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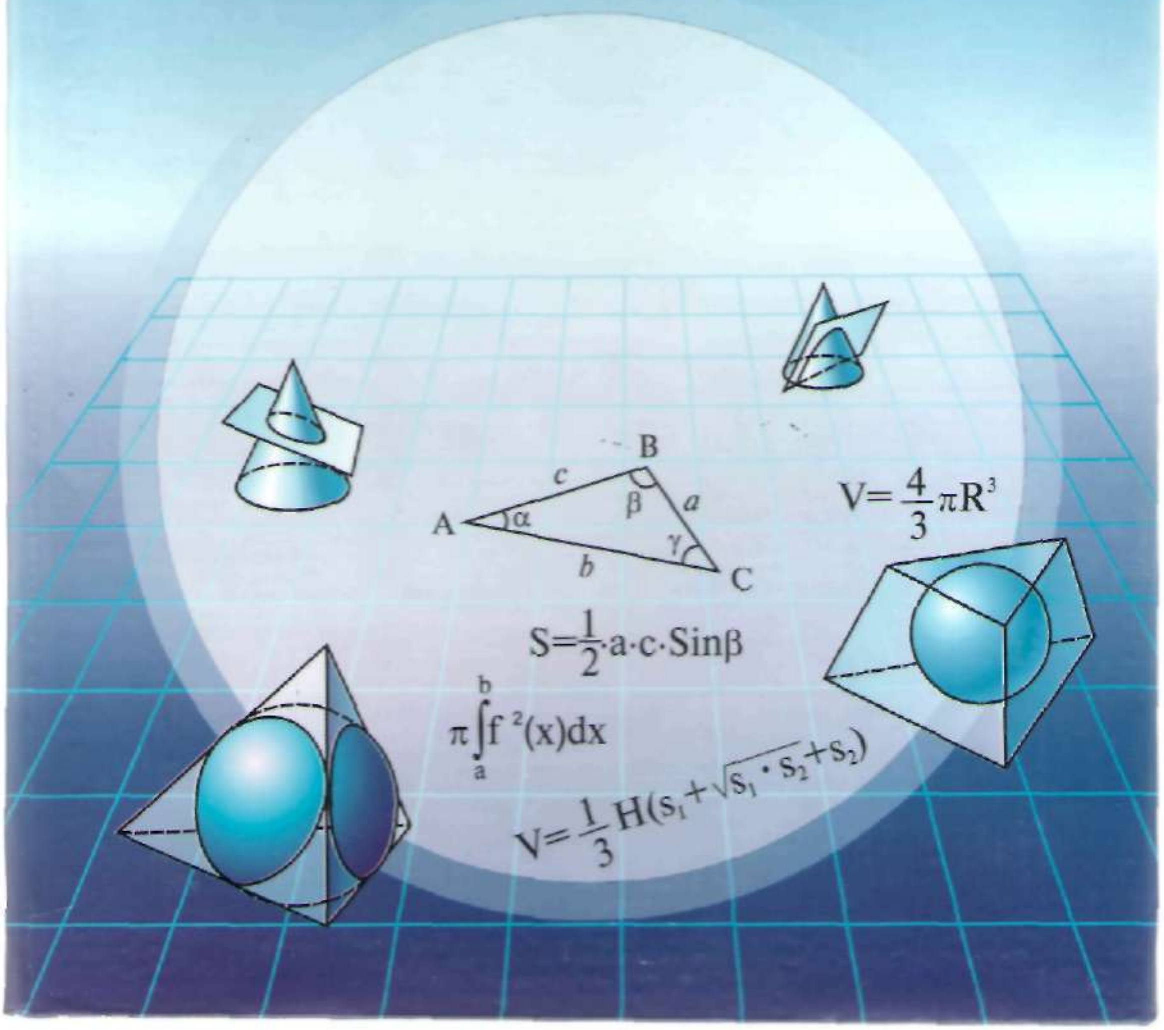
MATEMATIKADAN QO'LLANMA

II qism



O'ZBEKISTON RESPUBLIKASI OLIY
VA O'RTA MAXSUS TA'LIM VAZIRLIGI

F.R.USMONOV, R.J.ISOMOV, B.O.XO'JAYEV

MATEMATIKADAN QO'LLANMA

O'zbekiston Respublikasi oliy va o'rta maxsus ta'lim vazirligi o'quv qo'llanma sifatida tavsiya etgan

II qism

TOSHKENT – «NOSHIR» – 2009

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Ushbu o'quv qo'llanma mualliflarning 2006-yilda nashr etilgan «Matematikadan qo'llanma» kitobining II qismi bo'lib, geometriya fani bo'yicha akademik litsey, kasb-hunar kollejlari o'quvchilari uchun tasdiqlangan dasturlar asosida yozilgan. Qo'llanma 10 bobdan iborat bo'lib, geometriyaning planimetriya va stereometriya bo'limlari bo'yicha asosiy mavzularni o'z ichiga oladi. Geometrik tushunchalar, ta'riflar va teoremalar masalalarni yechish namunalari bilan bayon qilingan. Har bir bobda mustaqil yechish uchun testlar javoblari bilan keltirilgan.

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SO'ZBOSHI

O'zbekiston Respublikasi Prezidenti I.A.Karimovning «Yuksak ma'naviyat — yengilmas kuch» kitobida shunday satrlar bor: «...ota-bobolarimiz qadimdan bebaho boylik bo'lmish ilmu ma'rifat, ta'lim va tarbiyani inson kamoloti va millat ravnaqining eng asosiy sharti va garovi deb bilgan... Shuni unutmasligimiz kerakki, kelajagimiz poydevori bilim dargohlarida yaratiladi, boshqacha aytganda, xalqimizning ertangi kunining qanday bo'lishi farzandlarimizning bugun qanday ta'lim va tarbiya olishiga bog'liq».

Yoshlarning kamoloti borasidagi sa'y-harakatlar davlat siyosati darajasiga ko'tarilgan bo'lib, O'zbekiston Respublikasi hukumati tomonidan «Ta'lim to'g'risidagi qonun» va «Kadrlar tayyorlash milliy dasturi»da ifodalangan talablarga javob beradigan darsliklar, o'quv va uslubiy qo'llanmalar yaratish hozirgi kunning dolzarb masalasi bo'lib qolmoqda.

Mazkur qo'llanma 2006- yilda nashr qilingan «Matematikadan qo'llanma» kitobining ikkinchi qismidir. Kitobning birinchi qismi algebra va analiz asoslarining asosiy mavzularini o'z ichiga olgan bo'lsa, e'tiboringizga havola qilinayotgan ikkinchi qismida geometriyaning *planimetriya* va *stereometriya* bo'limlariga tegishli asosiy nazariy ma'lumotlar, masalalarni yechish usullari va 500 dan ko'proq masala va test topshiriqlari berilgan. Kitob 10 bobdan iborat. O'quvchilarda geometrik masalalarni yechish malakalarini shakllantirish maqsadida har bir bobning oxirida mustaqil ishlash uchun mo'ljallangan test topshiriqlari javoblari bilan keltirilgan.

Qo'llanmadan pedagog mutaxassislar tayyorlovchi universitetlar va institutlarning talabalari, akademik litsey, kasb-hunar kolleji o'qituvchi va o'quvchilari, matematika to'garaklarining rahbarlari, shuningdek, geometriya fani bo'yicha o'z bilimlarini mustaqil mustahkamlab, oliy o'quv yurtlariga kirishni niyat qilgan yoshlar foydalanishlari mumkin.

Mualliflar o'quvchilarning qo'llanma haqidagi tanqidiy fikr-mulohazalarini mamnuniyat bilan qabul qiladi.

GEOMETRIYA

Geometriya fani planimetriya va stereometriyaga bo'lib o'rganiladi. Geometriyaning barcha nuqtalari bilan bir tekislikka joylashgan tekis (yassi) shakllar va ularning xossalarini o'rganadigan qismi **planimetriya**, fazoviy, ya'ni barcha nuqtalari bir tekislikka joylashmagan geometrik shakllar (jismlar) va ularning xossalarini o'rganadigan qismi **stereometriya** deb ataladi.

Bizning buyuk allomalarimiz Abu Abdulloh Muhammad ibn Muso al-Xorazmiy (783—850), Ahmad Farg'oniy (797—861), Abu Rayhon Beruniy (973—1048), Abu Ali ibn Sino (980—1037), Mirzo Ulug'bek (1394—1449) fanning boshqa sohalari bilan bir qatorda matematikada ham chuqur iz qoldirdilar. Al-Xorazmiyning «Aljabr va al-muqobala» asarida «O'lchash haqida» bob bo'lib, unda geometrik ma'lumotlar berilgan. Xususan, al-Xorazmiy yozadi: to'g'ri burchakli uchburchakda, uning kichik tomonlari kvadradlarining yig'indisi, katta tomonining kvadratiga teng. O'tkir burchakli uchburchakda uning kichik tomonlari kvadratlarining yig'indisi katta tomoni kvadratidan katta bo'ladi, o'tmas burchakli uchburchakda kichik tomonlari kvadratlarining yig'indisi katta tomonning kvadratidan kichik bo'ladi.

«O'lchash haqida bob»da olim geometriyaga oid ko'plab amaliy masalalarini keltiradi va ularni yechish yo'llarini ko'rsatadi. Shulardan biri ushbu masaladir: «Uchburchakli yer maydonining tomonlari 10 va 10 gaz, asosi esa 12 gaz. Uning ichida (bir tomoni uchburchak asosida bo'lgan) kvadrat yer (maydoni) bor. Kvadratning tomoni uzunligini toping».

Ajdodlarimiz ijodi namunalari bilan o'quvchilarimizni, talabalarni, yoshlarimizni tanishtirib borish ularda milliy iftixor, vatanparvarlik tuyg'ularini yanada chuqurroq shakllantiradi, mustahkamlaydi. Prezidentimiz I.A.Karimov aytganidek, «Muhammad ibn Muso Xorazmiyning — o'nlik sanoq sistemasi, algebra va algoritm tushunchalarini dunyoda birinchi bo'lib ilm-fan sohasida joriy etgani va shu asosda aniq fanlar rivoji uchun o‘z vaqtida mustahkam asos yaratgani umum insoniy taraqqiyot rivojida katta ahamiyatga ega bo‘lganini barchamiz yaxshi bilamiz». ([1] 41- bet).

Qomusiy daho Abu Rayhon Beruniy fanning turli sohalariga oid 150 dan ortiq asarlar yozgan bo‘lib, shulardan 22 tasi matematikaga oiddir. Fan tarixchisi Sarton XI asrni «Beruniy asri» deb ta’riflaydi ([1], 41- bet).

Beruniyning geometriyaga taalluqli ishlari uning «Qonuni Ma’sudiy» asarining uchinchi maqolasida, «Astronomiya san’atidan boshlang‘ich ma’lumot beruvchi kitob» asarining birinchi bo‘limida bayon qilingan. Beruniy «Doiradagi vatarlarni unga ichki chizilgan siniq chiziqlarning hossalaridan foydalanib hisoblash haqida risola» nomli asarida Arximedning quyidagi teoremasini o‘ziga xos, yangi isbotini beradi. Shu teoremani keltiraylik: agar aylana yoyiga ichki chizilgan va teng bo‘lmagan ikki qismdan iborat siniq chiziqning katta qismiga shu yoyning o‘rtasidan perpendikular tushirilsa, u holda bu perpendikularning asosi berilgan siniq chiziqni teng ikki qismga ajratadi.

Abu Rayhon Beruniy bu teoremani ushbu masalani yechishga tatbiq etadi:

1- masala. Ma’lum uzunlikdagi tikka o‘sgan terak tanasi bir joydan sinib, singan bo‘lagining bir uchi esa singan joyida terak tanasiga ilinib qolgan. Agar terak bo‘yidan terak uchining yerga tekkan joyigacha masofa ma’lum bo‘lsa, terakni qanday balandlikda singanini aniqlang.

Beruniy hal qilgan quyidagi masala ham ahamiyatga molik:

2- masala. Kengligi ma’lum bo‘lgan daryoning ikki sohilida ma’lum balandlikdagi daraxtlar bor. Bu daraxtlarning uchlarida bo‘lgan ikki qush suv yuzida ko‘ringan baliq tomon uchib, baliqqa bir vaqtda yetib kelishdi. Baliq ko‘ringan joydan daryo sohiligacha va daraxtning uchlarigacha bo‘lgan masofalar topilsin.

Abu Ali ibn Sino nafaqat buyuk hakim, shu bilan birga mashhur matematik hamdir. Uning «Donishnoma», «Shifo kitobi» asarlarining kattagina qismi matematika, xususan geometriyaga bag‘ishlangan. Bu kitoblarda quyidagi kabi ma’lumotlarni ko‘rish mumkin.

1- teorema. Aylananing oltidan bir bo‘lagini tortib turuvchi vatar aylananing yarim diametriga tengdir.

2- teorema. Agar teng tomonli uchburchak doiraga ichki chizilgan bo‘lsa, u holda uning biror tomonining o‘ziga ko‘-

paytmasi doira yarim diametrining o'ziga ko'paytmasining uch baravariga tengdir.

3-teorema. Agar *BC* kesma aylananing o'ndan bir bo'lagini tortib turuvchi vatar bo'lsa, *CD* aylananing oltidan bir bo'lagining vatari bo'lib, *BC* ning davomida (*BC* to'g'ri chiziqda) aylana tashqarisida joylashgan bo'lsa, u holda *BC*ning *CD*ga nisbati *CD*ning *BD* ga nisbatiga tengdir, ya'ni $\frac{BC}{CD}=\frac{CD}{BD}$ bo'ladi.

4- teorema. Agar $\frac{BC}{CD}=\frac{CD}{BD}$ va *CD* aylananing oltidan bir bo'lagining vatari bo'lsa, u vaqtda *BC* hamma vaqt aylananing o'ndan bir bo'lagining vatari bo'ladi.

Shunday qilib, *C* nuqta *BD* kesmani o'rta va chet nisbatda bo'ladi («oltin kesim»). Demak $\frac{BC}{CD}$ nisbat muntazam o'nburchak tomonining unga tashqi chizilgan aylana radiusiga nisbati $\frac{CD}{BD}$ bilan birgalikda «oltin» proporsiyani hosil qiladi. «Oltin kesim»ning esa rassomchilik, arxitekturada keng tatbiqlari ma'lum.

Jahon fani taraqqiyotiga katta hissa qo'shgan qomusiy daholardan biri Abul-Abbos Ahmad ibn Muhammad ibn Kasir al-Farg'oniydir. Muhtaram Prezidentimiz aytganlaridek, «Ahmad Farg'oniyning *bebaho merosi o'z davri olimlari uchun dasturulamal bo'lib xizmat* qilgani tarixiy manbalar orqali yaxshi ma'lum» ([1], 41- bet). Uning, boshqa fanlar bilan bir qatorda, matematikadagi natijalari ham mashhurdir. Ma'lumki, Ptolomey stereometrik proyeksiyalarning asosiy xossalarini bayon etgan, ammo ularning qat'iy va nafis isbotlarini Ahmad Farg'oniy bergan. Binobarin, bu teoremalar (xossalar) Ptolomey — Farg'oniy teoremalari deb atalishi va ular universitetlarning matematika kurslaridan munosib joy olishi lozim [2].

PLANIMETRIYA

I bob. GEOMETRIYANING ASOSIY TUSHUNCHALARI

1- §. Eng sodda geometrik shakllar

Geometriyaning asosiy (ta'rifsiz qabul qilinadigan) tushunchalari: *nuqta*, *to'g'ri chiziq*, *tekislik* va *masofa* bo'lib, ularning xossalari aksiomalar (isbotsiz qabul qilinadigan tasdiqlar) orqali ta'riflanadi.

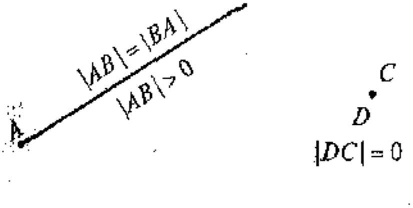
1.1. Nuqta. Qalamning o'tkir uchi, koinotdagi biror yoritgichning ko'rinishi, xaritadagi ma'lum shaharning belgisi va hokazolar nuqtaga misol bo'ladi. Nuqta hech qanday kattalikka ega emas.

1. Ikki nuqta orasidagi masofa har doim musbat va faqat nuqtalar ustma-ust tushgandagina 0 (nol)ga teng bo'ladi (1- *a*, *b* rasmlar).

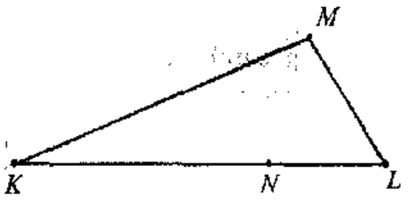
2. Bir nuqtadan ikkinchi nuqtagacha bo'lgan masofa ikkinchi nuqtadan birinchi nuqtagacha bo'lgan masofaga teng (1- *a* rasm).

3. Ikki nuqta orasidagi masofa bu nuqtalardan uchinchi nuqtagacha bo'lgan masofalar yig'indisidan katta emas (2- rasm). Umuman, ixtiyoriy *M* nuqta uchun $|KL| \leq |KM| + |ML|$. Agar $|KL| = |KN| + |NL|$ bo'lsa, *N* nuqta *K* va *L* nuqtalar orasida joylashgan deyiladi.

1.2. To'g'ri chiziq. Ustma-ust tushmagan ikki *A* va *B* nuqtalardan baravar uzoqlikdagi barcha nuqtalar to'g'ri chiziqni tashkil etadi (3-

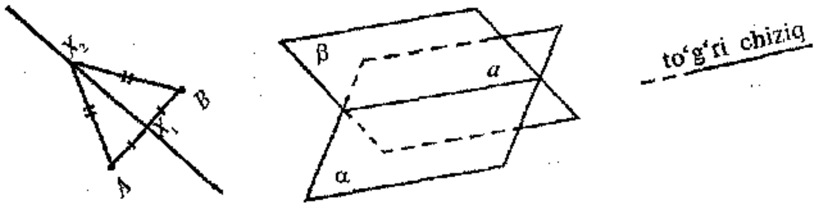


1- rasm.



$|KL| < |KM| + |ML|$, $|KL| = |KN| + |NL|$

2- rasm.



a) b) d)

3- rasm.

a rasm). Turli ikki tekislikning kesishgan (umumiy) qismi (chizig'i) ham to'g'ri chiziqqa misol bo'la oladi (3- *b* rasm). To'g'ri chiziq ikki tomonidan chegaralanmagan (3- *d* rasm) bo'ladi.

To'g'ri chiziqni qanday olmaylik, shu to'g'ri chiziqqa tegishli bo'lgan nuqtalar ham, tegishli bo'lmagan nuqtalar ham mavjud.

Har qanday ikki nuqtadan to'g'ri chiziq o'tkazish mumkin va faqat bitta.

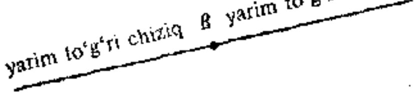
To'g'ri chiziq tekislikni ikkita yarim tekislikka ajratadi va u ikkala yarim tekislikning chegarasi deyiladi.

1.3. Nur. To'g'ri chiziq o'ziga tegishli biror nuqtasi bilan ikkita yarim to'g'ri chiziqqa ajraladi, ularning har biri ikkinchisini *to'ldiruvchi yarim to'g'ri chiziq* deyiladi (4- rasm).

Boshqacha qilib aytganda, biror *a* to'g'ri chiziq *B* nuqtasi bilan ikki yarim to'g'ri chiziqqa ajratilsa, ularning har biri boshi *B* nuqtadan iborat *nur* deb ham ataladi.

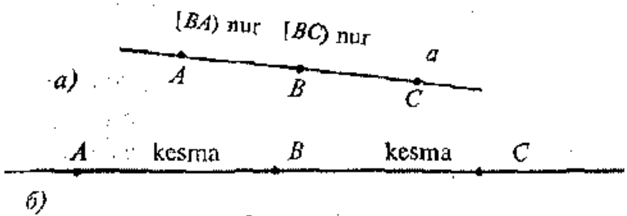
B nuqta bilan *a* to'g'ri chiziq ikkita yarim to'g'ri chiziqqa (nurga) ajralgan bo'lsa, ularning har biri *B* nuqtadan farqli nuqtalari *A* va *C* bilan [*BA*) va [*BC*) tarzda belgilanadi (5- *a* rasm).

1.4. Kesma. To'g'ri chiziqning berilgan ikki nuqtasi va ular orasidagi barcha nuqtalardan tashkil topgan qismi *kesma* deyiladi (5- rasm).



4- rasm.

Har bir kesma noldan katta tayin uzunlikka ega. Kesma uzunligi shu kesmaning har qanday nuqtasi ajratgan qismlari uzunliklarining yig'indisiga teng.



5- rasm.

6- rasm.

Agar ikki turli A va B nuqta orasidagi masofani $|AB|$ orqali belgilasak, *kesma uzunligining xossalarini quyidagicha ifodalash mumkin:*

1) $|AB| > 0$;

2) $|AB| = |BA|$;

3) $|AB| = |AC| + |BC|$ (bunda C nuqta A va B orasida).

Quyida ko'pchilik o'quvchilar uchun qiyinroq bo'lgan ba'zi masalalarni yechib ko'rsatamiz.

1- m a s a l a. To'g'ri chiziqdagi n ta turli nuqta nechta kesmani aniqlaydi (6- rasm)?

Y e c h i l i s h i. Barcha kesmalar sonini oddiy sanash yo'li bilan aniqlaymiz: Dastlab bir uchi A_1 nuqtada, ikkinchi uchi esa, qolgan nuqtalarning birida bo'lgan barcha kesmalarni sanasak, $n - 1$ son hosil bo'ladi. So'ngra bir uchi A_2 nuqtada, ikkinchi uchi esa, qolgan $(n - 2)$ ta nuqtalarning birida bo'lgan kesmalarni sanab, $n - 2$ ni aniqlaymiz va hokazo. Natijada barcha kesmalar soni:

$$(n - 1) + (n - 2) + \ldots + 3 + 2 + 1 = \frac{n \cdot (n - 1)}{2}.$$

J a v o b: $\frac{n \cdot (n - 1)}{2}$.

2- m a s a l a. Hech bir uchtasi bir to'g'ri chiziqda yotmagan n ta nuqta nechta kesmani aniqlaydi?

Y e c h i l i s h i. 1- masalaning yechimidek fikrlab, bu masalaning ham javobi $\frac{n \cdot (n - 1)}{2}$ ekanini aniqlaymiz.

J a v o b: $\frac{n \cdot (n - 1)}{2}$.

2- §. Burchaklar

2.1. Burchak tushunchasi. T a ' r i f. *Tekislikning umumiy boshlang'ich nuqtaga ega bo'lgan ikki nur va ular bilan chegaralangan qismi burchak deb ataladi. Nurlar — burchakning tomonlari, nurlarning umumiy uchi — burchak uchi deyiladi.*

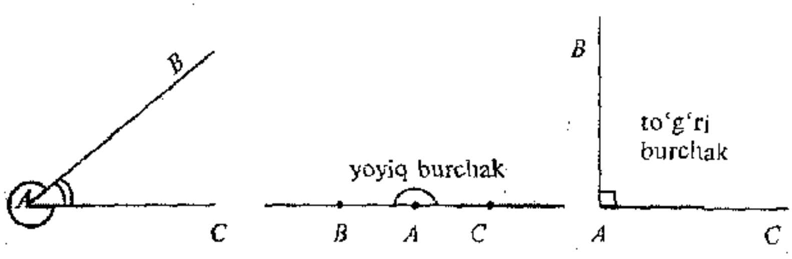
Bitta nuqtadan chiquvchi ikki nur tekislikni ikkita burchakka ajratadi (7- *a* rasm).

T a ' r i f. *Agar burchakni tashkil etuvchi nurlar bir-birini to'ldirib to'g'ri chiziq hosil qilsa, bunday burchak yoyiq burchak deyiladi* (7- *b* rasm).

Yozuvda burchaklarni belgilash uchun uning uchi va tomonlaridagi bittadan nuqta olinadi. Masalan, $\angle BAC$, bunda burchak uchiga qo'yilgan harf o'rtada yoziladi. Ba'zida burchak uchini yoki kattaligini bildiruvchi bitta ifoda bilan yoziladi. Masalan, $\angle A$, $\angle \alpha$ yoki ikki to'g'ri chiziq (yo nurlar) a va b lar orasidagi burchak $(a,\hat{}\,b)$ tarzida ham ifodalanadi.

2.2. Burchakning gradus o'lchovi. Burchaklarning o'lchov birligi yoyiq burchakning $\frac{1}{180}$ bo'lagiga teng bo'lgan burchak bo'lib, u 1 gradusli burchak hisoblanadi va $1°$ kabi belgilanadi. Bir gradusli burchakning $\frac{1}{60}$ qismiga teng bo'lgan burchak bir minutli burchak deyiladi va $1'$ kabi ifodalanadi. Bir minutli burchakning $\frac{1}{60}$ ulushi bir sekundli burchak deb atalib, $1''$ ko'rinishida ifodalanadi.

2.3. Burchakning radian o'lchovi. Burchaklarni o'lchashda gradus o'lchov bilan bir qatorda *radian o'lchov* deb ataluvchi o'lchov birli-



7- rasm.

gidan ham foydalaniladi. Yoyining uzunligi aylana radiusiga teng bo'lgan markaziy burchak kattaligi bir *radian* deb hisoblanadi. Burchakning radian o'lchovida yoyiq burchak π ($\pi \approx 3,14$) radianga teng.

π radian = $180°$; $45° = \frac{\pi}{4}$ radian;

1 radian $= \frac{180°}{\pi} \approx 57°17'45''$; $60° = \frac{\pi}{3}$ radian.

$1° = \frac{\pi}{180}$ radian $\approx 0,000291$ *radian;*

Burchaklar transportir yordamida graduslarda o'lchanadi.

Har bir burchak noldan katta tayin gradus o'lchoviga ega. Yoyiq burchak 180° ga teng. Burchakning gradus o'lchovi o'zining tomonlari orasidan o'tuvchi har qanday nur yordamida ajratilgan burchaklarning gradus o'lchovlari yig'indisiga teng.

2.4. To'g'ri, o'tkir va o'tmas burchaklar.

T a ' r i f. *Yoyiq burchakning yarmiga teng bo'lgan 90° li burchak to'g'ri burchak deyiladi* (7- *d* rasm). *To'g'ri burchakdan kichik burchak o'tkir burchak, to'g'ri burchakdan katta, ammo yoyiq burchakdan kichik bo'lgan burchak o'tmas burchak deb ataladi (9- rasm).*

2.5. Qo'shni burchaklar. Yoyiq burchakning uchidan chiqqan uchinchi yarim to'g'ri chiziq uni ikkita o'zaro qo'shni burchakka ajratadi. Demak, o'zaro qo'shni burchaklarning yig'indisi yoyiq burchak (180°) ga tengdir (8- rasm).

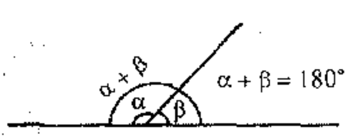
T a ' r i f. *Burchak uchidan chiqib, uni teng ikkiga bo'luvchi nur burchakning bissektrisasi deb ataladi* (10- rasm).

1- m a s a l a. Qo'shni burchaklardan biri ikkinchisidan 5 marta kichik bo'lsa, shu burchaklardan kattasini toping.

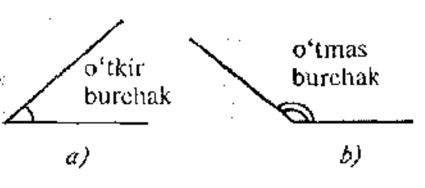
Y e c h i l i s h i. Masala shartiga ko'ra,

$x + 5x = 180° \Rightarrow 6x = 180° \Rightarrow x = 30°$; $5x = 150°$ (11- rasm).

J a v o b: 150°.

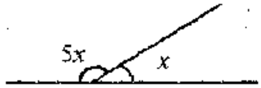


8- rasm.

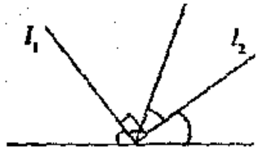


9- rasm.

10- rasm.



11- rasm.



12- rasm.

2- m a s a l a. O‘zaro qo‘shni burchaklarning bissektrisalari orasidagi burchak necha gradus $(l_1 \hat{,} l_2)$ (12- rasm)?

Y e c h i l i s h i. Qo‘shni burchaklarning bissektrisalari orasidagi burchak 90° ekanligiga oson ishonch hosil qilamiz. Chunki qo‘shni burchaklar yig‘indisi 180° ekanligidan ularning yarimlari yig‘indisi 90° bo‘ladi.

J a v o b: $(l_1 \hat{,} l_2) = 90°$.

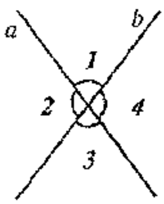
3- §. To‘g‘ri chiziqlarning tekislikda o‘zaro joylashuvi

To‘g‘ri chiziqlar bir-biri bilan kesishishi yoki kesishmasligi mumkin.

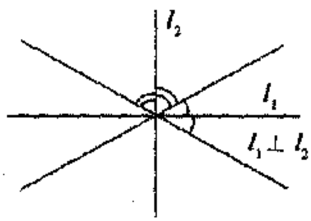
3.1. Vertikal burchaklar. Ikki to‘g‘ri chiziq (a va b) ning kesishishidan hosil bo‘lgan *1*, *2*, *3* va *4* burchaklardan o‘zaro qo‘shni bo‘lmaganlari bir-biriga *vertikal burchaklar* deyiladi (13- rasm). Bunda *1* burchak *3* ga, *2* burchak *4* ga vertikal.

Vertikal burchaklar bir-biriga tengdir.

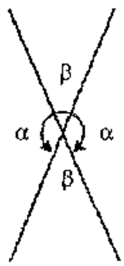
Agar vertikal burchaklar 90° dan bo‘lsa, ularni tashkil etuvchi to‘g‘ri chiziqlar (a va b) o‘zaro *perpendikular* deb ataladi va $a \perp b$ tarzida belgilanadi.



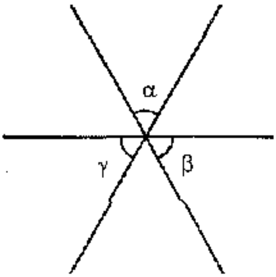
13- rasm.



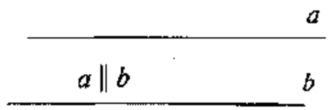
14- rasm.



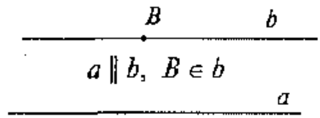
15- rasm.



16- rasm.



17- rasm.



18- rasm.

Ikki to‘g‘ri chiziqning kesishishidan hosil bo‘lgan vertikal burchaklarning bissektrisalari bir-biriga perpendikular bo‘ladi (14-rasm).

1- masala. Ikkita to‘g‘ri chiziqning kesishishidan hosil bo‘lgan burchaklardan uchtasining yig‘indisi 315° ga teng. Shu burchaklardan kichigini toping (15- rasm).

Yechilishi. $2\alpha + \beta = 315^\circ \Rightarrow \alpha + \alpha + \beta = 315^\circ \Rightarrow [\alpha + \beta = 180^\circ]$, $\alpha = 315^\circ - 180^\circ = 135^\circ$; $\beta = 45^\circ$.

Javob: 45°.

2- masala. Bir nuqtadan uchta to‘g‘ri chiziq o‘tkazilgan. $\alpha + \beta + \gamma$ ni toping (16- rasm).

Yechilishi. Vertikal burchaklarning o‘zaro tengligidan: $2\alpha + 2\beta + 2\gamma = 360^\circ \Rightarrow \alpha + \beta + \gamma = 180^\circ$.

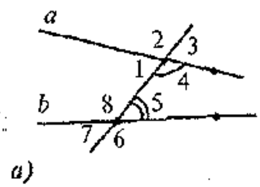
Javob: 180°.

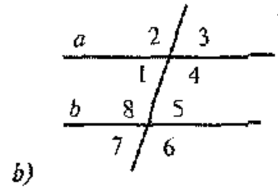
3.2. Parallel to‘g‘ri chiziqlar.

Ta’rif. *Agar tekislikdagi ikki a va b to‘g‘ri chiziq o‘zaro kesishmasa, ular parallel to‘g‘ri chiziqlar deyiladi va* $a \| b$ *kabi ifodalanadi* (17- rasm).

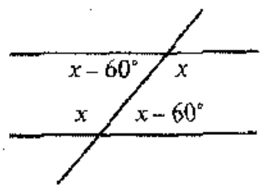
Berilgan *a* to‘g‘ri chiziqda yotmaydigan *B* nuqta orqali bu to‘g‘ri chiziqqa bittadan ortiq parallel to‘g‘ri chiziq o‘tkazish mumkin emas (18- rasm).

Ikki to‘g‘ri chiziqni uchinchi to‘g‘ri chiziq kesganda hosil bo‘lgan sakkizta burchak maxsus nomlarga ega: 1) $\angle 1$ va $\angle 8$, $\angle 4$ va $\angle 5$ — *ichki bir tomonli burchaklar*; 2) $\angle 1$ va $\angle 5$, $\angle 4$ va $\angle 8$ — *ichki almashinuvchi burchaklar*; 3) $\angle 3$ va $\angle 5$, $\angle 8$ va $\angle 2$, $\angle 1$ va $\angle 7$, $\angle 4$ va $\angle 6$ — *mos burchaklar*; 4) $\angle 2$ va $\angle 6$, $\angle 3$ va $\angle 7$ — *tashqi almashinuvchi burchaklar* deyiladi (19-*a* rasm).

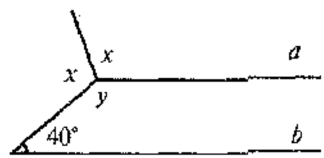




19- rasm.



20- rasm.



21- rasm.

3.3. To‘g‘ri chiziqlarning parallellik alomatlari.

Agar ikki a va b to‘g‘ri chiziqni uchinchi to‘g‘ri chiziq kesib o‘tganda:

1) mos burchaklar teng bo‘lsa, u holda a va b to‘g‘ri chiziqlar o‘zaro paralleldir;

2) ichki (yoki tashqi) almashinuvchi burchaklar teng bo‘lsa, u holda a va b to‘g‘ri chiziqlar o‘zaro paralleldir;

3) ichki bir tomonli burchaklar yig‘indisi 180° ga teng bo‘lsa, u holda a va b to‘g‘ri chiziqlar paralleldir (19- b rasm).

3- masala. Ikki parallel to‘g‘ri chiziqni uchinchi to‘g‘ri chiziq kesib o‘tganda hosil bo‘lgan ichki bir tomonli burchaklardan biri ikkinchisidan 60° kichik. Shu burchaklardan kattasini toping (20- rasm).

Yechilishi. Masala shartiga ko‘ra:

$$x + x - 60° = 180° \Rightarrow x = 120°.$$

Javob: 120°.

4- masala. 21- rasmda $a \parallel b$ bo‘lsa, x burchakni toping.

Yechilishi. $a \parallel b$ bo‘lgani uchun ichki bir tomonli burchaklar yig‘indisi 180° ga teng. $y + 40° = 180°$, bundan $y = 140°$ va $x = 110°$ ekani aniqlanadi.

Javob: 110°.

Mustaqil ishlash uchun test topshiriqlari

1. Biror to‘g‘ri chiziqqa tegishli 5 ta turli nuqta nechta kesmani aniqlaydi?

A) 4; B) 6; C) 7; D) 8; E) 10.

2. Hech bir uchtasi bir to‘g‘ri chiziqda yotmagan 6 ta turli nuqta nechta kesmani aniqlaydi?

A) 3; B) 6; C) sanoqsiz; D) 15; E) 10.

3. Hech qaysi uchtasi bir to‘g‘ri chiziqda yotmagan 7 ta nuqta orqali nechta turlicha to‘g‘ri chiziq o‘tkazish mumkin?

A) 28; B) 21; C) 42; D) 35; E) 14.

4. Hech qaysi uchtasi bir to‘g‘ri chiziqda yotmagan 9 ta nuqta orqali nechta turlicha to‘g‘ri chiziq o‘tkazish mumkin?

A) 9; B) 18; C) 72; D) 36; E) 24.

5. AB kesmada uchinchi C nuqta olindi. AB, AC, CA, CB nurlardan qaysi bir jufti ustma-ust tushadi?

A) AC va CB; B) AB va AC; C) AB va CB;
D) AC va CA; E) CA va CB.

6. AB nurda C nuqta belgilangan. $AB = 1,8$; $AC = 0,4$ bo‘lsa, BC kesmaning uzunligi qancha bo‘ladi?

A) 2,2; B) 1,4; C) 2,1; D) 2; E) aniqlab bo‘lmaydi.

7. Quyidagi iboralarning qaysi biri noto‘g‘ri?

A) Har qanday to‘g‘ri chiziqni olmaylik, tekislikda shu to‘g‘ri chiziqqa tegishli bo‘lgan nuqtalar ham, tegishli bo‘lmagan nuqtalar ham mavjud;

B) Har qanday ustma-ust tushmagan ikki nuqtadan bitta va faqat bitta to‘g‘ri chiziq o‘tkazish mumkin;

C) To‘g‘ri chiziqning ikki nuqtasi va ular orasidagi nuqtalardan iborat qismi kesma deyiladi.

D) To‘g‘ri chiziq biror O nuqtasi bilan boshi shu nuqtada bo‘lgan, qarama-qarshi yo‘nalgan ikki nurga ajraladi;

E) Koordinatalar tekisligida (3; 4) va (4; 3) sonlar juftliklari bitta nuqtani ifodalaydi.

8. Ushbu ta’riflarning qaysi biri noto‘g‘ri?

A) To‘g‘ri chiziq tekislikni ikki yarim tekislikka ajratadi, bunda (a) to‘g‘ri chiziq yarim tekisliklarning chegarasi deyiladi;

B) To'g'ri chiziqdagi uchta nuqta (*A*, *B*, *C*) uchta turli kesmani aniqlaydi;

C) Har bir kesma noldan katta son bilan o'lchanadigan tayin uzunlikka ega;

D) Istalgan yarim to'g'ri chiziqqa uning boshlang'ich nuqtasidan berilgan uzunlikdagi yagona kesmani qo'yish mumkin;

E) To'g'ri chiziqdagi turli 4 ta nuqta 4 ta kesmani aniqlaydi.

9. *AB* kesmada *C* nuqta olingan. *AB*, *AC*, *BC* kesmalar uchun quyidagi ifodalarning qaysi biri doim o'rinli?

A) $AB < AC + BC$; B) $AB = AC + BC$; C) $AB = AC$;

D) $AC = BC$; E) $AB = \frac{1}{2}(AC + CB)$.

10. *AB* kesmaning uzunligi 14. Bu kesmada *C* nuqta belgilangan. Agar *AC* kesma *BC* kesmadan 2 ga ortiq bo'lsa, *BC* kesmaning uzunligini toping.

A) 8; B) 6; C) 4; D) 10; E) 12.

11. *AC* kesmada *B* ichki nuqta bo'lib, $BC = 7{,}4$. *AB* kesmaning uzunligi *AC* kesma uzunligidan 3 marta kichik bo'lsa, *AC* ning uzunligini toping.

A) 11,2; B) 10,6; C) 10,8; D) 11,1; E) 12,1.

12. Uzunligi 32 sm bo'lgan *AB* kesma uch bo'lakka bo'lingan bo'lib, ikki chetki bo'laklarning o'rtalari orasidagi masofa 20 sm. O'rtasidagi kesmaning uzunligini toping.

A) 12 sm; B) 4 sm; C) 6 sm; D) 14 sm; E) 8 sm.

13. Uzunligi 15 bo'lgan *AB* kesma *C* nuqta bilan ikkiga bo'lingan bo'lib, ular uzunliklarining nisbati 2 : 3 kabi bo'lsa, *AC* va *BC* kesmalar uzunliklarini toping.

A) 12; 18; B) 4; 12; C) 6; 9; D) 14; 21; E) 4; 6.

14. Quyidagi iboralarning qaysi biri noto'g'ri?

A) Turli ikki nuqtadan faqat bitta to'g'ri chiziq o'tkazish mumkin;

B) Har bir kesma noldan katta tayin uzunlikka ega. Kesma uzunligi shu kesmaning har qanday nuqtasi ajratgan qismlari uzunliklarining yig'indisiga teng;

C) To'g'ri chiziq tekislikni ikkita yarim tekislikka ajratadi;

D) Bir to'g'ri chiziqda yotmagan uchta turli *A, B* va *C* nuqtalar uchun:

$$|AB| < |AC| + |BC|$$
$$|AC| < |AB| + |BC|$$
$$|BC| < |AB| + |AC|$$

munosabatlar o'rinlidir;

E) Bir to'g'ri chiziqdagi uchta turli *A*, *B* va *C* nuqtalar uchun $|BC| < |AB| - |AC|$ doim o'rinli.

15. Umumiy uchga ega bo'lgan *AB* va *AC* nurlarning birida (*A* nuqtadan farqli) *k* ta nuqta, ikkinchisida *n* ta nuqta belgilangan. Bir uchi *AC* nurdagi nuqtalarda, ikkinchi uchi esa *AB* nurdagi nuqtalarda bo'lgan barcha kesmalar sonini ifodalang.

A) $n \cdot k$; B) $n(k-1)$; C) $(n-1)k$; D) $\frac{n(k-1)}{2}$; E) $\frac{(n-1)k}{2}$.

16. Qo'shni burchaklarning kattaliklarining nisbati 7 : 3 kabi. Shu burchaklarning kichigini toping.

A) 63°; B) 51°; C) 57°; D) 48°; E) 54°.

17. Qo'shni burchaklardan biri ikkinchisidan to'rt marta kichik bo'lsa, shu burchaklardan kattasini toping.

A) 125°; B) 130°; C) 140°; D) 144°; E) 120°.

18. Qo'shni burchaklardan birining bissektrisasi burchakni 30° li burchaklarga ajratsa, ikkinchisining bissektrisasi qanday burchaklarga ajratadi?

A) 40°; 40°; B) 50°; 50°; C) 60°; 60°; D) 65°; 65°; E) 70°; 70°.

19. Ikki qo'shni burchaklarning ayirmasi 24° ga teng. Shu burchaklarning kichigini toping.

A) 72°; B) 68°; C) 82°; D) 76°; E) 78°.

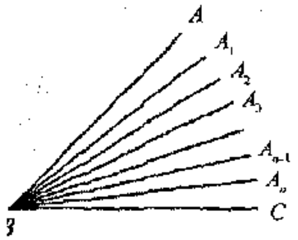
20. Qo'shni burchaklardan biri ikkinchisidan 32° katta. Shu burchaklardan kattasini toping.

A) 106°; B) 118°; C) 116°; D) 114°; E) 108°.

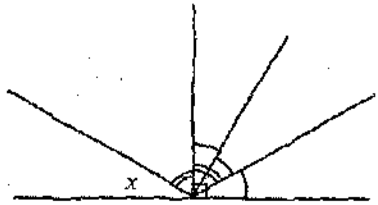
21. *ABC* burchakning *B* uchidan chiquvchi 6 ta nur uni nechta burchakka ajratadi (*ABC* burchakni qo'shib hisoblaganda)?

A) 16; B) 18; C) 19; D) 25; E) 28.

22. *ABC* burchakning *B* uchidan chiquvchi *n* ta nur uni nechta burchakka ajratadi (*ABC* burchakni qo'shib hisoblaganda) (22- rasm)?



22- rasm.



23- rasm.

A) $2n$; B) $\frac{n(n+1)}{2}$; C) $\frac{(n-1)n}{2}$; D) $\frac{(n+2)(n+1)}{2}$; E) $\frac{n^2}{2}$.

23. 23- rasmda o'zaro teng burchaklar bir xil sondagi yoychalar bilan belgilangan. x burchakning gradus o'lchovini toping.

A) 40°; B) 30°; C) 35°; D) 45°; E) 60°.

24. Quyidagi ta'rif va tasdiqlardan qaysi biri noto'g'ri?

A) Agar ikkita burchakning bitta tomoni umumiy bo'lsa, ular qo'shni burchaklar deyiladi;

B) Qo'shni burchaklarning yig'indisi 180° ga teng;

C) Agar ikki burchak o'zaro teng bo'lsa, u holda ularga qo'shni burchaklar ham o'zaro teng;

D) Agar burchak yoyiq bo'lsa, u holda uning gradus o'lchovi 180° bo'ladi;

E) To'g'ri burchakka qo'shni burchak to'g'ri burchak bo'ladi.

25. Biror nuqtadan o'tuvchi *n* ta to'g'ri chiziq bir-biri bilan kesishmaydigan* nechta burchak hosil qiladi?

A) n; B) $n+1$; C) $2n$; D) $\frac{3n}{2}$; E) $4n+1$.

26. Ikki parallel to'g'ri chiziqni uchinchisi kesib o'tganda hosil bo'lgan ichki bir tomonli burchaklardan biri ikkinchisidan 17 marta kichik. Shu burchaklardan kichigini toping.

A) 20°; B) 24°; C) 15°; D) 10°; E) 18°.

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*Bir-biri bilan kesishmaydigan burchaklar deb o'zaro ustma-ust tushmagan burchaklarga aytiladi.

27. Quyidagi tasdiqlardan qaysi biri noto‘g‘ri?

A) har bir burchak noldan katta tayin gradus o‘lchoviga ega;

B) yoyiq burchak 180° ga teng;

C) burchakning gradus o‘lchovi o‘zining tomonlari orasidan o‘tuvchi har qanday nur yordamida ajratilishidan hosil qilingan burchaklarning gradus o‘lchovlari yig‘indisiga teng;

D) ikki to‘g‘ri chiziqning kesishishidan hosil bo‘lgan ikki juft o‘zaro vertikal burchaklar kattaliklari teng bo‘lsa, u holda to‘g‘ri chiziqlar o‘zaro perpendikular bo‘ladi;

E) o‘zaro vertikal burchaklar yig‘indisi doim 180° ga teng.

28. Quyidagi iboralarning qaysi biri noto‘g‘ri?

A) Istalgan yarim to‘g‘ri chiziqqa uning boshlang‘ich nuqtasidan ma’lum uzunlikda yagona kesma qo‘yish mumkin.

B) Istalgan to‘g‘ri chiziq hosil qilgan tayin yarim tekislikka ma’lum gradus o‘lchovi 180° dan kichik yagona burchakni qo‘yish mumkin.

C) Berilgan to‘g‘ri chiziqda yotmaydigan nuqta orqali unga bittadan ortiq parallel to‘g‘ri chiziq o‘tkazish mumkin emas.

D) Qo‘shni burchaklarning yig‘indisi 180° ga teng.

E) Agar ikki to‘g‘ri chiziq o‘zaro parallel bo‘lmasa, ular o‘zaro perpendikular.

29. Ikkita to‘g‘ri chiziqning kesishishidan hosil bo‘lgan qo‘shni burchaklarning ayirmasi 40° ga teng. Shu burchaklardan kichigini toping.

A) 60°; B) 40°; C) 50°; D) 70°; E) 45°.

30. $a_1 \parallel a_2$ va $a_3 \parallel a_4$ bo‘lsa, x – ? (24- rasm).

A) 10°; B) 20°; C) 30°; D) 35°; E) 40°.

31. $a_1 \parallel a_2$, $a_3 \parallel a_4$, $a_1 \perp a_4$ bo‘lsa, 25- rasmdagi x ni toping.

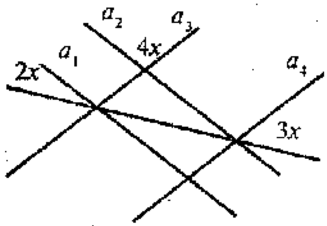
A) 30°; B) 45°; C) 55°; D) 60°; E) 70°.

32. $a_1 \parallel a_2$ bo‘lsa, 26- rasmdagi x burchakni toping.

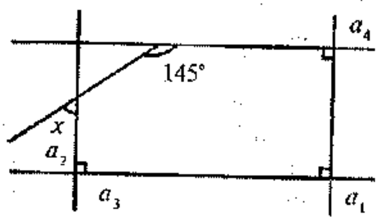
A) 20°; B) 30°; C) 35°; D) 40°; E) 45°.

33. $b_1 \parallel b_2$ bo‘lsa, 27- rasmdan foydalanib x burchakning gradus o‘lchovini toping.

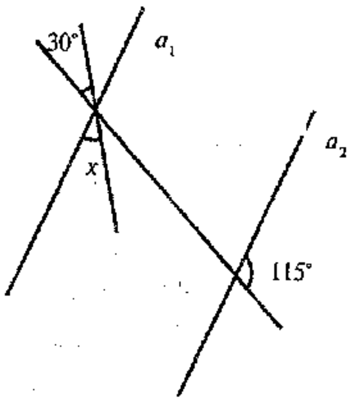
A) 45°; B) 55°; C) 60°; D) 70°; E) 75°.



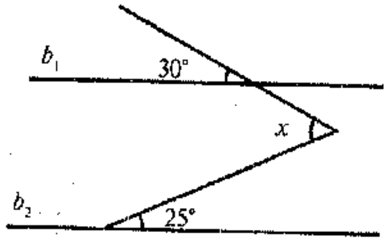
24- rasm.



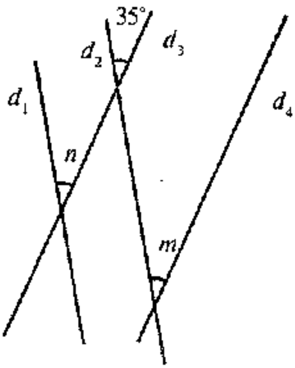
25- rasm.



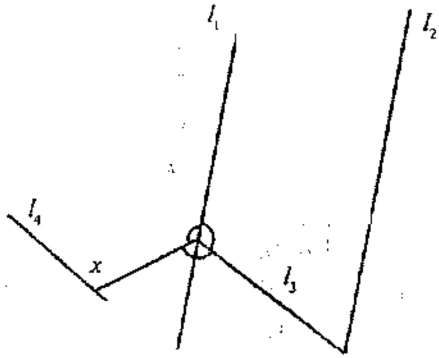
26- rasm.



27- rasm.



28- rasm.



29- rasm.

34. $d_1 \parallel d_2$ va $d_3 \parallel d_4$ bo'lsa, $m+n$ ni toping (28- rasm).
A) 35°; B) 40°; C) 60°; D) 65°; E) 70°

35. $l_1 \parallel l_2$ va $l_3 \parallel l_4$. x ning gradus qiymatini toping (29- rasm).
A) 80°; B) 90°; C) 100°; D) 110°; E) 120°.

36. Farqi 20° bo'lgan ikki qo'shni burchaklarning nisbatini toping.

A) $\frac{11}{7}$; B) $\frac{13}{3}$; C) $\frac{7}{5}$; D) $\frac{6}{5}$; E) $\frac{5}{4}$.

II bob. UCHBURCHAK. UCHBURCHAKLARDAGI METRIK MUNOSABATLAR

1- §. Uchburchak

T a ' r i f. *Bir to'g'ri chiziqda yotmagan uchta nuqtani bir-biri bilan uchta kesma orqali tutashtirish natijasida hosil bo'lgan shakl uchburchak deyiladi.*

1.1. Uchburchak turlari. Uchburchak burchaklariga qarab o'tkir burchakli (30- *a* rasm), to'g'ri burchakli (30- *b* rasm) yoki o'tmas burchakli (30- *d* rasm) bo'lishi mumkin. Uchburchak tomonlariga qarab turli tomonli (31- *a* rasm), teng yonli (31- *b* rasm) yoki teng tomonli bo'ladi (31- *d* rasm).

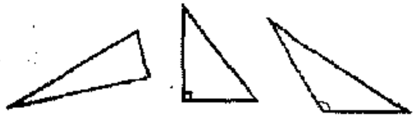
Uchburchakning uchlari (ba'zida ichki burchaklari) bosh harflar bilan (A, B, C...), tomonlari esa, qarshisidagi uchlariga mos ravishda kichik harflar bilan (a, b, c...) belgilanadi.

Uchburchakning uchala tomonlarining uzunliklari yig'indisi uning *perimetri* deyiladi. Uchburchakning perimetri P harfi bilan belgilansa, $P = a + b + c = AB + BC + CA$ kabi ifodalanadi.

Uchburchak ixtiyoriy tomonining uzunligi qolgan ikki tomoni uzunliklarining yig'indisidan kichik, ayirmasidan katta bo'ladi:

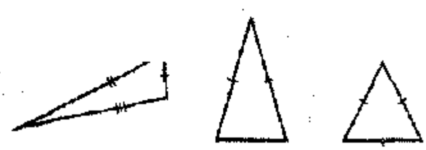
$$b - c < a < b + c; \quad a - c < b < a + c; \quad a - b < c < b + a.$$

Uchburchak tengsizligi deb ataluvchi bu tengsizliklardan uchta tomoni uzunliklari bo'yicha uchburchak yasash mumkinmi yoki yo'qligini aniqlashda foydalaniladi.



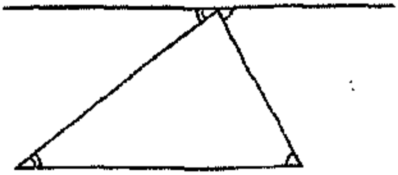
a) b) d)

30- rasm.



a) b) d)

31- rasm.



32- rasm.

33- rasm.

Masalan: 1) $a=2$; $b=2$; $c=2$ kesmalardan uchburchak yasab bo'ladi, ular uchun yuqoridagi tengsizliklar bajariladi.

2) $a=3$; $b=7$; $c=11$ kesmalardan uchburchak yasab bo'lmaydi, chunki ular uchun uchburchak tengsizligi bajarilmaydi.

Agar uchburchakning tomonlari uzunliklari o'sib borish tartibida joylashgan bo'lsa, ya'ni $a<b<c$ bo'lsa, u holda faqat eng katta tomon uchun uchburchak tengsizligining o'rinliligini tekshirish kifoya.

1- masala. a ning $\left(-1<a<\frac{1}{2}\right)$ qanday qiymatlarida uzunliklari mos ravishda 2, $1+a$ va $1-2a$ ga teng bo'lgan kesmalardan iborat uchburchak yasash mumkin?

Yechilishi. Berilgan kesmalardan uchburchak yasash mumkin bo'lishi uchun ularning ixtiyoriysi qolgan ikkitasining yig'indisidan kichik bo'lishi kerakligidan:

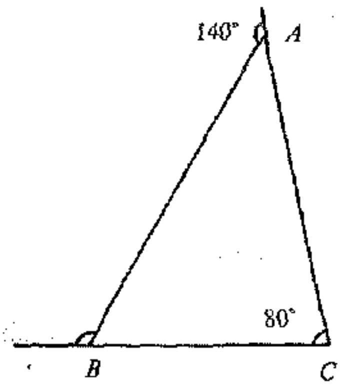
$$\begin{cases}1+a+1-2a>2\\1+a+2>1-2a\\2+1-2a>1+a\end{cases}\Rightarrow\begin{cases}-a>0\\3a>-2\\-3a>-2\end{cases}\Rightarrow\begin{cases}a<0\\a>-\frac{2}{3}\\a<\frac{2}{3}\end{cases}\Rightarrow -\frac{2}{3}<a<0\Rightarrow\left(-\frac{2}{3};0\right).$$

Javob: $a\in\left(-\frac{2}{3};0\right)$.

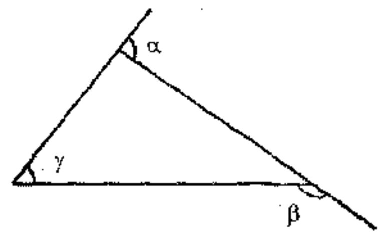
1.2. Uchburchakning tashqi burchagi.

Teorema. Uchburchak uchala burchagining yig'indisi 180° ga teng.

Bu teoremani 32- rasmdan va yoyiq burchakning 180°ga tengligidan foydalanib, o'zingiz isbotlashga harakat qiling.



34- rasm.



35- rasm.

Har qanday uchburchakning aqalli ikkita burchagi o'tkir bo'ladi.

Uchburchakning bir uchidagi *tashqi burchagi* deb uchburchakning shu uchidagi burchagiga qo'shni bo'lgan burchakka aytiladi (33- rasm).

Teorema. Uchburchakning tashqi burchagi o'ziga qo'shni bo'lmagan ichki burchaklari yig'indisiga teng.

Uchburchakning tashqi burchagi o'ziga qo'shni bo'lmagan istalgan ichki burchagidan katta.

2- masala. ABC uchburchakda A uchidagi tashqi burchagi 140° ga, C uchidagi ichki burchagi 80° ga teng. B uchidagi tashqi burchagini toping (34- rasm).

Yechilishi. A uchidagi tashqi burchagi 140° bo'lgani uchun shu uchidagi ichki burchagi 40° bo'ladi. Demak, $\angle B = 60°$ ekanligidan B uchidagi tashqi burchak 120° dir.

Javob: 120°.

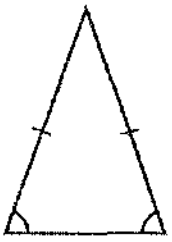
3- masala. Uchburchak ikki tashqi burchaklarining yig'indisi 240° ga teng. Uning shu burchaklarga qo'shni bo'lmagan ichki burchagi γ ni toping.

Yechilishi. Masalaning yechimini o'quvchi 35- rasmdagi chizmadan topishiga ishonamiz.

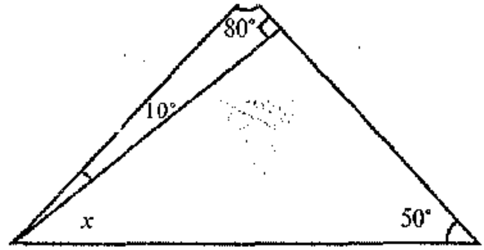
Javob: 60°.

1.3. Teng yonli uchburchak.

Ta'rif. *Agar uchburchakning ikki tomoni bir-biriga teng bo'lsa, u teng yonli uchburchak deyiladi* (36- rasm). *Uchinchi tomoni uchburchakning asosi deb ataladi.*



36- rasm.



37- rasm.

Teng yonli uchburchakning asosidagi burchaklari bir-biriga teng. Yon tomonlarning umumiy nuqtasi teng yonli uchburchakning *uchi* deyiladi.

4- masala. Teng yonli uchburchakning uchidagi burchagi 80° ga teng. Yon tomonga o'tkazilgan balandlik va asos orasidagi burchakni toping (37- rasm).

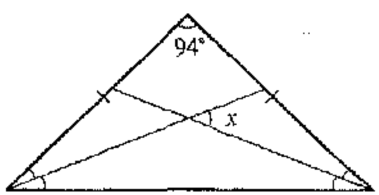
Yechilishi. Uchburchakning asosidagi burchaklari 50° li bo'lgani uchun undan 10° ni ayirib, 40° ni aniqlaymiz.

Javob: 40°.

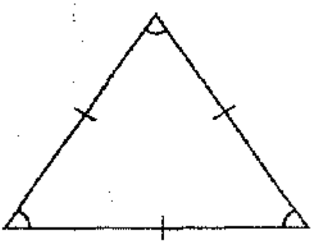
5- masala. Teng yonli uchburchakning uchidagi burchagi 94°. Asosidagi burchaklarining bissektrisalari kesishishidan hosil bo'lgan o'tkir burchakni toping. (38- rasm).

Yechilishi. Berilgan ABC teng yonli uchburchakning asosidagi burchaklari 43° dan bo'lgani va u burchaklarning bissektrisalari bilan hosil bo'lgan teng yonli uchburchakning tashqi burchagi $x° = 21{,}5° +$ $+ 21{,}5° = 43°$ ga teng.

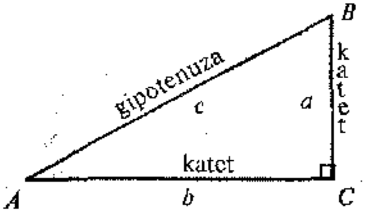
Javob: 43°.



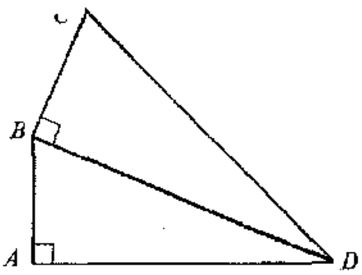
38- rasm.



39- rasm.



40- rasm.



41- rasm.

1.4. Teng tomonli (muntazam) uchburchak.

Ta'rif. *Uchala tomonlari o'zaro teng bo'lgan uchburchak teng tomonli uchburchak deyiladi* (39- rasm).

Muntazam uchburchakning ichki burchaklari ham bir-biriga teng bo'lib, ular 60° dandir.

1.5. To'g'ri burchakli uchburchak. Ma'lumki, uchburchakning biror burchagi to'g'ri bo'lsa, u *to'g'ri burchakli uchburchak* deyiladi. To'g'ri burchakli uchburchakning to'g'ri burchagini tashkil etuvchi tomonlari uning *katetlari* deyilib, uchinchi (to'g'ri burchak qarshisidagi) tomoni *gipotenuzasi* deb ataladi (40- rasm).

Teorema (Pifagor teoremasi). **To'g'ri burchakli uchburchakning a va b katetlari kvadratlarining yig'indisi c gipotenuzasi kvadratiga teng: $a^2+b^2=c^2$.**

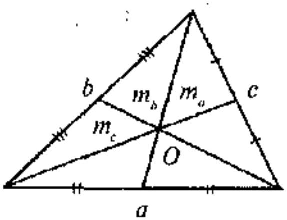
6- masala. 41- rasmda tasvirlangan to'g'ri burchakli ABC va BCD uchburchaklarda $|AB| = |BC| = 1$, $|AD| = 2$ bo'lsa, CD kesma uzunligini toping.

Yechilishi. Pifagor teoremasiga ko'ra ABD uchburchakda $|BD| = \sqrt{|AB|^2 + |AD|^2} = \sqrt{1^2+2^2} = \sqrt{5}$; BCD uchburchakda $|CD| = \sqrt{|BC|^2 + |BD|^2} = \sqrt{\left(\sqrt{5}\right)^2 + 1^2} = \sqrt{6}$.

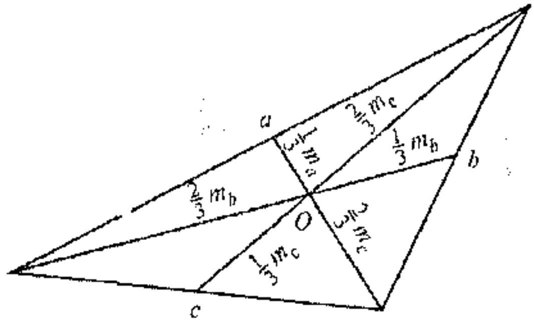
Javob: $\sqrt{6}$.

2- §. Uchburchakning asosiy elementlari

Uchburchakning uchta tomoni, uchta burchagidan tashqari uchta medianasi, uchta bissektrisasi, uchta balandligi, tashqi va ichki chi-



42- rasm.



43- rasm.

zilgan aylanalar radiuslari, perimetri, o'rta chizig'i hamda yuzi kabi asosiy elementlari bo'lib, ularning ixtiyoriy uchtasi (ulardan kamida biri chiziqli) ma'lum bo'lsa, qolganlarini topish mumkin.

2.1. Uchburchakning medianalari.

T a ' r i f. *Uchburchakning uchini uning qarshisidagi tomon o'rtasi bilan tutashtiruvchi kesma uchburchakning medianasi deyiladi* (odatda, medianalar m_a, m_b, m_c tarzda belgilanadi (42- rasm)).

Medianalar bir nuqtada kesishib, kesishish nuqtasida (uchidan hisoblaganda) 2 : 1 nisbatda bo'laklarga bo'linadi. Medianalarning kesishish nuqtasi *O uchburchak tekisligining og'irlik markazi* deyiladi (42–43- rasmlar).

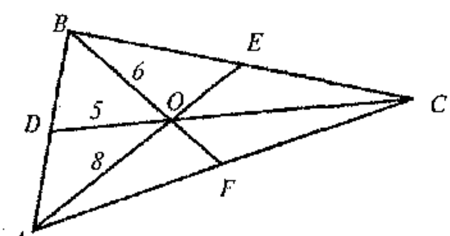
Tomonlari a, b va c bo'lgan uchburchakning medianalarini

$$m_a=\frac{1}{2}\sqrt{2b^2+2c^2-a^2}\ ;\ m_b=\frac{1}{2}\sqrt{2a^2+2c^2-b^2}\ ;\ m_c=\frac{1}{2}\sqrt{2b^2+2a^2-c^2}$$

formulalar yordamida topish mumkin.

Uchburchakning medianasi uning yuzini teng ikkiga ajratadi.

Uchburchakning uchala medianasi uni bir xil yuzli bir-birini qoplamaydigan oltita uchburchakka bo'ladi.



$BF=9$
$AE=12$
$CD=15$
AB — ?

14- rasm.

1- masala. Uchburchakning medianalari 9, 12 va 15 ga teng. Uzunligi 15 bo'lgan mediana tushirilgan tomon uzunligini toping.

Yechilishi. 44- chizmada O nuqta medianalarning kesishish nuqtasi bo'lib, $|OB|=6$, $|OD|=5$, $|OA|=8$. ΔBOA to'g'ri burchakli bo'lgani uchun uning gipotenuzasi $|BA|=2\cdot|OD|=10$.

Javob: $|AB|=10$.

2.2. Uchburchakning bissektrisalari.

Ta'rif. *Uchburchak burchagini teng ikkiga bo'luvchi kesma uning bissektrisasi deyiladi.*

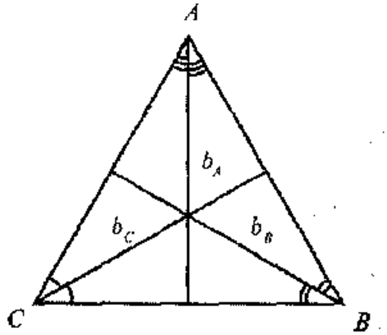
Uchburchakning uchala bissektrisasi bir nuqtada kesishadi va ular mos ravishda b_A, b_B va b_C kabi belgilanadi (45- rasm).

Bissektrisalarning kesishish nuqtasi uchburchakka ichki chizilgan aylana markazi bo'ladi, chunki bu nuqta uchburchak tomonlaridan baravar uzoqlashgandir.

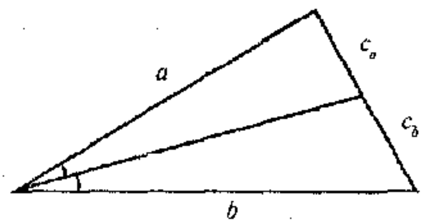
Uchburchakning bissektrisasi qarshisidagi tomonini qolgan ikki tomonlarining nisbati kabi bo'laklarga ajratadi (46- rasm):

$$\frac{a}{b}=\frac{c_a}{c_b}.$$

Agar b_A bissektrisaning A burchak qarshisidagi tomonida hosil qilgan kesmalari mos ravishda a_b va a_c bo'lsa, $b_A=\sqrt{b\cdot c-a_b\cdot a_c}$ formula orqali topiladi. Shunga o'xshash $b_B=\sqrt{a\cdot c-b_a\cdot b_c}$, $b_C=\sqrt{a\cdot c-c_a\cdot c_b}$ formulalardan ham foydalanish mumkin. To-



45- rasm.



46- rasm.

monlarning uzunliklari a, b va c bo'lgan uchburchakda bissektrisalarning uzunliklari mos ravishda ushbu formulalar orqali topiladi:

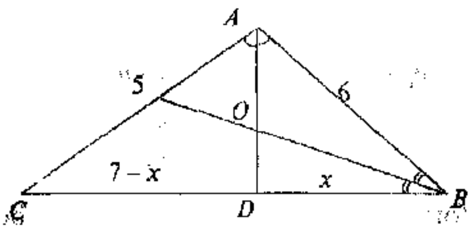
$$b_A = \frac{2}{b+c}\sqrt{b \cdot c \cdot p(p-a)} \text{ va } b_A = \frac{1}{b+c}\sqrt{bc(b+c)^2 - a^2};$$

$$b_B = \frac{2}{a+c}\sqrt{a \cdot c \cdot p(p-b)} \text{ va } b_B = \frac{1}{a+c}\sqrt{ac(a+c)^2 - b^2};$$

$$b_C = \frac{2}{a+b}\sqrt{a \cdot b \cdot p(p-c)} \text{ va } b_C = \frac{1}{a+b}\sqrt{ab(a+b)^2 - c^2};$$

bu yerda $p = \frac{a+b+c}{2}$.

2- m a s a l a. Tomonlarining uzunliklari 5, 6 va 7 bo'lgan uchburchakda katta tomoni qarshisidagi burchak bissektrisasi ichki chizilgan aylana markazida burchak uchidan hisoblaganda qanday nisbatdagi bo'laklarga ajraladi (47- rasm)?



47- rasm.

Y e c h i l i s h i. Bissektrisaning xossasiga ko'ra $\frac{5}{6} = \frac{7-x}{x}$, bundan $5x = 42 - 6x \Rightarrow 11x = 42 \Rightarrow x = \frac{42}{11}$. ΔABD da $\frac{|AO|}{|OD|} = \frac{6}{x}; \frac{6}{\frac{42}{11}} = \frac{|AO|}{|OD|} = \frac{11}{7}$.

J a v o b: $\frac{11}{7}$.

2.3. Uchburchakning balandliklari.

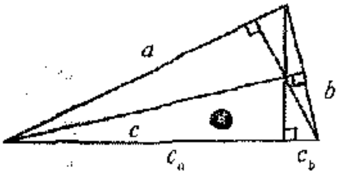
T a ' r i f. *Uchburchakning uchidan qarshisidagi tomonigacha bo'lgan masofani ifodalovchi kesma uning balandligi deyiladi.*

Uchburchakning uchala balandligi bir nuqtada kesishadi (48-rasm).

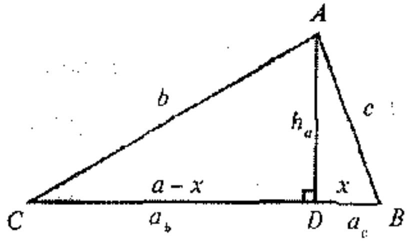
Har bir balandlik tushirilgan tomonda hosil bo'lgan bo'laklar orasida quyidagi munosabat o'rinli:

$$a^2 - c_a^2 = b^2 - c_b^2.$$

Bunda a, b — uchburchak tomonlari, c_a va c_b kesmalar esa uchburchakning uchinchi c tomonida balandlik hosil qilgan bo'laklar bo'-



48- rasm.



49- rasm.

lib, ular uchburchakning a va b tomonlarining c tomonidagi *proyeksiyalari* deyiladi.

Tomonlarining uzunliklari a, b, c bo'lgan uchburchakning balandliklari:

$$h_a = \frac{2}{a}\sqrt{p(p-a)(p-b)(p-c)},\quad h_b = \frac{2}{b}\sqrt{p(p-a)(p-b)(p-c)},$$

$$h_c = \frac{2}{c}\sqrt{p(p-a)(p-b)(p-c)}$$

formulalar yordamida hisoblanadi, bunda $p = \frac{a+b+c}{2}$.

3- masala. Tomonlarining uzunliklari a, b, c bo'lgan uchburchakning balandliklarini topish formulasini keltirib chiqaring.

Yechilishi. ABC uchburchakda h_a ni topaylik (49- rasm). To'g'ri burchakli uchburchak ADB da:

$$h_a^2 = c^2 - x^2 \tag{1}$$

hamda ADC uchburchakdan:

$$h_a^2 = b^2 - (a-x)^2. \tag{2}$$

Bu tengliklardan:

$$c^2 - x^2 = b^2 - (a-x)^2 \text{ yoki } c^2 - x^2 = b^2 - a^2 + 2ax - x^2.$$

Demak, $x = \frac{c^2 + a^2 - b^2}{2a}$ yoki $x^2 = \frac{(c^2+a^2-b^2)^2}{4a^2}$ ekan.

x^2 ning bu qiymatini (1) tenglikka qo'yib, $h_a^2 = c^2 -$

$$-\frac{(c^2+a^2-b^2)^2}{4a^2} = \frac{4a^2c^2-(c^2+a^2-b^2)^2}{4a^2} = \frac{(2ac+c^2+a^2-b^2)(2ac-c^2-a^2+b^2)}{4a^2}$$

ni hosil qilamiz.

So'nggi ifodada quyidagicha shakl almashtiramiz: $2ac + c^2 + a^2 - b^2 = (a+c)^2 - b^2 = (a+c-b)(a+c+b)$ va $2ac - c^2 - a^2 + b^2 = b^2 - (a^2 + c^2 - 2ac)^2 = b^2 - (a-c)^2 = (b-a+c)(b+a-c)$, bulardan

$$h_a^2 = \frac{1}{2a}\sqrt{(a+b+c)\cdot(a+c-b)(a+b-c)(b+c-a)} \qquad (3)$$

ni keltirib chiqaramiz.

Agar ABC uchburchakning perimetrini $2p$ bilan belgilasak, u holda:

$$a + c - b = 2p - b - b = 2p - 2b = 2(p-b)$$
$$a + b - c = 2p - c - c = 2p - 2c = 2(p-c)$$
$$b + c - a = 2p - a - a = 2p - 2a = 2(p-a)$$

bo'lib, bularni (3) ifodaga qo'yib topamiz:

$$h_a = \frac{1}{2a}\sqrt{2p\cdot 2(p-b)\cdot 2(p-c)\cdot 2(p-a)}$$

yoki

$$h_a = \frac{2}{a}\sqrt{p(p-b)(p-c)(p-a)}.$$

Shunga o'xshash:

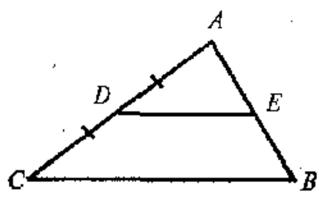
$$h_b = \frac{2}{b}\sqrt{p(p-a)(p-b)(p-c)}$$

va

$$h_c = \frac{2}{c}\sqrt{p(p-a)(p-b)(p-c)}.$$

Bu formulalar ABC uchburchak o'tmas burchakli bo'lganda ham shaklini o'zgartirmaydi.

2.4. Uchburchakning o'rta chizig'i. Uchburchakning ixtiyoriy ikki tomonining o'rtalarini tutashtiruvchi kesma uning *o'rta chizig'i* deb ataladi. Uchburchakda uchta o'rta chiziq bo'lib, ularning har biri uchinchi tomonga parallel va uning yarmiga teng bo'ladi (50- rasm):

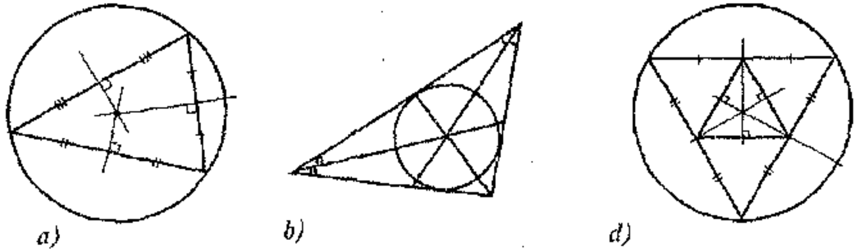


50-rasm.

$$DE \parallel BC, \ |DE| = \frac{1}{2}|BC|.$$

2.5. Uchburchakning to'rt ajoyib nuqtasi.

1- ajoyib nuqtasi medianalarning kesishish nuqtasidir. Uning ajoyibligi uchburchak tekisligining og'irlik markazi hisoblanadi.



51- rasm.

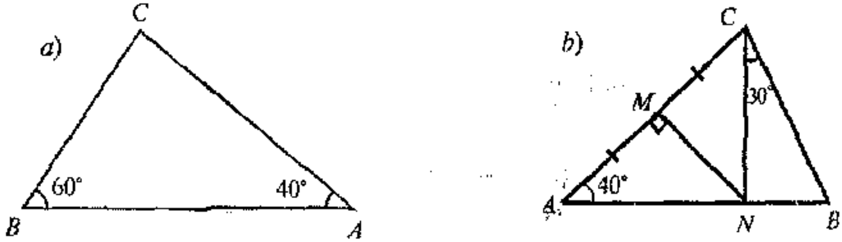
2- ajoyib nuqtasi uchburchak tomonlari o'rta perpendikularlarining kesishish nuqtasi. Bu nuqta uchburchakka tashqi chizilgan aylana markazidir (51- *a* rasm).

3- ajoyib nuqtasi uchburchakning uchala bissektrisasining kesishish nuqtasi. Bu nuqta uchburchakka ichki chizilgan aylana markazidir (51- *b* rasm).

4- ajoyib nuqtasi uchburchak balandliklarining kesishish nuqtasidir. Uni uchburchakning *ortomarkazi* deyilib, u uchburchakning har bir uchidan qarshi tomonga parallel chizilgan to'g'ri chiziqlardan hosil bo'lgan yangi uchburchakka tashqi chizilgan aylana markazi bo'ladi (51- *d* rasm).

1- m a s a l a. 52-*a* rasmdagi ABC uchburchakda ikki burchagi mos ravishda 60° va 40° ga teng bo'lsa, uning AB, AC va BC tomonlarini uzunliklarini o'sib borish tartibida yozing.

Y e c h i l i s h i. $\triangle ABC$ da uchinchi burchagi 80° bo'lgani uchun hamda har bir burchak qarshisidagi tomon uzunligi burchak kattaligiga proporsionalligini nazarda tutib, $|BC| < |AC| < |AB|$ ekanligini bilamiz.



52- rasm.

Javob: $|BC|$, $|AC|$, $|AB|$.

2-masala. 52- *b* rasmda $\triangle ABC$ da MN kesma AC tomonining o'rta perpendikulari ($N \in [AB]$). $\angle CAN = 40^\circ$, $\angle NCB = 30^\circ$ ekani ma'lum bo'lsa, $\triangle BCN$ ning perimetrini aniqlang, bunda $|AB| = c$, $|BC| = a$.

Yechilishi. $\triangle ANC$ teng yonli bo'lgani uchun $\angle A = \angle C = 40^\circ$. Demak, $\angle ACB = \angle ABC = 70^\circ$ ekanligidan $\triangle ABC$ teng yonlidir. $P_{\triangle CNB} = |CN| + |NB| + |BC|$; $|CN| = |AN|$, $BN = a - |AN|$. Shuning uchun $P_{\triangle CNB} = |AN| + a - |AN| + c = a + c$.

Javob: $a + c$.

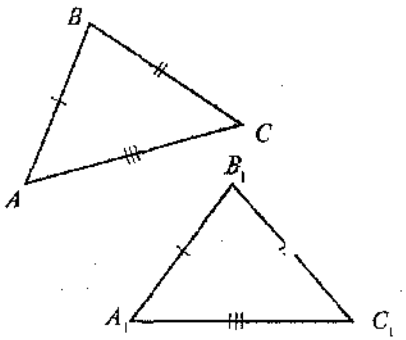
3- §. Uchburchaklarning tengligi

3.1. Uchburchaklarning tenglik alomatlari. ABC va $A_1B_1C_1$ uchburchaklarning o'zaro tengligini ularning chiziqli elementlari va burchaklarini taqqoslab bilish mumkin. Bunda taqqoslanayotgan mos elementlarining soni umumiy holda uchtadan kam bo'lmasligi, shu bilan birga ularning kamida bittasi chiziqli (ya'ni tomon, balandlik, ichki yoki tashqi aylana radiusi va hokazo) bo'lishi shart.

Uchburchaklar tengligining uch alomati quyida keltiriladi.

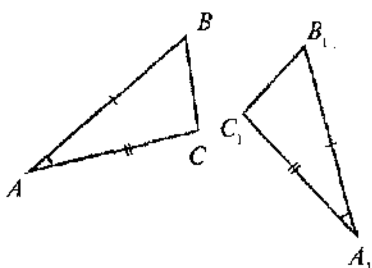
1. Agar bir uchburchakning uchta tomoni ikkinchi uchburchakning uchta tomoniga mos ravishda teng bo'lsa, bunday uchburchaklar o'zaro teng bo'ladi (53- rasm).

2. Agar bir uchburchakning ikki tomoni va ular orasidagi burchagi ikkinchi uchburchakning ikki tomoni va ular orasidagi burchagiga



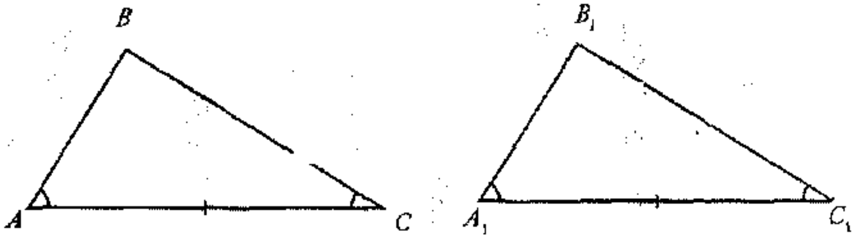
$$\left.\begin{aligned} |AB| &= |A_1B_1| \\ |BC| &= |B_1C_1| \\ |AC| &= |A_1C_1| \end{aligned}\right\} \Leftrightarrow \triangle ABC = \triangle A_1B_1C_1$$

53- rasm.



$$\left.\begin{aligned} |AB| &= |A_1B_1| \\ |AC| &= |A_1C_1| \\ \angle A &= \angle A_1 \end{aligned}\right\} \Leftrightarrow \triangle ABC = \triangle A_1B_1C_1$$

54- rasm.



$$\left.\begin{array}{l}|AC| = |A_1C_1| \\ \angle A = \angle A_1 \\ \angle C = \angle C_1\end{array}\right\} \Leftrightarrow \Delta ABC = \Delta A_1B_1C_1$$

55- rasm.

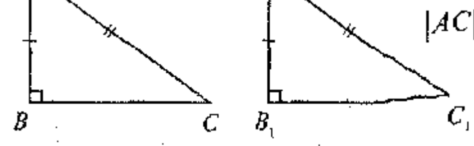
mos ravishda teng bo'lsa, bunday uchburchaklar teng bo'ladi (54- rasm).

3. ***Agar bir uchburchakning bir tomoni va unga yopishgan ikki*** **burchagi ikkinchi uchburchakning mos tomoni va unga yopishgan ikki burchagiga teng bo'lsa, bunday uchburchaklar teng bo'ladi** (55- rasm).

3.2. To'g'ri burchakli uchburchaklarning tenglik alomatlari. To'g'ri burchakli uchburchaklarning o'zaro teng yoki tengmasligini taqqoslashda ularning ikkitadan elementlarini taqqoslash kifoya (ma'lumki, elementlardan kamida bittasi chiziqli bo'lishi shart), chunki uchinchi element sifatida ularning to'g'ri burchaklari hisoblanadi.

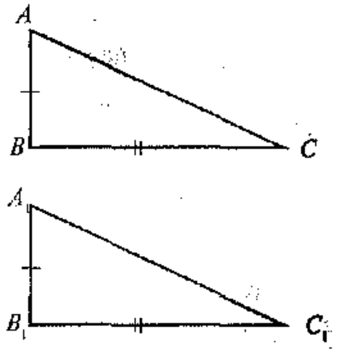
Masalan, to'g'ri burchakli uchburchakning gipotenuzasi va bir kateti ikkinchi to'g'ri burchakli uchburchakning gipotenuzasi va bir katetiga mos ravishda teng bo'lsa, bunday uchburchaklar o'zaro teng bo'ladi (56- rasm).

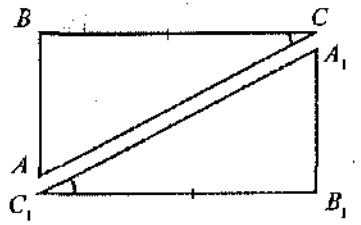
56-, 57-, 58- rasmlarda o'zaro teng to'g'ri burchakli uchburchaklar juftliklari tasvirlangan.



$$\left.\begin{array}{l}|AB| = |A_1B_1| \\ |AC| = |A_1C_1|\end{array}\right\} \Leftrightarrow \Delta ABC = \Delta A_1B_1C_1$$

56- rasm.





$$\left.\begin{array}{l}|AB| = |A_1B_1| \\ |AC| = |A_1C_1|\end{array}\right\} \Leftrightarrow \Delta ABC = \Delta A_1B_1C_1 \qquad \left.\begin{array}{l}|BC| = |B_1C_1| \\ \angle C = \angle C_1\end{array}\right\} \Leftrightarrow \Delta ABC = \Delta A_1B_1C_1$$

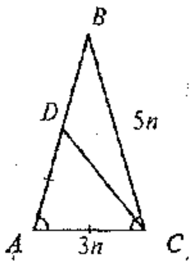
57- rasm. **58- rasm.**

1- m a s a l a. Teng yonli ABC uchburchakda $\angle A = \angle C$, $|AB| : |AC| = 5 : 3$ kabi va $|AB| - |AC| = 3$. Uchburchakning perimetrini hisoblang (59- rasm).

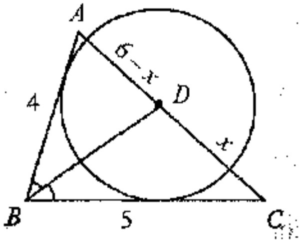
Y e c h i l i s h i. $|AC| = |AD|$; $|AB| - |AD| = 5n - 3n = 2n$; $2n = 3$; $n = 1{,}5$. $p = 2 \cdot 5n + 3n = 13n = 13 \cdot 1{,}5 = 19{,}5$.

J a v o b: 19,5.

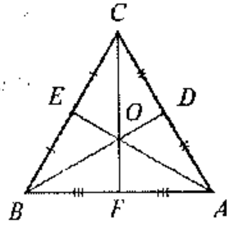
2- m a s a l a. 60- rasmdagi ABC uchburchakning AB, BC va AC tomonlari mos ravishda 4, 5 va 6 ga teng. AB va BC tomonlarga urinadigan aylananing markazi D nuqta AC tomonda ajratgan kesmalarning uzunliklari ko'paytmasini toping.



59- rasm.



60- rasm.



61- rasm.

Yechilishi. Burchak tomonlariga urinuvchi aylana markazi shu burchak bissektrisasida yotadi. ABC uchburchakning B burchagi bissektrisasi xossasiga ko'ra: $\frac{6-x}{x}=\frac{4}{5} \Rightarrow 30-5x=4x \Rightarrow x=\frac{30}{9}$;

$6-x=6-3\frac{1}{3}=2\frac{2}{3}$. $(6-x)x=\frac{8}{3}\cdot\frac{10}{3}=8\frac{8}{9}$.

Javob: $8\frac{8}{9}$.

3- masala. 61- rasmdagi ABC uchburchakning BD, AE va CF medianalari O nuqtada kesishadi. AOD uchburchakning yuzi 2,8 ga teng, BFC uchburchakning yuzini toping.

Yechilishi. Ma'lumki, uchburchakning uchala medianasi uni 6 ta bir-birini qoplamaydigan bir xil yuzli (tengdosh) uchburchaklarga ajratadi. Shunga ko'ra: $S_{\Delta BFC}=3S_{\Delta AOD}=3\cdot 2{,}8=8{,}4$.

Javob: 8,4.

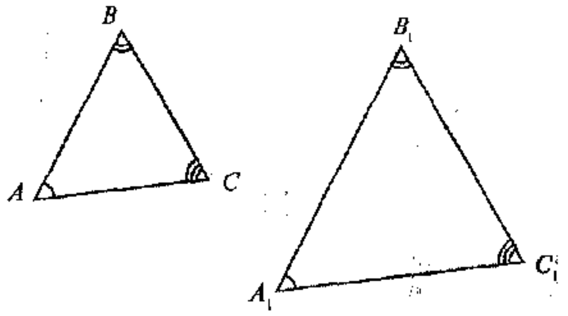
4- §. Uchburchaklarning o'xshashligi

4.1. O'xshashlik almashtirishi. Agar ABC uchburchakni $A_1B_1C_1$ uchburchakka almashtirishda nuqtalar orasidagi masofalar bir xil son marta o'zgarsa, bunday almashtirish *o'xshashlik almashtirishi* deyiladi va bu holda ABC uchburchak $A_1B_1C_1$ uchburchakka o'xshash deyilib, $\Delta ABC \sim \Delta A_1B_1C_1$ kabi ifodalanadi. Bunday almashtirish natijasida ABC uchburchakning ixtiyoriy X va Y nuqtalari $A_1B_1C_1$ uchburchakning X_1 va Y_1 nuqtalariga almashgan bo'lsa, u holda $|X_1Y_1|=k\cdot|XY|$ bo'ladi, bunda k soni *o'xshashlik koeffitsiyenti* deyilib, u ABC va $A_1B_1C_1$ uchburchaklardan biri ikkinchisidan k marta katta yoki kichikligini bildiradi (62- rasm):

$$|A_1B_1|=k\cdot|AB|;$$
$$|A_1C_1|=k\cdot|AC|;$$
$$|B_1C_1|=k\cdot|BC|;$$

$$k=\frac{|A_1B_1|}{|AB|}=\frac{|A_1C_1|}{|AC|}=\frac{|B_1C_1|}{|BC|};\quad \Delta ABC\sim\Delta A_1B_1C_1.$$

O'xshash uchburchaklarda burchaklar bir-biriga teng bo'lib ($\angle A=\angle A_1$; $\angle B=\angle B_1$; $\angle C=\angle C_1$), o'zaro teng burchaklar qarshi-



62-rasm.

sidagi tomonlar mos ravishda proporsional $\left(\frac{|AB|}{|A_1B_1|}=\frac{|AC|}{|A_1C_1|}=\frac{|BC|}{|B_1C_1|}\right)$ bo'ladi.

4.2. Uchburchaklar o'xshashligining uch alomati. Ikki uchburchakning o'zaro o'xshash, yoki o'xshash emasligini tekshirish uchun alohida alomatlar mavjud.

1- alomat. **Agar bir uchburchakning ikki burchagi ikkinchi uchburchakning ikki burchagiga teng bo'lsa, bunday uchburchaklar o'xshash bo'ladi.**

2- alomat. **Agar bir uchburchakning ikki tomoni ikkinchi uchburchakning ikki tomoniga proporsional bo'lsa va bu tomonlar tashkil etgan burchaklar teng bo'lsa, bunday ikkita uchburchak o'xshash bo'ladi.**

3- alomat. **Agar bir uchburchakning tomonlari ikkinchi uchburchak tomonlariga proporsional bo'lsa, bunday ikkita uchburchak o'xshash bo'ladi.**

Quyida ikki o'xshash uchburchakning mos tomonlaridan proporsiya tuzish uslubiga to'xtalamiz.

1. O'xshash uchburchaklarda o'zaro teng bo'lgan burchaklar aniqlanadi.
2. Teng burchaklar qarshisidagi tomonlar o'zaro proporsional hisoblanadi.
3. Bir-biriga teng nisbatlarni (proporsiyalarni) tuzishda kichik kesmaning kattaga yoki kattaning kichikka nisbati olinganda unga teng nisbatni tuzishda ham shuni nazarda tutish lozim.

4.3. To'g'ri burchakli uchburchaklarning o'xshashligi. To'g'ri burchakli uchburchakda bitta burchagi to'g'riligini inobatga olib, uchburchaklar o'xshashligining uch alomatini to'g'ri burchakli uchburchakka moslashtirish mumkin. Masalan:

1. To'g'ri burchakli uchburchaklar o'xshash bo'lishi uchun ularning bittadan o'tkir burchaklari teng bo'lishi yetarli.

2. To'g'ri burchakli uchburchaklarning katetlari mos ravishda proporsional bo'lsa, ular o'xshash bo'ladi.

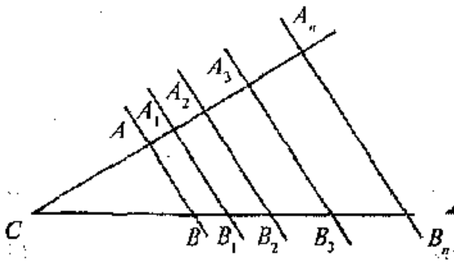
3. To'g'ri burchakli uchburchaklarning bir kateti bilan gipotenuzasi ikkinchi to'g'ri burchakli uchburchakning bir kateti bilan gipotenuzasiga proporsional bo'lsa, ular o'zaro o'xshash bo'ladi.

4.4. Proporsional kesmalar. Ikki kesmaning *nisbati* deb shu kesmalarning uzunliklari nisbatiga aytiladi.

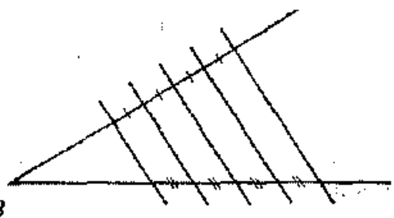
Agar ma'lum AB, CD, EF, MN kesmalar uchun: $\frac{|AB|}{|CD|}=\frac{|EF|}{|MN|}$ nisbatlarining tengligidan AB va CD kesmalar EF va MN kesmalarga *proporsional* deyiladi. Buning ma'nosi: AB kesma uzunligi CD kesma uzunligidan qancha marta katta bo'lsa, EF kesma uzunligi ham MN kesma uzunligidan shuncha marta katta bo'ladi.

T e o r e m a (Fales teoremasi). **Yoyiq burchakdan farqli burchak tomonlarini kesuvchi o'zaro parallel to'g'ri chiziqlar burchak tomonlarida proporsional kesmalar ajratadi** (63- rasm).

63- rasmda CA kesma CB kesmaga, A_1A_2 kesma B_1B_2 ga va hokazo proporsional hisoblanadi. Ya'ni $\frac{|CA|}{|CB_1|}=\frac{|A_1A_2|}{|B_1B_2|}=\frac{|A_2A_3|}{|B_2B_3|}=\ldots$ tengliklar o'rinlidir.



63- rasm.



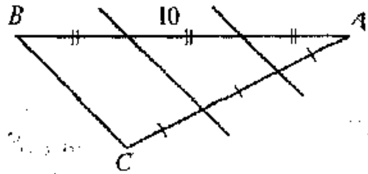
64- rasm.

Agar kesuvchi parallel to‘g‘ri chiziqlar orasidagi masofalar teng bo‘lsa, burchak tomonlarida hosil bo‘ladigan kesmalar ham (bir tomonda yotuvchi) bir xil uzunlikda bo‘ladi (64- rasm). Chunki proporsiyani tashkil etuvchi nisbatlar suratlarining (yoki maxrajlarining) bir-biriga tengligidan maxrajlari (suratlari) tengligi ham kelib chiqadi.

Yuqoridagi so‘nggi tushunchadan ixtiyoriy kesmani teng bo‘laklarga bo‘lish uslubi kelib chiqadi.

1- m a s a l a. *Uzunligi 10 birlik* bo‘lgan *AB* kesmani teng uch bo‘lakka ajrating.

Y e c h i l i s h i. *AB* kesmani biror uchidan chiqarilgan, *AB* to‘g‘ri chiziqda yotmagan *AC* nurda, *A* nuqtadan boshlab ketma-ket uchta bir xil uzunlikdagi kesmalar qo‘yib, uchinchi kesmaning *C* oxirini *B* nuqta bilan tutashtiramiz va *AC* ning bo‘linish nuqtalaridan *BC* ga parallel to‘g‘ri chiziqlar chizamiz (65- rasm), ular *AB* kesmani uchta teng bo‘lakka bo‘ladi. Bu kesmalarning har biri *AB* ning uchdan biriga tengligi ravshan.

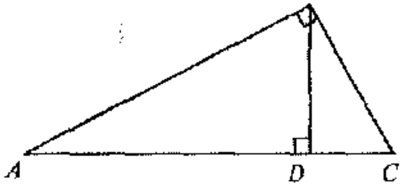


65- rasm.

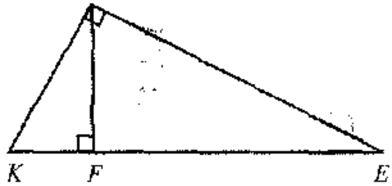
4.5. To‘g‘ri burchakli uchburchakdagi proporsional kesmalar.

To‘g‘ri burchakli uchburchakning gipotenuzasiga tushirilgan balandlik uni shunday ikki bo‘lakka ajratadiki, ularning har biri mos katetning gipotenuzadagi proyeksiyasi deyiladi. 66- rasmda *AD* kesma *AB* katetning proyeksiyasi, *DC* kesma esa *BC* katetning proyeksiyasidir.

1- t e o r e m a. **To‘g‘ri burchakli uchburchakda katetning uzunligi butun gipotenuza va undagi shu katet proyeksiyasi uzunliklari orasida o‘rta proporsional miqdordir** (66- rasm):



66- rasm.



67- rasm.

$$|AB|^2 = |AC| \cdot |AD| \text{ ва } |BC|^2 = |AC| \cdot |CD|.$$

2- t e o r e m a. **To‘g‘ri burchakli uchburchakning gipotenuzaga tushirilgan balandligining uzunligi gipotenuzada hosil bo‘lgan kesmalar uzunliklari orasida o‘rta proporsional miqdordir** (67- rasm):

$$LF^2 = KF \cdot FE.$$

Bu teoremalarning natijalari sifatida bir qancha proporsiyalar tuzib, to‘g‘ri burchakli uchburchakda proporsional kesmalarni ko‘rish mumkin:

$$|BC|^2 = |AC| \cdot |DC| \Leftrightarrow \frac{|BC|}{|AC|} = \frac{|DC|}{|BC|} \Leftrightarrow \frac{|BC|}{|DC|} = \frac{|AC|}{|BC|} \text{ kabi.}$$

$|AB|^2 = |AD| \cdot |AC|$ va $|LF|^2 = |KF| \cdot |FE|$ tengliklardan kerakli proporsiyalar tuzib, ulardagi noma’lum kesma uzunligi topilib, masalalar yechiladi.

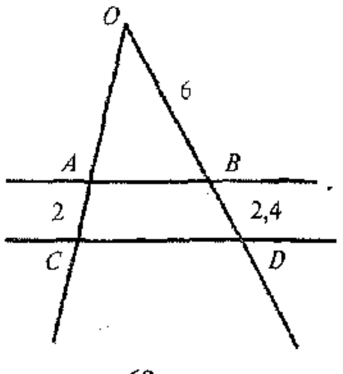
Umuman, keltirilgan teoremalarning isboti chizmadagi o‘xshash uchburchaklardan foydalanishga asoslangan.

1- m a s a l a. $(AB) \parallel (CD)$; $|OB| = 6$; $|BD| = 2{,}4$; $|AC| = 2$ bo‘lsa, $|OA|$ — ? (68- rasm).

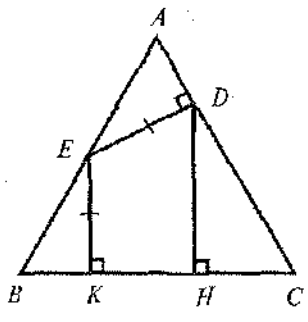
Y e c h i l i s h i. OCD va OAB uchburchaklarning o‘xshashligidan:

$$\frac{|OD|}{|OB|} = \frac{|OC|}{|OA|} \Rightarrow \frac{8{,}4}{6} = \frac{|OA| + 2}{|OA|} \Leftrightarrow 1{,}4\ |OA| = |OA| + 2 \Rightarrow |OA| = 5.$$

J a v o b: 5.



68- rasm.



69- rasm.

2- m a s a l a. Teng tomonli ABC uchburchakda (69- rasm) $|EK| = |ED|$; $(EK) \perp (BC)$; $(DH) \perp (BC)$, $\angle DEK = 120°$; $DC = 4\sqrt{3}$ sm bo'lsa, $|AD|$ necha santimetr?

Y e c h i l i s h i . Agar berilgan muntazam uchburchakning tomonini a bilan belgilasak, $\Delta BEK = \Delta EDA$ ekanligidan $|BE| = |EA|$. ΔEDA da 30° li burchak qarshisidagi tomon $|AD| = \frac{|AE|}{2} = \frac{a}{4}$ ekanligi aniqlanadi. Demak, $\frac{a}{4} = \frac{1}{3} \cdot 4 \cdot \sqrt{3} = \frac{4\sqrt{3}}{3}$.

J a v o b: $\frac{4\sqrt{3}}{3}$.

3- m a s a l a. To'g'ri burchakli uchburchakning gipotenuzasi 13 ga, gipotenuzaga tushirilgan balandlik 6 ga teng. Katta katetning gipotenuzadagi proyeksiyasini toping.

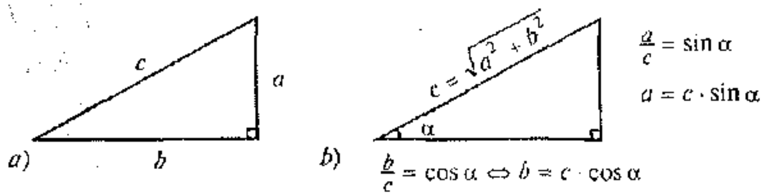
Y e c h i l i s h i. To'g'ri burchakli uchburchakda gipotenuzaga tushirilgan balandlik gipotenuzada hosil bo'lgan kesmalar orasida o'rta proporsional miqdordir. Shunga asosan, $6^2 = x(13 - x) \Rightarrow 36 = 13x - x^2 \Rightarrow x^2 - 13x + 36 = 0 \Rightarrow x = 4$.

J a v o b: $x = 4$.

5- §. Uchburchakdagi metrik munosabatlar

Uchburchakka oid masalalarni yechishda masala shartida berilgan va so'ralgan elementlarni bog'lovchi formulalar, tushunchalar *metrik munosabatlar* deyiladi.

Uchburchakning so'ralgan noma'lum elementlarini topish uchun uning 3 ta elementi (ulardan kamida bittasi chiziqli) berilgan bo'lishi lozim. Xususiy hollarda, ya'ni teng yonli yoki to'g'ri burchakli uchburchaklarga doir masalalarda 2 ta elementi, muntazam uchburchaklarga oid masalalarda 1 ta elementi berilsa yetarli.



70- rasm.

5.1. To'g'ri burchakli uchburchakdagi metrik munosabatlar. Ma'lumki, to'g'ri burchakli uchburchak gipotenuzasining kvadrati katetlar kvadratlarining yig'indisiga teng (70- *a* rasm).

To'g'ri burchakli uchburchak tomonlarining nisbati e'tiborga loyiq bo'lib, ular alohida-alohida nomlangan: $\frac{a}{c}$ nisbat α burchakning *sinusi*, $\frac{b}{c}$ nisbat esa shu burchakning *kosinusi*, $\frac{a}{b}$ nisbat α burchakning *tangensi*, $\frac{b}{a}$ nisbat esa α burchakning *kotangensi* deb ataladi va quyidagicha yoziladi:

$$\frac{a}{c} = \sin\alpha \iff a = c\cdot\sin\alpha; \quad \frac{b}{c} = \cos\alpha \iff b = c\cdot\cos\alpha;$$

$$\frac{a}{b} = \operatorname{tg}\alpha \iff a = b\cdot\operatorname{tg}\alpha; \quad \frac{b}{a} = \operatorname{ctg}\alpha \iff b = a\cdot\operatorname{ctg}\alpha \quad (70\text{- } b \text{ rasm})$$

kabi ifodalanadi va ulardan masalalar echishda keng foydalaniladi.

1- m a s a l a. To'g'ri burchakli uchburchakning c gipotenuzasi va o'tkir burchagi α berilgan. Katetlarni, ularning gipotenuzaga tushirilgan proyeksiyalarini va gipotenuzaga tushirilgan balandlikni toping (71- rasm).

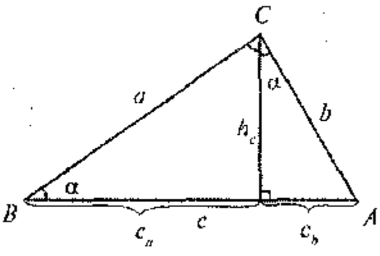
Y e c h i l i s h i . Katetlari: $a = c\cdot\cos\alpha$; $b = c\cdot\sin\alpha$. Katetlarning gipotenuzadagi proyeksiyalari:

$$c_a = a\cdot\cos\alpha; \quad c_b = b\cdot\sin\alpha;$$

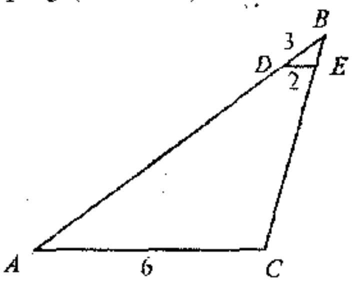
balandlik: $h_c^2 = a\cdot b\cos\alpha\cdot\sin\alpha \Rightarrow h_c = \sqrt{\frac{a\cdot b\sin 2\alpha}{2}}$.

J a v o b : $a = c\cdot\cos\alpha$, $b = c\cdot\sin\alpha$, $h_c = \sqrt{\frac{a\cdot b\sin 2\alpha}{2}}$

2- m a s a l a. ABC uchburchakda $|AC| = 6$, $|DB| = 3$ va $DE = 2$ $(AC \parallel DE)$. AB tomonining uzunligini toping (72- rasm).



71- rasm.



72- rasm.

Y e c h i l i s h i . *ABC* va *BDE* uchburchaklarning o'xshashligidan:

$$\frac{|AC|}{|DE|}=\frac{|AB|}{|DB|} \Rightarrow \frac{6}{2}=\frac{|AB|}{3} \Rightarrow |AB|=9.$$

J a v o b: $|AB|=9$.

5.2. Kosinuslar teoremasi. Ikki tomoni (a va b) va ular orasidagi burchagi (α) berilgan uchburchakning uchinchi tomonini topishda quyidagi teoremadan foydalaniladi.

T e o r e m a (kosinuslar teoremasi). **Uchburchakning istalgan tomonining kvadrati qolgan ikki tomoni kvadratlari yig'indisidan shu ikki tomon bilan ular orasidagi burchak kosinusining ikkilangan ko'paytmasini ayirish natijasiga teng:**

$$a^2=b^2+c^2-2bc\ \cos\alpha,$$
$$b^2=a^2+c^2-2ac\ \cos\beta,$$
$$c^2=a^2+b^2-2ab\ \cos\gamma.$$

E s l a t m a. Burchakning o'tkir yoki o'tmasligiga, ya'ni burchak kosinusining musbat yoki manfiyligiga bog'liq ravishda b^2+c^2 va $2bc\cos\alpha$ lar orasidagi ishora (+) yoki (–) bo'lishi mumkin.

3- m a s a l a. Uchburchakning b va c tomonlari orasidagi burchak 30° ga teng. Uchburchakning uchinchi tomoni 12 ga teng bo'lsa hamda uning tomonlari $c^2=b^2+12b+144$ shartni qanoatlantirsa, c ning qiymatini toping.

Y e c h i l i s h i . Kosinuslar teoremasidan γ burchakni topamiz: $\alpha=30°$; $a=12$; $c^2=b^2+a^2-2ab\cos\gamma=b^2+144+12b \Rightarrow -2\cos\gamma=1 \Rightarrow$

$\Rightarrow \cos\gamma=-\frac{1}{2} \Rightarrow \gamma=120°$. $\alpha=30°$; $\gamma=120°$ bo'lgani uchun $\beta=30°$, ya'ni uchburchak teng yonli, bundan $a=b=12$ kelib chiqadi. Demak, $c^2=a^2+a^2+12a=3\cdot144$; $c=12\sqrt{3}$.

J a v o b: $12\sqrt{3}$.

5.3. Sinuslar teoremasi. Masalalar yechishda ma'lum elementlar sifatida uchburchakning ikki burchagi va bir tomoni ma'lum bo'lib, qolganlarini topish talab qilinsa, «sinuslar teoremasi» dan foydalanish qulay.

T e o r e m a (sinuslar teoremasi). **Uchburchakning tomonlari qarshisidagi burchaklarning sinuslariga proporsional:**

$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}.$$

4- masala. ABC uchburchakda $\angle BAC=45°$; $\angle ACB=30°$ va $|BC|=14\sqrt{2}$ ga teng. AB tomon uzunligini toping.

Yechilishi. Sinuslar teoremasiga ko'ra:

$$\frac{c}{\sin C}=\frac{a}{\sin A}\Rightarrow c=a\cdot\frac{\sin C}{\sin A}\Rightarrow c=\frac{\sin 30°}{\sin 45°}\cdot a;$$

$$c=\frac{\frac{1}{2}}{\frac{\sqrt{2}}{2}}\cdot 14\cdot\sqrt{2}=14;\ \ c=14.$$

Javob: 14.

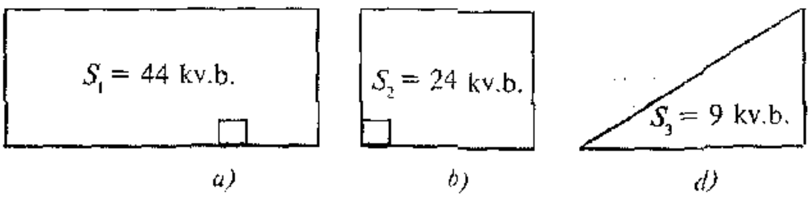
Topshiriq. $\frac{a}{\sin\alpha}=\frac{a}{\sin\beta}=\frac{a}{\sin\gamma}=2R$ (R uchburchakka tashqi chizilgan aylana radiusi) ekanligini isbotlang.

6- §. Uchburchak yuzini hisoblash formulalari

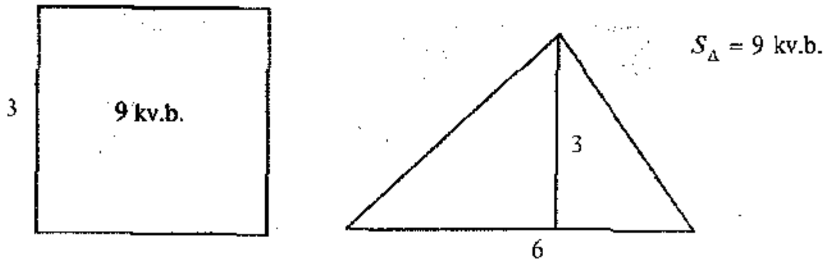
6.1. Yuz tushunchasi. Geometrik shakllarning yuzi (sathi, sirti) qanday o'lchanadi? Shakllarning yuzlari qanday taqqoslanadi? Quyida shu savollarga javob olasiz.

Biror ixtiyoriy shakl tanlab (73- *a* rasm), uning yuzini topish masalasini ko'raylik.

O'lchov birligi sifatida biror kvadrat tanlanib, shunday kvadratlarni bir-biriga taqab soha sirtiga «qo'yib» chiqiladi, natijada sirtni to'liq qoplagan barcha kvadratlar soni shaklning yuzini ifodalovchi



73- rasm.



74- rasm.

sonni bildiradi. Masalan, 73- rasmdagi *a* shaklning S_1 yuzi taxminan 44 kvadrat birlikka, 73- *b* shaklning S_2 yuzi 24 kvadrat birlikka, 73- *d* shaklning yuzi esa taqriban 9 kvadrat birlikka tengligi ko'rsatilgan. Yuzlarni o'lchashda birlik kvadrat tomoni 1 santimetr bo'lsa, santimetr kvadrat va h.k. o'lchov birliklardan foydalanish mumkin.

Demak:

1. Har qanday shaklning yuzi uni qoplaydigan birlik kvadratlar soni bilan ifodalanadi.
2. Teng shakllar teng yuzlarga ega.
3. Agar shakl bir-birini qoplamaydigan shakllardan tashkil topgan bo'lsa, uning yuzi tashkil qiluvchi shakllar yuzlarining yig'indisiga teng.

Teng yuzlarga ega bo'lgan shakllar tengdosh shakllar deyiladi. Yuzlar odatda S harfi bilan ifodalanadi. 74- rasmda kvadrat bilan uchburchak o'zaro tengdosh, chunki ularning yuzlari teng.

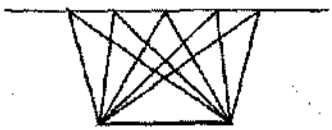
6.2. Uchburchakning yuzi. Uchburchakning balandlik tushirilgan tomonini odatda uchburchakning *asosi* deyiladi.

T e o r e m a. **Uchburchakning yuzi uning asosi bilan balandligi ko'paytmasining yarmiga teng:**

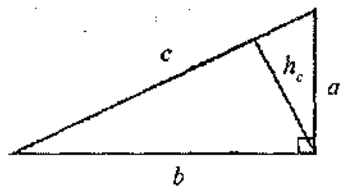
$$S_\Delta = \frac{1}{2} a \cdot h_a = \frac{1}{2} b \cdot h_b = \frac{1}{2} c \cdot h_c.$$

1- n a t i j a. Bir xil asosli va bir xil balandlikka ega bo'lgan barcha uchburchaklar o'zaro *tengdosh uchburchaklar* deyiladi (75- rasm).

To'g'ri burchakli uchburchakning yuzi uning katetlari ko'paytmasining yarmiga yoki gipotenuzasi bilan unga tushirilgan balandlik ko'paytmasining yarmiga teng (76- rasm):



75- rasm.



76- rasm.

$$S_{\Delta ABC} = \frac{1}{2} a \cdot b = \frac{1}{2} c \cdot h_c .$$

Uchta tomoni uzunliklari bo'yicha uchburchakning yuzi Geron formulasi orqali topiladi:

$$S = \sqrt{p(p-a)(p-b)(p-c)}, \text{ бунда } p = \frac{a+b+c}{2}.$$

Ikki tomoni uzunliklari va ular orasidagi burchak kattaligiga ko'ra uchburchakning yuzi $h_b = a \cdot \sin\alpha$ bo'lgani uchun $S_\Delta = \frac{1}{2} b \cdot h_b = = \frac{1}{2} a \cdot b \sin\alpha$ formuladan topiladi (77- rasm).

1- masala. Yuzi S, tomonlarining uzunliklari a, b va c bo'lgan uchburchakka ichki chizilgan aylana radiusi r ni toping (78- rasm).

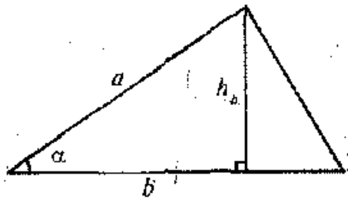
Yechilishi. Uchburchakning uchala tomoniga urinuvchi aylana unga *ichki chizilgan aylana* deyiladi. Berilgan ABC uchburchakning yuzi $S_{ABC} = S_{AOC} + S_{AOB} + S_{BOC}$ bo'lib, bundan $S_{ABC} = \frac{1}{2} b \cdot r + \frac{1}{2} c \cdot r + \frac{1}{2} a \cdot r = \frac{1}{2} r(b+c+a)$. Demak: $S = \frac{1}{2} r(a+b+c) \Rightarrow$

$$\Rightarrow r = \frac{2S}{a+b+c}.$$

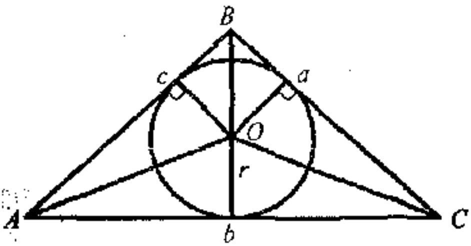
Javob: $r = \frac{2S}{a+b+c}$.

2- masala. Tomonlarining uzunliklari a, b va c bo'lgan uchburchakka tashqi chizilgan aylana radiusi $R = \frac{a \cdot b \cdot c}{4S}$ ekanini isbotlang (79- rasm).

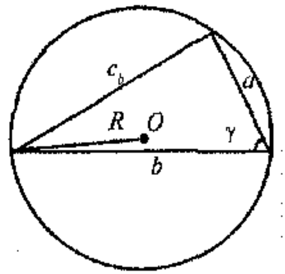
Isboti: Uchburchakning uchala uchidan o'tuvchi aylana uchburchakka *tashqi chizilgan aylana* deyiladi. Ma'lumki,



77- rasm.



78- rasm.



79- rasm.

$S = \frac{1}{2} a \cdot b \cdot \sin\gamma$. Sinuslar teoremasiga ko'ra $\frac{c}{\sin\gamma} = 2R \Rightarrow \sin\gamma = \frac{c}{2R}$ bo'lgani uchun $S = \frac{1}{2} a \cdot b \cdot \frac{c}{2R} \Rightarrow R = \frac{a \cdot b \cdot c}{4S}$.

1–2- masalalarda keltirib chiqarilgan formulalar ixtiyoriy ko'rinishdagi uchburchaklar uchun o'rinli bo'lib, quyida beriladigan formulalar to'g'ri burchakli, muntazam uchburchaklar uchun o'rinli ekanligini yodda tutish foydalidir.

To'g'ri burchakli uchburchakka ichki chizilgan aylana radiusi

$$r = \frac{a + b - c}{2},$$

tashqi chizilgan aylana radiusi

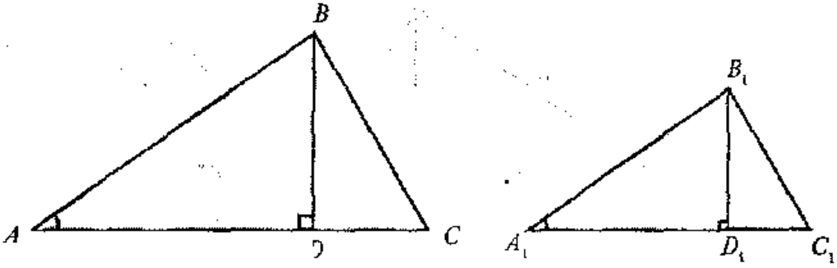
$$R = \frac{c}{2},$$

bunda c — gipotenuza.

Teng tomonli uchburchak uchun esa:

$$r = \frac{a\sqrt{3}}{6}, \quad R = \frac{a\sqrt{3}}{3},$$

bunda a — uchburchak tomoni.



80- rasm.

Muntazam uchburchakka tashqi chizilgan aylana radiusi ichki chizilgan aylana radiusidan ikki marta uzundir: $R = 2r$.

Shu bilan birga b — bissektrisa, m — mediana, h — balandligi bo'lsa,

$$r = \frac{1}{3}b = \frac{1}{3}m = \frac{1}{3}h; \quad R = \frac{2}{3}b = \frac{2}{3}m = \frac{2}{3}h$$

munosabatlar ham o'rinlidir.

6.3. O'xshash uchburchaklar yuzlarining nisbati.

Teorema. **Bittadan burchaklari teng bo'lgan ikki uchburchak yuzlarining nisbati teng burchaklarni tashkil etgan tomonlar uzunliklarining ko'paytmalari nisbati kabidir.**

80- rasmdagi ABC va $A_1B_1C_1$ uchburchaklarda $\angle A = \angle A_1$ bo'lib,

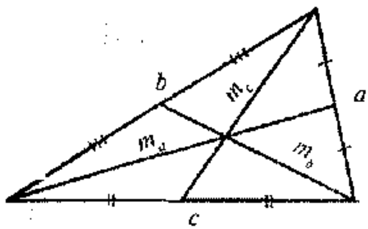
$$\frac{S_{ABC}}{S_{A_1B_1C_1}} = \frac{AC \cdot AB}{A_1C_1 \cdot A_1B_1}.$$

Teorema. **O'xshash uchburchaklar yuzlarining nisbati ularning chiziqli elementlarining uzunliklari kvadratlari nisbatiga teng:**

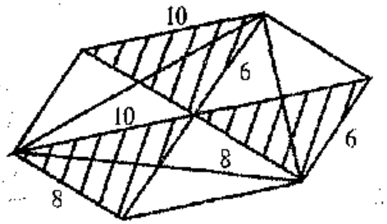
$$\frac{S_{ABC}}{S_{A_1B_1C_1}} = \frac{|AB|^2}{|A_1B_1|^2} = \frac{|AC|^2}{|A_1C_1|^2} = \frac{|BC|^2}{|B_1C_1|^2} = \frac{r^2}{R^2}.$$

1- masala. Medianalarining uzunliklari mos ravishda 9, 12 va 15 bo'lgan uchburchak yuzini toping (81- rasm).

Yechilishi. Masalaning yechimini topish uchun doim tayyor metrik munosabatlar bo'lavermaydi. Bunday hollarda masalani tahlil qilib, ma'lum va noma'lumlar orasidagi munosabatdan foydalanib, masalani yechish yo'li aniqlanadi. $m_c = 9$, $m_b = 12$, $m_a = 15$ lar



81- rasm.



82- rasm.

bilan uchburchak tomonlarini chizmada bog'lashga harakat qilaylik (82- rasm). Medianalarning umumiy kesishish nuqtasida hosil bo'ladigan bo'laklarining xossalarini bilgan holda 82- rasmdagi shtrixlangan bir-biriga teng uchburchaklar yuzlarining yig'indisi berilgan uchburchak yuziga tengligini asoslash mumkin. Geron formulasiga asosan tomonlari 6, 8, 10 bo'lgan uchburchakning yuzi 24 kv. b; $3 \cdot 24 = 72$ kv. b.

Javob: 72.

2- masala. Yuzi 48 ga teng bo'lgan ABC uchburchakning AC tomoni D nuqtada $|AD| : |DC| = 1 : 7$ kabi nisbatda bo'linadi. ABD uchburchakning yuzini toping.

Yechilishi. Yuzi 48 ga teng bo'lgan uchburchakning AC tomoni 8 ta teng bo'lakka bo'lingani uchun bo'linish nuqtalarini qarshi B uchi bilan tutashtirish natijasida 8 ta tengdosh uchburchaklar hosil bo'ladi. Ulardan biri bo'lgan ABD uchburchak yuzi $48 : 8 = 6$ ga tengdir.

Javob: 6.

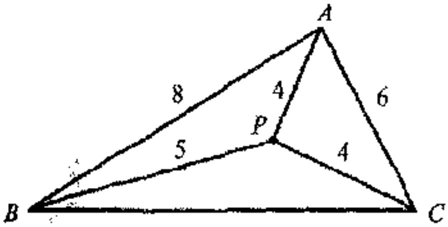
Mustaqil ishlash uchun test topshiriqlari

1. 83- rasmda $|AB| = 8$, $|AC| = 6$, $|BP| = 5$, $|AP| = |PC| = 4$. P uchburchak ichidagi bir nuqta bo'lsa, $|BC|$ ning eng katta butun son *qiymatini toping.*

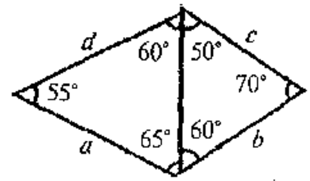
A) 5; B) 6; C) 7; D) 8; E) 9.

2. Tomonlarining uzunligi butun son bilan ifodalangan, bir tomoni 6 sm va qolgan ikki tomoni yig'indisi 15 sm ga teng bo'lgan nechta turli uchburchak yasash mumkin?

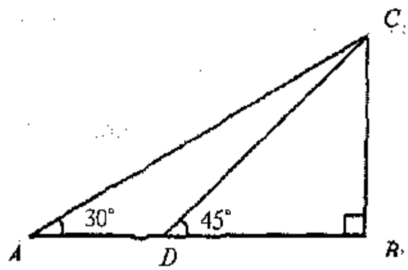
A) 2; B) 3; C) 4; D) 5; E) 6.



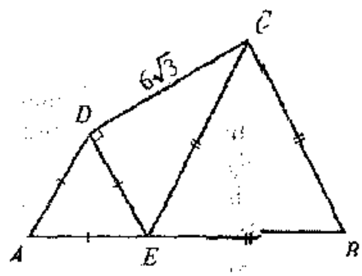
83- rasm.



84- rasm.



85- rasm.



86- rasm.

3. 84- rasmdagi to‘rtburchakning eng katta va eng kichik tomonlarini aniqlang.

A) a, b; B) d, b; C) d, a; D) a, c; E) b, c.

4. To‘g‘ri burchakli ABC uchburchakda $[AB] \perp [BC]$, $\angle A = 30°$, $\angle CDB = 45°$ bo‘lsa, $\frac{|AC|}{|DC|}$ ni aniqlang (85- rasm).

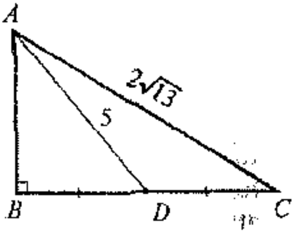
A) $\sqrt{2}$; B) $\sqrt{3}$; C) 2; D) 3; E) 1.

5. AED va BCE teng tomonli uchburchaklarda $[DE] \perp [DC]$ va $|DC| = 6\sqrt{3}$ sm bo‘lsa (86- rasm), $|AB|$ necha santimetr?

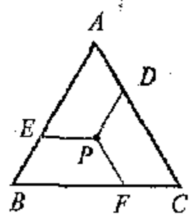
A) 12 sm; B) 15 sm; C) 18 sm; D) 20 sm; E) 24 sm.

6. 87- rasmda $|BD| = |DC|$, $|AD| = 5$ см, $|AC| = 2\sqrt{13}$ см bo‘lsa, $|AB|$ necha santimetr?

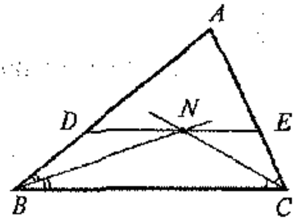
A) 8 sm; B) 6 sm; C) 3 sm; D) 4 sm; E) 2 sm.



87- rasm.



88- rasm.



89- rasm.

7. Tomoni 6 sm bo'lgan muntazam uchburchak ichidagi P nuqtadan (88- rasm) $(AB) \parallel (PD), (BC) \parallel (PE), (AC) \parallel (FP)$ lar o't-kazilgan. $|PD|+|PE|+|PF|$ yig'indi nimaga teng?

A) 12; B) 6; C) $6\sqrt{3}$; D) $3\sqrt{3}$; E) 3.

8. ABC uchburchakda N bissektrisalarining kesishgan nuqtasi, $(DE) \parallel (BC)$ va $|DB|+|EC|=8$ bo'lsa, $|DE|$ ni toping (89- rasm).

A) 4; B) 5; C) 6; D) 7; E) 8.

9. Tomonining uzunligi 12 sm bo'lgan muntazam uchburchak ichidagi ixtiyoriy nuqtadan uning tomonlarigacha bo'lgan masofalar yig'indisi necha santimetr?

A) 12 sm; B) $12\sqrt{3}$ sm; C) 6 sm; D) $6\sqrt{3}$ sm; E) $4\sqrt{3}$ sm.

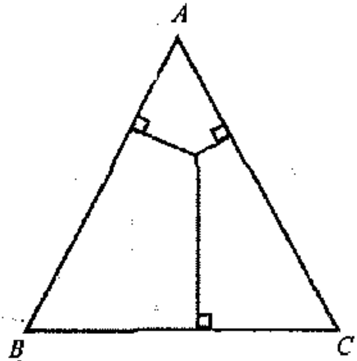
10. Musbat butun x son uchun, tomonlarining uzunliklari $x+8, 7-x$ va 10 bo'lgan nechta turli uchburchak yasash mumkin?

A) 10; B) 9; C) 7; D) 5; E) 4.

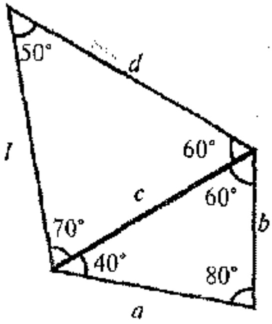
11. Muntazam uchburchakning ichidagi ixtiyoriy nuqtadan uning tomonlarigacha bo'lgan masofalar yig'indisi $\sqrt{3}$ ga teng. Uchburchakning yuzini toping (90- rasm).

A) 4; B) 3; C) $\sqrt{3}$;

D) $\frac{3\sqrt{3}}{4}$; E) Aniqlab bo'lmaydi.



90- rasm.



91- rasm.

12. 91- rasmdagi shaklga qarab eng uzun tomonni aniqlang.

A) *a*; B) *b*; C) *c*; D) *d*; E) *e*.

13. Tomonlari butun son bo‘lgan, bir tomoni 5 sm, qolgan tomonlari orasidagi farq 3 sm va eng uzun tomoni 12 sm dan kam bo‘lgan nechta turli uchburchak yasash mumkin?

A) 4; B) 5; C) 6; D) 7; E) 8.

14. Uchburchakning ikkita tomoni 0,8 va 1,9 ga teng. Uchinchi tomonining uzunligi butun son ekanligini bilgan holda shu tomonni toping.

A) 1; B) 2; C) bunday tomon mavjud emas; D) 3; E) 4.

15. Uchburchakning bir tomoni x ($x > 5$) sm, ikkinchi tomoni undan 3 sm qisqa, uchinchi tomoni esa birinchi tomonidan 2 sm uzun. Shu uchburchakning perimetrini toping.

A) $(3x - 1)$ sm; B) $(3x + 5)$ sm; C) $(3x + 3)$ sm;
D) $(3x + 2)$ sm; E) $(3x - 3)$ sm.

16. a ning qanday qiymatlarida uzunliklari mos ravishda $1 + a$, $1 - a$ va 1,5 bo‘lgan kesmalardan uchburchak yasash mumkin?

A) (–0,75; 0,75); B) (–0,5; 0,5); C) ∅; D) (–0,7; 0,7);
E) (–0,4; 0,4).

17. Uzunligi 1; 3; 5; 7; 9 ga teng bo‘lgan kesmalardan tomonlari har xil bo‘lgan nechta turli uchburchak yasash mumkin?

A) 4; B) 3; C) 5; D) 2; E) 6.

18. Teng yonli uchburchakning uchidagi burchagi 40° ga teng. Asosidagi burchakning bissektrisasi va shu burchak qarshisidagi tomon orasidagi burchakni toping.

A) 60°; B) 75°; C) 85°; D) 65°; E) 50°.

19. Perimetri 24 bo‘lgan uchburchakning balandligi uni perimetrlari 14 va 18 bo‘lgan ikkita uchburchakka ajratadi. Berilgan uchburchakning balandligini toping.

A) 10; B) 8; C) 6; D) 4; E) 3.

20. Uchburchakning tomonlari 4, 5 va 6 sm. 4 sm li tomonning 6 sm li tomondagi proyeksiyasi necha santimetr?

A) 1,25 sm; B) 1,5 sm; C) 2,25 sm; D) 2,5 sm; E) 3,5 sm.

21. Tomonlari 13, 14 va 15 bo'lgan uchburchakning eng kichik balandligini toping.

A) 11,2; B) 11,1; C) 11; D) 11,5; E) 11,6.

22. ABC uchburchakda A va B burchaklari bissektrisalari kesishishidan hosil bo'lgan kichik burchak 40° ga teng. Uchburchakning C burchagini toping.

A) 100°; B) 90°; C) 80°; D) 120°; E) 70°.

23. *Teng yonli to'g'ri burchakli uchburchakning o'tkir burchagi* bissektrisasi qarshisidagi katetni to'g'ri burchak uchidan hisoblaganda qanday nisbatda bo'ladi?

A) $\sqrt{2}:2$; B) 1 : 2; C) 2 : 1; D) $\sqrt{2}:1$; E) 2 : 3.

24. ABC to'g'ri burchakli uchburchakda AB gipotenuza, AD va BE bissektrisalar. Agar $|AB|=12$ va $|AD^2|+|BE^2|=169$ bo'lsa, DE ning uzunligini toping.

A) 5; B) 2,5; C) $\sqrt{28}$; D) 6; E) 4.

25. Uchburchak tomonlarining uzunliklari 6, 9 va 12. Uning katta burchagi bissektrisasining qarshi tomonda hosil qilgan katta kesmasining uzunligini toping.

A) 7,2; B) 4,8; C) 6,8; D) 8,4; E) 5,6.

26. ABC uchburchakda $AB=7$, $BC=9$ va $AC=12$ bo'lsa, AC tomonga tushirilgan h_b balandligini toping.

A) $\frac{5}{3}\sqrt{6}$; B) $\frac{8}{3}\sqrt{6}$; C) $\frac{7}{3}\sqrt{5}$; D) $\frac{8}{3}\sqrt{5}$; E) $\frac{8}{3}\sqrt{3}$.

27. 92- *a* rasmdagi noma'lum kesmaning uzunligini toping:

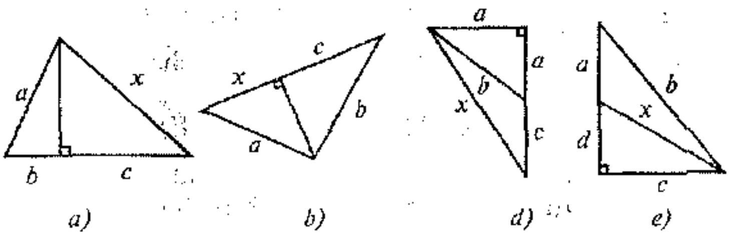
A) $x=\sqrt{a^2+b^2+c^2}$; B) $x=\sqrt{a^2-c^2-b^2}$;

C) $x=\sqrt{a^2-b^2+c^2}$; D) $x=\sqrt{b^2-a^2+c^2}$;

E) $x=\sqrt{c^2-a^2-b^2}$.

28. 92- *b* rasmdagi noma'lum kesmaning uzunligini toping:

A) $x=\sqrt{a^2+b^2-c^2}$; B) $x=\sqrt{b^2-c^2+a^2}$;



92- rasm.

C) $x = \sqrt{b^2 - a^2 + c^2}$; D) $x = \sqrt{a^2 - b^2 + c^2}$;

E) $x = \sqrt{c^2 - b^2 - a^2}$.

29. 92-*d* rasmdagi noma'lum kesmaning uzunligini toping:

A) $x = \sqrt{a^2 - b^2 - d^2}$; B) $x = \sqrt{d^2 - c^2 - b^2}$;

C) $x = \sqrt{c^2 - b^2 + d^2}$; D) $x = \sqrt{(c-d)^2 + b^2 + d^2}$;

E) $x = \sqrt{(c+d)^2 + b^2 - d^2}$.

30. 92- *e* rasmdagi noma'lum kesmaning uzunligini toping:

A) $x = \sqrt{d^2 + b^2 - (a+d)^2}$; B) $x = \sqrt{a^2 + b^2 + c^2}$;

C) $x = \sqrt{a^2 + b^2 + d^2}$; D) $x = \sqrt{d^2 + c^2}$;

E) $x = b^2 - a^2 - c^2 - d^2$.

31. *ABC* uchburchakning *BC* tomoniga *AD* to'g'ri chiziq shunday tushirilganki, $\angle CAD = \angle ACD$. *ABC* va *ABD* uchburchaklarning perimetrlari mos ravishda 37 va 24 ga teng. *AC* tomon uzunligini toping.

A) 7; B) 10; C) 13; D) 6,5; E) 5.

32. *ABC* uchburchakda *CD* bissektrisa, $AC = 5$, $CB = 7$, $AD = 3$ bo'lsa, *DB* kesma uzunligini toping.

A) 4; B) 4,1; C) 4,2; D) 4,3; E) 4,4.

33. Uchburchakning tomonlari 11 va 23 ga, uchinchi tomoniga tushirilgan medianasi 10 ga teng. Uchburchakning uchinchi tomonini toping.

A) 30; B) 15; C) 25; D) 28; E) 26.

34. Uchburchak burchaklarining kattaliklari nisbati 1 : 2 : 3 kabi, katta tomonining uzunligi 13 ga teng. Uchburchakning katta tomoniga tushirilgan balandligini toping.

A) $3{,}25\sqrt{3}$; B) 12; C) 8; D) $5\sqrt{3}$; E) 10.

35. Muntazam ABC uchburchakning perimetri 3 ga teng. Agar $AB_1 = 2AB$ va $AC_1 = 2AC$ bo'lsa, AB_1C_1 uchburchakning perimetrini toping.

A) 5; B) 6; C) 7; D) 8; E) 9.

36. Tomoni $\sqrt{3}$ ga teng bo'lgan muntazam uchburchakning ichidagi ixtiyoriy nuqtadan uning tomonlarigacha bo'lgan masofalar yig'indisi qanchaga teng bo'ladi?

A) 3; B) 1,5; C) $3\frac{\sqrt{3}}{2}$; D) nuqtaning vaziyatiga bog'liq;

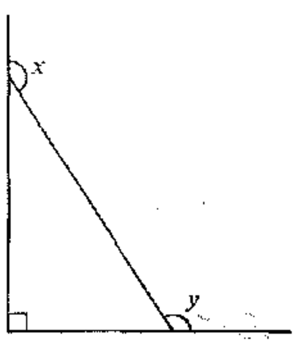
E) $2\frac{\sqrt{3}}{2}$.

37. 93- rasmdagi $x+y$ ni hisoblang.

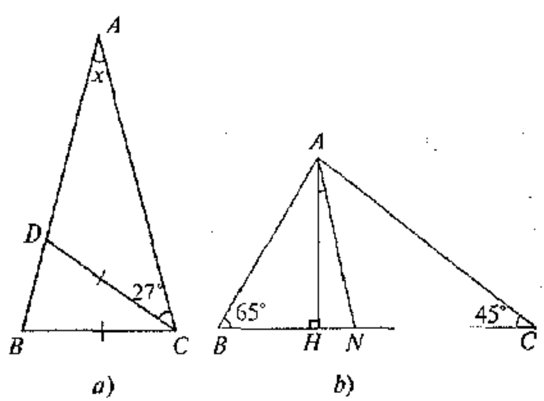
A) 90°; B) 180°; C) 225°; D) 240°; E) 270°.

38. $|AB| = |AC|$, $|DC| = |BC|$ va $\angle ACD = 27°$ bo'lsa, $\angle A$ ni toping (94- *a* rasm).

A) 30°; B) 42°; C) 60°; D) 72°; E) 84°.



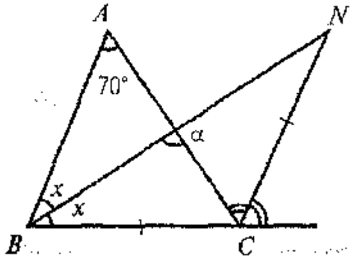
93- rasm.



94- rasm.

39. 94- *b* rasmda $AH \perp BC$, $\angle BAN = \angle NAC$, $\angle B = 65°$ va $\angle C = 45°$ bo‘lsa, $\angle NAH$ — ?

A) 10°; B) 15°; C) 20°; D) 25°; E) 30°.



95- rasm.

40. 95- rasmdagi x burchakni toping.

A) 45°; B) 60°; C) 75°; D) 80°; E) 35°.

41. Quyidagi iboralarning qaysi biri noto‘g‘ri?

A) Uchburchak ichki burchaklari yig‘indisi 180° ga teng;

B) Har qanday uchburchakning aqalli ikkita burchagi o‘tkir burchak bo‘ladi;

C) Uchburchakning tashqi burchagi o‘ziga qo‘shni bo‘lmagan ikkita ichki burchaklari yig‘indisiga teng;

D) Uchburchakning tashqi burchagi o‘ziga qo‘shni bo‘lmagan istalgan ichki burchagidan katta;

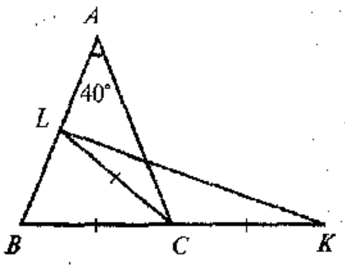
E) Uchburchak tashqi burchagi doim o‘tmas burchak bo‘ladi.

42. 95- rasmda $[BN]$ — bissektrisa, $[CN]$ — tashqi burchak bissektrisasi, $|BC| = |CN|$ va $\angle A = 70°$ bo‘lsa, α necha gradus?

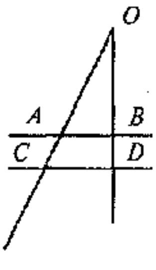
A) 45°; B) 60°; C) 75°; D) 90°; E) 105°.

43. 96- rasmda $\angle A = 40°$, $|CB| = |CL| = |CK|$ va $|AB| = |AC|$ bo‘lsa, $\angle CLK$ – ?

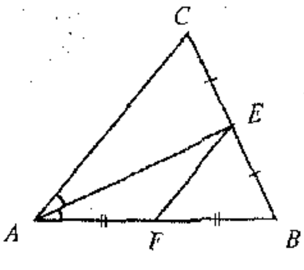
A) 20°; B) 25°; C) 30°; D) 35°; E) 40°.



96- rasm.



97- rasm.



98- rasm.

44. 97- rasmda $AB \parallel CD$; $OA=5$; $OB=4$; $OD=9$ bo'lsa, OC ni aniqlang.

A) 10,8; B) 10,5; C) 11,25; D) 11,3; E) 11.

45. 97- rasmda $AB \parallel CD$; $OA=3$; $OB=4$; $AC=1,5$ bo'lsa, BD ni aniqlang.

A) 3; B) 2; C) 2,5; D) 2,1; E) 26.

46. ABC uchburchakda AE — bissektrisa, E va F nuqtalar tegishli tomon o'rtalari. Agar $|AC|=8$, $|BC|=12$ bo'lsa, ABC uchburchakning perimetrini toping (98- rasm).

A) 18; B) 24; C) 25; D) 28; E) 36.

47. Tomonlari 10, 12 va 17 bo'lgan uchburchak bissektrisasining katta tomonida hosil qilgan kichik kesmasining uzunligini toping (99- rasm).

A) $7\frac{8}{11}$; B) $7\frac{6}{11}$; C) $8\frac{4}{11}$; D) $7\frac{7}{11}$; E) $7\frac{9}{11}$.

48. To'g'ri burchakli uchburchakda 30° li burchak qarshisidagi katetning gipotenuzaga nisbatini toping (100- rasm).

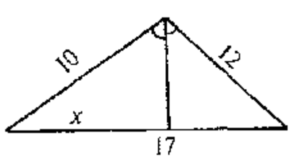
A) $\frac{1}{8}$; B) $\frac{1}{4}$; C) $\frac{2}{3}$; D) $\frac{1}{2}$; E) $\frac{3}{4}$.

49. 101- rasmda $BD=DC$; $AD=5$; $AC=2\sqrt{3}$ bo'lsa, AB necha santimetr bo'ladi?

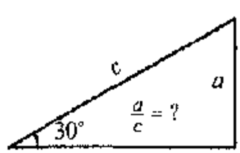
A) 8; B) 6; C) 4; D) 3; E) 2.

50. To'g'ri burchakli uchburchakning katetlari 9 va 12 ga teng. Kichik katetning gipotenuzadagi proyeksiyasini toping.

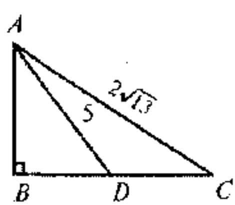
A) 6; B) $5\frac{2}{3}$; C) 5,4; D) 4,8; E) $6\frac{1}{3}$.



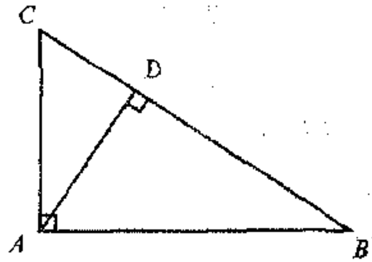
99- rasm.



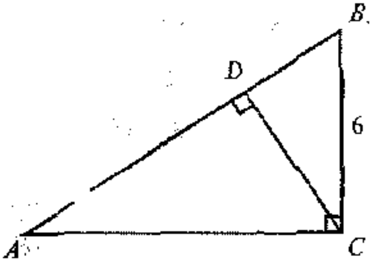
100- rasm.



101- rasm.



102- rasm.



103- rasm.

51. ΔABC da $\angle B = 90°$; $\angle C = 60°$. BB_1 balandlik 2 ga teng. $|AB|$ ni toping.

A) 4; B) 2; C) $2\sqrt{3}$; D) $2\sqrt{2}$; E) $\frac{4}{\sqrt{3}}$.

52. To'g'ri burchakli uchburchakning gipotenuzasi 8 ga, katetlaridan biri 4 ga teng. Ikkinchi katetning gipotenuzadagi proyeksiyasini toping (102-rasm).

A) 4; B) 3; C) 5; D) 7; E) 6.

53. $CB = 5$; $CD = 1{,}6$; AB^2 — ? (102- rasm).

A) 14; B) 16; C) 17; D) 18; E) 15.

54. ΔABC da $\angle BAC = 30°$, $|BC| = a$, $(BA) \perp (CD)$ bo'lsa, $|BD|$ ni toping.

A) $a\sqrt{3}$; B) $\frac{a}{3}$; C) $2a$; D) $\frac{a\sqrt{3}}{2}$; E) $\frac{2}{3}a$.

55. To'g'ri burchakli uchburchakning gipotenuzasi 25 ga, katetlaridan biri 10 ga teng. Ikkinchi katetning gipotenuzadagi proyeksiyasini toping (102- rasm).

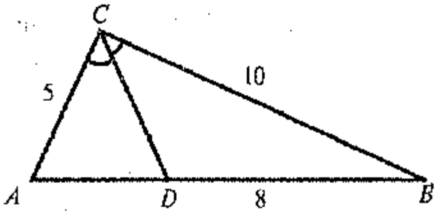
A) 14; B) 15,5; C) 18; D) 20,4; E) 21.

56. ABC uchburchakda $\angle C = 90°$; $CD - C$ uchidan tushirilgan balandlik, $|BC| = 6$ va $|AB| = 10$. AD ning uzunligini toping (103-rasm).

A) 4; B) 3,6; C) 4,2; D) 3,4; E) 3,8.

57. $|AC| = 5$, $|BC| = 10$, $|BD| = 8$ bo'lsa, CD bissektrisani toping (104- rasm).

A) $3\sqrt{2}$; B) $\sqrt{3}$; C) $\sqrt{5}$;
D) 2; E) 3.



104- rasm.

58. Teng yonli uchburchakda yon tomoni 17 sm, asosi esa 16 sm. Asosga tushirilgan balandlikni toping.
A) 15 sm; B) 14 sm;
C) 17 sm; D) 19 sm;
E) 16 sm.

59. Teng yonli to'g'ri burchakli uchburchakning asosi *a* ga teng. Uning yon tomonini toping.

A) $\frac{a}{2}$; B) $\frac{\sqrt{a}}{2}$; C) $\frac{a}{\sqrt{2}}$; D) $a\sqrt{2}$; E) $\frac{a}{\sqrt{3}}$.

60. Uchburchakning o'zaro 120° burchak hosil qiluvchi ikki tomoni uzunliklari farqi 1 dm ga teng. Agar uchinchi tomoni 13 dm ga teng bo'lsa, uchburchakning perimetrini toping.
A) 26 dm; B) 28 dm; C) 30 dm; D) 32 dm; E) 24 dm.

61. To'g'ri burchakli uchburchak gipotenuzasining shu gipotenuzaga tushirilgan medianaga nisbatini toping.
A) 3; B) 4; C) 2,5; D) 2; E) 1,5.

62. Tomonlari 10, 8 va 6 bo'lgan uchburchakning katta tomoniga o'tkazilgan medianasini toping.
A) 7; B) 6; C) 3; D) 4; E) 5.

63. To'g'ri burchakli uchburchakning burchaklaridan biri 60° ga, gipotenuzasiga tushirilgan medianasi 15 ga teng. Kichik katetning uzunligini toping.
A) 7,5; B) 10,5; C) 15; D) 12; E) 20.

64. To'g'ri burchakli uchburchakning bitta kateti 2 ga, bu katet qarshisidagi burchak 60° ga teng. Ikkinchi katetni toping.

A) $\sqrt{3}$; B) $2\sqrt{2}$; C) $\frac{2\sqrt{3}}{3}$; D) $\frac{\sqrt{2}}{2}$; E) $\frac{\sqrt{3}}{2}$.

65. Tomonlari 13, 14 va 15 sm bo'lgan uchburchakning eng kichik balandligi necha santimetr?
A) 11,2 sm; B) 11,1 sm; C) 11 sm; D) 11,5 sm; E) 11,6 sm.

66. Uchburchakning tomonlari 3, 5 va 6 ga teng. 5 ga teng bo'lgan tomon qarshisidagi burchakning kosinusini toping.

A) $-\frac{1}{2}$; B) $\frac{5}{18}$; C) $\frac{5}{9}$; D) $\frac{1}{2}$; E) $\frac{4}{9}$.

67. Uchburchakning tomonlari a, b va c ga teng. Bu uchburchakning tomonlari orasida $a^2 = b^2 + c^2 + bc$ munosabat o'rinli bo'lsa, uzunligi a ga teng tomon qarshisidagi burchakni toping.

A) 60°; B) 150°; C) 120°; D) 90°; E) 135°.

68. Uchburchak tomonlarining uzunliklari m, n va k ushbu $m^2 = n^2 + + k^2 + \sqrt{2}nk$ tenglikni qanoatlantiradi. Uzunligi m ga teng tomon qarshisidagi burchakni toping.

A) 45°; B) 150°; C) 120°; D) 90°; E) 135°.

69. Muntazam uchburchakning yuzi 64 ga teng. Uning perimetrini toping.

A) $16\sqrt[4]{27}$; B) $\frac{64}{3\sqrt{3}}$; C) 64; D) $64\sqrt{3}$; E) $\frac{40\sqrt{3}}{3}$.

70. Uchburchakning bir tomoni 17 ga, unga yopishgan burchaklari 103° va 47° ga teng. Uchburchakka tashqi chizilgan aylananing radiusini toping.

A) 8,5; B) $8{,}5\sqrt{3}$; C) $17\sqrt{2}$; D) 17; E) $17\sqrt{3}$.

71. Kichik tomoni $\sqrt{3}$ ga teng bo'lgan uchburchakning burchaklari 1 : 2 : 3 kabi nisbatda bo'lsa, uchburchakning perimetrini toping.

A) $3+3\sqrt{3}$; B) $2+\sqrt{3}$; C) $1+\sqrt{3}$; D) $9+4\sqrt{3}$; E) $6+6\sqrt{3}$.

72. Uchburchakning asosi 22 ga, yon tomonlari 13 ga va 19 ga teng. Asosiga tushirilgan medianasini toping.

A) 18; B) 12; C) 16; D) 13; E) 14.

73. Uchburchakning tomonlari 11 va 23 ga, uchinchi tomoniga tushirilgan medianasi 10 ga teng. Uchburchakning uchinchi tomonini toping.

A) 30; B) 15; C) 25; D) 28; E) 26.

74. ABC uchburchakda CD – bissektrisa, $AC = 5$, $CB = 7$, $AB = 8$, AD – ?

A) 4; B) $3\frac{2}{3}$; C) $3\frac{1}{3}$; D) 4,3; E) 4,4.

75. ABC uchburchakda $|AB| = 5$ sm, $|AC| = 10$ sm va $\angle A = 45°$. Shu uchburchakning yuzi necha kvadrat santimetr?

A) $\frac{5\sqrt{2}}{2}$ sm²; B) $10\sqrt{2}$ sm²; C) $50\sqrt{2}$ sm²; D) $25\sqrt{2}$ sm²;

E) $\frac{25\sqrt{2}}{2}$ sm².

76. Perimetrlari 24 va 36 bo'lgan ikki o'xshash uchburchakdan birining yuzi ikkinchisinikidan 10 ga ortiq. Kichik uchburchakning yuzini toping.

A) 20; B) 14; C) 11; D) 8; E) 8,5.

77. Ikkita o'xshash uchburchakning perimetrlari 18 va 36 ga, yuzlarining yig'indisi 30 ga teng. Katta uchburchakning yuzini toping.

A) 20; B) 24; C) 21; D) 18; E) 25.

78. Yuzlari 8 va 32 bo'lgan ikki o'xshash uchburchak perimetrlarining yig'indisi 48 ga teng. Kichik uchburchakning perimetrini toping.

A) 12; B) 16; C) 20; D) 9,6; E) aniqlab bo'lmaydi.

79. ΔABC ning tomonlari $\Delta A_1B_1C_1$ ning mos tomonlaridan $2\sqrt{3}$ marta katta. ΔABC ning yuzi $\Delta A_1B_1C_1$ ning yuzidan necha marta katta?

A) 12; B) 6; C) $2\sqrt{3}$; D) $4\sqrt{3}$; E) $6\sqrt{3}$.

80. $A_1B_1C_1$ va $A_2B_2C_2$ uchburchaklar o'xshash. $\Delta A_2B_2C_2$ ning yuzi $\Delta A_1B_1C_1$ ning yuzidan 9 marta katta. $\Delta A_1B_1C_1$ ning 3 ga teng bo'lgan tomoniga mos bo'lgan $\Delta A_2B_2C_2$ ning tomonini toping.

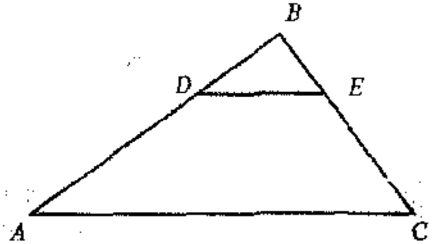
A) 9; B) 27; C) 12; D) 6; E) 18.

81. Uchburchakning asosiga parallel to'g'ri chiziq uning yuzini teng ikkiga bo'lsa, asosidan boshlab hisoblaganda, uning yon tomonlarini qanday nisbatda bo'ladi?

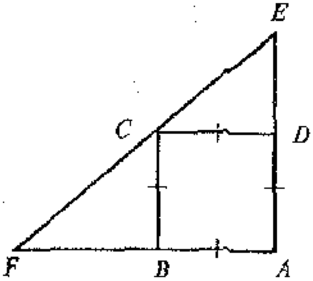
A) $(\sqrt{2}-1):1$; B) $1:1$; C) $\frac{1}{2}:1$; D) $(\sqrt{3}-1):1$;

E) $(2\sqrt{2}-1):1$.

82. Uchburchak tomonlarining o'rtalarini tutashtirib, perimetri 65 ga teng bo'lgan uchburchak hosil qilindi. Berilgan uchburchakning perimetrini toping.



105- rasm.



106- rasm.

A) 32,5; B) 260; C) 75; D) 195; E) 130.

83. 105- rasmdagi ABC uchburchakda $|AB| = 8$ va $|AD| = 2$ bo'lsa, ABC va DBE uchburchaklar yuzlarining nisbatini toping.

A) 2; B) $2\frac{1}{5}$; C) $1\frac{5}{7}$; D) $1\frac{7}{9}$; E) $1\frac{8}{9}$.

84. 106- rasmdagi $ABCD$ kvadratning A uchidan AE va AF to'g'ri chiziqlar, C uchidan esa BD diagonalga parallel bo'lgan CF to'g'ri chiziq o'tkazilgan. Agar kvadratning yuzi 3 ga teng bo'lsa, AFE uchburchakning yuzini toping.

A) 5; B) 6; C) 7; D) 9; E) 8.

85. Perimetri 48 ga teng bo'lgan uchburchakning har bir tomoni to'rtta teng kesmalarga bo'lindi. Bo'linish nuqtalari tomonlarga parallel kesmalar bilan tutashtirilgan. Bu kesmalar uzunliklarining yig'indisini toping.

A) 72; B) 96; C) 24; D) 144; E) 36.

86. Teng yonli uchburchakka ichki chizilgan aylananing markazi uning asosiga tushirilgan balandligini, uchidan boshlab hisoblaganda, 5 va 3 ga teng kesmalarga ajratadi. Uchburchakning asosini toping.

A) 14; B) 12; C) 10; D) 9; E) 8.

87. ABC uchburchak aylanaga ichki chizilgan. $AB = 24$ va aylana markazi shu tomondan 5 birlik masofada yotsa, aylananing radiusini toping.

A) 13; B) 12; C) 11; D) 10; E) 9.

88. Teng yonli to'g'ri burchakli uchburchak R radiusli aylanaga ichki chizilgan. Boshqa aylana bu uchburchakning katetlariga va birinchi aylanaga urinadi. Shu aylananing radiusini toping.

A) $2R(\sqrt{2}-1)$; B) $\frac{2}{3}R$; C) $R(\sqrt{2}-1)$; D) $\frac{\sqrt{2}}{2}R$; E) $\frac{\sqrt{3}}{4}R$.

89. To'g'ri burchakli uchburchakka ichki chizilgan aylana urinish nuqtasida katetlardan birini to'g'ri burchak uchidan boshlab hisoblaganda 6 va 10 ga teng kesmalarga ajratadi. Uchburchakning perimetrini toping.

A) 82; B) 81; C) 80; D) 75; E) 74.

90. ABC uchburchakning C uchidagi tashqi burchagi 90°, A uchidagi tashqi burchagi 150°ga va kichik tomonning uzunligi 12,5 ga teng. Shu uchburchakka tashqi chizilgan aylananing diametrini toping.

A) 23; B) 24; C) 25; D) 26; E) 28.

III bob. TO'RTBURCHAKLAR VA ULARNING XOSSALARI

1- §. To'rtburchak va uning asosiy elementlari

Hech bir uchtasi bir to'g'ri chiziqda yotmagan to'rt nuqta o'zaro kesishmaydigan to'rtta kesma bilan tutashtirilsa, to'rtburchak shakli hosil bo'ladi (107- rasm).

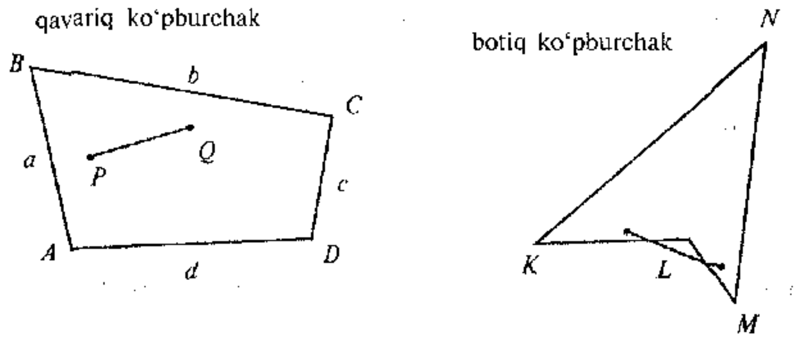
Berilgan nuqtalar *to'rtburchakning uchlari*, ularni tutashtiruvchi kesmalar uning *tomonlari* deb ataladi. Bir tomoniga tegishli uchlari *to'rtburchakning qo'shni uchlari,* qo'shni bo'lmaganlari *qarama-qarshi uchlari* deyiladi.

To'rtburchakning bir uchidan chiquvchi tomonlari *qo'shni tomonlar* deyiladi. Umumiy uchga ega bo'lmagan tomonlar *qarama-qarshi tomonlar* deb ataladi.

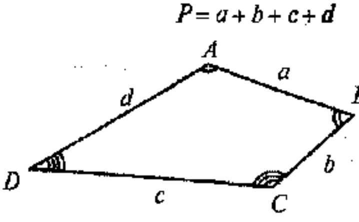
To'rtburchakning ixtiyoriy ikki nuqtasini tutashtiruvchi kesma uning tomonlarini kesib o'tmasa, bunday to'rtburchak *qavariq to'rtburchak* deyiladi, aks holda *botiq to'rtburchak* deb ataladi.

Quyida faqat qavariq to'rtburchaklarning xossalari bayon etiladi.

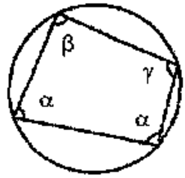
To'rtburchakning barcha tomonlari uzunliklarining yig'indisi uning *perimetri* deyiladi (108- rasm).



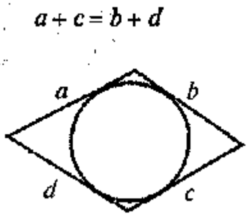
107- rasm.



108- rasm.



109- rasm.



110- rasm.

To'rtburchakning uchlari (burchaklari) odatda A, B, C, D kabi belgilanadi. Uning bitta tomoniga tegishli bo'lmagan uchlarini tutashtiruvchi kesma to'rtburchakning diagonali bo'lib, ular odatda d_1 va d_2 kabi belgilanadi.

To'rtburchakning barcha ichki burchaklarining yig'indisi 360° ga teng ($\angle A + \angle B + \angle C + \angle D = 360°$). Uning bir tomoniga yopishmagan ichki burchaklari o'zaro *qarama-qarshi burchaklari* deyiladi.

Agar diagonallarining uzunligi d_1, d_2 va ular orasidagi burchak φ ma'lum bo'lsa, qavariq to'rtburchakning yuzi $S = \frac{1}{2}d_1 \cdot d_2 \sin\varphi$ ifoda orqali topiladi.

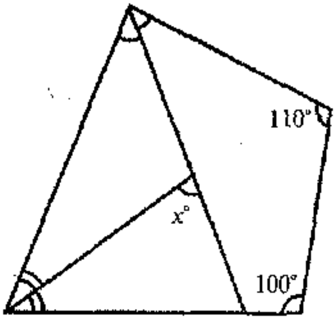
108- rasmda $\angle A$ bilan $\angle C$ hamda $\angle D$ bilan $\angle B$ o'zaro qarama-qarshi burchaklardir.

To'rtburchakning barcha uchlaridan o'tuvchi aylana unga tashqi chizilgan aylana hisoblanadi. Bu holda to'rtburchak aylanaga *ichki chizilgan* deyiladi (109- rasm).

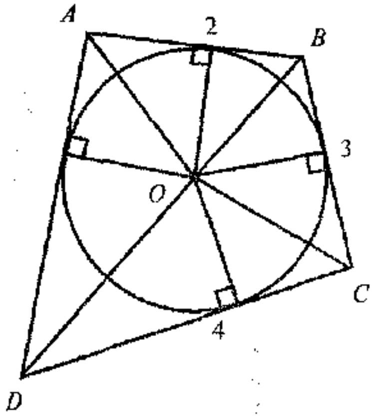
To'rtburchakning barcha tomonlariga urinuvchi aylana unga *ichki chizilgan aylana* deb ataladi. Bu holda to'rtburchak aylanaga *tashqi chizilgan* deyiladi (110- rasm).

To'rtburchakka ichki yoki tashqi aylana chizishni har doim ham iloji bo'lavermaydi. Qachon iloji borligini quyidagi teoremalar asoslaydi:

1- t e o r e m a. Agar to'rtburchakning qarama-qarshi burchaklarining yig'indisi 180° bo'lsa, u holda bu to'rtburchakka tashqi aylana chizish mumkin va, aksincha, agar to'rtburchakka tashqi aylana chizish mumkin bo'lsa, u holda uning qarama-qarshi burchaklari *yig'indisi 180° ga teng bo'ladi.*



111- rasm.



112- rasm.

2- t e o r e m a. Agar to'rtburchakning qarama-qarshi tomonlari yig'indisi bir-biriga teng bo'lsa, bu to'rtburchakka ichki aylana chizish mumkin va, aksincha, to'rtburchakka ichki aylana chizish mumkin bo'lsa, u holda uning qarama-qarshi tomonlari yig'indilari bir-biriga teng bo'ladi.

1- m a s a l a . 111- rasmda berilganlarga ko'ra x ni toping.

Y e c h i l i s h i. To'rtburchakning ichki burchaklari yig'indisi 360° ligini bilgan holda, qolgan burchaklari yig'indisining yarmi

$x = \frac{150^\circ}{2} = 75^\circ.$

J a v o b: 75°.

2- m a s a l a. To'rtburchakning uchta ketma-ket tomonlarining uzunliklari 2, 3 va 4 ga, unga ichki chizilgan aylananing radiusi 1,2 ga teng bo'lsa, to'rtburchakning yuzini toping (112- rasm).

Y e c h i l i s h i. Aylanaga tashqi chizilgan to'rtburchakning qarama-qarshi tomonlari yig'indisi bir-biriga tengligidan uning 4-tomonining uzunligi 3 ga tengligi ma'lum bo'ladi.

$$S_{ABCD} = S_{AOB} + S_{BOC} + S_{COD} + S_{DOA};$$

$$S_{ABCD} = \frac{1,2}{2}(3 + 2 + 3 + 4) = 7,2 \text{ kv. birl.}$$

J a v o b: 7,2 kv.birl.

2- §. To'g'ri to'rtburchak va kvadrat

Ta'rif. *To'rtala burchagi ham to'g'ri burchak bo'lgan to'rtburchak to'g'ri to'rtburchak deb ataladi* (113- rasm).

To'g'ri to'rtburchakning qo'shni tomonlari mos ravishda a va b bo'lsa, uning perimetri $P = 2(a + b)$ bo'ladi.

To'g'ri to'rtburchakning barcha tomonlari o'zaro teng bo'lsa, u kvadrat deb ataladi (114- rasm).

Tomoni a ga teng bo'lgan kvadratning perimetri $4a$ ga teng.

To'g'ri to'rtburchak quyidagi xossalarga ega:

1. To'g'ri to'rtburchakning qarama-qarshi tomonlari parallel va o'zaro bir-biriga teng.

2. To'g'ri to'rtburchakning diagonallari bir-biriga teng.

3. To'g'ri to'rtburchakning diagonallari kesishish nuqtasida teng ikkiga bo'linadi va kesishish nuqtasi uning uchlaridan barobar uzoqlikda yotadi.

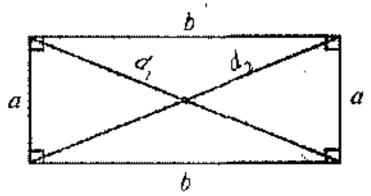
4. To'g'ri to'rtburchakka tashqi aylana chizish mumkin: bu aylananing markazi uning diagonallari kesishish nuqtasida bo'lib, diametri diagonalga teng bo'ladi.

To'g'ri to'rtburchakning yuzi uning ikkala o'lchamining ko'paytmasiga teng: $S = ab$.

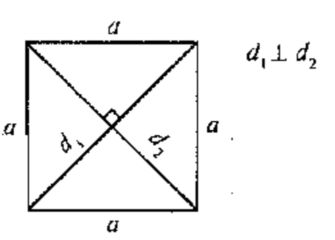
Kvadrat to'g'ri to'rtburchak bo'lgani uchun yuqoridagi xossalar kvadrat uchun ham o'rinli. Ulardan tashqari kvadrat quyidagi xossalarga ega:

5. Kvadrat diagonallari o'zaro perpendikulardir. Kvadratning yuzi uning tomoni kvadratiga teng: $S = a^2$ kv. birlik.

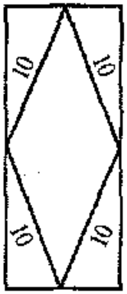
6. Diagonali d ga teng bo'lgan kvadratning yuzi $S = \frac{d^2}{2}$ ga teng.



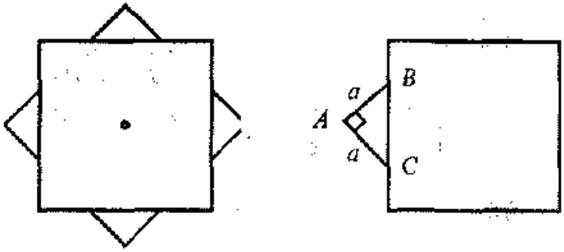
113- rasm.



114- rasm.



115- rasm.



a) b)

116- rasm.

7. Kvadratga ichki aylana chizish mumkin: bu aylana markazi uning diagonallari kesishgan nuqtada yotadi va uning diametri kvadrat tomoniga teng bo'ladi.

1- m a s a l a . To'g'ri to'rtburchakka uchlari uning tomonlarining o'rtalari bilan ustma-ust tushadigan to'rtburchak ichki chizilgan. Ichki chizilgan to'rtburchakning perimetri 40 ga teng. To'g'ri to'rtburchak tomonlarining nisbati 8 : 6 kabi bo'lsa, uning perimetrini toping (115- rasm).

Y e c h i l i s h i. Ichki chizilgan to'rtburchak teng tomonli bo'lgani uchun uning bir tomonining uzunligi 40 : 4 = 10. To'g'ri to'rtburchak tomonlarining nisbati 8 : 6 ga teng bo'lgani va ichki to'rtburchakning tomoni 10 ga teng bo'lgani uchun uning tomonlarining uzunligi 16 va 12 ekani ma'lum bo'ladi. Demak, to'g'ri to'rtburchakning perimetri 56 ga teng.

J a v o b: 56.

2- m a s a l a. Tomonlari 1 ga teng bo'lgan ikkita kvadrat ustma-ust qo'yilganidan keyin, ulardan biri kvadratlarning umumiy simmetriya markaziga nisbatan 45° ga burildi. Hosil bo'lgan shaklning perimetrini toping (116- *a* rasm).

Y e c h i l i s h i. Teng yonli, to'g'ri burchakli ΔABC ning katetini a deb belgilasak (116- *b* rasm), izlangan perimetr $P = 16a$ bo'ladi.

$$16a = 4 + 8a - 4a\sqrt{2} \Leftrightarrow 8a + 4a\sqrt{2} = 4 \Leftrightarrow 2a + a\sqrt{2} = 1,$$

$$a(2+\sqrt{2}) = 1 \Leftrightarrow a = \frac{1}{2+\sqrt{2}} \Leftrightarrow a = \frac{2-\sqrt{2}}{2},$$

$$P = 16 \cdot \frac{2-\sqrt{2}}{2} \Leftrightarrow P = 16 - 8\sqrt{2}.$$

Javob: $16 - 8\sqrt{2}$.

3- §. Trapetsiya va uning xossalari

Ta'rif. *Ikkita tomoni bir-biriga parallel bo'lib, qolgan ikkitasi parallel bo'lmagan to'rtburchak trapetsiya deb ataladi* (117- *a* rasm). Trapesiyaning parallel tomonlari uning *asoslari* deb ataladi. Parallel bo'lmagan tomonlari esa *yon tomonlari* deyiladi. Yon tomonlarining o'rtasini tutashtiruvchi kesma trapetsiyaning *o'rta chizig'i* deb ataladi (117- *e* rasm).

Yon tomonlaridan biri asoslariga perpendikular bo'lgan trapetsiya *to'g'ri burchakli trapetsiya* deb, yon tomonlari bir-biriga teng bo'lgan trapetsiya *teng yonli trapetsiya* deb ataladi (117- *b, d* rasm).

Trapetsiyalar quyidagi xossalarga ega:

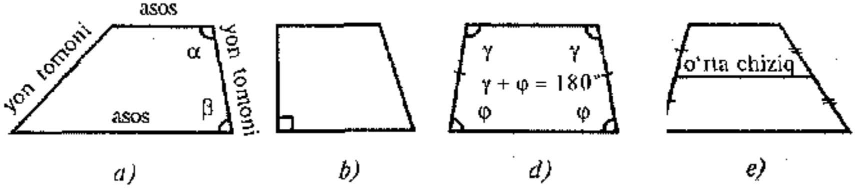
1) Trapetsiyaning o'rta chizig'i asoslariga parallel bo'lib, ular yig'indisining yarmiga teng: $l = \frac{a+b}{2}$.

2) Trapetsiyaning yuzi uning asosiga o'tkazilgan balandlikning o'rta chiziqqa ko'paytirilganiga teng.

$$S = l \cdot h; \; l = \frac{a+b}{2} \text{ bo'lgani uchun } S = \frac{a+b}{2} \cdot h.$$

3) Trapetsiyaning yon tomoniga yopishgan ikkita ichki burchaklarining yig'indisi 180° ga teng: $\alpha + \beta = 180°$ (117- *a* rasm).

4) Trapetsiyaning diagonali uning asoslari bilan bir xil burchak hosil qiladi.



117- rasm.

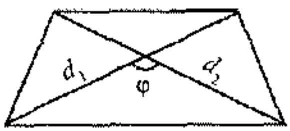
5) Teng yonli trapetsiyaning bitta asosiga yopishgan burchaklari teng.

6) Teng yonli trapetsiyaning qarama-qarshi burchaklari yig'indisi 180° ga teng (117- *d* rasm).

7) Teng yonli trapetsiyaga tashqi aylana chizish mumkin.

8) Teng yonli trapetsiyaga ichki aylana chizish mumkin bo'lsa, u holda uning yon tomoni o'rta chizig'iga teng bo'ladi.

9) Trapetsiyaga ichki chizilgan aylana mavjud bo'lsa, bu aylananing diametri trapetsiyaning balandligiga teng.



118- rasm.

10) Trapetsiya uchun ham barcha to'rtburchaklar uchun to'g'ri bo'lgan yuzni hisoblash qoidasi o'rinli. Trapetsiyaning yuzi uning diagonallari ko'paytmasining yarmini ular orasidagi burchak sinusiga ko'paytirilganiga teng (118- rasm):

$$S = \frac{1}{2} d_1 \cdot d_2 \cdot \sin\varphi .$$

1- m a s a l a. Teng yonli trapetsiyaning diagonali uning o'tkir burchagini teng ikkiga bo'ladi. Asoslarining nisbati 2 : 5 kabi bo'lib, perimetri 132 sm bo'lsa, trapetsiya o'rta chizig'ining uzunligini toping.

Y e c h i l i s h i. Trapetsiyaning diagonali o'tkir burchagining bissektrisasi ekanligidan uning yon tomonlari kichik asosiga tengligi ma'lum bo'ladi (119- rasm). $\frac{a}{b} = \frac{2}{5} \Leftrightarrow b = \frac{5}{2}a;\ 132 = 3a + \frac{5}{2}a \Leftrightarrow$ $\Leftrightarrow a = 24$ sm, $b = 60$ sm ekan. Demak, $l = 42$ sm.

J a v o b: 42 sm.

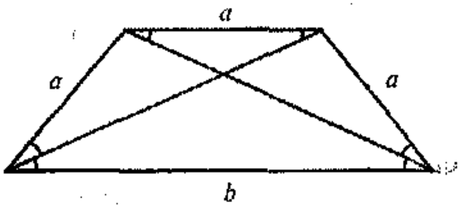
2- m a s a l a. Asoslarining nisbati 1 : 8 kabi bo'lgan to'g'ri burchakli trapetsiyaning bir burchagi 135°, o'rta chizig'i esa 18 sm. Shu trapetsiyaning kichik yon tomonini toping (120- rasm).

Y e c h i l i s h i: $\frac{a}{b} = \frac{1}{8} \Leftrightarrow b = 8a.$ O'rta chiziq $l = \frac{a+b}{2} = 18 \Leftrightarrow$

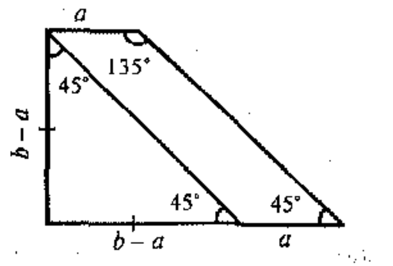
$\Leftrightarrow \frac{a+8a}{2} = 18.$ Demak: $a = 4$ sm va $b = 32$ sm. $b - a = 28$ sm.

J a v o b: 28 sm.

3- m a s a l a. Asoslari *a* va *b* bo'lgan trapetsiyaning o'tkir burchaklari yig'indisi 90° bo'lsa, uning asoslarining o'rtalarini tutashtiruvchi kesmaning uzunligini toping.



119- rasm.



120- rasm.

Y e c h i l i s h i. Trapetsiyaning yon tomonlarini davom ettirib, hosil qilingan to‘g‘ri burchakli uchburchakda izlangan kesma uzunligi gipotenuzaga o‘tkazilgan mediana bo‘lagi uzunligi sifatida aniqlanadi. Trapetsiyaning asoslari a va b bo‘lsa, $x = \frac{b}{2} - \frac{a}{2} \Leftrightarrow x = \frac{b-a}{2}$ bo‘ladi.

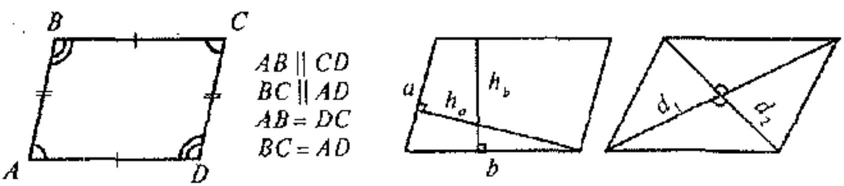
J a v o b: $\frac{b-a}{2}$.

4- §. Parallelogramm va uning xossalari

T a ’ r i f. *Qarama-qarshi tomonlari juft-jufti bilan bir-biriga parallel bo‘lgan to‘rtburchak parallelogramm deb ataladi* (121- rasm).

Parallelogramm quyidagi xossalarga ega:

1. Parallelogrammning qarama-qarshi tomonlari bir-biriga teng.
2. Parallelogrammning bir tomoniga yopishgan ichki burchaklari yig‘indisi 180° ga teng.
3. Parallelogrammning qarama-qarshi burchaklari bir-biriga teng.



121- rasm.

4. Parallelogrammning diagonali uni o'zaro teng ikki uchburchakka ajratadi.

5. Parallelogrammning diagonallari kesishish nuqtasida teng ikki bo'lakka bo'linadi.

6. Parallelogramm diagonallari bilan bir-birini qoplamaydigan to'rtta tengdosh uchburchakka ajraladi.

Parallelogrammning yuzini quyidagi formulalardan biri yordamida hisoblash mumkin (121- rasm):

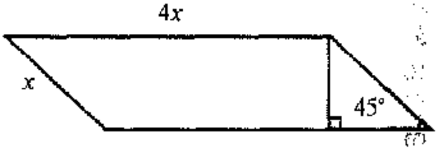
$$S = a \cdot h_a = b \cdot h_b; \; S = a \cdot b \cdot \sin\alpha; \; S = \frac{d_1 \cdot d_2 \cdot \sin\varphi}{2}.$$

Bu yerda a va b — parallelogrammning tomonlari, h_a va h_b — parallelogrammning tomonlariga tushirilgan balandliklar, α — parallelogrammning a va b tomonlari orasidagi burchak; d_1 va d_2 — parallelogrammning diagonallari, φ — ular orasidagi burchak.

Parallelogramm diagonallari kvadratlarining yig'indisi uning hamma tomonlari kvadratlarining yig'indisiga teng:

$$d_1^2 + d_2^2 = 2(a^2 + b^2).$$

Parallelogramm mavzusida masalalar yechishda masala shartidagi ma'lum elementlar soni uchtadan kam bo'lmasligi kerak (uchtadan kamida bittasi chiziqli element). Noma'lum elementlarni topishda parallelogrammning yuqoridagi xossalaridan foydalaniladi.



122- rasm.

1- masala. Parallelogrammning bir tomoni ikkinchi tomonidan 4 marta katta, perimetri $20\sqrt{2}$ sm, o'tkir burchagi 45° ga teng. Parallelogrammning yuzini hisoblang (122- rasm).

Yechilishi. Masala shartiga ko'ra:

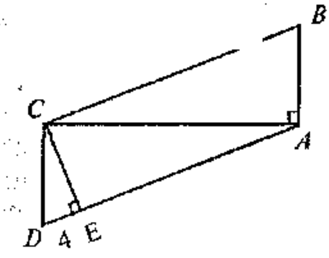
$$2(x + 4x) = 20\sqrt{2} \Rightarrow 10x = 20\sqrt{2} \Rightarrow x = 2\sqrt{2};$$

$$h = x \cdot \sin 45°; \; h = 2\sqrt{2} \cdot \frac{\sqrt{2}}{2} = 2;$$

$$S = h \cdot 4x = 2 \cdot 4 \cdot 2\sqrt{2} = 16\sqrt{2}.$$

Javob: $S = 16\sqrt{2}$.

2- m a s a l a. $ABCD$ parallelogrammda $AC \perp CD$, $CE \perp AD$, $|AE| = 16$ va $|DE| = 14$. Parallelogrammning yuzini toping (123- rasm).



123- расм.

Y e c h i l i s h i. To'g'ri burchakli uchburchak ACD da $|CE| = \sqrt{4 \cdot 16} = 8$; $S_{ABCD} = 20 \cdot 8 = 160$.

J a v o b: 160 kv. birl.

3- m a s a l a. Parallelogrammning digonallari $6\sqrt{2}$ va $8\sqrt{2}$ ga teng. Uning tomonlari kvadratlarining yig'indisini toping.

Y e c h i l i s h i. Ma'lumki, diagonallari d_1 va d_2, tomonlari a va b bo'lgan parallelogrammda $d_1^2 + d_2^2 = 2(a^2 + b^2)$. Shuning uchun $2(a^2 + b^2) = (6\sqrt{2})^2 + (8\sqrt{2})^2 = 72 + 128 = 200$.

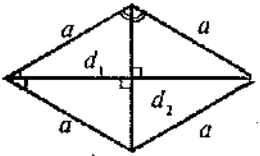
J a v o b: 200.

5- §. Romb va uning xossalari

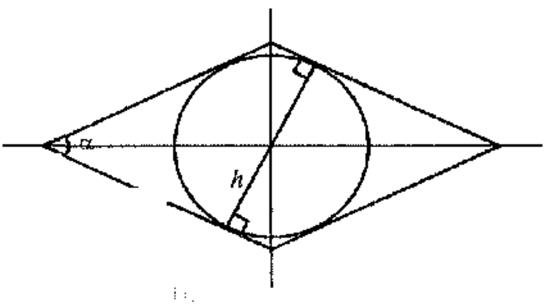
T a ' r i f. *Barcha tomonlari bir-biriga teng bo'lgan parallelogramm romb deb ataladi.*

Romb uchun parallelogrammning barcha xossalari o'rinli bo'lib, ulardan tashqari rombning o'ziga xos quyidagi xossalari mavjud:

1. Rombning diagonallari o'zaro perpendikulardir (124- rasm).
2. Rombning diagonallari uning simmetriya o'qlaridir.



124- rasm.



125- rasm

3. Rombning diagonallari uning burchaklari uchun bissektrisadir.

4. Romb diagonallarining kesishish nuqtasi uning simmetriya markazidir.

5. Rombga doim ichki aylana chizish mumkin bo'lib, uning diametri rombning balandligiga teng (125- rasm). Rombning yuzini hisoblash uchun formulalar:

$$S = a \cdot h; \; S = a^2 \cdot \sin\alpha; \; S = \frac{d_1 \cdot d_2}{2}.$$

Romb bilan bog'liq masalalarni yechishda, uning ikki elementi berilgan bo'lsa (ikkisidan kamida biri chiziqli element), so'ralgan elementi ma'lum formulalar yordamida hisoblab topiladi.

1- m a s a l a. Rombning balandligi 5 ga, diagonallari ko'paytmasi 80 ga teng. Uning perimetrini toping.

Y e c h i l i s h i. Rombning perimetri $P = 4x$ (x — romb tomoni).

Yuzini topish formulalari orqali: $h \cdot x = \frac{d_1 \cdot d_2}{2} \Rightarrow 5x = \frac{80}{2} \Rightarrow x = 8.$

Demak, $P = 4x \Rightarrow P = 32.$

J a v o b: $P = 32.$

2- m a s a l a. Agar rombning tomoni 10 ga, burchaklaridan biri esa 150° ga teng bo'lsa, uning yuzi qanchaga teng bo'ladi?

Y e c h i l i s h i. Ma'lumki, rombning yuzini topish formulalaridan biri $S = a^2 \sin\alpha$ ga asosan

$$S = 10^2 \sin 30° = 100 \cdot \frac{1}{2} = 50.$$

J a v o b: 50 kv. birlik.

3- m a s a l a. Rombning tomoni 10 ga teng. Agar uning balandligi 4 ga uzaytirilsa, yuzi 50% ga ortadi. Rombning yuzini aniqlang.

Y e c h i l i s h i. Ma'lumki, rombning yuzi: $S_{romb} = 10h$. Masala shartiga ko'ra

$$10h + \frac{h \cdot 10}{100} \cdot 50 = (h + 4) \cdot 10 \Leftrightarrow 15 \cdot h = 10h + 40 \Leftrightarrow 5h = 40 \Leftrightarrow h = 8.$$

Demak, $S_{romb} = 10 \cdot 8 = 80.$

J a v o b: $S_{romb} = 80$ kv.birlik.

Mustaqil ishlash uchun test topshiriqlari

1. Quyidagi tasdiqlarning qaysi biri noto'g'ri?

A) To'rtburchak uning ixtiyoriy tomonidan o'tuvchi to'g'ri chiziqqa nisbatan bitta yarim tekislikda yotsa, u qavariq to'rtburchak deb ataladi.

B) To'rtburchak tomonlarining yig'indisi uning perimetri deyiladi.

C) To'rtburchakning barcha ichki burchaklari yig'indisi ularga mos barcha tashqi burchaklarining yig'indisiga teng.

D) Har qanday to'rtburchak tomonlarining o'rtalarini ketma-ket tutashtirishdan hosil qilingan to'rtburchakning qarama-qarshi tomonlari o'zaro ham parallel, ham tengdir.

E) To'rtburchakning barcha ichki burchaklari yig'indisi 360° ga teng.

2. Aylanaga tashqi chizilgan $ABCD$ to'rtburchakda $AB = 6$, $AD = 4$, $DC = 3$ bo'lsa, BC tomon uzunligini toping.

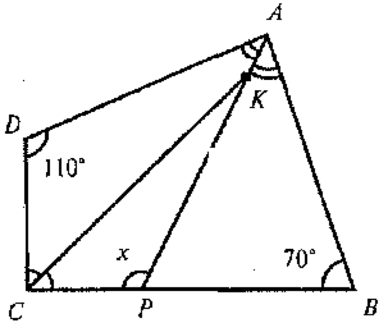
A) 4; B) 4,5; C) 5; D) 5,5; E) 6.

3. Diagonallari uzunligi 18 va 14 bo'lgan to'rtburchakning tomonlari o'rtalarini tutashtirishdan hosil bo'lgan to'rtburchakning perimetrini toping.

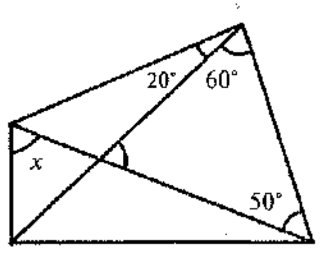
A) 26; B) 28; C) 32; D) 34; E) 36.

4. $ABCD$ to'rtburchakda AP va CK bissektrisalar, $\angle D = 110°$ va $\angle B = 70°$ bo'lsa, $\angle APC$ ni toping (126- rasm).

A) 160°; B) 150°; C) 140°; D) 130°; E) 120°.



126- rasm.



127- rasm.

5. 127- rasmda berilganlarga ko'ra x ni aniqlang.

A) 40°; B) 45°; C) 60°; D) 75°; E) 80°.

6. Perimetri 8 sm bo'lgan to'rtburchak bir tomonining uzunligi qolgan tomonlaridan mos ravishda 3 mm, 4 mm, 5 mm katta bo'lsa, uning eng katta tomonining uzunligini toping.

A) 21 mm; B) 23 mm; C) 25 mm; D) 27 mm; E) 26 mm.

7. Perimetri 66 sm bo'lgan to'rtburchakning bir tomoni ikkinchisidan 8 sm uzun, shu tomoni uchinchisidan 8 sm qisqa, to'rtinchi tomoni esa ikkinchisidan uch marta uzun bo'lsa, to'rtburchak eng kichik tomonining uzunligini toping.

A) 5 sm; B) 6 sm; C) 7 sm; D) 8 sm; E) 9 sm.

8. $ABCD$ qavariq to'rtburchakda $\angle A = \angle B = \angle C$ va $\angle D = 135°$ bo'lsa, $\angle A, \angle B, \angle C$ larni toping.

A) 75°; B) 85°; C) 70°; D) 90°; E) 60°.

9. Burchaklari kattaligi 1, 2, 4, 5 sonlariga proporsional bo'lgan qavariq to'rtburchakning eng katta burchagini toping.

A) 110°; B) 120°; C) 135°; D) 150°; E) 160°.

10. To'rtburchakning uchta ketma-ket tomonlarining uzunliklari 2, 3 va 4 ga, unga ichki chizilgan aylananing radiusi 1,2 ga teng bo'lsa, to'rtburchakning yuzini toping.

A) 7,2; B) 8,6; C) 7,8; D) 6,8; E) 8,2.

11. To'rtta nuqta aylanani yoylarining uzunligi maxraji 3 ga teng bo'lgan geometrik progressiya tashkil etuvchi bo'laklarga ajratadi. Shu nuqtalarni ketma-ket tutashtirish natijasida hosil bo'lgan to'rtburchaklarning diagonallari orasidagi kichik burchakni toping.

A) 30°; B) 45°; C) 60°; D) 70°; E) 75°.

12. To'rtburchakning bir diagonali uni perimetrlari 25 va 27 bo'lgan ikkita uchburchakka ajratadi. Agar to'rtburchakning perimetri 32 ga teng bo'lsa, o'tkazilgan diagonalning uzunligini toping.

A) 6; B) 8; C) 10; D) 11; E) 10,5.

13. Tomonlari 72 va 8 bo'lgan to'g'ri to'rtburchakka tengdosh kvadratning tomonini toping.

A) 36; B) 28; C) 24; D) 18; E) 26.

14. Kvadratning tomonini necha marta kamaytirganda yuzi 4 marta kamayadi?

A) 5; B) 2,5; C) 3; D) 4; E) 2.

15. To‘g‘ri to‘rtburchakning eni 5 ga teng, bo‘yi undan 7 ga ortiq. To‘g‘ri to‘rtburchakning perimetrini toping.

A) 32; B) 34; C) 24; D) 26; E) 30.

16. To‘g‘ri to‘rtburchakning eni 7 sm, bo‘yi undan 2 marta ortiq. To‘g‘ri to‘rtburchakning perimetrini toping.

A) 22 sm; B) 20 sm; C) 34 sm; D) 30 sm; E) 42 sm.

17. Agar kvadratning tomoni 5 marta qisqartirilsa, uning yuzi necha marta kamayadi?

A) 5; B) 10; C) 20; D) 25; E) 7,5.

18. Agar to‘g‘ri to‘rtburchakning tomonlari 4 marta orttirilsa, uning yuzi necha marta ortadi?

A) 4; B) 8; C) 12; D) 16; E) 32.

19. Agar to‘g‘ri to‘rtburchakning tomonlari 4 marta kamaytirilsa, uning yuzi necha marta kichiklashadi?

A) 16; B) $\sqrt{2}$; C) 4; D) 8; E) $2\sqrt{2}$.

20. To‘g‘ri to‘rtburchakning yuzi 400 ga, tomonlarining nisbati 4 : 1 ga teng. To‘rtburchakning perimetrini hisoblang.

A) 100; B) $100\sqrt{2}$; C) 200; D) $50\sqrt{2}$; E) 120.

21. To‘g‘ri to‘rtburchakning perimetri 60 ga teng, bir tomoni boshqa tomonidan 6 ga ortiq. To‘g‘ri to‘rtburchakning yuzini toping.

A) 196; B) 216; C) 108; D) 144; E) 180.

22. To‘g‘ri to‘rtburchakning katta tomoni 12 ga, diagonallarining kesishgan nuqtasidan katta tomonigacha bo‘lgan masofa 3 ga teng. To‘g‘ri to‘rtburchakning yuzini toping.

A) 96; B) 54; C) 48; D) 72; E) 64.

23. Diagonallari 8 va 14 bo‘lgan to‘rtburchaklarning eng katta yuzi qancha bo‘lishi mumkin?

A) 64; B) 62; C) 56; D) 52; E) 49.

24. 18 ta gugurt cho‘pidan ularni sindirmay eng katta yuzli to‘g‘ri to‘rtburchak yasalgan. Shu to‘rtburchakning yuzini toping.

A) 16; B) 20; C) 24; D) 28; E) 30.

25. To‘g‘ri to‘rtburchakning perimetri 32 ga, qo‘shni tomonlarining ayirmasi 2 ga teng. Ularning tomonlarini toping.

A) 8 va 6; B) 12 va 10; C) 10 va 8; D) 9 va 7; E) 11 va 9.

26. *ABCD* to‘g‘ri to‘rtburchakning *A* burchagi bissektrisasi *BC* tomonini uzunliklari $BM = 16$ sm va $MC = 14$ sm bo‘lgan ikki qismga ajratadi. To‘g‘ri to‘rtburchakning yuzini toping.

A) 500 sm^2; B) 420 sm^2; C) 480 sm^2; D) 510 sm^2; E) 460 sm^2.

27. Trapetsiyaning o‘rta chizig‘i 9 sm, asoslaridan biri ikkinchisidan 6 sm qisqa. Trapetsiyaning katta asosini toping.

A) 15 sm; B) 18 sm; C) 14 sm; D) 12 sm; E) 10 sm.

28. Trapetsiyaning kichik asosi 4 sm. O‘rta chizig‘i katta asosidan 4 sm qisqa. Trapetsiyaning o‘rta chizig‘ini toping.

A) 6 sm; B) 10 sm; C) 8 sm; D) 9 sm; E) 12 sm.

29. Teng yonli trapetsiyaning perimetri 36 sm, o‘rta chizig‘i esa 10 sm. Shu trapetsiyaning yon tomonini toping.

A) 10 sm; B) 8 sm; C) 9 sm; D) 13 sm; E) 12 sm.

30. Teng yonli trapetsiyaning asoslari 7 va 13 ga, o‘tmas burchagi 135° ga teng. Shu trapetsiyaning yuzini hisoblang.

A) 60; B) 30; C) $10\sqrt{3}$; D) 136,5; E) 120.

31. Teng yonli trapetsiyaning asoslari 10 va 20 ga, asosidagi burchagi 60° ga teng. Shu trapetsiyaning yuzini hisoblang.

A) $500\sqrt{3}$; B) $75\sqrt{3}$; C) $25\sqrt{3}$; D) $250\frac{\sqrt{3}}{3}$; E) 150.

32. Asoslari 12 va 16 ga, o‘tmas burchagi 120° ga teng bo‘lgan teng yonli trapetsiyaning yuzini hisoblang.

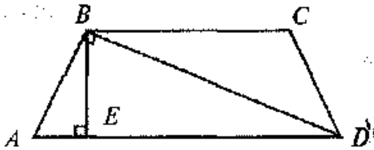
A) $56\sqrt{3}$; B) $\frac{56}{\sqrt{3}}$; C) $28\sqrt{3}$; D) 14; E) 42.

33. Trapetsiya diagonallaridan birining uzunligi 27 sm, ikkinchisi esa diagonallar kesishish nuqtasida 10 va 8 sm li kesmalarga ajraladi. Birinchi diagonal qanday uzunlikdagi kesmalarga bo‘lingan?

A) 20 va 7 sm; B) 14 va 13 sm; C) 18 va 9 sm;
D) 12 va 15 sm; E) 10 va 17 sm.

34. 128- rasmda *ABCD* teng yonli trapetsiya. Unda $\angle ABD = 90°$; $BE \perp AD$; $|AE| = 4$ va $|ED| = 9$. Trapetsiyaning balandligi *BE* ni toping.

A) 4; B) 5; C) 6;
D) 7; E) 8.



128- rasm.

35. Ushbu ta'riflar va tasdiqlardan qaysi biri noto'g'ri?

A) Faqat ikki qarama-qarshi tomonlarigina parallel bo'lgan to'rtburchak trapetsiya deb ataladi.

B) Trapetsiya qarama-qarshi burchaklarining yig'indisi 180° ga teng.

C) Trapetsiyaning parallel tomonlari uning asoslari, qolganlari yon tomonlari deyiladi. Yon tomonlari o'zaro teng bo'lsa, trapetsiya teng yonli deb ataladi.

D) Trapetsiyaning o'rta chizig'i asoslariga parallel bo'lib, ular yig'indisining yarmiga teng.

E) Trapetsiya diagonallarining o'rtalarini tutashtiruvchi kesma asoslariga parallel va ular ayirmasining yarmiga teng.

36. Teng yonli trapetsiyaning burchaklaridan biri 120° bo'lsa, shu trapetsiya kichik asosining yon tomoniga nisbati qancha bo'ladi?

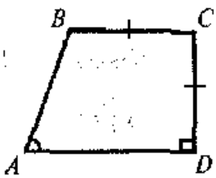
A) 2; B) $\frac{1}{2}$; C) aniqlab bo'lmaydi; D) 1,5; E) 3.

37. *ABCD* to'g'ri burchakli trapetsiyada $|BC| = |DC| = 3$ см va $\angle A = 60°$ bo'lsa (129- rasm), $|AB|$ necha santimetr?

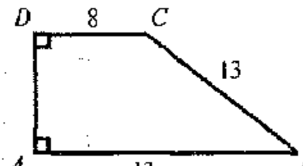
A) $(1+\sqrt{3})$ sm; B) $(\sqrt{2}+2)$ sm; C) $(3+\sqrt{3})$ sm; D) $2\sqrt{3}$ sm; E) $3\sqrt{3}$ sm.

38. *ABCD* to'g'ri burchakli trapetsiyada $|AB| = |BC| = 13$ sm, $|DC| = 8$ sm bo'lsa, shu trapetsiyaning yuzini toping (130- rasm).

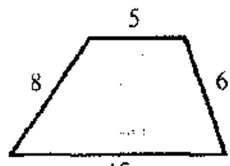
A) 64 sm²; B) 76 sm²; C) 92 sm²; D) 116 sm²; E) 126 sm².



129- rasm.



130- rasm.



131- rasm.

39. 131- rasmdagi trapetsiyaning yuzini aniqlang.

A) 24; B) 18; C) 32; D) 36; E) 48.

40. O'tkir burchagi $60°$ bo'lgan teng yonli trapetsiyaning perimetri 50 ga teng bo'lib, asoslari $1:2$ kabi nisbatda bo'lsa, uning katta asosini toping.

A) 20; B) 18; C) 22; D) 24; E) 16.

41. Trapetsiya asoslarining uzunligi 28 va 12 ga teng. Trapetsiya diagonallarining o'rtalarini tutashtiruvchi kesmaning uzunligini toping.

A) 8; B) 10; C) 6; D) 9; E) 7.

42. To'g'ri burchakli trapetsiyaning o'tkir burchagi $45°$ ga, perimetri 2 ga teng. Balandligi qanday bo'lganda shu trapetsiyaning yuzi eng katta bo'ladi?

A) 0,5; B) $\frac{\sqrt{2}}{2}$; C) $\sqrt{2}-0,5$; D) $\sqrt{2}-1$; E) $2-\sqrt{2}$.

43. $ABCD$ trapetsiyaning AC diagonali yon tomoniga perpendikular hamda DAB burchakning bissektrisasida yotadi. Agar $AC=8$ va $\angle DAB=60°$ bo'lsa, trapetsiyaning o'rta chizig'ini toping.

A) $4\sqrt{3}$; B) $3\sqrt{3}$; C) $2,5\sqrt{3}$; D) $2\sqrt{3}$; E) $1,5\sqrt{3}$.

44. Teng yonli trapetsiyaning asoslari 20 va 12 ga teng bo'lib, unga tashqi chizilgan aylananing markazi katta asosda yotadi. Trapetsiyaning diagonalini toping.

A) $8\sqrt{5}$; B) $6\sqrt{5}$; C) 16; D) 12; E) $4\sqrt{5}$.

45. Teng yonli trapetsiyaning diagonallari o'zaro perpendikular. Trapetsiyaning katta asosi $18\sqrt{2}$ ga, kichik asosi $6\sqrt{2}$ ga teng. Shu trapetsiyaning yuzini toping.

A) $264\sqrt{2}$; B) $238\sqrt{2}$; C) 290; D) 288; E) 248.

46. Teng yonli trapetsiyaning yon tomoni va kichik asosi b ga, katta asosiga yopishgan burchagi α ga teng. Shu trapetsiyaning yuzini toping.

A) $2b^2\sin\alpha$; B) $b^2\sin2\alpha$; C) $\frac{1}{2}b\sin\alpha$; D) $b^2\sin\alpha\cos^2\frac{\alpha}{2}$;

E) $2b^2\sin\alpha\cos^2\frac{\alpha}{2}$.

47. Parallelogrammning diagonali tomonlari bilan 20° va 50° li burchaklar tashkil etadi. Parallelogrammning katta burchagini toping.

A) 100°; B) 110°; C) 120°; D) 130°; E) 150°.

48. Parallelogramm burchaklaridan ikkitasining ayirmasi 70° ga teng. Shu burchaklarni toping.

A) 45°, 125°; B) 65°, 135°; C) 75°, 105°; D) 55°, 125°;
E) 60°, 130°.

49. Tomonlari 4 va 8 m bo'lgan parallelogrammning yuzi 16 m^2. Parallelogrammning o'tmas burchagini toping.

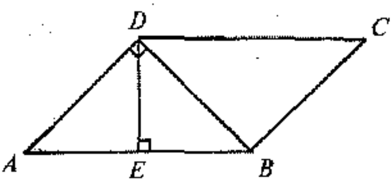
A) 120°; B) 150°; C) 135°; D) 105°; E) 160°.

50. *ABCD* parallelogrammda $AD = 3$ sm, $S_{ABCD} = 12$ sm^2, DE – ? (132- rasm).

A) 2 sm; B) 2,2 sm; C) 2,3 sm; D) 2,1 sm; E) 2,4 sm.

51. *ABCD* parallelogrammda $AD \parallel BC$, $AB \parallel DC$, $DB = 4$ sm, $S_{ABCD} = 12$ sm^2, AB – ? (132- rasm).

A) 6 sm; B) 7 sm; C) 5,5 sm;
D) 5 sm; E) 6,5 sm.



132- rasm.

52. Parallelogramm qo'shni tomonlarining yig'indisi 10 ga, ayirmasi esa 6 ga teng. Shu parallelogramm diagonallari kvadratlarining yig'indisini toping.

A) 120; B) 20; C) 136; D) 64; E) 32.

53. Parallelogrammning 5 ga teng bo'lgan diagonali uning 12 ga teng bo'lgan tomoniga perpendikular. Parallelogrammning perimetrini toping.

A) 50; B) 34; C) 100; D) 48; E) 68.

54. Parallelogrammning tomonlaridan biri ikkinchisidan 4 marta katta. Agar uning perimetri $20\sqrt{2}$ ga, o'tkir burchagi 45° ga teng bo'lsa, yuzini toping.

A) $8\sqrt{2}$; B) $32\sqrt{2}$; C) 16; D) 8; E) $16\sqrt{2}$.

55. Parallelogrammning burchaklaridan biri 150° ga teng. Uning 6 ga teng bo'lgan diagonali tomoniga perpendikular. Parallelogrammning perimetrini toping.

A) 36; B) 48; C) $12(2+\sqrt{3})$; D) 36; E) $36\sqrt{3}$.

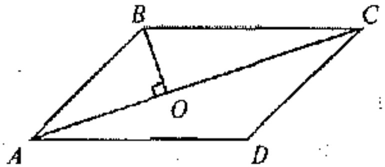
56. Parallelogrammning o'tkir burchagi 60° ga teng. Uning kichik diagonali katta tomoni bilan 30° li burchak tashkil qiladi. Parallelogrammning katta tomoni 20 ga teng. Uning yuzini toping.

A) $100\sqrt{2}$; B) 85; C) $95\sqrt{3}$; D) $100\sqrt{3}$; E) $110\sqrt{3}$.

57. *ABCD* parallelogrammda $AD = DB$, $S_{ABCD} = 32$ sm^2, DE — ? (132- rasm).

A) 4 sm; B) 4,5 sm;

C) 3 sm; D) 3,5 sm; E) 5 sm.



133- rasm.

58. *ABCD* parallelogrammda $(OB) \perp (AC)$, $|AO| = 8$, $|OC| = 6$ va $|BO| = 4$ (133- rasm). S_{ABCD} — ?

A) 50; B) 28; C) 56; D) 52; E) 32.

59. Parallelogrammning tomonlari *a* va *b* ga, o'tmas burchagi α ga teng. Parallelogrammning yuzini hisoblash uchun quyidagi ifodalardan qaysi biri to'g'ri?

A) $ab\cos\alpha$; B) $\frac{1}{2}ab\cos\alpha$; C) $ab\sin\alpha$; D) $\frac{ab}{2\sin\alpha}$;

E) $\frac{1}{2}ab\sin\alpha$.

60. Perimetri 60 ga teng bo'lgan parallelogrammning balandliklari nisbati 2:3 kabi bo'lsa, uning katta tomoni uzunligini toping.

A) 20; B) 18; C) 15; D) 13; E) 12.

61. Parallelogramm o'tkir burchagining bissektrisasi uning diagonallarini 3,2 va 8,8 bo'lgan kesmalarga ajratadi. Agar parallelogrammning perimetri 30 ga teng bo'lsa, uning katta tomonini toping.

A) 12; B) 11; C) 10; D) 9; E) 8.

62. 1) Rombning tomoni 10 ga, kichik diagonali 12 ga teng. Rombning yuzini hisoblang.

A) 98; B) 96; C) 94; D) 102; E) 92.

63. Rombning tomoni 6 ga, o'tkir burchagi 30° ga teng. Uning diagonallarining ko'paytmasini toping.

A) 27; B) 18; C) 42; D) 36; E) 28.

64. 1) Rombning diagonallari 3 : 4 kabi nisbatda, yuzi esa 384 ga teng. Rombning tomonini toping.

A) 18; B) 20; C) 24; D) 28; E) 30.

65. Rombning tomoni 6 ga, yuzi 18 ga teng. Rombning o'tmas burchagini toping.

A) 135°; B) 120°; C) 150°; D) 140°; E) 165°.

66. Tomoni 4 sm bo'lgan rombga ichki chizilgan aylananing radiusi 1 sm. Rombning o'tkir burchagi kosinusini toping.

A) $\frac{3}{4}$; B) $\frac{2\sqrt{2}}{3}$; C) $\frac{4}{5}$; D) $\frac{\sqrt{3}}{2}$; E) $\frac{\sqrt{5}}{3}$.

67. Uzunligi 2π ga teng aylana o'tkir burchagi 30° bo'lgan rombga ichki chizilgan. Rombning perimetrini toping.

A) 2; B) 10; C) 8; D) 4; E) 16.

68. Balandligi 28 ga teng bo'lgan rombga ichki chizilgan doiraning yuzini hisoblang.

A) 198π; B) 190π; C) 192π; D) 200π; E) 196π.

69. Rombning $3\sqrt{3}$ ga teng bo'lgan balandligi tomonini teng ikkiga bo'ladi. Rombning perimetrini toping.

A) $12\sqrt{3}$; B) 24; C) 36; D) $36\sqrt{3}$; E) 48.

70. Rombning tomonlari a ga, o'tmas burchagi α ga teng. Rombning yuzini hisoblash uchun quyida keltirilgan ifodalardan qaysi biri to'g'ri?

A) $a^2 \cos\alpha$; B) $0{,}5a^2 \cos\alpha$; C) $0{,}5a^2 \sin\alpha$;

D) $a^2 \sin\alpha$; E) $\frac{a}{\sin\alpha}$.

71. Perimetri $2p$ ga, diagonallarining yig'indisi m ga teng bo'lgan rombning yuzini toping.

A) $\frac{m^2+p^2}{2}$; B) $\frac{m^2-p^2}{2}$; C) $\frac{m^2+p^2}{4}$; D) $\frac{m^2-p^2}{4}$; E) $\frac{m^2p^2}{4}$.

72. Agar rombning bir diagonalini 10% uzaytirib, ikkinchi diagonali 15% qisqartirilsa, rombning yuzi qanday o'zgaradi?

A) 5% ortadi; B) o'zgarmaydi; C) 6,5% kamayadi;

D) 5,65% kamayadi; E) 6,5% ortadi.

73. Yuzi Q ga teng bo'lgan doiraga o'tmas burchagi 150° bo'lgan romb tashqi chizilgan. Rombning yuzini hisoblang.

A) $\frac{8Q}{\pi}$; B) $\frac{4Q}{\pi}$; C) $2Q\pi$; D) $\frac{6Q}{\pi}$; E) $\frac{3}{2}\pi Q$.

74. Rombning tomoni 10 sm. Agar uning balandligi 4 sm ga uzaytirilsa, yuzi 50% ga ortadi. Rombning yuzini (sm^2) toping.

A) 40; B) 60; C) 80; D) 100; E) 50.

75. Tomoni 16 ga va o'tkir burchagi 30° ga teng rombga ichki chizilgan aylananing diametrini toping.

A) 6; B) 7; C) 8; D) 9; E) 10.

76. To'g'ri burchakli uchburchakning burchaklaridan biri 60° ga teng. Bu uchburchakka romb shunday ichki chizilganki, 60° li burchak umumiy, rombning qolgan uchlari uchburchakning tomonlarida yotadi. Agar rombning tomoni $\frac{\sqrt{12}}{5}$ ga teng bo'lsa, berilgan uchburchakning katta katetini toping.

A) 1,8; B) 2,4; C) $\frac{3\sqrt{3}}{5}$; D) $\frac{6\sqrt{3}}{5}$; E) 2,2.

77. Romb tomonining uning diagonallari bilan tashkil qilgan burchaklari nisbati 5:4 kabi. Rombning o'tmas burchagini toping.

A) 96°; B) 100°; C) 120°; D) 130°; E) 150°.

78. Diagonallari 32 va 24 ga teng bo'lgan rombning o'tmas burchagi kotangensini toping.

A) $-\frac{5}{21}$; B) $-\frac{7}{24}$; C) $-\frac{24}{7}$; D) $-\frac{7}{16}$; E) $-\frac{3}{7}$.

79. Romb diagonallari 6 va 8 ga teng. Unga ichki chizilgan doira yuzining romb yuziga nisbatini toping.

A) $3\pi : 4$; B) $3\pi : 8$; C) $6\pi : 11$; D) $9\pi : 25$; E) $6\pi : 25$.

80. Diagonali orqali ikkita muntazam uchburchakka ajraladigan rombga ichki chizilgan aylananing radiusi r ga teng. Rombning yuzini toping.

A) $2r^2\sqrt{3}$; B) $4r^2$; C) $\frac{4r^2\sqrt{3}}{3}$; D) $4r^2\sqrt{2}$; E) $\frac{8r^2\sqrt{3}}{3}$.

IV bob. KO'PBURCHAKLAR

1- §. Qavariq ko'pburchaklar

1.1. Siniq chiziq.

Ta'rif. *A_1, A_2, ..., A_n nuqtalar va ularni tutashtiruvchi A_1A_2, A_2A_3, ..., $A_{n-1}A_n$ kesmalardan iborat shakl $A_1A_2A_3$... A_n siniq chiziq deyiladi* (134- rasm).

Agar siniq chiziq o'zi bilan kesishmasa, *sodda siniq chiziq* deyiladi (135- rasm). Aks holda *yulduzsimon siniq chiziq* deb ataladi (136- rasm).

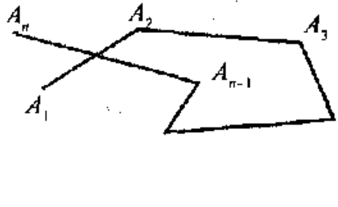
Siniq chiziqning boshlang'ich nuqtasi bilan oxirgi nuqtasi ust-ma-ust tushsa, *yopiq siniq chiziq* deyiladi (137- rasm).

A_1, A_2, ..., A_n nuqtalar *siniq chiziqning uchlari,* A_1A_2, A_2A_3, ..., $A_{n-1}A_n$ kesmalar *siniq chiziqning bo'g'inlari* deyiladi.

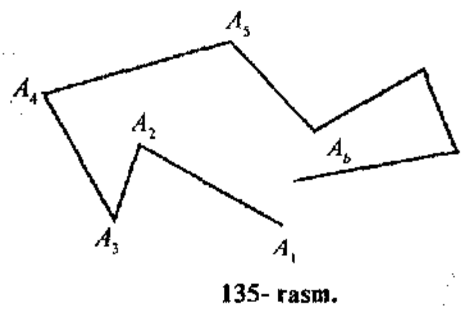
1.2. Ko'pburchak. Ta'rif. *Sodda yopiq siniq chiziq bilan uning ichki sohasining birlashmasi ko'pburchak deyiladi* (137-, 138- rasmlar).

Siniq chiziqning uchlari *ko'pburchakning uchlari*, bo'g'inlari esa *ko'pburchakning tomonlari* deb ataladi.

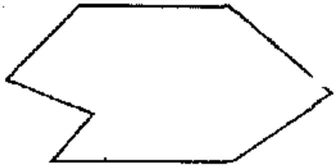
Agar ko'pburchak ixtiyoriy tomonini ichiga olgan to'g'ri chiziqqa nisbatan bitta yarim tekislikda yotsa, u *qavariq ko'pburchak* deyiladi (138- rasm). Aks holda *botiq ko'pburchak* deb ataladi (137- rasm).



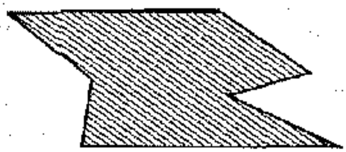
134- rasm.



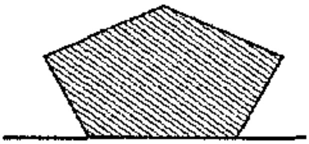
135- rasm.



136- rasm.



137- rasm.



138- rasm.

Ko'pburchakning diagonali deb uning bir tomoniga tegishli bo'lmagan ikki uchini tutashtiruvchi kesmaga aytiladi. n burchakning har bir uchidan $(n-3)$ ta diagonal chiqadi. Demak, n ta uchidan $n(n-3)$ ta diagonal chiqadi. Bunda har bir diagonal ikki marta hisobga olinganiga e'tibor bersak, n burchakning barcha diagonallari soni: $\frac{n(n-3)}{2}$ formula bilan hisoblanadi $(n > 3)$. Agar ko'pburchak diagonallari soni ma'lum bo'lsa, shu formula asosida uning tomonlari sonini ham aniqlashimiz mumkin.

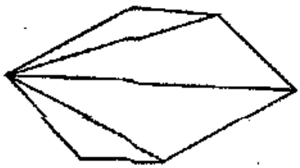
1- masala. To'qqizburchakning nechta diagonali bor?

Yechilishi: $\frac{n(n-3)}{2}$; $n = 9$; $\frac{9(9-3)}{2} = 27$.

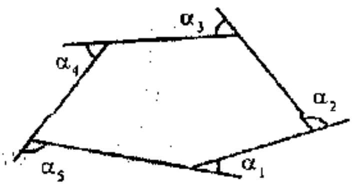
Javob: 27 ta.

Ko'pburchakning *ichki burchagi* (yoki uchidagi burchagi) deb uning bir uchidan chiqqan ikki tomoni orasidagi burchakka aytiladi.

Ko'pburchakning barcha ichki burchaklari yig'indisini topaylik. Ma'lumki, uchburchakning uchala ichki burchaklari yig'indisi 180° ga teng. Ko'pburchakning bir uchidan chiquvchi barcha diagonallari uni $(n-2)$ ta uchburchakka ajratadi. Shuning uchun ko'p-



139- rasm.



140- rasm.

burchakning barcha ichki burchaklari yig‘indisi $180°(n-2)$ ga tengdir (139- rasm).

Ko‘pburchakning ichki burchagiga qo‘shni burchak uning *tashqi burchagi* deyiladi (140- rasm).

Ko‘pburchakning barcha tashqi burchaklari yig‘indisi 360° ga tengligini isbotlang.

I s b o t i: n burchakning har bir uchidagi ichki va bitta tashqi burchagining yig‘indisi 180° ga, n ta uchidagi shunday burchaklari yig‘indisi $180° \cdot n$ ga teng. Demak, barcha tashqi burchaklari yig‘indisi: $180° \cdot n - 180°(n-2) = 180° \cdot n - 180° \cdot n + 360° = 360°$.

2- m a s a l a. Ko‘pburchakning diagonallari soni tomonlari sonidan 2,5 marta ko‘p. Ko‘pburchakning tomonlari nechta?

Y e c h i l i s h i. n burchakning diagonallari soni $\frac{n(n-3)}{2}$ ga teng. Masala shartiga ko‘ra: $2{,}5n = \frac{n(n-3)}{2}$, bundan $5n = n^2 - 3n \Rightarrow$ $\Rightarrow 8n = n^2 \Rightarrow n = 8$.

J a v o b: $n = 8$.

3- m a s a l a. Qavariq 20 burchakning diagonallari nechta?

Y e c h i l i s h i. Ma’lumki, n burchakning diagonallari soni $\frac{n(n-3)}{2}$. Bunda $n = 20$ bo‘lgani uchun $\frac{20(20-3)}{2} = 170$.

J a v o b: 170 ta.

4- m a s a l a. Agar ko‘pburchakning diagonallari soni 90 ta bo‘lsa, uning tomonlari nechta bo‘ladi?

Y e c h i l i s h i. Tomonlari soni n ta bo‘lsin, u $\frac{n(n-3)}{2} = 90$ tenglamaning natural echimiga teng bo‘ladi. Shuning uchun $n^2 - 3n - 180 = 0$ ni yechib, $n = 15$ ekanini aniqlaymiz.

J a v o b: $n = 15$.

Umuman, 3- masaladagi shart bilan berilgan masalada $n(n-3)=$ $=2d$ (d — ko'pburchak diagonallari soni) tenglama natural yechimga ega bo'lmasa, «diagonallari soni d bo'lgan qavariq ko'pburchak mavjud emas» deyiladi.

5- m a s a l a. Qavariq beshburchakning barcha ichki burchaklarining yig'indisi necha gradus?

Y e c h i l i s h i. Beshburchakning bir uchidan chiqqan barcha diagonallari uni uchta uchburchakka bo'ladi. 3 ta uchburchak ichki burchaklarining yig'indisi $3 \cdot 180° = 540°$ ga teng.

J a v o b: 540°.

6- m a s a l a. Barcha ichki burchaklari yig'indisi 1600° bo'lgan ko'pburchak tomonlari soni nechta?

Y e c h i l i s h i. Masala yechimga ega bo'lishi uchun $180° \cdot (n-2)=$ $=1600°$ tenglama natural yechimga ega bo'lishi kerak. O'sha yechim masala savoliga javob bo'ladi. Agar tenglama natural yechimga ega bo'lmasa, masala ham yechimga ega bo'lmaydi, ya'ni ichki burchaklari yig'indisi 1600° bo'lgan ko'pburchak mavjud emas.

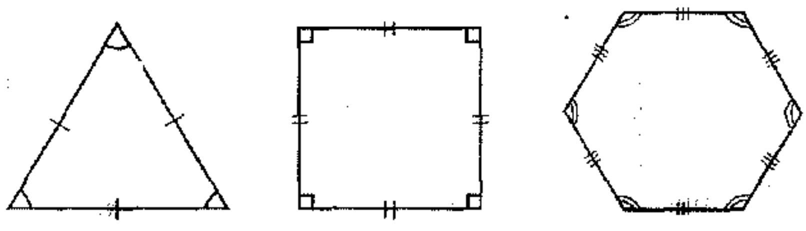
J a v o b: Unday ko'pburchak mavjud emas.

2- §. Muntazam ko'pburchaklar

T a ' r i f. *Hamma tomonlari o'zaro teng va barcha burchaklari bir-biriga teng bo'lgan qavariq ko'pburchak muntazam ko'pburchak deb ataladi.*

141- rasmda muntazam uchburchak, teng tomonli to'rtburchak (kvadrat) hamda muntazam oltiburchak tasvirlangan.

2.1. Muntazam ko'pburchakka ichki va tashqi chizilgan aylanalarning mavjudligi. Har qanday muntazam ko'pburchakda uning uch-



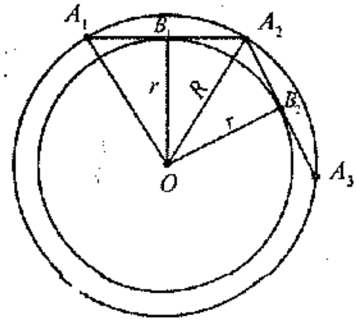
141- rasm.

laridan baravar uzoqlikda yotgan nuqta (tashqi chizilgan aylana markazi O) mavjud (142- rasm).

Ixtiyoriy muntazam ko'pburchakning tomonlaridan bir xil masofada yotuvchi (ko'pburchakka ichki chizilgan aylana markazi O) nuqta mavjud.

T e o r e m a. Har qanday muntazam *(142- rasm)* ***ko'pburchakka tashqi* aylana chizish mumkin.**

T e o r e m a. Ixtiyoriy muntazam ko'pburchakka ichki aylana chizish mumkin (142- rasm).



142- rasm.

$[A_1O]=[A_2O]= ... = R$ — tashqi chizilgan aylana radiusi.

$[B_1O]=[B_2O]= ... = r$ — ichki chizilgan aylana radiusi.

$(O;\ R)$ — tashqi aylana. $(O;\ r)$ — ichki aylana.

Tashqi va ichki chizilgan aylanalar *o'zaro konsentrik aylanalar*; ularning umumiy markazi *muntazam ko'pburchakning markazi*, aniqrog'i, *simmetriya markazi* deyiladi.

2.2. Muntazam ko'pburchakdagi metrik munosabatlar.

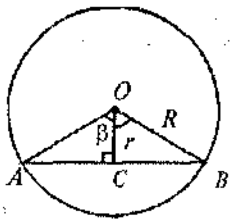
1) Muntazam n burchakning ichki burchagi $\alpha_n = \dfrac{180^\circ(n-2)}{n}$ ga teng. Xususan: $\alpha_3 = 60^\circ$, $\alpha_4 = 90^\circ$, $\alpha_6 = 120^\circ$.

Tomonining uzunligi a_n ga teng bo'lgan muntazam n burchakka tashqi chizilgan aylana radiusi R va ichki chizilgan aylana radiusi r bo'lsa (143- rasm),

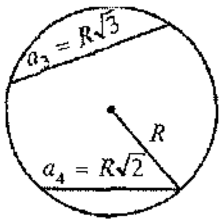
$$a_n = 2R\cdot\sin\frac{180^\circ}{n} \Rightarrow R = \frac{a_n}{2\sin\frac{180^\circ}{n}};\quad r = OC = \frac{a_n}{2\operatorname{tg}\frac{180^\circ}{n}}$$

bo'ladi.

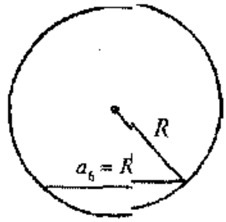
Haqiqatan ham, ΔAOB da $\beta = \angle AOC = \frac{1}{2}\angle AOB = \frac{1}{2}\cdot\frac{360^\circ}{n} = \frac{180^\circ}{n}$ bo'lib, uchburchakdagi metrik munosabatlardan foydalanib yuqoridagi ifodalar keltirib chiqariladi.



143- rasm.



144- rasm.



145- rasm.

Xususan, $n = 3$ da muntazam uchburchak uchun $\beta = \frac{180^\circ}{3} = 60^\circ$; $R = \frac{a_3}{2\sin 60^\circ} = \frac{a_3}{\sqrt{3}} \Rightarrow a_3 = R\sqrt{3}$ (144- rasm); $r = \frac{a_3}{2\,\mathrm{tg}\,60^\circ} = \frac{a_3}{2\sqrt{3}} \Rightarrow a_3 = 2\sqrt{3}\,r$ bo'ladi.

$n = 4$ da, ya'ni muntazam to'rtburchak uchun $\beta = \frac{180^\circ}{4} = 45^\circ$ bo'lib,

$$R = \frac{a_4}{2\sin 45^\circ} = \frac{a_4}{\sqrt{2}} \Rightarrow a_4 = R \cdot \sqrt{2} \text{ (144- rasm),}$$

$$r = \frac{a_4}{2\,\mathrm{tg}\,45^\circ} = \frac{a_4}{2} \Rightarrow a_4 = 2r \text{ bo'ladi.}$$

$n = 6$ da, ya'ni muntazam oltiburchak uchun $\beta = \frac{180^\circ}{6} = 30^\circ$

bo'lib, $R = \frac{a_6}{2\sin 30^\circ} = \frac{a_4}{2 \cdot \frac{1}{2}} = a_6 \Rightarrow R = a_6$ (145- rasm),

$$r = \frac{a_6}{2\,\mathrm{tg}\,30^\circ} = \frac{a_6\sqrt{3}}{2} \Rightarrow a_6 = \frac{2r}{\sqrt{3}} \text{ bo'ladi.}$$

T e o r e m a. Muntazam ko'pburchakning yuzi uning perimetri bilan ichki chizilgan aylana radiusi ko'paytmasining yarmiga teng:

$$S = \frac{1}{2} p \cdot r .$$

Teorema. ***Muntazam n burchakning S_n yuzi:***

$$S_n = \frac{1}{2} nR^2 \sin\frac{360°}{n}$$

ga teng, bunda R — tashqi chizilgan aylana radiusi.

2.3. Muntazam ko'pburchaklarning o'xshashligi. Muntazam qavariq n burchaklar o'zaro o'xshashdir, ya'ni o'xshashlik almashtirish bilan ularning birini ikkinchisiga akslantirish mumkin.

O'xshash shakllarda o'xshashlik koeffitsiyenti mos chiziqli o'lchamlari nisbatiga teng. Muntazam n burchakda chiziqli o'lchamlari ularning perimetrlari, xususan tomonlar uzunliklari, ichki va tashqi chizilgan aylana radiuslari uzunliklari bo'la oladi.

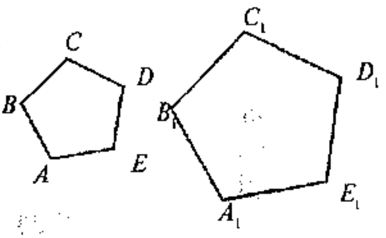
Agar ikki muntazam n burchak perimetrlari P_n, P_n', tomonlari a_n, a_n', ichki chizilgan aylanalar radiuslari r, r' va tashqi chizilgan aylanalar radiuslari R, R' bo'lsa,

$$\frac{P_n'}{P_n} = \frac{a_n'}{a_n} = \frac{r'}{r} = \frac{R'}{R} = k$$

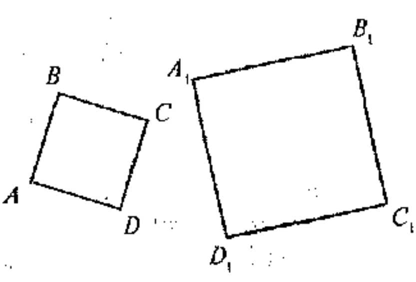
bo'lib, bunda k o'xshashlik koeffitsiyenti hisoblanadi.

O'xshash ko'pburchaklarning birini ikkinchisiga akslantirishda mantiqan fikrlab k yoki $\frac{1}{k}$ koeffitsiyentdan foydalaniladi.

Masalan, $\frac{A_1B_1}{AB} = k$ bo'lsa, $A_1B_1C_1D_1E_1$ ko'pburchakni $ABCDE$ ga o'xshash almashtirishda $\frac{1}{k}$ koeffitsiyentdan foydalaniladi, aks holda $ABCDE$ ko'pburchakni $A_1B_1C_1D_1E_1$ ko'pburchakka k koeffitsiyent bo'yicha akslantirish bajariladi (146-, 147- rasmlar).



146- rasm.



147- rasm.

1- m a s a l a. Aylanaga ichki chizilgan muntazam uchburchak perimetri 18 ga teng bo'lsa, shu aylanaga ichki chizilgan muntazam to'rtburchak perimetrini toping.

Y e c h i l i s h i: Masala shartiga ko'ra muntazam uchburchak tomoni $a_3 = 6$ ga teng, ya'ni $R = \frac{6}{\sqrt{3}}$ bo'lib, $a_4 = R \cdot \sqrt{2} = \frac{6}{\sqrt{3}} \cdot \sqrt{2} =$

$= 2\sqrt{6}$; $P_4 = 4a_4 = 8\sqrt{6}$.

J a v o b: $P_4 = 8\sqrt{6}$.

2- m a s a l a. Muntazam uchburchakka ichki chizilgan aylana radiusini tashqi chizilgan aylana radiusiga nisbatini toping.

Y e c h i l i s h i: $\frac{r}{R} = ?$ $r = \frac{a_3}{2\,\mathrm{tg}\,60^\circ} = \frac{a_3}{2\sqrt{3}}$; $R = \frac{a_3}{2\sin 60^\circ} = \frac{a_3}{\sqrt{3}}$;

$$\frac{r}{R} = \frac{\frac{a_3}{2\sqrt{3}}}{\frac{a_3}{\sqrt{3}}} = \frac{a_3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{a_3} = \frac{1}{2}.$$

J a v o b: $\frac{r}{R} = \frac{1}{2}$.

3- m a s a l a. *R radiusli aylanaga ichki chizilgan* muntazam sakkizburchak tomoni a_8 ning R orqali ifodasini toping.

Yechilishi: $R = \frac{a_8}{2\sin\frac{180}{8}}$ dan $a_8 = 2R \cdot \sin 22{,}5^\circ = 2R \cdot \sqrt{\frac{1-\cos 45^\circ}{2}} =$

$= 2R \cdot \sqrt{\frac{2-\sqrt{2}}{4}} = R\sqrt{2-\sqrt{2}}$.

J a v o b: $a_8 = R\sqrt{2-\sqrt{2}}$.

Mustaqil ishlash uchun test topshiriqlari

1. Ichki burchaklari yig'indisi uning har bir uchidan bittadan olingan tashqi burchaklari yig'indisidan 6 marta katta bo'lgan ko'pburchakning tomoni nechta?

A) 16; B) 10; C) 15; D) 12; E) 14.

2. Agar qavariq ko'pburchak ichki burchaklarining yig'indisi tashqi burchaklari yig'indisidan 4 marta katta bo'lsa, uning tomonlari nechta?

A) 5; B) 6; C) 10; D) 8; E) 12.

3. α_3, α_4 va α_5 mos ravishda uchburchak, to'rtburchak va beshburchak tashqi burchaklarining yig'indilari. Quyidagi munosabatlardan qaysi biri o'rinli?

A) $\alpha_3 < \alpha_4 < \alpha_5$; B) $\alpha_3 = \alpha_4 < \alpha_5$; C) $\alpha_3 < \alpha_4 = \alpha_5$;
D) $\alpha_3 = \alpha_5 < \alpha_4$; E) $\alpha_3 = \alpha_4 = \alpha_5$.

4. Qavariq beshburchak burchaklaridan ikkitasi to'g'ri burchak, qolganlari o'zaro 2 : 3 : 4 kabi nisbatda. Beshburchakning katta burchagini toping.

A) 90°; B) 120°; C) 150°; D) 110°; E) 160°.

5. Har bir ichki burchagi 150° bo'lgan qavariq ko'pburchakning nechta tomoni bor?

A) 5; B) 7; C) 10; D) 12; E) 15.

6. Qavariq o'nburchakning nechta diagonali bor?

A) 30; B) 36; C) 35; D) 24; E) 54.

7. Qavariq ko'pburchakning diagonallari soni tomonlari sonidan 12 ta ko'p. Ko'pburchakning tomonlari nechta?

A) 5; B) 6; C) 8; D) 9; E) 10.

8. 148- chizmada $\angle A = x° + 10°$, $\angle B = x°$, $\angle C = x° - 5°$, $\angle D = x° - 10°$, $\angle E = x° + 5°$. Shu ko'pburchakning eng katta burchagi eng kichik burchagidan necha gradus ortiq?

A) 5°; B) 8°; C) 10°; D) 15°; E) 20°.

9. 149- chizmada BE bissektrisa, $\angle ADC = 100°$ va $\angle BCE = 40°$ bo'lsa, $\angle BED$ — ?

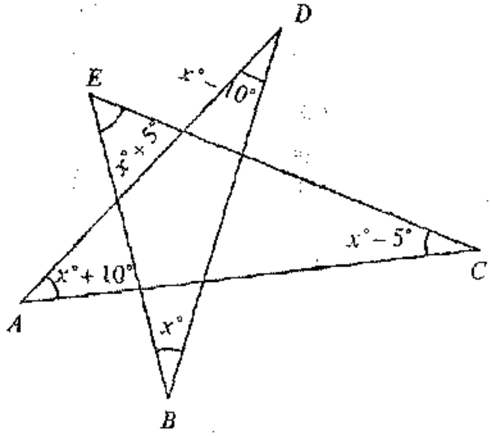
A) 50°; B) 60°; C) 70°; D) 75°; E) 80°.

10. Quyidagi tasdiqlarning qaysi biri noto'g'ri?

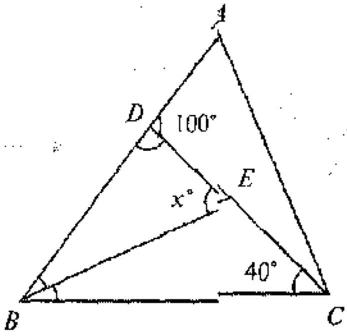
A) Qavariq n burchakning diagonallari soni $\frac{n(n-3)}{2}$ ga teng.

B) Qavariq n burchakning barcha ichki burchaklari yig'indisi $(n-2) \cdot 180°$.

C) Qavariq n burchakning barcha tomonlari yig'indisi uning perimetri deyiladi.



148- rasm.



149- rasm.

D) Qavariq n burchakning barcha tashqi burchaklari yig'indisi 360° ga teng.

E) Barcha tomonlari bir-biriga teng bo'lgan qavariq ko'pburchakning diagonallari ham o'zaro teng bo'ladi.

11. n ning qanday qiymatida qavariq n burchakning tomonlari soni diagonallari soniga teng bo'ladi?

A) 4; B) 5; C) 6; D) 7; E) 8.

12. Agar qavariq ko'pburchakning barcha ichki burchaklari bilan bitta tashqi burchagi yig'indisi $\frac{23\pi}{2}$ ga teng bo'lsa, uning tomonlari nechta?

A) 11; B) 12; C) 13; D) 14; E) 15.

13. Aylanaga ichki chizilgan muntazam uchburchakning tomoni a ga teng. Shu aylanaga ichki chizilgan kvadrat tomonini toping.

A) $a\sqrt{\frac{2}{3}}$; B) $a\sqrt{3}$; C) $a\sqrt{2}$; D) $a\sqrt{\frac{3}{2}}$; E) $a\sqrt{6}$.

14. Radiusi 4 bo'lgan aylanaga muntazam uchburchak ichki chizilgan bo'lib, bu uchburchak tomoniga kvadrat yasalgan. Kvadratga tashqi chizilgan aylana radiusini toping.

A) $2\sqrt{3}$; B) $2\sqrt{6}$; C) $4\sqrt{3}$; D) $3\sqrt{2}$; E) $\sqrt{3-\sqrt{2}}$.

15. R radiusli aylanaga ichki chizilgan muntazam o'n ikki burchak tomonining (a_{12}) ifodasini yozing.

A) $R\sqrt{2-\sqrt{2}}$; B) $R\sqrt{3-\sqrt{3}}$; C) $R\sqrt{2-\sqrt{3}}$;

D) $R\sqrt{3-\sqrt{2}}$; E) $R(1-\sqrt{3})$.

16. Bir muntazam n burchakka ichki va tashqi chizilgan aylanalar radiuslari r_1 va R_1 ga teng, ikkinchi muntazam n burchakka ichki chizilgan aylana radiusi r_2 ga teng. Unga tashqi chizilgan aylananing radiusi nimaga teng?

A) $\frac{r_1}{R_1 \cdot r_2}$; B) $\frac{r_2}{R_1 \cdot r_1}$; C) $\frac{R_1 \cdot r_1}{r_2}$; D) $\frac{R_1 \cdot r_2}{r_1}$; E) $\frac{r_1 \cdot r_2}{R_1}$.

17. Muntazam oltiburchakka tashqi chizilgan aylananing radiusi 12 ga teng. Uning kichik diagonalini toping.

A) $12\sqrt{2}$; B) $12\sqrt{3}$; C) $6\sqrt{5}$; D) $8\sqrt{5}$; E) $9\sqrt{5}$.

18. Kichik diagonali $12\sqrt{3}$ bo'lgan muntazam oltiburchakka tashqi chizilgan aylananing radiusini toping.

A) $4\sqrt{3}$; B) $6\sqrt{3}$; C) 12; D) 14; E) $8\sqrt{3}$.

19. Aylanaga ichki chizilgan muntazam oltiburchakning tomoni 20 ga teng. Shu aylanaga kvadrat ham ichki chizilgan. Kvadratga ichki chizilgan doiraning yuzini toping.

A) 400π; B) 300π; C) 150π; D) 200π; E) 250π.

20. Muntazam oltiburchakka tashqi chizilgan aylananing radiusi $\sqrt{3}$ bo'lsa, unga ichki chizilgan aylananing radiusini toping.

A) 1,5; B) $\frac{\sqrt{3}}{2}$; C) $\frac{\sqrt{6}}{2}$; D) 1,2; E) 1.

21. Muntazam oltiburchakka tashqi chizilgan aylananing uzunligi 4π ga teng. Shu ko'pburchakning yuzini toping.

A) 6; B) $\sqrt{3}$; C) $6\sqrt{3}$; D) $4\sqrt{3}$; E) 12.

22. Muntazam ko'pburchakning perimetri 60 ga, unga ichki chizilgan aylananing radiusi 8 ga teng. Shu ko'pburchakning yuzini hisoblang.

A) 240; B) 480; C) 120; D) 60; E) 180.

23. Radiusi R ga teng bo'lgan aylanaga ichki chizilgan muntazam oltiburchakning tomonini toping.

A) R; B) $\frac{2R}{\sqrt{3}}$; C) $\sqrt{32}$; D) $\sqrt{2R}$; E) $\frac{R}{2}$.

24. Radiusi R ga teng bo'lgan aylanaga tashqi chizilgan muntazam oltiburchakning tomonini toping.

A) $\frac{\sqrt{3}R}{2}$; B) $\sqrt{3}R$; C) $\frac{4}{3}R$; D) $\frac{3}{4}R$; E) $\frac{2}{\sqrt{3}}R$.

25. R radiusli aylanaga tashqi chizilgan muntazam o'n ikki burchakning tomonini toping.

A) $\frac{2\sqrt{32}}{3}$; B) $\frac{2\sqrt{2-\sqrt{2}R}}{\sqrt{2+\sqrt{2}}}$; C) $1{,}2R$; D) $2(2-\sqrt{3})R$;

E) $1{,}5R$.

26. Muntazam oltiburchakka tashqi chizilgan aylananing radiusi $5\sqrt{3}$ ga teng. Uning parallel tomonlari orasidagi masofani toping.

A) 10; B) 12; C) 15; D) 16; E) 17.

27. Muntazam oltiburchakning kichik diagonali $6\sqrt{3}$ ga teng. Shu oltiburchakka tashqi chizilgan aylananing uzunligini toping.

A) 8π; B) 9π; C) 12π; D) 16π; E) 18π.

28. Muntazam n burchakning ichki burchagi tashqi burchagidan 5 marta katta bo'lsa, bu ko'pburchakning diagonallari soni nechta?

A) 32; B) 35; C) 36; D) 42; E) 54.

29. Qavariq ko'pburchakning diagonallari uning tomonlaridan 12 ta ko'p. Ko'pburchakning tomonlari nechta?

A) 5; B) 6; C) 8; D) 9; E) 10.

30. Eng kichik burchagi $50°$ bo'lgan biror qavariq ko'pburchakning ichki burchaklari arifmetik progressiyani tashkil qiladi. Arifmetik progressiya ayirmasi $10°$ bo'lsa, bu ko'pburchakning tomoni eng ko'pi bilan nechta bo'lishi mumkin?

A) 30 ta; B) 27 ta; C) 24 ta; D) 5 ta; E) 3 ta.

V bob. AYLANA VA DOIRA

1- §. Aylana

1.1. Aylananing asosiy elementlari va xossalari.

Ta'rif. *Aylana deb tekislikning ma'lum O nuqtasidan barobar uzoqlikda joylashgan barcha nuqtalari to'plamiga aytiladi.*

O nuqta *aylana markazi*, markazni aylananing ixtiyoriy nuqtasi bilan tutashtiruvchi kesma *aylana radiusi* deyiladi.

Aylananing tenglamasi. Markazi $O(a;\ b)$ nuqtada, radiusi r bo'lgan aylananing tenglamasi

$$(x-a)^2+(y-b)^2=r^2$$

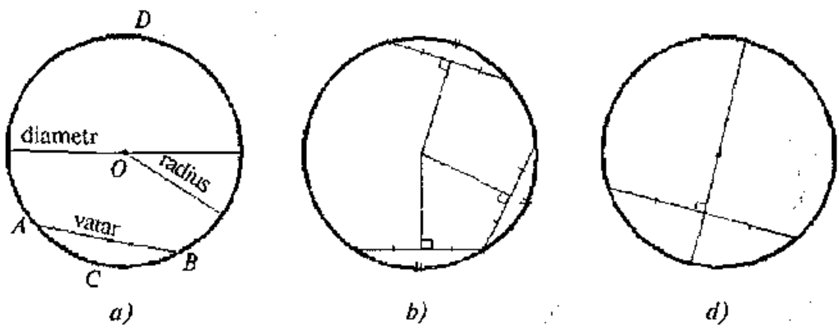
ko'rinishda bo'ladi. Chunki $(O,\ r)$ aylanadagi ixtiyoriy $N(x;\ y)$ nuqta bilan $O(a;\ b)$ nuqtalar orasidagi masofa ifodasi $\sqrt{(x-a)^2+(y-b)^2}$ bo'lib, u aylana radiusi r ga teng.

Xususan, aylana markazi $O(0;\ 0)$ koordinatalar boshida bo'lsa, bunday aylana tenglamasi

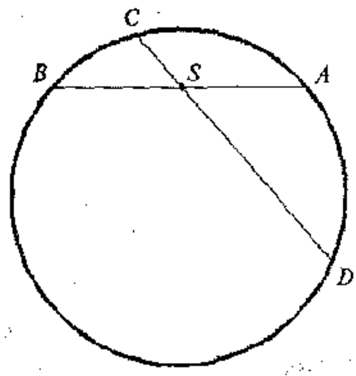
$$x^2+y^2=r^2$$

ko'rinishda bo'ladi.

Aylananing ixtiyoriy ikki nuqtasini tutashtiruvchi kesma uning *vatari* deb ataladi (150- rasm). Aylana markazidan o'tuvchi eng katta



150- rasm.



151- rasm.

vatar aylananing *diametri* deyilib, uning uzunligi ikki radius uzunligiga tengdir (150- *a* rasm).

Vatarlarning xossalari

1. Vatarning o'rta perpendikulari diametr orqali o'tadi (150- *d* rasm).

2. Teng vatarlar aylana markazidan bir xil masofada yotadi (150- *b* rasm).

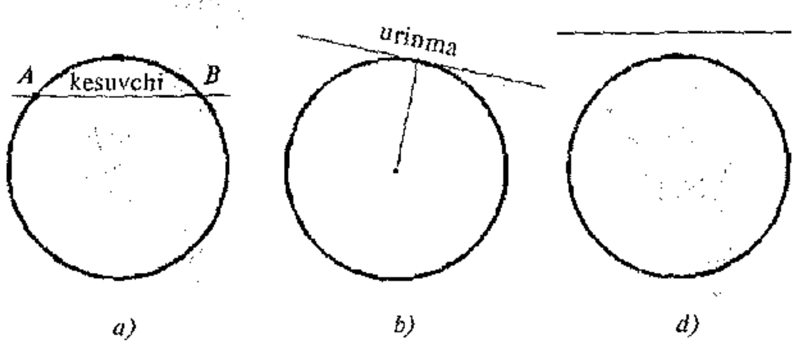
3. Turli vatarlardan kattasi aylana markaziga yaqin joylashgan bo'ladi.

4. Biror S nuqtada kesishuvchi ikki AB va CD vatarlar uchun quyidagi munosabat o'rinlidir (151- rasm).

$$|CS|\cdot|DS|=|AS|\cdot|BS|.$$

1.2. To'g'ri chiziq bilan aylananing vaziyati. To'g'ri chiziq bilan aylana tekislikda o'zaro 152- rasmdagidek, ya'ni ikki umumiy nuqtaga ega, bitta umumiy nuqtaga ega va umuman umumiy nuqtaga ega bo'lmagan vaziyatda bo'lishi mumkin.

I holda to'g'ri chiziq aylanani *kesuvchi* deyiladi (152- *a* rasm). II holda aylanaga *urinuvchi* – *urinma* deb ataladi va ularning umumiy nuqtasi to'g'ri chiziq bilan aylananing *urinish nuqtasi* hisoblanadi (152- *b* rasm).



152- rasm.

Aylanaga undan tashqaridagi A nuqtadan ikkita AB va AC urinmalar o'tkazish mumkin. Bunda A nuqtadan urinish nuqtalarigacha bo'lgan masofalar o'zaro tengdir: $|AB| = |AC|$ (153- rasm).

O'zaro urinuvchi aylana bilan to'g'ri chiziqning urinish nuqtalariga o'tkazilgan radius urinmaga perpendikulardir (153, 154- rasmlar).

Aylanadagi A nuqtadan aylanaga bitta va faqat bitta urinma o'tkazish mumkin (154- rasm). Yuqoridagi tasdiqlardan birinchisining isboti to'g'ri burchakli AOB va AOC uchburchaklarning o'zaro tengligidan kelib chiqadi: $\Delta AOB = \Delta AOC$, chunki $|OB| = |OC| = r$ va AO umumiy gipotenuza. Shunga ko'ra $|AB| = |AC|$.

Biror aylanaga undan tashqaridagi A nuqtadan AD urinma hamda aylanani C va B nuqtalarda kesuvchi AC to'g'ri chiziq o'tkazilgan bo'lsa, ular uchun

$$|AD|^2 = |AB| \cdot |AC|$$

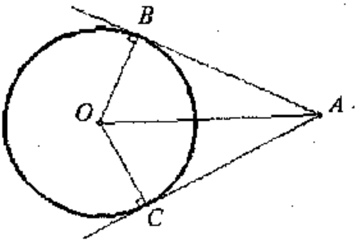
tenglik doim o'rinli bo'ladi (155- rasm).

1.3. Tekislikdagi ikki aylananing o'zaro vaziyati.

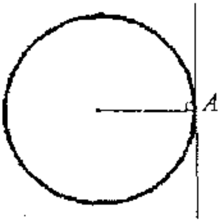
1. Ikki aylana o'zaro umumiy nuqtaga ega bo'lmagan hol. Umumiy markazga ega bo'lgan turli radiusli aylanalar *konsentrik aylanalar* deyiladi (156- rasm).

2. Bitta umumiy nuqtaga ega bo'lgan hol. Bu holda ikki aylana o'zaro *urinuvchi* deyilib, umumiy nuqta aylanalarning *urinish nuqtasi* deb ataladi. Aylanalarning urinish nuqtasi ularning O_1O_2 *markazlar chizig'ida* yotadi (157- *a*, *b* rasmlar).

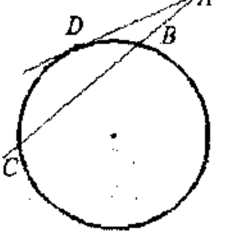
3. Ikkita umumiy nuqtaga ega bo'lishi mumkin. Bu holda ular ikki nuqtada bir-biri bilan kesishgan hisoblanadi (158- *a*, *b* rasmlar).



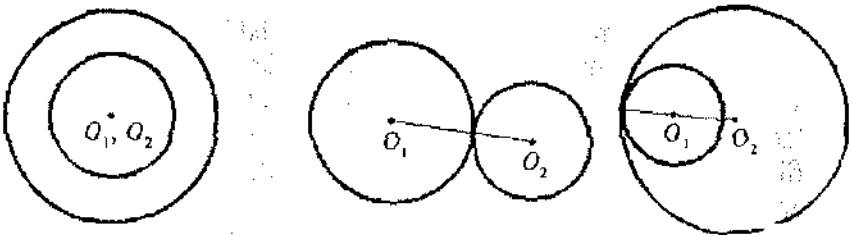
153- rasm.



154- rasm.

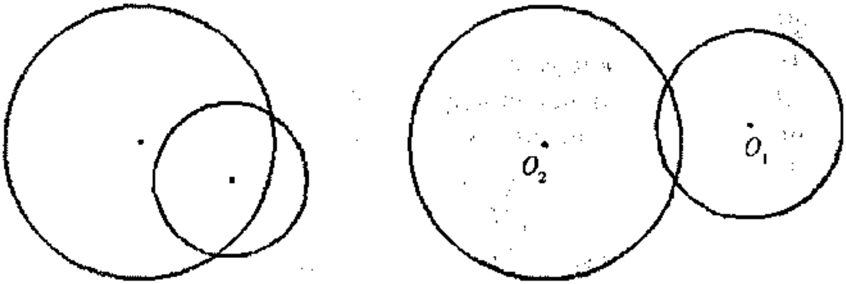


155- rasm.



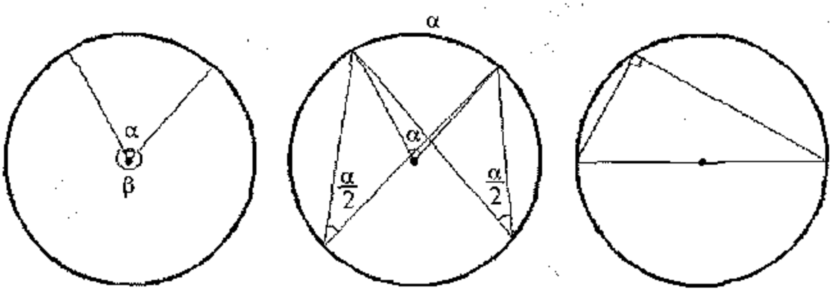
a) *b)*

156- rasm. **157- rasm.**



a) *b)*

158- rasm.



a) *b)* *d)*

159- rasm.

1.4. Markaziy burchak. Aylanaga ichki chizilgan burchak. Urinma bilan vatar orasidagi burchak. T a ' r i f. *Markaziy burchak deb uchi aylana markazida bo'lgan va tomonlari radiuslardan tashkil topgan burchakka aytiladi* (159- *a* rasm).

Ta'rifga asoslanib chizilgan burchak aylanada ikkita turli markaziy burchak tashkil etib, ulardan biri burchak tomonlari aylanada ajratgan yoylardan biriga tiralsa, ikkinchisi ikkinchi yoyga tiralgan hisoblanadi. Markaziy burchak o'zi tiralgan yoyning gradus o'lchovi bilan o'lchanadi.

Uchi aylanada yotgan va tomonlari vatarlardan tashkil topgan burchak aylanaga *ichki chizilgan burchak* deyiladi (159- *b* rasm). Burchak ichidagi aylana yoyi ichki chizilgan burchakning tiralgan yoyi hisoblanadi. Bitta yoyga tiralgan barcha ichki chizilgan burchaklar o'zaro teng va ular shu yoyga tiralgan markaziy burchakning yarmi bilan o'lchanadi.

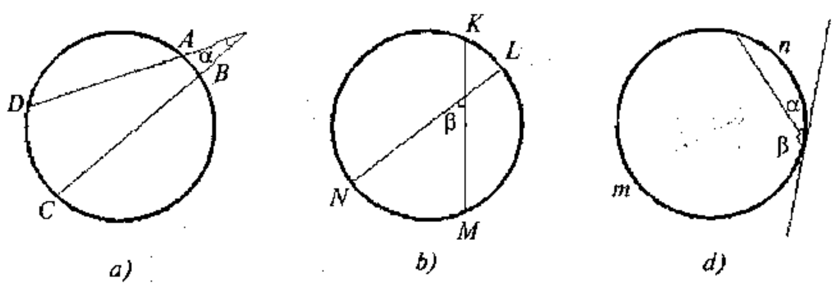
Diametrga tiralgan ichki chizilgan burchak to'g'ri burchakdir (159- *d* rasm).

Uchi aylanadan tashqarida joylashgan, tomonlari aylanani kesuvchi bo'lgan burchak kattaligi uning tomonlari orasidagi aylana yoylarining gradus o'lchovlari ayirmasining yarmi bilan o'lchanadi (160- rasm):

$$\alpha = \frac{\breve{CD} - \breve{AB}}{2}.$$

Uchi aylana ichida, tomonlari aylanani kesuvchi β burchak kattaligi kesuvchilarning oralaridagi aylana yoylarining gradus o'lchovlari yig'indisining yarmiga teng: $\beta = \frac{\breve{KL} + \breve{MN}}{2}$ (160- *b* rasm).

Urinma bilan vatar orasidagi burchak vatar ajratgan mos yoy gradus o'lchovining yarmi bilan o'lchanadi: $\alpha = \frac{1}{2}n$; $\beta = \frac{1}{2}m$ (160- *d* rasm).



160- rasm.

1.5. Aylananing uzunligi. Aylana l uzunligining diametriga nisbati har qanday aylana uchun taxminan 3,14... ga teng.

Bu nisbatdan aylana uzunligi uchun $l \approx 2 \cdot R \cdot 3,14$; $3,14 \approx \pi$ boʻlgani uchun $l = 2\pi R$ formula kelib chiqadi.

1.6. Aylana yoyining uzunligi. Aylana butun yoyining gradus oʻlchovi 360° boʻlgani uchun, 1° li yoyining uzunligi: $l_{1^\circ} = \frac{2\pi R}{360^\circ} = \frac{\pi R}{180^\circ}$ ga, n° li yoyining uzunligi esa: $l_{n^\circ} = \frac{\pi R}{180^\circ} \cdot n^\circ$ ga teng.

1- m a s a l a. Aylananing berilgan nuqtasidan diametr va radiusga teng vatar oʻtkazilgan. Diametr va vatar orasidagi burchakni toping.

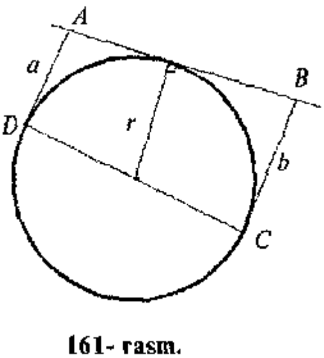
Y e c h i l i s h i. Vatarning ikkinchi uchini aylana markazi bilan tutashtirsak, teng tomonli uchburchak hosil boʻladi. Demak, izlangan burchak 60° ekan.

2- m a s a l a. Aylanaga urinuvchi toʻgʻri chiziq uning diametri uchlaridan a va b masofalarda yotadi. Aylana diametrining uzunligini toping (161- rasm).

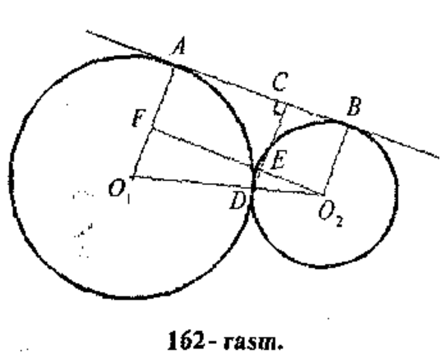
Y e c h i l i s h i. Urinish nuqtasiga oʻtkazilgan r radius $ABCD$ trapetsiya uchun oʻrta chiziq boʻladi: $r = \frac{a+b}{2}$. Shunga asosan $|DC| = 2r = a + b$.

J a v o b: $a + b$.

3- m a s a l a. Radiuslari 2 va 6 boʻlgan ikki aylana oʻzaro tashqi ravishda urinadi. Aylanalarning urinish nuqtasidan ularning umumiy tashqi urinma toʻgʻri chizigʻigacha masofani toping (162- rasm).



161- rasm.



162- rasm.

Y e c h i l i s h i. Kichik aylana markazidan AB urinmaga parallel chizsak, o'zaro o'xshash to'g'ri burchakli O_2ED va O_2FO_1 uchburchaklar hosil bo'ladi. Aylanalarning umumiy urinish nuqtasi D ularning markazlar chizig'i O_1O_2 da yotishi va izlanayotgan masofa DC kesmaning uzunligiga tengligiga e'tibor bering. Shuning uchun:

$$|ED| : |O_1F| = 2 : 8, |O_1F| = 4, \text{ chunki } |O_2B| = |EC| = |AF| = 2.$$

$$|ED| = 4 \cdot \frac{1}{4} = 1; |DC| = |ED| + |EC| = 1 + 2 = 3.$$

J a v o b: 3.

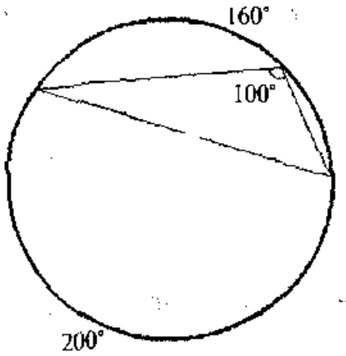
4- m a s a l a. Aylana vatari uni gradus o'lchovlari 4 : 5 kabi nisbatda bo'lgan ikki yoyga ajratadi. Kichik yoyning ixtiyoriy nuqtasidan vatar qanday burchak ostida ko'rinadi?

Y e c h i l i s h i. Masala shartiga ko'ra: $4\alpha + 5\alpha = 360° \Rightarrow \alpha = 40°$ bo'lgani uchun yoylarning gradus o'lchovlari 160° va 200° dir. 200° li yoyga tiralgan (izlangan burchak) ichki chizilgan burchak 100° ekanligi ma'lum bo'ladi (163- rasm).

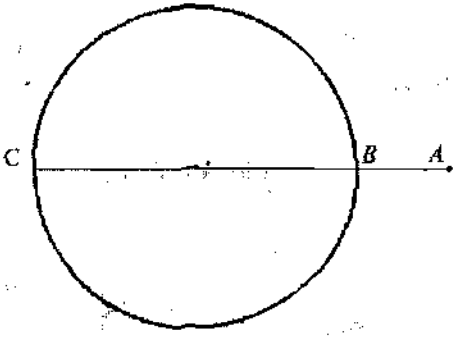
J a v o b: 100°.

5- m a s a l a. Aylana tashqarisidagi A nuqtadan uning eng yaqin va eng uzoqdagi nuqtalarigacha bo'lgan masofalar mos ravishda 3 va 7 ga teng bo'lsa, aylananing uzunligini toping (164- rasm).

Y e c h i l i s h i: Ma'lumki, bu uchala nuqta A nuqtadan va aylana markazidan o'tuvchi AC kesuvchida yotadi. Demak, $|BC| = 2R =$



163- rasm.



164- rasm.

$=|AC|-|AB|=4$, bundan $R=2$ va aylananing uzunligi 4π ekanligi ma'lum bo'ladi.

J a v o b: 4π.

6- m a s a l a. Radiusi 2 ga teng bo'lgan aylananing uzunligi radiusi 5 ga teng bo'lgan aylana yoyi uzunligiga teng bo'lsa, shu yoyga tiralgan markaziy burchakni toping.

Y e c h i l i s h i. Masala shartiga ko'ra $2\pi R = \frac{\pi R'}{180}\cdot n^\circ$, bunda $R=2$, $R'=5$ bo'lgani uchun $4\pi = \frac{5\pi}{180^\circ}\cdot n^\circ \Rightarrow 4\cdot 36^\circ \Rightarrow n^\circ = 144^\circ$.

J a v o b: 144°.

2- §. Doira

T a ' r i f. *Tekislikning aylana bilan chegaralangan sohasi doira deb ataladi* (165- rasm).

Doiraning barcha nuqtalari uni chegaralovchi aylana markazidan radiusi qadar masofadan uzoqda yotmaydi.

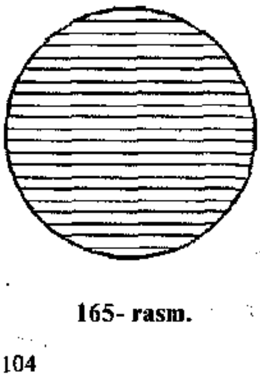
Shuning uchun markazi $O(a; b)$ nuqtada, radiusi r ga teng bo'lgan doira

$$(x-a)^2+(y-b)^2 \le r^2$$

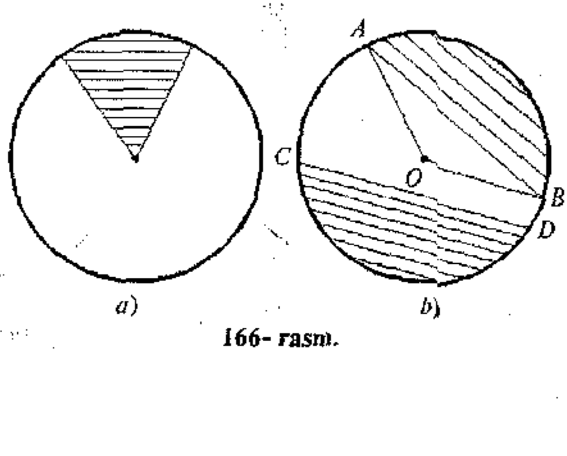
tengsizlik bilan ifodalanadi.

Xususan, markazi koordinatalar tekisligining boshida, radiusi r ga teng bo'lgan doira $x^2+y^2 \le r^2$ tengsizlik bilan ifodalanadi.

Aylananing markazi, radiusi, vatari va diametri tegishli doiraning markazi, radiusi, vatari va diametri bo'ladi. Radiusi R bo'lgan



165- rasm.



166- rasm.

doiraning yuzi $S = \pi R^2$ kabi ifodalanib, uni doiraga ichki (yoki tashqi) chizilgan muntazam ko'pburchaklar yuzlari ketma-ketligining tomonlar soni cheksizlikka intilgandagi limiti sifatida keltirib chiqarish mumkin.

Doiraning ikki radiusi bilan chegaralangan qismi *doira sektori* deyiladi (166-*a* rasm).

n° li yoyga ega bo'lgan sektor yuzi $S = \frac{\pi r^2}{360^\circ} \cdot n^\circ$ formula bilan hisoblanadi.

Doiraning vatari ajratgan bo'laklari *doiraviy segment* deb ataladi (166-*b* rasm).

Doiraviy segmentning yuzi uni yarim doiradan katta yoki kichikligiga qarab ikki xil yo'l bilan aniqlanadi. Masalan, *AB* vatar bilan ajratilgan kichik doiraviy segmentning yuzi kichik doiraviy sektor yuzidan *AOB* uchburchak yuzini ayirib aniqlansa, katta doiraviy segmentning yuzi topish uchun unga mos sektor yuziga *AOB* uchburchak yuzi qo'shiladi (166-*b* rasm).

1-masala. Markazi koordinatalar boshida bo'lib, (–6; –8) *nuqtadan o'tuvchi aylana bilan chegaralangan doira yuzini toping.*

Yechilishi. Doira radiusi 10 ga tengligi oson aniqlanadi. Demak, $S_{\text{doira}} = 100\pi$.

Javob: 100π.

Mustaqil ishlash uchun test topshiriqlari

1. Aylananing $\frac{1}{3}$ bo'lagi necha gradusli yoy bo'ladi?

A) 60°; B) 90°; C) 120°; D) 150°; E) 180°.

2. Soatning minut mili 20 minutda necha gradusli yoy "chizadi"?

A) 150°; B) 120°; C) 90°; D) 60°; E) 75°.

3. Radiuslari uzunliklarining nisbati 1 : 3 kabi bo'lgan aylanalar uzunliklari nisbatini toping.

A) 1 : 3; B) 2 : 3; C) 1 : 4; D) 1 : 5; E) π : 25.

4. *Radiuslari 25 va 35 bo'lgan aylanalar bir-biriga urinadi.* Tashqi va ichki urinishlar yuz bergan hollar uchun aylanalar markazlari orasidagi masofani toping.

A) 40;35; B) 60;10; C) 35;10; D) 35;15; E) 15;20.

5. Markaziy burchakka mos yoy aylananing $\frac{1}{6}$ qismiga teng. Shu markaziy burchakni toping.

A) 45°; B) 60°; C) 90°; D) 30°; E) 120°.

6. Radiusi 12 ga va markaziy burchagi 105° ga teng bo'lgan doiraviy sektorning yoyi aylana shakliga keltirilgan. Shu aylananing radiusini aniqlang.

A) 3,5; B) 3,2; C) 4,5; D) 4; E) 4,2.

7. Radiusi 5 ga teng bo'lgan aylana yoyining uzunligi radiusi 2 ga teng aylana uzunligiga teng bo'lsa, hosil bo'lgan markaziy burchakni toping.

A) 120°; B) 150°; C) 144°; D) 135°; E) 148°.

8. Aylananing uzunligi radiusi 4 ga va markaziy burchagi 120° ga teng yoy uzunligiga teng. Aylananing radiusini toping.

A) $\frac{2\sqrt{2}}{3}$; B) $\frac{2\sqrt{2}}{\sqrt{3}}$; C) $2\frac{2}{3}$; D) $1\frac{1}{3}$; E) 2.

9. Aylananing markaziy burchagi 100°, u tiralgan yoy uzunligi 10 sm bo'lsa, aylana radiusi necha santimetr ($\pi = 3$ deb olinsin)?

A) 5; B) 6; C) 3; D) 2; E) 8.

10. Radiusi 1 ga teng aylana uchta yoyga bo'lingan. Ularga mos markaziy burchaklar 1; 2 va 6 sonlariga proporsional. Yoylardan eng kattasining uzunligini toping.

A) $\frac{4\pi}{3}$; B) $\frac{3\pi}{4}$; C) $\frac{2\pi}{9}$; D) $\frac{5\pi}{9}$; E) $\frac{4\pi}{9}$.

11. Aylananing uzunligi $6\sqrt{3}$ ga teng bo'lgan vatari 120° li yoyni tortib turadi. Aylananing uzunligini toping.

A) 12π; B) 10π; C) 13π; D) 14π; E) 9π.

12. Uzunligi 30π ga teng bo'lgan aylananing 60° li yoyni tortib turuvchi vatarining uzunligi qancha?

A) 12; B) 16; C) 15; D) 13; E) 17.

13. Aylanani *AB* vatar ikkita yoyga ajratadi. Bu yoylarning nisbati 4 : 5 kabi. *AB* vatar katta yoyning ixtiyoriy nuqtasidan qanday burchak ostida ko'rinadi?

A) 100°; B) 95°; C) 80°; D) 85°; E) 90°.

14. Ushbu iboralarning qaysi biri noto‘g‘ri?

A) Uchi aylanada yotgan, tomonlari aylana vatarlaridan iborat burchak aylanaga ichki chizilgan burchak deyiladi;

B) Aylanaga ichki chizilgan burchak o‘zi tiralgan yoyning yarmi bilan o‘lchanadi;

C) Bitta yoyga tiralgan barcha ichki chizilgan burchaklar o‘zaro tengdir;

D) Diametrga tiralgan barcha ichki chizilgan burchaklar to‘g‘ri burchakdir;

E) Diametrning yarmiga tiralgan ichki burchak 45° bo‘ladi.

15. Quyidagi ta’rif va tasdiqlardan qaysinisi noto‘g‘ri?

A) Berilgan nuqtadan ma’lum masofada yotuvchi nuqtalar to‘plami aylana deyiladi;

B) Aylana uzunligining diametriga nisbati aylanaga bog‘liq emas, ya’ni har qanday ikkita aylana uchun ham bir xildir;

C) Aylana uzunligi $l = 2\pi R$ formula bilan hisoblanadi;

D) $n°$ li yoy uzunligi $l_{n°} = \frac{\pi}{180°} n°$ formula bilan hisoblanadi;

E) $\pi \approx 3,14...$ irratsional sondir.

16. Uzunligi 30π ga teng bo‘lgan aylananing 60° li yoyining uzunligini toping.

A) 12π; B) 6π; C) 5π; D) 3π; E) 4π.

17. Aylanani AB vatar ikkita yoyga ajratadi. Bu yoylarning nisbati $4:5$ kabi. AB vatar kichik yoyning ixtiyoriy nuqtasidan qanday burchak ostida ko‘rinadi?

A) 100°; B) 95°; C) 80°; D) 85°; E) 90°.

18. Doiraga ichki chizilgan to‘g‘ri to‘rtburchakning tomonlari 12 va 16 ga teng. Doiraning yuzini toping.

A) 100π; B) 95π; C) 80π; D) 85π; E) 90π.

19. Aylanaga ichki chizilgan to‘g‘ri to‘rtburchakning tomonlari 32 va 24 ga teng. Aylananing uzunligini toping.

A) 20π; B) 10π; C) 40π; D) 12π; E) 24π.

20. Doiraning yuzi 36π bo‘lsa, shu doiraga tashqi chizilgan kvadrat yuzini toping.

A) 121; B) 100; C) 169; D) 150; E) 144.

21. Teng yonli to'g'ri burchakli uchburchakning yuzi 8 ga teng bo'lsa, shu uchburchakka tashqi chizilgan aylana uzunligini hisoblang.

A) 4π; B) $3\sqrt{2}\pi$; C) $4\sqrt{2}\pi$; D) 3π; E) 5π.

22. *P* nuqta radiusi 6 sm bo'lgan aylananing markazidan 12 sm uzoqlikda joylashgan. *P* nuqtadan urinma va aylananing markazidan o'tadigan kesuvchi o'tkazilgan. Urinma va kesuvchi orasidagi burchakni toping.

A) 75°; B) 65°; C) 60°; D) 30°; E) 45°.

23. Har birining diametri 50 ga teng bo'lgan uchta quvur suv o'tkazish qobiliyati shu uchta quvurnikiga teng bo'lgan bitta quvur bilan almashtiriladi. Katta quvurning diametrini toping.

A) $50\sqrt{2}$; B) 100; C) $50\sqrt{3}$; D) 70; E) 75.

24. Doiraning radiusi *r* ga teng. 90° li yoyga mos keladigan segmentning yuzini toping.

A) $\frac{\pi r^2}{8}$; B) $\frac{r^2}{2}(\pi-2)$; C) $\frac{\pi r^2}{4}$; D) $\frac{r^2}{8}(\pi-2)$;

E) $\frac{r^2}{4}(\pi-2)$.

25. Uzunligi *m* ga teng bo'lgan vatar 90° li yoyga tiraladi. Hosil bo'lgan segmentning yuzini toping.

A) $\frac{\pi m^2}{8}$; B) $\frac{m^2}{8}(\pi-2)$; C) $\frac{m^2(\pi-\sqrt{3})}{4}$; D) $\frac{\pi m^2}{4}$;

E) $\frac{m^2}{4}(\pi-\sqrt{2})$.

26. Aylananing *A* nuqtasidan o'tkazilgan *AB* va *AC* vatarlarning uzunliklari mos ravishda 5 va 12 ga teng. Agar ularning ikkinchi uchlari tutashtirilsa, yuzi 15 ga teng uchburchak hosil bo'ladi. *AB* va *AC* vatarlar orasidagi o'tkir burchakni toping.

A) 60°; B) 45°; C) 30°; D) 20°; E) 15°.

27. Ikkita doiraning umumiy vatari 60° va 120°li yoylarni tortib turadi. Kichik doira yuzining katta doira yuziga nisbatini toping.

A) 2:3; B) 1:2; C) 1:3; D) 1:4; E) 2:5.

28. Radiusi *R* ga teng bo'lgan doiraning markazidan bir tomonda ikkita bir-biriga parallel vatar o'tkazildi. Bu vatarlardan biri 120°li, ikkinchisi 60°li yoyni tortib turadi. Parallel vatarlar orasida joylashgan kesimning yuzini toping.

A) $\frac{\pi R^2}{3}$; B) $\frac{\pi R^2}{4}$; C) $\frac{3\pi R^2}{8}$; D) $\frac{\pi R^2}{6}$; E) $\frac{\pi R^2}{8}$.

29. Aylanadan tashqaridagi nuqtadan ikkita kesuvchi o'tkazildi. Birinchi kesuvchining aylana ichidagi qismi 47 ga, tashqi qismi esa 9 ga teng. Ikkinchi kesuvchining ichki qismi tashqi qismidan 72 ga katta. Ikkinchi kesuvchining uzunligini toping.

A) 88; B) 84; C) 82; D) 80; E) 76.

30. To'g'ri chiziq doiraning aylanasini uzunliklarining nisbati 1:3 kabi bo'lgan ikki yoyga ajratadi. Bu chiziq doiraning yuzini qanday nisbatda bo'ladi?

A) 1 : 9; B) $\frac{\pi-2}{3\pi+2}$; C) $\frac{\pi+2}{3\pi+2}$; D) $\frac{\pi+4}{2\pi+1}$; E) 4 : 9.

31. Aylana markazidan turli tomonda uzunliklari 36 va 48 ga teng bo'lgan parallel vatarlar o'tkazilgan. Ular orasidagi masofa 42 ga teng bo'lsa, aylananing radiusini toping.

A) 34; B) 32; C) 30; D) 28; E) 26.

32. Radiusi 12 ga va markaziy burchagi 105° ga teng bo'lgan doiraviy sektorning yoyi aylana shakliga keltirilgan. Shu aylananing radiusini toping.

A) 4,5; B) 4,2; C) 4; D) 3,5; E) 3,2.

33. Aylanadan tashqaridagi nuqtadan o'tkazilgan ikki urinmaning urinish nuqtalari, aylanani 1:9 nisbatdagi ikkita yoyga ajratadi. Urinmalar orasidagi burchakni toping.

A) 144°; B) 120°; C) 110°; D) 72°; E) Aniqlab bo'lmaydi.

34. $3x + y = 10$ va $2x - 3y - 36 = 0$ to'g'ri chiziqlarning kesishish nuqtasi markazi koordinatalar boshida bo'lgan aylanaga tegishli. Aylananing radiusini toping.

A) 13; B) 12; C) 10; D) 8; E) 6.

35. $M(3; -1)$ nuqtadan $x^2 + 2x + y^2 - 4y = 11$ aylanagacha bo'lgan masofani toping.

A) 2,5; B) 2; C) 1,5; D) 1; E) 0,5.

36. $x^2+y^2=25$ va $(x-8)^2+y^2=25$ aylanalarning umumiy vatarini o'z ichiga olgan to'g'ri chiziq tenglamasini tuzing.

A) $x=4$; B) $y=4$; C) $y=x+1$; D) $y=3x-4$; E) $y=2x+3$.

37. Koordinata tekisligida $x^2+y^2\le 4|y|$ tengsizlik bilan berilgan shaklning yuzini toping.

A) 16π; B) 12π; C) 8π; D) $6{,}5\pi$; E) 4π.

38. $x^2+y^2+4x-6y-3=0$ tenglama bilan berilgan aylana markazining koordinatalarini toping.

A) (2; –3); B) (–2; 3); C) (4; –6); D) (–4; 6); E) (4; –3).

39. Asosidagi burchagi α ga teng bo'lgan teng yonli uchburchakka ichki va tashqi chizilgan aylanalar radiuslarining nisbatini toping.

A) $\sin 2\alpha\cdot \mathrm{tg}\frac{\alpha}{2}$; B) $\mathrm{tg}\alpha\cdot\sin\frac{\alpha}{2}$; C) $\cos\frac{\alpha}{2}\cdot\mathrm{ctg}2\alpha$;

D) $\sin 2\alpha\cdot\mathrm{tg}^2\frac{\alpha}{2}$; E) $\cos 2\alpha\cdot\mathrm{ctg}^2\frac{\alpha}{2}$.

40. Doiraga ichki chizilgan uchburchakning bir tomoni uning diametriga teng. Doiraning yuzi 289π ga, uchburchak tomonidan birining uzunligi 30 ga teng. Shu uchburchakka ichki chizilgan doiraning yuzini toping.

A) 64π; B) 36π; C) 25π; D) 20π; E) 16π.

41. Teng yonli uchburchakning balandligi 20, asosining yon tomonga nisbati 4:3 kabi. Shu uchburchakka ichki chizilgan aylananing radiusini toping.

A) 12; B) 10; C) 8; D) 6; E) 4.

42. Radiusi 4 ga teng bo'lgan doiraga gipotenuzasi 26 ga teng bo'lgan to'g'ri burchakli uchburchak tashqi chizilgan. Shu uchburchakning perimetrini toping.

A) 64; B) 60; C) 58; D) 56; E) 52.

43. Aylanaga ichki chizilgan uchburchakning bir tomoni uning markazidan, qolgan ikki tomoni esa markazdan 3 va $3\sqrt{3}$ ga teng masofada o'tadi. Aylananing radiusini toping.

A) 12; B) 9; C) 8; D) 6; E) 3.

44. To'g'ri burchakli trapetsiyaning asoslari 6 va 4 ga teng. Unga ichki chizilgan aylananing uzunligini toping.

A) 2π; B) $2{,}4\pi$; C) 3π; D) $4{,}8\pi$; E) 6π.

45. O'tkir burchagi 30° bo'lgan teng yonli trapetsiyaga aylana ichki chizilgan. Aylana uzunligining trapetsiya perimetriga nisbatini toping.

A) 2π; B) π; C) $\frac{\pi}{2}$; D) $\frac{\pi}{4}$; E) $\frac{\pi}{8}$.

46. To'rtburchak uchta ketma-ket tomonlarining uzunliklari 2, 3 va 4 ga, unga ichki chizilgan aylananing radiusi 1,2 ga teng bo'lsa, to'rtburchak yuzini toping.

A) 6,8; B) 7,2; C) 7,8; D) 8,2; E) 8,6.

47. Tomonlari 1, 2, 3 va 4 bo'lgan to'rtburchakka ichki va tashqi aylanalar chizilgan. Uning kichik diagonalini toping.

A) $\sqrt{\frac{140}{11}}$; B) $\sqrt{\frac{55}{7}}$; C) $2\sqrt{2}$; D) $2\sqrt{\frac{15}{7}}$; E) 2,5.

48. *Doiraga ichki chizilgan muntazam uchburchakning yuzi unga* ichki chizilgan kvadrat yuzidan 18,5 ga kam. Shu doiraga ichki chizilgan muntazam oltiburchakning yuzini toping.

A) $9\sqrt{3}+6\sqrt{2}$; B) $8\sqrt{3}+15$; C) $13{,}5+12\sqrt{3}$; D) $12{,}5+13\sqrt{3}$;

E) $24\sqrt{3}+27$.

49. Ikkita o'zaro urinadigan aylanaga umumiy tashqi urinma o'tkazilgan. Urinma aylanalarning markazlari chizig'i davomi bilan markazlardan 24 sm va 72 sm uzoqlikdagi nuqtada kesishadi. Aylanalarning radiuslarini toping.

A) 12; 36; B) 20; 28; C) 16; 32; D) 15; 33; E) 14; 34.

50. Radiuslari 5 sm, 10 sm, 15 sm bo'lgan uchta aylana bir-biriga tashqi urinadi. Bu aylanalar markazlari orqali o'tuvchi aylana radiusini toping.

A) 12,5; B) 7,5; C) 10; D) 15; E) 8,5.

VI bob. VEKTORLAR

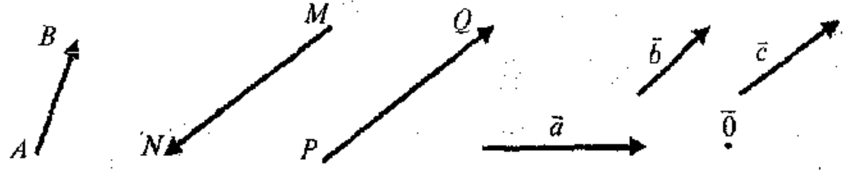
1-§. Vektor tushunchasi

1.1. Skalyar va vektor kattaliklar. Matematika, fizika kabi fanlarni o'rganishda ikki xil: 1) o'zining faqat son qiymati bilan ifodalanuvchi ***skalyar kattaliklar***; 2) son qiymatidan tashqari yo'nalishi ham muhim bo'lgan ***vektor kattaliklar*** uchraydi.

Vaqt, masofa, yuz, hajm kabi kattaliklar *skalyar kattaliklarga*, kuch, bosim, tezlik, tezlanish, parallel ko'chirish kabi kattaliklar *vektor kattaliklarga* misol bo'la oladi.

Chizmada *vektor kattaliklar* (bundan keyin *vektorlar* deb yuritiladi) yo'naltirilgan kesmalar ko'rinishida tasvirlanadi. Vektorlar kuch, bosim yoki tezlik mazmunidan qat'i nazar, ular matematikada erkin vektor sifatida o'rganiladi.

Kesmaning bir uchi *vektorning boshi*, yo'nalish qo'yilgan ikkinchi uchi esa *vektorning oxiri* sifatida qaraladi. Vektorlar kesmalar kabi uchlarini ifodalovchi harflar bilan $\overline{AB}$, $\overline{MN}$, $\overline{PQ}$ tarzida yoziladi. Bu ifodalardagi birinchi harf vektor boshini, ikkinchisi esa vektor oxirini bildiradi. Agar vektorning boshi va oxirini ko'rsatish shart bo'lmasa, uni bitta kichik harf bilan $\vec{a}$, $\vec{b}$, $\vec{c}$, ... tarzida ifodalash ham mumkin. Agar vektorning boshi bilan oxiri ustma-ust tushsa, bunday vektor **nol vektor** deyiladi. Nol vektorning yo'nalishi aniqmas bo'lib, $\overline{0}$ kabi ifodalanadi (167- rasm).



167- rasm.

Vektorni tashkil etuvchi kesma uzunligi *vektorning uzunligi* hisoblanadi va $\left|\overrightarrow{AB}\right|$ yoki $|\vec{a}|$ kabi belgilanadi.

1.2. O'zaro teng vektorlar. *Uzunliklari teng va bir xil yo'nalgan vektorlar o'zaro teng vektorlar deyiladi.*

Uzunliklari bir xil, yo'nalishlari esa qarama-qarshi bo'lgan vektorlar qarama-qarshi vektorlar deyiladi. $\vec{a}$ vektorga qarama-qarshi bo'lgan vektor $-\vec{a}$ tarzida belgilanadi.

168- rasmda bir-biriga teng $\overrightarrow{AB}$ va $\overrightarrow{CD}$ vektorlar hamda ularga qarama-qarshi $\overrightarrow{MN}$ vektor tasvirlangan.

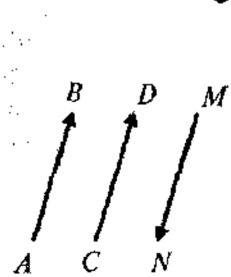
O'zaro teng vektorlar $\overrightarrow{AB}=\overrightarrow{CD}$ kabi, bir xil yo'nalgan vektorlar $\overrightarrow{AB}\uparrow\uparrow\overrightarrow{CD}$ ko'rinishda, qarama-qarshi yo'nalgan vektorlar $\overrightarrow{CD}\downarrow\uparrow\overrightarrow{MN}$ tarzida ifodalanadi.

Ta'rif. *Bir to'g'ri chiziqda yotuvchi yoki o'zaro parallel to'g'ri chiziqlardagi vektorlar kollinear vektorlar deb ataladi. Aks holda nokollinear vektorlar deyiladi.*

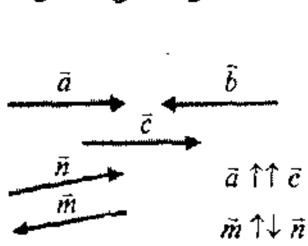
Kollinear vektorlar bir xil yoki qarama-qarshi yo'nalgan bo'lishi mumkin (169- rasm).

Koordinatalar tekisligida boshi koordinatalar boshida bo'lgan ($\overrightarrow{OA}$) vektor oxirining koordinatalari *vektor koordinatalari* deyiladi va $\overrightarrow{OA}\,(a;\,b)$ tarzida yoziladi (170- rasm).

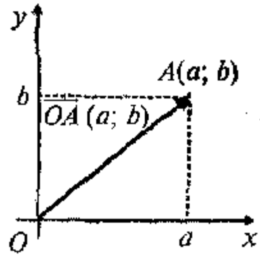
Ixtiyoriy nol bo'lmagan $\overrightarrow{CD}$ vektorning koordinatalari boshi koordinata boshida bo'lgan va ushbu vektorga teng bo'lgan $\overrightarrow{OB}$ vek-



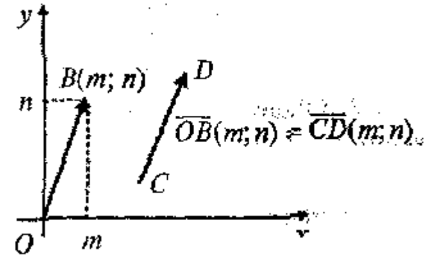
168- rasm.



169- rasm.



170- rasm.



171- rasm.

torning koordinatalari kabidir (171- rasm). Bu holda $\overline{CD}$ vektor O nuqtadan qo'yilgan deyiladi.

Umuman, boshi $A(x_n; y_n)$ va oxiri $B(x_m; y_m)$ nuqtalarda bo'lgan $\overline{AB}$ vektor koordinatalarini topish uchun uning oxiri koordinatalaridan boshining koordinatalarini mos ravishda ayiriladi:

$$\overline{AB}\,(x_m - x_n;\; y_m - y_n).$$

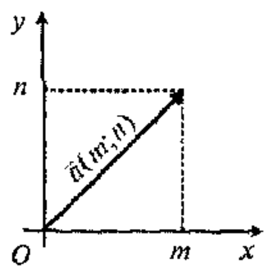
Vektorning koordinatalari uning koordinata o'qlaridagi *proyeksiyalari* deb ham ataladi. Bunda vektor proyeksiyalarining ifodalari manfiy son ham bo'lishi mumkin.

1.3. Koordinatalari bilan berilgan vektorning uzunligi (moduli).

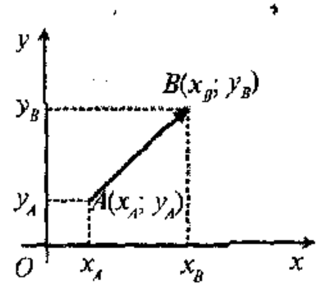
172- rasmda tasvirlangan $\bar{a}\,(m;\, n)$ vektorning uzunligi (moduli)

$$|\bar{a}| = \sqrt{m^2 + n^2},$$

$\overline{AB}\,(x_B - x_A;\, y_B - y_A)$ vektorning uzunligi (173- rasm) esa



172- rasm.



173- rasm.

$$\left|\overline{AB}\right| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$

kabi ifodalanadi.

1- m a s a l a. Boshi koordinata boshida bo'lgan bir xil $|a|$ modulli (uzunlikdagi) barcha vektorlar oxirlari to'plamining algebraik ifodasini yozing.

Y e c h i l i s h i. Izlangan nuqtalar to'plami markazi koordinata boshida bo'lgan $|a|$ radiusli aylanadan iboratdir. Uning barcha nuqtalarining $(x;\ y)$ koordinatalari $|a|^2 = x^2 + y^2$ tenglamani qanoatlantiradi.

J a v o b: $|a|^2 = x^2 + y^2$.

2- m a s a l a. $A(2;\ 4)$ va $B(-2;\ -4)$ nuqtalar aniqlagan vektor uzunligini toping.

Y e c h i l i s h i. Ma'lumki, $\left|\overline{AB}\right| = \left|\overline{BA}\right|$. Demak,

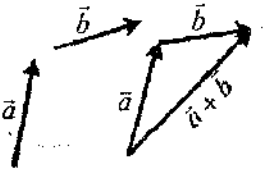
$$\left|\overline{AB}\right| = \sqrt{(-2-2)^2 + (-4-4)^2} = \sqrt{16+64} = \sqrt{80} = 4\sqrt{5}.$$

J a v o b: $\left|\overline{AB}\right| = \left|\overline{BA}\right| = 4\sqrt{5}.$

2-§. Vektorlar ustida amallar

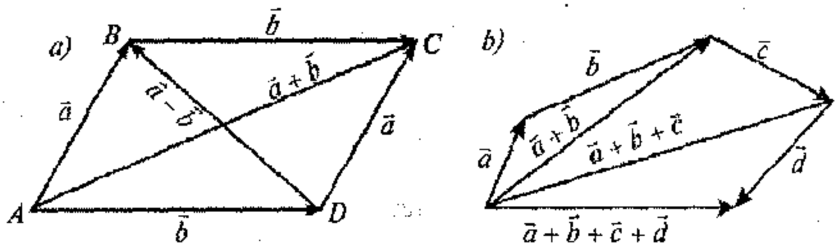
2.1. Vektorni vektorga qo'shish. Ikkita $\vec{a}$ va $\vec{b}$ vektorni birbiriga qo'shish uchun ulardan birining oxiridan ikkinchisi qo'yiladi va yig'indi vektor sifatida boshi birinchi vektor boshida, oxiri ikkinchi vektor oxirida bo'lgan vektor olinadi (174- rasm). Vektorlarni qo'shishning bu qoidasi *uchburchak qoidasi* deyiladi.

Umumiy A uchga ega bo'lgan $\vec{a}$ va $\vec{b}$ vektorlar yig'indisi shu vektorlarga yasalgan parallelogrammning shu uchdan chiqqan diagonali bilan tasvirlanadi (*parallelogramm qoidasi*). Haqiqatan ham, uchburchak qoidasiga ko'ra $\overline{AC} = \overline{AB} + \overline{BC} = \vec{a} + \vec{b}$ yoki $\overline{AC} = \overline{AD} + \overline{DC} = \vec{b} + \vec{a}$ (175- *a* rasm).



174- rasm.

Ikkita qo'shiluvchi vektorlar uchun ko'rib chiqilgan vektorlarni qo'shish tushunchasini istalgan chekli sondagi vektorlar uchun ham

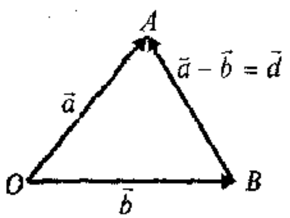


175- rasm.

umumlashtirish mumkin. 175- *b* rasmda $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ vektorlarning yig'indisi tasvirlangan.

Koordinatalari bilan berilgan $\vec{a}(x_a; y_a)$ va $\vec{b}(x_b; y_b)$ vektorlarning yig'indisi $\vec{a}+\vec{b}=\vec{c}$ vektorning koordinatalari qo'shiluvchi vektorlarning mos koordinatalarining yig'indisiga teng: $\vec{c}\,(x_a + x_b; y_a + y_b)$.

2.2. Vektorlar ayirmasi. Ikki ($\vec{a}$ va $\vec{b}$) vektorlarning ayirmasi ($\vec{a} - \vec{b}$ yoki $\vec{b} - \vec{a}$) ni topish uchun kamayuvchi vektorga ayriluvchi vektorning qarama-qarshisi qo'shiladi ($\vec{a}+(-\vec{b})$ yoki $\vec{b}+(-\vec{a})$). Agar $\vec{a}-\vec{b}=\vec{d}$ desak, u holda $\vec{a}=\vec{b}+\vec{d}$. Bundan vektorlarni ayirish qoidasi kelib chiqadi (176- rasm): umumiy *O* nuqtaga qo'yilgan $\vec{a}$ va $\vec{b}$ vektorlar uchun $\vec{d}=\vec{a}-\vec{b}$ ayirma kamayuvchi $\vec{a}$ va ayriluvchi $\vec{b}$ vektorlarning oxirlarini tutashtiruvchi va ayriluvchi vektordan kamayuvchi vektorga yo'nalgan bo'ladi.



176- rasm.

Koordinatalari bilan berilgan $\vec{a}(x_a; y_a)$ va $\vec{b}(x_b; y_b)$ vektorlarning ayirmasi bo'lgan $\vec{d}$ vektorning koordinatalari kamayuvchi vektor koordinatalaridan ayiriluvchi vektor koordinatalarini mos ravishda ayirib topiladi, ya'ni $\vec{a}-\vec{b}=\vec{d}$ bo'lsa, $\vec{d}(x_a - x_b; y_a - y_b)$ bo'ladi.

Vektorlar yig'indisining *xossalari*:

Har qanday $\vec{a}, \vec{b}$ va $\vec{c}$ vektorlar uchun quyidagi tengliklar o'rinlidir:

1. $\vec{a}+\vec{b}=\vec{b}+\vec{a}$ **(o'rin almashtirish xossasi);**

2. $(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$ **(guruhlash xossasi).**

1-masala. ABC uchburchakda $|AB|=3, |BC|=4, \angle B=90°$. $|\overline{BA}|-|\overline{BC}|$ va $|\overline{BA}-\overline{BC}|$ larni toping.

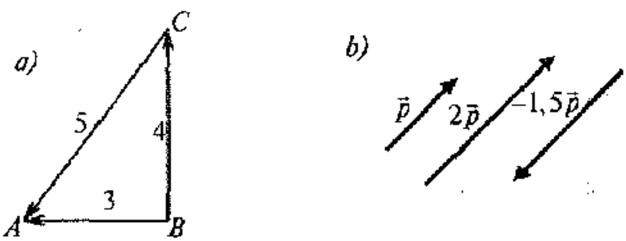
Yechilishi: 1) Masala shartiga ko'ra $|\overline{BA}|=|\overline{AB}|=3$, $|\overline{BC}|=|\overline{CB}|=4$. Demak, $|\overline{BA}|-|\overline{BC}|=3-4=-1$. 2) Ikki vektorni ayirish qoidasiga ko'ra $|\overline{BA}-\overline{BC}|=|\overline{CA}|$. Qaralayotgan uchburchak Misr uchburchagi bo'lib, $|\overline{CA}|=5$ (177- *a* rasm).

Javob: -1 va 5.

2.3. Vektorni songa ko'paytirish. Noldan farqli $\vec{p}$ *vektorning* $\lambda \neq 0$ *songa ko'paytmasi deb*, $\vec{p}$ vektorga kollinear, uzunligi $|\lambda|\cdot|\vec{p}|$ ga teng bo'lgan, $\lambda>0$ da $\vec{p}$ vektor bilan bir xil yo'nalgan, $\lambda<0$ bo'lganda esa unga qarama-qarshi yo'nalgan $\vec{q}=\lambda\vec{p}$ vektorga aytiladi.

177- *b* rasmda $\vec{p}, 2\vec{p}, -1,5\vec{p}$ vektorlar tasvirlangan.

Koordinatalari bilan berilgan $\vec{p}(x_p; y_p)$ vektorni biror λ songa ko'paytirishdan hosil bo'lgan $\vec{q}=\lambda\vec{p}$ vektorning koordinatalari $\vec{p}$ vektorning har bir koordinatasini λ ga ko'paytirib topiladi: $\vec{q}(\lambda x_p; \lambda y_p)$.



177- rasm.

2- m a s a l a. k ning qanday qiymatlarida $\bar{p}(k;\ 1)$ va $\bar{q}(4;\ k)$ vektorlar o'zaro kollinear bo'ladi?

Y e c h i l i s h i. Ma'lumki, $\bar{p}$ va $\bar{q}$ vektorlar o'zaro kollinear bo'lsa, u holda shunday λ soni topiladiki, bunda $\lambda \cdot k = 4,\ \lambda = k$ bo'ladi. Bundan $k^2 = 4$ va $k_1 = -2,\ k_2 = 2$ kelib chiqadi.

J a v o b: –2; 2.

Vektorni songa ko'paytirish ta'rifidan ularning *parallellik sharti* kelib chiqadi. Agar $\bar{b} = \lambda\vec{a}$ bo'lsa, u holda $a(x_1;\ y_1)$ va $b(x_2;\ y_2)$ vektorlarning koordinatalari bir-biri bilan $x_2 = \lambda x_1$ va $y_2 = \lambda y_1$ kabi bog'liqligidan $\frac{x_2}{x_1} = \frac{y_2}{y_1} = \lambda$ kelib chiqadi. Bu esa o'zaro kollinear vektorlarning mos koordinatalari proporsional bo'lishini bildiradi.

2.4. Vektorni ikkita nokollinear vektor bo'yicha yoyish. Har qanday $\bar{c}$ vektorni, berilgan ikki $\bar{a}$ va $\bar{b}$ nokollinear vektor bo'yicha $\bar{c} = \lambda\bar{a} + \mu\vec{b}$ yoyilma ko'rinishida tasvirlash mumkin. Yoyilma koeffitsiyentlari λ va μ yagona tarzda aniqlanadi.

3- m a s a l a. $\vec{m}(-1;2),\ \bar{p}(4;-2)$ va $\bar{l}(2;-3)$ vektorlar berilgan. $\bar{a} = \bar{m} + 2\bar{l}$ vektorni $\vec{m}$ va $\bar{p}$ vektorlar orqali ifodalang.

Y e c h i l i s h i. Masala shartiga ko'ra: $\bar{a}(-1+4;\ 2-6) \Rightarrow \bar{a}(3;-4)$.

Endi $\bar{a}$ vektorni nokollinear $\vec{m}$ va $\bar{p}\left(\text{chunki} -\frac{1}{4} \neq -\frac{2}{2}\right)$ bo'yicha $\bar{a} = \lambda\vec{m} + \mu\bar{p}$ yoyilmasidagi λ va μ koeffitsientlarni aniqlaymiz:

$$\begin{cases} 3 = -1 \cdot \lambda + 4 \cdot \mu, \\ -4 = 2 \cdot \lambda - 2 \cdot \mu \end{cases} \Rightarrow \begin{cases} -2\lambda + 8\mu = 6, \\ 2\lambda - 2\mu = -4 \end{cases} \Rightarrow \left[\begin{array}{l} \lambda = -\frac{5}{3}, \\ \mu = \frac{1}{3}. \end{array} \right.$$

Demak, $\bar{a} = -\frac{5}{3}\vec{m} + \frac{1}{3}\bar{p}$.

J a v o b: $\bar{a} = -\frac{5}{3}\vec{m} + \frac{1}{3}\bar{p}$.

2.5. Vektorlarning skalar ko'paytmasi. Yuqorida ko'rilgan vektorlar ustidagi amallar natijasida yana vektorlar hosil bo'lishini ko'rdik.

Vektorni vektorga skalar ko'paytirilganda ko'paytma songa (skalarga) teng bo'ladi.

Ta'rif. *Nol bo'lmagan $\vec{a}$ va $\vec{b}$ vektorlarning skalar ko'paytmasi deb, ularning uzunliklari (modullari) va bu vektorlar orasidagi burchak kosinusi ko'paytmasiga aytiladi,* ya'ni:

$$\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos(\vec{a},\hat{\ }\vec{b}).$$

Ixtiyoriy ikki nol bo'lmagan $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak deganda ularni bir nuqtadan qo'yib hosil qilingan burchakka aytiladi.

Skalyar ko'paytma ta'rifidan uning quyidagi ***xossalari*** kelib chiqadi:

1. $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$ (o'rin almashtirish qonuni);
2. $(\lambda\vec{a})\,\vec{b}=\lambda(\vec{a}\vec{b})$, bunda $\lambda\in R$ (guruhlash qonuni);
3. $(\vec{a}+\vec{b})\cdot\vec{c}=\vec{a}\vec{c}+\vec{b}\vec{c}$ (taqsimot qonuni);
4. $\vec{a}\cdot\vec{b}=0$ bo'lsa, $\vec{a}=\vec{0}$ yoki $\vec{b}=\vec{0}$ yoki $\vec{a}\perp\vec{b}$ bo'ladi.

$\vec{a}\cdot\vec{a}=|\vec{a}|^2$ skalar ko'paytma *skalar kvadrat* deb ataladi.

Vektorlarning skalar ko'paytmasidan foydalanib, ular orasidagi burchakni topish mumkin:

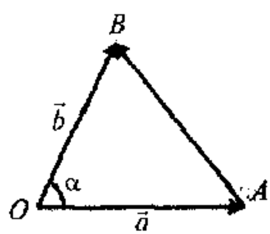
$$\vec{a}\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|\cos(\vec{a},\hat{\ }\vec{b})\Rightarrow\cos(\vec{a},\hat{\ }\vec{b})=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}.$$

Koordinatalari bilan berilgan $a(x_1;\ y_1)$ va $b(x_2;\ y_2)$ vektorlarning skalar ko'paytmasi $\vec{a}\cdot\vec{b}=x_1x_2+y_1y_2$.

I s b o t i. Isbotni umumiy hol uchun qilamiz. Ixtiyoriy O nuqtadan $\vec{a}$ va $\vec{b}$ vektorlarni qo'yib ($\overrightarrow{OA}=\vec{a}$ va $\overrightarrow{OB}=\vec{b}$), ΔOAB da kosinuslar teoremasiga asosan (178- rasm):

$$|\overrightarrow{AB}|^2=|\overrightarrow{OA}|^2+|\overrightarrow{OB}|^2-2|\overrightarrow{OA}||\overrightarrow{OB}|\cos\alpha.$$

$\overrightarrow{AB}=\vec{b}-\vec{a}$, $\overrightarrow{OA}=\vec{a}$, $\overrightarrow{OB}=\vec{b}$ bo'lgani uchun:



178- rasm.

$$\left|\vec{b}-\vec{a}\right|^2=|\vec{a}|^2+\left|\vec{b}\right|^2-2\vec{a}\vec{b} \Rightarrow \vec{a}\vec{b}=\frac{1}{2}\left(|\vec{a}|^2+\left|\vec{b}\right|^2-\left|\vec{b}-\vec{a}\right|^2\right). \quad (*)$$

$\vec{a}, \vec{b}$ va $\vec{b}-\vec{a}$ vektorlar mos ravishda $(x_1;\ y_1)$, $(x_2;\ y_2)$ va $(x_2-x_1;\ y_2-y_1)$ koordinatalarga ega bo'lgani uchun: $|\vec{a}|^2=x_1^2+y_1^2$, $\left|\vec{b}\right|^2=x_2^2+y_2^2$, $\left|\vec{b}-\vec{a}\right|^2=(x_2-x_1)^2+(y_2-y_1)^2$ bo'ladi. Bularni (*) ga qo'yib soddalashtirilsa, $\vec{a}\cdot\vec{b}=x_1x_2+y_1y_2$ hosil bo'ladi.

Shunga ko'ra: $\cos(\vec{a},\hat{}\vec{b})=\dfrac{x_1x_2+y_1y_2}{\sqrt{x_1^2+y_1^2}\cdot\sqrt{x_2^2+y_2^2}}$.

***Xulosa**. Agar $\vec{a}\perp\vec{b}$ bo'lsa, $x_1x_2+y_1y_2=0$ bo'ladi.*

4- m a s a l a. $\vec{a}(-4;7)$ va $\vec{b}(7;4)$ vektorlarning skalar ko'paytmasini toping.

Y e c h i l i s h i: $\vec{a}\cdot\vec{b}=-4\cdot7+7\cdot4=0$. Demak, $\vec{a}\perp\vec{b}$.

J a v o b: $\vec{a}\cdot\vec{b}=0$.

5- m a s a l a. Agar $\vec{p}$ va $\vec{q}$ o'zaro perpendikular birlik vektorlar bo'lsa, $\vec{a}=2\vec{p}-3\vec{q}$ va $\vec{b}=\vec{p}+5\vec{q}$ vektorlarning skalar ko'paytmasini toping.

Y e c h i l i s h i: $\vec{a}\cdot\vec{b}=(2\vec{p}-3\vec{q})(\vec{p}+5\vec{q})=2|\vec{p}|^2+7|\vec{p}||\vec{q}|\cos90^\circ-$ $-15|\vec{q}|^2=2-15=-13$.

J a v o b: -13.

6- m a s a l a. Tomonining uzunligi *a* ga teng bo'lgan *ABCD* kvadratdagi $\overrightarrow{AB}$ va $\overrightarrow{BC}$ hamda $\overrightarrow{AB}$ va $\overrightarrow{BD}$ vektorlar juftining skalar ko'paytmalarini toping.

Y e c h i l i s h i: $\left|\overrightarrow{AB}\right|=\left|\overrightarrow{BC}\right|=a$, $\left|\overrightarrow{BD}\right|=a\sqrt{2}$ bo'lgani uchun hamda $\left|\overrightarrow{AB}\right|$ va $\left|\overrightarrow{BC}\right|$ vektorlar orasidagi burchak to'g'ri burchak, $\left|\overrightarrow{AB}\right|$ va $\left|\overrightarrow{BD}\right|$ vektorlar orasidagi burchak 45° ekanligidan:

1) $\overrightarrow{AB}\cdot\overrightarrow{BC}=\left|\overrightarrow{AB}\right|\left|\overrightarrow{BC}\right|\cos 90^\circ=a\cdot a\cdot 0=0$.

2) $\overrightarrow{AB}\cdot\overrightarrow{BD}=\left|\overrightarrow{AB}\right|\left|\overrightarrow{BD}\right|\cos 45^\circ=a\cdot a\sqrt{2}\cdot\frac{\sqrt{2}}{2}=a^2$.

J a v o b: 0; a^2.

2.6. Parallel ko'chirish. *Tekislikdagi biror shaklning har bir nuqtasini bir xil yo'nalishda ma'lum masofaga siljitish shaklni parallel ko'chirish deyiladi.*

Har bir $\vec{c}(a;b)$ vektor parallel ko'chirishni aniqlaydi, chunki tekislikning har bir nuqtasini vektor yo'nalishi bo'yicha vektorning uzunligi qadar siljitilsa, tekislik $\vec{c}(a;b)$ vektor bo'yicha parallel ko'chgan deyiladi. Agar $A(x;\ y)$ nuqta $\vec{c}(a;b)$ vektor bo'yicha parallel ko'chirish natijasida $A'(x';y')$ nuqtaga o'tsa, uning koordinatalari

$$x'=x+a;\ \ y'=y+b$$

kabi ifodalanadi.

7- m a s a l a. Parallel ko'chirish $x'=x+2,\ y'=y-1$ formulalar bilan berilsa, (0;2) nuqta qanday nuqtaga ko'chadi?

Y e c h i l i s h i. (0;2) nuqta ko'chadigan nuqtaning koordinatalarini $(x';\ y')$ desak, u holda $x'=0+2=2; y'=2-1=1$.

J a v o b: (2;1) nuqtaga ko'chadi.

2.7. Vektorlar yordamida masalalar yechish.

1. K e s m a o' r t a s i n i n g k o o r d i n a t a l a r i n i t o p i s h. Uchlari $A(x_1;\ y_1)$ va $B(x_2;\ y_2)$ nuqtalarda bo'lgan AB kesmaning o'rtasi C nuqtaning $(x;\ y)$ koordinatalarini A va B nuqtalar koordinatalari orqali ifodasini topish talab qilingan bo'lsin (179-rasm).

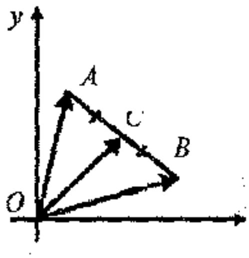
Bu masalani yechishda dastlab $\overrightarrow{OC}=\frac{1}{2}(\overrightarrow{OA}+\overrightarrow{OB})$ tenglik o'rinli ekanligini ko'rsatamiz. Vektorlarni qo'shishning uchburchak qoidasiga ko'ra

$$\overrightarrow{OC}=\overrightarrow{OA}+\overrightarrow{AC},$$

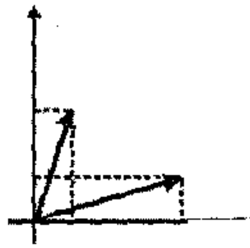
$$\overrightarrow{OC}=\overrightarrow{OB}+\overrightarrow{BC}.$$

Bu tengliklarni qo'shib,

$$2\overrightarrow{OC}=\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{AC}+\overrightarrow{BC}$$



179- rasm.



180- rasm.

tenglikka ega bo'lamiz. Bunda C nuqta AB kesmaning o'rtasi bo'lganli gi sababli $\overrightarrow{AC}+\overrightarrow{BC}=\vec{0}$. Demak,

$$\overrightarrow{OC}=\frac{1}{2}(\overrightarrow{OA}+\overrightarrow{OB}).$$

O nuqtaning koordinatalari (0; 0) ekanligini e'tiborga olib, vektorlar yig'indisini va vektorni songa ko'paytirishni A va B nuqtalarning koordinatalari orqali ifodalab, $\overrightarrow{OC}\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$ ga ega bo'lamiz.

Bundan $x=\frac{x_1+x_2}{2}$, $y=\frac{y_1+y_2}{2}$ kelib chiqadi. Shunday qilib, kesma o'rtasining har bir koordinatasi uning oxirlari mos koordinatalari yig'indisining yarmiga teng.

2. Koordinatalar tekisligida ikki nuqta orasidagi masofa. Tekislikda $C(x_1;\ y_1)$ va $D(x_2;\ y_2)$ nuqtalar berilgan bo'lsin. Bu ikki nuqta orasidagi $d=|\overrightarrow{CD}|$ masofani C va D nuqtalar koordinatalari orqali ifodalaymiz. $\overrightarrow{CD}$ vektorning koordinatalari $(x_2-x_1;\ y_2-y_1)$. Demak, bu vektorning uzunligi

$$|\overrightarrow{CD}|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}.$$

Shunday qilib,

$$d=|\overrightarrow{CD}|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}.$$

$M(x_0;\ y_0)$ nuqtadan $Ax+By+C=0$ tenglama bilan berilgan to'g'ri chiziqqacha bo'lgan masofa

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

formula bilan topiladi.

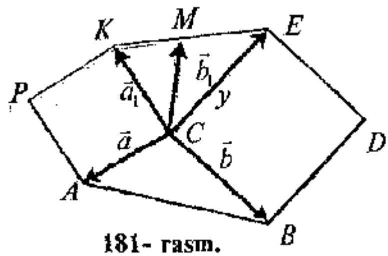
8- m a s a l a. Uchlari $M(4; 0)$, $N(12; -2)$ va $K(5; -9)$ nuqtalarda bo'lgan uchburchakning perimetrini toping.

Y e c h i l i s h i.

$|MN| = \sqrt{64+4} = \sqrt{68} = 2\sqrt{17}$; $|NK| = \sqrt{49+49} = 7\sqrt{2}$ va $|MK| = \sqrt{1+81} = \sqrt{82}$ bo'lgani uchun $|MN| + |NK| + |MK| = 2\sqrt{17} + 7\sqrt{2} + \sqrt{82}$.

J a v o b: $2\sqrt{17} + 7\sqrt{2} + \sqrt{82}$.

9- m a s a l a. ABC uchburchakning tomonlariga undan tashqarida $ACKP$ va $BCED$ kvadratlar chizilgan bo'lib, M nuqta KE kesmaning o'rtasi bo'lsa, $\overline{CM} \perp \overline{AB}$ ekanligini isbotlang (181- rasm).

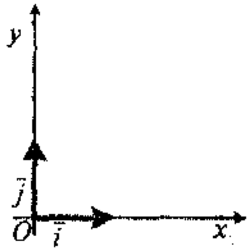


181- rasm.

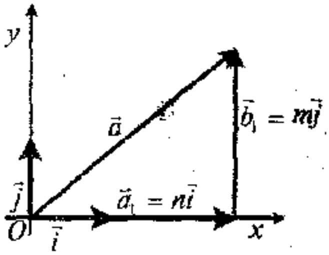
I s b o t i. $\overline{CA} = \bar{a}$, $\overline{CK} = \bar{a}_1$, $\overline{CB} = \bar{b}$, $\overline{CE} = \bar{b}_1$ bo'lsin. Unda $\overline{CM} = \frac{1}{2}(\bar{a}_1 + \bar{b}_1)$, $\overline{AB} = \bar{b} - \bar{a}$. $\overline{CM}$ va $\overline{AB}$ vektorning skalar ko'paytmasi $\overline{CM} \cdot \overline{AB} = -\frac{1}{2}(\bar{a}_1\bar{b} - \bar{a}_1\bar{a} + \bar{b}_1\bar{b} - \bar{b}_1\bar{a})$. $\bar{a} \perp \bar{a}_1$ ва $\bar{b} \perp \bar{b}_1$ bo'lgani uchun hamda ΔACE va ΔBCK larning ikki tomonlari va ular orasidagi burchaklari tengligiga asosan o'zaro teng bo'lgani sababli, $\bar{a} \cdot \bar{a}_1 = 0$; $\bar{b} \cdot \bar{b}_1 = 0$, ya'ni $\bar{a} \cdot \bar{a}_1 - \bar{b} \cdot \bar{b}_1 = 0$ bo'ladi.

Demak, $\overline{CM} \cdot \overline{AB} = 0$, bundan $\overline{CM} \perp \overline{AB}$ kelib chiqadi.

2.8. Vektorni koordinata o'qlari bo'yicha yoyish. Uzunligi 1 ga teng bo'lgan $\bar{e}(|\bar{e}| = 1)$ vektor *birlik vektor* deyiladi. Koordinatalar yarim o'qlariga yo'nalishdosh bo'lgan birlik vektorlar *koordinata*



182- rasm.



183- rasm.

vektorlari yoki *ortlar* deyiladi. Odatda abssissalar o'qidagi birlik vektor $\vec{i}(1;0)$, ordinatalar o'qidagi birlik vektor esa $\vec{j}(0;1)$ kabi belgilanadi (182- rasm).

Ixtiyoriy $\vec{a}(a_1;b_1)$ vektorni $\vec{i}$ va $\vec{j}$ birlik vektorlar orqali yoyilmasi $\vec{a}=n\vec{i}+m\vec{j}$ tarzida bo'lishini ko'rsatamiz.

$(a_1;\ b_1)$ juftlik koordinata boshidan qo'yilgan $\vec{a}$ vektor oxirining koordinatalari bo'lib, $\vec{a}_1=n\,\vec{i}$, $\vec{b}=m\vec{j}$ va $a=\vec{a}_1+\vec{b}_1$ ekanligidan $\vec{a}=n\vec{i}+m\vec{j}$ ekanligi kelib chiqadi (183- rasm).

10- m a s a l a. Agar $\vec{i}$ va $\vec{j}$ o'zaro perpendikular birlik vektorlar bo'lsa, $\vec{a}=2\vec{i}+\vec{j}$ vektorning uzunligini toping.

Y e c h i l i s h i. $a_1=2, b_1=1$ bo'lganligidan:

$$|\vec{a}|=\sqrt{2^2+1^2}=\sqrt{5}.$$

J a v o b: $|\vec{a}|=\sqrt{5}$.

2.9. Berilgan vektor yo'nalishidagi birlik vektor. Agar noldan farqli $\vec{a}$ vektorni o'zining uzunligi $|\vec{a}|$ ga bo'lsak, u holda hosil bo'lgan $\frac{\vec{a}}{|\vec{a}|}$ vektor $\vec{a}$ yo'nalishiga ega bo'lib, uzunligi 1 ga teng bo'lishi ravshan. Haqiqatan ham, agar $\frac{\vec{a}}{|\vec{a}|}=\vec{e}$ deb belgilasak, u holda

$$|\vec{e}| = \frac{|\vec{a}|}{|\vec{a}|} = 1.$$

Agar $\vec{a}(x_1; y_1)$ bo'lsa, $\vec{e}$ birlik vektorning koordinatalari $x = \frac{x_1}{|a|};\ y = \frac{y_1}{|a|}$ tengliklar bilan aniqlanadi

11-masala. $\vec{a} = -3\vec{i} + 4\vec{j}$ vektor yo'nalishidagi birlik vektor $\vec{e}(x; y)$ ning koordinatalarini toping.

Yechilishi. Berilgan vektor koordinatalari (–3; 4) ekanligidan $|\vec{a}| = \sqrt{(-3)^2 + 4^2} = \sqrt{25} = 5$.

$$x = -\frac{3}{5};\ y = \frac{4}{5}.$$

Javob: $\left(-\frac{3}{5}; \frac{4}{5}\right)$.

Mustaqil ishlash uchun test topshiriqlari

1. $\vec{a}\left(1; -\frac{4}{3}\right)$ vektor berilgan. $3\vec{a}$ vektorning modulini toping.

A) 4,5; B) 3,5; C) 5; D) 5,5; E) 6.

2. $\vec{b}(0; -2)$ ва $\vec{c}(-3; 4)$ vektorlar berilgan. $\vec{a} = 3\vec{b} - 2\vec{c}$ vektorning koordinatalarini toping.

A) (0; 8); B) (3; –6); C) (6; –6); D) (6; –14); E) (–6; –8).

3. $\vec{a}(5; 1)$ ва $\vec{b}(-2; 3)$ vektorlar berilgan. $|\vec{a} + \vec{b}|$ ni hisoblang.

A) 5; B) 3; C) 4; D) 2; E) 1.

4. $\vec{a}$ (2; 5) va $\vec{b}$ (m; –6) vektorlar m ning qanday qiymatida o'zaro perpendikular bo'ladi?

A) 14; B) 16; C) 15; D) –15; E) 14.

5. $\vec{a}$ ва $\vec{b}$ vektorlar berilgan. $|\vec{a}| = 3$, $|\vec{b}| = 4$ bo'lib, $\vec{a}$ ва $\vec{b}$ orasidagi burchak 60° ga teng. λ ning qanday qiymatida $(\vec{a} - \lambda\vec{b}) \perp \vec{a}$ bo'ladi?

A) 1; B) 2; C) 3; D) 1,5 E) 2,5.

6. Rombda o‘zaro teng bo‘lgan nechta juft vektorlar mavjud?

A) 3 ta; B) 5 ta; C) 2 ta; D) 4 ta; E) 1 ta.

7. $A(2; -8)$ va $B(5; -4)$ nuqtalar berilgan. $\overline{AB}$ vektorning modulini toping.

A) 4; B) 5; C) 3; D) 6,5; E) –7.

8. $A(2; 2)$, $B(3; -1)$, $C(2; 8)$ nuqtalar berilgan. Shunday $D(x; y)$ nuqtani topingki, $\overline{AB} = \overline{CD}$ bo‘lsin.

A) (3; 5); B) (–3, –5); C) (3; –5); D) (–3; 5); E) (5; 3).

9. $\vec{a}(9; 6)$ va $\vec{b}(4; 7)$ vektorlarning ayirmasi $\vec{c}(n; m)$ vektorning koordinatalarini toping.

A) $n = 5$; $m = 1$; B) $n = 5$; $m = 4$; C) $n = 5$; $m = -1$;
D) $n = -5$; $m = 1$; E) $n = -5$; $m = -1$;

10. Agar $\bar{a} = 2\vec{i} + 3\vec{j}$ ва $b = 2\vec{j}$ bo‘lsa, $\bar{p} = 2\bar{a} - 3\vec{b}$ vektorning koordinatalarini ko‘rsating.

A) (–4; 12); B) (–4; 0); C) (4; 0); D) (2; –6); E) (–2; 4).

11. Agar $\bar{a}(-6; 8)$ vektor berilgan bo‘lib, $|\lambda a| = 5$ bo‘lsa, λ ni toping.

A) $\frac{1}{2}$; B) $\pm\frac{1}{2}$; C) $-\frac{5}{6}$; D) $\frac{5}{6}$; E) $\pm\frac{5}{14}$

12. $\vec{a}(2; -4)$, $\vec{b}(1; 2)$, $\vec{c}(1; -2)$ va $\vec{d}(-2; -4)$ vektorlardan qaysilari kollinear vektorlar:

A) $\bar{a}, \bar{c}$ ва $\bar{b}, \bar{d}$; B) $\bar{b}, \bar{c}$; C) $\bar{a}, \bar{d}$; D) $\bar{a}, \bar{b}$;

E) kollinearlari yo‘q.

13. $|\bar{a}| = 3$, $|\vec{b}| = 4$ hamda $\vec{a}$ ва $\vec{b}$ vektorlar $\frac{\pi}{3}$ ga teng burchak tashkil qiladi. $\vec{c} = 3\vec{a} + 2\vec{b}$ vektorning uzunligini toping.

A) $\sqrt{217}$; B) 12; C) 17; D) $\sqrt{221}$; E) 13.

14. Agar $|\vec{a}| = 3$ ва $|\vec{b}| = 5$ bo‘lsa, $\vec{a} + \lambda\vec{b}$ ва $\vec{a} - \lambda\vec{b}$ vektorlar λ ning qanday qiymatlarida perpendikular bo‘ladi?

A) $-\frac{3}{5} < \lambda < \frac{3}{5}$; B) $-\frac{3}{5}$; C) $\frac{3}{5}$; D) $\pm\frac{3}{5}$; E) $\frac{5}{3}$.

15. $2\vec{a} = 2\vec{i} + \vec{j}$ ва $\vec{b} = -\vec{i} + 2\vec{j}$ vektorlarda yasalgan parallelogrammning diagonallari orasidagi burchakni toping.

A) $\arccos\frac{1}{\sqrt{21}}$; B) $\frac{\pi}{6}$; C) $\arccos\frac{2}{\sqrt{21}}$; D) $\frac{\pi}{2}$; E) $\frac{\pi}{3}$.

16. m ning qanday qiymatlarida $\vec{a}(m; m-3)$ vektorning uzunligi 3 dan kichik bo'ladi?

A) $-2 < m < 2$; B) $0 < m < 3$; C) $-1 < m < 3$; D) $-1 < m < 2$;
E) $-1 < m < 1$;

17. $\vec{a}\,(7; 3)$ va $\vec{b}\,(-2; -5)$ vektorlar orasidagi burchakni toping.

A) 30°; B) 45°; C) 60° D) 135°; E) 150°.

18. $\vec{a}$ ва $\vec{b}$ vektorlarning uzunliklari mos ravishda 11 va 23 ga teng bo'lib, bu vektorlar ayirmasining uzunligi 30 ga teng. $\vec{a}$ ва $\vec{b}$ vektorlar yig'indisining uzunligini toping.

A) 34; B) 64; C) 42; D) 50; E) 20.

19. $ABCD$ to'g'ri to'rtburchakda $AD = 12$, $CD = 5$ va O nuqta diagonallarning kesishish nuqtasi bo'lsa, $\left|\overline{AB} + \overline{AD} - \overline{DC} - \overline{OD}\right|$ ni toping.

A) 6,5; B) 7; C) 6; D) 13; E) 17.

20. Teng yonli ABC uchburchakda M nuqta AC asosning o'rtasi bo'lib, $AB = 5$, $BM = 4$ bo'lsa, $\left|\overline{MB} - \overline{MC} + \overline{BA}\right|$ ni toping.

A) 9; B) 6; C) 5; D) 4,5; E) 3.

21. $\vec{a}(-1; -1)$, $\vec{b}(-1; 1)$, $\vec{c}(-5; 3)$ va $\vec{d}(-5; 2)$ vektorlarning qaysilari $\vec{m}(-3; 2)$ va $\vec{n}(2; -1)$ vektorlardan yasalgan parallelogrammning diagonallari bo'ladi?

A) $\vec{a}$, $\vec{b}$; B) $\vec{a}$, $\vec{c}$; C) $\vec{a}$, $\vec{d}$; D) $\vec{c}$, $\vec{d}$; E) $\vec{b}$, $\vec{c}$.

22. $\vec{a}\,(3; 4)$ vektor yo'nalishidagi birlik vektorni toping.

A) $\vec{e}(0,6; 0,8)$; B) $\vec{e}(6; 16)$; C) $\vec{e}(1; 0)$; D) $\vec{e}(0; 1)$; E) $\vec{e}(2; 16)$.

23. $\vec{a}(2;3)$, $\vec{b}(3;-2)$ va $\vec{c}(4;19)$ vektorlar uchun $\vec{c}=m\vec{a}+n\vec{b}$ tenglik o'rinli bo'lsa, mn ko'paytmaning qiymatini toping.

A) –10; B) –12; C) 6; D) –8; E) 10.

24. $\vec{a}(3;\ 1)$ va $\vec{b}(1;\ 3)$ vektorlarda qurilgan parallelogramm diagonallarining uzunliklari yig'indisini toping.

A) $8\sqrt{2}$; B) $6\sqrt{2}$; C) 8; D) 6; E) $2\sqrt{2}$.

25. $\vec{a}\ (1;\ 2)$, $\vec{b}\ (3;\ -5)$ vektorlardan qurilgan uchburchak yuzini hisoblang.

A) 13; B) 6,5; C) 7; D) 5,5; E) 4,7.

26. $\vec{m}$ va $\vec{n}$ nokollinear vektorlar bo'lib, $|\vec{m}|=|\vec{n}|=4$ bo'lsa, $(\vec{m}+\vec{n})$ bilan $(\vec{m}-\vec{n})$ qanday burchak tashkil etadi?

A) 30°; B) 45°; C) 60°; D) 75°; E) 90°.

27. Agar $\vec{m}$ va $\vec{n}$ o'zaro perpendikular birlik vektorlar bo'lsa, $\vec{a}=2\vec{m}+\vec{n}$ vektorning uzunligini toping.

A) 2; B) $2\sqrt{2}$; C) 3; D) $\sqrt{5}$; E) $3\sqrt{3}$.

28. Agar $\vec{m}$ va $\vec{n}$ 120° li burchak tashkil etuvchi birlik vektorlar bo'lsa, $2\vec{m}+4\vec{n}$ va $\vec{m}-\vec{n}$ vektorlar orasidagi burchakni toping.

A) 60°; B) 90°; C) 120°; D) 135°; E) 150°.

29. $\vec{a}$ va $\vec{b}$ vektorlar 45° li burchak tashkil qiladi va $\vec{a}\cdot\vec{b}=4$. Shu vektorlarga qurilgan uchburchakning yuzini hisoblang.

A) 4; B) 2; C) $2\sqrt{2}$; D) $4\sqrt{2}$; E) 8.

30. Agar $\vec{m}$ bilan $\vec{n}$ vektorlar 30° li burchak tashkil etib, $\vec{m}\cdot\vec{n}=\sqrt{3}$ bo'lsa, ularga qurilgan parallelogrammning yuzini hisoblang.

A) 2; B) $\frac{\sqrt{3}}{2}$; C) $2\sqrt{3}$; D) 1; E) 1,5.

31. $\vec{a}$ va $\vec{b}$ vektorlar 120° li burchak hosil qiladi. Agar $|\vec{a}|=3$, $|\vec{b}|=5$ bo'lsa, $|\vec{a}-\vec{b}|$ ning qiymatini toping.

A) 2; B) 4; C) 6; D) 7; E) 10.

32. To'rtburchakning $M(2; -4)$, $N(-4; 0)$ va $P(2; -2)$ uchlari berilgan. Agar $\overrightarrow{MN} = 4\overrightarrow{QP}$ bo'lsa, Q uchining koordinatalarini toping.

A) (–7; 1); B) (–7; –1); C) (3,5; –3); D) (7; 1); E) (–2; –4).

33. Agar $|\vec{a}| = 3$, $|\vec{b}| = 5$ bo'lsa, x ning qanday qiymatlarida $\vec{a} + x\vec{b}$ va $\vec{a} - x\vec{b}$ vektorlar o'zaro perpendikular bo'ladi?

A) $\frac{3}{5}$; B) $-\frac{3}{5}$; C) $\pm\frac{3}{5}$; D) $\pm\frac{5}{3}$; E) $\frac{5}{3}$.

VII bob. STEREOMETRIYANING ASOSIY TUSHUNCHALARI

1-§. Stereometriyaning mantiqiy tuzilishi

1.1. Boshlang'ich *tushunchalar*. Xuddi planimetriyadagi kabi stereometriya quyidagi mantiqiy tarzda tuzilgan.

1. Asosiy, ta'rifsiz qabul qilingan tushunchalar keltiriladi.
2. Asosiy tushunchalarning xossalarini ifodalovchi aksiomalar beriladi.
3. Boshqa geometrik tushunchalarning ta'riflari asosiy tushunchalar yordamida beriladi.
4. Ta'riflar va aksiomalar asosida teoremalar isbotlanadi.

Stereometriyaning asosiy tushunchalari to'rtta: nuqta, to'g'ri chiziq, tekislik va masofa.

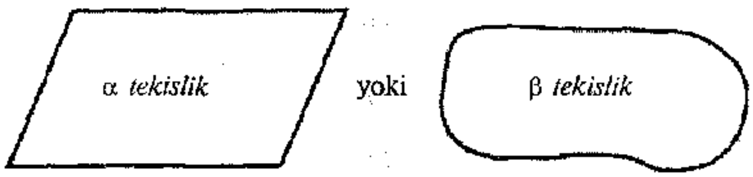
Tekislikni chizmada parallelogramm yoki biror tekis sirt shaklida tasvirlaymiz (184- rasm) va yunon alifbosidagi α, β, γ, ... harflari bilan belgilaymiz. Nuqtalar, to'g'ri chiziqlar xuddi planimetriyadagidek belgilanadi.

1 - m a s a l a. Quyidagi tasdiqlarning qaysi biri planimetriya aksiomasi hisoblanadi?

A) berilgan nuqtadan ma'lum to'g'ri chiziqqa faqat bitta perpendikular o'tkazish mumkin;

B) A dan B gacha bo'lgan masofa B dan A gacha bo'lgan masofaga teng;

C) siniq chiziq uzunligi uning oxirlari orasidagi masofadan katta;



184- rasm.

D) bir to‘g‘ri chiziqda yotmagan istalgan uchta nuqtadan bitta va faqat bitta aylana o‘tkazish mumkin;

E) *teng ko‘pburchaklarning yuzlari teng.*

Y e c h i l i s h i. Ma’lumki, yuqoridagi tasdiqlardan faqat ikkinchisi (*B*) aksioma bo‘lib, qolganlari isbottalab tushunchalardir.

1.2. Stereometriya aksiomalari. Stereometriya aksiomalarida asosiy *tushunchalar — nuqta*, to‘g‘ri *chiziq*, *tekislik* va masofaning asosiy xossalari ifodalanadi.

Dastlabki beshta aksioma tegishlilik tushunchasi bilan bog‘liq.

1-aksioma. Hech bo‘lmaganda bitta to‘g‘ri chiziq va hech bo‘lmaganda bitta tekislik mavjud. Har bir to‘g‘ri chiziq va har bir tekislik nuqtalarning bo‘sh bo‘lmagan to‘plami bo‘lib, bu to‘plam fazoning o‘zidan iborat emas.

2-aksioma. Istalgan ikkita turli nuqtadan bitta va faqat bitta to‘g‘ri chiziq o‘tadi.

3-aksioma. Tekislikning ikkita turli nuqtasidan o‘tuvchi to‘g‘ri chiziq shu tekislikda yotadi (185- rasm).

4-aksioma. Bir to‘g‘ri chiziqqa tegishli bo‘lmagan uchta nuqtadan bitta va faqat bitta tekislik o‘tadi (186- rasm).

5-aksioma. Agar ikkita turli tekislik bitta umumiy nuqtaga ega bo‘lsa, ular shu nuqtadan o‘tuvchi to‘g‘ri chiziq bo‘yicha kesishadi (187- rasm).

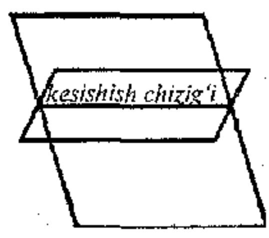
6-aksioma. Ixtiyoriy ikkita *A* va *B* nuqta uchun *A* dan *B* gacha bo‘lgan *masofa* deb ataluvchi nomanfiy kattalik mavjud. $|AB|$ masofa *A* va *B* nuqtalar faqat ustma-ust tushgandagina nolga teng.

7-aksioma. *A* nuqtadan *B* nuqtagacha bo‘lgan masofa *B* nuqtadan *A* nuqtagacha bo‘lgan masofaga teng:

$$|AB| = |BA|.$$

185- rasm.

186- rasm.



187- rasm.

8-aksioma. Ixtiyoriy uchta A, B, C nuqta uchun A dan C gacha bo‘lgan masofa A dan B gacha va B dan C gacha bo‘lgan masofalar yig‘indisidan katta emas:

$$|AC| \leq |AB| + |BA|.$$

9-aksioma. Har bir tekislik uchun planimetriyadan ma’lum tartib aksiomasi, tekislikning harakatchanligi aksiomasi va parallel to‘g‘ri chiziqlar aksiomasi bajariladi.

2- m a s a l a. Nima uchun uch oyoqli har qanday stol albatta turg‘un, to‘rt oyoqli stol esa doim bunday emasligini tushuntiring.

Y e c h i l i s h i. 4-aksiomaga asosan har qanday uch oyoqli stol oyoqlarining uchlari aniqlagan tekislik doim mavjud (stol turg‘un). Stolning oyoqlari to‘rtta bo‘lganida oyoqlarining ixtiyoriy uchtasi uchlari aniqlagan tekislikka to‘rtinchisining uchi tegishli bo‘lsa, stol turg‘un, aks holda turg‘un bo‘lmaydi.

1.3. Aksiomalardan kelib chiqadigan natijalar.

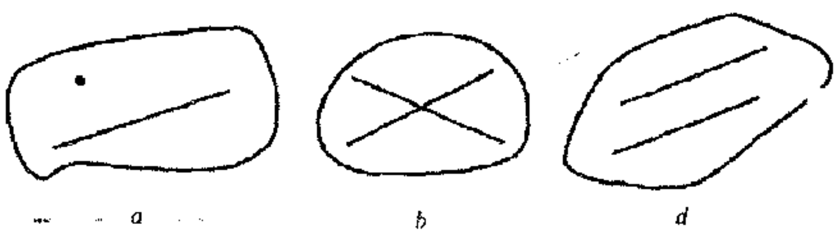
1-natija. To‘g‘ri chiziq va unga tegishli bo‘lmagan nuqtadan bitta va faqat bitta tekislik o‘tkazish mumkin.

2-natija. Ikkita kesishuvchi to‘g‘ri chiziq orqali bitta va faqat bitta tekislik o‘tkazish mumkin.

3-natija. O‘zaro parallel ikkita turli to‘g‘ri chiziqdan faqat bitta tekislik o‘tkazish mumkin (188- rasm).

3- m a s a l a. Bir tekislikka tegishli bo‘lmagan to‘rtta nuqta berilgan. Ulardan hech qanday uchtasi bir to‘g‘ri chiziqqa tegishli emasligini isbotlang.

I s b o t i. Agar nuqtalarning biror uchtasi bir to‘g‘ri chiziqqa tegishli bo‘lganda edi, u to‘g‘ri chiziq va to‘rtinchi nuqta orqali



a *b* *d*

188- rasm.

tekislik o'tardi va bu masalaning shartiga to'g'ri kelmaydi. Demak, ulardan hech qanday uchtasi bir to'g'ri chiziqda yotmaydi.

2-§. To'g'ri chiziqlar va tekisliklarning parallelligi

2.1. Fazoda parallel to'g'ri chiziqlar.

1-ta'rif. *Fazoda ikki to'g'ri chiziq bir tekislikda yotsa va kesishmasa, ular parallel to'g'ri chiziqlar deyiladi.*

2-ta'rif. *Kesishmaydigan va bir tekislikda yotmaydigan to'g'ri chiziqlar ayqash to'g'ri chiziqlar deyiladi* (189- rasm).

Teorema. To'g'ri chiziqdan tashqaridagi nuqtadan shu to'g'ri chiziqqa parallel to'g'ri chiziq o'tkazish mumkin va faqat bitta.

Teorema. Uchinchi to'g'ri chiziqqa parallel ikki to'g'ri chiziq o'zaro paralleldir (190- rasm).

2.2. To'g'ri chiziq bilan tekislikning parallelligi.

Ta'rif. *Agar to'g'ri chiziq bilan tekislik kesishmasa, ular o'zaro parallel deyiladi.*

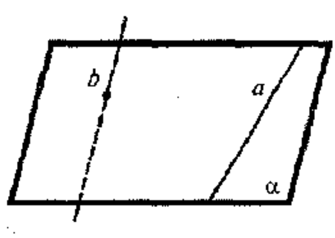
Teorema. Agar tekislikda yotmagan to'g'ri chiziq shu tekislikdagi biror to'g'ri chiziqqa parallel bo'lsa, bu to'g'ri chiziq tekislikning o'ziga ham parallel bo'ladi.

2.3. Tekisliklarning parallelligi.

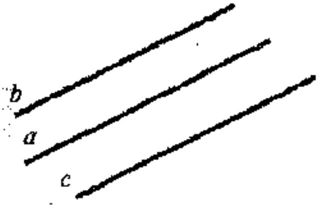
Ta'rif. *Agar ikki tekislik o'zaro kesishmasa, ular parallel tekisliklar deyiladi.*

1-teorema. Agar bir tekislikdagi kesishuvchi ikki to'g'ri chiziq ikkinchi tekislikdagi kesishuvchi ikki to'g'ri chiziqqa mos ravishda parallel bo'lsa, bu tekisliklar parallel bo'ladi.

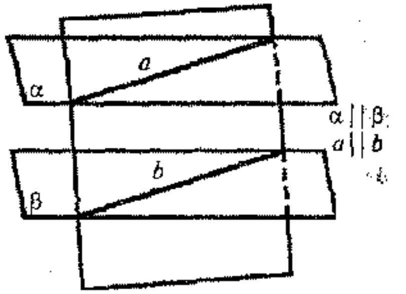
2-teorema. Tekislikdan tashqaridagi nuqtadan berilgan tekislikka parallel qilib bitta va faqat bitta tekislik o'tkazish mumkin.



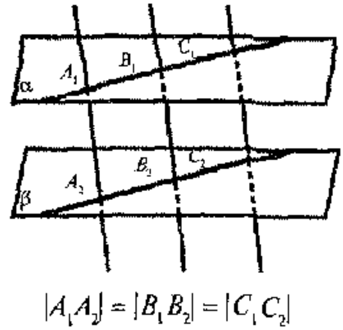
189- rasm.



190- rasm.



191- rasm.



192- rasm.

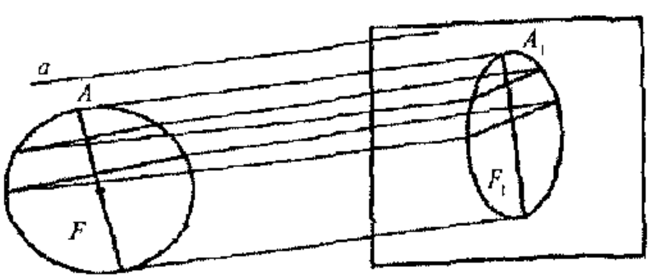
Parallel tekisliklarning xossalari.

1. Agar ikki parallel tekislik uchinchi tekislik bilan kesishsa, u holda kesishish to'g'ri chiziqlari parallel bo'ladi (191- rasm).

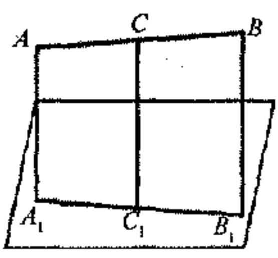
2. Ikkita parallel tekislik orasiga joylashgan parallel to'g'ri chiziqlarning kesmalari teng (192- rasm).

2.4. Fazoviy shakllarning tekislikda tasvirlanishi.

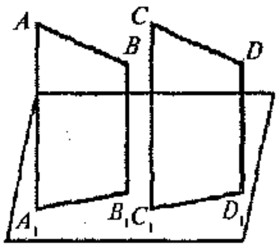
Parallel proyeksiya. Proyeksiya tekisligi hisoblanadigan biror α tekislikni kesuvchi qandaydir *a* to'g'ri chiziqni tanlab, fazoviy *F* shaklning ixtiyoriy *A* nuqtasidan o'tuvchi *a* ga parallel to'g'ri chiziqning α tekislik bilan kesishish nuqtasi A_1 nuqta *A* nuqtaning α tekislikdagi *parallel proyeksiyasi* (tasviri) deyiladi. *F* shaklning barcha nuqtalarini shu tariqa akslantirib, uning α tekisligidagi tasvirining parallel proyeksiyasini hosil qilamiz (193- rasm).



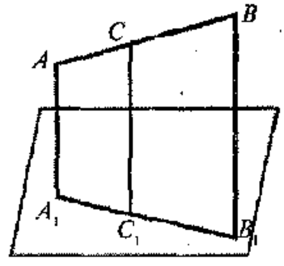
193-рacм.



194- rasm.



195- rasm.



196- rasm.

Parallel proyeksiyaning xossalari.

1. Parallel proyeksiyada to'g'ri chiziq to'g'ri chiziqqa almashadi (194- rasm).

2. Parallel to'g'ri chiziqlar proyeksiyasi parallel to'g'ri chiziqdir (195- rasm).

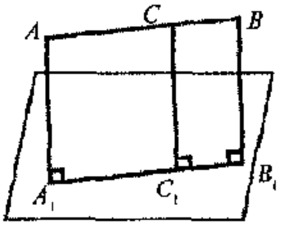
3. Bitta to'g'ri chiziq yoki parallel to'g'ri chiziqlar kesmalarining nisbati parallel proyeksiyada saqlanadi (196- rasm).

Agar proyeksiya yo'nalishi proyeksiya tekisligiga perpendikular bo'lsa, proyeksiya *ortogonal proyeksiya* deyiladi (197- rasm).

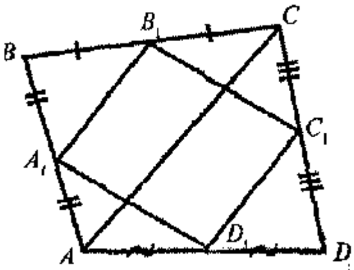
1-masala. Fazoviy to'rtburchak (fazoviy to'rtburchak uchlari bir tekislikda yotishi shart emas) tomonlarining o'rtalari parallelogramm uchlari bo'lishini isbotlang (198- rasm).

Yechilishi. A_1B_1 kesma ABC uchburchakning o'rta chizig'i, C_1D_1 kesma esa ACD uchburchakning o'rta chizig'i bo'lgani uchun:

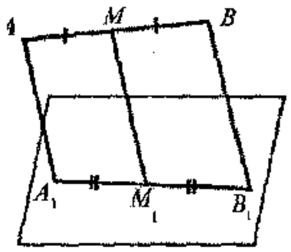
$$A_1B_1 \parallel AC;\ |A_1B_1| = \frac{1}{2}\ |AC|\ \text{va}\ C_1D_1 \parallel AC,\ |C_1D_1| = \frac{1}{2}\ |AC|.$$



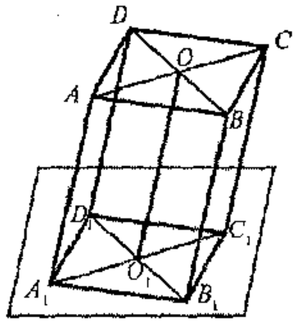
197- rasm.



198- rasm.



199- rasm.



200- rasm.

Demak, $A_1B_1 \parallel C_1D_1$ va $|A_1B_1| = |C_1D_1|$. Xuddi shunday, $C_1B_1 \parallel A_1D_1$ va $|S_1B_1| = |A_1D_1|$ ekanligi asoslanadi. Natijada, to'rtburchak $A_1B_1C_1D_1$ ning parallelogrammligi isbotlandi.

2- masala. 199- rasmda AB kesma biror tekislikka parallel proyeksiyalangan, M nuqta AB kesmaning o'rtasi. Agar $AA_1 = 5$, $BB_1 = 7$ bo'lsa, MM_1 kesmaning uzunligini toping.

Yechilishi. MM_1 kesmaning ABB_1A_1 trapetsiyaning o'rta chizig'i ekanligini asoslab, $|MM_1| = \frac{1}{2}(|AA_1| + |BB_1|) = 6$ ekanligini aniqlaymiz.

Javob: $|MM_1| = 6$.

Masalani kesma proyeksiya tekisligini kesib o'tgan hol uchun ham yechishga harakat qiling.

3- masala. $ABCD$ parallelogramm o'zi bilan kesishmaydigan α tekislikka parallel proyeksiyalangan. $A_1B_1C_1D_1$ — uning proyeksiyasi. Agar $|AA_1| = 2$; $|BB_1| = 3$; $|CC_1| = 8$ bo'lsa, DD_1 kesmaning uzunligini toping (200- rasm).

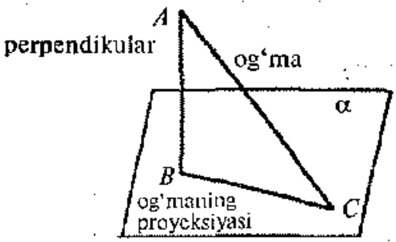
Yechilishi. AA_1CC_1 hamda DD_1B_1B trapetsiyalarda OO_1 o'rta chiziq bo'lib, uning uzunligi $|OO_1| = 5$ ga tengdir. Bundan:

$|DD_1| + |BB_1| = 2|OO_1|$; $|DD_1| + 3 = 10$; $|DD_1| = 7$.

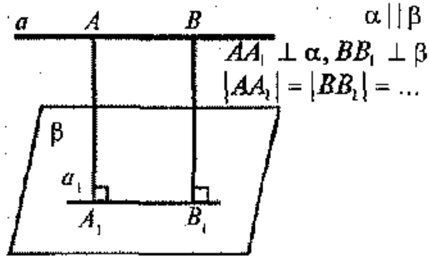
Javob: 7.

3-§. To'g'ri chiziqlar va tekisliklarning perpendikularligi

3.1. Fazoda to'g'ri chiziqlarning perpendikularligi. Tekislikdagidek, to'g'ri burchak ostida kesishgan ikki to'g'ri chiziq o'zaro perpendikular *to'g'ri chiziqlar* deyiladi.



201- rasm.



202- rasm.

1-teorema. Perpendikular to‘g‘ri chiziqlarga mos ravishda parallel bo‘lgan kesishuvchi to‘g‘ri chiziqlarning o‘zlari ham perpendikulardir.

2-teorema. Agar to‘g‘ri chiziq tekislikdagi kesishuvchi ikkita to‘g‘ri chiziqqa perpendikular bo‘lsa, bu to‘g‘ri chiziq shu tekislikka perpendikular bo‘ladi.

3-teorema. Tekislikda berilgan nuqta orqali unga perpendikular bitta va faqat bitta to‘g‘ri chiziq o‘tkazish mumkin.

4-teorema. Agar tekislik ikkita parallel to‘g‘ri chiziqdan biriga perpendikular bo‘lsa, u holda u ikkinchisiga ham perpendikulardir.

5-teorema. Bitta tekislikka perpendikular ikki to‘g‘ri chiziq **o‘zaro paralleldir.**

3.2. Perpendikular va og‘ma. Fazodagi A nuqtadan biror α tekislikka o‘tkazilgan perpendikular to‘g‘ri chiziqning tekislik bilan kesishgan B nuqtasi *perpendikularning asosi* deyiladi (201- rasm). AB kesma α tekislikka *perpendikular* deyiladi va bu kesmaning uzunligi A nuqtadan α tekislikkacha bo‘lgan *masofa* hisoblanadi.

A nuqtani tekislikning ixtiyoriy nuqtasi bilan tutashtiruvchi, perpendikulardan farqli kesma α tekislikka *og‘ma* deyiladi (201-rasm).

Og‘maning tekislik bilan kesishish nuqtasi uning *asosi* deyiladi. Bitta nuqtadan tekislikka o‘tkazilgan perpendikular va og‘maning asoslarini tutashtiruvchi kesma *og‘maning proyeksiyasi* deb ataladi.

To‘g‘ri chiziqdan unga parallel tekislikkacha bo‘lgan masofa deb, shu to‘g‘ri chiziqning istalgan nuqtasidan tekislikkacha bo‘lgan eng qisqa masofaga aytiladi (202-rasm).

Parallel tekisliklar orasidagi *masofa* deb, ulardan birining istalgan nuqtasidan ikkinchisigacha bo'lgan masofa tushuniladi.

Tekislikka undan tashqaridagi biror nuqtadan perpendikular va og'malar o'tkazilgan bo'lsa:

1) katta og'maning proyeksiyasi ham katta va, aksincha, proyeksiyasi katta og'maning o'zi ham kattadir;

2) teng og'malarning proyeksiyalari ham teng va aksincha, proyeksiyalari teng bo'lgan og'malar ham tengdir.

To'g'ri chiziq bilan tekislik orasidagi burchak deb, to'g'ri chiziq bilan uning tekislikdagi proyeksiyasi orasidagi burchakka aytiladi.

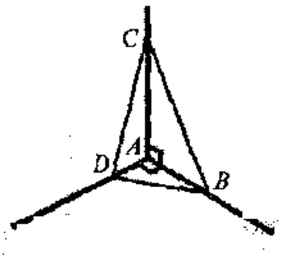
1- masala. AB, AC va AD to'g'ri chiziqlar juft-juft perpendikular (203- rasm). Agar $|AB| = 3$, $|BC| = 7$, $|AD| = 1{,}5$ bo'lsa, $|CD|$ ni toping.

Yechilishi. Chizmadagi ABC va ACD to'g'ri burchakli uchburchaklarga Pifagor teoremasini tatbiq etib, $|CD|$ ni topamiz. ΔABC da $|AC| = \sqrt{7^2 - 3^2} = \sqrt{40}$; ΔACD da $|CD| = \sqrt{40 + 1{,}5^2} = \sqrt{42{,}25} = 6{,}5$.

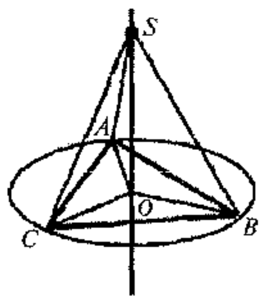
Javob: $|CD| = 6{,}5$.

2- masala. Uchburchakka tashqi chizilgan aylana markazidan uchburchak tekisligiga perpendikular to'g'ri chiziq o'tkazilgan. Bu to'g'ri chiziqning har bir nuqtasi uchburchakning uchlaridan baravar uzoqlikda yotishini isbotlang (204- rasm).

Yechilishi. ABC berilgan uchburchak, O nuqta unga tashqi chizilgan aylana markazi, S — perpendikulardagi ixtiyoriy nuqta. $OA = OB = OC = R$ uchburchakka tashqi chizilgan aylana radiusi.



203- rasm.



204- rasm.

OA, *OB*, *OC* kesmalar mos ravishda *SA*, *SB*, *SC* og'malarning proyeksiyalaridir. Proyeksiyalari teng bo'lgan og'malarning tengligidan: $SA = SB = SC$. Shuni isbotlash talab qilingan edi.

3.3. Uch perpendikular haqidagi teorema.

Teorema. Tekislikda og'maning asosidan uning proyeksiyasiga perpendikular qilib o'tkazilgan to'g'ri chiziq og'maning o'ziga ham perpendikular. Aksincha, tekislikdagi to'g'ri chiziq og'maga perpendikular bo'lsa, u og'maning proyeksiyasiga ham perpendikular bo'ladi (205- rasm).

Kesishuvchi ikki tekislikning kesishish chizig'iga perpendikular tekislik bilan kesganda hosil bo'lgan kesishish chiziqlari orasidagi burchak berilgan *tekisliklar orasidagi burchak* deyiladi va bu burchak kesishuvchi tekisliklarning *chiziqli burchagi* deb ataladi (206- rasm).

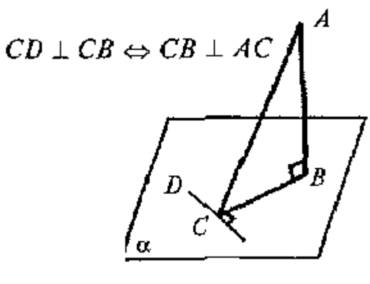
3- masala. Uchburchakka ichki chizilgan aylana markazidan uchburchak tekisligiga o'tkazilgan perpendikularning har bir nuqtasi uchburchak tomonlaridan baravar uzoqlikda turishini isbotlang.

Yechilishi. *K*, *M*, *N* nuqtalar *ABC* uchburchak tomonlarining aylana bilan urinish nuqtalari; $OK = OM = ON = r$; $OK \perp AB$, $OM \perp BC$, $ON \perp AC$ (207- *a* rasm). Uch perpendikular haqidagi teoremaga ko'ra $SK \perp AB$, $SM \perp BC, SN \perp AC$. To'g'ri burchakli

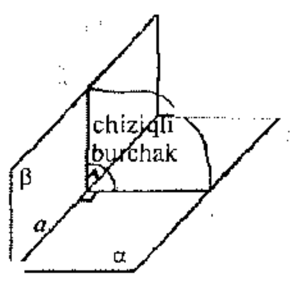
ΔSOK да $SK = \sqrt{r^2 + SO^2}$,

ΔSOM да $SM = \sqrt{r^2 + SO^2}$,

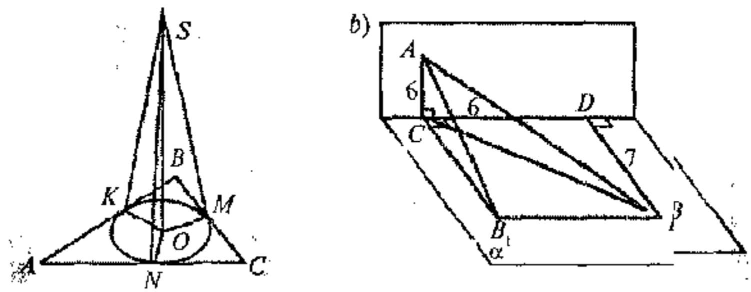
ΔSON да $SN = \sqrt{r^2 + SO^2}$.



205- rasm.



206- rasm.



207- rasm.

Shunday qilib, $SK = SM = SN$.

3.4. Tekisliklarning perpendikularlik alomati.

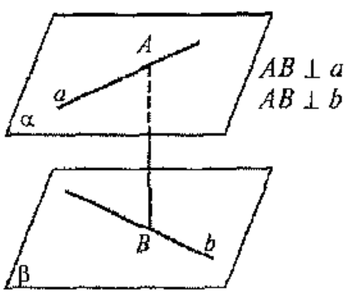
Ta'rif. *Kesishuvchi ikki tekislikning chiziqli burchagi 90° bo'lsa, bunday tekisliklar o'zaro perpendikular tekisliklar deb ataladi.*

Teorema. Agar tekislik boshqa bir tekislikka perpendikular to'g'ri chiziq orqali o'tsa, bu tekisliklar perpendikulardir.

4- masala. Ikki perpendikular tekislikda yotuvchi A va B nuqtalardan tekisliklar kesishgan to'g'ri chiziqqa AC va BD perpendikularlar tushirilgan. Agar $|AC| = 6$, $|BD| = 7$, $|CD| = 6$ bo'lsa, AB kesmaning uzunligini toping (207- *b* rasm).

Yechilishi. To'g'ri burchakli uchburchak ACB_1 da $|CB_1| = |BD| = 7$. $|AB_1| = \sqrt{6^2 + 7^2} = \sqrt{85}$. AB_1B to'g'ri burchakli uchburchakda $|AB|^2 = |AB_1|^2 + |B_1B|^2$; $|AB| = \sqrt{85 + 36} = \sqrt{121} = 11$.

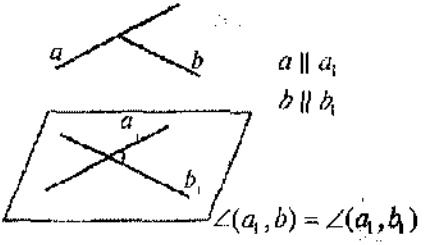
Javob: $|AB| = 11$.



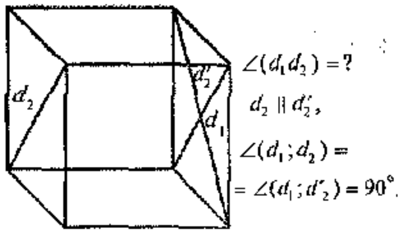
208-rasm.

3.5. Ayqash to'g'ri chiziqlar. Ayqash to'g'ri chiziqlarning *umumiy perpendikulari* deb, oxirlari shu to'g'ri chiziqlarda bo'lib, ularning har biriga perpendikular bo'lgan kesmaga aytiladi.

Ayqash to'g'ri chiziqlar bitta va faqat bitta umumiy perpendikularga ega. Umumiy perpendikularning uzunligi *ayqash to'g'ri chiziqlar orasidagi masofa* deyiladi (208- rasm).



209- rasm.



210- rasm.

Ta'rif. *Ayqash to'g'ri chiziqlar orasidagi burchak deb, ularning har biriga parallel bo'lgan kesishuvchi to'g'ri chiziqlar orasidagi burchakka aytiladi (209- rasm).*

5- m a s a l a. Kubning yoqlaridagi o'zaro ayqash diagonallari orasidagi burchakni toping (210- rasm).

Y e c h i l i s h i. Kub diagonallaridan birini vertikal qirrasi aniqlangan vektor bilan parallel ko'chirib, bir yog'idagi ikki diagonalini hosil qilamiz. Ma'lumki, ular o'zaro perpendikulardir. Demak, ayqash diagonallar orasidagi burchak 90° ekan.

J a v o b: 90°.

Mustaqil ishlash uchun test topshiriqlari

1. Boshlari umumiy bo'lgan, hech qanday uchtasi bir tekislikda yotmaydigan to'rtta nur berilgan bo'lsa, shu nurlardan har ikkitasi bir tekislikda yotadigan nechta turli tekislik o'tkazish mumkin?

A) 3 ta; B) 4 ta; C) 5 ta; D) 6 ta; E) 7 ta.

2. Boshlari umumiy bo'lgan, hech qanday uchtasi bir tekislikda yotmaydigan n ta nur berilgan bo'lsa, shu nurlardan har ikkitasi bir tekislikda yotadigan nechta turli tekislik o'tkazish mumkin?

A) $n - 1$; B) $(n - 1)/2$; C) $n(n - 1)$; D) $n(n - 1)/2$; E) $(2n - 1)/2$.

3. Uchta har xil tekislik umumiy nuqtaga ega. Bu tekisliklar juft-juft kesishganda nechta turli to'g'ri chiziq hosil bo'lishi mumkin?

A) 3 ta; B) 4 ta; C) 5 ta; D) 6 ta; E) aniqlab bo'lmaydi.

4. Hech bir uchtasi bir to'g'ri chiziqda yotmaydigan 6 ta nuqta nechta kesmani aniqlaydi?

A) 6 ta; B) 8 ta; C) 11 ta; D) 14 ta; E) 15 ta.

5. Fazodagi hech bir uchtasi bir to'g'ri chiziqda yotmaydigan n ta nuqta nechta kesmani aniqlaydi?

A) $2n+2$; B) $(n-1)n$; C) $n(n-1)/2$; D) $3n$; E) $(n-1)/2$.

6. Quyidagi fikrlarning qaysi biri noto'g'ri?

A) to'g'ri chiziq va unda yotmaydigan nuqta orqali bitta va faqat bitta tekislik o'tkazish mumkin;

B) to'g'ri chiziqning ikkita nuqtasi tekislikka tegishli bo'lsa, u holda to'g'ri chiziqning o'zi ham shu tekislikka tegishli bo'ladi;

C) tekislik va unda yotmaydigan to'g'ri chiziq yo kesishmaydi, yoki bitta nuqtada kesishadi;

D) bitta to'g'ri chiziqda yotmaydigan uchta nuqtadan bitta va faqat bitta tekislik o'tkazish mumkin;

E) hech bir uchtasi bir to'g'ri chiziqda yotmaydigan to'rtta nuqta orqali faqat 2 ta tekislik o'tkazish mumkin.

7. Kubning bir yog'ida yotmagan uchtadan olingan uchlaridan o'tuvchi tekisliklar sonini aniqlang.

A) 16 ta; B) 18 ta; C) 20 ta; D) 22 ta; E) 24 ta.

8. Kubning qirralari o'rtalarining bir yoqda yotmagan har bir uchtasi orqali o'tuvchi tekisliklar sonini aniqlang.

A) 32 ta; B) 26 ta; C) 24 ta; D) 20 ta; E) 16 ta.

9. Quyidagi jumlalarning qaysi biri noto'g'ri?

A) ikki bo'g'inli siniq chiziq tekis (yassi) shakldir;

B) uch bo'g'inli yopiq siniq chiziq tekis shakldir;

C) yopiq siniq chiziqning bo'g'inlari soni eng kamida uchta bo'ladi;

D) uch bo'g'inli yopiq siniq chiziq ABC da $AC < AB + BC$ bo'ladi;

E) uch bo'g'inli siniq chiziq yassi (tekiC) shakldir.

10. Quyidagi mulohazalarning qaysi biri noto'g'ri?

A) berilgan to'rtta nuqtaning ixtiyoriy ikkitasidan o'tuvchi to'g'ri chiziq qolgan ikkitasidan o'tuvchi to'g'ri chiziq bilan kesishmasa, bu nuqtalar bir tekislikda yotmaydi;

B) agar AB va CD to'g'ri chiziqlar bir tekislikda yotmasa, AC va BD to'g'ri chiziqlar ham bir tekislikda yotmaydi;

C) to'rtta nuqta bir tekislikda yotmasa, ulardan hech qanday uchtasi bir to'g'ri chiziqda yotmaydi;

D) A nuqtada kesishuvchi ikki to'g'ri chiziqning har birini kesib o'tuvchi, lekin A nuqtadan o'tmaydigan barcha to'g'ri chiziqlar bir tekislikda yotadi;

E) to'g'ri chiziq va unda yotmaydigan nuqta orqali kamida bitta tekislik o'tkazish mumkin.

11. Kubning o'zaro parallel necha juft qirralari mavjud?

A) 18 ta; B) 16 ta; C) 12 ta; D) 9 ta; E) 20 ta.

12. Kubning o'zaro parallel necha juft yoqlari bor?

A) 6 ta; B) 4 ta; C) 3 ta; D) 8 ta; E) 12 ta.

13. Kubning o'zaro ayqash to'g'ri chiziqlarni aniqlovchi qirralari necha juft?

A) 24 ta; B) 20 ta; C) 16 ta; D) 12 ta; E) 10 ta.

14. Fazoviy to'rtburchak tomonlarining o'rtalari qanday shaklning uchlari bo'ladi?

A) to'g'ri to'rtburchak; B) romb; C) trapetsiya;

D) kvadrat; E) parallelogramm.

15. N nuqtada $AN : NB = 1 : 3$ kabi nisbatda bo'linuvchi AB kesmaning α tekislikdagi parallel proyeksiyasi A_1B_1 (AB kesma α tekislik bilan kesishmaydi). Agar $|AA_1| = 3$, $|BB_1| = 5$ bo'lsa, $|NN_1|$ ni toping.

A) 4,5; B) 5; C) 5,5; D) 3,5; E) 4.

16. ABC uchburchakni AB to'g'ri chiziqqa parallel tekislik AC tomonini A_1 nuqtada, BC tomonini B_1 nuqtada kesadi. Agar $|AB| = 15$, $|AA_1| : |AC| = = 2 : 3$ bo'lsa, $|A_1B_1|$ ni toping.

A) 9; B) 7; C) 5; D) 3,5; E) 5.

17. O'rtasi N nuqta bo'lgan AB kesma α tekislikni kesib o'tadi. A_1B_1 kesma AB kesmaning α tekislikdagi parallel proyeksiyasi. Agar $|AA_1| = 3{,}6$, $|BB_1| = 4{,}8$ bo'lsa, NN_1 kesmaning uzunligini toping.

A) 1,6; B) 0,6; C) 0,8; D) 1,2; E) 0,4.

18. Quyidagi tasdiqning qaysi biri noaniq?

A) har qanday uchta nuqta orqali tekislik o'tkazish mumkin;

B) AB va AC to'g'ri chiziqlarni kesuvchi va A nuqtadan o'tmaydigan barcha to'g'ri chiziqlar bir tekislikda yotadi;

C) DE kesma biror tekislikni kesib o'tsa, DE to'g'ri chiziq ham shu tekislikni kesib o'tadi;

D) AB to'g'ri chiziq biror α tekislikni kesib o'tsa, AB kesma ham shu tekislikni kesadi;

E) kubni turli tekisliklar bilan kesib, kesimda uchburchak, muntazam uchburchak, kvadrat, to'g'ri to'rtburchak hosil qilish mumkin.

19. Quyidagi iboralarning qaysi biri noaniq?

A) o'zaro parallel ikki to'g'ri chiziqdan biri biror tekislikka parallel bo'lsa, ikkinchisi ham shu tekislikka parallel bo'ladi;

B) o'zaro kesishuvchi ikki tekislikning har biri biror to'g'ri chiziqqa parallel bo'lsa, ularning kesishish chizig'i ham shu to'g'ri chiziqqa parallel bo'ladi;

C) berilgan tekislikka undan tashqaridagi biror nuqtadan unga cheksiz ko'p parallel to'g'ri chiziqlar o'tkazish mumkin;

D) berilgan to'g'ri chiziqqa tashqaridagi biror nuqtadan unga cheksiz ko'p parallel tekislik o'tkazish mumkin;

E) bir tekislikka parallel bo'lgan ikki to'g'ri chiziq o'zaro paralleldir.

20. $ABCD$ kvadrat va uning tekisligida yotmagan E nuqta berilgan. Agar $AB = 10$ sm bo'lsa, AE, BE, CE va DE kesmalarning o'rtalarini tutashtirishdan hosil bo'lgan to'rtburchakning perimetrini toping.

A) 20 sm; B) 16 sm; C) 18 sm; D) 22 sm; E) 25 sm.

21. Har bir qirrasi 6 sm dan bo'lgan $SABC$ tetraedrni SB qirrasining o'rtasidan o'tib, SA va SC qirralariga parallel tekislik bilan kesishdan hosil bo'lgan kesim perimetrini toping.

A) 6 sm; B) 8 sm; C) 9 sm; D) 12 sm; E) 18 sm.

22. Qirrasi l ga teng bo'lgan $ABCDA_1B_1C_1D_1$ kubning AB hamda AD qirralarining o'rtasidan o'tib, CC_1 qirrasiga parallel bo'lgan tekislik bilan kesimi perimetrini toping.

A) $l\sqrt{2}$; B) $\frac{l(\sqrt{2}+2)}{2}$; C) $l\sqrt{2}+2$; D) $l\sqrt{2}+2$; E) $l(\sqrt{2}+2)$.

23. Ushbu tasdiqlarning qaysi biri noaniq?

A) $\alpha \parallel \beta$ bo'lsa, α tekislikka tegishli har bir to'g'ri chiziq β tekislikka parallel bo'ladi;

B) uchinchi tekislikka parallel bo'lgan ikki tekislik o'zaro paralleldir;

C) agar to'g'ri chiziq yoki tekislik o'zaro parallel tekisliklardan birini kessa, ikkinchisini ham kesadi;

D) ayqash to'g'ri chiziqlar orqali faqat bir juft o'zaro parallel tekislik o'tkazish mumkin;

E) biror tekislikdagi ikki to'g'ri chiziq ikkinchi tekislikka parallel bo'lsa, u holda tekisliklar ham o'zaro paralleldir.

24. $ABCDA_1B_1C_1D_1$ kubning AB qirrasidagi X nuqta uni $AX:BX=$ $=2:3$ kabi nisbatda bo'ladi. Kubning X nuqtadan o'tuvchi va AA_1C tekislikka parallel tekislik bilan kesimini yasang. $AB=a$ bo'lganda kesim perimetrini toping.

A) $2a+\frac{6\sqrt{2}}{5}a$; B) $a+\frac{6\sqrt{2}}{5}a$; C) $2a+4\sqrt{2}a$; D) $2a+\frac{3\sqrt{2}}{5}a$;

E) $2a$.

25. Tomoni 3 ga teng bo'lgan muntazam uchburchak uchlarining har biridan 2 birlik masofada joylashgan nuqtadan uchburchak tekisligigacha bo'lgan masofani toping.

A) 1; B) $\sqrt{2}$; C) $\sqrt{6}$; D) $\sqrt{3}$; E) 2.

26. Tekislikdan a masofada yotgan nuqtadan tekislik bilan 45° va 30° li burchaklar, o'zaro esa to'g'ri burchak tashkil etadigan ikkita og'ma o'tkazilgan. Og'malarning asoslari orasidagi masofani toping.

A) $a\sqrt{2}$; B) $a\sqrt{3}$; C) $a\sqrt{5}$; D) $a\sqrt{6}$; E) $a\sqrt{7}$.

27. Tekislikka o'tkazilgan perpendikular va og'ma orasidagi burchak 30°, perpendikularning uzunligi 10 ga teng. Og'maning uzunligini toping.

A) 20; B) $10\sqrt{3}$; C) $20\sqrt{3}$; D) $20/\sqrt{3}$; E) $20\sqrt{2}$.

28. Tekislikka tushirilgan perpendikular va og'ma orasidagi burchak 60° va og'maning uzunligi $20\sqrt{3}$. Perpendikularning uzunligini toping.

A) 10; B) 40; C) $10\sqrt{3}$; D) $5\sqrt{3}$; E) 20.

29. Bitta nuqtadan tekislikka og'ma va perpendikular o'tkazilgan. Og'maning uzunligi 10, perpendikularniki 6. Og'maning tekislikdagi proyeksiyasining uzunligini toping.

A) 4; B) 2; C) 8; D) 5; E) 3.

30. α tekislik va uni kesib o'tmaydigan $|AB| = 13$ kesma berilgan. Agar kesmaning uchlaridan α tekislikkacha bo'lgan masofalar $|AA_1| = 5$, $|BB_1| = 8$ bo'lsa, AB kesma yotuvchi to'g'ri chiziqning α tekislik bilan tashkil qilgan burchak sinusini toping.

A) 5/13; B) 8/13; C) 2/13; D) 3/13; E) 4/13.

31. Quyidagi mulohazalarning qaysi biri noto'g'ri?

A) agar to'g'ri chiziq tekislikka parallel bo'lsa, uning barcha nuqtalari tekislikdan bir xil uzoqlikda yotadi;

B) berilgan ikki parallel to'g'ri chiziqni kesib o'tuvchi hamma to'g'ri chiziqlar bir tekislikda yotadi;

C) fazoviy to'rtburchak tomonlarining o'rtalari parallelogrammning uchlari bo'ladi;

D) ikki ayqash to'g'ri chiziq orqali o'zaro parallel tekisliklar o'tkazish mumkin;

E) bitta tekislikka perpendikular ikki tekislik o'zaro paralleldir.

32. AB kesmaning A oxiridan tekislik o'tkazilgan. Shu kesmaning B oxiridan va C nuqtasidan tekislikni B_1 va C_1 nuqtalarda kesuvchi parallel to'g'ri chiziqlar o'tkazilgan. Agar $|AB| = 8$ va $|CC_1| : |AC| = 3 : 4$ bo'lsa, $|BB_1|$ kesmaning uzunligini toping.

A) 3; B) 5; C) 4; D) 6; E) 8.

33. Muntazam ABC uchburchakning AC tomoni orqali tekislik o'tkazilgan. Uchburchakning BD medianasi tekislik bilan $60°$ li burchak tashkil etadi. AB to'g'ri chiziq bilan tekislik orasidagi burchakning sinusini toping.

A) 1/2; B) 1/4; C) 3/4; D) 3 /2; E) 2 /2.

34. Nuqtadan tekislikka uzunliklari 10 va 15 bo'lgan og'malar tushirilgan. Birinchi og'maning tekislikdagi proyeksiyasi 7 bo'lsa, ikkinchi og'maning proyeksiyasini toping.

A) $\sqrt{170}$; B) $\sqrt{171}$; C) $\sqrt{172}$; D) $\sqrt{173}$; E) $\sqrt{174}$.

35. α va β tekisliklar $45°$ li burchak ostida kesishadi. α tekislikdagi A nuqtadan β tekislikkacha bo'lgan masofa 2 ga teng. A nuqtadan tekisliklarning kesishish chizig'igacha bo'lgan masofani toping.

A) 2; B) $2\sqrt{2}$; C) 3; D) 1; E) $2\sqrt{3}$.

36. Nuqtadan tekislikka ikkita og'ma o'tkazilgan. Og'malar 3 : 5 ga teng nisbatda bo'lib, ularning proyeksiyalari 33 va 17 ga teng. Og'malarning uzunligini toping.

A) $10\sqrt{2}; 5\sqrt{2}$; B) $25\sqrt{2}; 10\sqrt{2}$; C) $15\sqrt{2}; 5\sqrt{2}$; D) $25\sqrt{2}; 5\sqrt{2}$;

E) $10\sqrt{2}; 15\sqrt{2}$.

37. Berilgan nuqtadan tekislikka ikkita og'ma va perpendikular tushirildi. Og'malarning proyeksiyalari 27 va 15 ga teng hamda ulardan biri ikkinchisidan 6 ga uzun bo'lsa, perpendikularning uzunligini toping.

A) 30; B) 39; C) 45; D) 33; E) 36.

38. Uzunliklari 10 sm va 15 sm bo'lgan ikki kesma uchlari o'zaro parallel tekisliklarda yotadi. Birinchi kesmaning tekislikdagi proyeksiyasi $\sqrt{19}$ sm bo'lsa, ikkinchi kesmaning proyeksiyasi necha santimetr bo'ladi?

A) 12; B) 11; C) 10; D) 13; E) 9 sm.

39. Tekislikdan a masofada joylashgan nuqtadan tekislikka ikkita og'ma tushirilgan. Og'malarning har biri bilan tekislik orasidagi burchak 45° ga teng. Agar og'malar orasidagi burchak 60° ga teng bo'lsa, og'malarning asoslari orasidagi masofa qancha?

A) $2a$; B) $a\sqrt{3}$; C) $a\sqrt{2}$; D) $1{,}5a$; E) $2\sqrt{2}a$.

40. Quyidagi mulohazalarning qaysi biri noto'g'ri?

A) agar tekislik ikki parallel tekislikdan biriga perpendikular bo'lsa, u holda bu tekislik ikkinchi tekislikka ham perpendikular bo'ladi;

B) tekislikdagi kesishuvchi ikki chiziqning har biriga perpendikular bo'lgan to'g'ri chiziq tekislikning o'ziga ham perpendikular bo'ladi;

C) fazodagi ikki to'g'ri chiziq uchinchi to'g'ri chiziqqa perpendikular bo'lsa, ular o'zaro paralleldir;

D) agar tekislikdagi to'g'ri chiziq tekislikka tushirilgan og'maga perpendikular bo'lsa, bu to'g'ri chiziq og'maning proyeksiyasiga ham perpendikulardir;

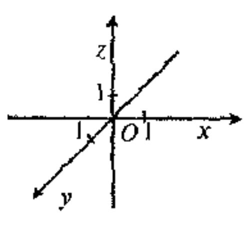
E) ikki parallel tekislikni uchinchi tekislik bilan kesganda hosil bo'lgan to'g'ri chiziqlar o'zaro parallel bo'ladi.

VIII bob. FAZODA TO'G'RI BURCHAKLI KOORDINATALAR SISTEMASI VA VEKTORLAR

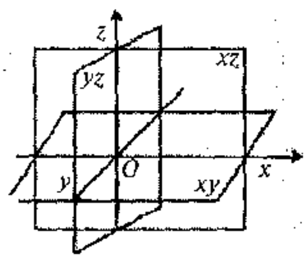
1-§. Fazoda dekart koordinatalar sistemasi va koordinatalar metodi

1.1. Koordinatalar sistemasi. Umumiy koordinatalar boshiga, bir xil birlik kesmaga ega bo'lgan, har biri qolgan ikkitasiga perpendikular uchta Ox, Oy, Oz koordinatali to'g'ri chiziqlar fazoda *to'g'ri burchakli Dekart koordinatalar sistemasini* tashkil etadi, ya'ni shunday sistema asosida fazodagi ixtiyoriy nuqtaning o'rnini sonlarning tartiblangan uchligi (x, y, z) orqali aniq ifodalash mumkin (211-rasm). Ox, Oy, Oz koordinatali to'g'ri chiziqlar mos ravishda koordinatalar sistemasining *abssissalar o'qi*, *ordinatalar o'qi* hamda *applikatalar o'qi* deb ataladi. Ularning juftliklari aniqlagan (xy), (yz), (zx) tekisliklar *koordinata tekisliklari* deyilib, ular fazoni teng sakkiz qismga (oktantga) ajratadi (212- rasm). Koordinatali fazoda tanlangan biror A nuqtaning o'rnini aniq ifodalovchi sonlarning tartiblangan $(x; y; z)$ uchligidagi birinchi «x» son A nuqtaning abssissasi, «y» son ordinatasi, «z» son applikatasi deyilib, ular birgalikda A nuqtaning koordinatalari hisoblanadi va $A(x; y; z)$ tarzida ifodalanadi. x, y, z larning ishoralariga qarab A nuqta qaysi oktantga tegishliligini hamda mos ravishda (xy), (xz) va (yz) tekisliklardan qancha masofada yotishini bilish mumkin. $O(0; 0; 0)$ nuqta *koordinatalar boshi* deyiladi.

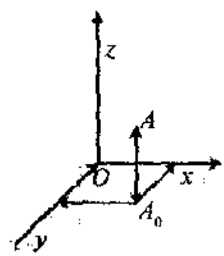
Koordinatali fazodagi biror A nuqtaning koordinatalari quyidagicha aniqlanadi. A nuqtaning (xy) tekislikdagi ortogonal proyeksiyasi A_0



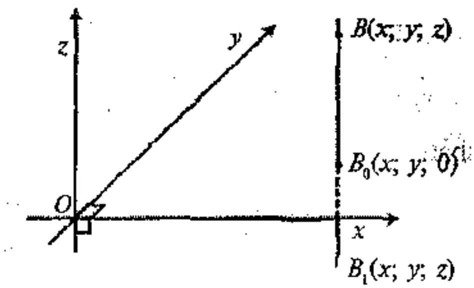
211-rasm.



212-rasm.



213-rasm.



214-расм.

nuqtaning koordinatalari (abssissasi va ordinatasi) mos ravishda A nuqta uchun ham abssissa va ordinata bo'ladi. AA_0 kesmaning uzunligini ifodalovchi son A nuqtaning qaysi yarim fazoda joylashganiga bog'liq holda tegishli ishora bilan uning applikatasi hisoblanadi (213- rasm).

Koordinatalari bilan berilgan biror $B(x;\ y;\ z)$ nuqtani koordinatali fazoda belgilash uchun, dastlab (xy) tekislikda izlanayotgan nuqtaning abssissasi va ordinatasiga ko'ra, uning ortogonal proyeksiyasi $B_0(x;\ y;\ 0)$ nuqta belgilanadi va $z=|B_0B|$ ni ishorasiga bog'liq holda ($B_0B \perp (xy)$) to'g'ri chiziqda B_1 nuqtaning o'rni aniqlanadi (214- rasm).

1.2. Koordinatalar metodi. Ko'pgina hollarda biror xususiyatga ega bo'lgan nuqtalar to'plamini aniqlashda koordinalar metodidan foydalanish qulaydir. Eng sodda masalalarda izlanayotgan nuqtalar to'plami to'g'ri chiziq, kesma, aylana, sfera bo'lishi mumkin. Masalan, R radiusli sfera berilgan nuqtadan musbat R masofada joylashgan fazodagi nuqtalar to'plamidir. To'g'ri burchakli $Oxyz$ koordinatalar sistemasida markazi koordinatalar boshida bo'lgan sfera

$$x^2 + y^2 + z^2 = R^2$$

tenglama bilan, markazi $O_1(a;\ b;\ c)$ nuqtada bo'lgan sfera esa

$$(x-a)^2 + (x-b)^2 + (z-c)^2 = R^2$$

tenglama bilan aniqlanadi. Shu sfera bilan chegaralangan shar

$$(x-a)^2 + (x-b)^2 + (z-c)^2 \le R^2$$

tengsizlik bilan ifodalanadi.

Koordinatalari bilan berilgan, ikki $A(x_1;\ y_1;\ z_1)$ va $B(x_2;\ y_2;\ z_2)$ nuqtalar orasidagi masofa $|AB| = d$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2},$$

AB kesmaning o'rtasi $C(x_3;\ y_3;\ z_3)$ nuqtaning koordinatalari

$$x_3 = \frac{x_1+x_2}{2},\quad y_3 = \frac{y_1+y_2}{2},\quad z_3 = \frac{z_1+z_2}{2},$$

AB kesmani berilgan λ nisbatda bo'luvchi $D(x;\ y;\ z)$ nuqtaning koordinatalari

$$x = \frac{x_1+\lambda x_2}{1+\lambda},\ y = \frac{y_1+\lambda y_2}{1+\lambda},\ z = \frac{z_1+\lambda z_2}{1+\lambda}$$

formulalari bilan topiladi.

Uchlari $A(x_1;\ y_1;\ z_1)$, $B(x_2;\ y_2;\ z_2)$, $C(x_3;\ y_3;\ z_3)$ nuqtalarda bo'lgan uchburchak medianalari kesishish nuqtasining koordinatalari

$$x = \frac{x_1+x_2+x_3}{3},\ y = \frac{y_1+y_2+y_3}{3},\ z = \frac{z_1+z_2+z_3}{3}$$

ifodalar bilan aniqlanadi.

$M_0(x_0;\ y_0;\ z_0)$ nuqtadan $Ax + By + Cz + D = 0$ tenglama bilan berilgan tekislikkacha bo'lgan masofa

$$d = \frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}$$

formula bilan topiladi.

1-masala. $A(1;\ 2;\ 3)$ nuqtadan teng uzoqlikda joylashgan va koordinatalar boshidan o'tuvchi nuqtalar to'plamining koordinatalarini qanoatlantiruvchi tenglamani yozing.

Yechilishi. Masala shartini qanoatlantiruvchi barcha nuqtalar to'plami markazi A nuqtada bo'lib, radiusi

$$R = |AO| = \sqrt{(1-0)^2+(2-0)^2+(3-0)^2} = \sqrt{14}$$

ga teng bo'lgan ($O(0;\ 0;\ 0)$ — koordinatalar boshi) sferaga tegishlidir. Bu sfera tenglamasi

$$(x-1)^2+(y-2)^2+(z-3)^2 = 14\ .$$

Javob: $(x-1)^2+(y-2)^2+(z-3)^2 = 14$.

2-masala. $A(x;\ 0;\ 0)$ nuqta $B(1;\ 2;\ 3)$ va $C(-1;\ 3;\ 4)$ nuqtalardan teng uzoqlikda joylashgani ma'lum bo'lsa, x ni toping.

Yechilishi. $|AB| = |AC|$ bo'lgani uchun

$(x-1)^2+4+9=(x+1)^2+9+16 \Leftrightarrow x^2-2x+1+4=x^2+2x+1+16 \Rightarrow$
$\Rightarrow x=-3.$

Javob: –3.

3-masala. Oz o'qida shunday N nuqtani topingki, undan $A(2; -3; 1)$ nuqtagacha bo'lgan masofa 7 ga teng bo'lsin.

Yechilishi. Oz o'qidagi nuqtaning abssissasi va ordinatasi nolga teng bo'lgani uchun, ya'ni $N(0; 0; z)$ ekanligidan

$$|AN|=\sqrt{2^2+(-3)^2+(1-z)^2}=7.$$

Bundan $4+9+1-2z+z^2=49 \Leftrightarrow z^2-2z-35=0 \Rightarrow \left[\begin{array}{l} z_1=-5 \\ z_2=7 \end{array}\right.$. Shunday qilib, A nuqtadan 7 birlik masofada yotuvchi Oz o'qidagi nuqtalar $N_1(0; 0; -5)$ va $N_2(0; 0; 7)$.

Javob: $N_1(0; 0; -5)$ va $N_2(0; 0; 7)$.

4-masala. (xz) tekislikka nisbatan $A(1; 2; 3)$ nuqtaga simmetrik bo'lgan nuqtani toping.

Yechilishi. Uchala koordinatasi musbat sonlar bo'lgan nuqta birinchi oktantda joylashgan bo'lib, bu nuqtani (xz) tekislikka nisbatan simmetrik ko'chirish natijasida u faqat ordinatasi qarama-qarshi ishora bilan farqlanuvchi A_2 nuqtaga ko'chadi, ya'ni ularning ordinatalari o'zaro qarama-qarshi sonlardir.

Javob: $A_1(1; -2; 3)$.

5-masala. Uchlari $A(2; 3; -1)$, $B(3; 0; -1)$, $C(1; 1; 1)$ nuqtalarda bo'lgan uchburchak medianalari kesishgan nuqtaning koordinatalarini toping.

Yechilishi. Uchburchak medianalarining kesishish nuqtasini $D(x; y; z)$ desak,

$$x=\frac{x_1+x_2+x_3}{3}=\frac{2+3+1}{3}=2,$$

$$y=\frac{y_1+y_2+y_3}{3}=\frac{3+0+1}{3}=\frac{4}{3},$$

$$x=\frac{y_1+y_2+y_3}{3}=\frac{-1-1+1}{3}=-\frac{1}{3},$$

Javob: $\left(2;\frac{4}{3};-\frac{1}{3}\right)$.

2-§. Fazoda vektorlar

Tekislikdagi vektorlar bilan bog'liq tushunchalarning bir qanchasini fazodagi vektorga qo'llash mumkin. Aniqrog'i, tekislikdagi vektorga berilgan ta'rif, vektorlar ustida amallar va ular bo'ysunadigan qonuniyatlarni fazodagi vektorlar uchun ham umumlashtirib aytish mumkin. Masalan, fazodagi vektor ta'rifi xuddi tekislikdagi kabi bo'lib, farqi shundaki, uning koordinatasi ikki son bilan emas, balki uchta son orqali ifodalanadi.

2.1. Vektorning koordinatalari. Boshi $A(x_1; y_1; z_1)$ nuqtada va oxiri $B(x_2; y_2; z_2)$ nuqtada bo'lgan $\overline{AB} = \vec{a}$ vektorning koordinatalari:

$$x = x_2 - x_1, \ y = y_2 - y_1, \ z = z_2 - z_1$$

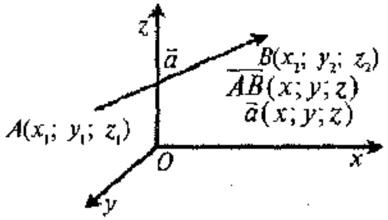
sonlardir. Har bir vektor o'z koordinatalari bilan $\overline{AB}(x; y; z) = \vec{a}(x; y; z)$ kabi ifodalanadi (215- rasm).

1-masala. Boshi $C(5; -4; 0)$ nuqtada va oxiri $D(-3; 0; 7)$ nuqtada bo'lgan $\overline{CD}$ vektorni koordinatalari bilan yozing.

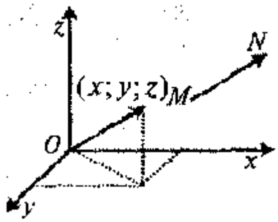
Yechilishi. $x = -3 - 5 = -8$, $y = 0 - (-4) = 4$, $z = 7 - 0 = 7$ sonlarning $(-8; 4; 7)$ uchligi $\overline{CD}$ vektor koordinatalaridir, ya'ni $\overline{CD}(-8; 4; 7)$.

Javob: $\overline{CD}(-8; 4; 7)$.

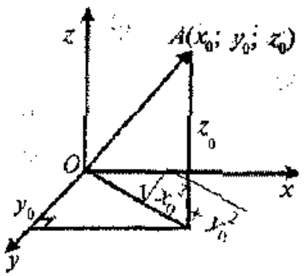
Har qanday $\overline{MN}(x; y; z)$ vektor boshi $O(0; 0; 0)$ nuqtada va oxiri koordinatalari $x; y; z$ bo'lgan vektorga tengligiga oson ishonch hosil qilish mumkin. Bu esa har qanday vektorni koordinatalar boshidan qo'yish mumkinligini bildiradi (216- rasm).



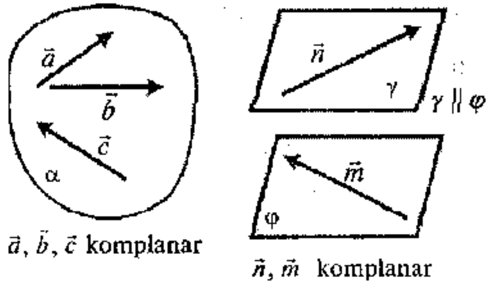
215- rasm.



216- rasm.



217- rasm.



218- rasm.

2.2. Vektorning uzunligi. 217- rasmdagi $\overline{OA}(x_0;\ y_0;\ z_0)$ vektorning uzunligini topaylik. Katetlarining uzunligi x_0 va y_0 bo'lgan to'g'ri burchakli uchburchak gipotenuzasi $\sqrt{x_0^2+y_0^2}$ ga tengligi ma'lum. Katetlarining uzunligi $\sqrt{x_0^2+y_0^2}$ hamda z_0 bo'lgan to'g'ri burchakli uchburchak gipotenuzasining uzunligi: $\left|\overline{OA}\right| = \sqrt{x_0^2+y_0^2+z_0^2}$.

Faqat nol vektorning uzunligi nolga teng bo'lib, boshqa vektorlarning uzunligi doim musbat songa tengdir.

Bir to'g'ri chiziqda yoki parallel to'g'ri chiziqlarda yotuvchi nol bo'lmagan ikki vektor *kollinear vektorlar* deyiladi.

Ikki $\vec{a}(x_1;y_1;z_1)$ va $\vec{b}(x_2;y_2;z_2)$ vektorlar kollinear bo'lishi uchun ularning proyeksiyalari proporsional bo'lishi zarur va yetarlidir:

$$\frac{x_1}{x_2}=\frac{y_1}{y_2}=\frac{z_1}{z_2}.$$

Bir tekislikdagi vektorlar yoki o'zaro parallel tekisliklarda yotuvchi vektorlar *komplanar vektorlar* deyiladi (218- rasm).

2- masala. Uzunligi $\sqrt{234}$ bo'lgan $\overline{OA}(-a;3a;4a)$ vektorning koordinatalarini toping.

Yechilishi. $\overline{OA}=\sqrt{a^2+9a^2+16a^2}=\sqrt{26a^2}$;

$$26a^2=234;\ a^2=9;\ a=\pm3;\ \overline{OA}(-3;9;12);$$

Javob: $(-3;9;12),\ (3;-9;-12)$.

3-masala. $\vec{a}(1; 1; 0)$ va $\vec{b}(1; 1; 2)$ vektorlar o'zaro kollinearmi? $\vec{c}(-2; 2; -4)$ va $\vec{d}(1; -1; 2)$ vektorlar-chi?

Yechilishi. Kollinear vektorlarning mos koordinatalari o'zaro proporsional bo'lishi ma'lum. Shunga ko'ra $\vec{a}$ va $\vec{b}$ vektorlar nokollinear, $\vec{c}$ va $\vec{d}$ vektorlar esa kollinear vektorlardir.

4-masala. $n(3; -5; 0)$ va $t(-1; 2; 0)$ vektorlar o'zaro komplanarmi?

Yechilishi. Ikkala vektorning uchinchi koordinatalari 0 ga tengligi ularning o'zaro bir-biriga komplanarligini bildiradi.

3-§. Vektorlar ustida amallar

Fazodagi vektorlar ustidagi amallar xuddi tekislikdagi vektorlar ustidagi amallar kabi ta'riflanadi.

3.1. Vektorlarning yig'indisi. $\vec{a}(x_1; y_1; z_1)$ va $\vec{b}(x_2; y_2; z_2)$ vektorlarning yig'indisi shunday vektorki, uning koordinatalari $(x_1 + x_2; y_1 + y_2; z_1 + z_2)$, ya'ni, $\vec{a} + \vec{b} = \vec{c}$ uchun

$$\vec{c}(x_1 + x_2;\ y_1 + y_2;\ z_1 + z_2).$$

Ixtiyoriy $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar uchun quyidagi tengliklar doim o'rinlidir:

1) $\vec{a} + \vec{b} = \vec{b} + \vec{a}$;

2) $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$.

Yig'indisi nol vektorga teng bo'lgan ikki vektor *o'zaro qarama-qarshi vektorlar* deyiladi.

x, y, z korrdinatalarning ixtiyoriy qiymatlarida $\vec{a}(x; y; z)$ va $\vec{b}(-x, -y, -z)$ vektorlar o'zaro qarama-qarshi vektorlardir.

1-masala. $\vec{a}(3; 1; -2)$ va $\vec{b}(2; -2; 5)$ vektorlarning yig'indisini toping.

Yechilishi. $\vec{a} + \vec{b} = \vec{c}$ desak, $\vec{c}(3 + 2; 1 + (-2); -2 + 5) = \vec{c}(5; -1; 3)$.

Javob: $\vec{c}(5; -1; 3)$.

3.2. Vektorlarning ayirmasi. $\vec{a}(x_1; y_1; z_1)$ va $\vec{b}(x_2; y_2; z_2)$ vektorlarning ayirmasi $(\vec{a} - \vec{b})$ deb, shunday $\vec{d}$ vektorga aytiladiki, uning ayriluvchi $\vec{b}$ vektor bilan yig'indisi $\vec{b}+\vec{d}$ kamayuvchi $\vec{a}$ vektorga teng bo'ladi:

$$\vec{a}-\vec{b}=\vec{d} \Rightarrow \vec{a}=\vec{b}+\vec{d},$$

bundan

$$\vec{d}\,(x_1-x_2;\ y_1-y_2;\ z_1-z_2).$$

2- m a s a l a. Agar $\vec{n}(4;1;5)$ va $\vec{m}(3;5;-1)$ bo'lsa, $\vec{k}=\vec{n}-\vec{m}$ ayirmaning koordinatalarini toping.

Y e c h i l i s h i. $\vec{k}\,(4-3;\ \ 1-5;\ \ 5-(-1))=\vec{k}\,(1;-4;\,6)$.

J a v o b: (1; –4; 6)

3.3. Vektorni songa ko'paytirish. $a(x;\ y;\ z)$ vektorni λ songa ko'paytmasi $\lambda\vec{a}$ yoki $\vec{a}\lambda$ vektorga teng bo'lib, uning koordinatasi $\vec{a}(\lambda x;\lambda y;\lambda z)$ kabidir.

Har qanday $\vec{a},\vec{b}$ vektorlar va λ, μ sonlar uchun quyidagi xossalar o'rinlidir:

1) $\lambda\vec{a}=\vec{a}\lambda$;

2) $\lambda(\vec{a}+\vec{b})=\lambda\vec{a}+\lambda\vec{b}$;

3) $(\lambda+\mu)\vec{a}=\lambda\vec{a}+\mu\vec{a}$;

Shuningdek, $|\lambda\vec{a}|=|\lambda|\cdot|\vec{a}|$ tenglik ham har doim bajariladi.

3- m a s a l a. $\vec{a}(5;-2;3)$, $\lambda=0{,}2$ bo'lsa, $\lambda\vec{a}$ vektorning koordinatalarini toping.

Y e c h i l i s h i. $0{,}2\,\vec{a}\,(5;-2;3)=\vec{c}\,(0{,}2\cdot5;\ \ 0{,}2\cdot(-2);\,0{,}2\cdot3)\Rightarrow$

$\Rightarrow\vec{c}\,(1;-0{,}4;\,0{,}6)$.

J a v o b: (1; –0,4; 0,6).

3.4. Vektorlarning skalyar ko'paytmasi. Fazoda berilgan vektorlar skalyar ko'paytmasining ta'riflanishi va bu tushuncha bilan bog'liq masalalarning yechilish usullari tekislikdagi kabidir.

4- m a s a l a. m ning qanday qiymatida $\vec{a}(1;m;-2)$ va $\vec{b}(m;3;-4)$ vektorlar o'zaro perpendikular bo'ladi?

Y e c h i l i s h i. Vektorlarning perpendikularlik shartidan foydalanamiz:

$$1\cdot m+m\cdot 3+(-2)\cdot(-4)=0 \Leftrightarrow 4m+8=0 \Rightarrow m=-2.$$

J a v o b: $m=-2$.

5- m a s a l a. $\vec{a}(-1;-1;0)$ va $\vec{b}(1;2;2)$ vektorlar orasidagi burchakni toping.

Y e c h i l i s h i. Koordinatalari bilan berilgan ikki vektor orasidagi burchak kosinusini topish formulasidan foydalanamiz. Bu burchakni α desak,

$$\cos\alpha=\frac{(-1)\cdot 1+(-1)\cdot 2+0\cdot 2}{\sqrt{1+1+0}\cdot\sqrt{1+4+4}}=\frac{-3}{3\sqrt{2}}=-\frac{\sqrt{2}}{2}\Rightarrow \alpha=135^{\circ}.$$

J a v o b: 135°.

3.5. Fazoda vektorni koordinata o'qlari bo'yicha yoyish.

Uzunligi birga teng bo'lgan vektorning birlik vektor deyilishini bilamiz, ya'ni $\vec{e}(x_1;y_1;z_1)$ birlik vektor bo'lsa, $|\vec{e}|=\sqrt{x_1^2+y_1^2+z_1^2}=1$.

Fazoda tanlangan to'g'ri burchakli koordinatalar sistemasida ham $\vec{i}(1;0;0), \vec{j}(0;1;0)$ va $\vec{k}(0;0;1)$ vektorlar **birlik vektorlar** yoki **ortlar** deb ataladi (219- rasm).

Har qanday $\vec{a}(x_1;\ y_1;\ z_1)$ vektorni birlik vektorlar bilan yagona tarzda $\vec{a}=x_1\vec{i}+y_1\vec{j}+z_1\vec{k}$ ko'rinishda ifodalash mumkin va bu ifoda $\vec{a}$ ***vektorning koordinata o'qlari bo'yicha yoyilmasi*** deyiladi.

Umuman, uchta $\overline{OA}, \overline{OB}$ va $\overline{OC}$ komplanar vektorlar berilgan bo'lsa, shunday a, b, c haqiqiy sonlar topiladiki, ular yordamida ixtiyoriy $\overline{ON}$ vektorni

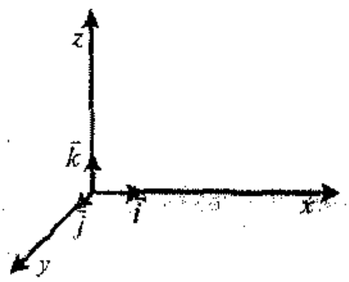
$$\overline{ON}=a\cdot\overline{OA}+b\cdot\overline{OB}+c\cdot\overline{OC}$$

ko'rinishdagi yagona chiziqli yoyilma tarzida ifodalash mumkin.

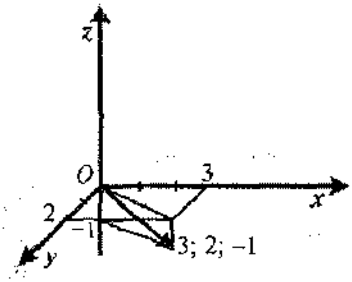
6- m a s a l a. $\vec{a}=3\vec{i}+2\vec{j}-\vec{k}$ vektorning uzunligini toping (220- rasm).

Y e c h i l i s h i. Berilgan $\vec{a}$ vektorning koordinata o'qlari bo'yicha yoyilmasida $x_1=3;\ y_1=2;\ z_1=-1$ ekanligidan

$$|\vec{a}|=\sqrt{x_1^2+y_1^2+z_1^2}=\sqrt{3^2+2^2+(-1)^2}=\sqrt{14}.$$



219- rasm.



220- rasm.

Javob: $\sqrt{14}$.

Agar vektorlarning koordinata o'qlari bo'yicha yoyilmalari ma'lum bo'lsa, u holda vektorlar ustidagi chiziqli amallarni ularning proyeksiyalari ustidagi arifmetik amallar bilan almashtirish mumkin. Masalan, agar $\vec{a} = x_1\vec{i} + y_1\vec{j} + z_1\vec{k}$, $\vec{b} = x_2\vec{i} + y_2\vec{j} + z_2\vec{k}$ bo'lsa, u holda

$$\lambda\vec{a} = \lambda x_1\vec{i} + \lambda y_1\vec{j} + \lambda z_1\vec{k};$$

$$\vec{a} \pm \vec{b} = (x_1 \pm x_2)\vec{i} + (y_1 \pm y_2)\vec{j} + (z_1 \pm z_2)\vec{k}.$$

7-masala. $\vec{a} = 2\vec{i} + 3\vec{j} + 5\vec{k}$ va $\vec{b} = 3\vec{i} - 2\vec{j} + 5\vec{k}$ vektorlar ayirmasining koordinatalarini toping.

Yechilishi.

$$\vec{a} - \vec{b} = (2-3)\vec{i} + (3-(-2))\vec{j} + (5-5)\vec{k} = -\vec{i} + 5\vec{j}.$$

Javob: (−1; 5; 0).

8-masala. $\vec{a}(-2;3;2\sqrt{3})$ vektor bilan bir xil yo'nalishdagi birlik vektorning koordinatalarini toping.

Yechilishi. Berilgan vektorning uzunligini topamiz:

$$|\vec{a}| = \sqrt{4+9+12} = 5.$$

Birlik vektorni $\vec{e}(x;y;z)$ desak,

$$x = -\frac{2}{5};\ y = \frac{3}{5};\ z = \frac{2\sqrt{3}}{5}.$$

J a v o b: $\left(-\frac{2}{5};\ \frac{3}{5};\ \frac{2\sqrt{3}}{5}\right)$.

9- m a s a l a. Parallel ko'chirishda $A\,(2;\ 1;\ -1)$ nuqta $A'(1;-1;0)$ nuqtaga o'tsa, koordinatalar boshi qanday nuqtaga o'tadi?

Y e c h i l i s h i. Parallel ko'chirish formulalaridan (VI bob, 2-§) $a = x' - x,\ \ b = y' - y,\ \ c = z' - z$. Masala shartiga ko'ra $a = 1 - 2 = -1$; $b = -1 - 1 = -2;\ \ c = 0 - (-1) = 1$.

Bunday parallel ko'chirishda $O\,(0;\,0;\,0)$ nuqta $O(0 + a;\,0 + b;0 + c)$ nuqtaga ko'chadi. Demak, $O(0;\,0;\,0) \to O'(-1;-2;1)$.

J a v o b: Koordinatalar boshi $O'(-1;-2;1)$ nuqtaga ko'chadi.

Mustaqil ishlash uchun test topshiriqlari

1. $N\,(0;\ y;\ 0)$ nuqta $A\,(0;\ 2;\ 0)$ va $B\,(3;\ 1;\ 0)$ nuqtalardan teng uzoqlikdaligi ma'lum bo'lsa, y ni toping.

A) 1; B) 1,5; C) –1,5; D) 2; E) –3.

2. Agar kesmaning bir uchi $A(1;\ -5;\ 4)$, o'rtasi $C\,(4;\ -2;\ 3)$ nuqtada bo'lsa, ikkinchi uchining koordinatalarini toping.

A) (6; 5; 3); B) (7; –1; 2); C) (7; 1; 2); D) (5; 4; 6); E) (7; 3; 1).

3. Koordinatalar boshiga nisbatan (1; 2; 3) nuqtaga simmetrik bo'lgan nuqtani ko'rsating.

A) (–1; 2; 3); B) (–1; –2; 3); C) (1; 2; –3); D) (1; –2; 3);
E) (–1; –2; –3).

4. Oxy tekislikka nisbatan $(a;\ b;\ c)$ nuqtaga simmetrik bo'lgan nuqtani toping.

A) $(-a;\ b;\ c)$; B) $(-a;\ -b;\ c)$; C) $(a;\ b;\ -c)$; D) $(a;\ -b;\ c)$;
E) $(-a;\ -b;\ -c)$.

5. Quyidagi nuqtalardan qaysi biri (yz) tekislikda yotadi?

A) (2; –3; 0); B) (2; 0; –5); C) (1; 0; –4); D) (0; 9; –7); E) (1; 0; 0).

6. Oy o'qqa nisbatan (2; 3; –5) nuqtaga simmetrik bo'lgan nuqtani toping.

A) (–2; 3; –5); B) (–2; 3; 5); C) (–2; –3; –5); D) (–2; –3; 5); E) (2; 3; 5).

7. Uchlari $A\,(4;\ 5;\ 1)$, $B\,(2;\ 3;\ 0)$ va $C\,(2;\ 1;\ -1)$ nuqtalarda joylashgan uchburchakning BD medianasi uzunligini toping.

A) 1; B) $\sqrt{2}$; C) 3; D) 2; E) $\sqrt{5}$.

8. $A(2;\ -1;\ 0)$ va $B(-2;\ 3;\ 2)$ nuqtalar berilgan. Koordinata boshidan AB kesmaning o'rtasigacha bo'lgan masofani toping.

A) $\sqrt{2}$; B) $-\sqrt{2}$; C) $2\sqrt{2}$; D) 2; E) 1.

9. $A(-3;\ 8;\ 3\sqrt{33})$ nuqtadan Ox o'qqacha bo'lgan masofani toping.

A) 17; B) 18; C) 19; D) 21; E) 23.

10. AB kesmaning o'rtasi Ox o'qida yotadi. Agar $A(-3;\ m;\ 5)$, $B(2;\ -2;\ n)$ bo'lsa, m va n ni toping.

A) $m = 2;\ n = -5$; B) $m = -5;\ n = 2$; C) $m = -2;\ n = 4$;
D) $m = 1;\ n = 4$; E) $m = 3;\ n = -4$.

11. Agar $\vec{a}(6;2;1)$ va $\vec{b}(0;-1;2)$ bo'lsa, $\vec{c} = 2\vec{a} - \vec{b}$ vektorning uzunligini toping.

A) 13; B) 14; C) 15; D) $6\sqrt{2}$; E) 9.

12. Agar $\vec{p}(-1;2;8)$ va $\vec{q}(3;-2;1)$ bo'lsa, $\vec{m} = \vec{q} - \vec{p}$ vektorning uzunligini toping.

A) 9; B) 8; C) $\sqrt{14}$; D) 6; E) 10.

13. *$\vec{a}(x;\ 1;\ 2)$ vektorning uzunligi 3 ga teng. x ning qiymatini* toping.

A) 2; B) ± 2; C) 0; D) 1; E) -1.

14. y ning qanday qiymatlarida $\vec{b} = 12\vec{i} - y\vec{j} + 15\vec{k}$ vektorning uzunligi 25 ga teng bo'ladi?

A) 14; B) 16; C) 14 va -14; D) 2; E) 16 va -16.

15. z ning qanday qiymatlarida $\vec{c} = 2\vec{i} - 9\vec{j} + z\vec{k}$ vektorning uzunligi 11 ga teng bo'ladi?

A) 6; B) ± 6; C) 4; D) ± 5; E) 7.

16. $A(1;\ 0;\ 1)$, $B(-1;\ 1;\ 2)$ va $C(0;\ 2;\ -1)$ nuqtalar berilgan. Koordinatalar boshi O nuqtada joylashgan. Agar $\overline{AB} + \overline{CD} = \overline{0}$ bo'lsa, $\overline{OD}$ vektorning uzunligini toping.

A) 4; B) 2; C) 9; D) 3; E) 6.

17. $\overline{AB}$ $(-3; 0; 2)$ va $\overline{AC}$ $(7; -2; 2)$ vektorlar ABC uchburchakning tomonlaridir. Shu uchburchakning AN medianasi uzunligini toping.

A) 2,5; B) 1,5; C) 36; D) 32; E) 3.

18. Agar $|\overline{AB}| = |\overline{AC}| = |\overline{AB} + \overline{AC}| = 4$ bo'lsa, $|\overline{CB}|$ ning qiymatini toping.

A) $4\sqrt{2}$; B) $4\sqrt{3}$; C) $2\sqrt{3}$; D) 4,5; E) $\frac{3\sqrt{3}}{2}$.

19. $\vec{a}\,(1; -2; 3)$ vektorning oxiri $B\,(2; 0; 4)$ nuqta bo'lsa, bu vektor boshining koordinatalarini toping.

A) (1; 2; 1); B) (–1; 2; 1); C) (1; –2; 1); D) (1; 2; –1); E) (–1; 2; –1).

20. $\vec{a}\,(-2; 6; 3)$ vektorga yo'nalishdosh bo'lgan birlik vektorning koordinatalarini toping.

A) $\left(\frac{2}{3}; \frac{6}{7}; \frac{3}{7}\right)$; B) (–1; –3; –1); C) $\left(-\frac{1}{3}; 1; \frac{1}{2}\right)$; D) $\left(-\frac{2}{7}; \frac{6}{7}; \frac{3}{7}\right)$;

E) $\left(-\frac{2}{3}; 2; 1\right)$.

21. $\vec{a}(2; -3; 4)$ va $\vec{b}(-2; -3; 1)$ vektorlarning skalyar ko'paytmasini toping.

A) 9; B) 17; C) 13; D) 4; E) 36.

22. m va n ning qanday qiymatlarida $\vec{a}\,(15; m; 1)$ va $\vec{b}\,(18; 12; n)$ vektorlar kollinear bo'ladi?

A) $m = 5$; $n = 6$; B) $m = 10$; $n = 1,2$; C) $m = 10$; $n = 12$;
D) $m = 5$; $n = 0,6$; E) $m = 1$; $n = 1,2$.

23. $\vec{a}\,(-1; 2; 3)$ va $\vec{b}\,(5; x; -1)$ vektorlar berilgan. x ning qanday qiymatida $\vec{a}\vec{b} = 2$ tenglik o'rinli bo'ladi?

A) 3; B) 4; C) –5; D) 5; E) –4.

24. $\vec{a} = m\vec{i} + 3\vec{j} + 4\vec{k}$, $\vec{b} = 4\vec{i} + m\vec{j} - 7\vec{k}$ vektorlar m ning qanday qiymatida o'zaro perpendikular bo'ladilar.

A) 4; B) 3; C) –4; D) –3; E) 5,5.

25. $A\,(1; 3; 0)$, $B\,(2; 3; -1)$ va $C\,(1; 2; -1)$ nuqtalar berilgan. $\vec{CA}$ va $\vec{CB}$ vektorlar orasidagi burchak kattaligini toping.

A) 30°; B) 45°; C) 60°; D) 90°; E) 120°.

26. $|\vec{a}|=1$, $|\vec{b}|=|\vec{c}|=2$, $\widehat{\vec{a}\vec{b}}=\widehat{\vec{b}\vec{c}}=60^\circ$ ekanligi ma'lum bo'lsa, $(\vec{a}+\vec{b})\cdot\vec{c}$ ni hisoblang.

A) 2; B) 3; C) 4; D) 8; E) 12.

27. $\vec{a}$ va $\vec{b}$ vektorlar $\vec{c}$ vektorga perpendikular bo'lib, $\widehat{\vec{a}\vec{b}}=120^\circ$. $|\vec{a}|=|\vec{b}|=|\vec{c}|=1$ bo'lsa, $(\vec{a}-\vec{b}+\vec{c})(\vec{a}-\vec{c})$ skalyar ko'paytmaning qiymatini toping.

A) 3; B) $\frac{1}{3}$; C) 2; D) $\frac{1}{2}$; E) $-\frac{1}{2}$.

28. $\vec{a}\,(1;\,2;\,3)$ vektor berilgan. Boshi $A\,(1;\,1;\,1)$ nuqtada, oxiri Oxy tekislikdagi B nuqtada bo'lgan $\vec{a}$ vektorga kollinear vektorning koordinatalarini toping.

A) (0; 1; 2); B) $\left(-\frac{1}{3};\,-1\frac{1}{3};\,-1\right)$; C) (1; 1; 0);

D) $\left(-\frac{1}{3};\,-1\frac{1}{3};\,0\right)$; E) (2; 3; 4).

29. $\vec{a},\vec{b},\vec{c}$ birlik vektorlar juft-jufti bilan 60° li burchak tashkil qiladi. $\vec{a}$ va $\vec{b}-\vec{c}$ vektorlar orasidagi burchakni toping.

A) 30°; B) 45°; C) 60°; D) 90°; E) 120°.

30. $ABCDA_1B_1C_1D_1$ kubning qirrasi 2 ga teng. $\overrightarrow{AC}\cdot\overrightarrow{C_1A}$ ning qiymatini toping.

A) 4; B) $2\sqrt{2}$; C) $\sqrt{2}$; D) 8; E) –8.

IX bob. KO‘PYOQLAR

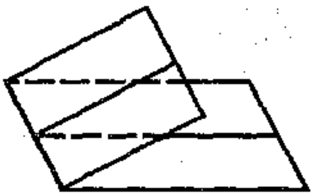
1-§. Ko‘p yoqli burchak

1.1. Ikki yoqli burchak. Umumiy chegaraga ega bo‘lgan ikki yarim tekislik *ikki yoqli burchakni* tashkil etadi (221- rasm).

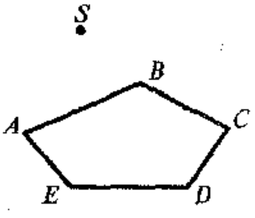
Yarim tekisliklar ikki yoqli burchakning *yoqlari*, ularning umumiy chegarasi esa ikki yoqli burchakning *qirrasi* deyiladi. Ikki yoqli burchakning qirrasiga perpendikular tekislik uning yoqlarini ikkita yarim to‘g‘ri chiziq bo‘yicha kesadi. Bu yarim to‘g‘ri chiziqlarning kesuvchi tekislikda hosil qilgan burchagini *ikki yoqli burchakning chiziqli burchagi* deyiladi. Har qanday ikki yoqli burchak o‘zining chiziqli burchagining kattaligi bilan o‘lchanadi.

1.2. Ko‘p yoqli burchak. Uch yoqli burchak. $ABCDE$ ko‘pburchak va uning tekisligiga tegishli bo‘lmagan S nuqta berilgan bo‘lsin (222- rasm). Boshi S nuqtada bo‘lgan va $ABCDE$ ko‘pburchakdan o‘tuvchi barcha nurlar birlashmasi *ko‘p yoqli burchak* deyiladi (223- rasm).

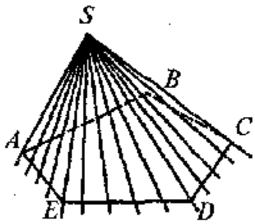
Bunda kattaligi (0°; 180°) oraliqda bo‘lgan ASB, BSC, CSD, ... burchaklar ko‘p yoqli burchakning *tekis burchaklari* yoki *yoqlari*, tekis burchaklarning umumiy tomonlari ko‘p yoqli burchakning *qirrasi*, barcha qirralarning umumiy nuqtasi ko‘p yoqli burchakning *uchi*, ko‘p yoqli burchakning yoqlari tashkil etgan ikki yoqli burchak *ko‘p yoqli burchakning ikki yoqli burchagi* deyiladi.



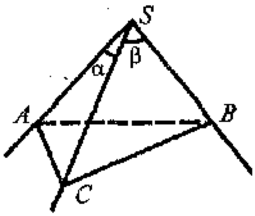
221- rasm.



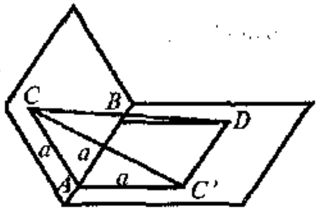
222- rasm.



223- rasm.



224- rasm.



225- rasm.

Ko'p yoqli burchaklar yoqlarining soniga bog'liq ravishda *uch yoqli burchak, to'rt yoqli burchak* va hokazo deb atalishi mumkin. Ko'p yoqli burchakni ifodalashda dastlab uchining ifodasi, so'ngra ko'pburchak uchlarining ifodasi yozilib, *SABCDE* tarzida belgilanadi.

Uch yoqli va ko'p yoqli burchaklar tekis burchaklarining xossalari:

1- xossa. Uch yoqli burchakning har bir tekis burchagi qolgan ikki tekis burchagi yig'indisidan kichik (224- rasm).

Agar uch yoqli burchakning tekis burchaklari kattaliklari $\alpha = \angle ASC, \beta = \angle CSB$ ва $\gamma = \angle ASB$ bilan belgilangan bo'lsa, $\alpha < \beta+\gamma, \beta < \alpha+\gamma, \gamma < \alpha+\beta$ bo'ladi.

Umuman, yuqoridagi tasdiq har qanday qavariq ko'p yoqli burchak uchun o'rinlidir. Har qanday qavariq ko'p yoqli burchakning istalgan tekis burchagi qolgan tekis burchaklarning yig'indisidan kichik.

1-xossadan $\beta > \alpha - \gamma; \alpha > \beta - \gamma; \gamma > \alpha - \beta$ natija kelib chiqadi.

2- xossa. Uch yoqli burchak barcha tekis burchaklarining yig'indisi 360° dan kichik, ya'ni $\alpha + \beta + \gamma < 360°$.

Har qanday ko'p yoqli burchak barcha tekis burchaklarining yig'indisi 360° dan kichik.

1- masala. 120° li ikki yoqli burchakning qirrasida A va B nuqtalar berilgan. Ularning har biridan ikki yoqli burchakning turli yoqlarida AB ga perpendikular qilib AC va BD kesmalar chizilgan. Agar $|AB| = |AC| = |BD| = a$ bo'lsa, CD kesmaning uzunligini toping.

Yechilishi. Masalada berilgan va so'ralgan kesmalarni bir-biriga uchburchak tarzida «bog'lab», noma'lum kesmani topamiz (225- rasm).

$AC \perp AB$, $\triangle CAC'$ da $|AC| = |AC'| = a$. Kosinuslar teoremasiga ko'ra

$$|CC'|^2 = 2a^2 - 2a^2\cos 120^\circ;\ \cos 120^\circ = -\sin 30^\circ = -\frac{1}{2}.$$

Demak, $|CC'|^2 = \sqrt{3}a$; to'g'ri burchakli $\triangle DC'C$ dan $|CD| =$ $= \sqrt{3a^2 + a^2} = 2a$

Javob: $|CD| = 2a$.

2- masala. $SABC$ uch yoqli burchakning SC qirrasidagi ikki yoqli burchagi to'g'ri, SB qirrasidagi ikki yoqli burchagi 60° ga teng $\left(\gamma < \frac{\pi}{2}\right)$. CSB tekis burchagi esa γ ga teng (226- rasm). Qolgan ikki tekis burchak $\alpha = \angle ASB$ va $\beta = \angle CSA$ ni toping.

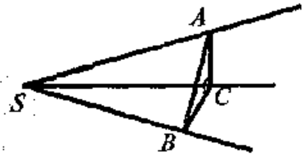
Yechilishi. SA qirraning A nuqtasidan SB qirraga AB perpendikular va SC qirraga AC perpendikular tushiramiz. Uch perpendikular haqidagi teoremaga ko'ra CB kesma SB qirraga perpendikular.

To'g'ri burchakli SAB, SCB, ASC va ABC uchburchaklardan quyidagilarni hosil qilamiz:

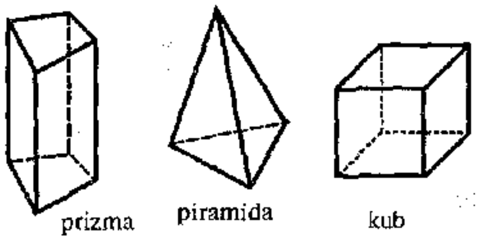
$$\mathrm{tg}\alpha = \frac{|AB|}{|SB|} = \frac{|BC|}{\cos\varphi} : \frac{|BC|}{\mathrm{tg}\gamma} = \frac{\mathrm{tg}\gamma}{\cos\varphi},$$ bundan $\mathrm{tg}\alpha = 2\mathrm{tg}\gamma \Rightarrow \alpha = \mathrm{arctg}(2\mathrm{tg}\gamma)$.

$$\mathrm{tg}\beta = \frac{|AC|}{|SC|} = |BC| : \mathrm{tg}\varphi : \frac{|BC|}{\sin\gamma} = \mathrm{tg}\varphi \cdot \sin\gamma,$$ bundan $\mathrm{tg}\beta = \sqrt{3}\sin\gamma \Rightarrow \beta =$ $= \mathrm{arctg}(\sqrt{3}\sin\gamma)$.

Javob: $\alpha = \mathrm{arctg}(2\mathrm{tg}\gamma)$, $\beta = \mathrm{arctg}(\sqrt{3}\sin\gamma)$.



226-rasm.



227-rasm.

1.3. Ko'p yoqli sirt. Prizma, piramida, kub kabi geometrik shakllar (jismlar)ning har biri ko'pburchaklardan tashkil topgan ko'pyoqli sirtga ega bo'lgan ko'pyoqdir.

T a ' r i f. *Ko'pburchaklardan tashkil topgan sirtning ixtiyoriy nuqtasi ko'pburchaklardan birining nuqtasi bo'lsa, yo faqat ikkitasining umumiy nuqtasi bo'lsa, yoki sirtdagi ko'p yoqli burchaklardan faqat birining uchi bo'lsa, bunday sirt* ***sodda ko'pyoqli sirt*** *deyiladi.*

Ko'pyoqli sirtni tashkil qiluvchi ko'pburchaklar uning *yoqlari* deyiladi; bu ko'pburchaklar tomonlarini ko'p yoqli sirtning *qirralari*, uchlarini esa ko'p yoqli sirtning *uchlari* deyiladi.

Agar ko'p yoqli sirtning har bir qirrasi uning ikkita yog'iga tegishli bo'lsa, u holda sirt *yopiq ko'p yoqli sirt* deyiladi. Kubning sirti yopiq ko'p yoqli sirtga misol bo'ladi. Piramida yoki prizmaning yon sirti yopiq bo'lmagan ko'p yoqli sirtga misol bo'la oladi.

2-§. Ko'pyoq

2.1. Ko'pyoq

T a ' r i f. *Yopiq ko'p yoqli sirt bilan chegaralangan jism* ***ko'pyoq*** *deyiladi.*

Ko'pyoqning bir yog'iga tegishli bo'lmagan ikki uchini tutashtiruvchi kesma *ko'pyoqning diagonali* deyiladi.

Ko'pyoqning bir juft diagonali aniqlagan tekislik bilan kesimi *ko'pyoqning diagonal kesimi* deb ataladi.

1- m a s a l a. 5 ta yog'i va 6 ta uchi bo'lgan ko'pyoqning nechta qirrasi bo'ladi?

Y e c h i l i s h i. Odatda bu turdagi masalalarni, bir necha xil ko'pyoqlikni ko'z oldingizga keltirib, ularning har birini uchlari, yoqlari va qirralari sonini hisoblab yechasiz. Aslida bu va bunga o'xshash jiddiy masalalarni yechishda $U + Y = Q + 2$ ifodadan foydalanish qulaydir. Bu ifoda mashhur Dekart—Eyler teoremasining ifodasi bo'lib, undagi U — qavariq ko'pyoqning uchlari sonini, Y — yoqlari sonini, Q — esa qirralari sonini bildiradi.

Demak, 5 ta yog'i va 6 ta uchi bo'lgan ko'pyoqning barcha qirralari soni 9 ta ekan.

J a v o b: 9 ta.

2- m a s a l a. Kubning nechta diagonali mavjud?

Y e c h i l i s h i. Kubning har bir uchida uch yoqli burchak bo'lib, bu burchaklarning uchi kubning sakkizta uchidan faqat bittasigina

228-расм.

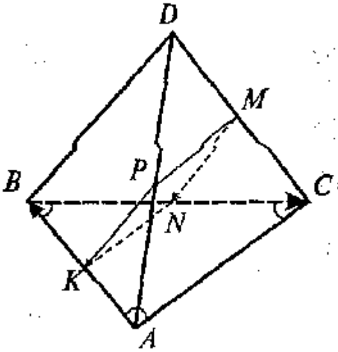
bilan umumiy yoqda yotmaydi. Shuni hisobga olib kub diagonallarining soni 4 ta ekanini aniqlaymiz.

J a v o b: 4 ta.

2.2. Muntazam ko‘pyoqlar.

T a ’ r i f. *Agar ko‘pyoqning: a) yoqlari tomonlari soni bir xil bo‘lgan muntazam ko‘pburchaklardan iborat bo‘lsa; b) har bir uchida bir xil sondagi qirralar uchrashsa, bunday ko‘pyoq muntazam ko‘pyoq deyiladi.*

Muntazam ko‘pyoqning faqat besh turi borligi isbotlangan. Ular: tetraedr, geksaedr (kub), oktaedr, dodekaedr va ikosaedr (228- rasm).



229-расм.

3- m a s a l a. Muntazam $DABC$ tetraedrda M, N, K va P nuqtalar mos ravishda DC, BC, AB va DA qirralarining o‘rtalari. Agar tetraedrning qirrasi 4 ga teng bo‘lsa, $\overline{MN}\cdot\overline{PK}+\overline{AB}\cdot\overline{BC}$ ni hisoblang (229- rasm).

Y e c h i l i s h i. $MNKP$ to‘rtburchak parallelogrammdir (uchburchak o‘rta chizig‘ining xossasidan foydalanib isbotlanadi). Shuning uchun $\overline{MN}\cdot\overline{PK}=4$ ($MN \parallel PK$ bo‘lgani uchun). $\left|\overline{AB}\right|=\left|\overline{BC}\right|=4$ va $\widehat{\overline{AB},\overline{BC}}=120^\circ$ bo‘lgani uchun $\left|\overline{AB}\right|\cdot\left|\overline{BC}\right|=4\cdot 4\cos 120^\circ=-8$. D e m a k, $\overline{MN}\cdot\overline{PK}+\overline{AB}\cdot\overline{BC}=-4$.

J a v o b: -4.

2.3. Hajm tushunchasi. Hajmlarning umumiy xossalari.

Sirtlarning yuzlarini o‘lchash masalasi kabi har bir jismga (xususan, ko'pyoqqa) uning hajmi deb ataladigan aniq bir musbat miqdor V ni mos qo'yish mumkinki, bunda quyidagi xossalar bajarilsin:

1) qirrasining uzunligi uzunlik o'lchov birligi uchun qabul qilingan kubning hajmi hajmlarning o'lchov birligidir;

2) teng jismlarning (xususan, ko'pyoqlarning) hajmlari tengdir;

3) agar jism (ko'pyoq) ixtiyoriy ikkitasining umumiy ichki nuqtalari bo'lmagan bir nechta bo'laklar (ko'pyoqlar) birlashmasidan iborat bo'lsa, u holda jismning (ko'pyoqning) hajmi barcha bo'laklar hajmlarining yig'indisiga teng bo'ladi.

Har bir jism (ko'pyoq) aniq hajmga ega.

3-§. Prizma

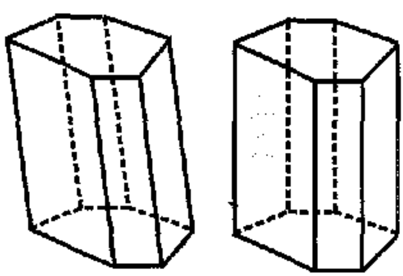
3.1. To'g'ri va og'ma prizma.

Ta'rif. *Ikki yog'i parallel tekisliklarda yotuvchi n burchaklar (ko'pburchaklar), qolgan n ta yog'i parallelogrammlar bo'lgan ko'pyoq **prizma** deyiladi* (230- rasm).

Parallel tekisliklardagi *n* burchaklar *prizmaning asoslari*, qolgan yoqlari esa *yon yoqlari* deb ataladi. Uchlari asos tekisliklariga tegishli bo'lgan perpendikular *prizmaning balandligi* deyiladi.

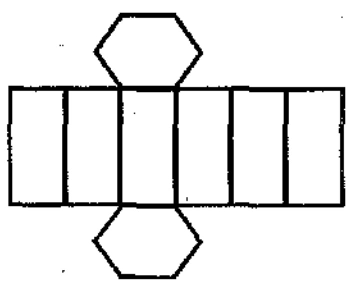
Yon qirralari asos tekisligiga perpendikular bo'lgan prizma to'g'ri prizma, aks holda og'ma prizma bo'ladi.

Asosi muntazam ko'pburchak bo'lgan to'g'ri prizma *muntazam prizma* deb ataladi. 231- rasmda muntazam prizma yoyilmasi tasvirlangan.



og'ma prizma   to'g'ri prizma

230-rasm.



prizma yoyilmasi

231-rasm.

1- m a s a l a. Prizmaning yoqlari eng kamida nechta bo'lishi mumkin? Bunday prizmada nechta uch, nechta qirra bo'ladi?

Y e c h i l i s h i. Ma'lumki, prizmaning yoqlari soni eng kichik bo'lishi uchun uning asoslari uchburchakdan tashkil topgan bo'lishi kerak. Bu holda prizma 5 ta yoqqa, 6 ta uchga, 9 ta qirraga ega.

2- m a s a l a. To'rt burchakli muntazam prizmaning diagonali 25 sm ga, yon yog'ining diagonali 20 sm ga teng. Prizmaning balandligini toping (232- rasm).

Y e c h i l i s h i. d_1 – prizma diagonali, d_2 – prizma yon yog'ining diagonali.

To'g'ri burchakli A_1BC uchburchakda $d_1^2 = d_2^2 + a^2$ (a – asos tomoni), bundan $a = \sqrt{25^2 - 20^2} = 15$ см. To'g'ri burchakli A_1AB uchburchakda $H = AA_1 = \sqrt{d_2^2 - a^2} = \sqrt{400 - 225} = \sqrt{175} = 5\sqrt{7}$ sm.

J a v o b: $H = 5\sqrt{7}$ sm.

3.2. Parallelepiped va uning xossalari.

T a' r i f. *Asosi parallelogramm bo'lgan prizma parallelepiped deyiladi* (233- *rasm*).

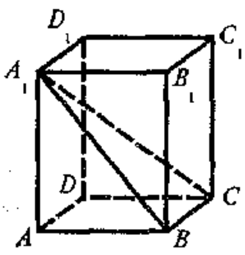
Parallelepiped og'ma yoki to'g'ri bo'lishi mumkin.

T a' r i f. *To'g'ri parallelepipedning asosi to'g'ri to'rtburchak bo'lsa, u to'g'ri burchakli parallelepiped deb ataladi* (234-*rasm*).

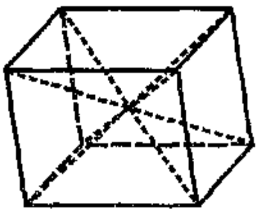
Xossalari:

1. Parallelepipedning qarama-qarshi yoqlari o'zaro teng va paralleldir.

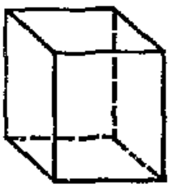
2. Parallelepipedning barcha diagonallari bir nuqtada kesishadi va bu nuqtada teng ikkiga bo'linadi (233- rasm).



232-rasm.



233-rasm.



234-rasm.

To‘g‘ri burchakli parallelepipedning barcha yoqlari to‘g‘ri to‘rtburchaklardir. Uning bir uchidan chiquvchi uchta qirrasi to‘g‘ri burchakli parallelepipedning *o‘lchovlari* deyiladi.

T a ’ r i f. *Uchala o‘lchovi bir-biriga teng bo‘lgan to‘g‘ri burchakli parallelepiped kub deb ataladi* (234- *rasm*).

Kubning barcha yoqlari o‘zaro teng kvadratlardir.

T e o r e m a. To‘g‘ri burchakli parallelepiped diagonali uzunligining kvadrati uning uchala o‘lchovi kvadratlarining yig‘indisiga teng.

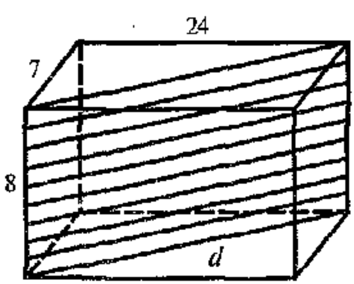
N a t i j a. To‘g‘ri burchakli parallelepipedning barcha diagonallarining uzunliklari teng.

3- m a s a l a. To‘g‘ri burchakli parallelepiped asosining tomonlari 7 va 24 sm, balandligi esa 8 sm. Diagonal kesimining yuzini aniqlang (235- rasm).

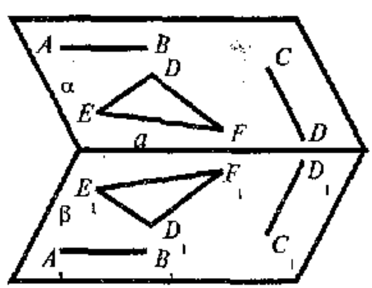
Y e c h i l i s h i. Parallelepiped asosining diagonali $d = \sqrt{7^2 + 24^2} =$ $= \sqrt{625} = 25$ см. Demak, diagonal kesimning yuzi $S = 8 \cdot 25 = 200$ см^2.

J a v o b: 200 sm^2.

3.3. Ko‘pburchak ortogonal proyeksiyasining yuzi. α va β tekisliklar bir-biri bilan a to‘g‘ri chiziq bo‘yicha kesishsin, $(\alpha^\wedge\beta) = \varphi$, $0^\circ < \varphi < 90^\circ$ bo‘lsin (236- rasm). β tekislikni α tekislikka ortogonal proyeksiyalaganda: 1) AB kesmaning $(AB \parallel a)$ uzunligi uning A_1B_1 proyeksiyasi uzunligiga teng: $|AB| = |A_1B_1|$; 2) $a \perp CD$ kesmaning C_1D_1 proyeksiya tekisligi uchun $|C_1D_1| = |CD| \cdot \cos\varphi$ bo‘ladi.



235-rasm.



236-rasm.

T e o r e m a. Uchburchakning tekislikdagi ortogonal proyeksiyasining yuzi proyeksiyalanuvchi uchburchak yuzini uchburchak tekisligi bilan uning proyeksiyasi orasidagi burchak kosinusiga ko'paytirilganiga teng.

$$S_{\Delta E_1D_1F_1} = S_{\Delta EDF} \cdot \cos\varphi.$$

Har qanday ko'pburchakni uchburchaklarga ajratish mumkin, shuning uchun teorema ko'pburchak uchun ham to'g'ridir.

4- m a s a l a. Kubning qirrasi a ga teng. Agar tekislik kub asosining tomonidan o'tsa hamda asos bilan 30° burchak tashkil etsa, kub kesimining yuzini toping (237- rasm).

Y e c h i l i s h i. $a^2 = x\cos 30^\circ \Rightarrow x = \dfrac{a^2}{\cos 30^\circ} \Rightarrow x = \dfrac{2a^2}{\sqrt{3}}.$

J a v o b: $\dfrac{2a^2}{\sqrt{3}}.$

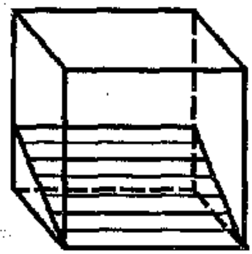
3.4. Prizma sirtining yuzi.

Ko'pyoq barcha yoqlarining yuzlari yig'indisi ko'pyoq sirtining *yuzi* deyiladi.

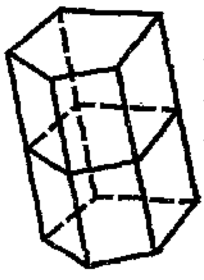
Prizmaning sirti ikki asos yuzlari yig'indisiga yon sirti yuzini qo'shilganiga teng:

$$S_{pr} = 2S_{asos} + S_{yon}.$$

Prizmani uning biror yon qirrasiga perpendikular tekislik bilan (barcha yon qirralarini kesuvchi) kessak, kesimda hosil bo'lgan ko'pburchak prizmaning *perpendikular kesimi* deyiladi. Prizmaning har bir yon yog'i, asosi prizmaning yon qirrasidan, balandligi esa



237-rasm.



238-rasm.

perpendikular kesim tomonidan iborat parallelogrammdir. Shuning uchun prizma yon sirtining yuzi perpendikular kesim perimetri bilan yon qirrasining ko'paytmasiga teng (238- rasm). Xususan, to'g'ri prizma *yon sirtining yuzi* asosining perimetri bilan prizma balandligining (bu holda yon qirrasi prizma balandligiga teng) ko'paytmasiga teng.

5- m a s a l a. Uchburchakli og'ma prizmaning yon qirralaridan biridagi ikki yoqli burchak 120° ga teng; prizmaning shu qirrasidan boshqa yon qirralarigacha bo'lgan masofalar 16 sm va 14 sm. Agar prizmaning yon qirrasi 20 sm bo'lsa, yon sirtining yuzini toping (239- rasm).

Y e c h i l i s h i. $\angle ABC = 120^\circ$; $|AB| = 16$; $|BC| = 14$. Kosinuslar teoremasidan $AC = 26$ sm ekanini aniqlaymiz. Demak, yon sirti $S_{yon} = 20 \cdot (14 + 16 + 26) = 20 \cdot 56 = 1120$ sm².

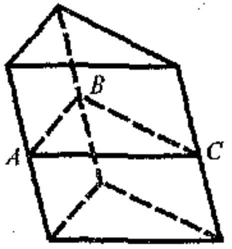
J a v o b: 1120 sm².

6- m a s a l a. Prizmaning asosi tomoni $2\sqrt{5}$ bo'lgan muntazam oltiburchak, yon yoqlari kvadratlardan iborat. Prizmaning katta diagonalini toping (240- rasm).

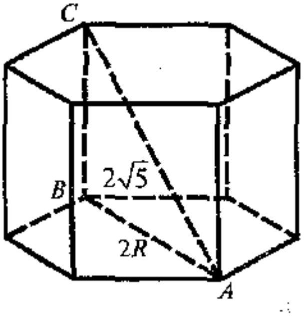
Y e c h i l i s h i. Tomoni $2\sqrt{5}$ ga teng bo'lgan muntazam oltiburchakning katta diagonali ($a_6 = R$ bo'lgani uchun) $2R$ ga, ya'ni $4\sqrt{5}$ ga teng. Prizmaning katta diagonali uzunligi $|AC|$ ni ABC to'g'ri burchakli uchburchakdan topamiz:

$$|AC|^2 = (2\sqrt{5})^2 + (4\sqrt{5})^2 = 20 + 80 = 100.$$

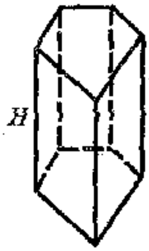
J a v o b: $|AC| = 10$ см .



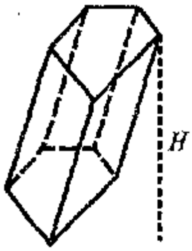
239-rasm.



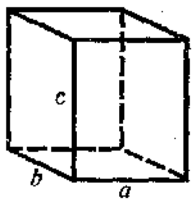
240-rasm.



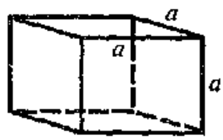
241- rasm.



242- rasm.



243- rasm.



244- rasm.

3.5. Prizmaning hajmi. Har qanday prizmaning hajmi asosining yuzi bilan balandligining ko'paytmasiga teng (241-, 242- rasmlar).

$$V_{\text{prizma}} = S_{\text{asos}} \cdot H \quad (H - \text{prizma balandligi}).$$

Xususan, har qanday parallelepipedning hajmi asosining yuzi bilan balandligining ko'paytmasiga teng:

$$V_{\text{p-ped}} = S_{\text{asos}} \cdot H\ .$$

To'g'ri burchakli parallelepipedning hajmi uning bir uchidan chiqqan uchala qirrasining ko'paytmasiga teng (243- rasm):

$$V_{\text{p-ped}} = a \cdot b \cdot c\ .$$

Kubning hajmi uning qirrasining kubiga teng (244- rasm):

$$V_{\text{kub}} = a^3\ .$$

7- m a s a l a. Uchburchakli muntazam prizmaning balandligi 8 ga, asosining yuzi $9\sqrt{3}$ ga teng. Prizma yon yog'ining diagonalini toping.

Y e c h i l i s h i. Yuzi $9\sqrt{3}$ ga teng bo'lgan muntazam uchburchakning tomoni 6 ga teng. Tomonlari 8 va 6 bo'lgan to'g'ri to'rtburchakning diagonali 10 ga teng.

J a v o b: 10.

8- m a s a l a. Prizmaning jami qirralari soni 36 ga teng bo'lsa, uning nechta yon yog'i bor?

Y e c h i l i s h i. Agar prizmaning asosi n burchakdan iborat bo'lsa, uning barcha qirralari soni $3 \cdot n = 36$ bo'ladi. Demak, $n = 12$ ga teng. Bundan prizmaning yon yoqlari soni 12 ta ekanligi ma'lum bo'ladi.

J a v o b: 12 ta.

4-§. Piramida. Kesik piramida

4.1. Piramida.

Ta'rif. *Yoqlaridan biri ixtiyoriy ko'pburchak, qolgan yoqlari umumiy uchga ega bo'lgan uchburchaklardan iborat ko'pyoq piramida deyiladi* (245- rasm).

Piramida yon yoqlarining umumiy uchi *piramidaning uchi* deyiladi. Piramida uchi qarshisidagi yoq uning *asosi* deb ataladi. Piramida uchidan uning asosigacha bo'lgan masofani ifodalovchi kesma piramidaning *balandligi* deb hisoblanadi.

Agar piramidaning yon qirralari asos tekisligi bilan bir xil burchak tashkil etsa, u holda piramidaning balandligi uning asosiga tashqi chizilgan aylananing markaziga tushadi.

Agar piramidaning yon yoqlari asos tekisligi bilan bir xil burchak tashkil etsa, u holda piramidaning balandligi uning asosiga ichki chizilgan aylananing markaziga tushadi. Shu bilan birga:

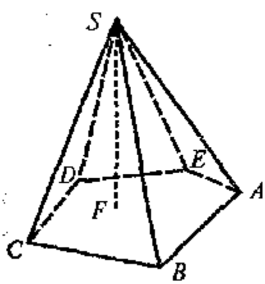
$S_{asos} = S_{yon} \cdot \cos\alpha$, *bunda* α — *asosdagi ikki yoqli burchak.*

4.2. Muntazam piramida.

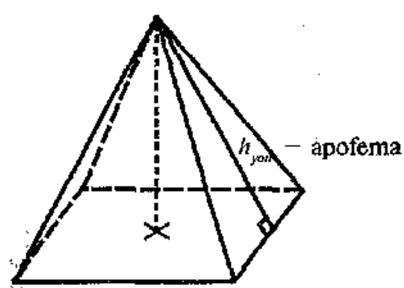
Ta'rif. *Agar piramidaning asosi muntazam ko'pburchak bo'lib, uning balandligi asosi markaziga tushsa, u muntazam piramida deb ataladi* (245- rasm).

Muntazam piramidaning barcha yon yoqlari o'zaro teng bo'lgan teng yonli uchburchaklardan iborat (bundan uning barcha yon qirralari o'zaro tengligi kelib chiqadi).

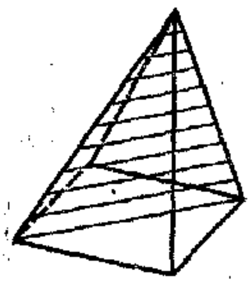
Muntazam piramida yon yog'ining asosiga tushirilgan balandligi uning *apofemasi* deyiladi (246- rasm).



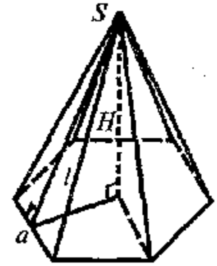
245- rasm.



246- rasm.



247- rasm.



248- rasm.

Muntazam piramida yon sirtining yuzi asosining perimetri bilan apofemasi ko'paytmasining yarmiga teng:

$$S_{yon} = \frac{1}{2} P \cdot h_{yon}.$$

Bunda P — asos perimetri, h_{yon} — apofemasi.

Piramida to'la sirtining yuzi uning yon sirtining yuzi bilan asosi yuzining yig'indisiga teng:

$$S_{to'la} = S_{yon} + S_{asos}.$$

Piramidaning bir yog'ida yotmagan ikki yon qirrasidan o'tuvchi tekislik bilan kesimi piramidaning *diagonal kesimi* deyiladi (247- rasm).

1- masala. Muntazam piramidaning asosi tomoni a ga teng bo'lgan oltiburchakdir. Piramidaning balandligi H ga teng bo'lsa, yon sirtining yuzini toping (248- rasm).

Yechilishi. Tomoni a ga teng bo'lgan muntazam oltiburchakka ichki chizilgan aylana radiusi $\frac{\sqrt{3}a}{2}$ ga teng bo'lgani uchun muntazam piramidaning apofemasi

$$l = \sqrt{H^2 + \left(\frac{\sqrt{3}a}{2}\right)^2} = \frac{1}{2}\sqrt{4H^2 + 3a^2}$$

bo'lib, yon sirti:

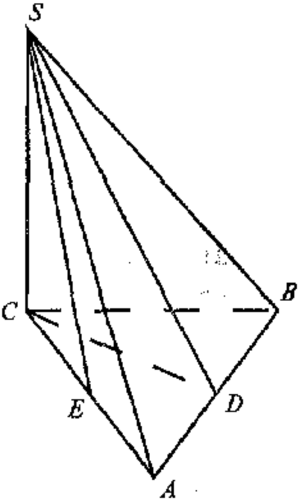
$$S_{yon} = \frac{1}{2} 6 \cdot a \cdot \frac{1}{2}\sqrt{4H^2 + 3a^2} = \frac{3a}{2}\sqrt{4H^2 + 3a^2}$$

Javob: $S_{yon} = \frac{3a}{2}\sqrt{4H^2 + 3a^2}$.

O'quvchida masalalarni mustaqil va optimal usullar bilan yechish malakasining shakllanishida nazariy ma'lumotlarni yaxshi o'zlashtirishdan tashqari qo'yilgan masalani turli usullarda yechib ko'rish va yechimni tahlil qila bilish ham katta ahamiyatga ega.

Namuna sifatida ushbu masalani turli usullar bilan yechilishini ko'rib chiqamiz.

2- masala. *SABC* piramidaning asosi teng tomonli *ABC* uchburchak bo'lib, $AB = BC = AC = a$. *SC* qirra *ABC* uchburchak tekisligiga perpendikular va $SC = h$. *E* nuqta *AC* tomonning *D* nuqta esa *AB* tomonning o'rtasi. *CD* va *CE* to'g'ri chiziqlar orasidagi burchakni va masofani toping (249- rasm).



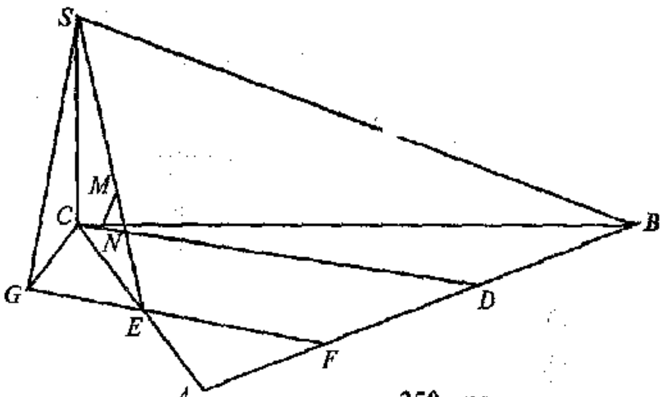
249- rasm.

Yechilishi. Avvalo, masala yechilishini rasmiylashtirishning ushbu qoidalariga rioya qilishni tavsiya etamiz:

1. Masalada berilganlarni, topilishi kerak bo'lgan kattaliklarni (miqdorlarni) yozish, belgilashlar kiritish.
2. Masala mazmuniga mos shakl (shakllar) chizish.
3. Zarur hollarda shaklda qo'shimcha yasashlar bajarish; shaklning u yoki bu qismlarini alohida qilib chizib olish.
4. Masalani yechishning rejasini aniq tasavvur etish.

Qaralayotgan masalani 3 xil usulda yechish mumkin. Har bir usul rejasini keltiramiz.

1- usul. *SE* va *CD* — ayqash chiziqlar orasidagi burchakni topishda shu chiziqlarda yotuvchi $\overrightarrow{SE}$ va $\overrightarrow{CD}$ vektorlar orasidagi burchak kosinusi formulasidan foydalaniladi. Buning uchun esa shu vektorlarning koordinatalarini topish kerak. *SE* va *CD* ayqash chiziqlar orasidagi masofa *CD* ning ixtiyoriy nuqtasidan *CD* ga parallel va *SE* dan o'tuvchi tekislikkacha bo'lgan masofaning o'zidir. Tekislikda yotuvchi 3 ta nuqtaning koordinatalarini bilgan holda uning tenglamasini tuzish va nuqtadan tekislikkacha bo'lgan masofa formulsidan foydalanish mumkin. Masalani 1-usulda yechish ana shu rejaga asoslangan.



250- rasm.

2- u s u l. *SE* va *CD* ayqash chiziqlar orasidagi *MN* masofa ularga o'tkazilgan umumiy perpendikularning uzunligidir. $\vec{MN}$ va $\vec{SE}$, $\vec{MN}$ va $\vec{CD}$ vektorlarning skalyar ko'paytmasi nolga teng. Bundan $\vec{MN}$ vektor koordinatalarini, binobarin, uning uzunligini topish mumkin (250- rasm).

3- u s u l. Bu usulda qo'shimcha yasash qilinadi: $CD \parallel FG$ o'tkaziladi va ΔSGE qaraladi. ΔSGE dan kosinuslar teoremasiga ko'ra, izlanayotgan burchakni topish mumkin. *SE* va *CD* to'g'ri chiziqlar orasidagi masofani *CSGE* va *SCGE* piramidalarning hajmlari tengligidan topish mumkin, chunki *CSGE* piramidaning balandligi ayni shu masofaga tengdir.

5. Masala yechish jarayonida foydalanilayotgan teoremalar, formulalarni aytib o'tish.

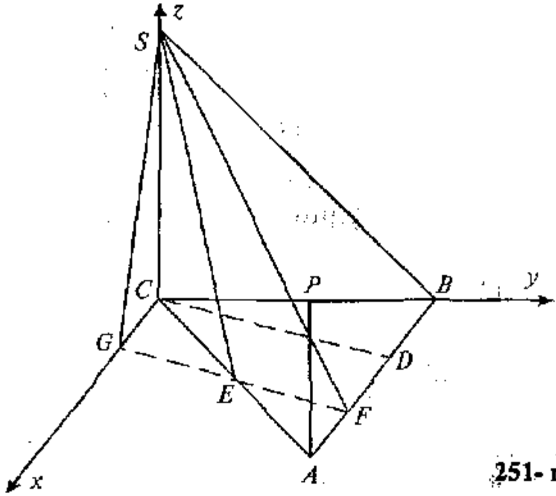
6. Chiqargan xulosangiz, olgan natijangiz qat'iy asoslanishi, isbotlanishi lozim. Bu — geometriyada (umuman, matematikada!) masala yechish jarayonining eng nozik, eng muhim «nuqtasi», joyi, jihati, o'ziga xosligidir!

7. Olingan javobni tahlil qilish; parametrlarning u yoki bu qiymatlarida javobning qanday bo'lishini aniqlash.

(Har bir masalada bu bandlarning hammasi bo'lishi shart emas, albatta).

Endi masalani yechishga kirishaylik,

B e r i l g a n: *SABC* — piramida, $AB = BC = AC = a$, $CS = h$, $CS \perp ABC$, $CE = CA$, $AD = DB$.



251- rasm.

B e l g i l a s h l a r:

SE va *CD* to'g'ri chiziqlar orasidagi burchakni φ bilan, ular orasidagi masofani *d* bilan belgilaylik.

T o p i s h k e r a k:

$\varphi(SE \wedge CD) = \varphi = ?$

$d(SE, CD) = d = ?$

M a s a l a n i n g y e c h i l i s h i.

1- u s u l. *CS* qirra piramidaning asos tekisligiga — $\triangle ABC$ tekisligiga perpendikularligi masala yechishda koordinatalar usulidan foydalanishni taqozo etadi, bu usulni qo'lashga imkon beradi. Koordinatalar sistemasini quyidagicha kiritish ma'qul. *C* nuqtani koordinata boshi sifatida olish va aplikatlar o'qini *CS* qirra, ordinatalar o'qini esa *CB* qirra bo'ylab yo'naltirish. U holda absissalar o'qi, tabiiyki, *C* nuqtadan *BCS* uchburchak tekisligiga perpendikular bo'lib o'tadi (251- rasm).

Kiritilgan bu koordinatalar sistemasida *C; S; B; A; D; E* nuqtalarning koordinatalarini topish oson:

$C(0; 0; 0)$, $\qquad S(0; 0; h)$

$B(0; a; 0)$, $\qquad A\left(\frac{a\sqrt{3}}{2}; \frac{a}{2}; 0\right)$.

Masalan *A* nuqtaning koordinatalarini topish uchun $AP \perp BC$ o'tkazamiz. U holda $CP = PB = \frac{a}{2}$, $AP = \frac{a\sqrt{3}}{2}$.

Ma'lumki, $K_1(a_1; b_1; c_1)$, $K_2(a_2; b_2; c_2)$ va L nuqta K_1K_2 kesmaning o'rtasi, $K_1L = LK_1$ bo'lsa, u holda L nuqtaning koordinatalari $L\left(\frac{a_1+a_2}{2}; \frac{b_1+b_2}{2}; \frac{c_1+c_2}{2}\right)$ bo'ladi. Shunga ko'ra:

D nuqta AB ning o'rtasi bo'lgani uchun $D\left(\frac{a\sqrt{3}}{4}; \frac{3a}{4}; 0\right)$;

E nuqta AC ning o'rtasi bo'lgani uchun $E\left(\frac{a\sqrt{3}}{4}; \frac{a}{4}; 0\right)$ bo'ladi.

$\vec{SE}$ va $\vec{CD}$ vektorlarni kiritamiz va ularning koordinatalarini topamiz. Topilishi kerak bo'lgan φ burchak, aslida $\vec{SE}$ va $\vec{CD}$ vektorlar orasidagi burchakdir. $\vec{SE}$ ning koordinatalarini topish uchun E nuqtaning koordinatalaridan S nuqtaning mos koordinatalarini ayirish kerak. U holda $\vec{SE}\left(\frac{a\sqrt{3}}{4}; \frac{a}{4}; -h\right)$. Shunga o'xshash $\vec{CD}\left(\frac{a\sqrt{3}}{4}; \frac{3a}{4}; 0\right)$.

Ikki vektor orasidagi burchakning kosinusunu topish uchun bu vektorlarning skalyar ko'paytmasini ularning uzunliklari ko'paytmasiga bo'lish kerak:

$$\cos\varphi = \frac{(\vec{SE}, \vec{CD})}{|\vec{SE}|\cdot|\vec{CD}|}.$$

Vektorlarning skalyar ko'paytmasi ularning mos koordinatalari ko'paytmasining yig'indisiga teng bo'lgani uchun:

$$(\vec{SE}, \vec{CD}) = \frac{a\sqrt{3}}{4}\cdot\frac{a\sqrt{3}}{4} + \frac{a}{4}\cdot\frac{3a}{4} + 0\cdot(-h) = \frac{3a^2}{8}.$$

Vektorning uzunligi uning koordinatalari kvadratlarining yig'indisidan chiqarilgan kvadrat ildizga teng. Binobarin, $|\vec{SE}| = \sqrt{\frac{3a^2}{16} + \frac{a^2}{16} + h^2} = \frac{1}{2}\sqrt{a^2 + 4h^2}$.

CD kesma ΔABC ning balandligi bo'lgani uchun $|\vec{CD}| = \frac{a\sqrt{3}}{2}$.

Shunday qilib, $\cos\varphi = \frac{\sqrt{3}}{2}\cdot\frac{a}{\sqrt{a^2+4h^2}}$.

Mazmuniga ko'ra, $a>0$, $h>0$ bo'lgani uchun $\frac{a}{\sqrt{a^2+4h^2}}$ nisbat doimo 1 dan kichik. Demak, φ topiladi.

Xususan: 1) $a = 4\sqrt{2}$, $h = 2$ bo'lsa, $\varphi = 45°$;

2) $a = 8$, $h = 4\sqrt{2}$ bo'lsa, $\varphi = 60°$;

3) a - o'zgarmas bo'lib, $h \to \infty$ (h cheksizlikka intilsa) bo'lsa, u holda $\frac{a}{\sqrt{a^2+4h^2}} \to 0$ va demak, $\varphi \to 90°$.

4) a - o'zgarmas bo'lib, $h \to 0$ bo'lsa, u holda $\frac{a}{\sqrt{a^2+4h^2}} \to 1$ va demak, $\varphi \to 30°$.

Natijalarni ushbu jadval ko'rinishida ifodalaylik.

a	$4\sqrt{2}$	8	o'zgarmas	o'zgarmas	$\to\infty$	a
h	2	$4\sqrt{2}$	$\to\infty$	$\to 0$	o'zgarmas	a
φ	45°	60°	$\to 90°$	$\to 30°$	$\to 30°$	$\approx 67°14'$

Izoh. a va h parametrlarning o'zgarishiga qarab, ularga mos ravishda shaklning, burchak φ ning o'zgarishini qobiliyatli o'quvchi idrok qila bilishi zarur.

Endi SE va CD ayqash chiziqlar orasidagi $d(SE, CD)$ masofani hisoblaymiz.

Bu masofa SE orqali CD ga parallel qilib o'tkazilgan tekislik bilan CD orasidagi masofaga teng.

Shuni e'tiborga olib, E nuqtadan CD ga parallel EF to'g'ri chiziqni, SE va F nuqta orqali esa SEF tekislikni o'tkazamiz (251-rasm). $EF \parallel CD$, E nuqta AC kesmaning o'rtasi bo'lgani uchun F nuqta ham AD ning o'rtasi bo'ladi. Bundan $AF = \frac{a}{4}$. Demak, F nuqtaning koordinatalari $F\left(\frac{3\sqrt{3}}{8}a; \frac{5a}{8}; 0\right)$ bo'ladi. $d(SE, CD) = d(SEF, CD)$ ekanligidan bu masofa CD kesmaning ixtiyoriy nuqtasidan SEF tekislikkacha bo'lgan masofaga tengdir. CD kesmadagi nuqta sifatida C nuqtani olish qulay, albatta. Shunday qilib $d(SE, CD) = d(SEF, C)$.

Berilgan $M(x_0; y_0; z_0)$ nuqtadan berilgan $A_1x + B_1y + C_1z + D_1 = 0$ tekislikkacha (bu tekislikni α deb belgilaylik) bo'lgan masofa

$$d(M;\alpha) = \frac{|A_1x_0 + B_1y_0 + C_1z_0 + D_1|}{\sqrt{A_1^2 + B_1^2 + C_1^2}}$$

formula bo'yicha hisoblanishi ma'lum.

Demak, izlanayotgan masofani topish uchun SEF tekislik tenglamasini tuzish kerak. Bu tekislik tenglamasini

$$A_1x + B_1y + C_1z + D_1 = 0 \qquad (1)$$

ko'rinishida izlaymiz. Tekislik S, E, F nuqtalardan o'tgani uchun bu nuqtalarning koordinatalari tekislik tenglamasini qanoatlantiradi. Bundan noma'lum A_1, B_1, C_1, D_1 koeffitsiyentlarni topib olamiz:

$$\begin{cases} hC_1 + D_1 = 0, \\ \frac{a\sqrt{3}}{4}A_1 + \frac{a}{4}B_1 + D_1 = 0, \\ \frac{3\sqrt{3}}{8}A_1 + \frac{5a}{8}B_1 + D_1 = 0. \end{cases}$$

Bundan $A_1 = \sqrt{3}B_1$, $D_1 = \frac{a}{2}B_1$, $C_1 = \frac{a}{2h}B_1$.

Nihoyat, topilgan koeffitsiyentlarni (1) tenglamaga qo'yib, $2\sqrt{3}hx - 2hy + az - ah = 0$ tenglamani hosil qilamiz. Bu SEF tekislik tenglamasi. U holda, C nuqtaning koordinatalari $C(0; 0; 0)$ bo'lgani uchun $d(SEF, C) = \frac{ah}{\sqrt{a^2 + 16h^2}}$ bo'ladi.

Xususan: 1) $a = 4\sqrt{2}$, $h = 2$ bo'lsa, $d = \frac{2\sqrt{3}}{3}$;

2) $a = 8$, $h = 4\sqrt{2}$ bo'lsa, $d = \frac{4\sqrt{2}}{3}$;

3) a — o'zgarmas bo'lib, h $\to\infty$(h cheksizlikka intilsa) bo'lsa, u holda $d \to \frac{a}{4}$.

Javob: $\varphi = \arccos\left(\frac{\sqrt{3}}{2} \cdot \frac{a}{\sqrt{a^2 + 4h^2}}\right)$; $d = \frac{ah}{\sqrt{a^2 + 16h^2}}$.

Ko'rilgan usul geometrik masalalar yechishning koordinatalar usuli deyiladi.

2-usul. φ burchakni topish I usuldagi kabi bajariladi. Endi $d(SE, CD)$ ni topamiz.

$MN \perp SE$, $MN \perp CD$ bo'lsin (250-rasm). Biz $\overrightarrow{MN}$ vektorning koordinatalari va uzunligi $|\overrightarrow{MN}|$ ni topamiz. Ravshanki, $d(SE, CD) = \overrightarrow{MN}$.

$\overrightarrow{MN} = \overrightarrow{MS} + \overrightarrow{SC} + \overrightarrow{CN}$ ekanligi ayon.

$\overrightarrow{MS}$ va $\overrightarrow{SE}$, $\overrightarrow{SC}$ va $\overrightarrow{CS}$, $\overrightarrow{CN}$ va $\overrightarrow{CD}$ vektorlar kollinear; bulardan $\overrightarrow{MS}$ va $\overrightarrow{SE}$, $\overrightarrow{SC}$ va $\overrightarrow{CS}$ vektorlar qarama-qarshi, $\overrightarrow{CN}$ va $\overrightarrow{CD}$ vektorlar esa bir xil yo'nalgan. Binobarin, $\overrightarrow{MS} = -m\overrightarrow{SE}$, $\overrightarrow{SC} = -\overrightarrow{CS}$ va $\overrightarrow{CN} = n \cdot \overrightarrow{CD}$ deyish mumkin. U holda $\overrightarrow{MN} = -m\overrightarrow{SE} - \overrightarrow{CS} + n\overrightarrow{CD}$. Bu tenglikda $\overrightarrow{SE}$, $\overrightarrow{CS}$, $\overrightarrow{CD}$ vektorlarning koordinatalarini qo'yamiz:

$$\overrightarrow{MN} = -m\left(\frac{a\sqrt{3}}{4}; \frac{a}{4}; -h\right) - (0; 0; h) + n \cdot \left(\frac{a\sqrt{3}}{4}; \frac{3a}{4}; 0\right) =$$

$$= \left(\frac{a\sqrt{3}}{4}(-m+n); \frac{a}{4}(-m+3n); h(m-1)\right).$$

$MN \perp SE$, $MN \perp CD$ bo'lgani uchun mos vektorlarning skalyar ko'paytmasi nolga teng:

$$(\overrightarrow{MN}, \overrightarrow{SE}) = 0, \ (\overrightarrow{MN}, \overrightarrow{CD}) = 0.$$

Bu ikki tenglamadan m va n koeffitsiyentlarni topamiz:

$$\begin{cases} (-m+n) \cdot \frac{3a^2}{16} + \frac{a^2}{16} \cdot (-m+3n) - h^2(m-1) = 0; \\ (-m+n) \cdot \frac{3a^2}{16} + \frac{3a^2}{16} \cdot (-m+3n) = 0. \end{cases}$$

Tenglamalar sistemasini yechib, $m = 2n$ va $n = \frac{8h^2}{a^2+16h^2}$ ekanini topamiz.

Xususan, 1) $a = 4\sqrt{2}$, $h = 2$ bo'lsa, $n = \frac{1}{3}$ va $m = \frac{2}{3}$;

2) $a = 8$, $h = 4\sqrt{2}$ bo'lsa, $n = \frac{4}{9}$ va $m = \frac{8}{9}$.

Shunday qilib, $\overrightarrow{MN}=\left(\frac{na\sqrt{3}}{4};\frac{na}{4};(2n-1)h\right)$. U holda $d(SE,CD)=$ $=|\overrightarrow{MN}|=\sqrt{\frac{3a^2n^2}{16}+\frac{a^2n^2}{16}+(2n-1)^2h^2}=\sqrt{\frac{a^2n^2}{4}+(2n-1)^2h^2}$, bu yerga n ning $n=\frac{8h^2}{a^2+16h^2}$ qiymatini qo'yib hisoblash natijasida $d=\frac{ah}{\sqrt{a^2+16h^2}}$ ekanini olamiz.

3-usul. $CD\perp FG$ o'tkazamiz. Bundan $\varphi(SE,CD)=\varphi(SE,FG)$. ΔCGEda $CG=GE$ va $\angle CGE=120^\circ$ ekanligi ravshan (251-rasm). Sinuslar teoremasidan $CG=\frac{a}{2\sqrt{3}}$ tenglikni hosil qilamiz. U holda G nuqtaning koordinatalari $G\left(\frac{a}{2\sqrt{3}};0;0\right)$ bo'ladi. To'g'ri burchakli ΔCSE dan: $SE^2=CE^2+CS^2=\frac{a^2}{4}+h^2$; ΔSCG dan: $SG^2=\frac{a^2}{12}+h^2$.

S va G nuqtalarni tutashtirib, ΔSCG ni hosil qilamiz. Bu uchburchakdan kosinuslar teoremasiga ko'ra:

$$SG^2=GE^2+SE^2-2GE\cdot SE\cdot\cos\varphi.$$

Bu tenglikka mos qiymatlarni qo'yamiz. U holda:

$$\frac{a^2}{12}+h^2=\frac{a^2}{12}+\frac{a^2}{4}+h^2-2\cdot\frac{a}{2\sqrt{3}}\cdot\frac{1}{2}\sqrt{a^24h^2}\cdot\cos\varphi,$$

bundan $\cos\varphi=\frac{\sqrt{3}}{2}\cdot\frac{a}{\sqrt{a^2+4h^2}}$.

Endi $d(SE,\ CD)$ masofani hisoblaymiz. SEF va SGE ayni bir tekislik bo'lgani uchun $d(SE,\ CD)=d(C,\ SGE)$. Masofa d ni topish uchun $CSGE$ va $SCGE$ piramidalar hajmlarini hisoblaymiz va natijalarni tenglashtiramiz:

$$V_{CSGE}=\frac{1}{3}\cdot d\cdot S_{\Delta SGE}=\frac{1}{3}\cdot d\cdot\frac{1}{2}\cdot GE\cdot SE\cdot\sin\varphi=$$

$$=\frac{d}{6}\cdot\frac{a}{2\sqrt{3}}\cdot\frac{\sqrt{a^2+4h^2}}{2}\sin\varphi=\frac{ad\sqrt{a^2+4h^2}}{24\sqrt{3}}\sin\varphi=$$

$$=\frac{ad\sqrt{a^2+16h^2}}{48\sqrt{3}},\ \text{chunki}\ \sin\varphi=\frac{\sqrt{a^2+16h^2}}{2\sqrt{a^2+4h^2}}$$

$$V_{CSGE} = \frac{1}{3} \cdot CS \cdot S_{\Delta SGE} = \frac{1}{3} \cdot h \cdot \frac{1}{2} \cdot GC \cdot GE \cdot \sin 120° =$$

$$= \frac{h}{6} \cdot \frac{a^2}{12} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}a^2h}{144}.$$

Ammo $\frac{ad\sqrt{a^2+4h^2}}{24\sqrt{3}} = \frac{\sqrt{3}a^2h}{144}$, bundan $d = \frac{ah}{\sqrt{a^2+16h^2}}$

Qo‘yilgan masala yechildi.

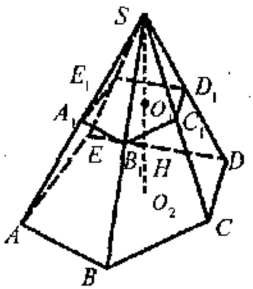
Javob: $\cos\varphi = \frac{\sqrt{3}}{2} \cdot \frac{a}{\sqrt{a^2+4h^2}}$; $d = \frac{ah}{\sqrt{a^2+16h^2}}$.

4.3. Kesik piramida. Piramidaning asosiga parallel tekislik bilan kesib, biror piramida olinsa, qolgan qismi *kesik piramida* deb ataladi (252, 253- rasmlar).

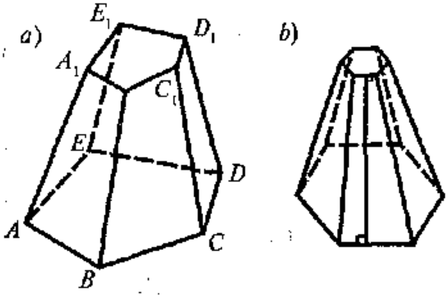
Ta’rif. *Ikki yog‘i parallel tekisliklardagi o‘xshash ko‘pburchaklardan, yon yoqlari esa trapetsiyalardan tashkil topgan ko‘pyoq* ***kesik piramida*** *deyiladi* (253- *rasm*).

Kesik piramidaning parallel yoqlari kesik piramida *asoslari* deb ataladi. $A_1B_1 \parallel AB, B_1C_1 \parallel BC, \ldots$ Shu bilan barcha $\frac{A_1B_1}{AB} = \frac{B_1C_1}{BC} = \ldots = \frac{H_1}{H}$ (bunda $H = SO$ piramida balandligi bo‘lsa, $H_1 = SO_1$ kesilgan kichik piramidaning balandligi) doimo o‘rinli bo‘ladi.

Kesik piramida asoslari orasidagi masofani ifodalovchi kesma uning ***balandligi*** deyiladi.



252- rasm.



253- rasm.

Piramidaning asosiga parallel bo'lgan kesuvchi tekislik berilgan piramidaga o'xshash kichik piramida ajratadi, ya'ni

$$SABCDE \sim SA_1B_1C_1D_1E_1.$$

Demak ikkala piramidaning mos chiziqli elementlari o'zaro proporsionaldir (252-rasm):

$$\frac{SA}{SA_1} = \frac{SB}{SB_1} = \ldots = \frac{SE}{SE_1} = \frac{AB}{A_1B_1} = \frac{BC}{B_1C_1} = \ldots = \frac{H}{H_1}.$$

$SABCD$ piramidaning sirti S, $SA_1B_1C_1D_1$ piramidaning sirti S_1 bo'lsa

$$\frac{S}{S_1} = \frac{(SB)^2}{(SB_1)^2} = \frac{(SA)^2}{(SA_1)^2} = \ldots = \frac{H^2}{H_1^2},$$

ya'ni asoslar yuzlarining nisbati ularning mos chiziqli elementlari kvadratlarining nisbatiga tengdir.

Agar kesik piramida muntazam piramidaning qismi bo'lsa, ***muntazam kesik piramida*** deb ataladi. Muntazam kesik piramidaning yon yog'i bir-biriga teng bo'lgan teng yonli trapetsiyalardan iboratdir. Shu trapetsiyalardan har birining balandligi kesik piramidaning *apofemasi* deyiladi (253- *b* rasm).

Muntazam kesik piramidaning yon sirtini topish uchun uning bir yon yog'ining yuzini topib, yon yoqlar soniga ko'paytiriladi. Natijada:

$$S_{yon} = \frac{1}{2}(P + p) \cdot h_{yon},$$

bunda P va p — kesik piramida asoslarining perimetrlari.

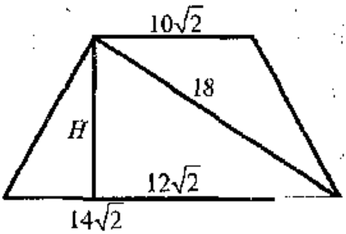
Demak, muntazam kesik piramida yon sirtining yuzi asoslar perimetrlari yig'indisining yarmi bilan apofemasining ko'paytmasiga teng.

2- m a s a l a. Muntazam to'rtburchakli kesik piramida asoslarining tomonlari 14 sm va 10 sm, diagonali 18 sm. Kesik piramidaning balandligi necha santimetr?

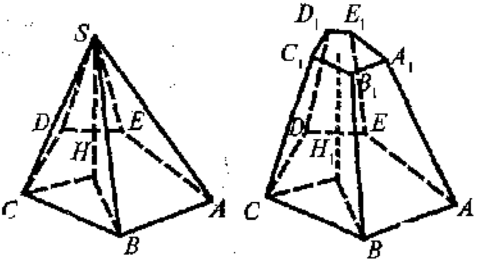
Y e c h i l i s h i. Tomonlari 10 sm va 14 sm bo'lgan asoslarning diagonallari mos ravishda $10\sqrt{2}$ va $14\sqrt{2}$ sm bo'ladi. Kesik piramidaning diagonal kesimida (254-rasm) Pifagor teoremasini qo'llasak,

$$H = \sqrt{18^2 - (12\sqrt{2})^2} = \sqrt{324 - 288} = \sqrt{36} = 6 \text{ см}.$$

J a v o b: 6 sm.



254- rasm.



a) *b)*

255- rasm.

4.4. Piramida va kesik piramidaning hajmi. Istalgan piramidaning hajmi asosining yuzi bilan balandligi ko'paytmasining uchdan biriga teng (255- *a* rasm):

$$V_{SABCDE} = \frac{1}{3} S_{ABCDE} \cdot H.$$

Istalgan kesik piramidaning hajmi shunday uchta piramidaning hajmiga tengki, ularning balandliklari kesik piramidaning balandligiga teng bo'lib, ulardan birining asosi kesik piramidaning ostki asosiga, ikkinchisi esa ustki asosiga, uchinchisiniki esa ikkala asos yuzlarining o'rta geometrigiga teng (255- *b* rasm).

$$V_{\text{k.piram.}} = \frac{1}{3} H_1 S_1 + \frac{1}{3} H_1 S_2 + \frac{1}{3} H_1 \sqrt{S_1 \cdot S_2} = \frac{1}{3} H_1 (S_1 + S_2 + \sqrt{S_1 \cdot S_2}),$$

bunda H_1 – kesik piramidaning balandligi.

$SABDE$ va $SA_1B_1D_1E_1$ piramidalarning o'xshashligidan ularning hajmlarining nisbati bir-biriga mos chiziqli elementlar kublarining nisbatiga tengdir, ya'ni:

$$\frac{V}{V_1} = \frac{|AB|^3}{|A_1B_1|^3} = \frac{|BC|^3}{|B_1C_1|^3} = \ldots = \frac{H^3}{H_1^3}.$$

3- m a s a l a. Muntazam to'rtburchakli piramidaning balandligi 9 ga, diagonal kesimining yuzi 36 ga teng. Piramidaning hajmini toping.

Y e c h i l i s h i. Piramida asosining tomonini x desak, uning diagonali $x\sqrt{2}$ bo'ladi. $36 = \frac{1}{2} \cdot 9 \cdot x\sqrt{2}$; bundan $x = 4\sqrt{2}$. $S_{\text{asos}} = x^2 = 32$.

$V_{\text{pir}} = \frac{1}{3} \cdot 32 \cdot 9 = 96$.

J a v o b: 96.

4- m a s a l a. Kesik piramida asoslarining yuzlari 96 va 24 ga, unga mos keluvchi butun piramidaning balandligi 16 ga teng. Kesik piramidaning hajmini toping.

Y e c h i l i s h i. Kesik piramidaning balandligini x desak, $\frac{V}{V_1}=\frac{H^3}{H_1^3}\Rightarrow\frac{V}{V_1}=\frac{16}{x^3}$; $V=\frac{1}{3}\cdot96\cdot16=32\cdot16$; $V_1=\frac{1}{3}\cdot24\cdot x=8x$; $\frac{32\cdot16}{8x}=$

$=\frac{16^3}{x^3}\Rightarrow x^2=64\Rightarrow x=8$. Demak, $V_1=64$.

$V_{\text{kesik pir}}=V-V_1=32\cdot16-64=448.$

J a v o b: 448.

Mustaqil ishlash uchun test topshiriqlari

1. 60° li ikki yoqli burchakning yoqlarida yotgan A va B nuqtalardan burchakning qirrasiga AA_1 va BB_1 perpendikular tushirilgan. Agar $|AA_1|=2, |BB_1|=3, |A_1B_1|=4$ bo'lsa, $|AB|$ ni toping.

A) 5; B) $\sqrt{20}$; C) $\sqrt{23}$; D) 6; E) $\sqrt{24}$.

2. Ikki yoqli burchakning biror yog'ida olingan A nuqtadan uning ikkinchi yog'igacha bo'lgan masofa A nuqtadan uning qirrasigacha bo'lgan masofadan ikki marta kichik bo'lsa, ikki yoqli burchakning kattaligini aniqlang.

A) 30°; B) 45°; C) 60°; D) 75°; E) 90°.

3. Kattaligi 100° bo'lgan ikki yoqli burchak yoqlariga perpendikular bo'lgan to'g'ri chiziqlar orasidagi burchakni toping.

A) 70°; B) 75°; C) 80°; D) 90°; E) 100°.

4. Ikki yoqli burchakning kattaligi 45°. Uning bir yog'idagi B nuqta qirrasidan 8 sm masofada bo'lsa, ikkinchi yog'idan qancha uzoqlikda bo'ladi?

A) $4\sqrt{2}$; B) $4\sqrt{3}$; C) 5; D) $5\sqrt{2}$; E) $5\sqrt{3}$.

5. Uch yoqli burchakning ikki tekis burchagi 70° va 80° bo'lsa, uchinchi tekis burchagi kattaligining chegaralarini yozing.

A) (0°; 150°]; B) (10°; 150°); C) (10°; 180°);

D) (180°; 150°); E) [0°; 150°].

6. Uch yoqli burchakning barcha tekis burchaklari to'g'ri burchak. Shu uch yoqli burchak ikki yoqli burchaklarining kattaligini toping.

A) 30°; B) 45°; C) 60°; D) 75°; E) 90°.

7. 1) 125°, 120°, 115°, 2) 80°, 50°, 30° va 3) 100°, 120°, 10°. Qaysi uchlik biror uch yoqli burchak tekis burchaklarining kattaliklari bo'lishi mumkin?

A) 1; B) 2; C) 3; D) 1 va 2; E) hech biri bo'la olmaydi.

8. Uchala tekis burchagi 60° dan bo'lgan uch yoqli burchakning bir qirrasida, uchidan a masofada bo'lgan nuqta belgilangan. Shu nuqtadan qarshisidagi yoqqacha bo'lgan masofani toping.

A) $\frac{a\sqrt{6}}{4}$; B) $\frac{a\sqrt{2}}{3}$; C) $\frac{a\sqrt{3}}{4}$; D) $\frac{a\sqrt{6}}{3}$; E) $a\sqrt{2}$.

9. Uch yoqli burchakning ikki tekis burchagi 60° dan bo'lib, uchinchisi 90°ga teng. Shu tekis burchagiga uning qarshisidagi qirraning og'ish burchagini toping.

A) 30°; B) 45°; C) 60°; D) 75°; E) 90°.

10. Ikki yoqli burchakning qirrasi bitta bo'lsa, yoqlari soni n ta bo'lgan ko'pyoqli burchakning qirralari soni nechta?

A) $\frac{n}{2}$ ta; B) $(n-1)$ ta; C) n ta; D) $n+1$ ta; E) $2n$ ta.

11. Kubning barcha qirralari uzunliklarining yig'indisi 48 ga teng. Kub sirtining yuzini toping.

A) 96; B) 24; C) 36; D) 48; E) 55.

12. Diagonali $\sqrt{3}$ ga teng bo'lgan kub sirtining yuzini toping.

A) 6; B) 3; C) 9; D) 4,5; E) 2.

13. Tomoni 4 ga teng bo'lgan kubning uchidan shu uch bilan umumiy nuqtaga ega bo'lmagan yog'idan simmetriya markazigacha bo'lgan masofani toping.

A) $2\sqrt{6}$; B) $2\sqrt{3}$; C) $2\sqrt{3}$; D) 3; E) 2.

14. To'rtburchakli muntazam prizma asosining yuzi 144 sm^2, balandligi 14 sm. Shu prizmaning diagonalini toping.

A) 18; B) 22; C) 16; D) $14\sqrt{2}$; E) $16\sqrt{2}$.

15. To'g'ri prizmaning balandligi 50 ga, asosining tomonlari 13, 37 va 40 ga teng. Prizmaning to'la sirtini toping.

A) 2730; B) 3900; C) 4500; D) 4740; E) 4980.

16. Muntazam to'rtburchakli prizma asosining tomoni 4 ga, balandligi $4\sqrt{6}$ ga teng. Prizmaning diagonali asos tekisligi bilan qanday burchak hosil qiladi?

A) 30°; B) 45°; C) 35°; D) 75°; E) 60°.

17. Uchburchakli to'g'ri prizma asosining tomonlari 36, 29 va 25 ga, to'la sirti esa 1620 ga teng. Prizmaning balandligini toping.

A) 20; B) 12,6; C) 10; D) 18; E) 15.

18. To'g'ri parallelepiped asosining tomonlari 8 va 4 ga teng bo'lib, ular 60° li burchak tashkil etadi. Parallelepipedning kichik diagonali $8\sqrt{3}$ ga teng bo'lsa, shu diagonalning asos tekisligi bilan tashkil etgan burchagini toping.

A) 60°; B) 30°; C) arctg2; D) $\arccos\frac{1}{\sqrt{3}}$; E) 45°.

19. Og'ma prizmaning yon qirrasi 20 ga teng va asos tekisligi bilan 30° burchak hosil qiladi. Prizmaning balandligini toping.

A) 12; B) $10\sqrt{3}$; C) 10; D) $10\sqrt{2}$; E) 15.

20. Kubning barcha qirralari yig'indisi 96. Kubning hajmini toping.

A) 256; B) 216; C) 384; D) 64; E) 512.

21. Kub yog'ining yuzi 2 marta orttirilsa, uning hajmi necha marta ortadi?

A) 2; B) 8; C) 4; D) $\sqrt{8}$; E) $\sqrt[3]{4}$.

22. To'g'ri parallelepiped asosining tomonlari $2\sqrt{2}$ va 5 sm bo'lib, o'zaro 45° li burchak tashkil etadi. Parallelepipedning kichik diagonali 7 sm. Uning hajmi qancha?

A) 60 sm³; B) 120 sm³; C) 80 sm³; D) 90 sm³; E) 100 sm³.

23. Uchburchakli to'g'ri prizmaning barcha qirralari bir xil uzunlikka ega, to'la sirti esa $8+16\sqrt{3}$ ga teng. Prizma asosiinng yuzini toping.

A) 4; B) $2\sqrt{6}$; C) $2\sqrt{3}$; D) 3; E) 8.

24. Piramidaning asosi tomonlari 6 va 8 ga teng bo'lgan to'g'ri to'rtburchakdan iborat. Piramidaning har bir yon qirrasi $5\sqrt{5}$ ga teng bo'lsa, balandligini toping.

A) 5; B) 10; C) 100; D) 25; E) 20.

25. Piramidaning asosi to'g'ri burchakli uchburchak bo'lib, gipotenuzasining uzunligi 10 ga teng. Piramidaning yon qirralari 13 ga teng bo'lsa, balandligini toping.

A) 11; B) 12; C) 10; D) 13; E) 9.

26. Piramidaning asosi gipotenuzasi uzunligi 2 ga teng bo'lgan to'g'ri burchakli uchburchakdan iborat. Piramidaning yon qirralari asos tekisligi bilan α burchak tashkil qiladi. Agar uning balandligi 5 ga teng bo'lsa, $tg\alpha$ ning qiymatini toping.

A) 1; B) 2; C) 3; D) 4; E) 5.

27. Muntazam to'rt burchakli piramidaning balandligi 6 sm, apofemasi 6,5 sm. Piramida asosining perimetrini toping.

A) 10; B) 12; C) 24; D) 20; E) 8.

28. Muntazam piramida yon sirtining yuzi 48 ga, apofemasi 8 ga teng. Piramida asosining perimetrini toping.

A) 6; B) 12; C) 8; D) 10; E) 14.

29. Muntazam to'rtburchakli piramidaning balandligi 24 ga, asosining tomoni 14 ga teng. Uning apofemasini toping.

A) 18; B) 27; C) 25; D) 21; E) 28.

30. Muntazam tetraedrning uchrashmaydigan (ayqash) qirralari orasidagi burchakni toping.

A) 60°; B) 90°; C) 45°; D) 120°; E) aniqlab bo'lmaydi.

31. Muntazam piramidaning asosi ichki burchaklarining yig'indisi 720° ga, tomoni 6 ga teng bo'lgan ko'pburchakdan iborat. Agar piramidaning yon qirrasi 10 ga teng bo'lsa, piramidaning balandligini aniqlang.

A) 8; B) 6; C) 9; D) 7; E) 6,2.

32. Muntazam to'rtburchakli piramidaning uchidagi tekis burchagi 60° ga teng. Shu piramidaning yon qirrasi va asosi orasidagi burchakni toping.

A) 15°; B) 30°; C) 45°; D) 60°; E) 75°.

33. Apofemasi 5 ga teng bo'lgan muntazam to'rtburchakli piramidaning to'la sirti 11 dan katta va 24 dan kichik. Piramida asosi tomonining uzunligi qanday oraliqda yotadi?

A) (0,5; 1,5); B) (1; 2); C) (1,5; 2,5); D) (2; 3); E) (1; 3).

34. Qirrasi 6 ga teng bo'lgan kub ustki asosining markazi quyi asosning uchlari bilan tutashtirilgan. Hosil bo'lgan piramidaning yon sirtini toping.

A) $36\sqrt{5}$; B) $18\sqrt{5}$; C) $48\sqrt{3}$; D) $36\sqrt{3}$; E) $72\sqrt{5}$.

35. Uchburchakli piramidaning yon qirralari o'zaro perpendikular hamda mos ravishda 4; 6 va 8 ga teng. Piramidaning hajmini toping.

A) 64; B) 48; C) 32; D) 24; E) aniqlab bo'lmaydi.

36. Piramidaning asosidagi barcha ikki yoqli burchakli 60°ga teng. Piramida yon sirtining yuzi 36 ga teng bo'lsa, asosining yuzi qanchaga teng bo'ladi?

A) 36; B) $18\sqrt{2}$; C) $18\sqrt{3}$; D) 18; E) 24.

37. Hajmi $8\sqrt{3}$ ga teng bo'lgan tetraedrning balandligini toping.

A) 3; B) 4; C) $2\sqrt{3}$; D) $3\sqrt{3}$; E) $4\sqrt{3}$.

38. Piramidaning balandligi 8 ga teng. Piramida uchidan 4 ga teng masofada asosga parallel tekislik o'tkazilgan. Hosil bo'lgan kesim yuzi 27 ga teng bo'lsa, piramida hajmining uning balandligiga nisbatini aniqlang.

A) 48; B) 21; C) 92; D) 54; E) 36.

39. To'g'ri parallelepipedning asosi diagonallarining nisbati 2 : 5 bo'lgan rombdan iborat. Parallelepipedning diagonallari 10 va 17 ga teng. Parallelepipedning hajmini toping.

A) 240; B) 300; C) 360; D) 480; E) 720.

40. Muntazam oltiburchakli piramidaning hajmi 13,5 ga, balandligi $\sqrt{3}$ ga teng. Piramida yon qirrasining asos tekisligi bilan tashkil qilgan burchakni toping.

A) 60°; B) 45°; C) 30°; D) $\operatorname{arctg}\frac{2}{3}$; E) $\operatorname{arctg}\frac{3}{4}$.

41. Muntazam to'rtburchakli kesik piramidaning diagonallari o'zaro perpendikular va ularning har biri 8 ga teng. Piramidaning balandligini toping.

A) 6; B) $4\sqrt{2}$; C) 4; D) $3\sqrt{2}$; E) $2\sqrt{2}$.

42. To'g'ri prizmaning asosi teng yonli uchburchak bo'lib, uning asosi 6 ga va asosga yopishgan burchakning sinusi 0,6 ga teng. Agar prizma asoslari yuzlarining yig'indisi uning yon sirtiga teng bo'lsa, prizmaning hajmini toping.

A) 5,75; B) 6,75; C) 7,2; D) 7,5; E) 8,2.

43. Uchburchakli piramidaning asosi tomonlari 4,4 va 2 ga teng bo'lgan uchburchakdan iborat. Piramidaning barcha yon yoqlari asos tekisligi bilan 60°li burchak tashkil etadi. Piramidaning hajmini toping.

A) 6; B) $2\sqrt{3}$; C) 3; D) $\sqrt{3}$; E) 1,5.

44. Muntazam to'rtburchakli piramidaning yon qirrasi $3\sqrt{2}$ ga, yon qirra va asos tekisligi orasidagi burchak 45°ga teng. Piramidaning hajmini toping.

A) 24; B) 18; C) $15\sqrt{2}$; D) $12\sqrt{2}$; E) $9\sqrt{2}$.

45. Piramidaning asosi to'g'ri burchakli uchburchakdan iborat. Uchburchakning katetlari 3 va 4 ga teng. Piramidaning yon yoqlari asos tekisligi bilan 60°li burchaklar hosil qiladi. Piramidaning to'la sirtini toping.

A) 15; B) 18; C) 20; D) 24; E) 30.

46. Muntazam to'rtburchakli piramidaning balandligi 8 ga, asosining tomoni 12 ga teng. Piramidaning yon yog'iga parallel holda asosining markazi orqali o'tgan kesimning yuzini hisoblang.

A) 45; B) 30; C) 50; D) 60; E) 72.

47. Oktaedrning qirrasi a ga teng. Uning to'la sirtini hisoblang.

A) $2a^2\sqrt{3}$; B) $a^2\sqrt{3}$; C) $\frac{2\sqrt{3}}{3}a^2$; D) $4a^2\sqrt{3}$; E) $\frac{\sqrt{3}}{3}a^2$.

48. $ABCDA_1B_1C_1D_1$ parallelepiped ostki asosining DB diagonali va ustki aosining unga qarama-qarshi uchi orqali tekislik o't-kazilgan. Bu tekislik parallelepipeddan C_1DBC piramidani ajratadi. Berilgan parallelepiped hajmining C_1DBC piramida hajmiga nisbatini toping.

A) 3:1; B) 4:1; C) 5:1; D) 6:1; E) 9:1.

49. Uchburchakli piramidaning asosidagi barcha ikki yoqli burchaklar 30°ga teng. Agar piramidaning balandligi 6 ga teng bo'lsa, uning asosiga ichki chizilgan doiraning radiusini toping.

A) $6\sqrt{3}$; B) 6; C) $2\sqrt{3}$; D) 3; E) 2.

50. Uchburchakli piramida asosining tomonlari 6; 8 va 10 ga teng. Piramidaning yon qirralari asos tekisligi bilan bir xil burchak hosil qiladi. Agar piramidaning balandligi 4 ga teng bo'lsa, yon qirrasi qanchaga teng bo'ladi?

A) $\sqrt{41}$; B) 3; C) 4; D) 5; E) $2\sqrt{10}$.

51. Muntazam uchburchakli prizmaning hajmi 16 ga teng. Asosidagi tomonning uzunligi qanday bo'lganda, prizmaning to'la sirti eng katta bo'ladi?

A) 2; B) 3; C) 4; D) 6; E) $3\sqrt{2}$.

52. Muntazam tetraedrning qirrasi 1 ga teng. Uning asosiga tashqi chizilgan aylananing markazidan uning yon yog'igacha bo'lgan masofani toping.

A) $\frac{2\sqrt{3}}{6}$; B) $\frac{\sqrt{6}}{9}$; C) $\frac{2\sqrt{2}}{5}$; D) $\frac{3\sqrt{2}}{8}$; E) $\frac{5\sqrt{6}}{6}$.

X bob. AYLANISH SIRTLARI VA JISMLARI

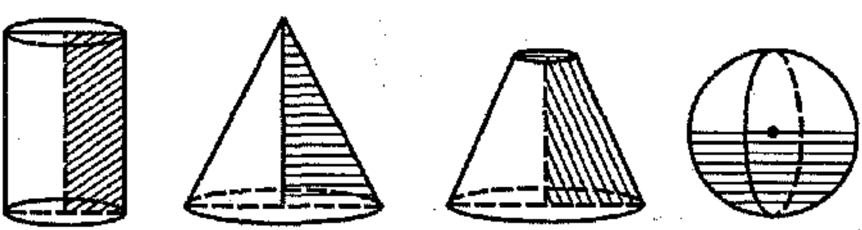
O'quvchilarga ma'lum bo'lgan silindr, konus, shar kabi geometrik shakllar umuman *aylanish jismlari* deb ataladi, sababi ularning har birini biror to'g'ri chiziq (o'q) atrofida biror yassi shaklni aylantirish natijasida «hosil bo'ladigan» shakl sifatida tasavvur etish mumkin. Masalan, to'g'ri to'rtburchakni simmetriya o'qi yoki biror tomoni atrofida aylantirib *to'g'ri doiraviy silindr* (256- *a* rasm), to'g'ri burchakli uchburchakni biror kateti atrofida aylanishidan *to'g'ri doiraviy konus* (256- *b* rasm), to'g'ri burchakli trapetsiyani kichik yon tomoni atrofida aylantirib, *kesik konus* (256- *d* rasm), yarim doirani diametri atrofida aylantirish bilan *shar* hosil qilish mumkin (256- *e* rasm).

Quyida aylanish jismlariga oid zarur tushunchalar tizimi bayon etiladi.

Ta'rif. *Biror to'g'ri chiziq (o'q) atrofida ikkinchi chiziqning aylanishidan hosil bo'lgan sirt aylanish sirti deyiladi* (257- *rasm*). *Bu holda o'q atrofida aylanadigan chiziq sirtning yasovchisi deb ataladi.*

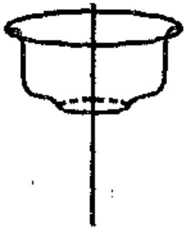
Ba'zi sodda aylanish sirtlarini ko'ramiz.

1) O'zaro ikki parallel to'g'ri chiziq biri ikkinchisi atrofida aylanib *silindrik sirt* hosil qiladi (258- rasm).

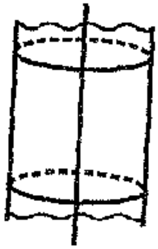


a) *b*) *d*) *e*)

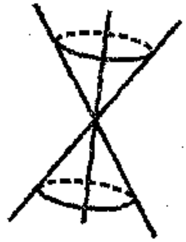
256- rasm.



257- rasm.



258- rasm.



259- rasm.

260- rasm.

2) Bir-biri bilan kesishuvchi ikki to'g'ri chiziqdan biri ikkinchisi atrofida aylanishi natijasida *konus sirt* hosil bo'ladi (259- rasm).

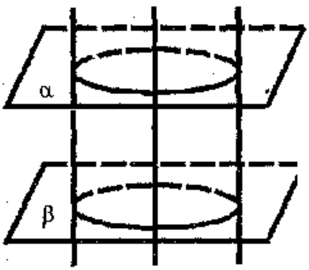
3) Yarim aylana (yoki aylana) o'z diametri atrofida aylanishidan sfera hosil bo'ladi (260- rasm).

Silindrik sirtni o'qiga perpendikular ikki tekislik bilan kesilsa, tekisliklar orasida hosil bo'lgan sirt *to'g'ri doiraviy silindrik sirt* deb ataladi (261- rasm).

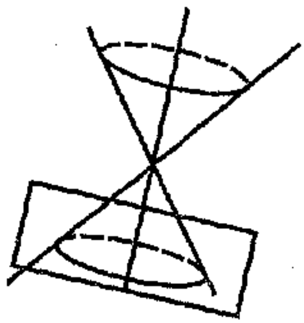
Konus sirt yasovchilarining kesishish nuqtasi sirtning *uchi* deyiladi. Konus sirtining o'qiga perpendikular tekislik uning uchidan kesuvchi tekislik tarafida to'g'ri doiraviy konus sirtini ajratadi (262- rasm).

Fazoning aylanish sirti bilan chegaralangan qismi *aylanish jismi* deyiladi.

Aylanish jismlarining o'ziga xos xususiyatlariga qarab, ularga boshqacha ta'rif berish mumkin.



261- rasm.



262- rasm.

1-§. Silindr

1.1. Silindrning ta'rifi. *Silindr deb o'zaro parallel tekisliklarda yotuvchi bir-biriga teng ikki doiraning mos nuqtalarini tutashtiruvchi o'zaro parallel barcha kesmalardan tashkil topgan jismga aytiladi* (263-*a rasm*).

Agar kesmalar doiralar yotgan tekisliklarga perpendikular bo'lsa, hosil bo'lgan silindr *to'g'ri silindr* deyiladi (263- *b* rasm).

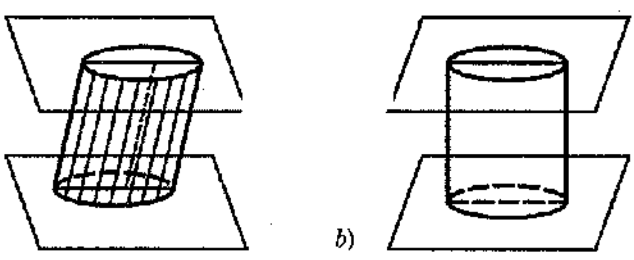
Doiralar silindrning *asoslari* deyiladi. To'g'ri silindr asoslari markazidan o'tuvchi to'g'ri chiziq silindrning *o'qi* deyiladi. Silindr o'qiga parallel bo'lib, asos aylanalarining nuqtalarini tutashtiruvchi kesma silindrning *yasovchisi* deb ataladi. Silindrning barcha yasovchilari o'zaro teng va paralleldir.

Maktab geometriyasida faqat to'g'ri silindr xossalari, ular bilan bog'liq masalalar o'rganiladi. Shu bois kelgusida to'g'ri silindr atamasi qisqa qilib *silindr* deb ataladi.

Silindrning sirti uning yon sirti va asos doiralarining yuzidan iboratdir.

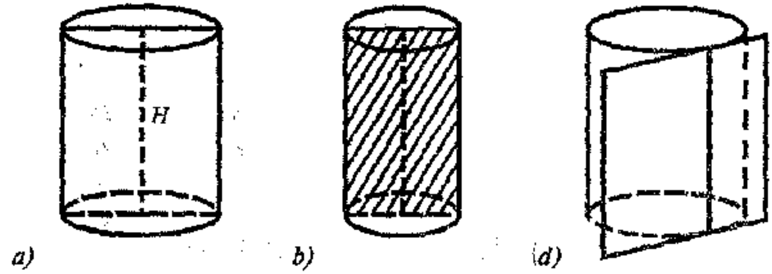
Silindr asosining radiusi ba'zida silindr radiusi (R — radius) deyiladi. Asoslari orasidagi masofa esa silindrning balandligi hisoblanadi (264- *a* rasm, H — balandlik).

Silindrni tekislik bilan turlicha kesib, kesimda doira, ellips, to'g'ri to'rtburchak va hokazo shakllarni hosil qilish mumkin. Silindr asosiga parallel tekislik uning yon sirtini asos aylanasiga teng aylana bo'yicha kesadi. Silindrni ixtiyoriy ikki yasovchisi orqali o'tib, kesuvchi tekislik kesimda to'g'ri to'rtburchak hosil qiladi.



b)

263- rasm.



264- rasm.

Silindrning o'qi orqali o'tgan tekislik bilan kesimi silindrning *o'q kesimi* deyiladi (264- *b* rasm).

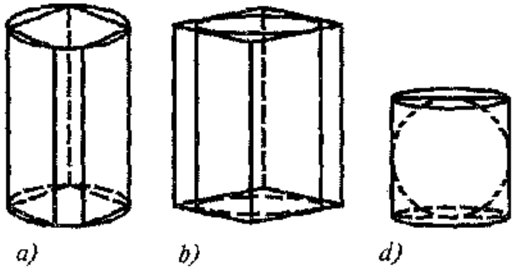
Agar tekislik silindr yon sirti bilan bitta umumiy kesmaga ega bo'lsa, bunday tekislik silindr yon sirtiga *urinuvchi tekislik* deb ataladi (264- *d* rasm).

Silindrga ichki chizilgan prizma deb shunday prizmaga aytiladiki, unda prizmaning asos tekisliklari silindrning asos tekisliklari bilan, prizmaning yon qirrasi esa silindr yasovchisi bilan ustma-ust tushadi (265- *a* rasm).

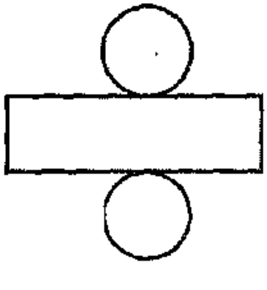
Silindrga tashqi chizilgan prizma deb, asoslari silindr asoslari bilan ustma-ust tushgan, yon yoqlari silindr yon sirtiga urinuvchi prizmaga aytiladi (265- *b* rasm).

Silindrga ichki chizilgan shar deb silindrning ikkala asosiga va barcha yasovchilariga urinuvchi sharga aytiladi (265- *d* rasm).

Silindr sirtini asos aylanalari va biror yasovchisi bo'yicha qirqib, uning tekislikdagi yoyilmasini hosil qilish mumkin (266- rasm).



265- rasm.



266- rasm.

1-masala. Balandligi h bo'lgan to'g'ri silindr o'q kesimining yuzi S ga teng bo'lsa, silindr asosining yuzini toping.

Yechilishi. Silindr asosining radiusi R desak, $2R \cdot h = S$; $R = \frac{S}{2h}$. Demak, $S_{asos} = \pi R^2 = \pi \frac{S^2}{4h^2}$.

Javob: $S_{asos} = \pi \frac{S^2}{4h^2}$.

1.2. Silindr sirtining yuzi.

Ta'rif. *Silindr sirtining yuzi unga ichki (tashqi) chizilgan muntazam prizma yoqlarini cheksiz ikkilantirish natijasida prizma sirtining yuzi intilgan songa aytiladi.*

Teorema. Silindr yon sirtining yuzi asos aylanasi uzunligi bilan balandligining ko'paytmasiga teng.

Silindr yon sirtining yoyilmasi o'lchamlari $2\pi R$ (asos aylanasi uzunligi) va H(silindr balandligi) bo'lgan to'g'ri to'rtburchak ekanligidan uning yuzi $2\pi R \cdot H$ ga teng. Demak, $S_{yon} = 2\pi R \cdot H$, bunda R — silindr radiusi, H — balandligi.

Silindr to'la sirtining yuzini hisoblash uchun uning yon sirtiga ikki asosining yuzlari qo'shiladi:

$$S_{to'la} = S_{yon} + 2S_{asos} = 2\pi R \cdot H + 2\pi R^2 = 2\pi R(H + R),$$

demak, $S_{to'la} = = 2\pi R(H + R)$.

2-masala. Radiusi 2,5, o'q kesimining diagonali 13 bo'lgan silindr sirtining yuzini toping.

Yechilishi. 267- rasmda masala shartiga ko'ra silindrning $ABCD$ o'q kesimida $AD = 2R = 5$, $AC = 13$.

ADC to'g'ri burchakli uchburchakda

$$CD = \sqrt{AC^2 - AD^2} = \sqrt{169 - 25} = 12.$$

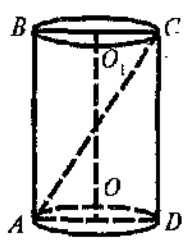
Demak, $AD = OO_1 = H = 12$. Endi silindr sirtining yuzini topamiz:

$S_{silindr} = 2\pi R(H + R) = 2\pi \cdot 2,5\ (12 + 2,5) = 72,5\,\pi$.

Javob: $72,5\pi$.

1.3. Silindrning hajmi.

Ta'rif. *Silindrning hajmi deb, unga ichki (tashqi) chizilgan muntazam prizmaning yoqlari soni cheksiz ikkilantirilganda uning hajmi intilgan songa aytiladi.*



267- rasm.

T e o r e m a. Silindr hajmi asosining yuzi bilan balandligi ko‘paytmasiga teng:

$$V_s = S_{as} \cdot H = \pi R^2 \cdot H.$$

3- m a s a l a. Silindrning hajmi 48π ga, yon sirtining yuzi esa 24π ga teng bo‘lsa, uning radiusi va balandligini toping.

Y e c h i l i s h i. Masala shartiga ko‘ra

$$\begin{cases} \pi R^2 \cdot H = 48\pi, \\ 2\pi R \cdot H = 24\pi \end{cases}$$

sistema hosil bo‘ladi. Sistemadagi birinchi tenglamani ikkinchisiga bo‘lib, $R = 4$ ekanini aniqlaymiz. Buni tenglamalardan biriga qo‘yib, $H = 3$ ni topamiz.

J a v o b: $R = 4$, $H = 3$.

4- m a s a l a. Ikki turli silindrning o‘q kesimlari tomonlari 4 va 6 bo‘lgan to‘g‘ri to‘rtburchakdan iborat bo‘lsa, ulardan qaysi birining sirti katta bo‘ladi?

Y e c h i l i s h i. Ma’lumki, radiusi R, balandligi h bo‘lgan silindr sirti:

1) agar $R = 2$ va $h = 6$ bo‘lsa, u holda $S = 32\pi$;

2) agar $R = 3$, $h = 4$ bo‘lsa, u holda $S = 42\pi$.

Demak, radiusi 3 bo‘lgan silindrning sirti katta bo‘ladi.

J a v o b: Radiusi 3 ga teng bo‘lgan silindrning sirti katta.

5- m a s a l a. To‘la sirtining yuzi 24π ga teng silindrning hajmi ko‘pi bilan qanchaga teng bo‘lishi mumkin?

Y e c h i l i s h i. Masala shartiga ko‘ra

$$S_{to‘la} = 2\pi R(H + R) = 24\pi,$$

bundan

$$H = \frac{12 - R^2}{R},$$

$$\pi R\, H = \pi R^2 \cdot \frac{12 - R^2}{R} = \pi(12R - R^3).$$

$R = x$ desak, $V_{silindr} = \pi(12x - x^3)$, silindrning hajmi x ning funksiyasi sifatida ifodalanadi:

$$V_{silindr} = V(x).$$

Bu funksiyani ekstremumga tekshiramiz:

$$V'(x) = \pi(12 - 3x^2), \quad V'(x) = 0.$$

Bunda $x < 2$ da $V'(x) > 0$, $x > 2$ da $V'(x) < 0$, binobarin $x = 2$ da funksiya eng katta qiymatga erishadi.

Shunday qilib, silindrning eng katta hajmi $R = 2$ da 16π ga teng bo'ladi.

Javob: 16π.

Geometriyaning shu turkumdagi ekstremal masalalarini yechishga klassik tengsizliklarning tatbiqi kitobning 1- ilovasida (216- bet) keltirilgan.

2-§. Konus

2.1. Konusning ta'rifi. Konus sirtni, umuman, konusni turlicha ta'riflash mumkin. Quyida shulardan eng soddasini beramiz.

Ta'rif. *To'g'ri burchakli uchburchakning biror kateti atrofida aylanishidan hosil bo'lgan jism konus deyiladi* (268- *rasm*).

268- rasmda ABC to'g'ri burchakli uchburchakning AC kateti atrofida aylanishidan hosil bo'lgan konus tasvirlangan. Bunda AC katet *konusning o'qi* va shu bilan birga *konus balandligi* hisoblanadi. Konus asosini hosil qiluvchi ikkinchi AB katet konus asos doirasining radiusi, ba'zida *konus radiusi* deyiladi. Konus uchi C ni asos aylanasining nuqtasi bilan tutashtiruvchi CB gipotenuza *konusning yasovchisi* deb ataladi. Konus yasovchisi bilan asos radiusi orasidagi burchak konusning *qiyalik burchagi* deyiladi.

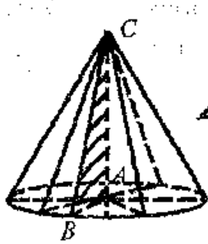
1-masala. Radiusi r bo'lgan konusning yasovchisi bilan balandligi orasidagi burchak α ga teng bo'lsa, uning balandligi va yasovchisini toping.

Yechilishi. Konus yasovchisini l, balandligini h bilan belgilasak, to'g'ri burchakli uchburchakdagi metrik munosabatdan foydalanib,

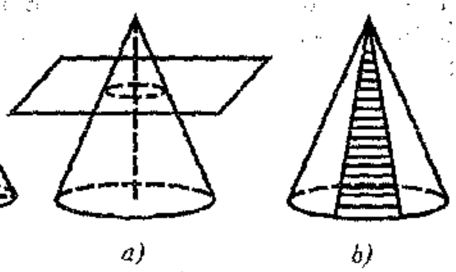
$l = \frac{r}{\sin\alpha}$; $h = r\,\mathrm{ctg}\alpha$ ekanligiga oson ishonch hosil qilish mumkin.

Javob: $l = \frac{r}{\sin\alpha}$; $h = r\,\mathrm{ctg}\alpha$.

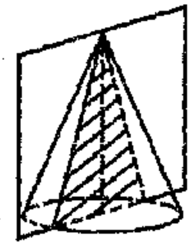
2.2. Konusning kesimlari. Konusni tekislik bilan turlicha kesib, kesimda: teng yonli uchburchak, doira, ellips, parabola va hokazolarni hosil qilish mumkin (269- rasm). Xususan: a) konus asosiga parallel tekislik bilan kesilsa, kesimda doira; b) ikki yasovchi orqali o'tuvchi tekislik bilan kesimda teng yonli uchburchak; d) konusning o'qi orqali o'tgan kesuvchi tekislik bilan kesimda eng katta teng yonli uchburchak hosil bo'lishini ko'rish mumkin (270- rasm).



268- rasm.



269- rasm.



270- rasm.

Konusning bitta yasovchisi orqali o'tuvchi tekislik konusning yon sirtiga *urinuvchi tekislik* deyiladi va bu tekislik shu yasovchisi tegishli bo'lgan o'q kesimiga perpendikular bo'ladi.

Konusga ichki chizilgan piramida deb, uchi konus uchida, asosidagi ko'pburchak konus asosiga ichki chizilgan piramidaga aytiladi. Bu holda piramidaning har bir yon qirrasi konusning yasovchisi bilan ustma-ust tushadi.

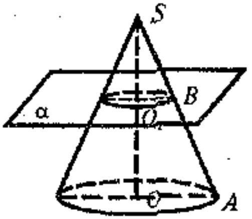
Konusga tashqi chizilgan piramida deb, uchi konus uchida, asosidagi ko'pburchak konus asosiga tashqi chizilgan piramidaga aytiladi. Bu holda piramidaning har bir yog'i konus yon sirtiga urinuvchi bo'lishiga e'tibor bering.

2-masala. Radiusi 4 bo'lgan konus asosiga parallel va balandligining o'rtasidan o'tuvchi tekislik bilan kesilgan. Kesimda hosil bo'lgan doira yuzi konus asosi yuzidan necha marta kichik bo'ladi?

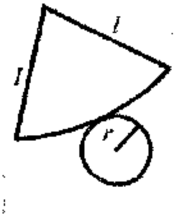
Yechilishi. Balandlik o'rtasidan asosiga parallel ravishda o'tkazilgan α tekislik konusdan o'ziga o'xshash konus ajratadi (271-rasm). Bu konuslarning o'q kesimlari o'xshash teng yonli uchburchaklar bo'lib, $SO \perp AO$, $SO_1 \perp BO_1$. Masala shartiga ko'ra $SO_1 = \frac{1}{2}SO$.

O'xshash shakllarda chiziqli o'lchamlar nisbati o'xshashlik koeffitsiyentiga teng, mos yuzlar nisbati esa bu koeffitsiyentning kvadratiga teng. Shunga ko'ra

$k = \frac{SO_1}{SO} = \frac{1}{2}; \frac{S_1}{S} = \frac{1}{4}$ (S_1 — kichik konus asosining, S — berilgan konus asosining yuzlari).



271- rasm.



272- rasm.

Berilgan konus asosining yuzi $S = \pi R^2 = 16\pi$. Shunday qilib, $\frac{S_1}{16} = \frac{1}{4} \Rightarrow S_1 = 4$. Demak, kesimdagi doira yuzi asos yuzidan 4 marta kichik.

2.3. Konus sirtining yuzi. Konus sirti uning yon sirti bilan asosining yuzi yig'indisiga teng. 272- rasmda konusning tekislikdagi yoyilmasi tasvirlangan. Yasovchisi l va radiusi r bo'lgan konusning yon sirtining yoyilmasi radiusi l, yoyi uzunligi $2\pi r$ bo'lgan sektordan iborat bo'ladi. Bu sektorning yuzi (ya'ni konus yon sirti) l radiusli doira yuzidan necha marta kichik bo'lsa, $2\pi r$ yoy uzunligi $2\pi l$ yoy uzunligidan shuncha marta kichik bo'ladi, ya'ni

$$\frac{\pi l^2}{S_{yon}} = \frac{2\pi l}{2\pi r} \Rightarrow S_{yon} = \frac{\pi l^2 \cdot 2\pi r}{2\pi l} = \pi r l. \text{ Demak, } S_{yon} = \pi r l.$$

So'nggi formulani (ifodani) konusga ichki yoki tashqi chizilgan muntazam piramida tomonlari sonini cheksiz ikkilantirganda piramida yon sirti intilgan son sifatida keltirib chiqarish ham mumkin.

Konusning (to'la) sirtini topish uchun uning yon sirtiga asosining yuzi qo'shiladi:

$$S_{konus} = S_{yon} + S_{as} = \pi r l + \pi r^2 = \pi r(l + r),$$

demak,

$$S_{konus} = \pi r(l + r).$$

3- m a s a l a. Balandligi 4, yasovchisi 5 bo'lgan konus yon sirtining yoyilmasi hosil qilgan sektor burchagini toping.

Y e c h i l i s h i. Konusning radiusi $r = \sqrt{5^2 - 4^2} = 3$ bo'lgani uchun uning asos aylanasining uzunligi $2\pi \cdot 3 = 6\pi$. Bu esa (yoyilma) sektor yoyining uzunligidir. 6π radiusi 5 bo'lgan aylana uzunligidan qancha marta kichik bo'lsa, izlanayotgan sektorning α burchagi 360° dan shuncha marta kichik bo'ladi:

$\alpha : 360^\circ = 6\pi : 10$, bundan $\alpha = 216^\circ$ ekanligi kelib chiqadi.

J a v o b: 216°.

4-m a s a l a. Konusning yasovchisi 8 bo'lib, asos tekisligi bilan 60° burchak tashkil etadi. Konusning sirtini toping.

Y e c h i l i s h i. $S_{konus} = S_{yon} + S_{as} = \pi rl + \pi r^2$, bunda r — konusning radiusi, l — yasovchi; $l = 8, r = \frac{l}{2} = \frac{8}{2} = 4$. Demak, $S_{konus} = \pi r(l + r) =$

$= \pi \cdot 4 \cdot 12 = 48\pi$.

J a v o b: 48π.

2.4. Konusning hajmi.

T e o r e m a. Konusning hajmi asosi yuzining balandligiga ko'paytmasining uchdan biriga teng.

Teoremaning isboti. Konusga ichki va tashqi chizilgan muntazam piramidalarning tomonlari soni cheksiz ikkilantirilganda ularning hajmlari konus hajmiga cheksiz yaqinlashadi. Shuning uchun radiusi r, balandligi H bo'lgan konusning hajmi $\frac{1}{3}\pi r^2 \cdot H$ shaklida ifodalanadi.

Demak, $V_{konus} = \frac{1}{3} S_{as} \cdot H = \frac{1}{3}\pi r^2 H$.

5-m a s a l a. Konusning o'q kesimi tomonlari 40, 40 va 48 bo'lgan uchburchak bo'lsa, konusning hajmini toping.

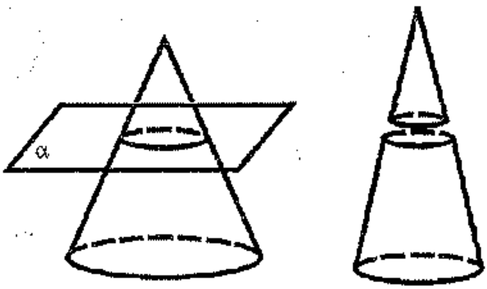
Y e c h i l i s h i. $V_{konus} = \frac{1}{3} S_{as} H$ ni aniqlash uchun r va H ni hisoblaymiz:

$r = \frac{48}{2} = 24$; $H = \sqrt{40^2 - 24^2} = \sqrt{1600 - 576} = \sqrt{1024} = 32$. U holda

$V = \frac{1}{3}\pi \cdot 576 \cdot 32 = 6144\pi$.

J a v o b: 6144π.

2.5. Kesik konus. Biror konus asosiga parallel tekislik bilan kesilsa, u ikkita jismga ajraladi. Ulardan biri berilgan konusga o'xshash kichik konus, ikkinchisi *kesik konus* deb ataladi (273- rasm).



273-rasm.

274-rasm.

Kesik konusning ikki asosi bo'lib, ular o'zaro parallel tekisliklarda yotuvchi doiralardir.

Kesik konus asoslari orasidagi masofa *kesik konusning balandligi* hisoblanadi. Kesik konusni to'g'ri burchakli trapetsiyaning kichik yon tomoni atrofida aylanishidan hosil bo'ladigan aylanish jismi kabi ta'riflash ham mumkin (274- rasm).

Konusni asosiga parallel tekislik bilan kesganda unga ichki va tashqi chizilgan muntazam piramidalar ham kesilib, kesik konusga ichki va tashqi chizilgan muntazam kesik piramidalar hosil bo'ladi. Shuning uchun kesik konusning sirti, hajmini ta'riflashda, ularning ifodalarini keltirib chiqarishda kesik piramidalarning tomonlarini cheksiz ikkilantirganda ularning sirtlari, hajmlari kesik konus sirti va hajmiga juda yaqin bo'ladi. Shularni hisobga olib, ular quyidagicha ifodalanadi:

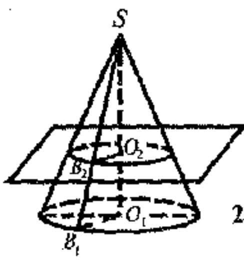
$$S_{k.k.yon} = \pi l(R + r); \quad S_{k.k.} = S_{k.k.yon} + S_{as1} + S_{as2} = \pi l(R + r) + \pi R^2 + \pi r^2;$$

$$V_{k.k.} = \frac{1}{3}\pi H(R^2 + R \cdot r + r^2),$$

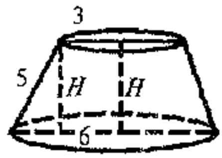
bunda l — kesik konusning yasovchisi, R — katta asosining radiusi, r — kichik asosining radiusi, H — kesik konusning balandligi.

6-masala. Asoslarining radiuslari r_1 va r_2, yasovchisi l bo'lgan kesik konus yon sirtining yuzi $S = \pi l(r_1 + r_2)$ kabi ifodalanishini isbotlang.

Isboti. 275- rasmdagi kesik konus asoslarining radiuslari $|O_1B_1| = r_1$, $|O_2B_2| = r_2$, $|B_1B_2| = l$ yasovchisi bo'lsa, kesik konusning yon sirti yasovchilari SB_1 va SB_2 bo'lgan konuslar yon sirtlarining ayirma-



275- rasm.



276- rasm.

siga tengdir. Agar $|SB_2| = x$ desak, $|SB_1| = x + l$ bo'ladi. $\Delta SB_2O_2 \sim \Delta SB_1O_1$ dan $\frac{x}{r_2} = \frac{x+l}{r_1}$, bundan $x = \frac{lr_2}{r_1 - r_2}$;

$$S_{\text{yon.k.k.}} = \pi r_1(l + x) - \pi r_2 x =$$

$$= \pi\left(r_1 l + \frac{r_1 \cdot r_2 l}{r_1 - r_2} - \frac{r_2^2 l}{r_1 - r_2}\right) = \pi l(r_1 + r_2).$$

Shunday qilib, $S_{\text{yon.k.k.}} = \pi l(r_1 + r_2)$ ekanligi isbotlandi.

7-masala. Yasovchisi 5, asoslarining radiuslari 3 va 6 bo'lgan kesik konusning hajmini toping (276- rasm).

Yechilishi. Ma'lumki, $V_{\text{k.k.}} = \frac{1}{3}\pi(R^2 + R \cdot r + r^2) \cdot H$. Demak,

$V_{\text{k.k.}} = \frac{1}{3}\pi(6^2 + 3 \cdot 6 + 3^2) \cdot 4 = \frac{1}{3}\pi \cdot 63 \cdot 4 = 84\pi.$

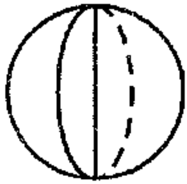
Javob: 84π.

Uchidan H_1 masofada konusni S_1 yuzli doira bo'yicha kesuvchi tekislik konusdan V_1 hajmli konus ajratsa,

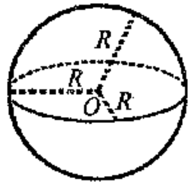
1) $\frac{S_1}{S} = \left(\frac{H_1}{H}\right)^2 = \left(\frac{l_1}{l}\right)^2$; 2) $\frac{V_1}{V} = \left(\frac{H_1}{H}\right)^3 = \left(\frac{l_1}{l}\right)^3$ o'rinli bo'ladi. Bunda S_1, H_1, V_1, l_1 – kichik konus asosining yuzi, balandligi, hajmi, yasovchisi, S, H, V, l berilgan konusning asosi yuzi, balandligi, hajmi va yasovchisi.

3-§. Shar va sfera

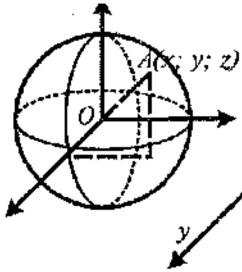
3.1. Shar va sferaning ta'rifi. Shar doiraning, xususan, yarim doiraning o'z diametri atrofida aylanishchidan hosil bo'ladi (277- rasm). Sharning sirti *sfera* deyiladi.



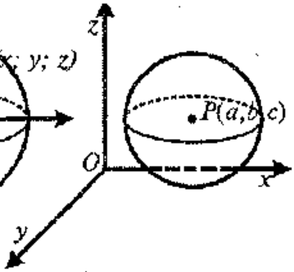
277- rasm.



278- rasm.



279- rasm.



280- rasm.

Ta'rif. *Fazoning biror O nuqtasidan bir xil R masofada bo'lgan barcha nuqtalar to'plami* ***sfera*** *deb ataladi* (278- rasm).

O nuqta *sfera markazi*, *R* masofa esa *sfera radiusi* deyiladi.

Sferaning ixtiyoriy ikki nuqtasini tutashtiruvchi kesma uning *vatari*, sfera markazidan o'tuvchi vatari esa *sferaning diametri* deb ataladi.

Markazi koordinatalar boshida bo'lgan *R* radiusli sferaning ixtiyoriy *A* (*x*; *y*; *z*) nuqtasi uchun $R^2 = x^2 + y^2 + z^2$ munosabat o'rinli. Bu ifoda markazi koordinatalar boshida bo'lgan *R* radiusli sfera tenglamasi hisoblanadi (279- rasm).

Umuman, markazi koordinatali fazoning biror *P*(*a*; *b*; *c*) nuqtasida bo'lgan *R* radiusli sferaning tenglamasi

$$(x-a)^2+(y-b)^2+(z-c)^2=R^2$$

ko'rinishda bo'ladi (280- rasm).

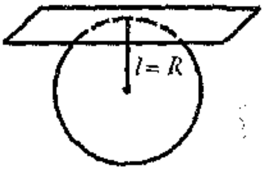
Sferaning tekislik bilan kesimi doimo aylanadan iborat bo'ladi. Sfera markazidan o'tuvchi tekislik uni eng katta aylana bo'yicha kesadi. Xususan, sfera bilan yagona umumiy nuqtaga ega bo'lgan tekislik sferaga shu nuqtada *urinuvchi tekislik* deyiladi (281- rasm).

Sferaga urinuvchi tekislik sfera chegaralagan sharga ham *urinuvchi tekislik* deyiladi, bu holda shar tekislikka urinuvchi shar deb ataladi.

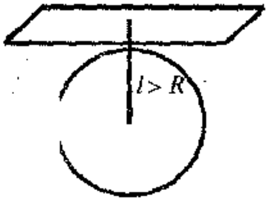
Fazoda *R* radiusli shar tekislik bilan o'zaro turli vaziyatda bo'lishi mumkin. Agar shar markazidan tekislikkacha masofani *l* bilan belgilasak:

1) $l = R$ bo'lsa, ular bitta umumiy nuqtaga ega bo'lib, bu nuqta tekislik bilan sharning *urinish nuqtasi* deyiladi (281- rasm).

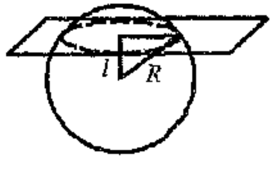
2) $l > R$ bo'lganda tekislik bilan shar umumiy nuqtaga ega bo'lmaydi (282- rasm).



281- rasm.



282- rasm.



283- rasm.

T e o r e m a. ***Urinish nuqtasiga o'tkazilgan radius urinma tekislikka perpendikular bo'ladi*** $(R \perp \alpha)$.

3) Agar $l < R$ bo'lsa, tekislik sharni (sferani) $r = \sqrt{R^2 - l^2}$ radiusli doira (aylana) bo'yicha kesadi (283- rasm). Markazdan o'tuvchi tekislik ($l = 0$) sharni (sferani) eng katta doira (aylana) hosil qilib kesadi.

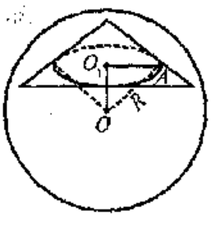
1 - m a s a l a. Uchburchak tekisligi radiusi 4 ga teng bo'lgan sharni uchburchak tomonlari sharga uringan holda kesadi (284- rasm). Shar markazidan uchburchak tekisligigacha bo'lgan masofa 3 ga teng bo'lsa, uchburchakka ichki chizilgan aylananing radiusi nechaga teng bo'ladi?

Y e c h i l i s h i. Kesimda hosil bo'lgan doira uchburchak tomonlariga urinuvchi, uchi shar markazida bo'lgan konus asosidir. Uchburchak tomoniga urinish nuqtasini A, kesim markazini O_1 desak, to'g'ri burchakli uchburchak AO_1O da $|O_1O| = 3$, $R = 4$, AO_1 konus asosi radiusi. $|AO_1| = \sqrt{R^2 - |OO_1|} = \sqrt{4^2 - 3^2} = \sqrt{7}$.

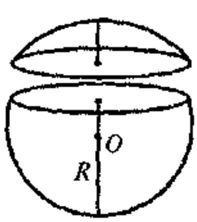
J a v o b: $\sqrt{7}$.

3.2. Shar bo'laklari.

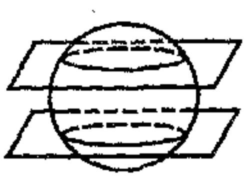
1. R radiusli sharni markazidan l *masofada* biror tekislik bilan kessak ($l < R$ bo'lganda), u asosi doira bo'lgan ikkita *shar segmenti*ga



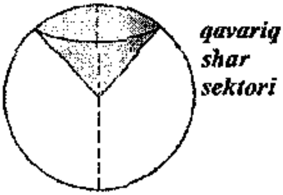
284- rasm.



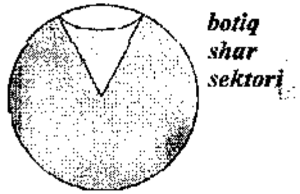
285- rasm.



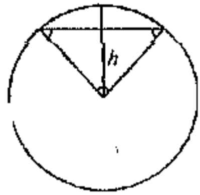
286- rasm.



287- rasm.



288- rasm.



289- rasm.

ajraladi (285- rasm). Kichik shar segmentining balandligi $H_1 = R - l$ bo'lsa, kattasining balandligi $H_2 = R + l$ ga tengdir.

2. Sharni o'zaro parallel ikki tekislik bilan kesganda tekisliklar orasidagi qismi *shar qatlami* deyiladi (286- rasm). Shar qatlamining sirti *shar kamari* deyiladi.

3. Shar segmenti va asosi shu segment asosidan, uchi esa shar markazida joylashgan konusdan tashkil topgan jism *shar sektori* deyiladi (287- rasm).

Doiraviy sektor yoyining gradus o'lchovi 90° dan kichik bo'lsa, qavariq shar sektori hosil bo'ladi. Qavariq shar sektori shar segmenti va konus birlashmasidan tashkil topgan (287- rasm).

Doiraviy sektor yoyining gradus o'lchovi 90° dan katta bo'lsa, botiq shar sektori hosil bo'ladi. U butun shardan qavariq shar sektorini olinganidan iborat (288- rasm).

2-masala. 30° li doiraviy sektorning R radiusi atrofida aylanishidan hosil bo'lgan shar sektoridan ajratilgan shar segmentining balandligini toping (289- rasm).

Yechilishi. Shar sektorining tarkibiy qismi bo'lgan konusning o'q kesimi muntazam uchburchak bo'lgani uchun uning balandligi

$$h = \sqrt{R^2 - \frac{R^2}{4}} = \frac{R\sqrt{3}}{2}; \quad h = \frac{R\sqrt{3}}{2}.$$

Shar segmentining balandligi $H = R - h = R - \frac{R\sqrt{3}}{2} = \frac{2R - R\sqrt{3}}{2} =$

$= \frac{R(2-\sqrt{3})}{2}.$

Javob: $\frac{R(2-\sqrt{3})}{2}$.

3.3. Shar va shar bo'laklarining hajmini va sirtining yuzini topish formulalari.

1. R radiusli sharning hajmi $V = \frac{4}{3}\pi R^3$. Shu shar sirtining (sferaning) yuzi $S = 4\pi R^2$.

2. R radiusli shardan kesilgan H balandlikli shar segmentining hajmi $V = \pi H^2\left(R - \frac{H}{3}\right)$. Sirtining yuzi esa $S = 2\pi RH$.

3. R radiusli sharning bo'lagi bo'lgan H balandlikli sektorning hajmi $V = \frac{2}{3}\pi R^2 \cdot H$ (H — segment balandligi). Bu sektor sirtining yuzini topish uchun sektor konusining yon sirtiga unga mos segment sirti qo'shiladi.

4. R radiusli shardan kesilgan H balandlikli shar qatlami bir doirasining radiusi r_1, ikkinchisiniki r_2 bo'lsa, uning hajmi $V = \frac{1}{6}\pi H^3 + \frac{1}{2}\pi(r_1^2 + r_1^2)H$. Yon sirti esa $S = 2\pi RH$ bo'ladi.

1-masala. Radiuslari 2; 3 va 4 bo'lgan metall sharlar eritilib, bitta shar quyildi. Shu sharning hajmini toping.

Yechilishi. $R_1 = 2$ bo'lsa, $V_1 = \frac{4}{3}\pi 2^3 = \frac{32}{3} = \pi$, $R_2 = 3$ bo'lsa, $V_2 = \frac{4}{3}\pi 3^3 = 36\pi$, $R = 4$ bo'lganda $V_3 = \frac{256}{3}\pi$ bo'ladi. $V = V_1 + V_2 + V_3 =$

$= \frac{32}{3}\pi + 36\pi + \frac{256}{3}\pi = 99\pi$.

Javob: 99π.

2-masala. Hajmi $\frac{9\pi}{16}$ ga teng bo'lgan shar sirtining yuzini toping.

Yechilishi. $V = \frac{4}{3}\pi R^3 = \frac{9\pi}{16} \Leftrightarrow \frac{27}{64} = R^3 \Leftrightarrow R = \frac{3}{4}$.

$S_{shar} = 4\pi R^2 \Rightarrow S = 4\pi\frac{9}{16} \Leftrightarrow S = \frac{9\pi}{4}$.

Javob: $\frac{9\pi}{4}$.

3-masala. Sharning diametriga perpendikular tekislik diametrni 3 sm va 9 sm li bo'laklarga ajratadi. Sharning hajmi qanday qismlarga ajraladi?

Y e c h i l i s h i. $R = 6$ radiusli shar tekislik bilan balandligi 3 sm va 9 sm li bo'laklarga ajraladi. Ularning hajmlari mos ravishda $V_1 = \pi 3^2 \cdot \left(6 - \frac{3}{3}\right) = 45\pi$ sm^3, $V_2 = \pi 9^2 \cdot \left(6 - \frac{9}{3}\right) = 243$ sm^3.

J a v o b: 45π sm^3, 243π sm^3.

Stereometrik masalalar ichida biror ko'pyoqqa, aylanish jismlariga ichki yoki tashqi chizilgan (joylashgan) shar va sfera bilan bog'liq masalalar ko'plab uchraydi.

1. Ko'pyoqqa tashqi chizilgan sfera deb, ko'pyoqning barcha uchlaridan o'tuvchi sferaga aytiladi. Bu holda ko'pyoq *sferaga ichki chizilgan* deyiladi.

2. Ko'pyoqqa ichki chizilgan sfera deb ko'pyoqning barcha yoqlariga urinuvchi sferaga aytiladi, bu holda ko'pyoq *sferaga tashqi chizilgan* deyiladi.

3. Silindr yoki kesik konusga ichki chizilgan sfera deb uning ikkala asosi va barcha yasovchilariga urinuvchi sferaga aytiladi. Bu holda silindr yoki kesik konus sferaga tashqi chizilgan deyiladi.

4. Konusga ichki chizilgan sfera deb uning asosiga va barcha yasovchisiga urinuvchi sferaga aytiladi.

5. Sferaga ichki chizilgan konus *deb asos aylanasi va uchi sfera*da bo'lgan konusga aytiladi, bu holda sfera *konusga tashqi chizilgan sfera* deyiladi.

6. Sferaga ichki chizilgan silindr yoki kesik konus deb asos aylanalari sferada yotgan silindr va kesik konusga aytiladi. Bu holda sfera silindr va kesik konusga *tashqi chizilgan sfera* deyiladi.

Mustaqil ishlash uchun test topshiriqlari

1. Balandligi 3 bo'lgan silindrning yon sirti yoyilganda yasovchisi hosil bo'lgan to'g'ri to'rtburchak diagonali bilan 60° li burchak tashkil qiladi. Silindrning hajmini toping.

A) $\frac{81}{4\pi}$; B) $\frac{81}{2\pi}$; C) $\frac{27}{4\pi}$; D) $\frac{27}{2\pi}$; E) $\frac{81}{\pi}$.

2. O'q kesimining yuzi Q ga teng bo'lgan silindr yon sirtining yuzini toping.

A) $2Q$; B) $Q\pi$; C) $\frac{Q}{\pi}$; D) $2Q\pi$; E) $\frac{2Q}{\pi}$.

3. Silindrning balandligi b ga, o'q kesimining diagonali d ga teng. osining radiusini toping.

A) $\sqrt{b^2+d^2}$; B) $\sqrt{b^2-d^2}$; C) $\frac{1}{2}\sqrt{d^2-b^2}$;

D) $\sqrt{d^2-b^2}$; E) $2\sqrt{d^2-b^2}$.

4. Silindr asosining radiusi uch marta orttirilsa, uning hajmi necha marta ortadi?

A) 3; B) 4; C) 6; D) 9; E) 3π.

5. Silindr yon sirtining yoyilmasi tomoni a ga teng bo'lgan kvadratdan iborat. Silindrning hajmini toping.

A) $\frac{a^3}{2\pi}$; B) $\frac{2\pi a^3}{3}$; C) $4\pi a^3$; D) πa^3; E) $\frac{a^3}{4\pi}$.

6. Silindr yon sirti yoyilmasining diagonali asos tekisligi bilan 45° li burchak tashkil qiladi. Silindrning yon sirti $144\pi^2$ ga teng bo'lsa, radiusini toping.

A) 12; B) $\sqrt{12}$; C) 6; D) 36; E) $2\sqrt{6}$.

7. Radiusi R, balandligi H bo'lgan silindr o'q kesimining diagonali uzunligini toping.

A) H^2+4R^2; B) $4R^2-H$; C) $\sqrt{H^2+4R^2}$;

D) $\sqrt{H^2+2R^2}$; E) $\sqrt{2R^2-H^2}$.

8. Silindrning to'la sirti 50 sm^2, yon sirti esa 30 sm^2 bo'lsa, silindrning radiusini toping.

A) $\frac{10}{\pi}$; B) $\sqrt{\frac{10}{\pi}}$; C) $\frac{\pi}{10}$; D) $\frac{\sqrt{\pi}}{10}$; E) $\sqrt{\frac{\pi}{10}}$.

9. Silindrning yon sirti yoyilmasi yuzi Q ga teng bo'lgan kvadrat shaklida bo'lsa, silindr asosining yuzini toping.

A) $\frac{Q}{4\pi}$; B) $\frac{Q}{2\pi}$; C) Q^2; D) $\frac{Q}{\pi}$; E) $\frac{Q}{\pi^2}$.

10. Silindr asoslaridan birining d diametri ikkinchisining markazidan α burchak ostida ko'rinadi. Shu silindrning to'la sirtini toping.

A) $\frac{\pi d^2}{2}\left(1+\operatorname{ctg}\frac{\alpha}{2}\right)$; B) $\frac{\pi d}{2}\left(1-\operatorname{ctg}\frac{\alpha}{2}\right)$; C) $\pi d(1-\operatorname{ctg}\alpha)$;

D) $\frac{\pi d}{2}\operatorname{ctg}\frac{\alpha}{2}$; E) $(1-\operatorname{ctg}\alpha)\frac{d}{\pi}$.

11. Balandligi 16 sm, radiusi 10 sm bo'lgan silindrning o'qidan 6 sm masofadagi tekislik bilan kesimining yuzini toping.

A) 144 sm^2; B) 225 sm^2; C) 256 sm^2; D) 169 sm^2; E) 625 sm^2.

12. Radiusi R va balandligi H bo'lgan silindr o'qiga parallel tekislik asos aylanasidan 60° li yoy kesib hosil qilgan kesimining yuzini toping.

A) $\frac{R\cdot H}{2}$; B) $R\cdot H$; C) $\frac{R\cdot H}{2}$; D) $\frac{\sqrt{3}}{2}R\cdot H$; E) $\sqrt{3}R\cdot H$.

13. *Silindr asosining yuzi 4 ga, yon sirtining yuzi* $12\sqrt{\pi}$ *ga teng.* Silindrning balandligini toping.

A) 3; B) 4; C) 2; D) 2,8; E) 3,2.

14. Silindrning balandligi 6 ga, asosining radiusi 5 ga teng. Uzunligi 10 ga teng bo'lgan kesmaning uchlari ikkala asos aylanalarida yotadi. Bu kesmadan o'qqacha bo'lgan eng qisqa masofani toping.

A) 3; B) 4; C) 4,5; D) 5; E) 5.

15. Radiusi 3 ga teng bo'lgan silindrning to'la sirti 28π dan kichik emas va 30π dan katta emas. Shu silindr balandligi qanday sonlar oralig'ida yotadi?

A) $\left[\frac{1}{2};\ 2\right]$; B) $\left[1;\ 2\frac{2}{3}\right]$; C) $[1;\ 2]$; D) $\left[\frac{5}{3};\ 2\right]$; E) $\left[1;\ \frac{5}{3}\right]$.

16. O'q kesimi tomoni 6 sm li teng tomonli uchburchak bo'lgan *konusning radiusini toping.*

A) 3; B) 4; C) 5; D) $\sqrt{3}$; E) 2.

17. l yasovchisi asos diametriga teng bo'lgan konusning balandligini toping.

A) $\frac{l\sqrt{2}}{2}$; B) $\frac{l\sqrt{3}}{2}$; C) $\frac{l}{2}$; D) $\frac{l}{4}$; E) $\frac{2l}{\sqrt{3}}$.

18. Konusning balandligi uning asosi diametridan ikki marta kichik bo'lsa, o'q kesimining uchidagi burchagini toping.

A) 45°; B) 60°; C) 90°; D) 30°; E) 120°.

19. Yasovchisi l bo'lib, asos tekisligi bilan 60° li burchak tashkil etuvchi konus asosining yuzini toping.

A) $\frac{\pi l^2}{2}$; B) $\frac{\pi l^3}{3}$; C) $\frac{\pi l^2}{5}$; D) $\frac{\pi l^2}{4}$; E) πl^2

20. Agar konus yon sirtining yoyilmasi $180°$ yoyli doiraviy sektor bo'lsa, konus o'q kesimi uchidagi burchagi necha gradus?

A) $30°$; B) $45°$; C) $60°$; D) $90°$ E) $75°$.

21. Konusning o'q kesimi teng tomonli uchburchak. To'la sirti 18 ga teng. Konus asosining yuzini toping.

A) 6; B) 12; C) $3\sqrt{2}$; D) 3; E) 4.

22. Asosining radiusi R ga teng va o'q kesimi to'g'ri burchakli uchburchakdan iborat konusning yon sirtini toping.

A) πR^2; B) $\sqrt{2}\pi R^2$; C) $\sqrt{3}\pi R^2$; D) $\frac{1}{2}\pi R^2$; E) $\frac{1}{3}\pi R^2$.

23. Konusning yasovchisi 100 ga, uning asos tekisligi bilan tashkil qilgan burchagining sinusi 0,6 ga teng. Konus o'q kesimining perimetrini toping.

A) 360; B) 320; C) 420; D) 340 E) 400.

24. Konusning o'q kesimi yuzi $16\sqrt{3}$ ga teng bo'lgan muntazam uchburchak. Konusning to'la sirtini toping

A) 48π; B) 44π; C) 46π; D) $48\sqrt{3}\pi$; E) 42π.

25. Asos aylanasining uzunligi $8\sqrt{\pi}$ ga, balandligi 9 ga teng bo'lgan konusning hajmini toping.

A) 16π; B) 24; C) 16; D) 48; E) 144.

26. Konusning o'q kesimi muntazam uchburchakdan, silindrniki esa kvadratdan iborat. Agar ularning hajmlari teng bo'lsa, to'la sirtlarining nisbatini toping.

A) $\sqrt{2}:\sqrt{3}$; B) $\sqrt[3]{3}:\sqrt[3]{2}$; C) $3:2$; D) $1:\sqrt[3]{3}$; E) $\sqrt[3]{9}:2$.

27. Asoslarining radiusi 2 va 7 ga, o'q kesimining diagonali 15 ga teng bo'lgan kesik konusning yasovchisini toping.

A) 6; B) 13; C) 4; D) 5; E) 12.

28. Asoslarining radiusi 2 va 7 ga, o'q kesimining diagonali 15 ga teng bo'lgan kesik konus yon sirtining yuzini toping.

A) 112π; B) 115π; C) 117π; D) 120π; E) 125π.

29. Kesik konus asoslari radiusi R va r. Yasovchisi asos tekisligi bilan $45°$ burchak tashkil etadi. Kesik konusning hajmini toping.

A) $\frac{1}{3}\pi\sqrt{R^3-r^3}$; B) $\frac{1}{3}\pi(R\cdot r)^3$; C) $\frac{1}{3}\pi(R-r)^3$; D) $\frac{1}{3}\pi(R+r)^3$;

E) $\frac{1}{3}\pi(R^3-r^3)$.

30. Kesik konus o'q kesimining yuzi asoslari yuzlarining ayirmasiga teng. Asoslarining radiuslari R va r bo'lsa, kesik konusning hajmini toping.

A) $\frac{1}{3}\pi(R^3 - r^3)$; B) $\frac{1}{3}\pi(R^3 + r^3)$; C) $\frac{1}{3}\pi H(r + R)^2$;

D) $\frac{1}{3}\pi(R + r)^3$; E) $\frac{1}{3}\pi(R - r)^3$.

31. Radiusi 13 ga teng bo'lgan shar tekislik bilan kesilgan. Agar shar markazidan kesimgacha masofa 10 ga teng bo'lsa, kesimning yuzini toping.

A) 69π; B) $3\sqrt{6}\pi$; C) 100π; D) 3; E) 9π.

32. Sirtining yuzi 16π ga teng bo'lgan sharning hajmini toping.

A) 69π; B) $3\sqrt{6}\pi$; C) 100π; D) 33π; E) $\frac{32\pi}{3}$.

33. Ikkita sfera yuzlarining nisbati 2 ga teng. Bu sferalar diametrlarining nisbatini toping.

A) 2; B) 4; C) 8; D) $\sqrt{2}$; E) $2\sqrt{2}$.

34. Tenglamasi $x^2 + y^2 + z^2 - 4x + 10z - 35 = 0$ bo'lgan sferaning radiusi uzunligini toping.

A) 5; B) 6; C) 7; D) 8; E) 4.

35. Radiusi $\sqrt[3]{2}$ bo'lgan shar yon sirti asosining yuzidan 3 marta katta bo'lgan konusga tengdosh. Konusning balandligini toping.

A) 4; B) 3; C) 5; D) 2; E) 6.

36. Radiuslari 15 ga va 20 ga teng bo'lgan ikki shar markazlari orasidagi masofa25 ga teng. Shar sirtlari kesishishidan hosil bo'lgan aylananing uzunligini toping.

A) 24π; B) 20π; C) 25π; D) 15π; E) 18π.

37. Tomoni 12,5 ga teng bo'lgan romb tomonlari shar sirtiga urinadi. Sharning radiusi 10 ga teng. Shar markazidan romb tekisligigacha masofa 8 bo'lsa, rombning yuzini toping.

A) 150; B) $\frac{\sqrt{481}}{2}$; C) 120; D) 135; E) $130\sqrt{3}$.

38. Silindrga shar ichki chizilgan. Silindrning hajmi 16π ga teng bo'lsa, sharning hajmini toping.

A) 30; B) 30π; C) $\frac{31\pi}{3}$; D) 32π; E) $\frac{32\pi}{3}$.

39. Sharga ichki chizilgan konusning balandligi 3 ga, asosining radiusi $3\sqrt{3}$ ga teng. Sharning radiusini toping.

A) 5; B) 6; C) $4\sqrt{3}$; D) $5\sqrt{2}$; E) 5,6.

40. Sharga tashqi chizilgan kesik konusning yasovchilari o'rtalaridan o'tuvchi tekislik bilan shu kesik konus hosil qilgan kesimning yuzi 4π ga teng. Kesik konusning yasovchisini toping.

A) 2; B) 4; C) 3; D) 5; E) 6.

41. Kesik konusga shar ichki chizilgan. Kesik konusning kichik asosining yuzi 36π ga, katta asosining yuzi esa 64π ga teng. Shar sirtining yuzini toping.

A) 172π; B) 100π; C) 144π; D) 156π; E) 192π.

42. Balandligi shar diametrining 0,1 qismiga teng bo'lgan shar segmentining hajmi shar hajmining qanday qismini tashkil etadi?

A) 0,028; B) 0,28; C) 2,08; D) 2,8; E) 0,8.

43. Agar shar sektori konusining radiusi 60 sm ga, shar radiusi esa 75 sm ga teng bo'lsa, shar sektorining hajmini toping.

A) 112000π; B) 12500π; C) 112500π; D) 1100π; E) 121500π.

44. Kubga ichki va tashqi chizilgan sferalar yuzlarining nisbatini toping.

A) 1:2; B) 1:3; C) 2:3; D) 3:4; E) 1:4.

45. Hajmi 432π ga teng bo'lgan silindrga ichki chizilgan shar sirtining yuzini toping.

A) 120π; B) 130π; C) 144π; D) 150π; E) 124π.

46. Muntazam oltiburchakli piramidaning apofemasi 5 ga, uning asosiga tashqi chizilgan doiraning yuzi 12π ga teng. Shu piramidaga ichki chizilgan sharning radiusini toping.

A) 3; B) 3,2; C) 1,5; D) 2,5; E) 2,4.

47. Piramidaning to'la sirti 60 ga teng, unga ichki chizilgan sharning radiusi 5 ga teng. Piramidaning hajmini toping.

A) 100; B) 80; C) 90; D) 120; E) 150.

48. Muntazam tetraedrning qirrasi 1 ga teng. Shu tetraedrga tashqi chizilgan shar radiusini toping.

A) $\frac{2\sqrt{2}}{3}$; B) $\frac{3\sqrt{2}}{8}$; C) $\frac{11\sqrt{2}}{24}$; D) $\frac{2\sqrt{3}}{5}$; E) $\frac{\sqrt{6}}{4}$.

49. Muntazam uchburchakli piramidaga konus ichki chizilgan. Agar piramidaning yon yoqlari bilan asosi 60°li burchak hosil qilib, piramidaning asosiga ichki chizilgan aylananing radiusi 16 ga teng bo'lsa, konusning yon sirtini toping.

A) 536π; B) 524π; C) 518π; D) 514π; E) 512π.

50. Muntazam to'rtburchakli prizmaga silindr ichki chizilgan. Silindr hajmining prizma hajmiga nisbatini toping.

A) $\frac{3\pi}{4}$; B) $\frac{2\pi}{3}$; C) $\frac{\pi}{4}$; D) $\frac{\pi}{3}$; E) $\frac{\pi}{5}$.

51. Muntazam to'rtburchakli piramida asosining tomoni 12 ga unga ichki chizilgan sharning radiusi 3 ga teng. Piramida yon sirtin toping.

A) 480; B) 360; C) 280; D) 240; E) 120.

52. Sharga ichki chizilgan konusning asosi sharning eng katta doirasidan iborat. Sharning hajmi konusning hajmidan necha marta katta?

A) 4; B) 3; C) 2,5; D) 2; E) 1,5.

53. Sharga ichki chizilgan konusning balandligi 3 ga, asosining radiusi $3\sqrt{3}$ ga teng. Sharning radiusini toping.

A) $5\sqrt{2}$; B) $4\sqrt{3}$; C) 6; D) 5,6; E) 5.

54. Sharga tashqi chizilgan kesik konusning yasovchilari o'rtalaridan o'tuvchi tekislik bilan shu kesik konus hosil qilgan kesimning yuzi 4π ga teng. Kesik konusning yasovchisini toping.

A) 6; B) 5; C) 4; D) 3; E) 2.

55. Teng tomonli silindrga radiusi 3 ga teng shar ichki chizilgan. Silindr va shar sirtlari orasida joylashgan jismning hajmini toping.

A) 18π; B) 24π; C) 27π; D) 12π; E) 21π.

56. Asosi a ga, asosidagi burchagi α ga teng bo'lgan teng yonli uchburchakni yon tomoni atrofida aylantirishdan hosil bo'lgan jismning hajmini toping.

A) $\frac{\pi a^3 \sin\alpha}{3}$; B) $\frac{\pi a^3 \sin^2\alpha}{6\cos\alpha}$; C) $\frac{\pi a^3 \operatorname{tg}\alpha}{2}$;

D) $\frac{\pi a^3 \cos\alpha}{6\sin^2\alpha}$; E) $\frac{\pi a^3 \sin 2\alpha}{12}$.

ILOVALAR

1- ilova

Klassik tengsizliklar va ekstremal masalalar

1. Klassik tengsizliklar.

$a_1, a_2, ..., a_n$ — musbat sonlar bo'lsin. Quyidagi kattaliklarni qaraymiz:

$$H_n = \frac{n}{\frac{1}{a_1}+\frac{1}{a_2}+...+\frac{1}{a_n}}, \quad G_n = \sqrt[n]{a_1 \cdot a_2 \cdot ... \cdot a_n},$$

$$A_n = \frac{a_1+a_2+...+a_n}{n}, \quad D_n = \sqrt{\frac{a_1^2+a_2^2+...+a_n^2}{n}},$$

Ular, mos ravishda: *o'rta garmonik, o'rta geometrik, o'rta arifmetik, o'rtacha kvadrat miqdorlar* deyiladi. Bu miqdorlar ushbu

$$H_n \le G_n \le A_n \le D_n$$

tengsizliklar bilan bog'langan.

$a_1 = a_2 = ... = a_n$ bo'lganda va faqat shu holda (*) da tenglik ishorasi «=» bo'ladi. (*) tengsizliklar **klassik tengsizliklar** deyiladi.

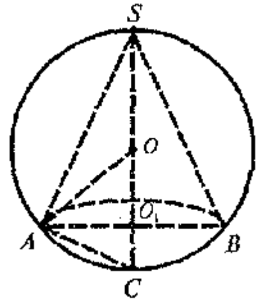
2. Klassik tengsizliklarning ekstremal masalalarni yechishga tatbiqi. Bu tatbiq quyidagi mulohazalarga asoslangan:

1. Aytaylik $a_1 + a_2 + ... + a_n = C_1$ — o'zgarmas son bo'lsin. U holda $G_n \le \frac{C_1}{n}$, demak, G_n miqdor $\frac{C_1}{n}$ dan oshib ketolmaydi; $a_1 = a_2 = ... = a_n$ bo'lganda va faqat shu holda $G_n = \frac{C_1}{n}$ bo'ladi. Bu holda G_n ning maksimumi haqida gapirish mumkin va bu maksimum qiymat $\frac{C_1}{n}$ ga teng bo'ladi.

2. Aytaylik $a_1 \cdot a_2 \cdot ... \cdot a_n = C_2$ — o'zgarmas son bo'lsin. U holda $\sqrt[n]{C_2} \le A_n$, demak, A_n miqdor $\sqrt[n]{C_2}$ dan kichik bo'lmaydi; $a_1 = a_2 = ... = a_n$ bo'lganda va faqat shu holda $A_n = \sqrt[n]{C_2}$ bo'ladi. Bu holda A_n ning minimumi haqida gapirish mumkin va bu minimum qiymat $\sqrt[n]{C_2}$ ga teng bo'ladi.

3. Ekstremumga oid masalalar yechish.

1-masala. R radiusli sharga eng katta hajmli konus ichki chizilgan. Konus balandligining asos radiusiga nisbatini toping.



290-rasm.

Yechilihsi. Masala shartiga mos shakl chizamiz (290-rasm). U holda

Berilgan:	Topish kerak:
$AO = OS = R$	$\frac{SO_1}{AO_1} = ?$

Endi, $SO_1 = h$, $AO = r$ kabi belgilangan miqdorlar, mos ravishda R radiusli sharga ichki chizilgan eng katta hajmdagi konusning balandligi va asosining radiusi deylik. U holda $V = \frac{1}{3}\pi r^2 h$ bo'ladi.

$CS = 2R$, $O_1C = 2R - h$ bo'lgani uchun to'g'ri burchakli ΔCAS dan $r^2 = (2R - h)h$, binobarin konusning hajmi $V(h) = \frac{\pi}{3}h^2(2R - h)$, $0 \le h \le 2R$, h ning funksiyasi sifatida ifodalanadi.

Shu bilan birga, $V(0) = V(2R) = 0$, ya'ni $V(h)$ funksiya o'zining eng katta qiymatiga (0; $2R$) oraliqda erishadi. Biz ko'radigan masalalarda, ko'pgina amaliy masalalarda bo'lgani kabi, ekstremumga tekshirilayotgan funksiya berilgan oraliqda faqat bitta statsionar nuqtaga: yoki maksimum nuqtasiga, yoki minimum nuqtasiga ega bo'ladi. Bunday hollarda shu berilgan oraliqdagi eng katta qiymatini funksiya maksimum nuqtasida qabul qiladi.

Oraliqdagi eng kichik qiymatini esa minimum nuqtasida qabul qiladi.

$f(h) = h^2(2R - h)$ funksiyani kiritamiz.

$f(h) = \frac{1}{2}(4R - 2h)\cdot h \cdot h \le \frac{1}{2}\left(\frac{4R-2h+h+h}{3}\right)^3 = \frac{1}{2}\cdot\frac{64R^3}{27} = \frac{32R^3}{27}$ va tenglikka $4R - 2h = h$, $3h = 4R$, $h = \frac{4R}{3}$ bo'lganda erishadi. Bunda biz $G_3 \le A_3$ *tengsizlikka teng kuchli bo'lgan* $a_1 \cdot a_2 \cdot a_3 \le \left(\frac{a_1+a_2+a_3}{3}\right)^3$ tengsizlikdan foydalandik. h ning ayni shu $\frac{4R}{3}$ qiymatida $V(h)$ hajm o'zining eng katta qiymatini oladi:

$$V\left(\frac{4R}{3}\right) = \frac{\pi}{3}\cdot\frac{32R^3}{27} = \frac{32\pi R^3}{81}.$$

Shu bilan birga, $r = \sqrt{\frac{2R}{3} \cdot \frac{4R}{3}} = \frac{2\sqrt{2}}{3} R$ va $\frac{h}{r} = \sqrt{2}$.

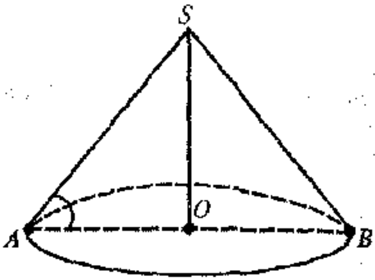
Javob: $\sqrt{2}$.

Ba'zan, oson isbot qilinadigan quyidagi tasdiqdan foydalanishga to'g'ri keladi:

Agar $f(x)$ funksiyaning biror oraliqdagi qiymatlari nomanfiy bo'lsa, u holda $f(x)$ va $(f(x))^n$ funksiyalar (bunda n natural son) eng katta (eng kichik) qiymatini ayni bir nuqtada qabul qiladi.

2-masala. Yasovchisi l ga teng bo'lgan eng katta hajmdagi konus asosi radiusining balandligiga nisbatini toping.

Yechilishi: Masala shartiga mos shakl chizib olamiz (291-rasm).



291-rasm.

Berilgan:	Topish kerak:
$AS = l$	$\frac{AO}{SO} = ?$

Endi, $AO = r$, $OS = h$, $\angle SAO = \alpha$, $0 < \alpha < \frac{\pi}{2}$ deylik. U holda ΔAOS dan: $h = l\sin\alpha$, $r = l\cos\alpha$.

Konusning hajmi

$$V(\alpha) = \frac{\pi}{3} l^3 \cos^2\alpha \cdot \sin\alpha$$

esa α ning funksiyasi bo'ladi. Konus yasovchisining asos tekisligiga og'ish burchagi α qanday bo'lganda $V(\alpha)$ eng katta qiymatga erishadi?

Buning uchun esa $f(\alpha) = \cos^2\alpha \cdot \sin\alpha$ funksiyaning $0 < \alpha < \frac{\pi}{2}$ oraliqdagi eng katta qiymatini topish lozim. $f(\alpha)$ va $f^2(\alpha)$ funksiyalar ayni bir α_0 da eng katta qiymatga erishgani uchun $f^2(\alpha)$ funksiyaning ekstremumga tekshirish qulay.

$$f^2(\alpha) = \cos^4\alpha \cdot \sin^2\alpha = \left(1 - \cos^2\alpha\right) \cdot \cos^2\alpha \cdot \cos^2\alpha =$$

$$= \frac{1}{2}(2 - 2\cos^2\alpha) \cdot \cos^2\alpha \cdot \cos^2\alpha \le \frac{1}{2}\left(\frac{2 - 2\cos^2\alpha + \cos^2\alpha + \cos^2\alpha}{3}\right)^3 = \frac{1}{2} \cdot \frac{8}{27} = \frac{4}{27}.$$

Shu bilan birga, tenglikka $2 - 2\cos^2\alpha = \cos^2\alpha$, $3\cos^2\alpha = 2$, $\cos\alpha = \sqrt{\frac{2}{3}}$, $\alpha = \alpha_0 = \arccos\sqrt{\frac{2}{3}}$ bo'lganda erishiladi va ayni shu α_0 burchak $V(\alpha)$ ga teng eng katta qiymatni beradi. $f(\alpha_0) = \frac{2}{3\sqrt{3}}$ bo'lgani uchun $V(\alpha_0) = = \frac{\pi}{3} l^3 \cdot \frac{2}{3\sqrt{3}} = \frac{2\sqrt{3}}{27}\pi l^3$.

Bunday hajmga ega konusning balandligi $h = \frac{l}{\sqrt{3}}$ ga, asosi radiusi esa $r = l \cdot \sqrt{\frac{2}{3}}$ ga teng.

Demak, $\frac{r}{h} = \sqrt{2}$.

Javob: $\sqrt{2}$.

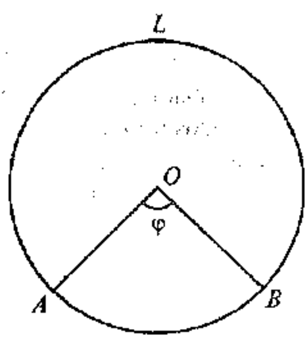
3-masala. Radiusi R bo'lgan doiradan qanday sektor kesib olinsa, doiraning qolgan qismidan eng katta hajmdagi konussimon idish (voronka) yasash mumkin? Bu idish asosi radiusining balandligiga nisbatini toping.

Yechilishi: Masala shartiga moslab shakllar chizib olamiz (292, 293-rasmlar).

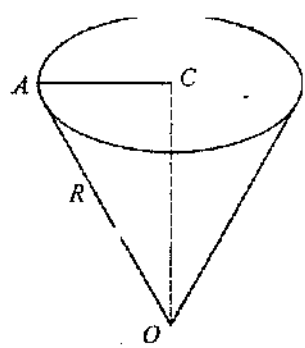
Berilgan:	Topish kerak:
$AO = OB = R$	$\angle AOB = ?$
	$\frac{AC}{OC} = ?$

Endi, $\angle AOB = \varphi$, $AC = r$, $OC = h$ deylik. U holda bu idishning yasovchisi berilgan doira radiusi R ga, idish asosining uzunligi esa ALB yoyga teng bo'ladi.

$\cup ALB = 2\pi R - R\varphi = R \cdot (2\pi - \varphi)$ ekanligi ravshan. Qulaylik uchun $2\pi - \varphi = x$ deylik, $0 \leq x \leq 2\pi$. Shu bilan birga, $R(2\pi - \varphi) = R \cdot x$; $2\pi r = Rx$, $r = \frac{Rx}{2\pi}$ munosabatlar o'rinli.



292-rasm.



293-rasm.

ΔACO dan $h=\sqrt{R^2-r^2}=R\cdot\sqrt{1-\frac{x^2}{4\pi^2}}=\frac{R}{2\pi}\sqrt{4\pi^2-x^2}$. Nihoyat, $V(x)=\frac{R^3\cdot x^2}{24\pi^2}\cdot\sqrt{4\pi^2-x^2}$. Odatdagidek, $f(x)=x^2\sqrt{4\pi^2-x^2}$ funksiyani kiritib, $f^2(x)$ funksiyani ekstremumga tekshiramiz.

$$f^2(x)=x^2\cdot x^2\cdot(4\pi^2-x^2)=\frac{1}{2}x^2\cdot x^2\cdot(8\pi^2-2x^2)\le$$

$$\le\frac{1}{2}\left(\frac{x^2+x^2+8\pi^2-2x^2}{3}\right)^3=\frac{1}{2}\cdot\frac{8^3\cdot\pi^6}{27}.$$

Demak, $f(x)\le\frac{16\pi^3}{3\sqrt{3}}$, bunda tenglikka $x^2=8\pi^2-2x^2$ bo'lganda erishiladi, ya'ni $3x^2=8\pi^2$, $x_0=2\pi\sqrt{\frac{2}{3}}$. Bu nuqta $f(x)$ ning, shuningdek $V(x)$ ning ham, maksimum nuqtasi bo'ladi, $V(0)=V(2\pi)=0$ ekani ravshan.

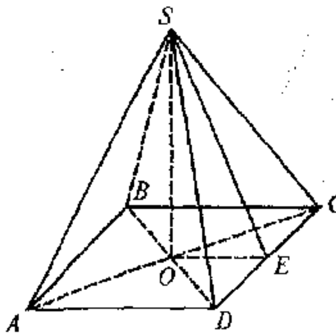
$$V(x_0)=\frac{R^3}{24\pi^2}\cdot\frac{16\pi^3}{3\sqrt{3}}=\frac{2\pi R^3}{9\sqrt{3}}.$$

U holda $r=\frac{R}{2\pi}\cdot 2\pi\sqrt{\frac{2}{3}}=R\sqrt{\frac{2}{3}}$,

$$h=R\cdot\sqrt{1-\frac{4\pi^2}{4\pi^2}\cdot\frac{2}{3}}=R\sqrt{\frac{1}{3}}.$$

Demak, $\frac{r}{h}=\sqrt{2}$; (yana $\sqrt{2}$).

Javob: $\varphi=2\pi\left(1-\sqrt{\frac{2}{3}}\right)$; $\sqrt{2}$.



294-rasm.

4-masala. Eng katta hajmli muntazam to'rtburchakli piramidaning yon sirti s ga teng. Asos tomonining piramida balandligiga nisbatini toping.

Yechilishi. Masala shartiga mos 294- rasmni chizamiz.

Berilgan:	Topish kerak:
$SABD$ – muntazam to'rtburchakli piramida, $S_{yon}=s$.	$\frac{AB}{OS}=?$

Endi $AB = BC = CD = DA = x$, $OS = h$ deylik. U holda $OE = \frac{x}{2}$, $S_{yon} = s = 2x \cdot SE$, bundan $SE = \frac{s}{2x}$. ΔSEO dan Pifagor teoremasiga ko'ra, $h = \frac{1}{2x}\sqrt{s^2 - x^4}$. U holda $V(x) = \frac{1}{3} \cdot S_{asos} \cdot h = \frac{x}{6}\sqrt{s^2 - x^4}$. Shu bilan birga, $V(0) = V\left(\sqrt{s}\right) = 0$; $V(x)$, va $f^4(x) = x^4 \cdot (s^2 - x^4)^2$ funksiyalar ayni bir nuqtada ekstremumga erishadi. Ammo

$$f^4(x) = \frac{2x^4}{2} \cdot (s^2 - x^4)(s^2 - x^4) \le \frac{1}{2}\left(\frac{2x^4 + s^2 - x^4 + s^2 - x^4}{3}\right)^3 = \frac{1}{2} \cdot \frac{8s^6}{27} = \frac{4s^6}{27},$$

demak, $f(x) \le \frac{s \cdot \sqrt{2s}}{\sqrt[4]{27}}$, bunda tenglikka $2x^4 = s^2 - x^4$, $3x^4 = s^2$, $x = \frac{\sqrt{s}}{\sqrt[4]{3}}$ bo'lganda erishadi. Shunday qilib, eng katta hajm $V\left(\frac{\sqrt{s}}{\sqrt[4]{3}}\right) = \frac{1}{6} \cdot \frac{s\sqrt{25}}{\sqrt[4]{27}} = \frac{\sqrt[4]{12}}{18} \cdot \left(\sqrt{s}\right)^3$ ga teng. Nihoyat, $h = \frac{1}{6} \cdot \frac{\sqrt[4]{3}}{\sqrt{5}} \cdot \sqrt{s^2 - \frac{s^2}{3}} = \frac{\sqrt[4]{3}}{2} \cdot \sqrt{\frac{2}{3}} \cdot \sqrt{s}$.

U holda $\frac{x}{h} = \frac{AB}{OS} = \sqrt{2}$.

Javob: $\sqrt{2}$

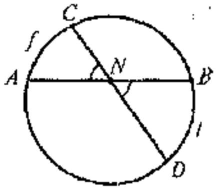
Geometrik masalalarni yechishda foydalaniladigan formulalar

Geometrik masalalarni tahlil qilib, undagi ma'lum (berilgan) va noma'lum (so'ralgan)lar orasidagi munosabatlarni algebraik ifodalasak, o'quvchiga tanish formulalar hosil bo'ladi. Quyida geometrik masalalarni yechishda ko'p foydalaniladigan formulalar tizimi ilova sifatida berilmoqda.

I. Nuqta, to'g'ri chiziq, kesma. 1. Hech bir uchtasi bir to'g'ri chiziqda yotmagan n ta nuqta $\frac{n(n-1)}{2}$ ta kesmani aniqlaydi.

2. Qavariq m burchakning diagonallari soni $\frac{m(m-1)}{2}$ ta.

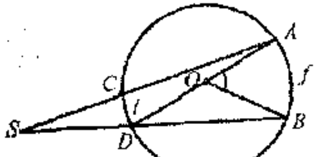
II. Doiradagi burchaklar va kesishuvchi kesmalar.



1. $|AN|\cdot|NB| = |CN|\cdot|ND|$.

2. $\angle ANC = \angle BND = \frac{1}{2}(\cup AfC + \cup BlD)$.

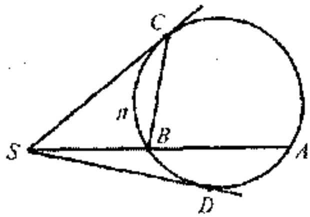
3. $\angle AOB = \frac{1}{2}\cup AfB$.



4. $\angle AOB = \cup AfB$.

5. $\angle ASB = \frac{1}{2}(\cup AfB - \cup ClD)$.

6. $|SC|\cdot|SA| = |SD|\cdot|SB|$.



7. $\angle BSC = \angle ASC = \frac{\cup BnC}{2}$.

8. $\angle CSD = \frac{1}{2}(\cup CAD - \cup CBD)$.

9. $|SC|^2 = |SA|\cdot|SB|,\ |CS| = |DS|$.

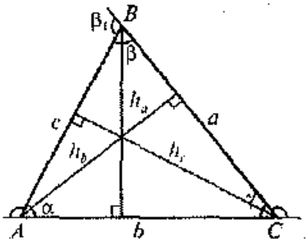
III. Uchburchakdagi metrik munosabatlar. Uchburchakning asosiy elementlari.

a, b, c — ΔABC tomonlarining uzunliklari;

α, β, γ — uchburchakning ichki burchaklari kattaliklari;

$P = a + b + c$ — uchburchakning perimetri;

$p = \frac{a+b+c}{2}$ — uchburchak yarimperimetri;



α_1, β_2, γ_3 — ΔABC tashqi burchaklarining kattaliklari;

h_a, h_b, h_c — mos ravishda uchburchakning a, b, c tomonlariga tushirilgan balandliklar uzunliklari;

MN — uchburchakning o'rta chizig'i;

R — uchburchakka tashqi chizilgan aylana radiusi;

r — uchburchakka ichki chizilgan aylana radiusi;

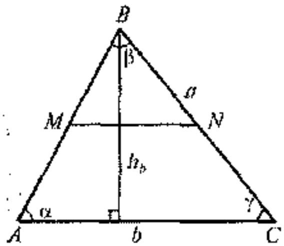
S — geometrik figuralarning yuzlari.

m_a, m_b, m_c — a, b, c tomonlarga o'tkazilgan medianalar uzunliklari;

b_A, b_B, b_C — A, B, C burchaklarining bissektrisalari uzunliklari.

1. Uchburchak ichki burchaklarining yig'indisi:

$$\alpha + \beta + \gamma = 180°.$$



2. Uchburchak tashqi burchaklari:

$\alpha + \alpha_1 = 180°$, $\beta + \beta_1 = 180°$, $\gamma + \gamma_1 = 180°$,

$\alpha_1 = \beta + \gamma$, $\beta_1 = \alpha + \gamma$, $\gamma_1 = \alpha + \beta$, $\alpha_1 + \beta_1 + \gamma_1 = 360°$.

3. Uchburchak tengsizligi:

$$\begin{cases} a+b>c \\ a+c>b \\ b+c>a, \end{cases} \qquad \begin{cases} |a-b|<c \\ |a-c|<b \\ |b-c|<a. \end{cases}$$

4. Sinuslar teoremasi: $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2R$.

5. Kosinuslar teoremasi:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha,$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta,$$
$$c^2 = a^2 + b^2 - 2ab\cos\gamma.$$

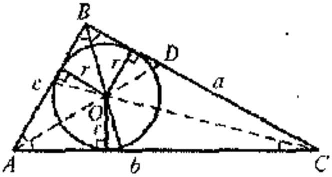
6. Uchburchakning yuzi:

$$S = \frac{1}{2}ah_a;\ S = \frac{1}{2}bh_b;\ S = \frac{1}{2}ch_c;$$

$$S = \frac{1}{2}bc\sin\alpha;\ S = \frac{1}{2}ac\sin\beta;\ S = \frac{1}{2}ab\sin\gamma;$$

$$S = \frac{abc}{4R};\ S = |MN|\cdot h_b;\ p = \frac{a+b+c}{2};$$

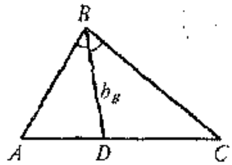
$$S = \sqrt{p(p-a)(p-b)(p-c)};\ S = p\cdot$$



IV. Uchburchakdagi ajoyib nuqtalar.
1. Bissektrissalar. Uchburchak bissektrisalarining kesishish nuqtasi unga ichki chizilgan aylana markazi bo'ladi.

$$r = \frac{2S}{a+b+c} = \frac{S}{p}$$

Uchburchak burchagi bissektrisasi o'zi tushgan tomonni qolgan tomonlarga proporsional bo'lakarga bo'ladi.



$$\frac{AD}{DC} = \frac{AB}{BC}.$$

$$b_A = \frac{2}{b+c}\sqrt{bcp(p-a)};\ b_B = \frac{2}{a+c}\sqrt{acp(p-b)};$$

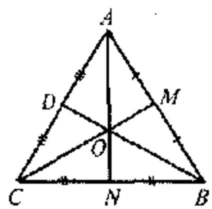
$$b_C = \frac{2}{a+b}\sqrt{abp(p-c)};\ p = \frac{a+b+c}{2}.$$

2. Medianalar. Uchburchakning medianalari bir nuqtada kesishadi va kesishish nuqtasida $\frac{1}{2}$ nisbatda bo'linadi, ya'ni

$$\frac{OD}{OB} = \frac{ON}{OA} = \frac{OM}{OC} = \frac{1}{2}.$$

$$m_a = \sqrt{b^2 + c^2 + 2bc\cdot\cos\angle A};\ m_b = \sqrt{a^2 + c^2 + 2ac\cdot\cos\angle B};$$

$$m_c = \sqrt{a^2 + b^2 + 2ab\cdot\cos\angle C};$$



$$m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2};\ m_b = \frac{1}{2}\sqrt{2a^2 + 2c^2 - b^2};$$

$$m_a = \frac{1}{2}\sqrt{2a^2 + 2b^2 - c^2};$$

$$m_a^2 + m_b^2 + m_c^2 = \frac{3}{4}(a^2 + b^2 + c^2);$$

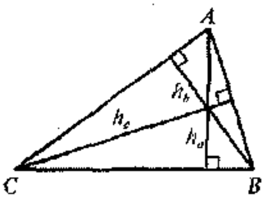
$$m = \frac{m_a + m_b + m_c}{2}.$$

$$S_{\Delta ABC} = \frac{4}{3}\sqrt{m(m-m_a)(m-m_b)(m-m_c)}.$$

Uchburchakning medianalarining kesishish nuqtasi uning «og'irlik markazi» deyiladi, ya'ni $\vec{OM} + \vec{OD} + \vec{ON} = \vec{0}$.

3. Balandliklar. Uchburchakning balandliklari h_a, h_b, h_c bir nuqtada kesishadi. Agar ΔABC ning har bir uchidan uning qarshisidagi tomoniga parallel to'g'ri chiziqlar chizsak, ular kesishib, tomonlarining o'rtalari

A, B, C nuqtalarda bo‘lgan shunday uchburchak hosil bo‘ladiki, u uchburchak uchun h_a, h_b, h_c kesmalar tomonlarining o‘rta perpendikulari bo‘ladi.



Uchburchak tomonlarining o‘rta perpendikularlari bir nuqtada kesishadi va bu nuqta *uchburchakka tashqi chizilgan aylana markazi* bo‘ladi.

$$R=\frac{abc}{4S};\ h_a=\frac{2S}{a};\ h_c=\frac{2S}{c};\ \frac{1}{r}=\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_c};$$

$$S=\frac{r(h_a+h_b+h_c)}{\sqrt{3}};\ S=\frac{1}{2}\sqrt{2h_a\cdot h_b\cdot h_c\cdot R}.$$

Har qanday uchburchak uchun:

$$h_a\le b_A\le m_a;\ h_b\le b_B\le m_b;\ h_c\le b_C\le m_c$$

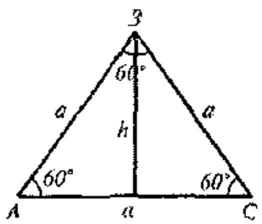
V. Teng tomonli (muntazam) uchburchak.

$|AB|=|AC|=|BC|=a$, $\alpha=\beta=\gamma=60°$,

$$h=b=m=\frac{\sqrt{3}}{2}a,$$

$$R=\frac{a}{\sqrt{3}},\ r=\frac{a}{2\sqrt{3}},\ R=2r,\ R=\frac{2}{3}h,$$

$$r=\frac{1}{3}h,\ S=\frac{a^2\sqrt{3}}{4}.$$



VI. To‘g‘ri burchkli uchburchak. a_c va b_c — a va b katetlarning gipotenuzadagi proyeksiyasi, $|AN|=b_c$, $NB=a_c$.

h_c — gipotenuzaga tushirilgan balandlik.

$a^2+b^2=c^2$ — Pifagor teoremasi.

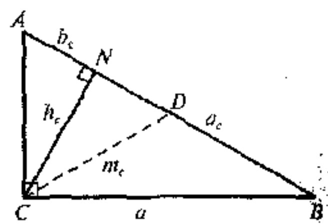
$c=a_c+b_c$, $|AD|=|BD|=|CD|=m_c$.

$$S=\frac{1}{2}ab;\ S=\frac{1}{2}ch_c;\ R=\frac{c}{2}$$

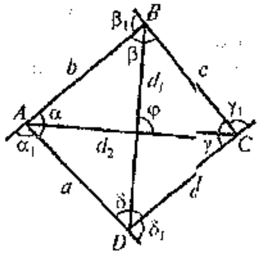
$$r=\frac{a+b-c}{2};\ 2(R+r)=a+b;\ a^2=ca_c;$$

$$b^2=cb_c;\ h_c=\sqrt{a_c\cdot b_c};$$

$$h_c=\frac{a\cdot b}{c};\ m_a=\frac{1}{2}\sqrt{4b^2+a^2};\ m_b=\frac{1}{2}\sqrt{4a^2+b^2};\ m_c=\frac{c}{2}.$$



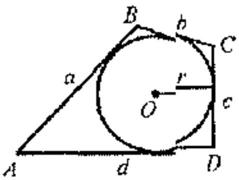
VII. Ixtiyoriy qavariq to'rtburchak. d_1 va d_2 — diagonallar uzunligi. φ — diagonallar orasidagi burchak.



$$S = \frac{1}{2} d_1 d_2 \sin\varphi\,,\ \alpha + \beta + \gamma + \delta = 360°\,,$$

$$\alpha_1 + \beta_1 + \gamma_1 + \delta_1 = 360°\,.$$

VIII. Aylanaga ichki va tashqi chizilgan to'rtburchaklar. 1. $a + c = b + d$ bo'lsa, to'rtburchakka ichki aylana chizish mumkin.



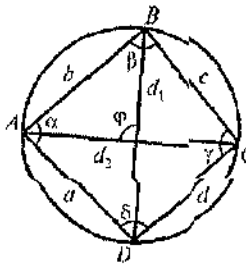
$$S = pr\,,\ p = \frac{a+b+c+d}{2}\,,$$

$$S = \sqrt{a \cdot b \cdot c \cdot d \sin^2\left(\frac{B+D}{2}\right)}\,.$$

Aylanaga ichki chizilgan to'rtburchakning yuzi: $S = \sqrt{a \cdot b \cdot c \cdot d}$.

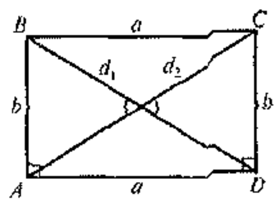
2. $\alpha + \gamma = 180°$, $\beta + \delta = 180°$ bo'lsa, to'rtburchakka tashqi aylana chizish mumkin.

Agar to'rtburchak diagonallarining ko'paytmasi qarama-qarshi tomonlari ko'paytmasining yig'indisiga teng bo'lsa, unga tashqi aylana chizish mumkin (Ptolomey teoremasi): $|AC| \cdot |BD| = |AB| \cdot |DC| + |BC| \cdot |AD|$.



$$S = \sqrt{(p-a)(p-b)(p-c)(p-d)}\,,\ p = \frac{a+b+c+d}{2}\,.$$

IX. To'g'ri to'rtburchak.

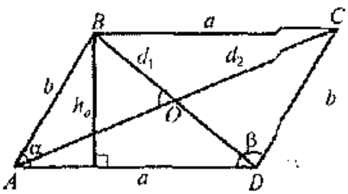


1. $\angle A = \angle B = \angle C = \angle D = 90°$,

$$d_1 = d_2 = d\,,\ d = \sqrt{a^2 + b^2}\,.$$

$$S = \frac{1}{2} d^2 \sin\varphi\,,\ S = ab\,,\ R = \frac{d}{2}\,.$$

X. Parallelogramm.



$|AO| = |OC|$, $|BO| = |OD|$,

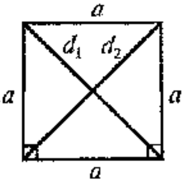
$$\alpha + \beta = 180°,\ d_1^2 + d_2^2 = 2(a^2 + b^2)\,;$$

$$S = ah_a\,,\ S = bh_b\,,$$

$S = ab\sin\alpha$; $S = \frac{1}{2}d_1 d_2 \sin\varphi$, $d_1^2 = a^2 + b^2 - 2ab\cos\alpha$,

$d_2^2 = a^2 + b^2 - 2ab\cos\beta$.

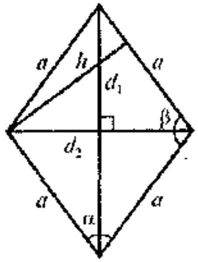
XI. Kvadrat.



$d_1 = d_2 = d$, $d_1 \perp d_2$, $d = \sqrt{2}a$, $S = a^2$,

$S = \frac{1}{2}d^2$, $R = \frac{d}{2}$, $r = \frac{a}{2}$.

XII. Romb.

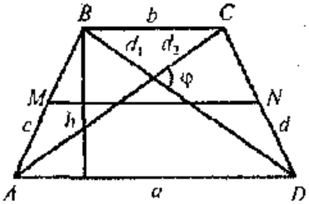


$d_1 \perp d_2$, $\alpha + \beta = 180°$, $S = ah$, $S = \frac{d_1 d_2}{2}$;

$S = a^2\sin\alpha$, $r = \frac{h}{2}$, $d_1^2 + d_2^2 = 4a^2$

$d_1 = 2a\cos\frac{\beta}{2}$, $d_2 = 2a\sin\frac{\beta}{2}$.

XIII. Trapetsiya.



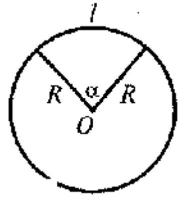
$|MN| = \frac{a+b}{2}$ — o'rta chizig'i, $S = \frac{(a+b)h}{2}$,

$S = |MN| \cdot h$, $S = \frac{1}{2}d_1 d_2 \sin\varphi$, $a + b = c + d$ bo'lsa, trapetsiyaga ichki aylana chizish mumkin.

$r = \frac{h}{2}$, $r = \frac{\sqrt{ab}}{2}$, $c = d$ bo'lsa, unga tashqi aylana chizish mumkin.

XIV. O'xshash figuralarning yuzlari.

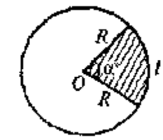
$\frac{S_1}{S_2} = \left(\frac{a_1}{a_2}\right)^2 = \left(\frac{h_1}{h_2}\right)^2 = \left(\frac{p_1}{p_2}\right)^2$, $S_1 \sim S_2$.



XV. Aylana va doira. 1. $d = 2R$, $C = 2\pi R$, $C = \pi d$ — aylana uzunligi. $l = \frac{\pi R\alpha°}{180°}$, $l = \alpha_{rad} R$, l — yoy uzunligi, $\alpha_{rad} = \frac{\pi\alpha°}{180°}$, $\alpha°$ — markaziy burchakning gradus o'lchovi.

$S = \pi R^2$, $S = \frac{1}{4}\pi d^2$ — doira yuzi.

2. $S_{sekt} = \frac{\pi R^2 \alpha^\circ}{360^\circ}$ — doiraviy sektor yuzi.



$S_{segm} = \frac{\pi R^2 \alpha^\circ}{360^\circ} \pm S_{\Delta AOB}$ — doiraviy segment yuzi yoki $S_{segm} = \frac{R^2}{2}(\alpha \pm \sin \alpha^\circ)$.

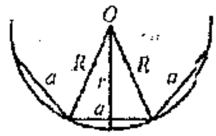
XVI. Ko'pburchaklar. Qavariq ko'pburchak ichki burchaklarining yig'indisi $180^\circ(n-2)$ ga teng, n — ko'pburchak tomonlarining soni.

Ko'pburchakning bitta burchagining gradus o'lchovi $\frac{180^\circ(n-2)}{n}$. Tashqi burchaklar yig'indisi 360°. Ko'pburchakning diagonallari soni — N: $N = \frac{n(n-3)}{2}$.

XVII. Muntazam ko'pburchaklar.

$S = \frac{1}{2} R^2 n \sin \frac{360^\circ}{n}$, $S = \frac{1}{2} a \cdot n \cdot r$.

$R = \frac{a}{2\sin\frac{180^\circ}{n}}$, $r = \frac{a}{2\operatorname{tg}\frac{180^\circ}{n}}$



XVIII. Ko'pyoqlar.

l — yon qirrasi uzunligi; P — asos perimetri uzunligi; S — asos yuzi; H — balandlik; P_{kes} — perpendikular kesim perimetri; S_{yon} — yon sirt yuzi; S_t — to'la sirt yuzi; S_{kes} — perpendikular kesim yuzi; V — hajm.

XIX. Kub. $S_{asos} = a^2$, $S_t = 6a^2$, $S_{yon} = 4a^2$;

$V = a^3$, $d_1 = \sqrt{2}a$, $d = \sqrt{3}a$;

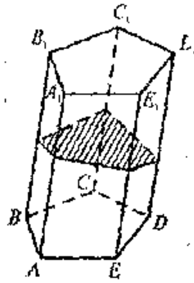
$R = \frac{a\sqrt{3}}{2}$, $r = \frac{1}{2}a$.

XX. Prizma.

$S_{yon} = P_{kes} \cdot l$,

$V = SH$,

$V = S_{kes} \cdot l$.



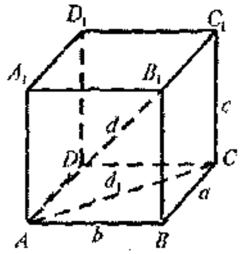
XXI. To'g'ri burchakli parallelepiped.

$d_1 = \sqrt{a^2 + b^2}$; $d = \sqrt{a^2 + b^2 + c^2}$;

$S_{asos} = a \cdot b$,

$S_{yon} = P \cdot l$, $S_t = Pl + 2S_{as}$,

$V = SH$, $V = abc$.



XXII. Piramida.

$V = \frac{1}{3}SH$, $S_{yon} = \frac{1}{2}(ah_a + bh_b + ch_c)$.

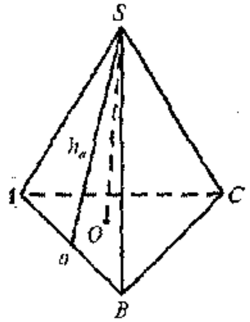
Muntazam piramida uchun:

$S_{yon} = \frac{1}{2}ph_a$, $S_t = \frac{1}{2}ph_a + S_{as}$.

Qirrasi a ga teng bo'lgan muntazam tetraedr uchun:

$S_{yon} = \frac{3\sqrt{3}}{4}a^2$, $S_t = 3\sqrt{3} \cdot a^2$, $V = \frac{a^3\sqrt{2}}{12}$,

$R = \frac{a\sqrt{6}}{4}$, $r = \frac{a\sqrt{6}}{12}$, $R = 3r$.



R — muntazam piramidaga tashqi, r — ichki chizilgan shar radiusi.

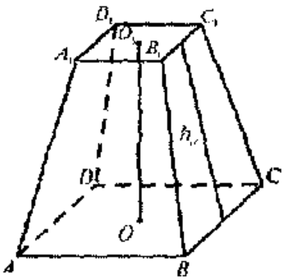
XXIII. Kesik piramida.

$V = \frac{1}{3}H(S_1 + \sqrt{S_1 \cdot S_2} + S_2)$, S_1 va S_2 asos yuzlari.

Muntazam kesik piramida:

$S_{yon} = \frac{1}{2}(p_1 + p_2)h_a$, $S_t = S_{yon} + S_1 + S_2$,

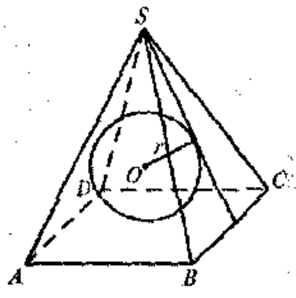
p_1 va p_2 — ostki va ustki asoslari perimetri.



XXIV. Ko'pyoqqa ichki chizilgan shar.

$V = \frac{1}{3} \cdot S_t \cdot r$.

V — ko'pyoq hajmi,
S_t — ko'pyoq to'la sirti,
r — shar radiusi.



XXV. O'xshash ko'pyoqlilar hajmlarining nisbati.

$$\frac{V_1}{V_2}=\left(\frac{a_1}{a_2}\right)^3=\left(\frac{H_1}{H_2}\right)^3=\left(\frac{P_1}{P_2}\right)^3=\left(\frac{l_1}{l_2}\right)^3, \quad V_1 \sim V_2.$$

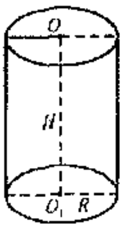
XXVI. Eyler formulasi. Har qanday qavariq ko'pyoq uchun $U+Y=Q+2$ tenglik o'rinli. Bunda U — ko'pyoqning uchlari soni, Y — yoqlari soni, Q — qirralari soni.

XXVII. Silindr.

$S_{yon}=2\pi R\cdot H,$

$S_t=2\pi R(R+H),$

$V=\pi R^2\cdot H.$

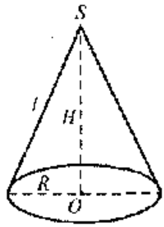


XXVIII. Konus.

$S_{yon}=\pi R\cdot l,$

$S_t=\pi R(R+l),$

$V=\frac{1}{3}\pi R^2 H.$

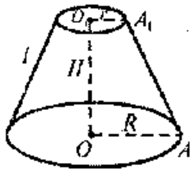


XXIX. Kesik konus.

$S_{yon}=\pi l(R+r),\ O_1A_1=r,\ OA=R.$

$S_t=\pi l(R+r)+\pi R^2+\pi r^2.$

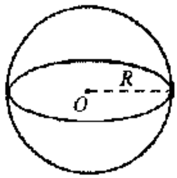
$V=\frac{1}{3}\pi H(R^2+R\cdot r+r^2).$



XXX. Shar.

$S=4\pi R^2;\ S=\pi d^2;$

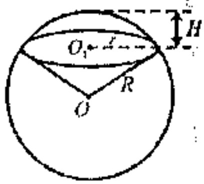
$V=\frac{4}{3}\pi R^3.$



XXXI. Shar segmenti.

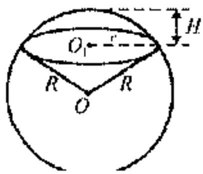
$S_{yon}=2\pi RH;\ S_t=2\pi RH+\pi r^2;$

$V=\pi H^2\left(R-\frac{1}{3}H\right).$



XXXII. Shar sektori.

$S_{yon}=\pi R(2H+r);\ V=\frac{2}{3}\pi R^2 H.$



TEST TOPSHIRIQLARINING JAVOBLARI

I bob

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
E	D	B	D	D	B	E	E	B	E	D	E	C	E	A	E	D	C	E	A
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36				
E	D	B	A	B	E	E	E	D	B	B	C	B	E	E	E				

II bob

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
D	B	B	A	C	D	D	E	D	E	C	D	E	B	A	A	A	B	D	C
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
A	A	D	A	A	C	C	D	E	A	C	C	A	A	B	B	E	B	A	E
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
E	E	A	C	B	D	A	D	C	C	A	E	C	D	E	B	A	A	C	B
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
D	E	C	C	A	C	C	E	A	D	A	B	A	C	E	D	B	B	A	A
81	82	83	84	85	86	87	88	89	90										
A	E	D	B	A	B	A	A	C	C										

III bob

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
C	C	C	A	E	B	B	A	D	A	B	C	C	E	B	E	D	D	A	A
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
B	D	C	B	D	C	D	C	B	B	B	C	D	C	B	C	D	E	E	A
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
A	D	A	A	D	E	B	D	B	E	D	C	A	E	C	D	A	C	C	B

61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
B	B	D	B	C	D	E	E	B	D	D	C	A	B	C	A	B	B	E	E

IV bob

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
E	C	E	E	D	C	C	E	C	E	B	C	A	B	C	D	B	C	D	A
21	22	23	24	25	26	27	28	29	30										
C	A	A	E	D	C	C	E	C	E										

V bob

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
C	B	A	B	B	A	C	D	B	A	A	C	C	E	D	C	A	A	C	E	C
22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42
D	C	E	B	C	C	D	B	B	C	D	A	C	D	A	C	B	A	B	C	B
43	44	45	46	47	48	49	50													
D	D	E	B	B	E	A	A													

VI bob

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
C	D	A	C	D	C	B	A	A	C	B	A	A	D	D	B	D	E	A	B
21	22	23	24	25	26	27	28	29	30	31	32	33							
E	A	A	B	D	E	D	C	B	D	D	C	C							

VII bob

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
D	D	A	E	C	E	E	C	E	E	A	C	A	E	D	C	B	A	E	A
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
C	E	E	A	A	D	D	C	C	D	E	D	C	E	B	D	E	A	C	C

VIII bob

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
E	C	E	C	D	C	A	E	C	A	A	A	B	E	B	D	E	B	A	D
21	22	23	24	25	26	27	28	29	30										
A	B	D	A	C	B	D	B	D	E										

IX bob

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
B	A	C	A	B	C	E	D	B	C	A	A	A	B	E	E	C	C	C	E
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
B	A	A	B	B	E	D	B	C	B	A	C	B	A	C	D	B	E	C	C
41	42	43	44	45	46	47	48	49	50	51	52								
B	B	D	B	B	A	A	D	A	A	C	B								

X bob

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	B	C	D	E	C	C	B	A	A	C	B	A	A	D	A	B	C	D	C
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
A	B	A	A	D	A	B	C	E	A	A	E	B	D	A	A	A	E	B	B
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56				
E	B	C	B	C	C	A	E	E	C	D	A	C	C	A	B				

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MUNDARIJA

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Botir Odilovich Xo'jayev

MATEMATIKADAN QO'LLANMA

O'zbekiston Respublikasi Oliy va o'rta maxsus ta'lim vazirligi o'quv qo'llanma sifatida tavsiya etgan

II qism

TOSHKENT – «NOSHIR» – 2009

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Musahhih ***A. Tillaxo'jayev***
Sahifalovchi ***S. Po'latov***

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