3}}{2}$

r va R – kubga ichki va tashqi chizilgan sharlarning radiuslari

• *Kub 9 ta simmetriya tekisligiga ega*

PIRAMIDA

• *Ixtiyoriy piramida*



$S_T = S_{as} + S_{yon}$

$V = \frac{1}{3}S_{as}H$

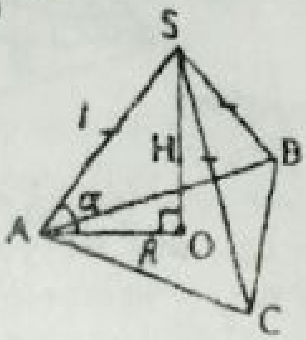
$V = \frac{1}{3}S_T \cdot r_{sh}$

r_{sh} – piramidaga ichki chizilgan sharning radiusi

• Masala yechishda ko'p hollarda piramidaning balandligi asosining qaysi nuqtasiga tushishi muhimdir.

Bunday hollarda quyidagi ma'lumotlardan foydalanish yaxshiroq bo'ladi.

I – hol. ***Masalada piramidaning barcha yon qirralari teng, yoki piramidaning barcha yon qirralari asos tekisligi bilan bir xil (α) burchak hosil qiladi deyilsa, uning balandligi asosiga tashqi chizilgan aylana markazига tushadi.***



$H^2 + R^2 = l^2$

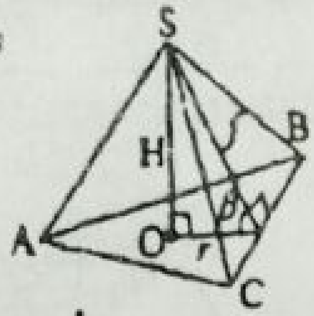
$Sin\alpha = \frac{H}{l}$

$\cos\alpha = \frac{R}{l}$

$tg\alpha = \frac{H}{R}$

* Agar biror ko'pburchakka tashqi aylana chizish mumkin bo'lmasa bu ko'pburchak bunday piramidaga asos bo'lolmaydi. (M-n; romb)

II – hol. *Piramidaning asosidagi barcha ikki yoqli burchaklari (β) teng, yoki piramidaning uchi uning tomonlaridan bir xil uzoqlikda bo'lsa, uning balandligi asosiga ichki chizilgan aylana markaziga tushadi.*



$H^2 + r^2 = f^2$

$Sin\beta = \frac{H}{f}$

$tg\beta = \frac{H}{r}$

$\cos\beta = \frac{r}{f} = \frac{S_{as}}{S_{yon}}$ | $S_{yon} = \frac{P_{as} \cdot f}{2}$

f - apofema,

P_{as} - piramida asosining perimetri

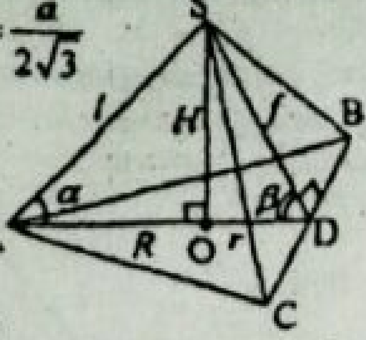
(To'g'ri to'rtburchak asos bo'lolmaydi)

Muntazam piramidalar

1) Muntazam uchburchakli piramida

$R=\frac{a}{\sqrt{3}}$ | $r=\frac{a}{2\sqrt{3}}$

$r=\frac{R}{2}$



f – apofema

l – yon qirra

α – yon qirra asos tek. or. burchak

β – asosidagi ikki yoqli burchak, yoki yon yoq va asos tek. or. burchak

$S_{asos}=\frac{a^2\sqrt{3}}{4}$ | $S_{yon}=\frac{1}{2}Pf=\frac{3}{2}af$

$S_T=S_{asos}+S_{yon}$ | $V=\frac{1}{3}S_{asos}H$

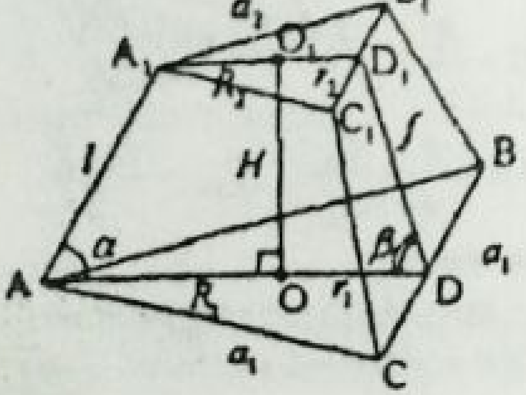
$H^2+R^2=l^2$ | $H^2+r^2=f^2$

$H^2=l^2-R^2=f^2-r^2$

$\sin\alpha=\frac{H}{l}$ | $\cos\alpha=\frac{R}{l}$ | $tg\alpha=\frac{H}{R}$

$\sin\beta=\frac{H}{f}$ | $\cos\beta=\frac{r}{f}$ | $tg\beta=\frac{H}{r}$

Muntazam uchburchakli kesik piramida



$S_1=\frac{a_1^2\sqrt{3}}{4}$ $S_2=\frac{a_2^2\sqrt{3}}{4}$

$S_{yon}=\frac{1}{2}(P_1+P_2)f=\frac{3}{2}(a_1+a_2)f$

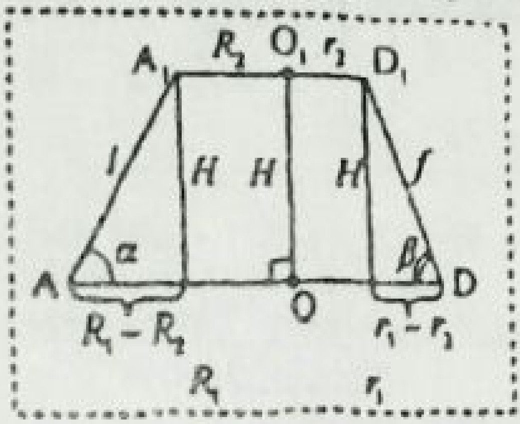
$S_T=S_1+S_2+S_{yon}$

$V=\frac{1}{3}H\left(S_1+\sqrt{S_1S_2}+S_2\right)$

$\frac{a_1}{a_2}=\frac{R_1}{R_2}=\frac{r_1}{r_2}$ | $\frac{S_1}{S_2}=\frac{a_1^2}{a_2^2}=\frac{R_1^2}{R_2^2}=\frac{r_1^2}{r_2^2}$

S_1, S_2 – piramida asoslarining yuzalari

* AA_1D_1D trapetsiya ajratib olinganda



$\sin\alpha=\frac{H}{l}$ | $\cos\alpha=\frac{R_1-R_2}{l}$

$tg\alpha=\frac{H}{R_1-R_2}$

$\sin\beta=\frac{H}{f}$ | $tg\beta=\frac{H}{r_1-r_2}$

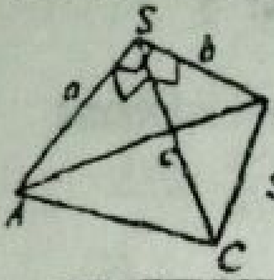
$\cos\beta=\frac{r_1-r_2}{f}=\frac{S_1-S_2}{S_{yon}}$

$(R_1-R_2)^2+H^2=l^2$

$(r_1-r_2)^2+H^2=f^2$

$l^2-(R_1-R_2)^2=f^2-(r_1-r_2)^2$

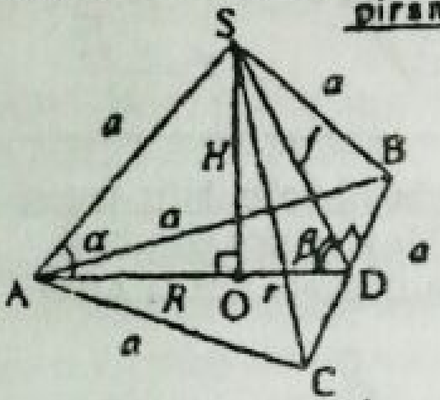
• Yon qirralari o'zaro perpendikulyar bo'lgan uchburchakli piramida



$V = \frac{abc}{6}$

$S_{yon} = \frac{1}{2}(ab + bc + ac)$

• Tetraedr – hamma qirralari teng muntazam uchburchakli piramida



$H = \frac{a\sqrt{6}}{3}$ | $S_T = a^2\sqrt{3}$ | $V = \frac{a^3\sqrt{2}}{12}$

• Tetraedr asosiga ichki va tashqi chizilgan aylanalar radiuslari

$r = \frac{a}{2\sqrt{3}} = \frac{a\sqrt{3}}{6}$ | $R = \frac{a}{\sqrt{3}} = \frac{a\sqrt{3}}{3}$

• Tetraedrga ichki va tashqi chizilgan sharlar radiuslari

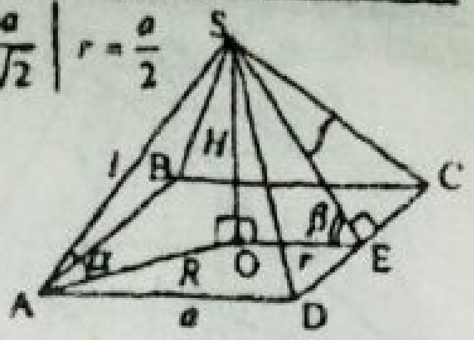
$r_{sh} = \frac{a\sqrt{6}}{12} = \frac{R_{sh}}{3}$ | $R_{sh} = \frac{a\sqrt{6}}{4} = 3r_{sh}$

• Yon qirra va asos tekisligi orasidagi burchak kosinusi; $\cos\alpha = \frac{1}{\sqrt{3}}$

• Asosidagi ikki yoqli burchak kosinusi; $\cos\beta = \frac{1}{3}$

Muntazam to'rtburchakli piramida

$R = \frac{a}{\sqrt{2}}$ | $r = \frac{a}{2}$



α – yon qirra va asos tek.or.burchak

β – asosidagi ikki yoqli (yakt yon yoq va asos tekisligi orasidagi) burchak

$Sin\alpha = \frac{H}{l}$ $\cos\alpha = \frac{R}{l}$ $tg\alpha = \frac{H}{R}$

$Sin\beta = \frac{H}{f}$ $\cos\beta = \frac{r}{f}$ $tg\beta = \frac{H}{r}$

$R^2 + H^2 = l^2$ $r^2 + H^2 = f^2$

$l^2 - R^2 = f^2 - r^2$

$S_{as} = a^2$ | $S_{yon} = \frac{1}{2}Pf$ | $S_T = S_{as} + S_{yon}$

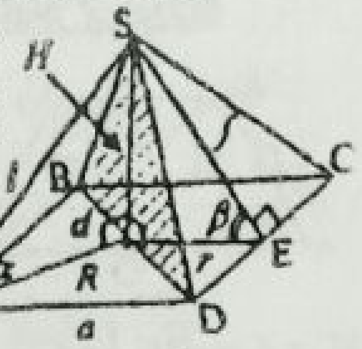
$V = \frac{1}{3}S_{as}H$ $V = \frac{1}{3}S_T \cdot r_{sh}$

r_{sh} – piramidaga ichki chizilgan sharning radiusi.

Diagonal kesim

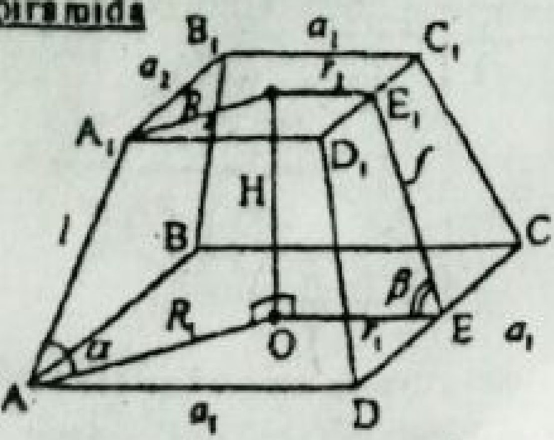
$d = \sqrt{2}a$

$d = 2l\cos\alpha$



$S_k = \frac{d \cdot H}{2}$ | $H = \frac{a\sqrt{2}}{2}tg\alpha$ | $H = l \cdot Sin\alpha$

Muntazam to'rtburchakli kesik piramida



$$\sin\alpha = \frac{H}{l} \quad\Big|\quad \cos\alpha = \frac{R_1 - R_2}{l}$$

$$tg\alpha = \frac{H}{R_1 - R_2} \quad\Big|\quad tg\beta = \frac{H}{r_1 - r_2}$$

$$\sin\beta = \frac{H}{f} \quad\Big|\quad \cos\beta = \frac{r_1 - r_2}{f} = \frac{S_1 - S_2}{S_{yon}}$$

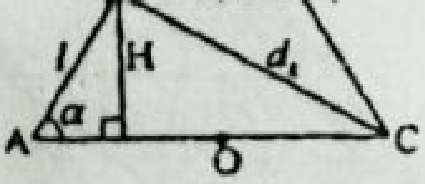
$$S_1 = a_1^2 \quad S_2 = a_2^2 \quad\Big|\quad S_{yon} = \frac{1}{2}(P_1 + P_2)f$$

$$S_T = S_1 + S_2 + S_{yon}$$

$$V = \frac{1}{3}H\left(S_1 + \sqrt{S_1 S_2} + S_2\right)$$

$$\frac{a_1}{a_2} = \frac{R_1}{R_2} = \frac{r_1}{r_2} \quad\Big|\quad \frac{S_1}{S_2} = \frac{a_1^2}{a_2^2} = \frac{R_1^2}{R_2^2} = \frac{r_1^2}{r_2^2}$$

Diagonal kesimi

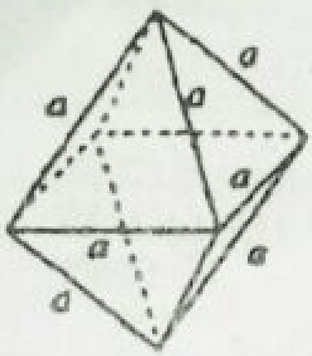


$$AC = d_1 = \sqrt{2}a_1 \quad\Big|\quad A_1C_1 = d_2 = \sqrt{2}a_2$$

d_1, d_2 – piramida asoslarining diagonallari.

$$S_k = \frac{d_1 + d_2}{2}H \quad\Big|\quad d_k^2 = \left(\frac{d_1 + d_2}{2}\right)^2 + H^2$$

Oktaedr – muntazam sakkizyoq



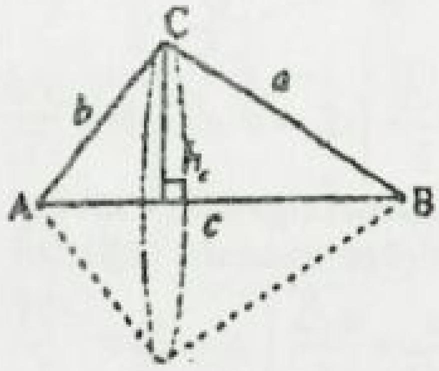
$$r_{ich} = \frac{a\sqrt{6}}{6}$$

$$R_{ta} = \frac{a\sqrt{2}}{2}$$

$$S_T = 2a^2\sqrt{3}$$

$$V = \frac{a^3\sqrt{2}}{3}$$

Uchburchakni uning biror tomoni atrofida aylantirishdan hosil bo'lgan jismning sirti va hajmi



c – tomon atrofida aylantirilganda:

$$S_c = \pi \cdot h_c(a + b) \quad\Big|\quad V_c = \frac{1}{3}\pi \cdot c \cdot h_c^2$$

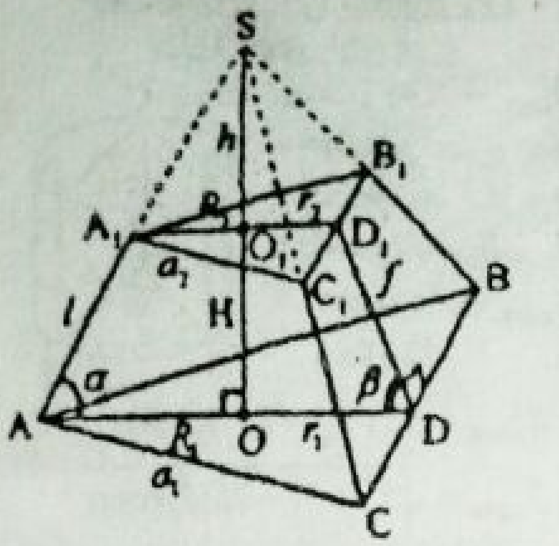
a – tomon atrofida aylantirilganda:

$$S_a = \pi \cdot h_a(b + c) \quad\Big|\quad V_a = \frac{1}{3}\pi \cdot a \cdot h_a^2$$

b – tomon atrofida aylantirilganda:

$$S_b = \pi \cdot h_b(a + c) \quad\Big|\quad V_b = \frac{1}{3}\pi \cdot b \cdot h_b^2$$

To'ldirilgan piramida



SABC – *katta (yoki to'la) piramida*

$SA_1B_1C_1$ – *kichik piramida*

H_T – *to'la piramidaning balandligi*

H – *kesik piramidaning balandligi*

h – kichik piramidaning balandligi

V_T – *to'la piramidaning hajmi*

V_1 – *kichik piramidaning hajmi*

V – *kesik piramidaning hajmi*

$$H_T = SO = H + h$$

$$H = OO_1 \qquad h = SO_1$$

$$V_T = V + V_1$$

$$V = \frac{1}{3}H\left(S_1 + \sqrt{S_1S_2} + S_2\right)$$

$$V_T = \frac{1}{3}S_1H_T \qquad V_1 = \frac{1}{3}S_2h$$

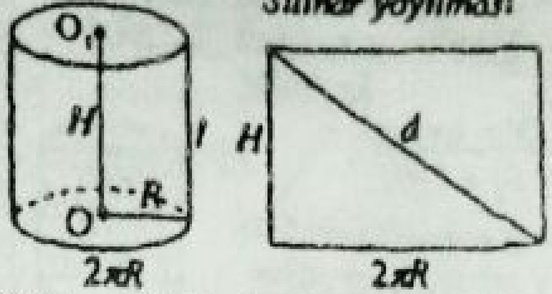
$$\frac{a_1}{a_2} = \frac{H_T}{h} = \frac{H+h}{h} = \frac{R_1}{R_2} = \frac{r_1}{r_2}$$

$$\frac{S_1}{S_2} = \frac{a_1^2}{a_2^2} = \frac{H_T^2}{h^2} = \frac{(H+h)^2}{h^2} = \frac{R_1^2}{R_2^2} = \frac{r_1^2}{r_2^2}$$

$$\frac{V_T}{V_1} = \frac{a_1^3}{a_2^3} = \frac{H_T^3}{h^3} = \frac{(H+h)^3}{h^3} = \frac{R_1^3}{R_2^3} = \frac{r_1^3}{r_2^3}$$

SILINDR

Silindr yoyilmasi



OO_1 – *silindr o'qi*, *H – balandligi*,

l – yasovchisi, *R – asosining radiusi*

S_{as} – *silindr asosining yuzi*

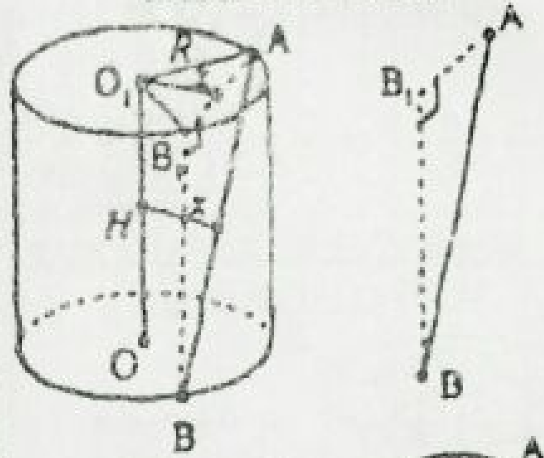
V – silindrning hajmi

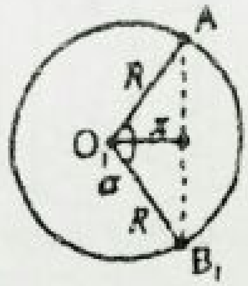
$$OO_1 = H = l \;\Big|\; S_{as} = \pi R^2 \;\Big|\; S_{yon} = 2\pi RH$$

$$S_T = 2S_{as} + S_{yon} = 2\pi R(R + H)$$

$$V = S_{as}H = \pi R^2 H$$

Silindr va kesma





$$AB_1^2 + H^2 = AB^2$$

$$x^2 + \left(\frac{AB_1}{2}\right)^2 = R^2$$

$$\sin\frac{\alpha}{2} = \frac{AB_1}{2R}$$

$$\cos\frac{\alpha}{2} = \frac{x}{R} \qquad tg\frac{\alpha}{2} = \frac{AB_1}{2x}$$

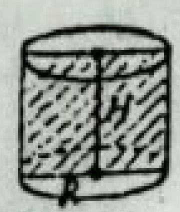
x – kesma va silindr o'qi or. masofa

Silindr va kesimlar

Silindrning o'qiga parallel kesimlar

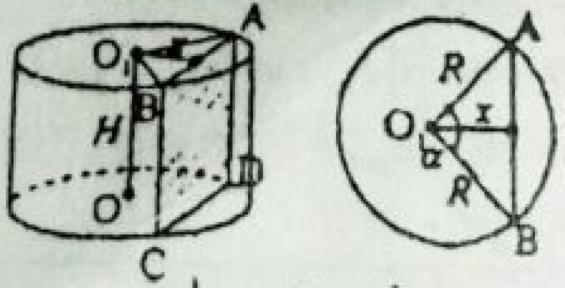
I. O'q kesim

$S_1 = 2RH$



Teng tomonli silindrda o'q kesimi kvadratdan iborat bo'ladi; $H = 2R$

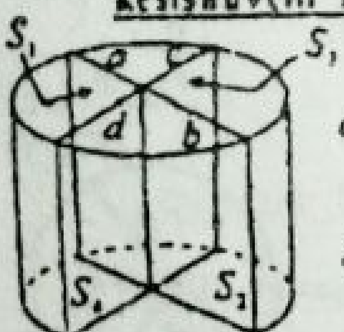
II. Silindr o'qidan biror x masofadan o'tuvchi kesim



$S_1 = AB \cdot H$ | $x^2 + \left(\frac{AB}{2}\right)^2 = R^2$

$\sin\frac{\alpha}{2} = \frac{AB}{2R}$ | $\cos\frac{\alpha}{2} = \frac{x}{R}$ | $tg\frac{\alpha}{2} = \frac{AB}{2x}$

III. Silindr o'qiga parallel o'zaro kesishuvchi kesimlar



$a \cdot b = c \cdot d =$
$= R^2 - x^2$

$S_1 \cdot S_2 = S_3 \cdot S_4$

Tekisliklar perpendikulyar bo'lsa

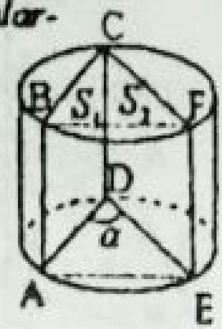
$S_{o'q} = \sqrt{S_1^2 + S_2^2 + S_3^2 + S_4^2}$ o'rinli.

x – silindr asosining markazidan tekisliklarning kesishish nuqtasigacha bo'lgan masofa

$S_{o'q}$ – silindrning o'q kesimi

IV. Silindr yasovchisidan o'tuvchi tekisliklar

S_{ABFE} – ABFE nuqtalardan o'tuvchi tekislik $\alpha = 90^\circ$ da, ya'ni $S_1 \perp S_2$ bo'lganda S_{ABFE} – o'q kesim bo'ladi

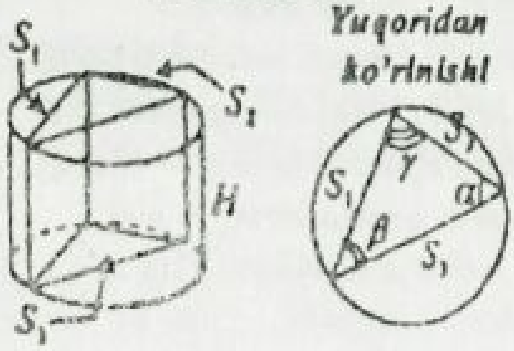


$S_{ABFE}^2 = S_1^2 + S_2^2 - 2S_1S_2\cos\alpha$

$S_{ABFE} = \sqrt{S_1^2 + S_2^2 - 2S_1S_2\cos\alpha}$

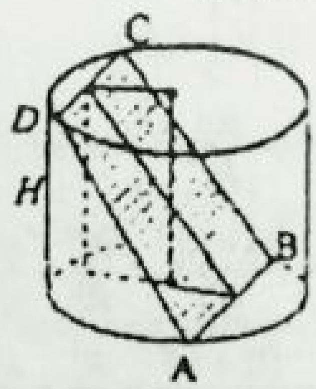
Qo'shimcha

Yuqoridan ko'rinishi



$\frac{S_1}{\sin\alpha} = \frac{S_2}{\sin\beta} = \frac{S_3}{\sin\gamma} = 2RH$

V. Silindr o'qini kesib o'tuvchi kesim



Kesim ABCD trapetsiyadan iborat bo'ladi.

KONUS

R – konus asosining radiusi

l – konus yasovchisi, *H* – balandligi

h – konus asosi markazidan uning yasovchisigacha bo'lgan masofa

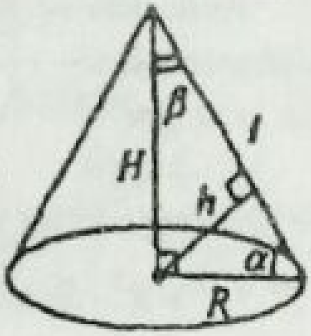
α – yasovchi va asos tek. or. burchak

β – yasovchi va balandlik or. burchak

$$\sin\alpha = \frac{H}{l} = \frac{h}{R}$$

$$\cos\alpha = \frac{R}{l} = \frac{S_{a}}{S_{yon}}$$

$$tg\alpha = \frac{H}{R}$$



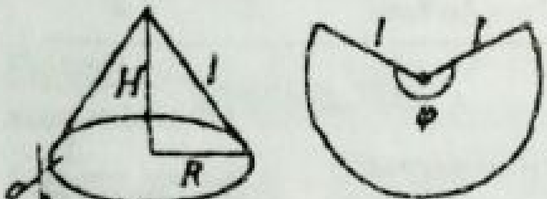
$$H^2 + R^2 = l^2 \mid R \cdot H = l \cdot h$$

$$S_{a} = \pi R^2 \qquad S_{yon} = \pi Rl$$

$$S_T = S_{a} + S_{yon} = \pi R(R + l)$$

$$V = \frac{1}{3}S_{a}H = \frac{1}{3}\pi R^2 H = \frac{1}{3}S_{yon} \cdot h$$

Konusning yoyilmasi

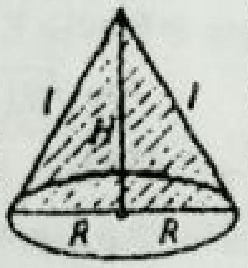


$$2\pi R = l \cdot \varphi \qquad \varphi = \frac{2\pi R}{l}$$

I. Konusning o'q kesimi

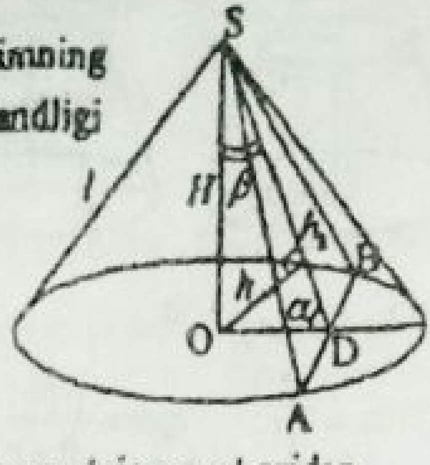
$$S_1 = RH$$

Teng tomonli konusning o'q kesimi muntazam uchburchakdan iborat bo'ladi.

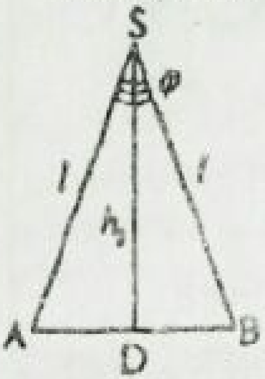


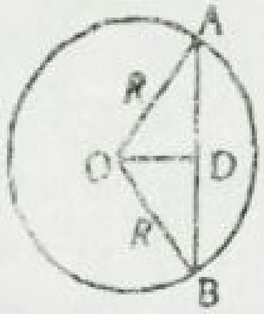
II. Konusning yasovchilari orqali o'tkazilgan kesim

h_1 – kesimning balandligi



h – konus asosining markazidan kesimgacha bo'lgan masofa

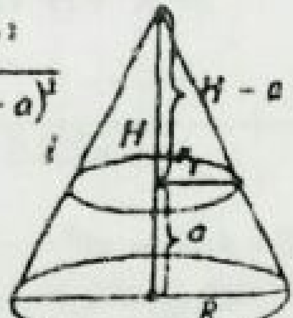




II. Konusning asosiga parallel qilib o'tkazilgan kesim

$$\frac{S_{a}}{S_1} = \frac{R^2}{r_1^2} = \frac{H^2}{(H-a)^2}$$

$$\frac{R}{r_1} = \frac{H}{H-a}$$



V – konusning to'liq hajmi

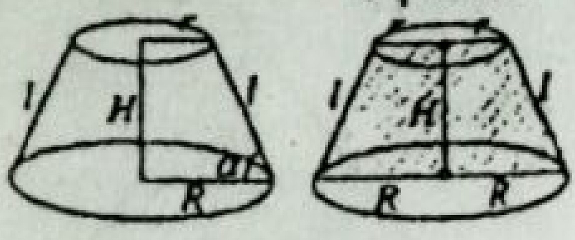
V_1 – konusning kesimdan yuqori qismining hajmi

$$\frac{V}{V_1} = \frac{R^3}{r_1^3} = \frac{H^3}{(H-a)^3}$$

a – konus asosidan kesimgacha bo'lgan masofa

Kesik konus

O'q kesimi



$$H^2+(R-r)^2=l^2$$

$$\sin\alpha=\frac{H}{l} \qquad tg\alpha=\frac{H}{R-r}$$

$$\cos\alpha=\frac{R-r}{l}=\frac{S_{as1}-S_{as2}}{S_{yon}}$$

$$S_{as1}=\pi R^2 \qquad S_{as2}=\pi r^2$$

$$S_{yon}=\pi l(R+r)$$

$$S_T=S_{as1}+S_{as2}+S_{yon}$$

$$S_T=\pi\left(R^2+r^2+l(R+r)\right)$$

$$V=\frac{1}{3}H\left(S_{as1}+\sqrt{S_{as1}S_{as2}}+S_{as2}\right)$$

$$V=\frac{1}{3}\pi H\left(R^2+Rr+r^2\right)$$

$$S_{o'q.k}=(R+r)\cdot H$$

SHAR

$$S=4\pi R^2=\pi d^2$$

$$V=\frac{4}{3}\pi R^3=\frac{\pi d^3}{6}$$

• Markazi $O(a;b;c)$ nuqtada bo'lgan R radiusli shar tenglamasi

$$(x-a)^2+(y-b)^2+(z-c)^2\le R^2$$

• Markazi $O(a;b;c)$ nuqtada bo'lgan R radiusli sfera tenglamasi

$$(x-a)^2+(y-b)^2+(z-c)^2=R^2$$

Shar kesimi. Shar va tekislik

R, r – shar va kesim radiuslari

f – shar markazidan kesimgacha masofa

α – sharning radiusi va tek. or. bur.

$$f^2+r^2=R^2 \quad S_k=\pi r^2$$

$$\sin\alpha=\frac{f}{R} \quad \cos\alpha=\frac{r}{R} \quad tg\alpha=\frac{f}{r}$$

• Agar biror ko'pburchakning tomonlari (M-n: uchburchak, to'rtburchak va hokazo) sharga urinsa, kesim shu ko'pburchakka ichki chizilgan doira bo'ladi va r ko'pburchakka ichki chizilgan doiraning radiusi bo'ladi.

Shar segmenti

r – shar segmenti asosining radiusi

H – sh. segmenti balandligi

S_{yon} – segment yon sirti

S_T – to'la sirti, V – segment hajmi

$$r=\sqrt{H(2R-H)}$$

$$S_{yon}=2\pi RH \qquad S_T=2\pi RH+\pi r^2$$

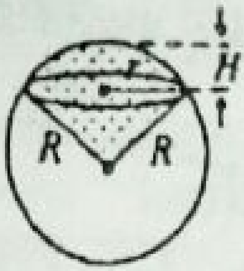
$$V=\frac{1}{3}\pi H^2(3R-H)$$

Shar sektori

$S_T = \pi R(2H + r)$

$V = \frac{2\pi}{3} R^2 H$

$V = \frac{\pi}{6} d^2 H$



d – sharning diametri

Shar halqasi

H – shar halqasi balandligi

R_1, R_2 – shar halqasining radiuslari

$S_{yon} = 2\pi RH$

$S_T = \pi(2RH + R_1^2 + R_2^2)$

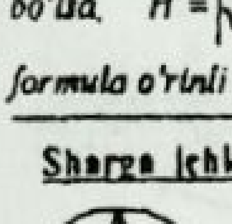
$H = \sqrt{R^2 - R_1^2} + \sqrt{R^2 - R_2^2}$

$V = \frac{1}{6}\pi H(3R_1^2 + 3R_2^2 + H^2)$

$V = \frac{1}{6}\pi H^3 + \frac{1}{2}\pi(R_1^2 + R_2^2)$

Eslatma: R_1 va R_2 – radiuslar shar markazidan bir tomonga joylashgan bo'lsa, $H = \left|\sqrt{R^2 - R_1^2} - \sqrt{R^2 - R_2^2}\right|$ formula o'rinli bo'ladi.

Sharga ichki chizilgan konus



$(H - R)^2 + R_k^2 = R^2$

$R = \frac{R_k^2 + H^2}{2H}$

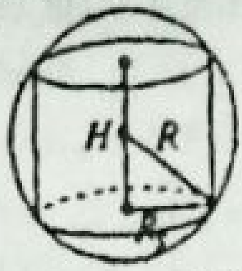
R – shar radiusi, H – kon. balandligi

R_k – konus asosining radiusi

Sharga ichki chizilgan silindr

R_S – silindr asosining radiusi

$R_S^2 + \frac{H^2}{4} = R^2$

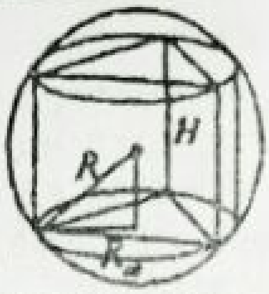


$H = \frac{2\sqrt{3}}{3} R$ bo'lganda sharga ichki chizilgan silindrning hajmi eng katta bo'ladi.

Sharga ichki chizilgan prizma

$R_{as}^2 + \left(\frac{H}{2}\right)^2 = R^2$

R_{as} – prizma asosiga tashqi chizilgan aylana radiusi



Sharga ichki chizilgan piramida

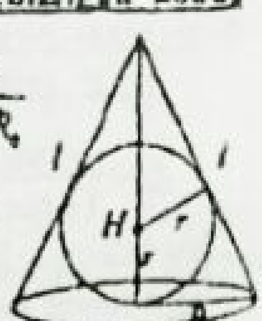
$(H - R)^2 + R_{as}^2 = R^2$

$R = \frac{R_{as}^2 + H^2}{2H}$

Sharga tashqi chizilgan konus

$\frac{l}{H - r} = \frac{R_k}{r} = \frac{H}{l - R_k}$

$\frac{R_k}{r} = \frac{H}{\sqrt{H^2 - 2Hr}}$

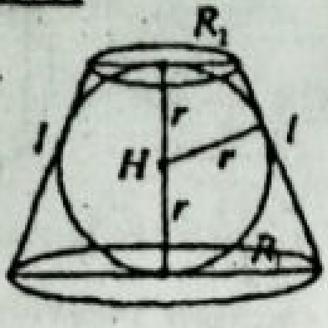


r – sharning radiusi

Sharga tashqi chizilgan kesik konus

$l = R_1 + R_2 = l_{o'r}$

$H = 2r \;\Big|\; r = \dfrac{H}{2}$

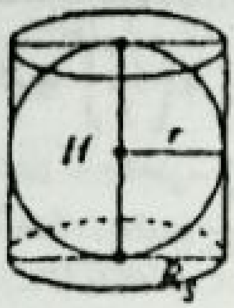


$l_{o'r}$ – trapetsiyaning o'rta chizig'i

Silindrga ichki chizilgan shar

$r = R_s = \dfrac{H}{2}$

Silindrning o'q kesimi kvadratdan iborat bo'ladi



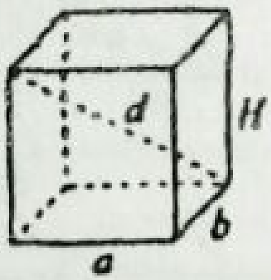
Shar va parallelepiped

R_{sh} – tashqi chizilgan sharning rad.

r_{sh} – ichki chizilgan sharning rad.

$R_{sh} = \dfrac{d}{2} \;\Big|\; r_{sh} = \dfrac{H}{2}$

$V = \dfrac{1}{3} S_T \cdot r_{sh}$



Muntazam piramidaga shar ichki chizilganda

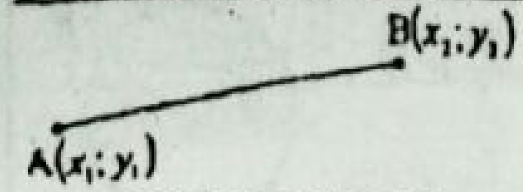
$$r_{sh} = \frac{\sin\alpha}{1+\cos\alpha} \cdot r_{ae}$$

r_{ae} – pir. asosiga ichki chizilgan aylananing radiusi.

α – pir. asosidagi ikki yoqli burchak

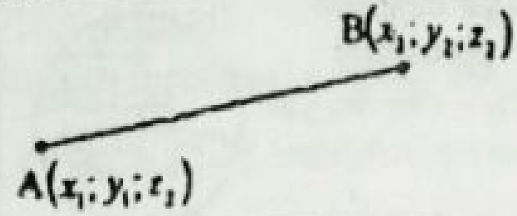
KOORDINATALAR SISTEMASI

• Ikki nuqta orasidagi masofa



$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ yoki

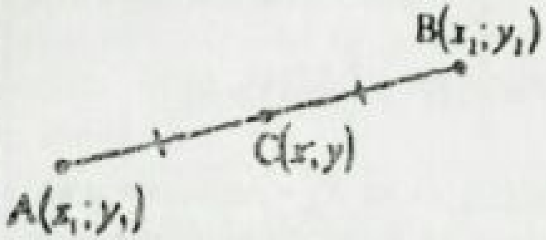
$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$



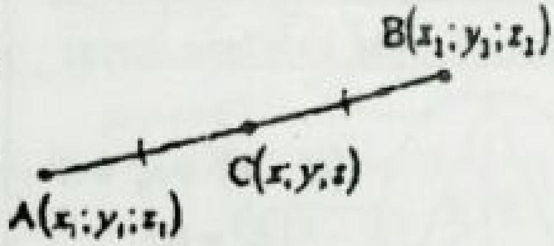
$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_2 - z_1)^2}$

• Kesma o'rtasining koordinatasi



$x = \dfrac{x_1 + x_2}{2} \;\Big|\; y = \dfrac{y_1 + y_2}{2}$



$x = \dfrac{x_1 + x_2}{2} \;\Big|\; y = \dfrac{y_1 + y_2}{2} \;\Big|\; z = \dfrac{z_1 + z_2}{2}$

XYZ koordinatalar sistemasidagi nuqtalarning yozilish tartibi

I) Koordinata o'qlaridagi nuqtalarning yozilishi

OX o'qidagi nuqtalar; $P(x;0;0)$

OY o'qidagi nuqtalar; $P(0;y;0)$

OZ o'qidagi nuqtalar; $P(0;0;z)$

II) Koordinata tekisliklaridagi nuqtalarning yozilishi

XY tekisligidagi nuqtalar. $P(x;y;0)$

YZ tekisligidagi nuqtalar; $P(0;y;z)$

XZ tekisligidagi nuqtalar; $P(x;0;z)$

*** A(x;y;z) nuqtaning koordinatalar sistemasidagi proyektsiyalari**

I) Koordinata o'qlaridagi pro-ri

OX o'qidagi – $A_x(x;0;0)$

OY o'qidagi – $A_y(0;y;0)$

OX o'qidagi – $A_z(0;0;z)$

II) Koordinata tekisliklaridagi pro-ri

XY tekisligidagi – $A_{xy}(x;y;0)$

YZ tekisligidagi – $A_{yz}(0;y;z)$

XZ tekisligidagi – $A_{xz}(x;0;z)$

*** A(x;y;z) nuqtaning koordinatalar fazosidagi simmetrik nuqtalari**

I) Koordinata o'qlariga nisbatan simmetrik nuqtalari

OX o'qiga nisbatan – $A'_x(x;-y;-z)$

OY o'qiga nisbatan – $A'_y(-x;y;-z)$

OZ o'qiga nisbatan – $A'_z(-x;-y;z)$

M-n: $B(-2;3;-1)$ nuqtaning OZ o'qiga nisbatan simmetrik nuqtasi $B'_z(2;-3;-1)$ ko'rinishda bo'ladi.

II) Koordinata tekisliklariga nisbatan simmetrik nuqtalari

XY tekisligiga nisbatan – $A'_{xy}(x;y;-z)$

YZ tekisligiga nisbatan – $A'_{yz}(-x;y;z)$

XZ tekisligiga nisbatan – $A'_{xz}(x;-y;z)$

M-n: $D(6;-2;1)$ nuqtaning YZ tekisligiga nisbatan simmetrik nuqtasi $D'_{yz}(-6;-2;1)$ ko'rinishda bo'ladi

III) Koordinatalar boshiga nisbatan simmetrik nuqtasi

$A'_0(-x;-y;-z)$

M-n: $N(-2;3;9)$ nuqtaning koordinatalar boshiga nisbatan simmetrigi $N'_0(2;-3;-9)$ ko'rinishda bo'ladi;

A(x;y;z) nuqtadan

I) Koordinata o'qlarigacha masofalar

OX o'qigacha ya'ni $A(x;y;z)$ nuqtadan $A_x(x;0;0)$ nuqtagacha

$$AA_x = \sqrt{y^2 + z^2}$$

OY o'qigacha: $AA_y = \sqrt{x^2 + z^2}$

OZ o'qigacha: $AA_z = \sqrt{x^2 + y^2}$

II) Koordinata tekisliklarigacha masofalar

XY tekisligigacha; ya'ni $A(x;y;z)$ nuqtadan $A_{xy}(x;y;0)$ gacha masofa

$$AA_{xy} = |z|$$

YZ tekisligigacha; $AA_{yz} = |x|$

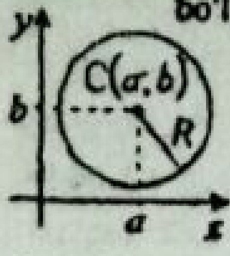
XZ tekisligigacha; $AA_{xz} = |y|$

III) Koordinatalar boshigacha, ya'ni A(x;y;z) nuqtadan O(0;0;0)

nuqtagacha; $AO = \sqrt{x^2 + y^2 + z^2}$

AYLANA TENGLAMASI

I) Markazi $C(a;b)$ nuqtada, radiusi R bo'lgan aylana tenglamasi



$$(x-a)^2+(y-b)^2=R^2$$

Doira tenglamasi

$$(x-a)^2+(y-b)^2\le R^2$$

Markazi $C(a;b;c)$ nuqtada bo'lgan

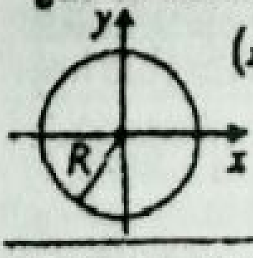
a) Sfera tenglamasi;

$$(x-a)^2+(y-b)^2+(z-c)^2=R^2$$

b) Shar tenglamasi;

$$(x-a)^2+(y-b)^2+(z-c)^2\le R^2$$

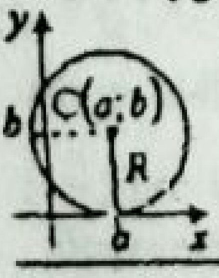
II) Markazi koordinatalar boshida, ya'ni markazi $C(0;0)$ nuqtada bo'lgan R radiusli aylana tenglamasi



$$(x-0)^2+(y-0)^2=R^2$$

$$x^2+y^2=R^2$$

III) Markazi $C(a;b)$ nuqtada bo'lgan OX o'qiga urinuvchi aylana

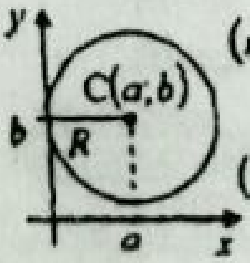


$$(x-a)^2+(y-b)^2=R^2$$

$$R=|b|$$

$$(x-a)^2+(y-b)^2=b^2$$

IV) Markazi $C(a;b)$ nuqtada bo'lgan OY o'qiga urinuvchi aylana

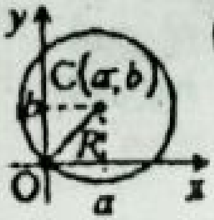


$$(x-a)^2+(y-b)^2=R^2$$

$$R=|a|$$

$$(x-a)^2+(y-b)^2=a^2$$

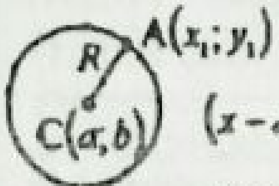
V) Markazi $C(a;b)$ nuqtada bo'lgan koordinatalar boshidan, ya'ni $O(0;0)$ nuqtadan o'tuvchi aylana



$$(x-a)^2+(y-b)^2=R^2$$

$$R=OC=\sqrt{a^2+b^2}$$

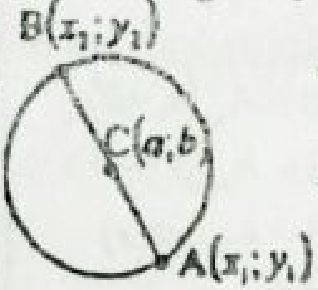
VI) Markazi $C(a;b)$ nuqtada bo'lgan va $A(x_1;y_1)$ nuqtadan o'tuvchi aylana tenglamasi



$$(x-a)^2+(y-b)^2=R^2$$

$$R=AC=\sqrt{(a-x_1)^2+(b-y_1)^2}$$

VII) Diametrining uchlari $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalarda bo'lgan aylana tenglamasi



$$a=\frac{x_1+x_2}{2}$$

$$b=\frac{y_1+y_2}{2}$$

$$R=\frac{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}}{2}$$

$$(x-a)^2+(y-b)^2=R^2$$

• To'g'ri chiziq va aylana

$$\begin{cases}(x-a)^2+(y-b)^2=R^2\\ y=kx+b\end{cases}$$

$D>0$ da kesishadi

$D=0$ da urinadi

$D<0$ da kesishmaydi

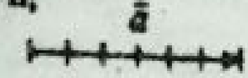
VEKTORLAR

Vektor – $\vec{a}$

Vektorning uzunligi, moduli yoki absolyut qiymati – $|\vec{a}|$ yoki a

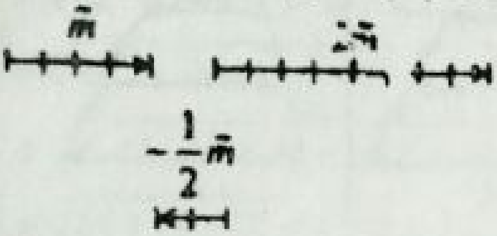
$$|\vec{a}| = \sqrt{\vec{a}^2} \quad \text{yoki} \quad a = \sqrt{\vec{a}^2}$$

M-n;



$$|\vec{a}| = a = 6$$

• Vektorni songa ko'paytirish



Kolleniar vektorlar

I) Parllel vektorlar

$$\vec{a} = 2\vec{b}$$

II) Qarama qarshi yo'nalgan vektorlar

$$\vec{a} = -\frac{6}{5}\vec{d}$$

III) Teng vektorlar

$$\vec{a} = \vec{p}$$

IV) Qarama qarshi vektorlar

$$\vec{a} = -\vec{q} \quad \text{va} \quad |\vec{a}| = |\vec{q}|$$

1 – tarif. Bir xil yoki qarama qarshi yo'nalgan vektorlar kolleniar vektorlar deyiladi.

2 – tarif. O'zaro parallel to'g'ri chiziqlarda yotuvchi vektorlar kolleniar vektorlar deyiladi.

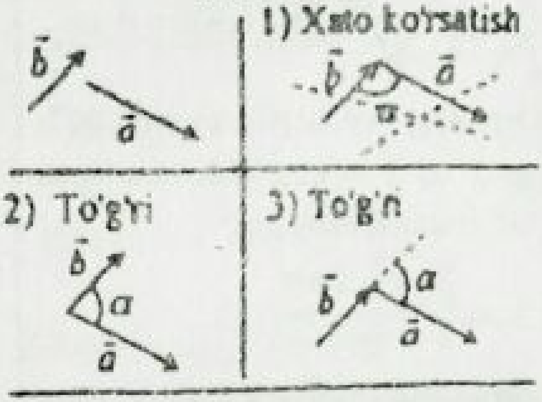
3 – tarif. Bir-biridan faqat biror son marta farq qiluvchi vektorlar kolleniar vektorlar deyiladi;

M-n; $\vec{a}$ va $\vec{b}$ vektorlar kolleniar vektorlar bo'lsa, $\vec{a} = \lambda \cdot \vec{b}$, hamda

1) $\lambda > 0$ bo'lganda vektorlar parallel

2) $\lambda < 0$ bo'lganda qarama qarshi yo'nalgan bo'ladi.

Vektorlar orasidagi burchakni ko'rsatish



Vektorlarning skalyar ko'paytmasi

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\alpha \quad \text{yoki}$$

$$\vec{a} \cdot \vec{b} = a \cdot b \cdot \cos\alpha$$

• $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchakning kosinusi

$$\cos\alpha = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$$

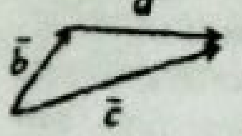
• Perpendikulyarlik sharti; $\alpha = 90°$

$$\vec{a} \cdot \vec{b} = 0$$

Vektorlarni qo'shish va ayirish

I) Uchburchak qoidasi

Qo'shish



$\vec{c} = \vec{a} + \vec{b}$

Ayirish

$\vec{d} = \vec{a} - \vec{b}$

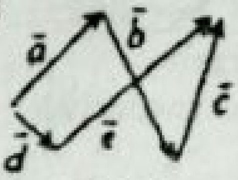
II) Parallelogramm qoidasi

$\vec{d}_1 = \vec{a} + \vec{b}$

$\vec{d}_2 = \vec{a} - \vec{b}$

* Qo'shimcha ma'lumotlar

1)



$\vec{a} + \vec{b} + \vec{c} = \vec{d} + \vec{e}$

2) Orasidagi burchagi α ga teng bo'lgan $\vec{a}$ va $\vec{b}$ vektorlardan qurilgan

a) Uchburchakning yuzi.

$$S_{\triangle} = \frac{|\vec{a}| \cdot |\vec{b}| \cdot \sin\alpha}{2}$$

b) Parallelogrammning yuzi.

$$S_{par} = |\vec{a}| \cdot |\vec{b}| \cdot \sin\alpha$$

3) Agar $\vec{a}$ va $\vec{b}$ vektorlar biror α burchak tashkil qilsa, hamda ular uchun $\vec{a} \cdot \vec{b} = \mu$ munosabat berilgan bo'lsa ularga qurilgan

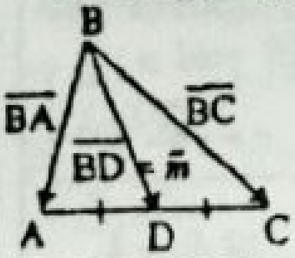
a) Uchburchakning yuzi:

$$S_{\triangle} = \frac{\vec{a} \cdot \vec{b} \cdot |tg\alpha|}{2} = \mu \cdot |tg\alpha|$$

b) Parallelogrammning yuzi.

$$S_{par} = \vec{a} \cdot \vec{b} \cdot |tg\alpha| = \mu \cdot |tg\alpha|$$

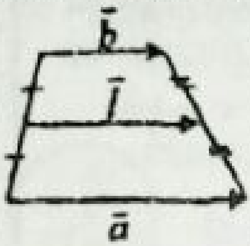
4) Ixtiyoriy uchburchak medianasi uchun quyidagi ma'lumot o'rinli



$$\overline{BD} = \frac{\overline{BA} + \overline{BC}}{2}$$

$$\vec{m} = \frac{\overline{BA} + \overline{BC}}{2}$$

5) Ixtiyoriy trapetsiya uchun



$$\vec{l} = \frac{\vec{a} + \vec{b}}{2}$$

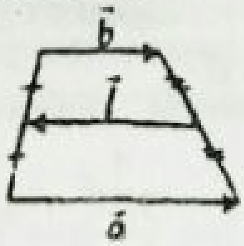
$$l = \frac{a + b}{2}$$

1) $\vec{a} = \lambda\vec{l}$

$$\lambda = \frac{|\vec{a}|}{|\vec{l}|}$$

2) $\vec{b} = \mu\vec{l}$

$$\mu = \frac{|\vec{b}|}{|\vec{l}|}$$



$$\vec{l} = -\frac{\vec{a} + \vec{b}}{2}$$

$$l = \frac{a + b}{2}$$

1) $\vec{a} = \lambda\vec{l}$

$$\lambda = -\frac{|\vec{a}|}{|\vec{l}|}$$

2) $\vec{b} = \mu\vec{l}$

$$\mu = -\frac{|\vec{b}|}{|\vec{l}|}$$

Vektorlarga doir misollar

1 – misol $|\vec{a}| = 4$, $|\vec{b}| = 6$, hamda $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak 60° bo'lsa, $2\vec{a} + \vec{b}$ vektorning modulini toping.

Yechilishi,

$$|2\vec{a} + \vec{b}| = \sqrt{(2\vec{a} + \vec{b})^2} =$$

$$= \sqrt{4\vec{a}^2 + 4\vec{a}\vec{b} + \vec{b}^2} =$$

$$= \sqrt{4a^2 + 4ab\cos\alpha + b^2} =$$

$$= \sqrt{4 \cdot 4^2 + 4 \cdot 4 \cdot 6 \cdot \cos 60^\circ + 6^2} =$$

$$= \sqrt{64 + 48 + 36} = \sqrt{148} = 2\sqrt{37}$$

---

2 – misol $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 8$ hamda $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak $\alpha = 60^\circ$, $\vec{a}$ va $\vec{c}$ vektorlar orasidagi burchak $\beta = 120^\circ$ bo'lsa, $\vec{a}$ va $(2\vec{b} + 3\vec{c})$ vektorlarning skalyar ko'paytmasini toping.

Yechilishi;

$$\vec{a}(2\vec{b} + 3\vec{c}) = 2\vec{a}\vec{b} + 3\vec{a}\vec{c} =$$

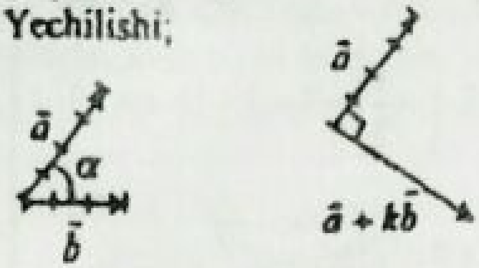
$$= 2ab\cos\alpha + 3ac\cos\beta = 2 \cdot 3 \cdot 4 \times$$

$$\times \cos 60^\circ + 3 \cdot 3 \cdot 8 \cdot \cos 120^\circ =$$

$$= 24 \cdot \frac{1}{2} + 72 \cdot \left(-\frac{1}{2}\right) = -24$$

3 – misol. $|\vec{a}| = 4$, $|\vec{b}| = 3$ hamda ular orasidagi burchak $\alpha = 60^\circ$ bo'lsa, k ning qanday qiymatida $\vec{a}$ vektor $(\vec{a} + k\vec{b})$ vektorga perpendikulyar bo'ladi?

Yechilishi;



Perpendikulyarlik shartiga ko'ra $\vec{a} \cdot (\vec{a} + k\vec{b}) = 0$ bo'lishi kerak.

$$\vec{a} \cdot (\vec{a} + k\vec{b}) = \vec{a}^2 + k \cdot \vec{a}\vec{b} =$$

$$= a^2 + k \cdot ab\cos\alpha =$$

$$= 4^2 + k \cdot 4 \cdot 3 \cdot \cos 60^\circ =$$

$$= 16 + k \cdot 12 \cdot \frac{1}{2} = 16 + 6k$$

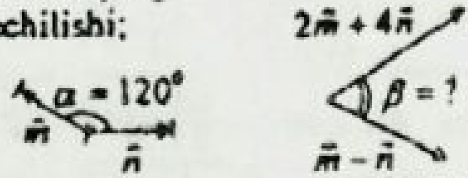
$16 + 6k = 0$ dan $k = -\frac{8}{3}$

yoki $k = -2\frac{2}{3}$ kelib chiqadi.

---

4 – misol. Agar $\vec{m}$ va $\vec{n}$ vektorlar $\alpha = 120^\circ$ li burchak hosil qiluvchi birlik vektorlar bo'lsa, $2\vec{m} + 4\vec{n}$ va $\vec{m} - \vec{n}$ vektorlar orasidagi burchakni toping.

Yechilishi;



$$\cos\beta = \frac{(2\vec{m} + 4\vec{n}) \cdot (\vec{m} - \vec{n})}{|2\vec{m} + 4\vec{n}| \cdot |\vec{m} - \vec{n}|}$$

1-ish;

$(2\bar{m} + 4\bar{n})\cdot(\bar{m} - \bar{n}) = 2\bar{m}^2 - 2\bar{m}\bar{n} +$

$+ 4\bar{m}\bar{n} - 4\bar{n}^2 = 2\bar{m}^2 + 2\bar{m}\bar{n} - 4\bar{n}^2 =$

$= 2m^2 + 2mn\cos\alpha - 4n^2 =$

$= 2\cdot 1^2 + 2\cdot 1\cdot 1\cdot\cos 120^\circ - 4\cdot 1^2 =$

$= 2 + 2\cdot\left(-\frac{1}{2}\right) - 4 = 2 - 1 - 4 = -3$

2-ish;

$|2\bar{m} + 4\bar{n}| = \sqrt{(2\bar{m} + 4\bar{n})^2} =$

$\sqrt{4\bar{m}^2 + 16\bar{m}\bar{n} + 16\bar{n}^2} =$

$= \sqrt{4m^2 + 16mn\cos 120^\circ + 16n^2} =$

$= \sqrt{4\cdot 1^2 + 16\cdot 1\cdot 1\cdot\left(-\frac{1}{2}\right) + 16\cdot 1^2} =$

$= \sqrt{4 - 8 + 16} = \sqrt{12} = 2\sqrt{3}$

3-ish;

$|\bar{m} - \bar{n}| = \sqrt{(\bar{m} - \bar{n})^2} =$

$= \sqrt{\bar{m}^2 - 2\bar{m}\bar{n} + \bar{n}^2} =$

$= \sqrt{m^2 - 2mn\cos 120^\circ + n^2} =$

$= \sqrt{1^2 - 2\cdot 1\cdot 1\cdot\left(-\frac{1}{2}\right) + 1^2} = \sqrt{3}$

4-ish;

$\cos\beta = \frac{-3}{2\sqrt{3}\cdot\sqrt{3}} = -\frac{1}{2}$

va $\beta = \arccos\left(-\frac{1}{2}\right) = 120^\circ$

<u>5 - misol.</u> Agar $\bar{m}$ va $\bar{n}$ vektrorlar $\alpha = 30^\circ$ burchak tashkil etsa va $\bar{m}\bar{n} = \sqrt{3}$ bo'lsa, ularga qurilgan uchburchakning yuzini hisoblang.

Yechilishi; 80 - betdagi ma'lumotga ko'ra;

$$S_\Delta = \frac{\bar{m}\bar{n}\cdot|tg\alpha|}{2} = \frac{\sqrt{3}\cdot|tg 30^\circ|}{2} = \frac{1}{2}$$

<u>6 - misol.</u> $\bar{a}$ va $\bar{b}$ vektorlar 135° li burchak tashkil qiladi, va $\bar{a}\bar{b} = 10$. Shu vektorlardan qurilgan parallelogrammning yuzini toping.

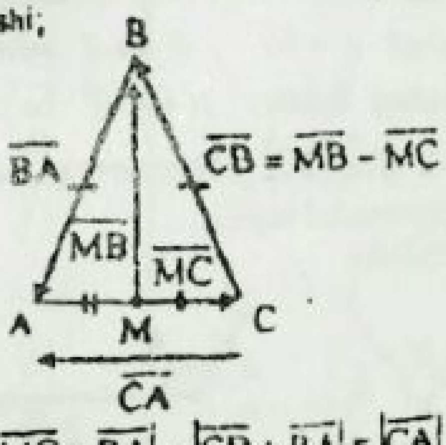
Yechilishi; 80 - betdagi ma'lumotga ko'ra:

$$S_{par} = \bar{a}\bar{b}|tg\alpha| = 10\cdot tg 135^\circ = 10$$

<u>7 - misol.</u> ABC teng yonli uchburchakda M nuqta AC asosning o'rtasi Agar AB = 5 va BM = 4 bo'lsa, $|\overline{MB} - \overline{MC} + \overline{BA}|$ ning qiymatini toping.

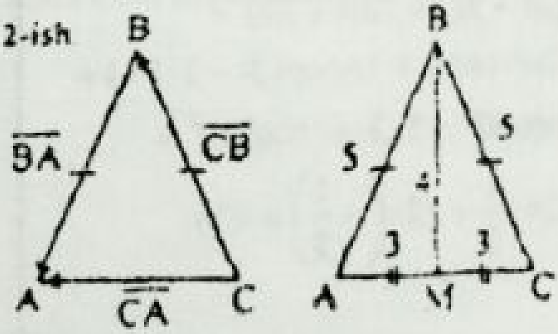
Yechilishi;

1-ish



$|\overline{MB} - \overline{MC} + \overline{BA}| = |\overline{CB} + \overline{BA}| = |\overline{CA}|$

2-ish



Demak, $|\overline{CA}| = AC = 3 + 3 = 6$

PROYEKTSIYALI VEKTORLAR

XOY koordinatalar tekisligida

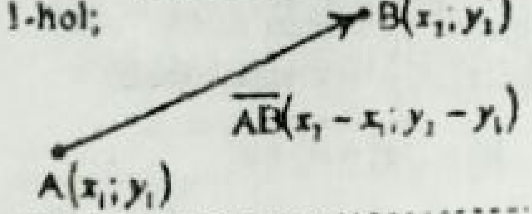
• **Vektorning ko'rinishi**

$\bar{a}(a_x;a_y)$ yoki $\bar{a}=a_x\bar{i}+a_y\bar{j}$

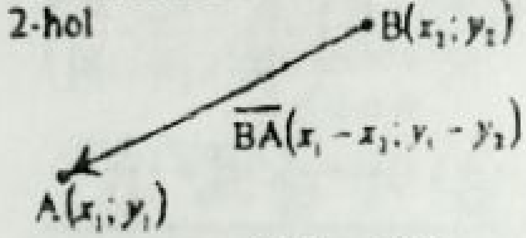
Bu yerda; $\bar{i}(1;0)$ va $\bar{j}(0;1)$ OX va OY o'qlaridagi o'zaro perpendikulyar birlik vektorlar, ya'ni; $\bar{i}\cdot\bar{j}=0$

• $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalardan vektor yasash

1-hol;



2-hol



Xulosa; $\overline{AB}=-\overline{BA}$

• Vektorlarni songa ko'paytirish

$$\lambda\cdot\bar{a}(a_x;a_y)=\lambda\cdot(a_x;a_y)=$$
$$=(\lambda a_x;\lambda a_y)$$

• $\bar{a}(a_x;a_y)$ vektorning uzunligi, moduli yoki absolyut qiymati

$$|\bar{a}|=\sqrt{a_x^2+a_y^2}$$

• Birlik vektor $\bar{e}(e_x;e_y)$

$$|\bar{e}|=\sqrt{e_x^2+e_y^2}=1 \text{ yoki}$$
$$e_x^2+e_y^2=1$$

XYZ koordinatalar fazosida

• **Vektorning ko'rinishi**

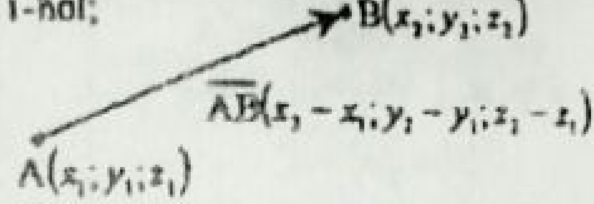
$\bar{a}(a_x;a_y;a_z)$ yoki $\bar{a}=a_x\bar{i}+a_y\bar{j}+a_z\bar{k}$

Bu yerda; $\bar{i}(1;0;0)$, $\bar{j}(0;1;0)$, $\bar{k}(0;0;1)$ OX, OY va OZ o'qlaridagi o'zaro perpendikulyar birlik vektorlar, ya'ni

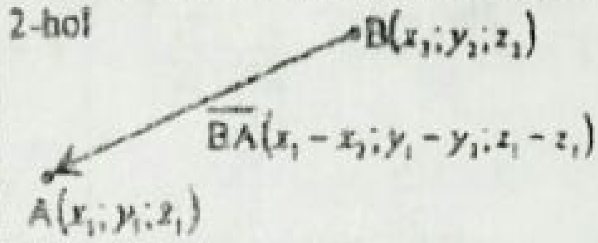
$\bar{i}\cdot\bar{j}=0$, $\bar{i}\cdot\bar{k}=0$ va $\bar{j}\cdot\bar{k}=0$

• $A(x_1;y_1;z_1)$ va $B(x_2;y_2;z_2)$ nuqtalardan vektor yasash

1-hol;



2-hol



Xulosa, $\overline{AB}=-\overline{BA}$

• Vektorlarni songa ko'paytirish

$$\lambda\cdot\bar{a}(a_x;a_y;a_z)=\lambda\cdot(a_x;a_y;a_z)=$$
$$=(\lambda a_x;\lambda a_y;\lambda a_z)$$

• $\bar{a}(a_x;a_y;a_z)$ vektorning uzunligi, moduli yoki absolyut qiymati

$$|\bar{a}|=\sqrt{a_x^2+a_y^2+a_z^2}$$

• Birlik vektor $\bar{e}(e_x;e_y;e_z)$

$$|\bar{e}|=\sqrt{e_x^2+e_y^2+e_z^2}=1 \text{ yoki}$$
$$e_x^2+e_y^2+e_z^2=1$$

• $\vec{a}(a_x;a_y)$ vektor yo'nalishidagi birlik $\vec{e}(e_x;e_y)$ vektorni topish

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$$

$$\vec{e} = \frac{\vec{a}(a_x;a_y)}{|\vec{a}|} = \left(\frac{a_x}{|\vec{a}|};\frac{a_y}{|\vec{a}|}\right)$$

$$e_x = \frac{a_x}{|\vec{a}|};\ e_y = \frac{a_y}{|\vec{a}|}$$

• $\vec{a}(a_x;a_y)$ vektorga qarama-qarshi yo'nalgan birlik $\vec{e}'(e'_x;e'_y)$ vektorni topish

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$$

$$\vec{e}' = -\frac{\vec{a}(a_x;a_y)}{|\vec{a}|} = \left(-\frac{a_x}{|\vec{a}|};-\frac{a_y}{|\vec{a}|}\right)$$

$$e'_x = -\frac{a_x}{|\vec{a}|};\ e'_y = -\frac{a_y}{|\vec{a}|}$$

• $\vec{a}(a_x;a_y)$ va $\vec{b}(b_x;b_y)$ vektorlarni qo'shish va ayirish

1) $\vec{a}(a_x;a_y) + \vec{b}(b_x;b_y) =$
$= (a_x;a_y) + (b_x;b_y) =$
$= (a_x + b_x;a_y + b_y)$

2) $\vec{a}(a_x;a_y) - \vec{b}(b_x;b_y) =$
$= (a_x;a_y) - (b_x;b_y) =$
$= (a_x - b_x;a_y - b_y)$

M-n: $\vec{a}(2;3)$, $b(-2;1)$, $\vec{a} - 2\vec{b} = ?$

$\vec{a} - 2\vec{b} = (2;3) - 2(-2;1) =$
$= (2;3) - (-4;2) = (6;1)$

• $\vec{a}(a_x;a_y;a_z)$ vektor yo'nalishidagi birlik $\vec{e}(e_x;e_y;e_z)$ vektorni topish

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$\vec{e} = \frac{\vec{a}(a_x;a_y;a_z)}{|\vec{a}|} = \left(\frac{a_x}{|\vec{a}|};\frac{a_y}{|\vec{a}|};\frac{a_z}{|\vec{a}|}\right)$$

$$e_x = \frac{a_x}{|\vec{a}|};\ e_y = \frac{a_y}{|\vec{a}|};\ e_z = \frac{a_z}{|\vec{a}|}$$

• $\vec{a}(a_x;a_y;a_z)$ vektorga qarama-qarshi yo'nalgan birlik $\vec{e}'(e'_x;e'_y;e'_z)$ vektorni topish

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

$$\vec{e}' = -\frac{\vec{a}(a_x;a_y;a_z)}{|\vec{a}|} = \left(-\frac{a_x}{|\vec{a}|};-\frac{a_y}{|\vec{a}|};-\frac{a_z}{|\vec{a}|}\right)$$

$$e'_x = -\frac{a_x}{|\vec{a}|};\ e'_y = -\frac{a_y}{|\vec{a}|};\ e'_z = -\frac{a_z}{|\vec{a}|}$$

• $\vec{a}(a_x;a_y;a_z)$ va $\vec{b}(b_x;b_y;b_z)$ vektorlarni qo'shish va ayirish

1) $\vec{a}(a_x;a_y;a_z) + \vec{b}(b_x;b_y;b_z) =$
$= (a_x;a_y;a_z) + (b_x;b_y;b_z) =$
$= (a_x + b_x;a_y + b_y;a_z + b_z)$

2) $\vec{a}(a_x;a_y;a_z) - \vec{b}(b_x;b_y;b_z) =$
$= (a_x;a_y;a_z) - (b_x;b_y;b_z) =$
$= (a_x - b_x;a_y - b_y;a_z - b_z)$

• $\vec{a}(a_x;a_y)$ vektorni $\vec{m}(m_x;m_y)$ va $\vec{n}(n_x;n_y)$ vektorlarga yoyish

$$\vec{a}=\lambda\vec{m}+\mu\vec{n} \text{ yoki}$$
$$(a_x;a_y)=\lambda(m_x;m_y)+\mu(n_x;n_y)$$
$$(a_x;a_y)=(\lambda m_x+\mu n_x;\lambda m_y+\mu n_y)$$
$$\begin{cases} a_x=\lambda m_x+\mu n_x \\ a_y=\lambda m_y+\mu n_y \end{cases}$$

• $\vec{a}(a_x;a_y)$ va $\vec{b}(b_x;b_y)$ vektorlarning <u>skalyar ko'paytmasi</u>

$$\vec{a}\cdot\vec{b}=a_xb_x+a_yb_y$$

• $\vec{a}(a_x;a_y)$ va $\vec{b}(b_x;b_y)$ vektorlar orasidagi burchak kosinusi

$$\cos\alpha=\frac{a_xb_x+a_yb_y}{\sqrt{a_x^2+a_y^2}\cdot\sqrt{b_x^2+b_y^2}}$$

• $\vec{a}(a_x;a_y)$ va $\vec{b}(b_x;b_y)$ vektorlarning <u>perpendikulyarlik sharti</u>

$$a_xb_x+a_yb_y=0$$

• $\vec{a}(a_x;a_y)$ va $\vec{b}(b_x;b_y)$ vektorlarning <u>kollenearlik sharti</u>

$$\frac{a_x}{b_x}=\frac{a_y}{b_y}=\lambda$$

a) Agar $\lambda>0$, ya'ni (+) bo'lsa, <u>vektorlar parallel</u> bo'ladi.

b) Agar $\lambda<0$, ya'ni (−) bo'lsa, <u>vektorlar qarama-qarshi</u> yo'nalgan bo'ladi.

• $\vec{a}(a_x;a_y;a_z)$ vektorni $\vec{m}(m_x;m_y;m_z)$ va $\vec{n}(n_x;n_y;n_z)$ vektorlarga yoyish

$$\vec{a}=\lambda\vec{m}+\mu\vec{n} \text{ yoki}$$
$$(a_x;a_y;a_z)=\lambda(m_x;m_y;m_z)+\mu(n_x;n_y;n_z)$$
$$=(\lambda m_x+\mu n_x;\lambda m_y+\mu n_y;\lambda m_z+\mu n_z)$$
$$\begin{cases} a_x=\lambda m_x+\mu n_x \\ a_y=\lambda m_y+\mu n_y \\ a_z=\lambda m_z+\mu n_z \end{cases}$$

• $\vec{a}(a_x;a_y;a_z)$ va $\vec{b}(b_x;b_y;b_z)$ vektorlarning <u>skalyar ko'paytmasi</u>

$$\vec{a}\cdot\vec{b}=a_xb_x+a_yb_y+a_zb_z$$

• $\vec{a}(a_x;a_y;a_z)$ va $\vec{b}(b_x;b_y;b_z)$ vektorlar orasidagi burchak kosinusi

$$\cos\alpha=\frac{a_xb_x+a_yb_y+a_zb_z}{\sqrt{a_x^2+a_y^2+a_z^2}\cdot\sqrt{b_x^2+b_y^2+b_z^2}}$$

• $\vec{a}(a_x;a_y;a_z)$ va $\vec{b}(b_x;b_y;b_z)$ vektorlarning <u>perpendikulyarlik sharti</u>

$$a_xb_x+a_yb_y+a_zb_z=0$$

• $\vec{a}(a_x;a_y;a_z)$ va $\vec{b}(b_x;b_y;b_z)$ vektorlarning <u>kollenearlik sharti</u>

$$\frac{a_x}{b_x}=\frac{a_y}{b_y}=\frac{a_z}{b_z}=\lambda$$

a) Agar $\lambda>0$, ya'ni (+) bo'lsa, <u>vektorlar parallel</u> bo'ladi.

b) Agar $\lambda<0$, ya'ni (−) bo'lsa, <u>vektorlar qarama-qarshi</u> yo'nalgan bo'ladi.

MASALALAR YECHISHGA YORDAM

I. SONLARGA DOIR MASALALAR

1-M. 345678910111213....686970 sonining raqamlari yig'indisini toping.

Sonni to'liqroq ko'rinishga keltiramiz

12345678910111213...678970

123456789 uchun 1+2+3+...+9 = 45

10111213...19 uchun 55

20212223...29 uchun 65

30313233...39 uchun 75

40414243...49 uchun 85

50515253...59 uchun 95

60616263...69 uchun 105

70 uchun 7 + 0 = 7

12 uchun 1 + 2 = 3

Jami;

45+55+65+75+85+95+105+7−3=529

2-M. $1\cdot 2\cdot 3\cdot 4\cdot 5\cdot ...\cdot n$ ko'paytmadagi nollar sonini topish.

$$\left[\frac{n}{5}\right]+\left[\frac{n}{5^2}\right]+\left[\frac{n}{5^3}\right]+\left[\frac{n}{5^4}\right]+...$$

Maxraj suratdan (n dan) katta bo'lguncha da'vom ettiriladi va barcha qavslarning butun qismlari qo'shib chiqiladi.

Butun qismga m-n: [2,154...] = 2.

3-M. 1 dan N gacha natural sonlar ichida a ga ham b ga ham bo'linmaydiganlarining soni

$$N+\left[\frac{N}{a\cdot b}\right]-\left[\frac{N}{a}\right]-\left[\frac{N}{b}\right]$$

4-M. a ga karrali N dan katta bo'lmagan barcha natural sonlar yig'indisini topish.

$$\frac{a\left(1+\left[\frac{N}{a}\right]\right)}{2}\cdot\left[\frac{N}{a}\right]$$

5-M Bir necha natural sonlarning yig'indisi a ga teng. Agar ularning har biridan b ni ayirib (har birini b ga orttirib) yig'indi hisoblansa, u c ga teng bo'ladi. Yig'indida nechta natural son bor?

$$a-bx=c \quad (a+bx=c)$$

6-M. Ko'paytmaning har bir hadi a ga ko'paytirildi (a ga bo'lindi). Natijada ko'paytma b marta ortdi (b marta kamaydi). Ko'paytmada nechta had qatnashgan?

$$a^x=b$$

7-M. Kitob betlarini sahifalash.

Agar kitobning oxirgi sahifasi uch xonali son bilan tugagan bo'lsa?

N – jami ishlatilgan raqamlar soni

x – kitobning necha betligi

$$x=\frac{N+110}{3}$$

8-M Biror sonning o'ng tomoniga 0 raqamini yozish (yoki 0 raqamini o'chirish) bu sonni 10 ga ko'paytirish (10 ga bo'lish) degani bo'ladi.

Ya'ni;
- Biror x sonining o'ng tomoniga 0 raqami yozilganda $10x$ bo'ladi.
- Biror x sonning o'ng tomonidan 0 raqami o'chirilganda $\frac{x}{10}$ bo'ladi.

9-M. Ikki xonali son. $\overline{xy}=10x+y$

x – o'nliklar xonasidagi raqam

y – birliklar xonasidagi raqam

a) Raqamlarining yig'indisi; $x+y$ | ayirmasi; $x-y$

b) Raqamlarining ko'paytmasi; xy | bo'linmasi; $\frac{x}{y}$

c) $\overline{xy}=10x+y$ ikki xonali sonning raqamlari o'rni almashsa; $\overline{yx}=10y+x$

10-M. Jami N ta turistdan *a* tasi ingliz tilini, *b* tasi nemis tilini, *c* tasi ikkala tilni bilsa, *d* tasi esa ikkala tilni ham bilmasa, quyidagi munosabat o'rinli bo'ladi;

$$N = a + b + d - c$$

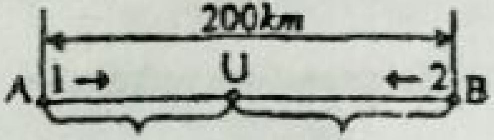
II. HARAKATGA DOIR MASALALAR

Formulalar; *υ*-tezlik, *S*-masofa, *t*-vaqt

$$S = \upsilon t \qquad \upsilon = \frac{S}{t} \qquad t = \frac{S}{\upsilon}$$

1-M. Orasidagi masofa 200 *km* bo'lgan A va B punktlardan bir-biriga qarab ikki turist bir vaqtning o'zida yo'lga chiqdi. Birinchisi avtobusda tezligi 40 *km/soat*, ikkinchisi avtomobilda. Agar ular 2 soatdan keyin uchrashgan bo'lishsa, avtomobilning tezligini toping.

Yechilishi;



U nuqta uchrashish joyini bildiradi

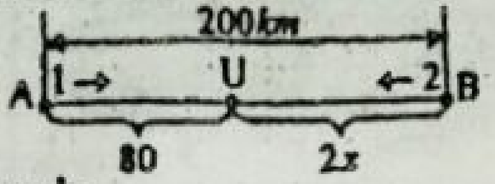
Quyidagi jadvalga ma'lumotlarni joylashtirib chiqamiz

	$\upsilon, \frac{km}{soat}$	*t, soat*	*S, km*
1 - turist	40	2	
2 - turist		2	

$S = \upsilon \cdot t$ formuladan foydalanib jadvalni to'ldiramiz

	$\upsilon, \frac{km}{soat}$	*t, soat*	*S, km*
1 - turist	40	2	80
2 - turist	*x*	2	2*x*

Rasmga faqat jadvaldagi masofalarni olib borib o'rnatamiz

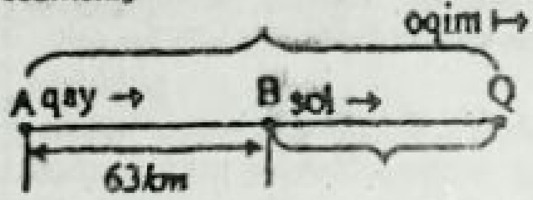


Demak;

$$80 + 2x = 200 \Rightarrow x = 60\ km/soat$$

2-M. Ikki pristan orasidagi masofa 63 *km*. Bir vaqtning o'zida oqim bo'ylab birinchi pristandan sol, ikkinchisidan motorli qayiq jo'natildi va motorli qayiq solni 3 soatda quvib yetdi. Agar daryo oqimining tezligi 3 *km/soat* bo'lsa, qayiqning turg'un suvdagi tezligini toping.

Yechilishi;



Q – qayiqning solni quvib yetish joyi

AQ – qayiqning solni quvib yetguncha bosib o'tgan masofasi

BQ – solning (yog'ochning) masofasi

Tezliklarni mulohaza qilamiz;

oqimning tezligi (o.t) – 3 *km/soat*

qayiqning turg'un (oqmaydigan) suvdagi tezligi (q.t) – *x* *km/soat*

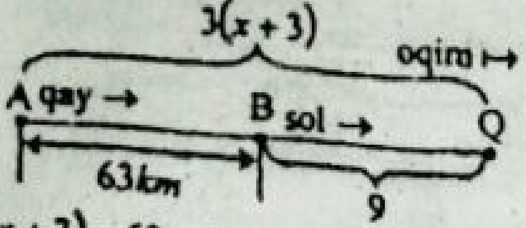
oqim bo'yicha tezlik (o.b.t) – *x* + 3

oqimga qarshi tezlik (o.q.t) – *x* – 3

Quyidagi jadvalni to'ldiramiz

	$\upsilon, \frac{km}{soat}$	*t, soat*	*S, km*
qayiq	*x* + 3	3	3(*x* + 3)
sol	3	3	9

Jadvaldagi masofalarni rasmga olib borib o'rnatamiz



$$3(x+3)=63+9 \Rightarrow x=21\ km/soat$$

<u>3-M.</u> Ikki shahardan bir vaqtning o'zida turli tezlik bilan ikki avtomobil bir-biriga qarab yo'lga chiqdi. Avtomobillarning har biri uchrashish joyigacha bo'lgan masofaning yarmini bosib o'tgandan keyin, haydovchilar tezligini 1,5 baravar oshirdi, natijada avtomobillar belgilangan muddatdan 1 soat oldin uchrashishdi. Harakat boshlangandan necha soatdan keyin avtomobillar uchrashishdi?

Yechilishi; Belgilangan vaqt x bo'lsin.

1-hol. Avtomobillarning har biri uchrashish joyigacha bo'lgan masofaning yarmini bosib o'tadi. Demak vaqt masofaga to'g'ri proportsional bo'lgani uchun mo'ljaldagi vaqtning ham yarmi ketadi, ya'ni $\frac{x}{2}$ soat ketadi, $\frac{x}{2}$ soat qoladi.

2-hol. Yo'lning qolgan qismida har bir haydovchi o'z tezligini 1,5 marta oshiradi. Vaqt tezlikka teskari proporsional bo'lgani uchun qolgan $\frac{x}{2}$ soat 1,5 marta kamayadi, ya'ni yo'lning qolgan qismiga $\frac{\frac{x}{2}}{1,5}=\frac{x}{3}$ soat vaqt ketadi.

Haydovchilar uchrashguncha jami:

$\frac{x}{2}+\frac{x}{3}=\frac{5x}{6}$ soat vaqt ketadi.

Haydovchilar belgilangan muddatdan 1 soat oldin uchrashgani uchun;

$$\frac{5x}{6}=x-1 \Rightarrow x=6 \text{ soat}$$

Javob; Avtomobillar $x=6$ soatdan so'ng uchrashishlari kerak bo'lgan va $6-1=5$ soatdan so'ng uchrashishgan.

<u>III. ISHGA DOIR MASALALAR</u>

<u>1-M.</u> $\frac{1}{t}=\frac{1}{t_1}+\frac{1}{t_2}$ formulaga doir.

Meshdagi suv Anvarning o'ziga 35 kunga, ukasi ikkalasiga esa 10 kunga yetadi. Meshdagi suv Anvarning ukasiga necha kunga yetadi?

Yechish; $t_1=35$, $t=10$, $t_2=?$

$\frac{1}{10}=\frac{1}{35}+\frac{1}{x}$ tenglamadan $x=14$ kun kelib chqadi.

<u>Ishga doir masalalar yechishga yordam</u>

A – butun (to'la) ish

x – ishchining ish unumi, ya'ni ishchining 1 soatda (1 kunda va hokozo) qancha ish bajarishi

t – ishni bajarishga ketga vaqt

$$t=\frac{A}{x} \quad A=x\cdot t \quad x=\frac{A}{t}$$

<u>1-M.</u> Ishchi har soatda 5 ta detal yasaydi. U 20 ta detalni qancha vaqtda yasaydi?

Yechish; $x=5$, $A=20$, $t=?$

$$t=\frac{A}{x}=\frac{20}{5}=4 \text{ soat}$$

Javob; 4 soatda

• Ba'zi bildiruvlar

• Butun ishning 40% i – 0,4A

• Butun ishning $\frac{5}{7}$ qismi – $\frac{5}{7}A$

• Ishchining ish unumi – x, u butun ishning 60% ini qancha vaqtda bajaradi?

Butun ish – A.

Butun ishning 60% i – 0,6A

$\frac{0,6A}{x}$ vaqtda bajaradi.

• x – birinchi ishchining ish unumi

y – ikkinchi ishchining ish unumi

t_1x – birinchi ishchining t_1 vaqtda qancha ish bajarishi

t_2y – ikkinchi ishchining t_2 vaqtda qancha ish bajarishi

$t\cdot(x+y)$ – ikkala ishchi birgalikda t vaqtda qancha ish bajarishi

$\frac{A}{x}$ – birinchi ishchining o'zi butun ishni qancha vaqtda bajarishi

$\frac{A}{y}$ – ikkinchi ishchining o'zi butun ishni qancha vaqtda bajarishi

$\frac{A}{x+y}$ – ikkala ishchi birgalikda butun ishni qancha vaqtda bajarishi

$A-t_1x$ – birinchi ishchining o'zi t_1 vaqt ishlagandan so'ng qolgan ish

$\frac{A-t_2y}{x+y}$ – ikkinchi ishchining o'zi t_2 vaqt ishlagandan so'ng qolgan ishni ikkala ishchi birgalikda qancha vaqtda bajarishi.

• Ishga doir masalalar

1-M. Bir ishchi buyurtmani 6 soatda, boshqasi esa 10 soatda bajaradi. Ular birgalikda 3 soat ishlaganlaridan keyin ishning qancha qismi bajarilmay qoladi?

Yechilishi;

1) $\frac{A}{x}=6 \Rightarrow x=\frac{A}{6}$

2) $\frac{A}{y}=10 \Rightarrow y=\frac{A}{10}$

$A-3(x+y)$ – ikkala ishchi birgalikda 3 soat ishlaganlaridan keyin qolgan ish

$$A-3(x+y)=A-3\left(\frac{A}{6}+\frac{A}{10}\right)=$$

$$=A-3\cdot\frac{4}{15}A=\frac{1}{5}A$$

Javob; Butun ishning $\frac{1}{5}$ qismi bajarilmay qoladi.

2-M. Usta muayyan ishni 12 kunda, uning shogirdi esa 30 kunda bajaradi. Agar 3 ta usta va 5 ta shogird birga ishlasalar, o'sha ishni necha kunda bajaradi?

Yechilishi;

1) $\frac{A}{x}=12 \Rightarrow x=\frac{A}{12}$

2) $\frac{A}{y}=30 \Rightarrow y=\frac{A}{30}$

3) $\frac{A}{3x+5y}=\frac{A}{3\cdot\frac{A}{12}+5\cdot\frac{A}{30}}=2,4$

Javob; 2,4 kunda bajaradi.

IV. ARALASHMALARGA DOIR MASALALAR

1-M. Qotishma mis va qo'rg'oshindan iborat. Qotishmaning 60% i mis bo'lib, mis qo'rg'oshindan 2 kg ko'p. Qotishmada qancha mis bor?

Yechish: Qotishmani biror idishda deb tasavvur qilamiz;

	mas.(kg)	foizi
Jami		100%
Mis	$x+2$	60%
Qo'rg'.	x	

Idishda chala yozilgan jadvalni to'ldiramiz.

	mas.(kg)	foizi
Jami	$2x+2$	100%
Mis	$x+2$	60%
Qo'rg'.	x	40%

Proportsiya tuzish uchun ikki qator ma'lumotni olish yetarli bo'lgani uchun eng qulayini tanlab olamiz.

$$x+2 \quad - \quad 60\%$$
$$x \quad - \quad 40\%$$
$$x\cdot 60=(x+2)\cdot 40$$
$$x=4kg$$

Mis; $x+2=4+2=6\ kg$

2-M. Massasi 400 g va kontsentratsiyasi 8% bo'lgan eritma massasi 600 g va kontsientratsiyasi 13% bo'lgan eritma bilan aralashtirildi. Hosil bo'lgan aralashma kontsentratsiyasini toping.

Yechish; 1-usul.

Quyidagi ma'lumotni qo'llash mumkin;

Massasi m_1 va kontsentratsiyasi a % bo'lgan eritma massasi m_2 va kontsentratsiyasi b % bo'lgan eritma bilan aralashtirilganda hosil bo'lgan yangi eritmaning necha foizligini (x) topish.

$$x=\frac{a\cdot m_1+b\cdot m_2}{m_1+m_2}$$

Bizning masalada,

$$m_1=400g\ ,\ a=8\%$$
$$m_2=600g\ ,\ b=13\%$$
$$x=\frac{8\cdot 400+13\cdot 600}{400+600}=11\%$$

Javob; 11% li aralashma hosil bo'ladi.

2 – usul; 1–eritma solingan idishga bor ma'lumotni joylaymiz;

	mas.(g)	foizi
Jami	400	100%
Suvi		
Kon.		8%

Jadvalning kon. qatorini to'ldirsa yetarli.

400 g ning 8% i $=0{,}08\cdot 400=32g$

	mas.(g)	foizi
Jami	400	100%
Suvi		
Kon.	32	8%

2-eritma uchun ham xuddi shunday yo'l bilan jadval to'ldiriladi.

	mas.(g)	foizi
Jami	600	100%
Suvi		
Kon.	78	13%

Ikkala eritma bir idishga solinganda bu eritmalarda faqat massalar qo'shiladi, foizlar esa qo'shilmaydi.

	mas.(g)	foizi
Jami	$600 + 400 =$ $= 1000$	100%
Suvi		
Kon.	$78 + 32 =$ $= 110$	x

Proportsiya tuzamiz.

$$1000 \; - \; 100\%$$

$$110 \; - \; x$$

$$x \cdot 1000 = 110 \cdot 100\%$$

$$x = 11\%$$

Javob; 11 % li aralashma hosil bo'ladi.

V. GEOMETRIK MASALALARGA YORDAM

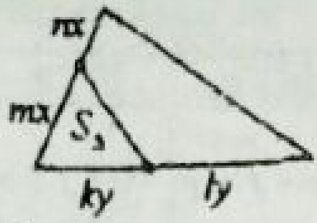
a) Planimetriyaga doir

1) Hech qaysi uchtasi bir to'g'ri chiziqda yotmaydigan n ta nuqtadan o'tkazish mumkin bo'lgan to'g'ri chiziqlar (kesmalar) soni;

$$\frac{n(n-1)}{2}$$

2) Uchburchakning ikki tomoni uning uchidan boshlab hisoblaganda $m:n$ va $k:l$ nisbatda bo'linganda,

S – umumiy katta uchburchakning yuzi



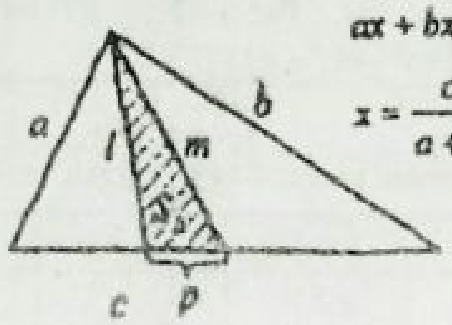
$$\frac{S_\Delta}{S} = \frac{mk}{(m+n)(k+l)}$$

$$S_\Delta = \frac{mk}{(m+n)(k+l)} S$$

3) Tomonlari a, b va c bo'lgan uchburchakning c tomoniga tushirilgan mediana va bissektrisasi orasidagi yuzani hamda c tomondan ajratgan kesma uzunligini hisoblash.

$$ax + bx = c$$

$$x = \frac{c}{a+b}$$



$$p = \frac{|a-b|x}{2} = \frac{|a-b| \cdot c}{2(a+b)}$$

$$S_\Delta = \frac{|a-b|}{2(a+b)} \cdot S$$

b) Stereometriyaga doir

O'lchamlari $a \times b \times c$ bo'lgan to'g'ri burchakli parallelepiped ichiga tomoni d bo'lgan kublardan eng ko'pi bilan nechta joylashtirish mumkin.

$$\left[\frac{a}{d}\right] \cdot \left[\frac{b}{d}\right] \cdot \left[\frac{c}{d}\right]$$

@ Ba'zi turdagi tenglama va tengsizliklarni yechish sxemasi

Bir tomoni modulli bo'lgan tenglama va tengsizliklar

$\lvert f\rvert<\varphi$	$\lvert f\rvert\le\varphi$	$\lvert f\rvert=\varphi$	$\lvert f\rvert\ge\varphi$	$\lvert f\rvert>\varphi$
$-\varphi<f<\varphi$ yoki $\begin{cases}-\varphi<f\\ f<\varphi\end{cases}$	$-\varphi\le f\le\varphi$ yoki $\begin{cases}-\varphi\le f\\ f\le\varphi\end{cases}$	$\begin{cases}\varphi\ge0\\ f=-\varphi,\ f=\varphi\end{cases}$ yoki $\begin{cases}f<0\\ f=-\varphi\end{cases}\cup\begin{cases}f\ge0\\ f=\varphi\end{cases}$	$f\le-\varphi,\ f\ge\varphi$	$f<-\varphi,\ f>\varphi$

Ikkala tomoni ham modulli bo'lgan tenglama va tengsizliklar

$\lvert f\rvert<\lvert\varphi\rvert$	$\lvert f\rvert\le\lvert\varphi\rvert$	$\lvert f\rvert=\lvert\varphi\rvert$	$\lvert f\rvert\ge\lvert\varphi\rvert$	$\lvert f\rvert>\lvert\varphi\rvert$
$f^2<\varphi^2$ yoki $(f-\varphi)\times$ $\times(f+\varphi)<0$	$f^2\le\varphi^2$ yoki $(f-\varphi)\times$ $\times(f+\varphi)\le0$	$f=\varphi$ va $f=-\varphi$	$f^2\ge\varphi^2$ yoki $(f-\varphi)\times$ $\times(f+\varphi)\ge0$	$f^2>\varphi^2$ yoki $(f-\varphi)\times$ $\times(f+\varphi)>0$

Bir tomoni kadrat ildizli bo'lgan tenglama va tengsizliklar

$\sqrt{f}<\varphi$	$\sqrt{f}\le\varphi$	$\sqrt{f}=\varphi$	$\sqrt{f}\ge\varphi$	$\sqrt{f}>\varphi$
$\begin{cases}\varphi>0\\ f\ge0\\ f<\varphi^2\end{cases}$	$\begin{cases}\varphi\ge0\\ f\ge0\\ f\le\varphi^2\end{cases}$	$\begin{cases}\varphi\ge0\\ f=\varphi^2\end{cases}$	$\begin{cases}\varphi<0\\ f\ge0\end{cases}\cup\begin{cases}\varphi\ge0\\ f\ge\varphi^2\end{cases}$	$\begin{cases}\varphi<0\\ f\ge0\end{cases}\cup\begin{cases}\varphi\ge0\\ f>\varphi^2\end{cases}$

Ikkala tomoni ham kvadrat ildizli bo'lgan tenglama va tengsizliklar

$\sqrt{f}<\sqrt{\varphi}$	$\sqrt{f}\le\sqrt{\varphi}$	$\sqrt{f}=\sqrt{\varphi}$	$\sqrt{f}\ge\sqrt{\varphi}$	$\sqrt{f}>\sqrt{\varphi}$
$0\le f<\varphi$ yoki $\begin{cases}f\ge0\\ f<\varphi\end{cases}$	$0\le f\le\varphi$ yoki $\begin{cases}f\ge0\\ f\le\varphi\end{cases}$	$\begin{cases}f=\varphi\\ f\ge0\end{cases}$ yoki $\begin{cases}f=\varphi\\ \varphi\ge0\end{cases}$	$f\ge\varphi\ge0$ yoki $\begin{cases}f\ge\varphi\\ \varphi\ge0\end{cases}$	$f>\varphi\ge0$ yoki $\begin{cases}f>0\\ \varphi\ge0\end{cases}$

@ Logarifmik tenglama va tengsizliklarni yechish sxemasi

$\log_\varphi f<0$	$\log_\varphi f\le 0$	$\log_\varphi f=0$	$\log_\varphi f\ge 0$	$\log_\varphi f>0$
$\begin{cases}(f-1)\times\\ \times(\varphi-1)<0\\ f>0\\ \varphi>0\end{cases}$	$\begin{cases}(f-1)\times\\ \times(\varphi-1)\le 0\\ f>0\\ \varphi>0,\varphi\ne 1\end{cases}$	$\begin{cases}f=1\\ \varphi>0,\varphi\ne 1\end{cases}$	$\begin{cases}(f-1)\times\\ \times(\varphi-1)\ge 0\\ f>0\\ \varphi>0,\varphi\ne 1\end{cases}$	$\begin{cases}(f-1)\times\\ \times(\varphi-1)>0\\ f>0\\ \varphi>0\end{cases}$

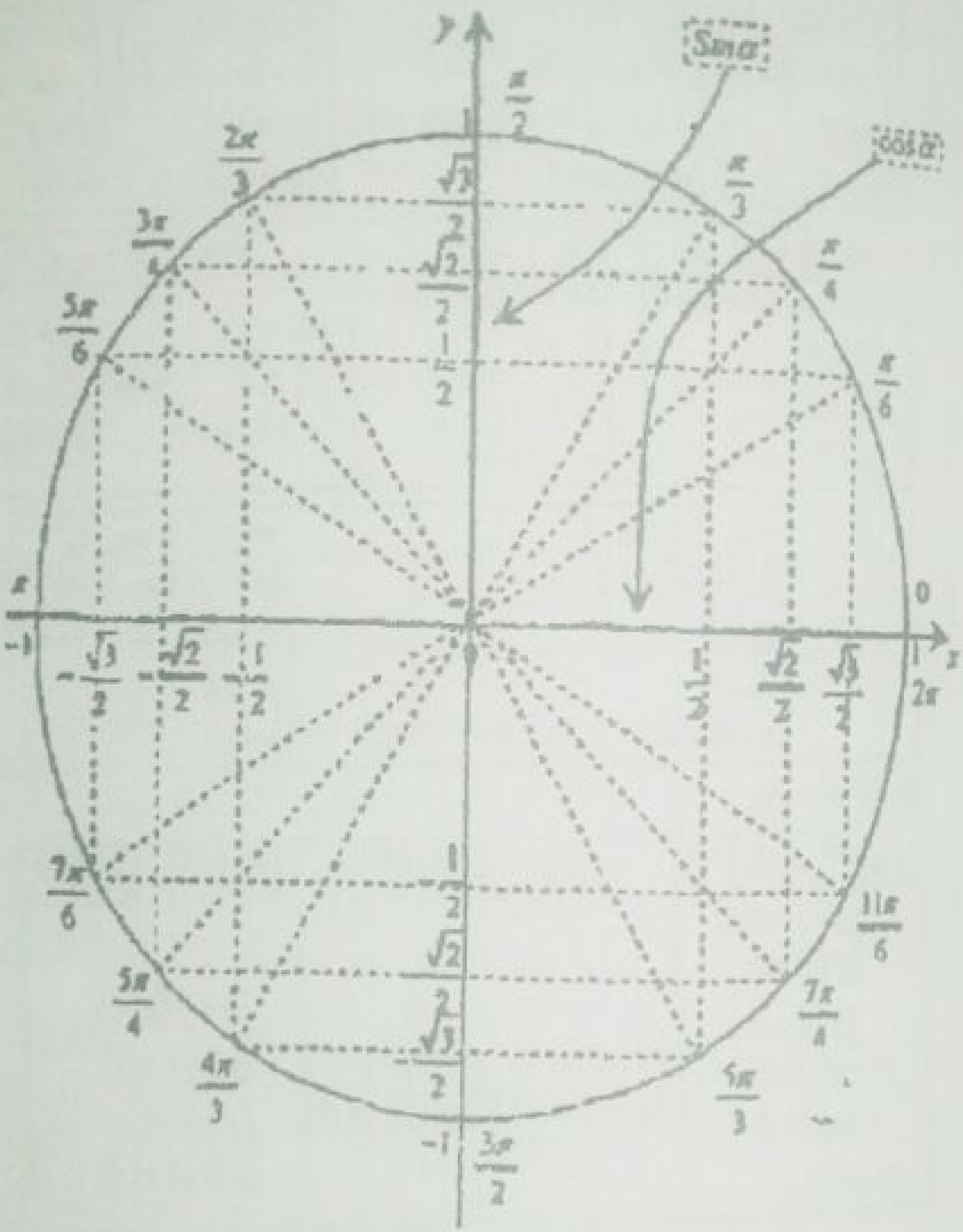
$\log_\varphi f_1<\log_\varphi f_2$	$\log_\varphi f_1\le\log_\varphi f_2$	$\log_\varphi f_1=\log_\varphi f_2$
$\begin{cases}(f_1-f_2)(\varphi-1)<0\\ f_1>0\\ f_2>0\\ \varphi>0\end{cases}$	$\begin{cases}(f_1-f_2)(\varphi-1)\le 0\\ f_1>0\\ f_2>0\\ \varphi>0,\varphi\ne 1\end{cases}$	$\begin{cases}f_1=f_2\\ f_1>0\\ \varphi>0,\varphi\ne 1\end{cases}$ yoki $\begin{cases}f_1=f_2\\ f_2>0\\ \varphi>0,\varphi\ne 1\end{cases}$

$\log_\varphi f_1\ge\log_\varphi f_2$	$\log_\varphi f_1>\log_\varphi f_2$
$\begin{cases}(f_1-f_2)(\varphi-1)\ge 0\\ f_1>0\\ f_2>0\\ \varphi>0,\varphi\ne 1\end{cases}$	$\begin{cases}(f_1-f_2)(\varphi-1)>0\\ f_1>0\\ f_2>0\\ \varphi>0\end{cases}$

@ Qo'shimcha;

$[f(x)]^{g(x)}=[f(x)]^{\varphi(x)}$	$[f(x)]^{g(x)}\ge[f(x)]^{\varphi(x)}$	$[f(x)]^{g(x)}\le[f(x)]^{\varphi(x)}$
1) $\begin{cases}g(x)=\varphi(x)\\ f(x)\ne 0\end{cases}$ 2) $\begin{cases}f(x)=0\\ g(x)>0\\ \varphi(x)>0\end{cases}$ 3) $\lvert f(x)\rvert=1$	$\begin{cases}0<f(x)<1\\ g(x)\le\varphi(x)\end{cases}\cup\begin{cases}f(x)>1\\ g(x)\ge\varphi(x)\end{cases}$	$\begin{cases}0<f(x)<1\\ g(x)\ge\varphi(x)\end{cases}\cup\begin{cases}f(x)>1\\ g(x)\ge\varphi(x)\end{cases}$

⑧ Sinα va cosα ning ba'zi burchaklardagi qiymatlarini birlik aylanadan aniqlanishi



$$tg\alpha = \frac{Sin\alpha}{\cos\alpha} \quad \text{va} \quad ctg\alpha = \frac{\cos\alpha}{Sin\alpha}$$

@ Uzunlik, yuza va hajm birliklari

Uzunlik birliklari orasidagi munosabatlar

$1mm = 10^{-1} sm = 10^{-2} dm = 10^{-3} m = 10^{-6} km$

$1sm = 10 mm = 10^{-1} dm = 10^{-2} m = 10^{-5} km$

$1dm = 10^{2} mm = 10 sm = 10^{-1} m = 10^{-4} km$

$1m = 10^{3} mm = 10^{2} sm = 10 dm = 10^{-3} km$

$1km = 10^{6} mm = 10^{5} sm = 10^{4} dm = 10^{3} m$

Yuza birliklari orasidagi munosabatlar

$1mm^2 = 10^{-2} sm^2 = 10^{-4} dm^2 = 10^{-6} m^2 = 10^{-12} km^2$

$1sm^2 = 10^{2} mm^2 = 10^{-2} dm^2 = 10^{-4} m^2 = 10^{-10} km^2$

$1dm^2 = 10^{4} mm^2 = 10^{2} sm^2 = 10^{-2} m^2 = 10^{-8} km^2$

$1m^2 = 10^{6} mm^2 = 10^{4} sm^2 = 10^{2} dm^2 = 10^{-6} km^2$

$1km^2 = 10^{12} mm^2 = 10^{10} sm^2 = 10^{8} dm^2 = 10^{6} m^2$

Hajm birliklari orasidagi munosabatlar

$1mm^3 = 10^{-3} sm^3 = 10^{-6} dm^3 = 10^{-9} m^3 = 10^{-18} km^3$

$1sm^3 = 10^{3} mm^3 = 10^{-3} dm^3 = 10^{-6} m^3 = 10^{-15} km^3$

$1dm^3 = 1 litr = 10^{6} mm^3 = 10^{3} sm^3 = 10^{-3} m^3 = 10^{-12} km^3$

$1m^3 = 10^{9} mm^3 = 10^{6} sm^3 = 10^{3} dm^3 = 10^{3} litr = 10^{-9} km^3$

$1km^3 = 10^{18} mm^3 = 10^{15} sm^3 = 10^{12} dm^3 = 10^{12} litr = 10^{9} m^3$

Massaning ba'zi o'lchov birliklari orasidagi munosabat

$1 g = 10^{-3} kg = 10^{-6} t$

$1 mg = 10^{-3} g = 10^{-6} kg = 10^{-9} t$

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